

1 二元关系

1.1

- (1) R_1 有对称性.
- (2) R_2 有对称性.
- (3) R_3 有传递性, 反自反性.
- (4) R_4 有自反性, 传递性, 反对称性.
- (5) R_5 有自反性, 对称性, 传递性.

1.2

- (1) aR_1b , 当且仅当 $ab \geq 0$, 有

$$\left\{ \begin{array}{l} \forall a \in \mathbb{Z}, aR_1a; \\ aR_1b \leftrightarrow ab \geq 0 \leftrightarrow bR_1a; \\ (-1)R_10, 0R_11, (-1) \not R_11. \end{array} \right.$$

- (2) aR_2b , 当且仅当 $a \geq b$, 有

$$\left\{ \begin{array}{l} \forall a \in \mathbb{Z}, aR_2a; \\ aR_2b, bR_2c \Rightarrow a \geq b \geq c \Rightarrow aR_2c; \\ 5R_21, 1 \not R_25. \end{array} \right.$$

- (3) aR_3b , 当且仅当 $ab > 0$, 有

$$\left\{ \begin{array}{l} aR_3b \Rightarrow ab > 0 \Rightarrow bR_3a; \\ aR_3b, bR_3c \Rightarrow ab > 0, bc > 0, ac = abcd/b^2 > 0 \Rightarrow aR_3c; \\ 0 \in \mathbb{Z}, 0 \not R_30. \end{array} \right.$$

1.3

- (1) $R_1 \circ R_2 = \{(c, d)\}$
- (2) $R_2 \circ R_1 = \{(a, d), (a, c)\}$
- (3) $R_1^2 = \{(a, a), (a, b), (a, d)\}$

$$(4) R_2^3 = \{(b, c), (b, d), (c, d)\}$$

1.4

证明.

$$\forall (a, c) \in R_1 \circ (R_2 \cap R_3), \text{ 则}$$

$$\exists (a, b) \in (R_2 \cap R_3), (b, c) \in R_1.$$

故

$$\begin{cases} (a, b) \in R_2 \\ (a, b) \in R_3 \end{cases} \Rightarrow \begin{cases} (a, c) \in R_1 \circ R_2 \\ (a, c) \in R_1 \circ R_3 \end{cases} \Rightarrow (a, c) \in ((R_1 \circ R_2) \cap (R_1 \circ R_3))$$

$$\text{即证 } R_1 \circ (R_2 \cap R_3) \subseteq (R_1 \circ R_2) \cap (R_1 \circ R_3).$$

□

1.5

证明.

$$(1) \forall x \in A, (x, x) \in I_A, \text{ 故 } (x, x) \in R', \text{ 即证 } R' \text{ 在 } A \text{ 上自反.}$$

$$(2) R \subseteq I_A \cup R \Rightarrow R \subseteq R'.$$

$$(3) \text{ 若有自反关系 } R'' \text{ 满足 } R \subseteq R'', \text{ 由自反性可得 } I_A \subseteq R'', \text{ 故}$$

$$R' = I_A \cup R \subseteq R''$$

即证 R' 为 R 的自反闭包.

□

1.6

(1) 证明. (a) 自反性:

$$\forall (a, b) \in \mathbb{N} \times \mathbb{N}, a + b = b + a \Rightarrow (a, b) \sim (a, b).$$

(b) 对称性:

$$\forall (a, b) \sim (c, d), \text{ 有 } a + d = b + c, c + b = a + d \Rightarrow (c, d) \sim (a, b).$$

(c) 传递性:

$$\forall (a, b) \sim (c, d), (c, d) \sim (e, f), \text{ 有 } a + d = b + c, c + f = d + e$$

故

$$a + f = (a + d) + (c + f) - d - c = (b + c) + (d + e) - d - c = b + e \Rightarrow (a, b) \sim (e, f).$$

□

(2)

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