

1 数论初步

1.1

证明.

(1)

$$\forall x|a, x|b \begin{cases} x > 0 & \xrightarrow{a>0, x|a} x \leq a \\ x < 0 & \xrightarrow{a>0} x < a \end{cases} \Rightarrow x < a \xrightarrow{a|a, a|b} (a, b) = a.$$

(2)

$$\left\{ \begin{array}{l} (a, b)|(a, b), (a, b)|b \\ \forall x|(a, b), x|b, \text{ 有 } x \leq (a, b). \text{ (证明同(1))} \end{array} \right. \Rightarrow ((a, b), b) = (a, b).$$

□

1.2

证明.

(1) 不妨假设 $\exists n > 0, (n, n+1) = d > 1$

$$\begin{aligned} (n, n+1) = d &\Rightarrow \exists x, y \in \mathbb{Z}, n = xd, n+1 = yd \\ &\Rightarrow 1 = (n+1) - n = (y-x)d > 0 \\ &\Rightarrow y > x, (y-x)d \geq d > 1 \\ &\Rightarrow \text{矛盾, 假设不成立.} \end{aligned}$$

(2) 可取 (n, k) , 证明如下

由推论 2.3, 取 $x = 1, a = n, b = k$, 有 $(n, k) = (n, n+k)$.

□

1.3

(1) $(314, 159) = 1$, 有解。由辗转相除法

$$314 = 159 * 1 + 155$$

$$159 = 155 * 1 + 4$$

$$155 = 4 * 38 + 3$$

$$4 = 3 * 1 + 1$$

即

$$1 = 4 - 3 * 1$$

$$= 4 - (155 - 4 * 38) * 1$$

$$= (159 - 155 * 1) * 39 - 155$$

$$= 159 * 39 - 155 * 40$$

$$= 159 * 39 - (314 - 159 * 1) * 40$$

$$= 159 * 79 - 314 * 40$$

即 $x = -40, y = 79$.

(2) $(3141, 1592) = 1$, 有解。由辗转相除法

$$3141 = 1592 * 1 + 1549$$

$$1592 = 1549 * 1 + 43$$

$$1549 = 43 * 36 + 1$$

即

$$1 = 1549 - 43 * 36$$

$$= (1592 - 43) - 43 * 36$$

$$= 1592 - 43 * 37$$

$$= 1592 - (1592 - 1549 * 1) * 37$$

$$= 1549 * 37 - 1592 * 36$$

$$= (3141 - 1592) * 37 - 1592 * 36$$

$$= 3141 * 37 - 1592 * 73$$

即 $x = 37, y = -73$.

1.4

证明.

□

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