1

1.1

(1) 不能构成映射，如 $x_1 = 0$，则 $x_2 = 0, 1, \ldots, 9$ 均满足.

(2) 能构成映射，$\forall\ y_1 \in \mathbb{R},\ \exists!\ y_2 = y_1^2$.

(3) 不能构成映射，如 $y_1 = 1$，则 $y_2 = \pm 1$ 均满足.

1.2

(1)

$$R_f = \{2, -2, 0\}$$

(2)

$$\begin{cases} |A| = 9 \\ |R_f| = 3 \end{cases} \quad \Rightarrow\ n = 3^9.$$

1.3

(1) 满射

$$\forall\ y \in \mathbb{Z}^+,\ \exists\ x = \pm(y-1) \in \mathbb{Z},\ f(x) = y.$$

(2) 既不是单射，又不是满射. 值域为 $\{0, 1, 2\} \subseteq \mathbb{Z} \cup \{0\}$

$$\forall\ y \in \mathbb{Z} \cup \{0\} \begin{cases} y \in \{0, 1, 2\},\ \exists\ x = 3k + y(k \in \mathbb{Z}),\ f(x) = y. \\ y \notin \{0, 1, 2\},\ \forall\ x \in \mathbb{Z},\ f(x) \neq y. \end{cases}$$

(3) f, g 均是双射.

$$\begin{cases} \forall\ y \in \mathbb{Z},\ \exists!\ x = y - 1 \in \mathbb{Z}, f(x) = y. \\ \forall\ y \in \mathbb{Z},\ \exists!\ x = y + 1 \in \mathbb{Z}, g(x) = y. \end{cases}$$

(4) 满射.

$$\forall\ y \in \{0, 1\},\ \exists\ x = 2k + y + 1(k \in \mathbb{Z}),\ f(x) = y.$$

(5) 既不是单射，又不是满射.

$$\begin{cases} \forall\ y \in \mathbb{Z},\ y < -16,\ \forall\ x \in \mathbb{Z},\ f(x) > y. \\ \exists\ y = -15,\ f(0) = f(-2) = y. \end{cases}$$

1.4

证明.

$$\begin{cases} f: A\times B \to B\times A,\ (a,b)\mapsto (b,a) \\ g: B\times A \to A\times B,\ (b,a)\mapsto (a,b) \end{cases}$$

则

$$f\circ g = Id_{A\times B},\quad g\circ f = Id_{B\times A},\quad g = f^{-1}.$$

即 f 为双射，即证 $|A\times B| = |B\times A|$. □

1.5

证明.

(1)

$$\forall\ f(x) = \sum_{i=0}^{n} a_i\cdot x^i,\ \exists!\ g(x) = \sum_{i=1}^{n} i\cdot a_i\cdot x^{i-1},\ \frac{d}{dx}f(x) = g(x).\ (a_i\in\mathbb{R})$$

值域为 $R[x]$，是满射不是双射.

$$\forall\ g(x) = \sum_{i=0}^{n} a_i\cdot x^i,\ \exists\ f(x) = a + \sum_{i=0}^{n} \frac{a_i}{i+1}\cdot x^{i+1},\ \frac{d}{dx}f(x) = g(x).\ (a, a_i\in\mathbb{R})$$

(2)

$$\forall\ f(x) = \sum_{i=0}^{n} a_i\cdot x^i,\ \exists!\ g(x) = \sum_{i=0}^{n} \frac{a_i}{i+1}\cdot x^{i+1},\ I(f(x)) = g(x).\ (a_i\in\mathbb{R})$$

值域为常数项为 0 的实系数多项式，既不是满射，也不是双射.

$$\forall\ g(x) = a\ (0\neq a\in\mathbb{R}),\ 若\exists\ f(x)\in R[x], 满足 I(f(x)) = g(x) = a\ I(f(x)) = g(x).$$

则

$$g(x) = \int_0^x f(t)\,dt \xrightarrow{x=0} g(0) = \int_0^0 f(t)\,dt = 0 \neq a.$$

矛盾.

□

1.6

证明. □

1.7

证明.

(1)

(2)

□

1.8

(1)

(2)

1.9

(1)

(2)

(3)

(4)

(5)

1.10

证明. □

1.11

(1)

(2)

1.12

(1)

(2)

(3)

(4)

1.13

(1)

(2)

1.14

(1)

(2)

(3)

1.15

(1)

(2)

1.16

证明. □

1.17

(1)

(2)

1.18

(1)

(2)

(3)

1.19

(1)

(2)