

# 概统作业 (Week 1)

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## 1 (P42, 习题 11)

(1) 基本事件共有  $|\Omega| = \binom{52}{10}$  个.

(1) 有 1 种花色: 基本事件有  $\binom{4}{1} \binom{13}{10} = 4 \binom{13}{10}$  个.

(2) 有 2 种花色: 基本事件有  $\binom{4}{2} \sum_{i=1}^9 \binom{13}{i} \binom{13}{10-i} = 6 \left[ \binom{26}{10} - 2 \binom{13}{10} \right]$  个.

(3) 有 3 种花色: 基本事件有

$$\begin{aligned} \binom{4}{3} \sum_{i=1}^8 \binom{13}{i} \left[ \sum_{j=1}^{9-i} \binom{13}{j} \binom{13}{10-i-j} \right] &= 4 \sum_{i=1}^8 \binom{13}{i} \left( \binom{26}{10-i} - 2 \binom{13}{10-i} \right) \\ &= 4 \sum_{i=1}^8 \left[ \binom{13}{i} \binom{26}{10-i} - 2 \binom{13}{i} \binom{13}{10-i} \right] \\ &= 4 \left[ \binom{39}{10} - 3 \binom{26}{10} + 3 \binom{13}{10} \right] \end{aligned}$$

即:

$$\begin{aligned} \mathbb{P}(A) &= 1 - \frac{4 \binom{39}{10} - 6 \binom{26}{10} + 4 \binom{13}{10}}{\binom{52}{10}} \\ &= 1 - \frac{4 \times 39 \times \cdots \times 30 - 6 \times 26 \times \cdots \times 17 + 4 \times 13 \times \cdots \times 4}{52 \times \cdots \times 43} \\ &\approx 0.8413 \end{aligned}$$

## 2 (P42, 习题 12)

记胜为 +, 负为-. 记甲在三局两胜制中胜利为事件 A, 甲在五局三胜制游戏中胜利为事件 B.

(1) 三局两胜制 (甲胜情况: +++, ++-, +-+, -++)

(a) 甲: +++

$$\mathbb{P}(a) = p^3.$$

(b) 甲: ++-

$$\mathbb{P}(b) = p^2(1-p).$$

(c) 甲:++

$$\mathbb{P}(c) = p(1-p)p = p^2(1-p).$$

(d) 甲:-++

$$\mathbb{P}(d) = (1-p)p^2.$$

$$\mathbb{P}(A) = \mathbb{P}(a) + \mathbb{P}(b) + \mathbb{P}(c) + \mathbb{P}(d) = p^2(3-2p).$$

(2) 五局三胜制 (甲胜情况:5+,4+,3+)

(a) 甲:5+

$$\mathbb{P}(a) = \binom{5}{5} p^5 = p^5.$$

(b) 甲:4+

$$\mathbb{P}(b) = \binom{4}{5} p^4(1-p) = 5p^4(1-p).$$

(c) 甲:3+

$$\mathbb{P}(c) = \binom{3}{5} p^3(1-p)^2 = 10p^3(1-p)^2.$$

$$\begin{aligned} \mathbb{P}(B) &= \mathbb{P}(a) + \mathbb{P}(b) + \mathbb{P}(c) & f(p) &= \mathbb{P}(B) - \mathbb{P}(A) \\ &= p^3[p^2 + 5p - 5p^2 + 10(p^2 - 2p + 1)] & &= p^3(6p^2 - 15p + 10) - p^2(3 - 2p) \\ &= p^3(6p^2 - 15p + 10) & &= 3p^2(2p^3 - 5p^2 + 4p - 1) \end{aligned}$$

则

$$f(0) = f(0.5) = f(1) = 0 \Rightarrow f(p) = 3p^2(2p-1)(p-1)^2 > 0, \mathbb{P}(B) > \mathbb{P}(A)$$

即五局三胜制对甲更有利.

### 3 (P42, 习题 14)

$|\Omega| = P_N^n = n! \cdot C_N^n$ . 分别用  $A, B, C$  表示上述 (1) - (3) 各事件.

(1) 即等价于将  $n$  个人和  $N-n$  个座位排成一排, 而且每个人不相邻

$$|A| = n! \cdot C_{N-n+1}^n \Rightarrow P(A) = \frac{|A|}{|\Omega|} = \frac{C_{N-n+1}^n}{C_N^n}.$$

(2) 将两个人视为一对, 则等价于将  $n/2$  对和  $N-n$  个座位排成一排, 而且每对不相邻. 若  $2|n$ , 有

$$|B| = n! \cdot C_{N-n+1}^{n/2} \Rightarrow P(B) = \frac{|B|}{|\Omega|} = \frac{C_{N-n+1}^{n/2}}{C_N^n}.$$

(3)

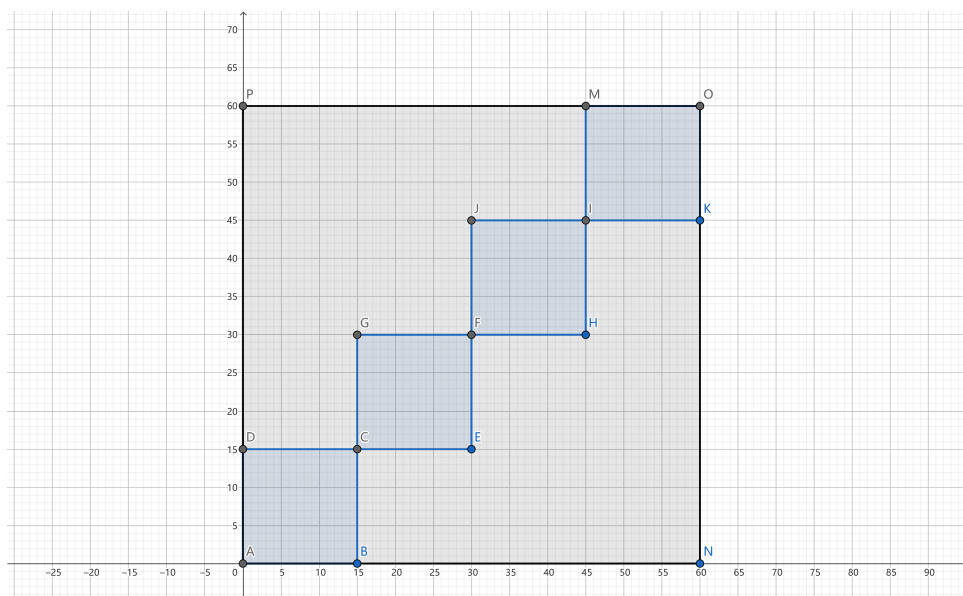
$$\begin{aligned} |C| &= \begin{cases} 2^n \cdot n! \cdot C_{N/2}^n & (N \equiv 0(\text{mod } 2)) \\ n \cdot 2^{n-1} \cdot (n-1)! \cdot C_{(N-1)/2}^{n-1} + 2^n \cdot n! \cdot C_{(N-1)/2}^n & (N \equiv 1(\text{mod } 2)) \end{cases} \\ \Rightarrow P(C) &= \begin{cases} \frac{2^n \cdot C_{N/2}^n}{C_N^n} & (N \equiv 0(\text{mod } 2)) \\ \frac{2^{n-1} \cdot C_{(N-1)/2}^{n-1} + 2^n \cdot C_{(N-1)/2}^n}{C_N^n} & (N \equiv 1(\text{mod } 2)) \end{cases} \end{aligned}$$

#### 4 (P42, 习题 18)

设甲到达时距 3 点  $x$  分钟, 乙到达时距 3 点  $y$  分钟, 以第  $i$  班公交车在 3 点后  $t_i$  ( $t_0 = 0$ ) 分钟到达.

$$\text{甲乙均上了第 } i \text{ 班车} \Leftrightarrow t_{i-1} < x, y \leq t_i, (i = 1, 2, 3, 4)$$

如图



$$P = \frac{S_{\text{蓝}}}{S_{\text{总}}} = \frac{4 \times 15 \times 15}{60 \times 60} = \frac{1}{4}$$

即概率为  $\frac{1}{4}$ .

#### 5 (P42, 习题 22)

除始发站外有 11 站,  $|\Omega| = 11^8$ . 分别用  $A, B, C$  表示上述 (1) – (3) 各事件.

(1)

$$|A| = 8! \cdot C_{11}^8 \Rightarrow P(A) = \frac{|A|}{|\Omega|} = \frac{8! \cdot C_{11}^8}{11^8}$$

(2)

$$|B| = C_{11}^1 \Rightarrow P(B) = \frac{|B|}{|\Omega|} = \frac{1}{11^7}$$

(3)

$$|C| = C_{11}^3 \cdot 5! \cdot C_8^5 \Rightarrow P(C) = \frac{|C|}{|\Omega|} = \frac{5! \cdot C_{11}^3 \cdot C_8^5}{11^8}$$