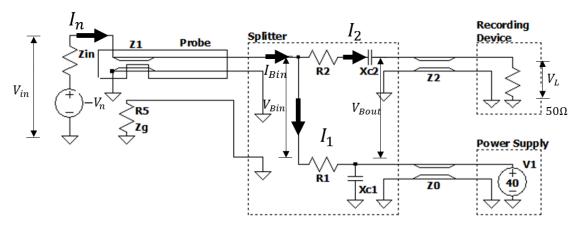
## Derivation of eq.9 and 10 Date 6/7/2024



Probe tip: equation for impedance and reflection boundary condition from probe tip

$$I_{in} = \frac{V_1^-(0) - V_1^+(0)}{k1Z_{01}}$$

$$V_{in} = V_1^+(0) + V_1^-(0)$$

$$I_{in} = \frac{V_{in} - V_n}{Z_{in}}$$

$$I_n = V_n \frac{1}{Z_{in} + Z_{01}}$$

$$\frac{V_1^-(0)-V_1^+(0)}{k1Z_{01}} = \frac{V_1^+(0)+V_1^-(0)-V_n}{Z_{in}}$$

$$-V_1^+(0)\left[\frac{1}{Z_{01}} + \frac{1}{Z_{in}}\right] + \frac{V_n}{Z_{in}} = V_1^-(0)\left[\frac{1}{Z_{in}} - \frac{1}{Z_{01}}\right]$$

$$V_1^+(0) = V_n \left[ \frac{Z_{01}}{Z_{in} + k1Z_{01}} \right] - V_1^-(0) \left[ \frac{Z_{01} - Z_{in}}{Z_{in} + Z_{01}} \right]$$

$$V_1^+(0) = V_n \left[ \frac{Z_{01}}{Z_{01} + Z_{in}} \right] + V_1^-(0)\Gamma_1; \Gamma_1 = \left[ \frac{Z_{in} - Z_{01}}{Z_{in} + Z_{01}} \right]$$

$$V_1^+(0) = V_1^-(0)\Gamma_1 + I_n k 1 Z_{01}; \Gamma_1 = \left[\frac{Z_{in} - Z_{01}}{Z_{01} + Z_{in}}\right]$$

$$V_1^+(l_{z01}) = V_1^-(l_{z01})e^{i2\gamma_{Z01}l_{z01}}\Gamma_1 + I_nk1Z_{01}e^{i\gamma_{Z01}l_{z01}}; \Gamma_1 = \left[\frac{Z_{in} - Z_{01}}{Z_{01} + Z_{in}}\right]$$

Probe wire

$$V_1^+(l_{z01})=V_1^+(0)e^{i\gamma_{Z01}l_{z01}}$$

$$V_1^-(l_{z01}) = V_1^-(0) e^{-i\gamma_{Z01} l_{z01}}$$

$$Z = Z_{01}$$

Wire Connect to Load

$$V_2^+(l_{z02}) = V_2^+(0)e^{-i\gamma_{Z02}l_{z02}}$$

$$V_2^-(l_{z02}) = V_2^-(0)e^{i\gamma_{Z02}l_{z02}}$$

$$Z = Z_{02}$$

Load

$$I_L = \frac{V_2^+(l_{z02}) - V_2^-(l_{z02})}{k2Z_{02}}$$

$$V_L = V_2^+(l_{z02}) + V_2^-(l_{z02})$$

$$I_L = \frac{V_L}{Z_L} \; ; \; Z_L = 50$$

$$\frac{V_2^+(l_{z02}) - V_2^-(l_{z02})}{k2Z_{02}} = \frac{V_2^+(l_{z02}) + V_2^-(l_{z02})}{Z_L}$$

$$-V_2^-(l_{z02})\left[\frac{1}{k2Z_{02}} + \frac{1}{Z_L}\right] = V_2^+(l_{z02})\left[\frac{1}{Z_L} - \frac{1}{k2Z_{02}}\right]$$

$$V_2^-(l_{z02}) = V_2^+(l_{z02}) \left[ \frac{Z_L - k2Z_{02}}{Z_L + k2Z_{02}} \right]$$

$$V_2^-(l_{z02}) = V_2^+(l_{z02})\Gamma_2; \Gamma_2 = \left[\frac{Z_L - k2Z_{02}}{Z_L + k2Z_{02}}\right]$$

$$V_2^-(0) = V_2^+(0)e^{-i2\gamma_{Z_{02}}l_{z_{02}}}\Gamma_2$$

Start solving for the frequency response

Voltage equation solving

$$I_n(R_1 + X_{c1}) = \frac{V_2^+(0) - V_2^-(0)}{Z_{02}}(R_2 + X_{c2}) + V_2^+(0) + V_2^-(0)$$

$$V_2^-(0) = V_2^+(0)e^{-i2\gamma_{Z_{02}}l_{z_{02}}}\Gamma_2$$

$$I_1(R_1+X_{c1}) = \frac{V_2^+(0) - V_2^+(0)e^{-i2\gamma_{Z_{02}}l_{z_{02}}}\Gamma_2}{Z_{02}}(R_2+X_{c2}) + V_2^+(0) + V_2^+(0)e^{-i2\gamma_{Z_{02}}l_{z_{02}}}\Gamma_2$$

$$I_1 = \frac{V_2^+(0) - V_2^+(0) e^{-i2\gamma_{Z_{02}} l_{z_{02}}} \Gamma_2}{Z_{02}} \frac{(R_2 + X_{c2})}{(R_1 + X_{c1})} + \frac{V_2^+(0) + V_2^+(0) e^{-i2\gamma_{Z_{02}} l_{z_{02}}} \Gamma_2}{(R_1 + X_{c1})}$$

$$I_1 = \frac{V_2^+(0)}{(R_1 + X_{c1})} \left( \frac{1 - e^{-i2\gamma_{Z_0 2} I_{Z_0 2}} \Gamma_2}{1} \frac{(R_2 + X_{c2})}{Z_{02}} + \frac{1 + e^{-i2\gamma_{Z_0 2} I_{Z_0 2}} \Gamma_2}{1} \right)$$

$$I_1 = \frac{V_2^+(0)}{(R_1 + X_{c1})} \left( \left[ 1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[ 1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2\gamma_{Z_{02}} l_{z_{02}}} \Gamma_2 \right)$$

$$V_2^+(0) = \frac{I_1(R_1 + X_{c1})}{\left(\left[1 + \frac{(R_2 + X_{c2})}{Z_{02}}\right] + \left[1 - \frac{(R_2 + X_{c2})}{Z_{02}}\right]e^{-i2\gamma_{Z02}I_{Z02}}\Gamma_2\right)}$$

Current equation solving

$$I_o = I_1 + I_2$$

$$I_2 = \frac{V_2^+(0) - V_2^-(0)}{Z_2}$$

$$I_o = I_1 + \frac{V_2^+(0) - V_2^+(0)e^{-i2\gamma_{Z02}I_{Z02}}\Gamma_2}{Z_2}$$

$$I_o = I_1 + V_2^+(0) \frac{1 - e^{-i2\gamma_{Z02}I_{Z02}}\Gamma_2}{Z_2}$$

$$I_o = I_1 \left( 1 + \frac{1 - e^{-i2\gamma_{Z_{02}} I_{z_{02}}} \Gamma_2}{Z_2} \frac{(R_1 + X_{c1})}{\left( \left[ 1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[ 1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2\gamma_{Z_{02}} I_{z_{02}}} \Gamma_2 \right) \right)$$

Reduced form check  $I_o = I_n \left( \frac{(Z_{02} + R_2 + R_1)}{Z_{02} + R_2} \right)$ 

\_\_\_\_\_\_

Current from input branch: plug in the probe tip equation

$$I_o = \frac{V_2^+(0)}{(R_1 + X_{c1})} \left( \left[ 1 + \frac{(R_2 + X_{c2})}{Z_{02}} \right] + \left[ 1 - \frac{(R_2 + X_{c2})}{Z_{02}} \right] e^{-i2\gamma_{Z02}l_{z02}} \Gamma_2 \right) + V_2^+(0) \frac{1 - e^{-i2\gamma_{Z02}l_{z02}} \Gamma_2}{Z_2}$$

$$I_o = \frac{V_1^+(0) - V_1^-(0)}{Z_1}$$

$$\frac{V_1^+(0)-V_1^-(0)}{Z_1} = V_2^+(0) \left\{ \frac{1}{(R_1+X_{c1})} \left( \left[1 + \frac{(R_2+X_{c2})}{Z_{02}}\right] + \left[1 - \frac{(R_2+X_{c2})}{Z_{02}}\right] e^{-i2\gamma_{Z02}l_{z02}} \Gamma_2 \right) + \frac{1 - e^{-i2\gamma_{Z02}l_{z02}} \Gamma_2}{Z_2} \right\}$$

$$V_1^+(l_{z01}) = V_1^-(l_{z01})e^{i2\gamma_{Z01}l_{z01}}\Gamma_1 + I_nk1Z_{01}e^{i\gamma_{Z01}l_{z01}}$$

$$\frac{V_1^-(0)\left(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1\right)+I_nZ_{01}e^{i\gamma_{Z01}l_{z01}}}{Z_1}=V_2^+(0)\left\{\frac{1}{(R_1+X_{c1})}\left(\left[1+\frac{(R_2+X_{c2})}{Z_{02}}\right]+\left[1-\frac{(R_2+X_{c2})}{Z_{02}}\right]e^{-i2\gamma_{Z02}l_{z02}}\Gamma_2\right)+\frac{1-e^{-i2\gamma_{Z02}l_{z02}}\Gamma_2}{Z_2}\right\}$$

Reduced form check  $I_n e^{i\gamma_{Z_{01}} l_{z_{01}}} = V_2^+(0) \left\{ \frac{Z_{02} + R_2 + R_1}{R_1 Z_{02}} \right\}$ 

Voltage I2 Branch: reintroduce the voltage equation

$$V_1^+(0) + V_1^-(0) = \frac{V_2^+(0) - V_2^+(0)e^{-i2\gamma_{Z_{02}}l_{z_{02}}}\Gamma_2}{Z_{02}}(R_2 + X_{c2}) + V_2^+(0) + V_2^+(0)e^{-i2\gamma_{Z_{02}}l_{z_{02}}}\Gamma_2$$

$$V_1^+(l_{z01}) = V_1^-(l_{z01})e^{i2\gamma_{Z01}l_{z01}}\Gamma_1 + I_nk1Z_{01}e^{i\gamma_{Z01}l_{z01}}$$

$$V_1^-(0)\left(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1+1\right)+I_nZ_{01}e^{i\gamma_{Z01}l_{Z01}}=\frac{V_2^+(0)(R_1+X_{c1})}{(R_1+X_{c1})}\left(\left[1+\frac{(R_2+X_{c2})}{Z_{02}}\right]+\left[1-\frac{(R_2+X_{c2})}{Z_{02}}\right]e^{-i2\gamma_{Z02}l_{Z02}}\Gamma_2\right)$$

Voltage I1 Branch

$$V_1^+(0) + V_1^-(0) = I_1(R_1 + X_{c1})$$
 Please recognize the term and don't use.

Eliminate V1

$$\begin{split} & \frac{V_1^-(0)\left(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1\right)}{Z_1} + I_n e^{i\gamma_{Z01}l_{Z01}} = V_2^+(0)\left\{\frac{1}{(R_1+X_{c1})}\left(\left[1+\frac{(R_2+X_{c2})}{Z_{02}}\right] + \left[1-\frac{(R_2+X_{c2})}{Z_{02}}\right]e^{-i2\gamma_{Z02}l_{z02}}\Gamma_2\right) + \frac{1-e^{-i2\gamma_{Z02}l_{z02}}\Gamma_2}{Z_2}\right\}\\ & V_1^-(0) = \frac{1}{(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1+1)}\frac{V_2^+(0)(R_1+X_{c1})}{(R_1+X_{c1})}\left(\left[1+\frac{(R_2+X_{c2})}{Z_{02}}\right] + \left[1-\frac{(R_2+X_{c2})}{Z_{02}}\right]e^{-i2\gamma_{Z02}l_{z02}}\Gamma_2\right) - \frac{I_nZ_{01}e^{i\gamma_{Z01}l_{z01}}}{e^{i2\gamma_{Z01}l_{z01}}\Gamma_1+1} \end{split}$$

$$(e^{i2\gamma_{\text{Z01}}I_{\text{Z01}}}\Gamma_1 + 1) \quad (R_1 + X_{c1}) \quad (\Gamma \quad Z_{02} \quad \Gamma \quad Z_{02} \quad Z_{02} \quad \Gamma \quad Z_{02} \quad$$

$$\frac{V_1^-(0)(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)}{Z_1} = \frac{(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)}{Z_1(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1+1)} \frac{V_2^+(0)(R_1+X_{c1})}{(R_1+X_{c1})} \left( \left[1+\frac{(R_2+X_{c2})}{Z_{02}}\right] + \left[1-\frac{(R_2+X_{c2})}{Z_{02}}\right] e^{-i2\gamma_{Z02}l_{Z02}}\Gamma_2 \right) - I_n e^{i\gamma_{Z01}l_{Z01}} \frac{(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)}{e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1+1} \\ I_n e^{i\gamma_{Z01}l_{Z01}} \left[1-\frac{(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)}{e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1+1}\right] = V_2^+(0) \left\{ \frac{1}{(R_1+X_{c1})} \left( \left[1+\frac{(R_2+X_{c2})}{Z_{02}}\right] + \left[1-\frac{(R_2+X_{c2})}{Z_{02}}\right] e^{-i2\gamma_{Z02}l_{Z02}}\Gamma_2 \right) \left[1-\frac{(R_1+X_{c1})(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)}{Z_1(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1+1)}\right] + \frac{1-e^{-i2\gamma_{Z02}l_{Z02}}\Gamma_2}{Z_2} \right\} \\ = \frac{1}{2} \left[ \frac{1}{(R_1+X_{c1})(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)} + \frac{1}{(R_1+X_{c1})(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)} \right] + \frac{1}{2} \left[ \frac{1}{(R_1+X_{c1})(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)} + \frac{1}{2} \left[ \frac{1}{(R_1+X_{c1})(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)} + \frac{1}{2} \left[ \frac{1}{(R_1+X_{c1})(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)} \right] + \frac{1}{2} \left[ \frac{1}{(R_1+X_{c1})(e^{i2\gamma_{Z01}l_{Z01}}\Gamma_1-1)} + \frac{1}{2} \left[ \frac{1}{(R_1+X_{c1}$$

Reduced form check

$$I_n e^{i\gamma_{Z_{01}}I_{z_{01}}} \left[ \frac{2}{e^{i2\gamma_{Z_{01}}I_{z_{01}}}\Gamma_1 + 1} \right] = V_2^+(0) \left\{ \frac{Z_{02} + R_2}{R_1 Z_{02}} \left[ 1 - \frac{R_1(e^{i2\gamma_{Z_{01}}I_{z_{01}}}\Gamma_1 - 1)}{Z_1(e^{i2\gamma_{Z_{01}}I_{z_{01}}}\Gamma_1 + 1)} \right] + \frac{1}{Z_2} \right\}$$

$$2I_{n}e^{i\gamma_{Z01}I_{z01}} = \frac{v_{2}^{+}(0)}{R_{1}Z_{1}Z_{02}} \left\{ (Z_{02} + R_{2})[Z_{1}(e^{i2\gamma_{Z01}I_{z01}}\Gamma_{1} + 1) - R_{1}(e^{i2\gamma_{Z01}I_{z01}}\Gamma_{1} - 1)] + R_{1}Z_{1}(e^{i2\gamma_{Z01}I_{z01}}\Gamma_{1} + 1) \right\}$$

$$2I_{n}e^{i\gamma_{Z01}I_{Z01}} = \frac{V_{2}^{+}(0)}{R_{1}Z_{1}Z_{02}} \left\{ Z_{1}Z_{02}e^{i2\gamma_{Z01}I_{201}}\Gamma_{1} + Z_{1}Z_{02} - e^{i2\gamma_{Z01}I_{201}}\Gamma_{1}Z_{02}R_{1} + R_{1}Z_{02} + Z_{1}R_{2}e^{i2\gamma_{Z01}I_{201}}\Gamma_{1} + R_{2}Z_{1} - R_{1}R_{2}e^{i2\gamma_{Z01}I_{201}}\Gamma_{1} + R_{1}R_{2} + R_{1}Z_{1}e^{i2\gamma_{Z01}I_{201}}\Gamma_{1} + R_{1}Z_{1}e^{i2\gamma_{Z01}I$$

$$2I_{n}e^{i\gamma_{Z_{01}}I_{Z_{01}}} = \frac{V_{2}^{+}(0)}{R_{1}Z_{1}Z_{02}} \left\{ \left( Z_{1}(R_{1} + R_{2} + Z_{2}) - R_{2} - R_{1}(R_{2} + Z_{02}) \right) e^{iZ\gamma_{Z_{01}}I_{Z_{01}}} \Gamma_{1} + Z_{1}(R_{1} + R_{2} + Z_{2}) + R_{1}(R_{2} + Z_{02}) \right\}$$

$$Z_1 = \frac{(R_2 + Z_2)R_1}{R_1 + R_2 + Z_2}$$

$$2I_n e^{i\gamma_{Z_{01}}I_{Z_{01}}} = \frac{V_2^+(0)}{R_1Z_1Z_{02}} \left\{ \left( (R_2 + Z_2)R_1 - R_2 - Z_{02} \right) e^{i2\gamma_{Z_{01}}I_{z_{01}}} \Gamma_1 + (R_2 + Z_2)R_1 + R_2 + Z_{02} \right\}$$

$$2I_n e^{i\gamma_{Z_{01}} I_{Z_{01}}} = \frac{V_2^+(0)(R_2 + Z_2)}{R_1 Z_1 Z_{02}} \{ (R_1 - R_1) e^{iZ\gamma_{Z_{01}} I_{z_{01}}} \Gamma_1 + (R_1 + R_1) \}$$

$$I_n e^{i\gamma_{Z_{01}} l_{Z_{01}}} = \frac{V_2^+(0)(R_2 + Z_2)}{Z_1 Z_{02}}$$

$$V_2^+(0) = I_n e^{-i\gamma_{Z01}l_{z01}} \frac{Z_1 Z_{02}}{(R_2 + Z_2)}$$

$$V_L = I_n e^{-i\gamma_{Z01} l_{z01}} e^{-i\gamma_{Z02} l_{z02}} \frac{Z_1 Z_{02}}{(R_2 + Z_2)}$$

\_\_\_\_\_\_

Re-Decoration

$$V_L = \frac{2Z_2I_n[1+\Gamma_2]e^{-i\gamma_{Z01}l_{z01}}e^{-i\gamma_{Z02}l_{z02}}}{\{AB+C\}}$$

$$A = ([R_2 + X_{c2} + Z_{02}] - [R_2 + X_{c2} - Z_{02}]e^{-i2\gamma_{Z02}l_{Z02}}\Gamma_2)/(R_1 + X_{c1})/Z_1$$

$$B = \left[ R_1 + X_{c1} + Z_{_1} - \left( R_1 + X_{c1} - Z_{_1} \right) e^{i2\gamma_{Z01} l_{z01}} \Gamma_1 \right]$$

$$C = [1 - e^{-i2\gamma_{Z02}l_{z02}}\Gamma_{2}][1 + e^{-i2\gamma_{Z01}l_{z01}}\Gamma_{1}]$$

Reduced form check

$$V_L = \frac{2Z_2 I_n e^{-i\gamma_{Z01} l_{z01}} e^{-i\gamma_{Z02} l_{z02}}}{\{AB+C\}}$$

$$A = \frac{([R_2 + Z_{02}])}{Z_1(R_1)}$$

$$B = \left[ R_1 + Z_1 - \left( R_1 - Z_1 \right) e^{i2\gamma_{Z01} l_{z01}} \Gamma_1 \right]$$

$$C = [1 + e^{-i2\gamma_{Z01}l_{z01}}\Gamma_1]$$

$$V_L = \frac{2Z_2I_ne^{-i\gamma_{Z01}I_{z01}}e^{-i\gamma_{Z02}I_{z02}}}{\left\{[R_2 + Z_{02}][R_1 + Z_1 - (R_1 - Z_1)e^{i2\gamma_{Z01}I_{z01}}\Gamma_1] + Z_1(R_1)[1 + e^{-i2\gamma_{Z01}I_{z01}}\Gamma_1]\right\}}Z_1(R_1)$$

$$V_L = \frac{2Z_2I_ne^{-i\gamma_{Z02}I_{z02}}e^{-i\gamma_{Z02}I_{z02}}}{\left\{R_2R_1 + R_2Z_1 + Z_1(R_1) + Z_{02}R_1 + Z_{02}Z_1 - Z_{02}\left(R_1 - Z_1\right)e^{i2\gamma_{Z02}I_{z02}\Gamma_1} - R_2\left(R_1 - Z_1\right)e^{i2\gamma_{Z02}I_{z02}\Gamma_1} + Z_1(R_1)e^{-i2\gamma_{Z02}I_{z02}\Gamma_1}\right\}}Z_1(R_1)$$

$$Z_1 = \frac{(R_2 + Z_2)R_1}{R_1 + R_2 + Z_2}$$

$$V_{\scriptscriptstyle L} = I_n e^{-i\gamma_{Z01} l_{z01}} e^{-i\gamma_{Z02} l_{z02}} \frac{Z_1 Z_{02}}{(R_2 + Z_2)}$$

Drive for circuit voltage to current scale

$$a = \frac{Z_1}{(R_2 + Z_2)}$$

$$Z_1 = \frac{(R_2 + Z_2)R_1}{R_1 + R_2 + Z_2}$$

$$R_2 = Z_1/a - Z_2$$

$$R_1 = Z_1(R_2 + Z_2)/(R_2 + Z_2 - Z_1)$$

$$R_1 = Z_1/(1-a)$$

#### Confirmation with simulation and experimental results

#### Calculation data

The formula given in previous section is plot according to setting parameter.

#### Simulation setup

LTspice Simulation is utilized for finding frequency response of the circuit where scale of circuit is varies and Zin is also varying. The simulation is set according to Fig. 1 where setting of R1 follow match condition while R2 is sweeping and when Zin is sweept the R1 and R2 is fixed.

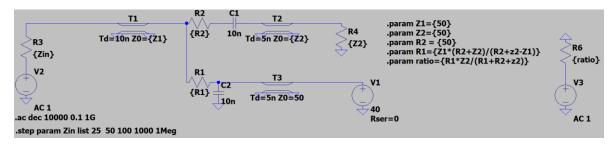


Figure 1 LTspice simulator where parameter is set according to actual circuit.

#### **Experiment setup**

The experiment was done by providing voltage signal to the circuit with constant sinewave amplitude and detect the signal with RF detector where sinewave is rectified to DC voltage as shown in Fig. 2. The RF detector characteristics are measured and normalized for every data because HP8471A only supports 100kHz-1.2GHz. The voltage source can perform frequency sweep where result is obtained. R1 and R2 resistors are hand adjustment within 5% error where setting of R1 follow match condition while R2 is sweeping. Zin sweeping relies on impedance converter where R1 and R2 are fixed.

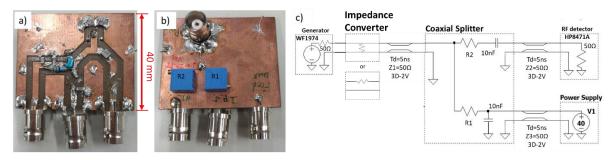


Figure 2 Experiment setup for frequency response measurement.

## Result

There are three result to compare performance of the formular 1 scale of the circuit, 2 Zin-dependence, 3 Group delay.

## Scale of the circuit

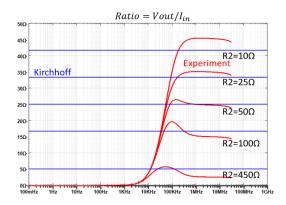


Figure 3 R2 sweep frequency response from actual circuit.

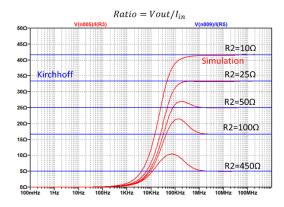


Figure 4 R2 sweep frequency response from simulation.

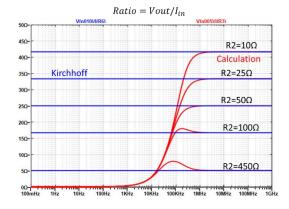


Figure 5 R2 sweep frequency response from calculation formular.

Frequency response of all three results shows good agreement. However, experiment in Fig. 3 shows deviation from ideal scale calculate from Kirchhoff law (blue curve). This deviation came from resistor error because the resistor value changes due to spring effect. This shows the fact that technician should fix the resistor with glue as shown in 500hms case. Result from simulation in Fig. 4 shows different in high-pass cut-off frequency from experiment and calculation result in Fig. 5 shows more realistic frequency response.

### Zin-dependence

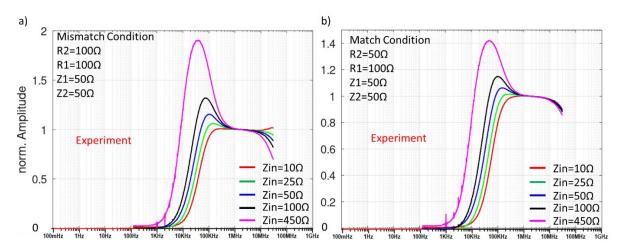


Figure 6 Zin sweep frequency response from actual circuit (a) mismatch and (b) match.

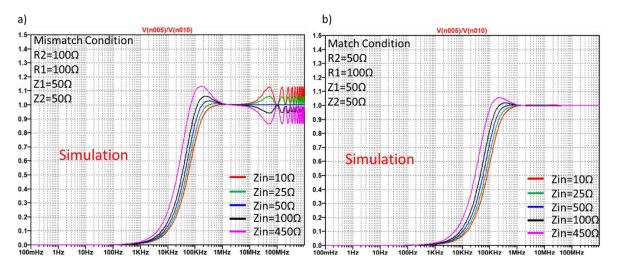


Figure 7 Zin sweep frequency response from simulation (a) mismatch and (b) match.

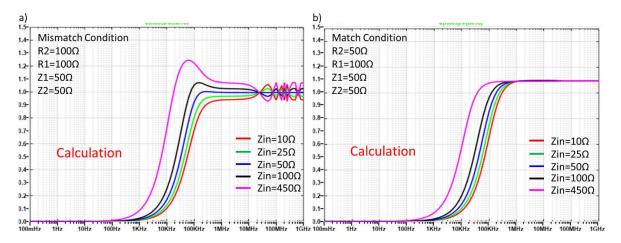


Figure 8 Zin sweep frequency response from formular(a) mismatch and (b) match.

The Zin sweep frequency response shows the difference between match and mismatch where in match case Zin is independence where reflection does not affect to the high pass region. In contrast, mismatch cases show strong Zin dependence. The experiment shown in Fig. 6 confirms the idea that the circuit will be independent from Zin when match condition is met. However, at the cutoff frequency overshoot appears to depend on the Zin. This is the good warning for technician to concern about the resonance peak. The simulation shown in Fig. 7 can roughly predict the overshoot appearance. In the case of formular equation, the prediction is roughly correct for (a) mismatch case where overshoot is predicted but wrong for (b) match case where overshoot disappears.

# **Group Delay calculation**

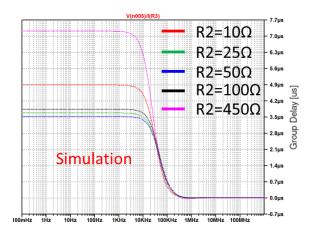


Figure 9 R2 sweep frequency group delay simulation.

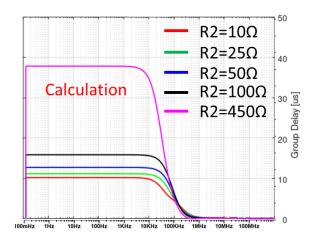


Figure 9 R2 sweep frequency group delay simulation.

In experiment, phase detection requires sensitive measurement especially for low frequency range where small delay must be detected from slow signal and network analyzer in the market only support 100kHz -3GHz range; therefore, result from simulation and formular only for discussion. The result from simulation and formular shows large difference where simulation shows delay in 10us range, but calculation shows delay in 50us range. For this reason, the phase both simulation and calculation result must be concern as they both have discrepancy from experiment. The delay appears in low frequency range, which is not use in photo-detachment measurement.

## Conclusion

The new formula proves the concept. With match condition, reflection will disappear, and the circuit becomes independent from the Zin. The result from simulation and experiment confirms that formular is derived correctly for scale. They also show that circuits become independent from Zin at high-pass region. However, formular and simulation is a prediction. Some differences appear where simulation shows less accurate cutoff decay characteristic and formular shows less accurate overshoot characteristic.

The most concern that the formular missed is that at match condition overshoot appear for different Zin value. For this reason, technician should protect recording device from damage, if some strong signal came at this overshoot frequency. However, this is subtle topic to put into the paper because the benefit of this circuit is flat frequency response at 1MHz – 1GHz range where the photo-detachment occupied this frequency range.