1　数论初步

1.1

证明.

(1)

$$\forall\ x|a,\ x|b \begin{cases} x > 0 & \xrightarrow{a>0,x|a}\ x \le a \\ x < 0 & \xrightarrow{a>0}\ x < a \end{cases} \Rightarrow\ x < a\ \xrightarrow{a|a,a|b}\ (a,b) = a.$$

(2)

$$\begin{cases} (a,b)|(a,b),\ (a,b)|b \\ \\ \forall\ x|(a,b),\ x|b,\ \text{有} x \le (a,b).\quad (\text{证明同}(1)) \end{cases} \Rightarrow\ ((a,b),b) = (a,b).$$

$\square$

1.2

证明.

(1) 不妨假设 $\exists\, n > 0,\ (n, n+1) = d > 1$

$$\begin{aligned} (n, n+1) = d &\Rightarrow \exists\, x, y \in \mathbb{Z},\ n = xd, n+1 = yd \\ &\Rightarrow 1 = (n+1) - n = (y-x)d > 0 \\ &\Rightarrow y > x,\ (y-x)d \ge d > 1 \\ &\Rightarrow \text{矛盾，假设不成立}. \end{aligned}$$

(2) 可取 (n, k)，证明如下

由推论 2.3，取 $x = 1,\ a = n,\ b = k$，有 $(n, k) = (n, n+k)$.

$\square$

1.3

(1) $(314, 159) = 1$，有解。由辗转相除法

$$314 = 159 * 1 + 155$$
$$159 = 155 * 1 + 4$$
$$155 = 4 * 38 + 3$$
$$4 = 3 * 1 + 1$$

即

$$
\begin{aligned}
1 &= 4 - 3 * 1 \\
&= 4 - (155 - 4 * 38) * 1 \\
&= (159 - 155 * 1) * 39 - 155 \\
&= 159 * 39 - 155 * 40 \\
&= 159 * 39 - (314 - 159 * 1) * 40 \\
&= 159 * 79 - 314 * 40
\end{aligned}
$$

即 $x = -40, y = 79.$

(2) $(3141, 1592) = 1$，有解。由辗转相除法

$$3141 = 1592 * 1 + 1549$$
$$1592 = 1549 * 1 + 43$$
$$1549 = 43 * 36 + 1$$

即

$$
\begin{aligned}
1 &= 1549 - 43 * 36 \\
&= (1592 - 43) - 43 * 36 \\
&= 1592 - 43 * 37 \\
&= \\
&= \\
&=
\end{aligned}
$$

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代数结构答案

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1.31

1.32