概统作业 (Week 2)

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1

设挑出第一箱为事件 A，挑出第二箱为事件 B，第 i 次取到为一等品为事件 M_i.

(1)

$$\begin{aligned} P(M_1) &= P(AM_1) + P(BM_1) \\ &= P(M_1|A) \cdot P(A) + P(M_1|B) \cdot P(B) \\ &= \frac{10}{50} \cdot \frac{1}{2} + \frac{18}{30} \cdot \frac{1}{2} \\ &= \frac{1}{10} + \frac{3}{10} \\ &= \frac{2}{5}. \end{aligned}$$

(2)

$$\begin{aligned} P(M_1M_2) &= P(AM_1M_2) + P(BM_1M_2) \\ &= P(M_1M_2|A) \cdot P(A) + P(M_1M_2|B) \cdot P(B) \\ &= \frac{10}{50} \cdot \frac{9}{49} \cdot \frac{1}{2} + \frac{18}{30} \cdot \frac{17}{29} \cdot \frac{1}{2} \\ &= \frac{9}{490} + \frac{51}{290} \\ &= \frac{276}{1421}. \end{aligned}$$

$$P(M_2|M_1) = \frac{P(M_1M_2)}{P(M_1)} = \frac{276}{1421} \cdot \frac{5}{2} = \frac{690}{1421}.$$

2

设方程有实根为事件 M，方程有重根为事件 N，$B = i$ 记为 B_i，$C = i$ 记为 C_i.

(1) $M \Leftrightarrow B^2 - 4C \geq 0$.

$$\begin{aligned} P(M) &= P(B_2C_1) + P(B_3(C_1 + C_2)) + P(B_4(C_1 + C_2 + C_3 + C_4)) + P(B_5) + P(B_6) \\ &= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{4}{6} + \frac{1}{6} + \frac{1}{6} \\ &= \frac{1}{36} + \frac{2}{36} + \frac{4}{36} + \frac{1}{3} \\ &= \frac{19}{36}. \end{aligned}$$

(2) $N \Leftrightarrow B^2 = 4C$.

$$\begin{aligned} P(N) &= P(B_2C_1) + P(B_4C_4) \\ &= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \\ &= \frac{1}{36} + \frac{1}{36} \\ &= \frac{1}{18}. \end{aligned}$$

3

Proof.

$$\begin{aligned} P(A|B) = P(A|B^C) \Leftrightarrow & \frac{P(AB)}{P(B)} = \frac{P(AB^C)}{P(B^C)} \\ \Leftrightarrow & P(AB) \cdot P(B^C) = P(AB^C) \cdot P(B) \\ \Leftrightarrow & P(AB) \cdot (1 - P(B)) = P(AB^C) \cdot P(B) \\ \Leftrightarrow & P(AB) = P(B) \cdot \left[P(AB) + P(AB^C)\right] \\ \Leftrightarrow & P(AB) = P(B) \cdot P(A) \cdot \left[P(B) + P(B^C)\right] \\ \Leftrightarrow & P(AB) = P(B) \cdot P(A). \end{aligned}$$

□

4

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) - (P(AB) + P(BC) + P(AC)) + P(AC \cap B) \\ &= 1 - (\frac{1}{8} + \frac{1}{8} + 0) + 0 \\ &= \frac{3}{4}. \end{aligned}$$

5

掷一个 6 面上写着 1 6 的骰子，定义:

$$A = \{\text{掷到 } 1 \text{ 或 } 2\},\ B = \{\text{掷到 } 2 \text{ 或 } 3\},\ C = \{\text{掷到 } 3 \text{ 或 } 4\}$$

有:

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(AB)}{\mathbb{P}(B)} = \frac{1}{2} > \mathbb{P}(A); \quad \mathbb{P}(B|C) = \frac{\mathbb{P}(BC)}{\mathbb{P}(C)} = \frac{1}{2} > \mathbb{P}(B); \quad \mathbb{P}(A|C) = \frac{\mathbb{P}(AC)}{\mathbb{P}(C)} = 0 < \mathbb{P}(A).$$