概统作业 (Week 6)

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1 (P116 T6)

(1) 由题意可得 $i,j\in\mathbb{N}^*,\ 1\le i<j,$ 有

$$P(X=i,Y=j)=\begin{cases}p^2(1-p)^{j-2} & (1\le i<j)\\ 0 & \text{其他}\end{cases}.$$

(2)

$$P(X=i)=p(1-p)^{i-1}.\quad (i\in\mathbb{N}^*)$$
$$P(Y=j)=(j-1)\cdot p^2(1-p)^{j-2}.\quad (j\in\mathbb{N}^*,\ j\ge 2)$$

2 (P116 T10)

(1)

$$F(x,y)=\int_{-\infty}^{x}du\int_{-\infty}^{y}dv\cos u\cos v=\begin{cases}0, & (x\le 0\text{或 }y\le 0)\\ \sin x\sin y, & (0<x<\dfrac{\pi}{2},\ 0<y<\dfrac{\pi}{2})\\ \sin x, & (0<x<\dfrac{\pi}{2},\ y\ge\dfrac{\pi}{2})\\ \sin y, & (x\ge\dfrac{\pi}{2},\ 0<y<\dfrac{\pi}{2})\\ 1. & (x\ge\dfrac{\pi}{2},\ y\ge\dfrac{\pi}{2})\end{cases}$$

(2)

$$\begin{aligned}P(0<X<\pi/4,\ \pi/4<Y<\pi/2)=&F(\pi/4,\pi/2)-F(\pi/4,\pi/4)-F(0,\pi/2)+F(0,\pi/4)\\ =&\frac{\sqrt{2}}{2}-\frac{1}{2}-0+0\\ =&\frac{\sqrt{2}-1}{2}.\end{aligned}$$

3 (P116 T9)

(1)

$$\begin{cases}F(+\infty,+\infty) &= a(b+\pi/2)(c+\pi/2)=1\\ F(-\infty,-\infty) &= a(b-\pi/2)(c-\pi/2)=0\\ F(0,-\infty) &= ab(c-\pi/2)=0\\ F(-\infty,0) &= ac(b-\pi/2)=0\end{cases}\Rightarrow\ a=\frac{1}{\pi^2},\ b=\frac{\pi}{2},\ c=\frac{\pi}{2}$$

(2)

$$\begin{aligned}P(X>0,Y>0)&=1-P(X\le 0)-P(Y\le 0)+P(X\le 0,Y\le 0)\\&=1-F(0,+\infty)-F(+\infty,0)+F(0,0)\\&=1-\frac{1}{2}-\frac{1}{2}+\frac{1}{4}\\&=\frac{1}{4}\end{aligned}$$

4 (P116 T5)

由题意可得

$$\begin{aligned}&P(X=-1,X+Y=0)=P(X=-1)\cdot P(X+Y=0)\\\Leftrightarrow&P(X=-1,Y=1)=P(X=-1)\cdot[P(X=1,Y=-1)+P(X=-1,Y=1)]\\\Leftrightarrow&a=(a+0.2)\cdot(a+b)\end{aligned}$$

又 $a+b+0.2+0.3=1$，有

$$\begin{cases}a+b=0.5\\a=(a+0.2)(a+b)\end{cases}\Rightarrow\ a=0.2,\ b=0.3.$$

5 (P117 T17)

(1) 对于 $X=x(0<x<1)$，有

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_1(x)}=\begin{cases}\dfrac{3y^2}{x^3}, & (0<y<x)\\[2ex] 0. & (\text{其他})\end{cases}\Rightarrow\ f(x,y)=f_{Y|X}(y|x)\cdot f_1(x)=\begin{cases}\dfrac{9y^2}{x} & (0<y<x<1)\\[2ex] 0. & (\text{其他})\end{cases}$$

又 $\forall\ x\notin(0,1),\ f_X(x)=0\Rightarrow f(x,y)=0$，故

$$f(x,y)=\begin{cases}\dfrac{9y^2}{x} & (0<y<x<1)\\[2ex] 0. & (\text{其他})\end{cases}$$

(2)

$$f_Y(y)=\int_{\mathbb{R}}f(x,y)dx=\int_y^1 f(x,y)dx=\begin{cases}-9y^2\ln y, & (0<y<1)\\ 0. & \text{其他}\end{cases}$$