

# 概统作业 (Week 3)

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## 1 (P45 第 37 题)

(1)

$$P(A) = P(AC) + P(A\bar{C}) = P(A|C) \cdot P(C) + P(A\bar{C}) \cdot P(\bar{C}) = 0.55.$$

(2)

$$P(B) = P(BC) + P(B\bar{C}) = P(B|C) \cdot P(C) + P(B\bar{C}) \cdot P(\bar{C}) = 0.5.$$

(3)

$$\begin{aligned} P(AB) &= P(ABC) + P(AB\bar{C}) \\ &= P(AB|C) \cdot P(C) + P(AB\bar{C}) \cdot P(\bar{C}) \\ &= P(A|C) \cdot P(B|C) \cdot P(C) + P(A|\bar{C}) \cdot P(B|\bar{C}) \cdot P(\bar{C}) \\ &= 0.9 \times 0.9 \times 0.5 + 0.2 \times 0.2 \times 0.5 \\ &= 0.425 \end{aligned}$$

(4) *Proof.*

$$P(A)P(B) = 0.55 \times 0.5 = 0.275 \neq P(AB).$$

□

## 2 (P45 第 38 题)

设第一次射中为事件  $A$ , 第二次射中为事件  $B$ , 第三次射中为事件  $C$ .

(1) 设恰有一次射中为事件  $M$

$$\begin{aligned} P(M) &= P(\overline{ABC}) + P(\overline{AB}\bar{C}) + P(\overline{A}BC) \\ &= 0.5 \times 0.4 \times 0.2 + 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.4 \times 0.8 \\ &= 0.26. \end{aligned}$$

(2) 设至少有一次射中为事件  $N$

$$P(N) = 1 - P(\overline{ABC}) = 0.96.$$

## 3 (P81 第 4 题)

设营收为  $X$  万元, 由题意可得  $X$  取值有 10, 5, 0, -2, 一天内发生故障的概率  $p = 0.2$ .

$$P(X = 10) = \binom{5}{0} p^0 (1-p)^5 = \frac{1024}{3125}.$$

$$P(X = 5) = \binom{5}{1} p^1 (1-p)^4 = \frac{256}{625}.$$

$$P(X=0) = \binom{5}{2} p^2 (1-p)^3 = \frac{128}{625}$$

$$P(X=-2) = 1 - P(X=10) - P(X=5) - P(X=0) = \frac{181}{3125}.$$

分布律如图

X	10	5	0	-2
P	$\frac{1024}{3125}$	$\frac{256}{625}$	$\frac{128}{625}$	$\frac{181}{3125}$

#### 4 (P81 第 8 题)

(1)

$$\begin{aligned} P(X \geq 1) &= 1 - P(X=1) - P(X=0) \\ &= 1 - 20 \cdot \binom{20}{1} \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^{19} - \binom{20}{0} \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{20} \\ &= 1 - \frac{4^{20}}{5^{19}} - \frac{4^{20}}{5^{20}} \\ &= \frac{5^{20} - 6 \times 4^{20}}{5^{20}}. \end{aligned}$$

(2) 设  $X=k$  时概率最大, 则有

$$\begin{cases} P(X=k) \geq P(X=k+1) \\ P(X=k) \geq P(X=k-1) \end{cases} \Rightarrow \begin{cases} \binom{20}{k} (1-p)^{20-k} p^k \geq \binom{20}{k+1} (1-p)^{19-k} p^{k+1} \\ \binom{20}{k} (1-p)^{20-k} p^k \geq \binom{20}{k-1} (1-p)^{21-k} p^{k-1} \end{cases}$$

解得

$$21p - 1 \leq k \leq 21p \Rightarrow 3.2 \leq k \leq 4.2 \xrightarrow{k \in \mathbb{N}} k = 4.$$

即  $X=4$  时概率最大.

#### 5 (P81 第 9 题)

设  $A$  出现的次数为  $X$ , 则  $X \in \{0, 1, 2, 3, 4\}$ .

(1)

$$\begin{aligned} P(B) &= P(B|X=1) \cdot P(X=1) + P(B|X \geq 2) \cdot P(X \geq 2) \\ &= 0.6 \cdot C_4^1 \cdot p^1 (1-p)^3 + 1 \cdot [1 - C_4^1 \cdot p^1 (1-p)^3 - C_4^0 \cdot p^0 (1-p)^4] \\ &= \frac{29763}{50000}. \end{aligned}$$

(2)

$$P(X=1|B) = \frac{P((X=1) \cap B)}{P(B)} = \frac{0.6 \cdot C_4^1 \cdot p^1 (1-p)^3}{29763/50000} = \frac{1372}{3307}.$$