

Machine Learning Foundations: Calculus

Terezija Semenski

Calculus formulas

Limits

Let f be a function.

The notation $\lim_{x \rightarrow a} f(x) = L$ explains that the functional values

$f(x)$ can be made arbitrarily close to L by requiring that x be sufficiently close to, but not equal to, a .

The derivative of the function f at a point

The derivative of the function f at a point $x = a$,

denoted by $f'(a)$, is $f'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$

provided if this limit exists.

If the limit exists, we say that f is differentiable at $x = a$.

Differentiation rules

The constant rule:

Let c be a constant. If $f(x) = c$, then $f'(x) = 0$

The power rule:

Let n be a positive integer.

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

The constant multiple rule:

For any function f and any constant c :

$$\frac{d}{dx}[cf(x)] = c \frac{d}{dx}[f(x)]$$

The sum rule:

For any functions f and g :

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

The product rule:

For any functions f and g :

$$\frac{d}{dx}(f(x)g(x)) = \frac{d}{dx}f(x)g(x) + f(x)\frac{d}{dx}g(x)$$

The quotient rule:

For any functions f and g :

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{d}{dx}f(x)g(x) - f(x)\frac{d}{dx}g(x)}{g^2(x)}$$

Chain rule

If $p = p(c)$ and $c = c(m)$ then $\frac{dp}{dc} \cdot \frac{dc}{dm} = \frac{dp}{dm}$

The power rule on a function chain

$$\frac{d}{dx}(f(x)^n) = n(f(x)^{n-1}) \cdot \frac{d}{dx}(f(x))$$

Integration rules

The power rule:

to integrate any real power of x (except -1):

$$\int x^n dx = \frac{1}{n+1} x^{n+1}$$

The constant multiple rule:

$$\int n f(x) dx = n \int f(x) dx$$

The sum rule:

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$