概统作业 (Week 15)

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1

此检验犯第二类错误的概率为

$$\alpha_{2\Psi}(\theta) = 1 - \beta_\Psi(\theta) = 1 - P_{\theta=\theta_1}\{X_1 = X_2 = X_3 = 1\} = 1 - \theta_1^6.$$

2

(1) 有最大似然函数

$$L(X_1, X_2, \dots, X_n; p) = \prod_{i=1}^{n} p(1-p)^{k-1} = p^{130}(1-p)^{233}$$

$$\ln L = 130 \cdot \ln p + 233 \cdot \ln(1-p)$$

令

$$\frac{\partial \ln L}{\partial p} = \frac{130}{p} + \frac{233}{p-1} = 0.$$

得

$$\hat{p} = \frac{130}{363} \approx 0.358$$

检验二阶导

$$\frac{\partial^2 \ln L}{\partial p^2} = -\frac{130}{p^2} - \frac{233}{(p-1)^2}$$

当 $p = \hat{p}$ 时，二阶偏导为负，故所求驻点为 L 极大值点，最大似然估计为

$$\hat{p} = \frac{130}{363} \approx 0.358$$

(2)

类别	1	2	3	4	5	6	≥ 7
$\widehat{E}$	46.54	29.879	19.182	12.315	7.906	5.076	9.102
O	48	31	20	9	6	5	11
$O - \widehat{E}$	1.46	1.121	0.818	-3.315	-1.906	-0.076	1.898

有检验

$$H_0 : P(X = k) = p \cdot (1-p)^{k-1}\ (k = 1, 2, \dots) \quad \leftrightarrow \quad H_1 : \exists\, k \in \mathbb{N}^*, P(X = k) \neq p \cdot (1-p)^{k-1}$$

构造统计量

$$Z = \sum \frac{(O - \widehat{E})^2}{\widehat{E}} \sim \chi^2_{k-r-2} \qquad (n \to \infty)$$

有拒绝域

$$W = \{Z > \chi^2_{k-r-1}(\alpha)\}$$

代入数据 $k = 7, r = 1, \alpha = 0.05$，得

$$\chi^2_{k-r-1}(\alpha) = \chi^2_7(0.05) = 14.067$$

$$Z = \sum \frac{\left(O - \widehat{E}\right)^2}{\widehat{E}} = 1.8715 < 14.067 \;\Rightarrow\; Z \notin W$$

故不能拒绝 H_0，即在显著性水平 $\alpha = 0.05$ 下认为 X 服从几何分布。

3

有检验

$$H_0 : p_{ij} = P(X = i)P(Y = j)\ (i = 1, \dots, a;\ j = 1, \dots, b) \quad \leftrightarrow \quad H_1 : \exists\ i, j \in \mathbb{N}^*, p_{ij} \neq P(X = i)P(Y = j)$$

取检验统计量

$$Z = \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{(n \cdot n_{ij} - n_{i.} \cdot n_{.j})}{n \cdot n_{i.} \cdot n_{.j}} \sim \chi^2_{(a-1)(b-1)} \qquad (n \to \infty)$$

拒绝域

$$W = \left\{Z > \chi^2_{(a-1)(b-1)}(\alpha)\right\}$$

代入数据 $a = 6,\ b = 2,\ \alpha = 0.05$，得

$$\chi^2_{(a-1)(b-1)}(\alpha) = \chi^2_6(0.05) = 12.592$$

$$Z = \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{(n \cdot n_{ij} - n_{i.} \cdot n_{.j})}{n \cdot n_{i.} \cdot n_{.j}} = 3.1922 < 12.592 \;\Rightarrow\; Z \notin W$$

故不能拒绝 H_0，即在显著性水平 $\alpha = 0.05$ 下认为两个班级的英语水平大致相等。