☑PTMS_HW4_PB21111668_姜宇曈

(补交)

① (1) $P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda}\quad(k=0,1\dots)$

其中X为每次产卵的数量

$$P(Y=n\mid X=k)=C_k^n\cdot p^n(1-p)^{k-n}\quad(n\leq k).$$

$$P(Y=n)=\sum_{k=n}^{\infty}P(Y=n\mid X=k)\cdot P(X=k)$$

(ps: 注意范围)

$$=\sum_{k=n}^{\infty}\frac{\lambda^k}{\cancel{k!}}e^{-\lambda}\cdot\frac{\cancel{k!}}{n!(k-n)!}p^n\cdot(1-p)^{k-n}$$

$$=\frac{(\lambda p)^n}{n!}e^{-\lambda}\sum_{k=n}^{\infty}\frac{\lambda^{k-n}}{(k-n)!}\cdot(1-p)^{k-n}$$

$$=\frac{(\lambda p)^n}{n!}e^{-\lambda}\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}\cdot(1-p)^k$$

$$=\frac{(\lambda p)^n}{n!}e^{-\lambda}\cdot e^{\lambda(1-p)}$$

$$=\frac{(\lambda p)^n}{n!}e^{-\lambda p}$$

∴ Y服从参数为λp的泊松分布

$$P(Z=m\mid X=k)=C_k^m(1-p)^m\cdot p^{k-m}$$

$$P(Z=m)=\sum_{k=m}^{\infty}P(Z=m\mid X=k)\cdot P(X=k)$$

$$=\sum_{k=m}^{\infty}C_k^m\cdot(1-p)^m\cdot p^{k-m}\frac{\lambda^k}{k!}e^{-\lambda}$$

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$$= \sum_{k=m}^{\infty} \frac{\cancel{k!}}{m!(k-m)!}(1-p)^m \cdot p^{k-m} \cdot \frac{\lambda^k}{\cancel{k!}} e^{-\lambda}$$

$$= \frac{\lambda^m e^{-\lambda} \cdot (1-p)^m}{m!} \sum_{k=m}^{\infty} \frac{\lambda^{k-m} \cdot p^{k-m}}{(k-m)!}$$

$$= \frac{(\lambda(1-p))^m e^{-\lambda}}{m!} e^{\lambda p}$$

$$= \frac{(\lambda(1-p))^m}{m!} e^{-\lambda(1-p)}$$

∴ Z服从以$\lambda(1-p)$为参数的泊松分布.

$X = Y + Z$

$$P(Y=n, Z=m) = P(X=m+n) \cdot P(Y=n \mid X=m+n)$$

$$= \frac{\lambda^{m+n}}{(m+n)!} e^{-\lambda} \cdot C_{m+n}^{n} p^n (1-p)^m$$

$$= \frac{\lambda^{m+n}}{n!\,m!} e^{-\lambda} \cdot p^n \cdot (1-p)^m$$

$$P(Y=n) \cdot P(Z=m) = \frac{(\lambda p)^n}{n!} e^{-\lambda p} \cdot \frac{(\lambda(1-p))^m}{m!} \cdot e^{-\lambda(1-p)}$$

$$= \frac{\lambda^{m+n}}{n!\,m!} e^{-\lambda} \cdot p^n (1-p)^m$$

∴ $P(Y=n, Z=m) = P(Y=n) \cdot P(Z=m)$.

∴ Y和Z相互独立

② $\frac{1}{4} = P(X=1) = F(1) - F(1^-) = 1 - (ax+b)\big|_{1^-}$

$= 1 - a - b$

又∵ $F(x)$ 右连续

∴ $\frac{1}{8} = F(-1) = F(-1^+) = ax+b\big|_{-1^+} = -a+b$.

∴ $\begin{cases} a = \frac{5}{16} \\ b = \frac{7}{16} \end{cases}$

③ (1) $1 = F(+\infty) = \int_{-\infty}^{+\infty} f(x)\,dx = a\int_{-\infty}^{+\infty} \frac{1}{1+x^2}\,dx$

$= a \arctan x \big|_{-\infty}^{+\infty} = a\pi$

∴ $a = \frac{1}{\pi}$

(2) $F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{x} \frac{1}{\pi}\frac{1}{1+x^2}\,dx = \frac{1}{\pi}\arctan x\big|_{-\infty}^{x}$

$= \frac{1}{\pi}\arctan x + \frac{1}{2}$ $\quad x \in (-\infty, +\infty)$

(3) $P(|X|<1) = P(-1<X<1)$ (由 $F(x)$ 连续)

$= F(1) - F(-1)$

$= \frac{1}{\pi}\arctan 1 + \frac{1}{2} - \left(\frac{1}{\pi}\arctan(-1) + \frac{1}{2}\right)$

$= \frac{1}{2}$

④ $X \sim U(1,4)$

∴ $f(x) = \begin{cases} \frac{1}{3} & 1 \le x \le 4 \\ 0 & 其他 \end{cases}$

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每次检测 $P(X>2)=\int_2^{+\infty} f(x)\,dx=\frac{2}{3}$

$P(Y\geq 2)=P(Y=2)+P(Y=3)$

$=C_3^2\cdot P(X>2)^2\cdot(1-P(X>2))^{3-2}+C_3^3\,P(X>2)^3\cdot(1-P(X>2))^{3-3}$

$=\frac{20}{27}$

5 (I) 参数为λ的指数分布：
密度函数：

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & (x>0)\\ 0 & (x\leq 0)\end{cases}$$

$X\sim EXP(\lambda)$

分布函数：

$$F(x)=\begin{cases}1-e^{-\lambda x} & (x\geq 0)\\ 0 & x<0\end{cases}\quad(\lambda>0)$$

(II) 代入λ=1，则

$$F(x)=\begin{cases}1-e^{-x} & (x\geq 0)\\ 0 & (x<0).\end{cases}$$

$P(X>2)=1-P(X\leq 2)=1-F(2)$
$=e^{-2}$

(2) $P(X>4 \mid X>2) = \dfrac{P(X>4,\ X>2)}{P(X>2)}$

$= \dfrac{P(X>4)}{P(X>2)} = \dfrac{1-F(4)}{e^{-2}} = \dfrac{e^{-4}}{e^{-2}}$

$= e^{-2}$

6) $F(x) = \begin{cases} 1-e^{-\frac{1}{5}x} & (x \geq 0) \\ 0 & (x<0) \end{cases}$

$P(X>10) = 1-P(X \leq 10) = 1-F(10) = 1-(1-e^{-\frac{1}{5}\times 10})$
$= e^{-2}$

$P(Y \geq 1) = 1-P(Y=0) = 1-C_5^0\, P(X>10)^0 (1-P(X>10))^5$
$= 1-(1-e^{-2})^5$
$= 0.517$