1. 取 $b\to a$，则 $b-a\to 0$，由 a 任意性知，$F(b)-F(a)$ 必 $\to 0$，故 $f(x)$ 存在

$F(x)=\int_{-\infty}^{x} f(x)dx=\int_0^x f(x)dx$，$F(b)-F(a)=\int_0^b f(x)dx-\int_0^a f(x)dx=\int_a^b f(x)dx$

故 $\int_a^b f(x)dx=\int_{a+\varepsilon}^{b+\varepsilon} f(x)dx$，即 $\int_a^{a+\varepsilon} f(x)dx=\int_b^{b+\varepsilon} f(x)dx$.

令 $\varepsilon\to 0$，则 $\int_a^{a+\varepsilon} f(x)dx=\varepsilon f(a)$，同理 $\int_b^{b+\varepsilon}=\varepsilon f(b)$

故 $f(a)=f(b)$，由 a、b 任意性知 $f(x)=C\ (0\le x\le 1)$

而 $\int_0^1 f(x)dx=C=1$，故 $X\sim U[0,1]$

2(1) $P(96<R<104)=\int_{96}^{104} f(x)dx$，$f(x)=\frac{1}{105-95}=\frac{1}{10}$，$P=0.8$.

(2) $P(96<R<104)=F(X=104)-F(X=96)$

对 $R\sim N(100,4)$ 标准化变换，即 $\frac{X-100}{2}\sim N(0,1)$

则 $P=\Phi(2)-\Phi(-2)=\Phi(2)-(1-\Phi(2))=2\Phi(2)-1=0.9544$

3.(1) 标准化变换，$\frac{X-1}{2}\sim N(0,1)$

$P(0\le X\le 4)=\Phi(\frac{3}{2})-(1-\Phi(\frac{1}{2}))=0.6247$

$P(X>2.4)=1-\Phi(0.7)=0.2420$

$P(|X|>2)=\Phi(-1.5)+1-\Phi(0.5)=2-\Phi(1.5)-\Phi(0.5)=0.3753$

(2) 即 $1-\Phi(\frac{C-1}{2})=2\Phi(\frac{C-1}{2})$，则 $\Phi(\frac{C-1}{2})=\frac{1}{3}$

故 $\frac{C-1}{2}=-0.43$，$C=0.14$

4.(1) $X\sim f(x)=1$. $y=g(x)=\frac{x}{1-x}=-1+\frac{1}{1-x}$，$1-x=\frac{1}{y+1}$，$x=\frac{y}{y+1}=h(y)$

故 $f_1(y)=\frac{1}{(y+1)^2}I_{[0,\frac{1}{2}]}(x)$

(2) 当 $x\in(a,1]$ 时，$Z=g(x)=x$，$x=h(y)=y$，故 $f_1(y)=f(x)\cdot 1=1$.

当 $x\in[0,a]$ 时，$Z=g(x)=0$，故 $P(Z=0)=a$. ∴分布函数 $F(x)=\begin{cases}0 & (x<0)\\ a & (0\le x<a)\\ x & (0\le a<1)\\ 1 & (x\ge 1)\end{cases}$

(3) 当 $x\in[0,b]$ 时，$W=g(x)=x^2+x$，故 $x=h(y)=\frac{\sqrt{1+4y}-1}{2}$

故 $f_1(y)=\frac{-1+\sqrt{1+4y}}{2}\times\frac{1}{\sqrt{1+4y}}$

当 $x\in(b,1]$ 时，$W=g(x)=x^2$，故 $x=h'(y)=\sqrt{y}$，故 $f_2(y)=\sqrt{y}\times\frac{1}{2\sqrt{y}}=\frac{1}{2}$

故 $f_3(y)=f_1(y)+f_2(y)=\frac{\sqrt{1+4y}-1}{2\sqrt{1+4y}}I_{[0,b^2+b]}(y)+\frac{1}{2}I_{(b^2,1]}(y)$

5. 每次取红球 $P_1=\frac{1}{6}$，黑球 $P=\frac{1}{3}$，白球 $P=\frac{1}{2}$

(1) $P(X=1|Z=0)=P(X=1,Z=0)/P(Z=0)$

$=\frac{1}{6}\times\frac{1}{3}\times C_2^1/(\frac{1}{6}\times\frac{1}{3}\times C_2^1+\frac{1}{3}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{6})$

$=4/9$

(2)

X \ Y	0	1	2
0	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{9}$
1	$\frac{1}{6}$	$\frac{1}{9}$	0
2	$\frac{1}{36}$	0	0

1. $F(\frac{1}{p}) - F(0) = F(\frac{2}{p}) - F(\frac{1}{p}) = \cdots \quad F(1) - F(\frac{p-1}{p})$

故 $F(\frac{1}{p}) = \frac{1}{p} \quad F(\frac{q}{p}) = qF(\frac{1}{p}) = F(\frac{q}{p})$

$\forall\, r \in \mathbb{Q} \cap [0,1]$ 可以表示为 $r = \frac{q}{p} \quad p, q \in \mathbb{N}$

故 $F(r) = r$ 由有理数的稠密性.

$\Rightarrow F(x) = x$