

1

1.1

- (1) 不能构成映射, 如 $x_1 = 0$, 则 $x_2 = 0, 1, \dots, 9$ 均满足.
- (2) 能构成映射, $\forall y_1 \in \mathbb{R}, \exists! y_2 = y_1^2$.
- (3) 不能构成映射, 如 $y_1 = 1$, 则 $y_2 = \pm 1$ 均满足.

1.2

- (1)

$$R_f = \{2, -2, 0\}$$

- (2)

$$\begin{cases} |A| = 9 \\ |R_f| = 3 \end{cases} \Rightarrow n = 3^9.$$

1.3

- (1) 满射

$$\forall y \in \mathbb{Z}^+, \exists x = \pm(y-1) \in \mathbb{Z}, f(x) = y.$$

- (2) 既不是单射, 又不是满射. 值域为 $\{0, 1, 2\} \subseteq \mathbb{Z} \cup \{0\}$

$$\forall y \in \mathbb{Z} \cup \{0\} \begin{cases} y \in \{0, 1, 2\}, \exists x = 3k + y (k \in \mathbb{Z}), f(x) = y. \\ y \notin \{0, 1, 2\}, \forall x \in \mathbb{Z}, f(x) \neq y. \end{cases}$$

- (3) f, g 均是双射.

$$\begin{cases} \forall y \in \mathbb{Z}, \exists! x = y - 1 \in \mathbb{Z}, f(x) = y. \\ \forall y \in \mathbb{Z}, \exists! x = y + 1 \in \mathbb{Z}, g(x) = y. \end{cases}$$

- (4) 满射.

$$\forall y \in \{0, 1\}, \exists x = 2k + y + 1 (k \in \mathbb{Z}), f(x) = y.$$

- (5) 既不是单射, 又不是满射.

$$\begin{cases} \forall y \in \mathbb{Z}, y < -16, \forall x \in \mathbb{Z}, f(x) > y. \\ \exists y = -15, f(0) = f(-2) = y. \end{cases}$$

1.4

证明.

$$\begin{cases} f: A \times B \rightarrow B \times A, (a, b) \mapsto (b, a) \\ g: B \times A \rightarrow A \times B, (b, a) \mapsto (a, b) \end{cases}$$

则

$$f \circ g = Id_{A \times B}, \quad g \circ f = Id_{B \times A}, \quad g = f^{-1}.$$

即 f 为双射, 即证 $|A \times B| = |B \times A|$. □

1.5

证明.

(1)

$$\forall f(x) = \sum_{i=0}^n a_i \cdot x^i, \exists! g(x) = \sum_{i=1}^n i \cdot a_i \cdot x^{i-1}, \frac{d}{dx} f(x) = g(x). (a_i \in \mathbb{R})$$

值域为 $R[x]$, 是满射不是双射.

$$\forall g(x) = \sum_{i=0}^n a_i \cdot x^i, \exists f(x) = a + \sum_{i=0}^n \frac{a_i}{i+1} \cdot x^{i+1}, \frac{d}{dx} f(x) = g(x). (a, a_i \in \mathbb{R})$$

(2)

$$\forall f(x) = \sum_{i=0}^n a_i \cdot x^i, \exists! g(x) = \sum_{i=0}^n \frac{a_i}{i+1} \cdot x^{i+1}, I(f(x)) = g(x). (a_i \in \mathbb{R})$$

值域为常数项为 0 的实系数多项式, 既不是满射, 也不是双射.

$$\forall g(x) = a (0 \neq a \in \mathbb{R}), \text{ 若 } \exists f(x) \in R[x], \text{ 满足 } I(f(x)) = g(x) = a \quad I(f(x)) = g(x).$$

则

$$a = g(x) = \int_0^x f(t) dt \xrightarrow{x=0} g(0) = \int_0^0 f(t) dt = 0 \neq a.$$

矛盾. □

1.6

证明. $\forall (b_{i1}, b_{i2}, \dots, b_{in}) \in S(B),$

$$\exists! f \in F, f: A \rightarrow B, \quad a_j \mapsto b_{ij}. \text{ 满足 } g(f) = (b_{i1}, b_{i2}, \dots, b_{in}).$$

即证 g 即是单射，又是满射，即 g 是从 F 到 $S(B)$ 的双射.

$$|F| = |S(B)| = |B|^n = m^n.$$

□

1.7

证明.

(1)

(2)

□

1.8

(1)

(2)

1.9

(1)

(2)

(3)

(4)

(5)

1.10

证明.

□

1.11

(1)

(2)

1.12

(1)

(2)

(3)

(4)

1.13

(1)

(2)

1.14

(1)

(2)

(3)

1.15

(1)

(2)

1.16

证明.

□

1.17

(1)

(2)

1.18

(1)

(2)

(3)

1.19

(1)

(2)