# Machine Learning Foundations: Calculus

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## Calculus formulas

Limits

Let f be a function.

The notation $\lim_{x \to a} f(x) = L$ explains that the functional values $f(x)$ can be made arbitrarily close to L by requiring that x be sufficiently close to, but not equal to, a.

The derivative of the function f at a point

The derivative of the function f at a point $x = a$ ,

denoted by $f'(a)$, is $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

provided if this limit exists.

If the limit exists, we say that f is differentiable at $x = a$.

## Differentiation rules

The constant rule:

Let c be a constant. If $f(x) = c$, then $f'(x) = 0$

The power rule:

Let $n$ be a positive integer.

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

The constant multiple rule:

For any function f and any constant c:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

The sum rule:

For any functions f and g:

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

The product rule:

For any functions f and g:

$$\frac{d}{dx}\left(f(x)\,g(x)\right) = \frac{d}{dx}f(x)\,g(x) + f(x)\,\frac{d}{dx}g(x)$$

The quotient rule:

For any functions f and g:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{d}{dx}f(x)\,g(x) - f(x)\,\frac{d}{dx}g(x)}{g^2(x)}$$

Chain rule

If $p = p(c)$ and $c = c(m)$ then $\dfrac{dp}{dc} \cdot \dfrac{dc}{dm} = \dfrac{dp}{dm}$

The power rule on a function chain

$$\frac{d}{dx}(f(x)^n) = n(f(x)^{n-1}) \cdot \frac{d}{dx}(f(x))$$

**Table of derivatives**

This table provides a table of common functions and their derivatives.

Throughout this table, a, b and k are constants, independent of x.

| $f(x)$ | $f'(x)$ |
|---|---|
| $e^x$ | $e^x$ |
| $e^{kx}$ | $ke^{kx}$ |
| $a^x$ | $(\ln a)a^x$ |
| $\ln(x)$ | $\frac{1}{x}$ |
| $\log_a x$ | $\frac{1}{x \ln a}$ |
| $\sin(x)$ | $\cos(x)$ |
| $\cos(x)$ | $-\sin(x)$ |
| $\tan(x)$ | $\sec^2(x)$ |
| $\cot(x)$ | $-\csc^2(x)$ |

## **Integration rules**

The power rule:

to integrate any real power of x (except –1):

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}$$

The constant multiple rule:

$$\int nf(x) \, dx = n \int f(x) \, dx$$

The sum rule:

$$\int [f(x) + g(x)] \, dx = \int f(x) \, dx + \int g(x) \, dx$$