概统作业 (Week 9)

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1 (P173 T16)

(1)

$$E(sgn(X)) = \int_{-2}^{1} sgn(x) \cdot f(x)dx = \frac{1}{3} \cdot \left(\int_{-2}^{0} sgn(x)dx + \int_{0}^{1} sgn(x)dx \right) = \frac{1}{3} \cdot (-1) = -\frac{1}{3}.$$

$$E(sgn^2(X)) = \int_{-2}^{1} sgn^2(x) \cdot f(x)dx = \frac{1}{3} \cdot \left(\int_{-2}^{0} sgn^2(x)dx + \int_{0}^{1} sgn^2(x)dx \right) = \frac{1}{3} \cdot 3 = 1.$$

因此

$$Var(sgn(X)) = E(sgn^2(X)) - (E(sgn(X)))^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

(2)

$$E(sgn(X) \cdot X) = \int_{-\infty}^{+\infty} sgn(x) \cdot x \cdot f(x)dx = 2\int_{0}^{+\infty} x \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = -\sqrt{\frac{2}{\pi}} \cdot e^{-\frac{x^2}{2}} \Big|_0^{+\infty} = \sqrt{\frac{2}{\pi}}.$$

2 (P173 T18)

(1)

$$f_Y(y) = P()f_{Y|X=1}(y) + f_{Y|X=2}(y) =$$

(2)

3 (P174 T24)

(1)

(2)

4 (P174 T26)

(1)

(2)

(3)

5 (P178 T55)

Proof.

(1)

(2)

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