



University of
South Australia

Neural networks Week 9

Perceptron

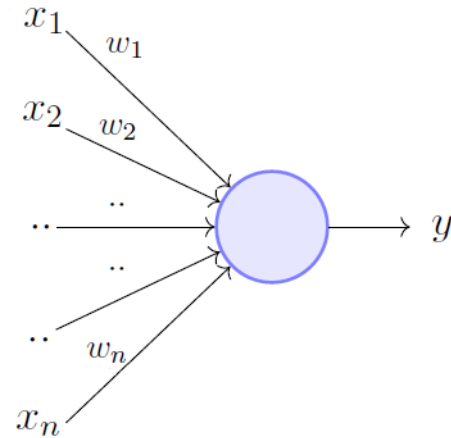
Dr Srećko Joksimović

Perceptron



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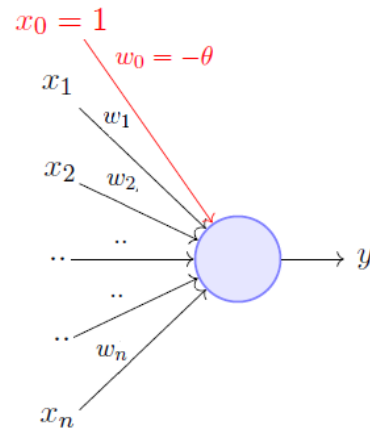
Quick reminder



$$y = 1 \quad \text{if} \quad \sum_{i=1}^n w_i * x_i \geq \theta$$
$$= 0 \quad \text{if} \quad \sum_{i=1}^n w_i * x_i < \theta$$

Rewriting the above,

$$y = 1 \quad \text{if} \quad \sum_{i=1}^n w_i * x_i - \theta \geq 0$$
$$= 0 \quad \text{if} \quad \sum_{i=1}^n w_i * x_i - \theta < 0$$



A more accepted convention,

$$y = 1 \quad \text{if} \quad \sum_{i=0}^n w_i * x_i \geq 0$$
$$= 0 \quad \text{if} \quad \sum_{i=0}^n w_i * x_i < 0$$

where, $x_0 = 1$ and $w_0 = -\theta$

Perceptron

- Single layer network
 - Contains only input and output nodes
- Activation function: $f = \text{sign}(w \bullet x)$
- Applying model is straightforward

$$Y = \text{sign}(0.3X_1 + 0.3X_2 + 0.3X_3 - 0.4)$$

$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

- $X_1 = 1, X_2 = 0, X_3 = 1 \Rightarrow y = \text{sign}(0.2) = 1$

Perceptron Learning Rule

- Initialize the weights (w_0, w_1, \dots, w_d)
- Repeat
 - For each training example (x_i, y_i)
 - ◆ Compute $f(w, x_i)$
 - ◆ Update the weights:

$$w^{(k+1)} = w^{(k)} + \lambda [y_i - f(w^{(k)}, x_i)] x_i$$

- Until stopping condition is met

Perceptron Learning Rule

- Weight update formula:

$$w^{(k+1)} = w^{(k)} + \lambda [y_i - f(w^{(k)}, x_i)] x_i ; \lambda: \text{learning rate}$$

- Intuition:

- Update weight based on error: $e = [y_i - f(w^{(k)}, x_i)]$
- If $y=f(x,w)$, $e=0$: no update needed
- If $y>f(x,w)$, $e=2$: weight must be increased so that $f(x,w)$ will increase
- If $y<f(x,w)$, $e=-2$: weight must be decreased so that $f(x,w)$ will decrease

Example of Perceptron Learning

$$w^{(k+1)} = w^{(k)} + \lambda[y_i - f(w^{(k)}, x_i)] \quad x_i = w^{(k)} + \lambda[y_i - \hat{y}_i^{(k)}] x_i$$

$$Y = \text{sign}\left(\sum_{i=0}^d w_i X_i\right)$$

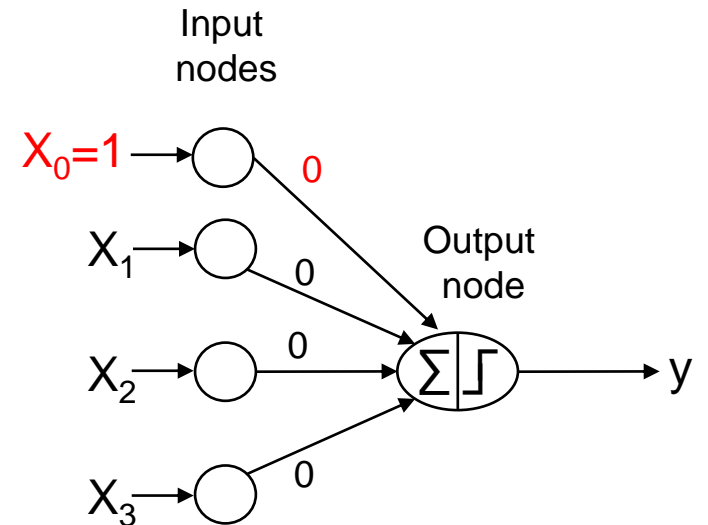
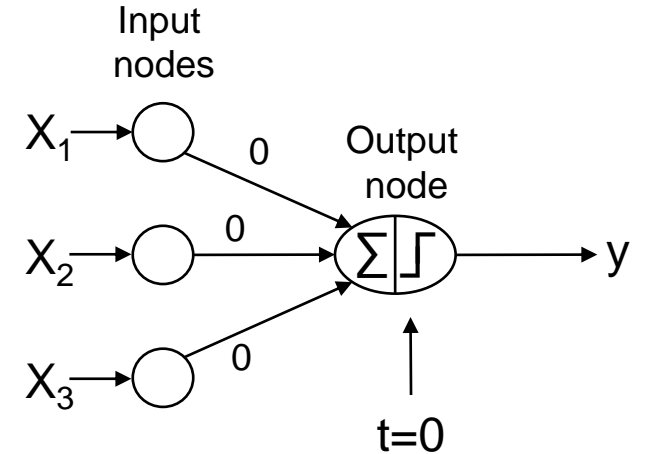
$$\lambda = 0.1$$

$$Y = \text{sign}(w_1 X_1 + w_2 X_2 + w_3 X_3 + w_0 X_0)$$

$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

X_1	X_2	X_3	Y
1	0	0	-1
1	0	1	1
1	1	0	1
1	1	1	1
0	0	1	-1
0	1	0	-1
0	1	1	1
0	0	0	-1

Epoch	w_0	w_1	w_2	w_3
0	0	0	0	0
1				
2				
3				
4				
5				
6				



Example of Perceptron Learning

$$w^{(k+1)} = w^{(k)} + \lambda[y_i - f(w^{(k)}, x_i)] \quad x_i = w^{(k)} + \lambda[y_i - \hat{y}_i^{(k)}] x_i$$

$$Y = \text{sign}\left(\sum_{i=0}^d w_i X_i\right)$$

$\lambda = 0.1$

$$Y = \text{sign}(w_1 X_1 + w_2 X_2 + w_3 X_3 + w_0 X_0)$$

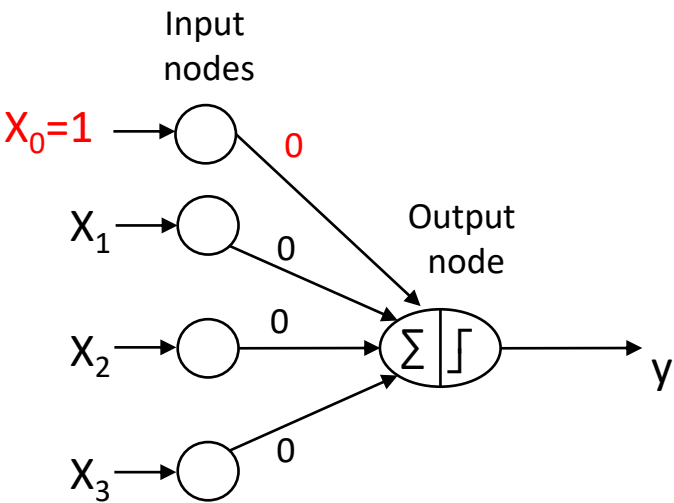
where $\text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

$$Y = \text{sign}(0 * 1 + 0 * 0 + 0 * 0 + 0 * 1)$$
$$= \text{sign}(0) = 1$$

where $\text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

X_1	X_2	X_3	Y
1	0	0	-1
1	0	1	1
1	1	0	1
1	1	1	1
0	0	1	-1
0	1	0	-1
0	1	1	1
0	0	0	-1

Epoch	w_0	w_1	w_2	w_3
0	0	0	0	0
1				
2				
3				
4				
5				
6				



Example of Perceptron Learning

$$w^{(k+1)} = w^{(k)} + \lambda[y_i - f(w^{(k)}, x_i)] \quad x_i = w^{(k)} + \lambda[y_i - \hat{y}_i^{(k)}] x_i$$

$$Y = \text{sign}(\sum_{i=0}^d w_i X_i)$$

$\lambda = 0.1$

$Y = \text{sign}(w_1X_1 + w_2X_2 + w_3X_3 + w_0X_0)$

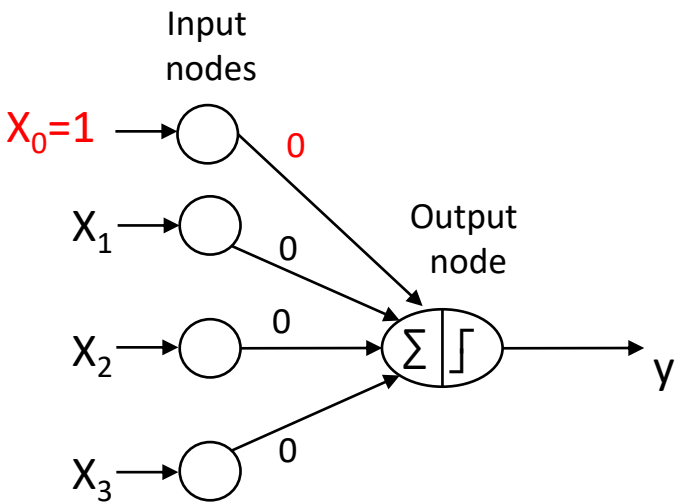
where $\text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

X ₁	X ₂	X ₃	Y
1	0	0	-1
1	0	1	1
1	1	0	1
1	1	1	1
0	0	1	-1
0	1	0	-1
0	1	1	1
0	0	0	-1

Epoch	w ₀	w ₁	w ₂	w ₃
0	0	0	0	0
1				
2				
3				
4				
5				
6				

$Y = \text{sign}(0 * 1 + 0 * 0 + 0 * 0 + 0 * 1)$
 $= \text{sign}(0) = 1$

where $\text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$



$w_0^1 = w_0^0 + 0.1 * [-1 - (1)] * X_0$
 $= 0 + 0.1 * [-2] * 1$
 $= -0.2$

$w_1^1 = w_1^0 + 0.1 * [-1 - (1)] * X_1$
 $= 0 + 0.1 * [-2] * 1$
 $= -0.2$

$w_2^1 = w_2^0 + 0.1 * [-1 - (1)] * X_2$
 $= 0 + 0.1 * [-2] * 0$
 $= 0$

$w_3^1 = w_3^0 + 0.1 * [-1 - (1)] * X_3$
 $= 0 + 0.1 * [-2] * 0$
 $= 0$

Example of Perceptron Learning

$$w^{(k+1)} = w^{(k)} + \lambda[y_i - f(w^{(k)}, x_i)] \quad x_i = w^{(k)} + \lambda[y_i - \hat{y}_i^{(k)}] x_i$$

$$Y = \text{sign}\left(\sum_{i=0}^d w_i X_i\right)$$

$$\lambda = 0.1$$

$$Y = \text{sign}(w_1 X_1 + w_2 X_2 + w_3 X_3 + w_0 X_0)$$

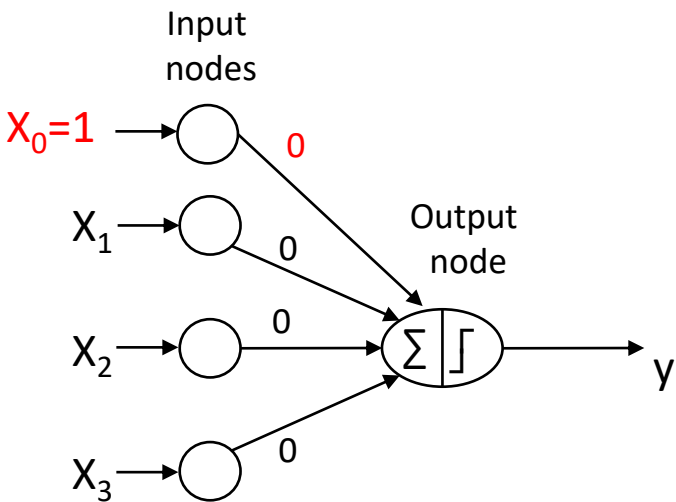
$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

X ₁	X ₂	X ₃	Y
1	0	0	-1
1	0	1	1
1	1	0	1
1	1	1	1
0	0	1	-1
0	1	0	-1
0	1	1	1
0	0	0	-1

$$Y = \text{sign}(-0.2 * 1 + 0 * 0 + 0 * 1 + (-0.2 * 1)) \\ = \text{sign}(-0.2 + 0 + 0 - 0.2) = \text{sign}(-0.4) = -1$$

$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Epoch	w ₀	w ₁	w ₂	w ₃
0	0	0	0	0
1	-0.2	-0.2	0	0
2				
3				
4				
5				
6				



Example of Perceptron Learning

$$w^{(k+1)} = w^{(k)} + \lambda[y_i - f(w^{(k)}, x_i)] \quad x_i = w^{(k)} + \lambda[y_i - \hat{y}_i^{(k)}] x_i$$

$$Y = \text{sign}(\sum_{i=0}^d w_i X_i)$$

$$\lambda = 0.1$$

$$Y = \text{sign}(w_1 X_1 + w_2 X_2 + w_3 X_3 + w_0 X_0)$$

$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

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1	1	1	1
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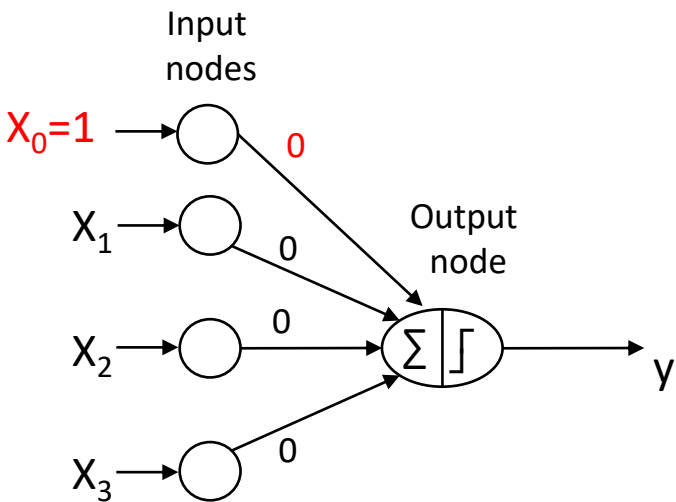
Epoch	w ₀	w ₁	w ₂	w ₃
0	0	0	0	0
1	-0.2	-0.2	0	0
2				
3				
4				
5				
6				

$$w_0^2 = w_0^1 + 0.1 * [1 - (-1)] * X_0 = -0.2 + 0.1 * (2) * 1 = 0$$

$$w_1^2 = w_1^1 + 0.1 * [1 - (-1)] * X_1 = -0.2 + 0.1 * (2) * 1 = 0$$

$$w_2^2 = w_2^1 + 0.1 * [1 - (-1)] * X_2 = 0 + 0.1 * (2) * 0 = 0$$

$$w_3^2 = w_3^1 + 0.1 * [1 - (-1)] * X_3 = 0 + 0.1 * (2) * 1 = 0.2$$



Example of Perceptron Learning

$$w^{(k+1)} = w^{(k)} + \lambda[y_i - f(w^{(k)}, x_i)] \quad x_i = w^{(k)} + \lambda[y_i - \hat{y}_i^{(k)}] x_i$$

$$Y = \text{sign}(\sum_{i=0}^d w_i X_i)$$

$$\lambda = 0.1$$

$$Y = \text{sign}(w_1 X_1 + w_2 X_2 + w_3 X_3 + w_0 X_0)$$

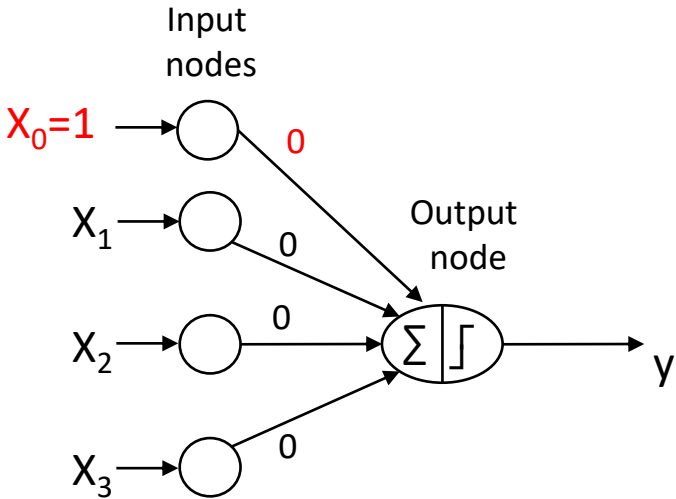
$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$$Y = \text{sign}(0 * 1 + 0 * 1 + 0.2 * 0 + 0 * 1) \\ = \text{sign}(0) = 1$$

$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

X ₁	X ₂	X ₃	Y
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1	1	0	1
1	1	1	1
0	0	1	-1
0	1	0	-1
0	1	1	1
0	0	0	-1

Epoch	w ₀	w ₁	w ₂	w ₃
0	0	0	0	0
1	-0.2	-0.2	0	0
2	0	0	0	0.2
3				
4				
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Example of Perceptron Learning

$$w^{(k+1)} = w^{(k)} + \lambda[y_i - f(w^{(k)}, x_i)] \quad x_i = w^{(k)} + \lambda[y_i - \hat{y}_i^{(k)}] x_i$$

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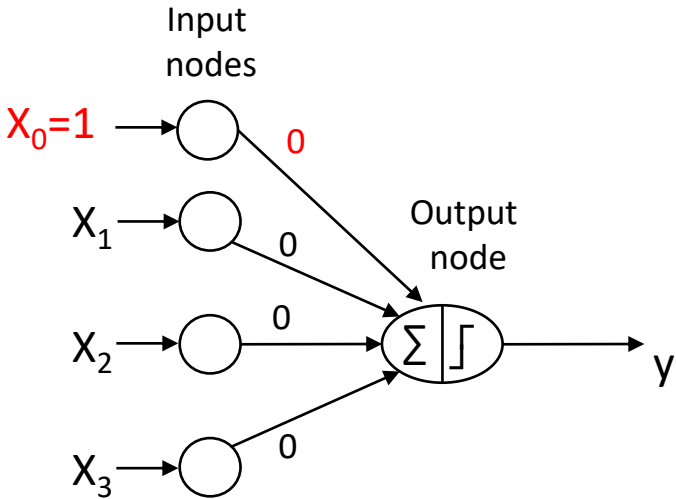
$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$$Y = \text{sign}(0 * 1 + 0 * 1 + 0.2 * 1 + 0 * 1) = \text{sign}(0.2) = 1$$

$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

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0	1	0	-1
0	1	1	1
0	0	0	-1

Epoch	w ₀	w ₁	w ₂	w ₃
0	0	0	0	0
1	-0.2	-0.2	0	0
2	0	0	0	0.2
3	0	0	0	0.2
4				
5				
6				



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$$w^{(k+1)} = w^{(k)} + \lambda[y_i - f(w^{(k)}, x_i)] \quad x_i = w^{(k)} + \lambda[y_i - \hat{y}_i^{(k)}] x_i$$

$$Y = \text{sign}(\sum_{i=0}^d w_i X_i)$$

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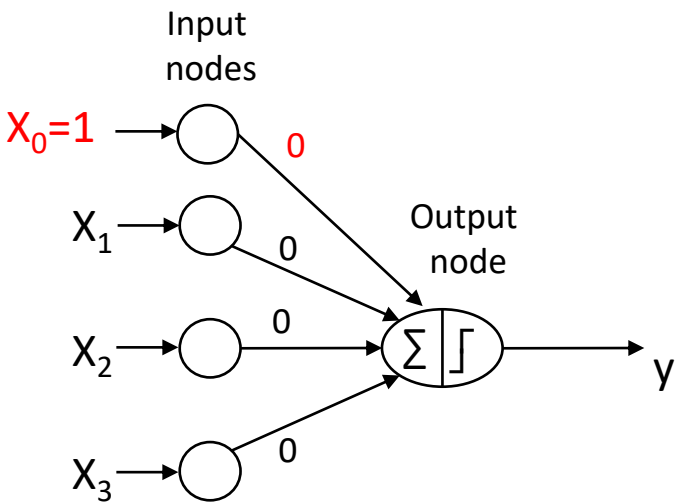
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$$Y = \text{sign}(0 * 0 + 0 * 0 + 0.2 * 1 + 0 * 1) = \text{sign}(0.2) = 1$$

$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

X ₁	X ₂	X ₃	Y
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1	0	1	1
1	1	0	1
1	1	1	1
0	0	1	-1
0	1	0	-1
0	1	1	1
0	0	0	-1

Epoch	w ₀	w ₁	w ₂	w ₃
0	0	0	0	0
1	-0.2	-0.2	0	0
2	0	0	0	0.2
3	0	0	0	0.2
4	0	0	0	0.2
5				
6				



$$w_0^5 = w_0^4 + 0.1 * [-1 - 1] * X_0 = 0 + 0.1 * (-2) * 1 = -0.2$$

$$w_1^5 = w_1^4 + 0.1 * [-1 - 1] * X_1 = 0 + 0.1 * (-2) * 0 = 0$$

$$w_2^5 = w_2^4 + 0.1 * [-1 - 1] * X_2 = 0 + 0.1 * (-2) * 0 = 0$$

$$w_3^5 = w_3^4 + 0.1 * [-1 - 1] * X_3 = 0.2 + 0.1 * (-2) * 1 = 0$$

Example of Perceptron Learning

$$w^{(k+1)} = w^{(k)} + \lambda[y_i - f(w^{(k)}, x_i)] \quad x_i = w^{(k)} + \lambda[y_i - \hat{y}_i^{(k)}] x_i$$

$$Y = \text{sign}(\sum_{i=0}^d w_i X_i)$$

$$\lambda = 0.1$$

$$Y = \text{sign}(w_1 X_1 + w_2 X_2 + w_3 X_3 + w_0 X_0)$$

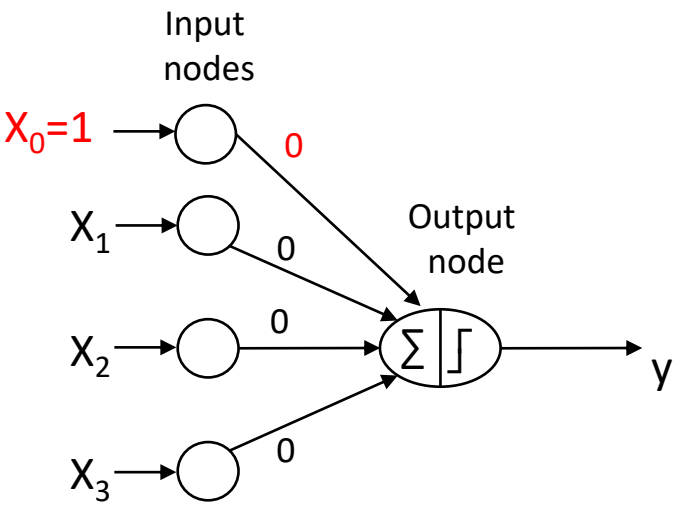
$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$$Y = \text{sign}(0 * 0 + 0 * 1 + 0 * 0 + (-0.2 * 1)) \\ = \text{sign}(-0.2) = -1$$

$$\text{where } \text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

X ₁	X ₂	X ₃	Y
1	0	0	-1
1	0	1	1
1	1	0	1
1	1	1	1
0	0	1	-1
0	1	0	-1
0	1	1	1
0	0	0	-1

Epoch	w ₀	w ₁	w ₂	w ₃
0	0	0	0	0
1	-0.2	-0.2	0	0
2	0	0	0	0.2
3	0	0	0	0.2
4	0	0	0	0.2
5	-0.2	0	0	0
6				



UPDATE?



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INFS 5100 Predictive Analytics

Q&A