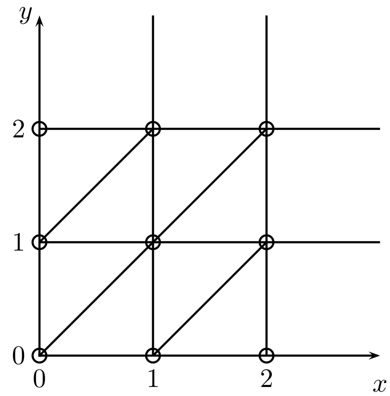
Tutorial 10 - Solutions

Exercise 1

Let $C = \{x, y\}$ be a set of clocks such that $c_x = 2$ and $c_y = 2$.

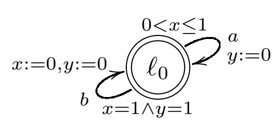
- Draw a picture with all regions for the clocks x and y.
  - Graphical representation of the regions for clocks x and y.



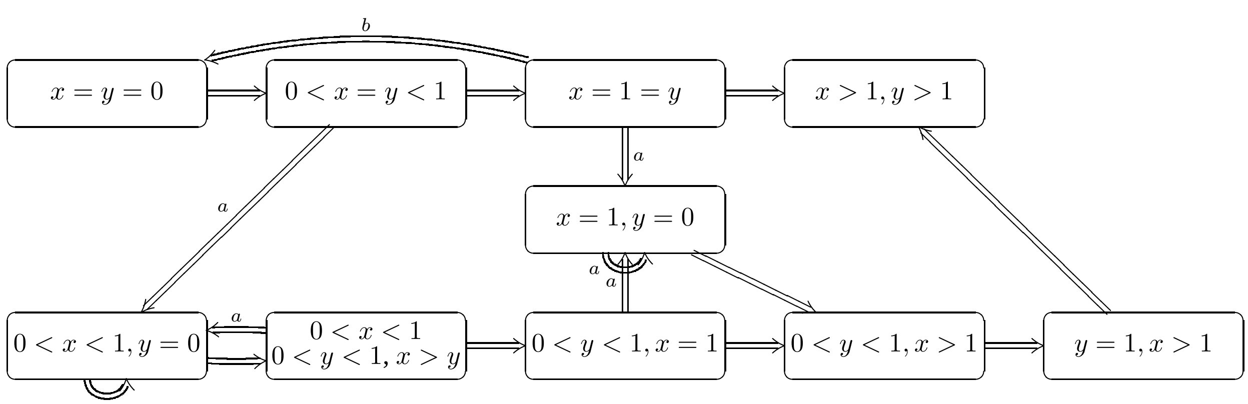
- How many different regions there are on the picture?
  - There are 9 corner points, 22 line segments, and 13 area regions.
- Select four different regions (corner point, line, two areas) and describe them via clock constraints.
  - Solution (for example): $[x = 0 \wedge y = 0]$, $[0 < x < 1 \wedge 1 < y < 2 \wedge x + 1 = y]$, $[0 < x < 1 \wedge 0 < y < 1 \wedge x < y]$, and $[1 < x < 2 \wedge 0 < y < 1 \wedge x > y + 1]$.
- Try to find a general formula which describes a number of regions for two clocks and arbitrary maximal constants c_x and c_y.
  - Solution: $(c_x + 1)(c_y + 1) + 5c_xc_y + 3(c_x + c_y) + 3$

Exercise 2*

Draw a region graph of the following timed automaton.



Since there is only one location ℓ_0, it is omitted in symbolic states of the region graph.

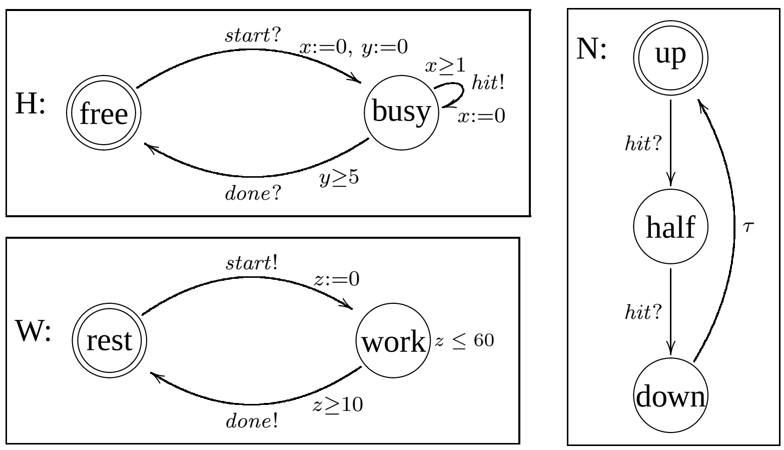


Using the region graph decide whether the following configurations are reachable from the initial configuration.

- (ℓ_0, v) where $v(x) = 0.7$ and $v(y) = 0.61$
  - Solution: Yes, since the symbolic state
  $$(\ell_0, [v]) = (\ell_0, 0 < x < 1 \wedge 0 < y < 1 \wedge x > y)$$
  is reachable from the initial symbolic state $(\ell_0, x = y = 0)$ of the region graph.
- (ℓ_0, v) where $v(x) = 0.2$ and $v(y) = 0.41$
  - Solution: No, since the symbolic state
  $$(\ell_0, [v]) = (\ell_0, 0 < x < 1 \wedge 0 < y < 1 \wedge x < y)$$
  is *not* reachable from the initial symbolic state $(\ell_0, x = y = 0)$ of the region graph.

Exercise 3

Consider the following network of timed automata from the lecture.



- Give an example of a timed trace in the network above.
  - A timed trace could be as follows:
  $$(20, \tau)(40, \tau)(60, \tau)(60, \tau)\cdots$$
  An example of a sequence of states could be:
  $\big((\text{free,rest,up}), [x = 0, y = 0, z = 0]\big) \xrightarrow{\tau} \big((\text{busy,work,up}), [x = 0, y = 0, z = 0]\big) \xrightarrow{20} \big((\text{busy,work,up}), [x = 20, y = 20, z = 20]\big) \xrightarrow{\tau} \big((\text{busy,work,half}), [x = 0, y = 20, z = 20]\big) \xrightarrow{40} \big((\text{busy,work,half}), [x = 40, y = 60, z = 60]\big) \xrightarrow{\tau} \big((\text{busy,work,down}), [x = 0, y = 60, z = 60]\big) \xrightarrow{\tau} \big((\text{free,rest,down}), [x = 0, y = 60, z = 60]\big) \xrightarrow{\tau} \big((\text{free,rest,up}), [x = 0, y = 60, z = 60]\big) \cdots$
- Which of the following properties are true?
  - A[] (W.rest $\vee$ $z \leq 100$) : **True**
  - E$\langle\rangle$ (W.rest $\wedge$ H.busy) : **False**
  - A$\langle\rangle$ W.rest : **True**
  - E[] H.busy : **False**
  - W.work $--$ $>$ W.rest : **True**