
Probe wire

$$V_1^+(l_{z01}) = V_1^+(0)e^{i\gamma_{z01}l_{z01}}$$

$$V_1^-(l_{z01}) = V_1^-(0)e^{-i\gamma_{z01}l_{z01}}$$

$$Z = Z_{01}$$

Wire Connect to Load

$$V_2^+(l_{z02}) = V_2^+(0)e^{-i\gamma_{z02}l_{z02}}$$

$$V_2^-(l_{z02}) = V_2^-(0)e^{i\gamma_{z02}l_{z02}}$$

$$Z = Z_{02}$$

Load

$$I_L = \frac{V_2^+(l_{z02}) - V_2^-(l_{z02})}{k2Z_{02}}$$

$$V_L = V_2^+(l_{z02}) + V_2^-(l_{z02})$$

$$I_L = \frac{V_L}{Z_L} ; Z_L = 50$$

$$\frac{V_2^+(l_{z02}) - V_2^-(l_{z02})}{k2Z_{02}} = \frac{V_2^+(l_{z02}) + V_2^-(l_{z02})}{Z_L}$$

$$-V_2^-(l_{z02}) \left[\frac{1}{k2Z_{02}} + \frac{1}{Z_L} \right] = V_2^+(l_{z02}) \left[\frac{1}{Z_L} - \frac{1}{k2Z_{02}} \right]$$

$$V_2^-(l_{z02}) = V_2^+(l_{z02}) \left[\frac{Z_L - k2Z_{02}}{Z_L + k2Z_{02}} \right]$$

$$V_2^-(l_{z02}) = V_2^+(l_{z02}) \Gamma_2 ; \Gamma_2 = \left[\frac{Z_L - k2Z_{02}}{Z_L + k2Z_{02}} \right]$$

$$V_2^-(0) = V_2^+(0)e^{-i2\gamma_{z02}l_{z02}} \Gamma_2$$

Start solving for the frequency response

Voltage equation solving

$$I_n(R_1 + X_{c1}) = \frac{V_2^+(0) - V_2^-(0)}{Z_{02}}(R_2 + X_{c2}) + V_2^+(0) + V_2^-(0)$$

$$V_2^-(0) = V_2^+(0)e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2$$

$$I_1(R_1 + X_{c1}) = \frac{V_2^+(0) - V_2^-(0)e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{Z_{02}}(R_2 + X_{c2}) + V_2^+(0) + V_2^-(0)e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2$$

$$I_1 = \frac{V_2^+(0) - V_2^-(0)e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{Z_{02}} \frac{(R_2 + X_{c2})}{(R_1 + X_{c1})} + \frac{V_2^+(0) + V_2^-(0)e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{(R_1 + X_{c1})}$$

$$I_1 = \frac{V_2^+(0)}{(R_1 + X_{c1})} \left(\frac{1 - e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{1} \frac{(R_2 + X_{c2})}{Z_{02}} + \frac{1 + e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{1} \right)$$

$$I_1 = \frac{V_2^+(0)}{(R_1 + X_{c1})} \left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2 \right)$$

$$V_2^+(0) = \frac{I_1(R_1 + X_{c1})}{\left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2 \right)}$$

Current equation solving

$$I_o = I_1 + I_2$$

$$I_2 = \frac{V_2^+(0) - V_2^-(0)}{Z_2}$$

$$I_o = I_1 + \frac{V_2^+(0) - V_2^-(0)e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{Z_2}$$

$$I_o = I_1 + V_2^+(0) \frac{1 - e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{Z_2}$$

$$I_o = I_1 \left(1 + \frac{1 - e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{Z_2} \frac{(R_1 + X_{c1})}{\left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2 \right)} \right)$$

$$\text{Reduced form check } I_o = I_n \left(\frac{(Z_{02} + R_2 + R_1)}{Z_{02} + R_2} \right)$$

Current from input branch : plug in the probe tip equation

$$I_o = \frac{V_2^+(0)}{(R_1 + X_{c1})} \left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2 \right) + V_2^+(0) \frac{1 - e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{Z_2}$$

$$I_o = \frac{V_1^+(0) - V_1^-(0)}{Z_1}$$

$$\frac{V_1^+(0) - V_1^-(0)}{Z_1} = V_2^+(0) \left\{ \frac{1}{(R_1 + X_{c1})} \left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2 \right) + \frac{1 - e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{Z_2} \right\}$$

$$V_1^+(l_{z01}) = V_1^-(l_{z01})e^{i\gamma Z_{01}l_{z01}}\Gamma_1 + I_n k_1 Z_{01} e^{i\gamma Z_{01}l_{z01}}$$

$$\frac{V_1^-(0)(e^{i2\gamma Z_{01}l_{z01}}\Gamma_1 - 1) + I_n Z_{01} e^{i\gamma Z_{01}l_{z01}}}{Z_1} = V_2^+(0) \left\{ \frac{1}{(R_1 + X_{c1})} \left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2 \right) + \frac{1 - e^{-i2\gamma Z_{02}l_{z02}}\Gamma_2}{Z_2} \right\}$$

$$\text{Reduced form check } I_n e^{iY_{Z01} l_{Z01}} = V_2^+(0) \left\{ \frac{Z_{02} + R_2 + R_1}{R_1 Z_{02}} \right\}$$

Voltage I2 Branch : reintroduce the voltage equation

$$V_1^+(0) + V_1^-(0) = \frac{V_2^+(0) - V_2^-(0) e^{-i2Y_{Z02} l_{Z02}} \Gamma_2}{Z_{02}} (R_2 + X_{c2}) + V_2^+(0) + V_2^-(0) e^{-i2Y_{Z02} l_{Z02}} \Gamma_2$$

$$V_1^+(l_{Z01}) = V_1^-(l_{Z01}) e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + I_n k1 Z_{01} e^{iY_{Z01} l_{Z01}}$$

$$V_1^-(0) (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1) + I_n Z_{01} e^{iY_{Z01} l_{Z01}} = \frac{V_2^+(0) (R_1 + X_{c1})}{(R_1 + X_{c1})} \left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2Y_{Z02} l_{Z02}} \Gamma_2 \right)$$

Voltage I1 Branch

$$V_1^+(0) + V_1^-(0) = I_1 (R_1 + X_{c1}) \text{ Please recognize the term and don't use.}$$

Eliminate V1

$$\frac{V_1^-(0) (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 - 1)}{Z_1} + I_n e^{iY_{Z01} l_{Z01}} = V_2^+(0) \left\{ \frac{1}{(R_1 + X_{c1})} \left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2Y_{Z02} l_{Z02}} \Gamma_2 \right) + \frac{1 - e^{-i2Y_{Z02} l_{Z02}} \Gamma_2}{Z_2} \right\}$$

$$V_1^-(0) = \frac{1}{(e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1)} \frac{V_2^+(0) (R_1 + X_{c1})}{(R_1 + X_{c1})} \left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2Y_{Z02} l_{Z02}} \Gamma_2 \right) - \frac{I_n Z_{01} e^{iY_{Z01} l_{Z01}}}{e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1}$$

$$\frac{V_1^-(0) (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 - 1)}{Z_1} = \frac{(e^{i2Y_{Z01} l_{Z01}} \Gamma_1 - 1)}{Z_1 (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1)} \frac{V_2^+(0) (R_1 + X_{c1})}{(R_1 + X_{c1})} \left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2Y_{Z02} l_{Z02}} \Gamma_2 \right) - I_n e^{iY_{Z01} l_{Z01}} \frac{(e^{i2Y_{Z01} l_{Z01}} \Gamma_1 - 1)}{e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1}$$

$$I_n e^{iY_{Z01} l_{Z01}} \left[1 - \frac{(e^{i2Y_{Z01} l_{Z01}} \Gamma_1 - 1)}{e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1} \right] = V_2^+(0) \left\{ \frac{1}{(R_1 + X_{c1})} \left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2Y_{Z02} l_{Z02}} \Gamma_2 \right) + \left[1 - \frac{(R_1 + X_{c1}) (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 - 1)}{Z_1 (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1)} \right] + \frac{1 - e^{-i2Y_{Z02} l_{Z02}} \Gamma_2}{Z_2} \right\}$$

Reduced form check

$$I_n e^{iY_{Z01} l_{Z01}} \left[\frac{2}{e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1} \right] = V_2^+(0) \left\{ \frac{Z_{02} + R_2}{R_1 Z_{02}} \left[1 - \frac{R_1 (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 - 1)}{Z_1 (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1)} \right] + \frac{1}{Z_2} \right\}$$

$$2I_n e^{iY_{Z01} l_{Z01}} = \frac{V_2^+(0)}{R_1 Z_1 Z_{02}} \{ (Z_{02} + R_2) [Z_1 (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1) - R_1 (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 - 1)] + R_1 Z_1 (e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + 1) \}$$

$$2I_n e^{iY_{Z01} l_{Z01}} = \frac{V_2^+(0)}{R_1 Z_1 Z_{02}} \{ Z_1 Z_{02} e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + Z_1 Z_{02} - e^{i2Y_{Z01} l_{Z01}} \Gamma_1 Z_{02} R_1 + R_1 Z_{02} + Z_1 R_2 e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + R_2 Z_1 - R_1 R_2 e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + R_1 R_2 + R_1 Z_1 e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + R_1 Z_1 \}$$

$$2I_n e^{iY_{Z01} l_{Z01}} = \frac{V_2^+(0)}{R_1 Z_1 Z_{02}} \{ (Z_1 (R_1 + R_2 + Z_2) - R_2 - R_1 (R_2 + Z_{02})) e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + Z_1 (R_1 + R_2 + Z_2) + R_1 (R_2 + Z_{02}) \}$$

$$Z_1 = \frac{(R_2 + Z_2) R_1}{R_1 + R_2 + Z_2}$$

$$2I_n e^{iY_{Z01} l_{Z01}} = \frac{V_2^+(0)}{R_1 Z_1 Z_{02}} \{ ((R_2 + Z_2) R_1 - R_2 - Z_{02}) e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + (R_2 + Z_2) R_1 + R_2 + Z_{02} \}$$

$$2I_n e^{iY_{Z01} l_{Z01}} = \frac{V_2^+(0) (R_2 + Z_2)}{R_1 Z_1 Z_{02}} \{ (R_1 - R_1) e^{i2Y_{Z01} l_{Z01}} \Gamma_1 + (R_1 + R_1) \}$$

$$I_n e^{iY_{Z01} l_{Z01}} = \frac{V_2^+(0) (R_2 + Z_2)}{Z_1 Z_{02}}$$

$$V_2^+(0) = I_n e^{-iY_{Z01} l_{Z01}} \frac{Z_1 Z_{02}}{(R_2 + Z_2)}$$

$$V_L = I_n e^{-iY_{Z01} l_{Z01}} e^{-iY_{Z02} l_{Z02}} \frac{Z_1 Z_{02}}{(R_2 + Z_2)}$$

Re-Decoration

$$V_L = \frac{2Z_2 I_n [1 + \Gamma_2] e^{-iY_{Z01} l_{Z01}} e^{-iY_{Z02} l_{Z02}}}{\{AB + C\}}$$

$$A = ([R_2 + X_{c2} + Z_{02}] - [R_2 + X_{c2} - Z_{02}]) e^{-i2Y_{Z02} l_{Z02} \Gamma_2} / (R_1 + X_{c1}) / Z_1$$

$$B = [R_1 + X_{c1} + Z_1 - (R_1 + X_{c1} - Z_1) e^{i2Y_{Z01} l_{Z01} \Gamma_1}]$$

$$C = [1 - e^{-i2Y_{Z02} l_{Z02} \Gamma_2}] [1 + e^{-i2Y_{Z01} l_{Z01} \Gamma_1}]$$

Reduced form check

$$V_L = \frac{2Z_2 I_n e^{-iY_{Z01} l_{Z01}} e^{-iY_{Z02} l_{Z02}}}{\{AB + C\}}$$

$$A = \frac{(R_2 + Z_{02})}{Z_1(R_1)}$$

$$B = [R_1 + Z_1 - (R_1 - Z_1) e^{i2Y_{Z01} l_{Z01} \Gamma_1}]$$

$$C = [1 + e^{-i2Y_{Z01} l_{Z01} \Gamma_1}]$$

$$V_L = \frac{2Z_2 I_n e^{-iY_{Z01} l_{Z01}} e^{-iY_{Z02} l_{Z02}}}{\{[R_2 + Z_{02}][R_1 + Z_1 - (R_1 - Z_1) e^{i2Y_{Z01} l_{Z01} \Gamma_1}] + Z_1(R_1)[1 + e^{-i2Y_{Z01} l_{Z01} \Gamma_1}]\}} Z_1(R_1)$$

$$V_L = \frac{2Z_2 I_n e^{-iY_{Z01} l_{Z01}} e^{-iY_{Z02} l_{Z02}}}{\{R_2 R_1 + R_2 Z_1 + Z_1(R_1) + Z_{02} R_1 + Z_{02} Z_1 - Z_{02}(R_1 - Z_1) e^{i2Y_{Z01} l_{Z01} \Gamma_1} - R_2(R_1 - Z_1) e^{i2Y_{Z01} l_{Z01} \Gamma_1} + Z_1(R_1) e^{-i2Y_{Z01} l_{Z01} \Gamma_1}\}} Z_1(R_1)$$

$$Z_1 = \frac{(R_2 + Z_2)R_1}{R_1 + R_2 + Z_2}$$

$$V_L = I_n e^{-iY_{Z01} l_{Z01}} e^{-iY_{Z02} l_{Z02}} \frac{Z_1 Z_{02}}{(R_2 + Z_2)}$$

Drive for circuit voltage to current scale

$$a = \frac{Z_1}{(R_2 + Z_2)}$$

$$Z_1 = \frac{(R_2 + Z_2)R_1}{R_1 + R_2 + Z_2}$$

$$R_2 = Z_1/a - Z_2$$

$$R_1 = Z_1(R_2 + Z_2)/(R_2 + Z_2 - Z_1)$$

$$R_1 = Z_1/(1 - a)$$

Result

There are three result to compare performance of the formular

1 scale of the circuit, 2 Zin-dependence, 3 Group delay.

Scale of the circuit

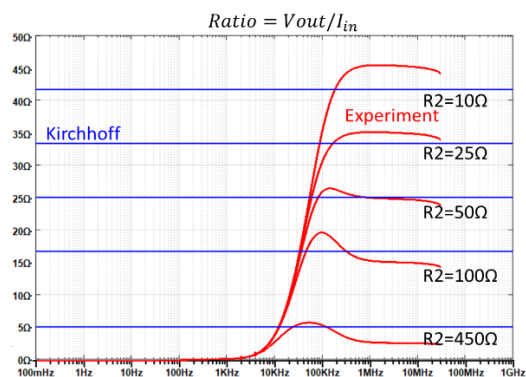


Figure 3 R2 sweep frequency response from actual circuit.

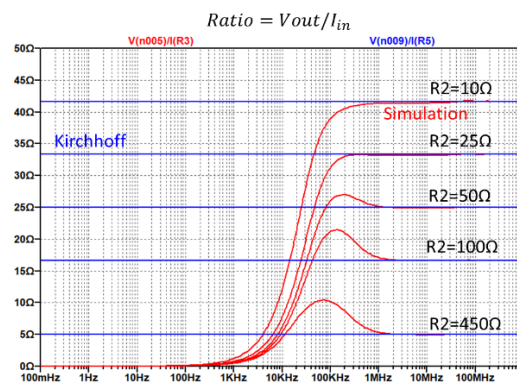


Figure 4 R2 sweep frequency response from simulation.

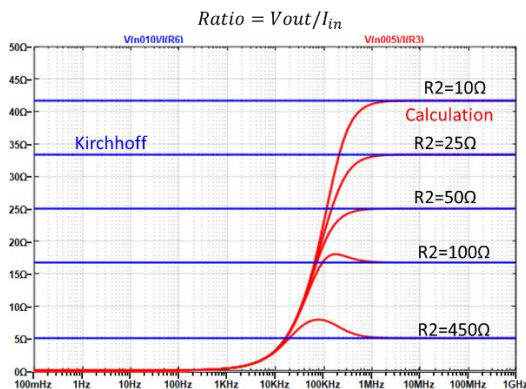


Figure 5 R2 sweep frequency response from calculation formular.

Frequency response of all three results shows good agreement. However, experiment in Fig. 3 shows deviation from ideal scale calculate from Kirchhoff law (blue curve). This deviation came from resistor error because the resistor value changes due to spring effect. This shows the fact that technician should fix the resistor with glue as shown in 50Ohms case. Result from simulation in Fig. 4 shows different in high-pass cut-off frequency from experiment and calculation result in Fig. 5 shows more realistic frequency response.

Zin-dependence

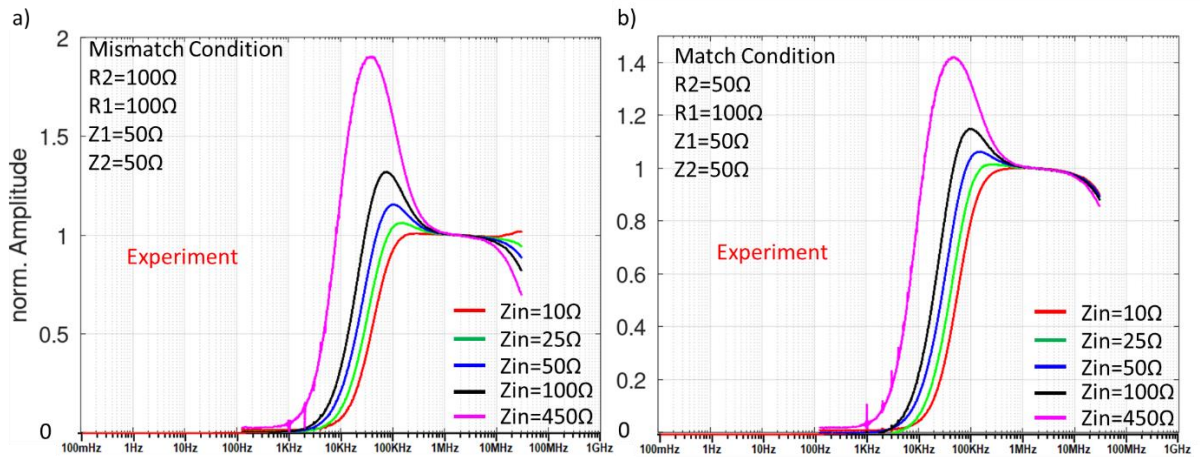


Figure 6 Zin sweep frequency response from actual circuit (a) mismatch and (b) match.

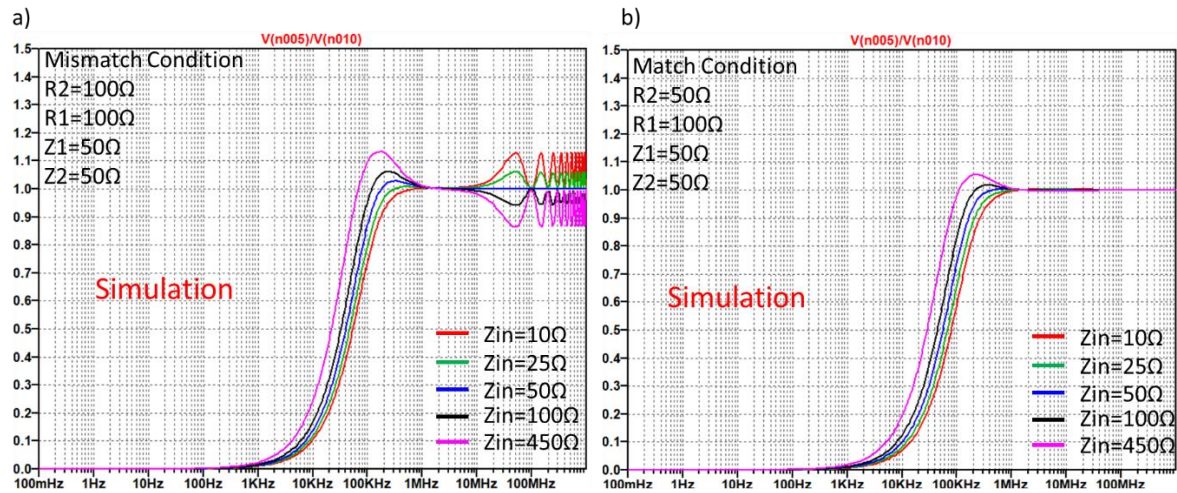


Figure 7 Zin sweep frequency response from simulation (a) mismatch and (b) match.

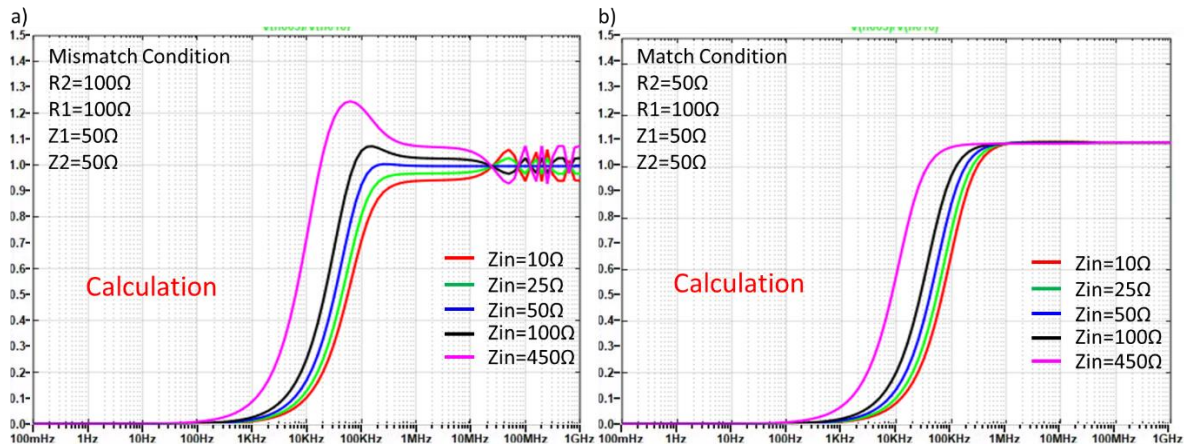


Figure 8 Z_{in} sweep frequency response from formular(a) mismatch and (b) match.

The Z_{in} sweep frequency response shows the difference between match and mismatch where in match case Z_{in} is independence where reflection does not affect to the high pass region. In contrast, mismatch cases show strong Z_{in} dependence. The experiment shown in Fig. 6 confirms the idea that the circuit will be independent from Z_{in} when match condition is met. However, at the cutoff frequency overshoot appears to depend on the Z_{in} . This is the good warning for technician to concern about the resonance peak. The simulation shown in Fig. 7 can roughly predict the overshoot appearance. In the case of formular equation, the prediction is roughly correct for (a) mismatch case where overshoot is predicted but wrong for (b) match case where overshoot disappears.

Group Delay calculation

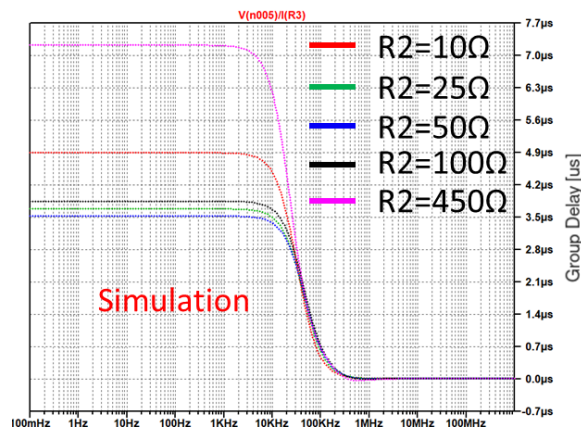


Figure 9 R_2 sweep frequency group delay simulation.

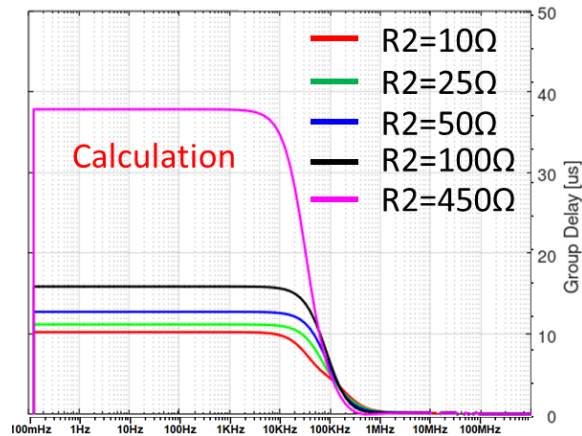


Figure 9 R2 sweep frequency group delay simulation.

In experiment, phase detection requires sensitive measurement especially for low frequency range where small delay must be detected from slow signal and network analyzer in the market only support 100kHz -3GHz range; therefore, result from simulation and formular only for discussion. The result from simulation and formular shows large difference where simulation shows delay in 10us range, but calculation shows delay in 50us range. For this reason, the phase both simulation and calculation result must be concern as they both have discrepancy from experiment. The delay appears in low frequency range, which is not use in photo-detachment measurement.

Conclusion

The new formula proves the concept. With match condition, reflection will disappear, and the circuit becomes independent from the Z_{in} . The result from simulation and experiment confirms that formular is derived correctly for scale. They also show that circuits become independent from Z_{in} at high-pass region. However, formular and simulation is a prediction. Some differences appear where simulation shows less accurate cutoff decay characteristic and formular shows less accurate overshoot characteristic.

The most concern that the formular missed is that at match condition overshoot appear for different Z_{in} value. For this reason, technician should protect recording device from damage, if some strong signal came at this overshoot frequency. However, this is subtle topic to put into the paper because the benefit of this circuit is flat frequency response at 1MHz – 1GHz range where the photo-detachment occupied this frequency range.