概统作业 (Week 2)

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1

设挑出第一箱为事件 A，挑出第二箱为事件 B

(1)

(2)

2

设方程有实根为事件 M，方程有重根为事件 N，$B=i$ 记为 B_i，$C=i$ 记为 C_i.

(1) $M \Leftrightarrow B^2-4C\geq 0$.

$$\begin{aligned}P(M)&=P(B_2C_1)+P(B_3(C_1+C_2))+P(B_4(C_1+C_2+C_3+C_4))+P(B_5)+P(B_6)\\&=\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{4}{6}+\frac{1}{6}+\frac{1}{6}\\&=\frac{1}{36}+\frac{2}{36}+\frac{4}{36}+\frac{1}{3}\\&=\frac{19}{36}.\end{aligned}$$

(2) $N \Leftrightarrow B^2=4C$.

$$\begin{aligned}P(N)&=P(B_2C_1)+P(B_4C_4)\\&=\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\\&=\frac{1}{36}+\frac{1}{36}\\&=\frac{1}{18}.\end{aligned}$$

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Proof.

$$\begin{aligned}P(A|B)=P(A|B^C)\Leftrightarrow&\frac{P(AB)}{P(B)}=\frac{P(AB^C)}{P(B^C)}\\\Leftrightarrow&P(AB)\cdot P(B^C)=P(AB^C)\cdot P(B)\\\Leftrightarrow&P(AB)\cdot(1-P(B))=P(AB^C)\cdot P(B)\\\Leftrightarrow&P(AB)=P(B)\cdot\left[P(AB)+P(AB^C)\right]\\\Leftrightarrow&P(AB)=P(B)\cdot P(A)\cdot\left[P(B)+P(B^C)\right]\\\Leftrightarrow&P(AB)=P(B)\cdot P(A).\end{aligned}$$

□

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$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) - (P(AB) + P(BC) + P(AC)) + P(AC \cap B) \\ &= 1 - (\frac{1}{8} + \frac{1}{8} + 0) + 0 \\ &= \frac{3}{4}. \end{aligned}$$

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