Petri Nets with time

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Plan

- **Petri nets with time**
- **T-time Petri Nets**
- **Other Semantics**
  - P-time Petri Nets
  - A-time Petri Nets
  - Strong vs Weak Semantics
- **Expressiveness and properties of TPN**
- **State Space of Time Petri Nets**
- **Model-checking of Time Petri Nets**
  - Temporal logics
  - Checking TPN with observers
- **Stopwatch Petri Nets**

Introduction

Timed constraints are added to Petri nets

in different ways

- date (point) [Ram74] : Timed Petri nets
- interval [Mer74] : Time Petri nets

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These constraints are associated with

- places : P-timed or P-time
- arcs (edge)
- transitions
- tokens...

Time Petri Nets

Several semantics

- strong semantics
- weak semantics

Time Petri Nets

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- strong semantics
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The main models:

- T-time Petri Nets with strong semantics[Mer74, BD91]
- P-time Petri Nets with strong semantics [KDC96]
- A-time Petri Nets with weak semantics[Han93, AN01, dFRA00]

The most widely used model is : T-time Petri Nets with strong semantics called Time Petri Nets (TPN).

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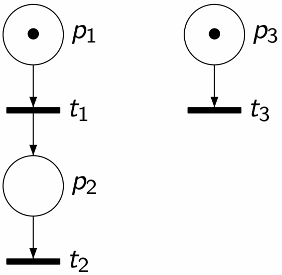
T-time Petri Nets (*T-TPN*)

T-TPN: Time constraints are associated with transitions

Example (Net of L. Gallon)



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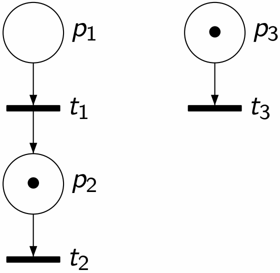

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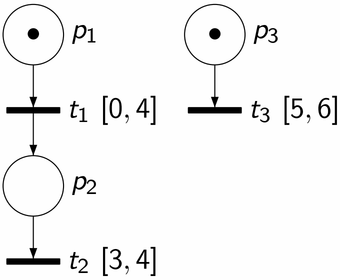

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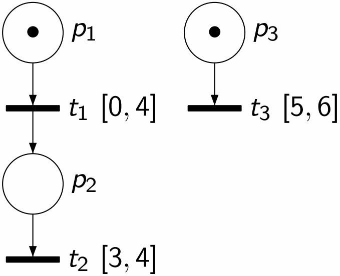
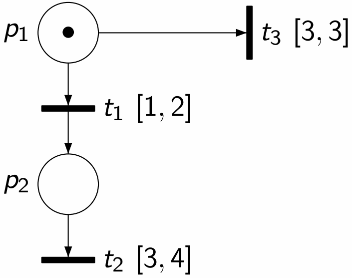


$$\begin{array}{l}\{p_1,p_3\}\\ \nu(t_1)=0\\ \nu(t_3)=0\end{array} \xrightarrow{\epsilon(4)} \begin{array}{l}\{p_1,p_3\}\\ \nu(t_1)=4\\ \nu(t_3)=4\end{array} \xrightarrow{t_1} \begin{array}{l}\{p_2,p_3\}\\ \nu(t_2)=0\\ \nu(t_3)=4\end{array} \xrightarrow{\epsilon(1)} \begin{array}{l}\{p_2,p_3\}\\ \nu(t_2)=1\\ \nu(t_3)=5\end{array} \xrightarrow{t_3} \cdots$$

Some examples

Example (Priority)



Is it possible to fire t3 ?

Some examples

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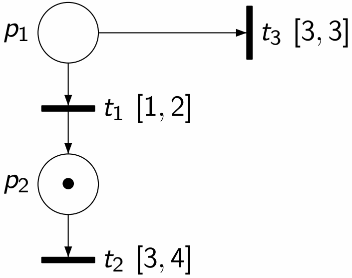

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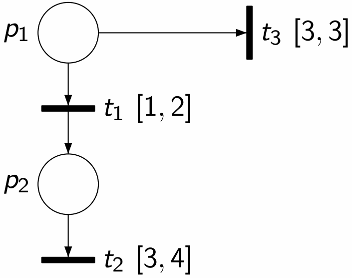

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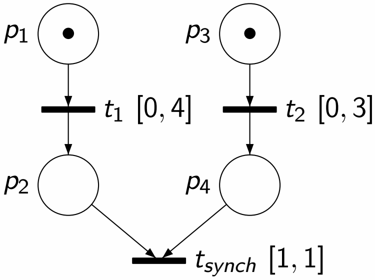

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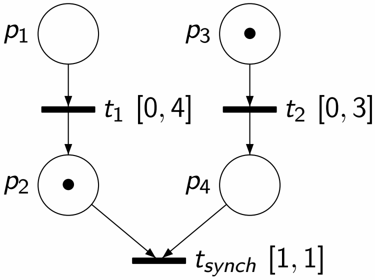

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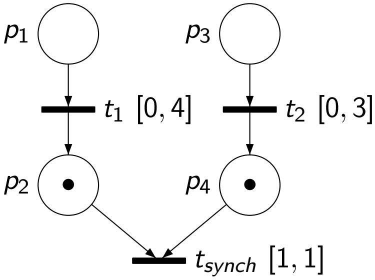

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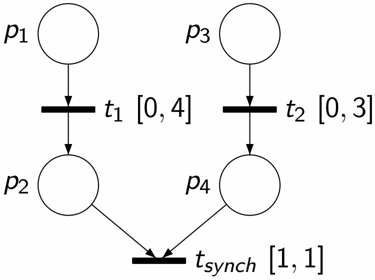

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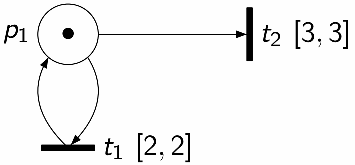

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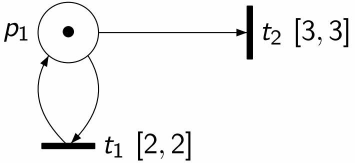

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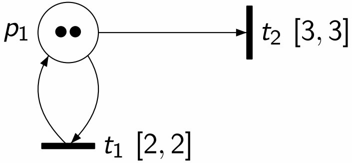

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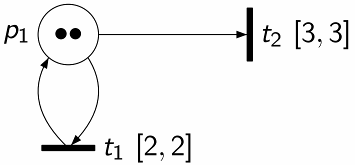

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A TPN : $\mathcal{N} = (\mathcal{P}, \mathcal{T}, {}^{\bullet}(.), (.)^{\bullet}, \mathcal{M}_{\prime}, \mathcal{I})$

Newly Enabled Transition

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Newly Enabled Transition

t' is Newly enabled by the firing of t from M :

$$\begin{aligned} \uparrow enabled(t', M, t) = \big(M - {}^{\bullet}t + t^{\bullet} \geq {}^{\bullet}t'\big) \wedge \\ \big((M - {}^{\bullet}t < {}^{\bullet}t') \vee (t' = t)\big) \end{aligned} \tag{1}$$

Definition and Semantics of *T-TPN*

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A *Time Petri Net* $\mathcal{N}$ is a tuple $(P, T, ^\bullet(.), (.)^\bullet, M_0, I)$
Remark : Often $I = [\alpha, \beta]$

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- M : a marking and
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- Un TPN generate a set of runs = alternation of discrete and continuous steps
- The semantics of a TPN N = Timed Transition System S_N

Formally

A *Time Petri Net* $\mathcal{N}$ is a tuple $(P, T, {}^\bullet(.), (.)^\bullet, M_0, I)$ where:

- $P = \{p_1, p_2, \cdots, p_m\}$ is a finite set of places
- $T = \{t_1, t_2, \cdots, t_n\}$ is a finite set of transitions and $P \cap T = \emptyset$;
- ${}^\bullet(.) \in (\mathbb{N}^P)^T$ is the backward incidence mapping; $(.)^\bullet \in (\mathbb{N}^P)^T$ is the forward incidence mapping;
- $M_0 \in \mathbb{N}^P$ is the initial marking;
- $I : T \to \mathcal{I}(\mathbb{Q}_{\geq 0})$ associates with each transition a firing interval;

Semantics of Time Petri Nets $\mathcal{N} = (P, T, {}^\bullet(.), (.)^\bullet, M_0, I)$

A state $Q = (M, \nu)$ of $\mathcal{N}$ is a pair with $M \in \mathbb{N}^P$ and $\nu \in \mathbb{R}_{\geq 0}^{enabled(M)}$.

The semantics of $\mathcal{N}$ is a Timed Transition System $S_{\mathcal{N}} = (Q, \{q_0\}, T, \rightarrow)$:

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$$(M,\nu) \xrightarrow{t} (M',\nu') \textit{ iff } \begin{cases} t \in enabled\,(M) \wedge M' = M - {}^\bullet t + t^\bullet \\ \nu(t) \in I(t), \\ \forall t' \in enabled\,(M')\,, \nu'(t') = \begin{cases} 0 \textit{ if } \uparrow\!enabled(t', M, t), \\ \nu(t') \textit{ otherwise.} \end{cases} \end{cases}$$

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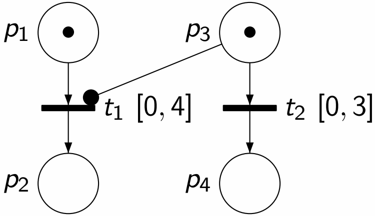
  - continuous transition relation is defined $\forall d \in \mathbb{R}_{\geq 0}$ by:

$$(M, \nu) \xrightarrow{d} (M, \nu') \ iff \begin{cases} \nu' = \nu + d \\ \forall t \in enabled\,(M)\,, \nu'(t) \in I(t)^{\downarrow} \end{cases}$$

Some particular arcs

Example (Logical inhibitor arc)



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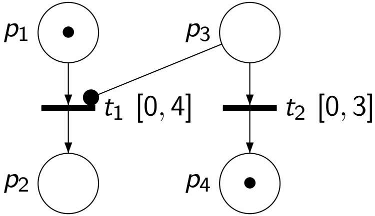

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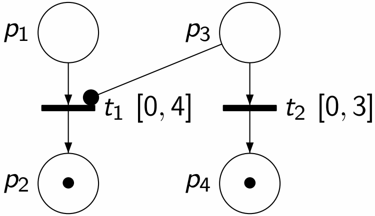

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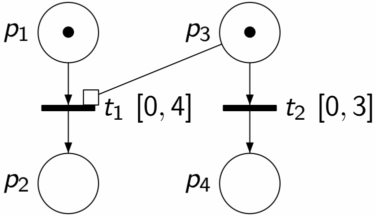

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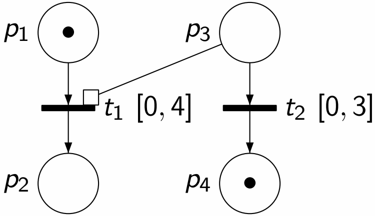

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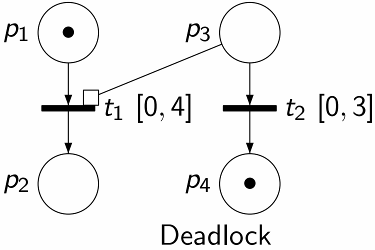

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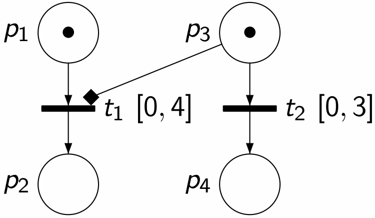

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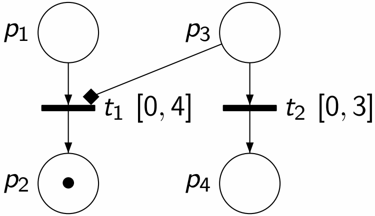

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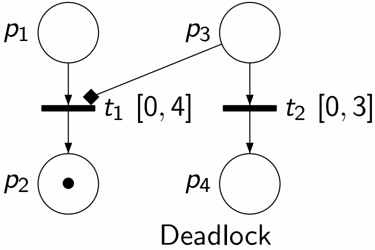

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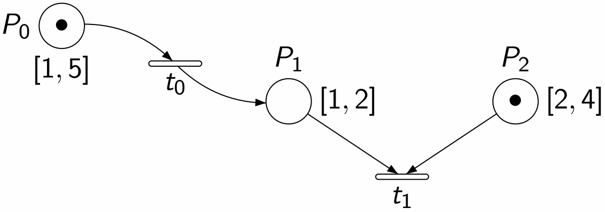

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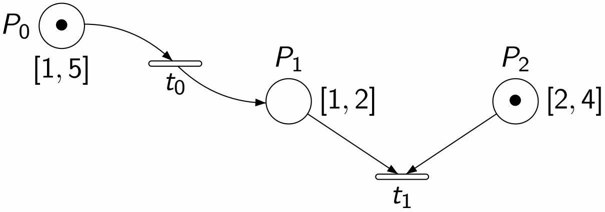

$$\begin{array}{c} \{p_0, p_2\} \\ \nu(p_0) = 0 \\ \nu(p_2) = 0 \end{array} \xrightarrow{\epsilon(3)} \begin{array}{c} \{p_0, p_2\} \\ \nu(p_0) = 3 \\ \nu(p_2) = 3 \end{array} \xrightarrow{t_0} \begin{array}{c} \{p_1, p_2\} \\ \nu(p_1) = 0 \\ \nu(p_2) = 3 \end{array} \xrightarrow{\epsilon(1)} \begin{array}{c} \{p_1, p_2\} \\ \nu(p_1) = 1 \\ \nu(p_2) = 4 \end{array} \xrightarrow{t_1}$$

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P-TPN: Time constraints are associated with places

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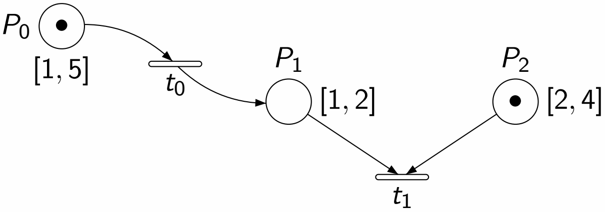


$$\begin{array}{c} \{p_0,p_2\} \\ \nu(p_0)=0 \\ \nu(p_2)=0 \end{array} \xrightarrow{\epsilon(3)} \begin{array}{c} \{p_0,p_2\} \\ \nu(p_0)=3 \\ \nu(p_2)=3 \end{array} \xrightarrow{t_0} \begin{array}{c} \{p_1,p_2\} \\ \nu(p_1)=0 \\ \nu(p_2)=3 \end{array} \xrightarrow{\epsilon(1)} \begin{array}{c} \{p_1,p_2\} \\ \nu(p_1)=1 \\ \nu(p_2)=4 \end{array} \xrightarrow{t_1}$$

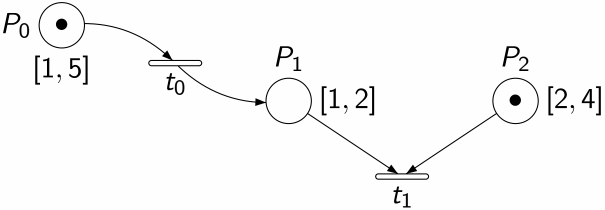
Weak semantics :

$$\begin{array}{c} \{p_0,p_2\} \\ \nu(p_0)=0 \\ \nu(p_2)=0 \end{array} \xrightarrow{\epsilon(3)} \begin{array}{c} \{p_0,p_2\} \\ \nu(p_0)=3 \\ \nu(p_2)=3 \end{array} \xrightarrow{t_0} \begin{array}{c} \{p_1,p_2\} \\ \nu(p_1)=0 \\ \nu(p_2)=3 \end{array} \xrightarrow{\epsilon(2)} \begin{array}{c} \{p_1,\hat{p}_2\} \\ \nu(p_1)=2 \\ \nu(p_2)=5 \end{array} \xrightarrow{\epsilon(\cdots)}$$

P-time Petri Nets (*P-TPN*)

P-TPN: Time constraints are associated with places

Example (A TPN $\in$ *P-TPN*)



$$\begin{array}{l}\{p_0,p_2\}\\ \nu(p_0)=0\\ \nu(p_2)=0\end{array} \xrightarrow{\epsilon(3)} \begin{array}{l}\{p_0,p_2\}\\ \nu(p_0)=3\\ \nu(p_2)=3\end{array} \xrightarrow{t_0} \begin{array}{l}\{p_1,p_2\}\\ \nu(p_1)=0\\ \nu(p_2)=3\end{array} \xrightarrow{\epsilon(1)} \begin{array}{l}\{p_1,p_2\}\\ \nu(p_1)=1\\ \nu(p_2)=4\end{array} \xrightarrow{t_1}$$

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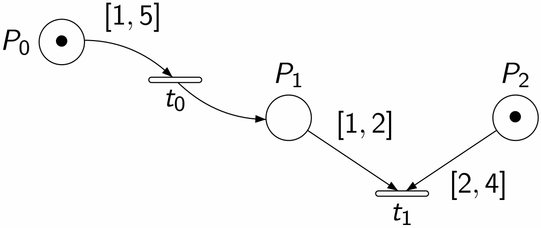
A-time Petri Nets (*A-TPN*)

Time constraints are associated with arcs (Place,transition)

Example (Un TPN $\in$ *A-TPN*)



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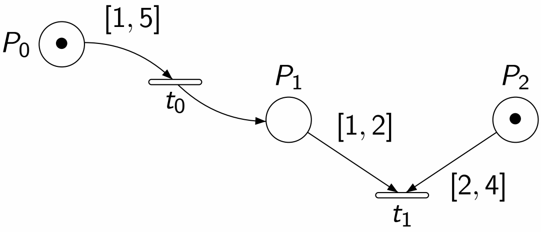

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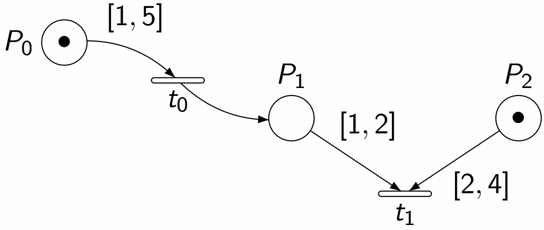

$$\begin{array}{c} \{p_0, p_2\} \\ \nu(p_0) = 0 \\ \nu(p_2) = 0 \end{array} \xrightarrow{\epsilon(3)} \begin{array}{c} \{p_0, p_2\} \\ \nu(p_0) = 3 \\ \nu(p_2) = 3 \end{array} \xrightarrow{t_0} \begin{array}{c} \{p_1, p_2\} \\ \nu(p_1) = 0 \\ \nu(p_2) = 3 \end{array} \xrightarrow{\epsilon(1)} \begin{array}{c} \{p_1, p_2\} \\ \nu(p_1) = 1 \\ \nu(p_2) = 4 \end{array} \xrightarrow{t_1}$$

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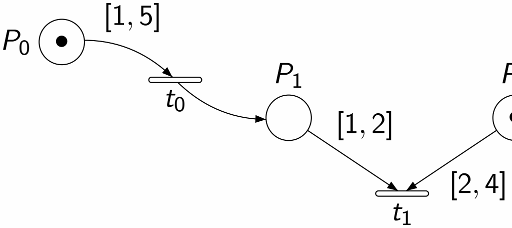

$$\begin{matrix}\{p_0,p_2\}\\ \nu(p_0)=0\\ \nu(p_2)=0\end{matrix} \xrightarrow{\epsilon(3)} \begin{matrix}\{p_0,p_2\}\\ \nu(p_0)=3\\ \nu(p_2)=3\end{matrix} \xrightarrow{t_0} \begin{matrix}\{p_1,p_2\}\\ \nu(p_1)=0\\ \nu(p_2)=3\end{matrix} \xrightarrow{\epsilon(1)} \begin{matrix}\{p_1,p_2\}\\ \nu(p_1)=1\\ \nu(p_2)=4\end{matrix} \xrightarrow{t_1}$$

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P-time Petri Nets (A-TPN)

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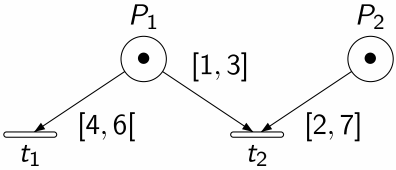

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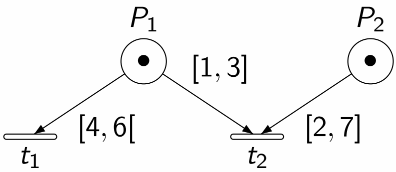

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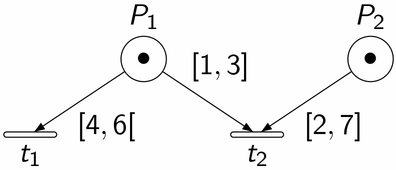

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Strong vs Weak Semantics

Let S, be a state of a Petri Nets, the strong semantics is defined by:

$$t \notin \mathit{firable}(S + d) \Rightarrow \forall d' \in [0, d] : t \notin \mathit{firable}(S + d') \qquad (2)$$

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With strong semantics, time elapsing cannot disable transition.

Plan

- **Petri nets with time**
- **T-time Petri Nets**
- **Other Semantics**
  - P-time Petri Nets
  - A-time Petri Nets
  - Strong vs Weak Semantics
- **Expressiveness and properties of TPN**
- **State Space of Time Petri Nets**
- **Model-checking of Time Petri Nets**
  - Temporal logics
  - Checking TPN with observers
- **Stopwatch Petri Nets**

Equivalence w.r.t. timed language and timed bisimulation.

Two Timed Transition Systems (TTS)

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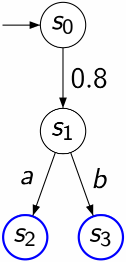

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- Both TTS accept the same timed language ie : the timed words $(a, 0.8)$ and $(b, 0.8)$

Equivalence w.r.t. timed language and timed bisimulation.

Two Timed Transition Systems (TTS)

- Both TTS accept the same timed language ie : the timed words $(a, 0.8)$ and $(b, 0.8)$
- These TTS are not timed bisimilar.

Comparison of the expressiveness of Time Petri Nets

Classes of TPN :

- Time Petri Nets where time is associated with places (*P-TPN*), arcs (*A-TPN*) or transitions (*T-TPN*).
- strong semantics ($\overline{T\text{-}TPN}$) or weak semantics ($\underline{T\text{-}TPN}$)
- safe nets (1-bounded)
- large or strict time constraints
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Comparison of expressiveness w.r.t. timed bisimilarity.

Results [BR08]

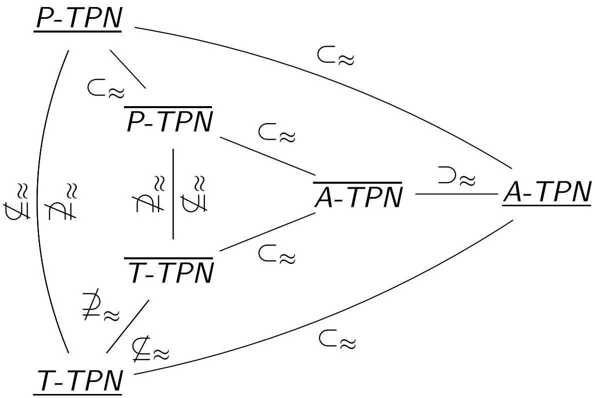

Figure: Comparison of expressiveness of TPN

Results [BR08]

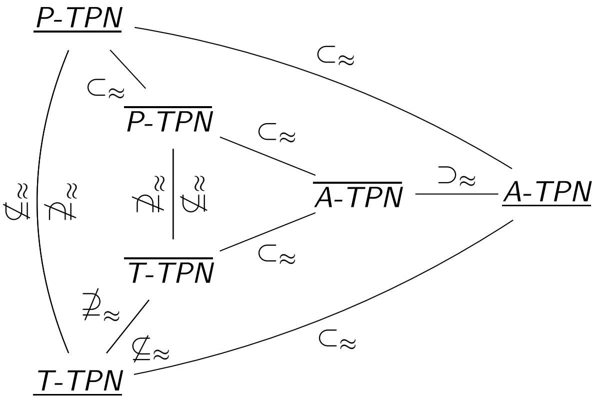

Figure: Comparison of expressiveness of TPN

However, *T-TPN* are easier than *A-TPN* for the modelling of synchronisations

Expressivité : $\overline{T\text{-}TPN}$ *vs* *TA* (automates temporisés)

Timed language acceptance

Timed Bisimulation

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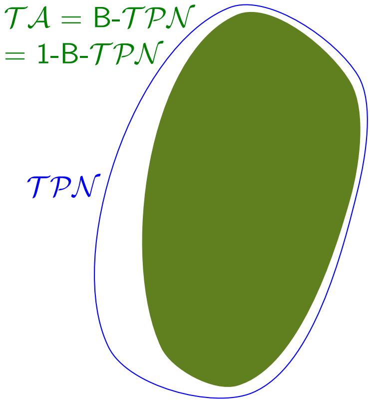

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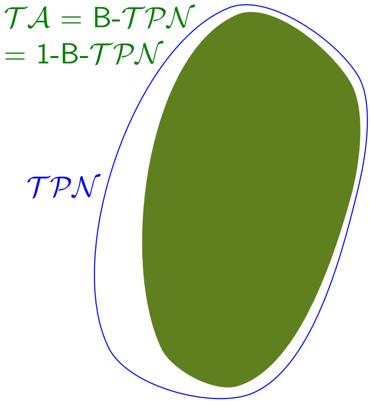

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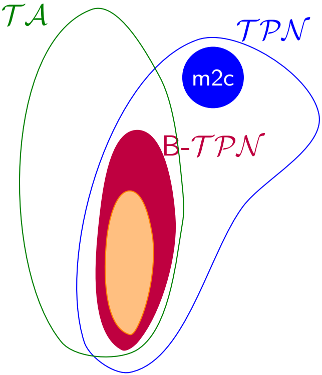

B-$\mathcal{TPN}(\leq, \geq)$
= 1-B-$\mathcal{TPN}(\leq, \geq)$

Main Decidability results

T-time semantics with strong semantics ($\overline{T\text{-}TPN}$).

Problem	$\overline{\text{T-TPN}}$ (not bounded)	$\overline{\text{T-TPN}}$ (bounded)
Boundedness		
k-Boundedness		
Accessibility empty language		
Universality language inclusion		
Model-checking de TCTL		

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Model-checking de TCTL	Undecidable	Decidable [CR06, BGR09]

Exercise

TPN *vs* PN

- How can we simulate the behaviour of a PN by aTPN ?

Plan

- Petri nets with time
- T-time Petri Nets
- Other Semantics
- Expressiveness and properties of TPN
- **State Space of Time Petri Nets**
- Model-checking of Time Petri Nets
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Model checking Problem

$\Rightarrow$ Explore the state space

Model checking Problem

$\Rightarrow$ Explore the state space

Problem

The state space of a TPN is infinite

Model checking Problem

$\Rightarrow$ Explore the state space

Problem

The state space of a TPN is infinite

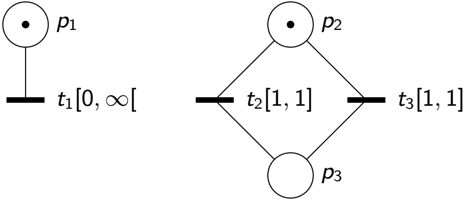
$\Rightarrow$ Group the states in equivalence classes (abstraction)

Exploration of the state space

- simulation

Exploration of the state space

- simulation



$$\begin{pmatrix} 1 & \vline & 0 \\ 1, & \vline & 0 \\ 0 & \vline & \times \end{pmatrix}$$

Exploration of the state space

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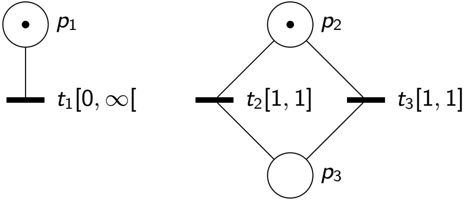

$$\begin{pmatrix} 1 & 0 \\ 1, & 0 \\ 0 & \times \end{pmatrix} \xrightarrow{0} \begin{pmatrix} 1 & \delta \\ 1, & \delta \\ 0 & \times \end{pmatrix}$$

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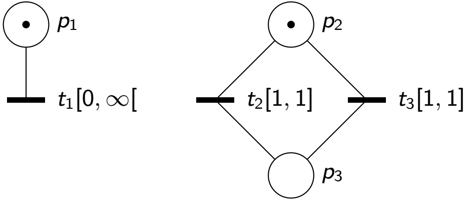

$$\left(\begin{array}{l|l}1 & 0\\ 1, & 0\\ 0 & \times\end{array}\right) \xrightarrow{0} \xrightarrow{0.7} \left(\begin{array}{l|l}1 & \delta\\ 1, & \delta\\ 0 & \times\end{array}\right)$$

Exploration of the state space

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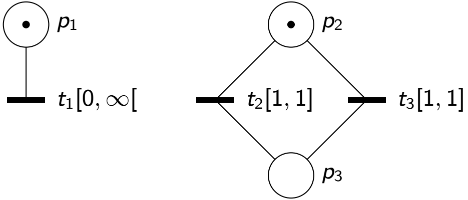

$$\left(\begin{array}{l|l} 1 & 0 \\ 1, & 0 \\ 0 & \times \end{array}\right) \quad \begin{array}{c} \xrightarrow{0} \\ \xrightarrow{0.7} \\ \cdots \\ \xrightarrow{1} \end{array} \quad \left(\begin{array}{l|l} 1 & \delta \\ 1, & \delta \\ 0 & \times \end{array}\right)$$

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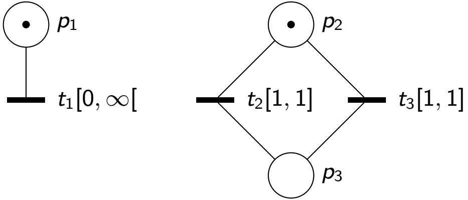

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$\rightarrow$ Infinity of branchings, states

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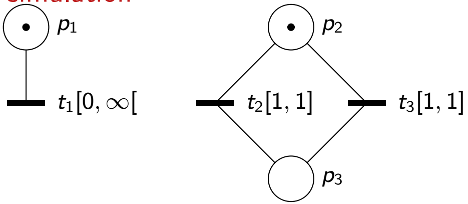

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Exploration of the state space

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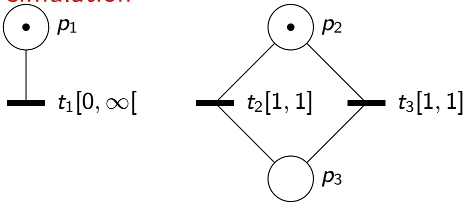

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- symbolic
  - symbolic states:

$$\begin{pmatrix} 1 \\ 1, x_1 = x_2 = 0 \\ 0 \end{pmatrix}$$

Exploration of the state space

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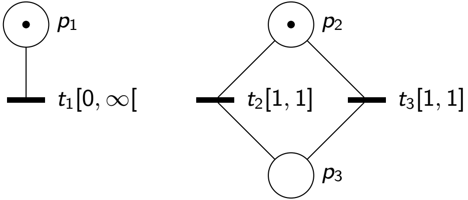

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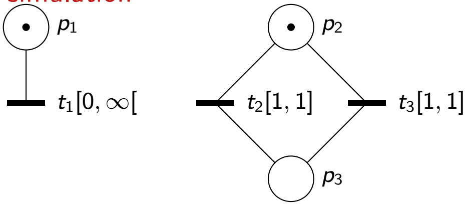

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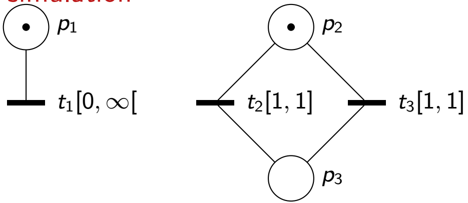

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$\rightarrow$ State class graph
$\rightarrow$ Regions graph
$\rightarrow$ Zones graph

State space computation

Definition (Symbolic state)

$\mathcal{Q} = (M, \mathcal{V})$

- M a marking,
- $\mathcal{V}$ a set of valuation such that M exists.

Basic Algorithm for the state space computation

Basic Algorithm

The set of state to explore: $Waiting \leftarrow \mathcal{Q}_0 = (M_0, \mathcal{V}_0)$.

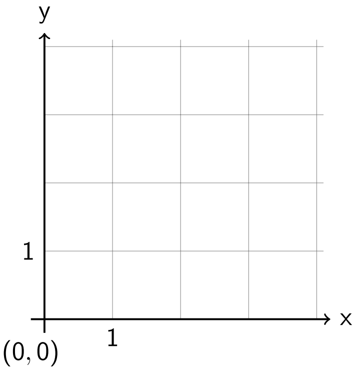
The set of explored states: $Visited \leftarrow \emptyset$

While $Waiting \neq \emptyset$

- $\mathcal{Q} \leftarrow pop(Waiting)$
- Computation of the fireable transitions from $\mathcal{Q}$: $firable(\mathcal{Q})$
- for all transition $t \in firable(\mathcal{Q})$
  - Compute the successor of $\mathcal{Q}$ by the firing of t : $next(\mathcal{Q}, t)$
  - if $next(\mathcal{Q}, t) \notin (Waiting \cup Visited)$
    then $Waiting \leftarrow Waiting \cup next(\mathcal{Q}, t)$
- $Visited \leftarrow Visited \cup \mathcal{Q}$

Region Abstraction [ACD90]

Idea: group clock valuations into equivalence classes: regions
all clock valuations of a region r should



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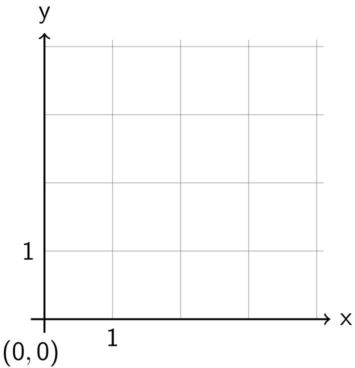

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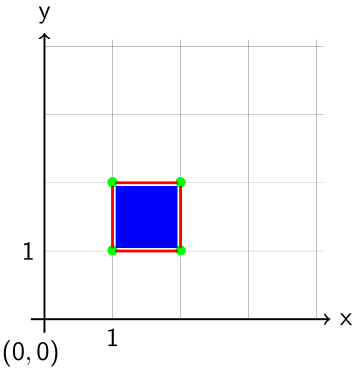

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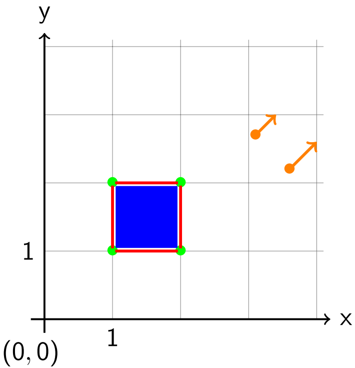

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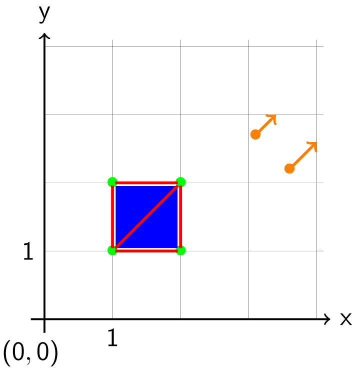

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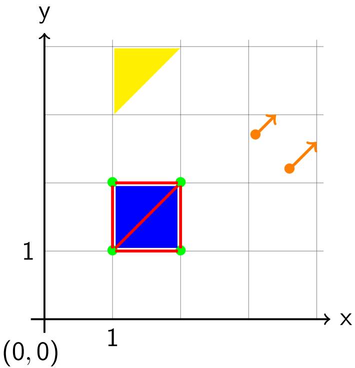

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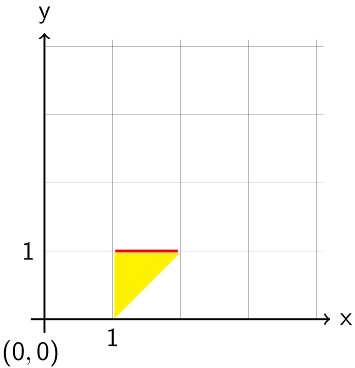

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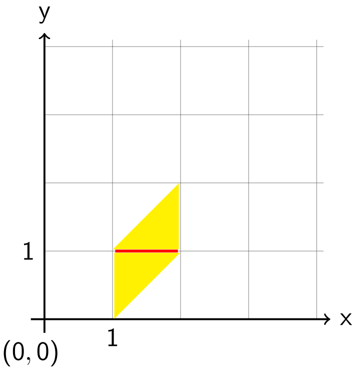

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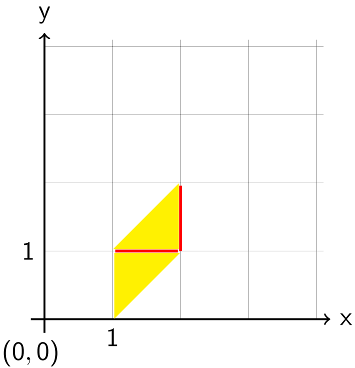

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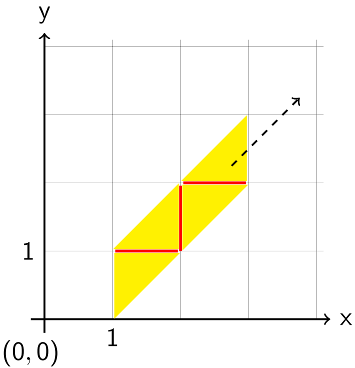

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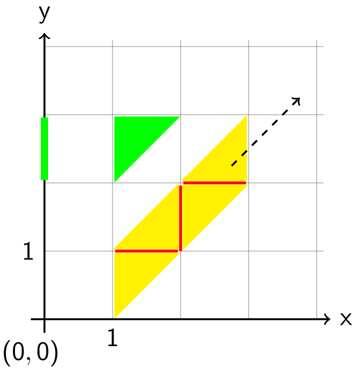

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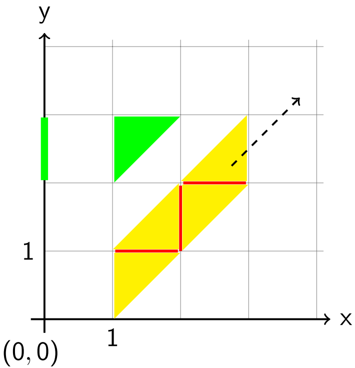

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Still infinitely many regions

Zone Graph

Region Graph → theoretical method

Zone Graph

Region Graph $\rightarrow$ theoretical method

Zone graph

- Grouping of regions
- Good results in practice

Definition (Zone)

$\mathcal{Z} = (M, Z)$ where M is a marking and Z is a polyhedron.

$$Z = \begin{cases} \forall i, j \text{ t.q. } t_i, t_j \in enabled(M), \\ x_i \leq z_i, \\ x_j \leq z_j, \\ x_i - x_j \leq z_{ij} \end{cases}$$

(x_i represents the clock associated with the transition i)

$\Rightarrow$ Difference Bound Matrix (DBM)

Computation of the state space

$\mathcal{Z}_0 = (M_0, Z_0)$
Computation of the states reachable by time elapsing (futur)

Computation of the state space

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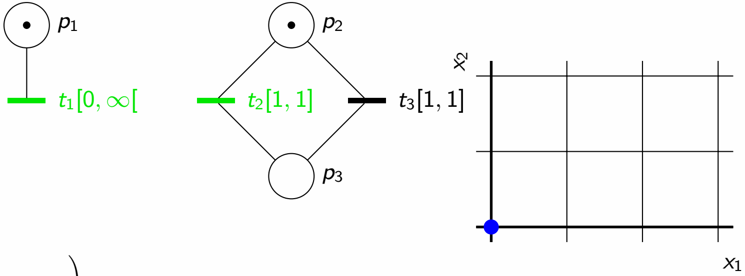
Example



$$\left(\begin{pmatrix}1\\1\\0\end{pmatrix}, x_1 = x_2 = 0\right)$$

$\overrightarrow{Z_0} \cap Inv\,(M_0)$, with $Inv\,(M_0) = \wedge_{t_i}\{x_i \leq \beta_i\}$, $t_i \in enabled\,(M_0)$

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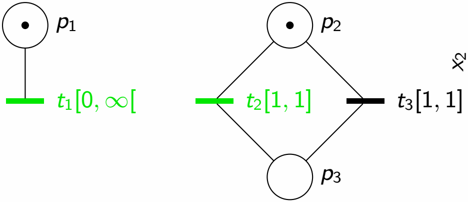

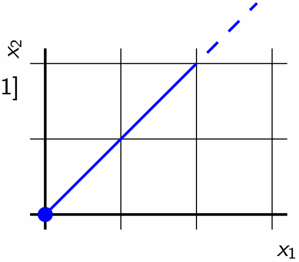

$$\left(\begin{pmatrix}1\\1\\0\end{pmatrix}, x_1 = x_2 = 0\right) \rightarrow \left(\begin{pmatrix}1\\1\\0\end{pmatrix}, x_1 = x_2 \in [0, \infty[\right)$$

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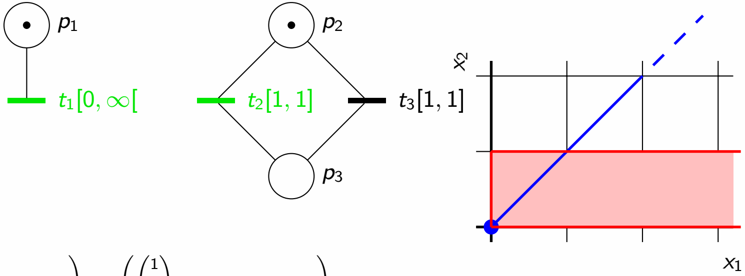

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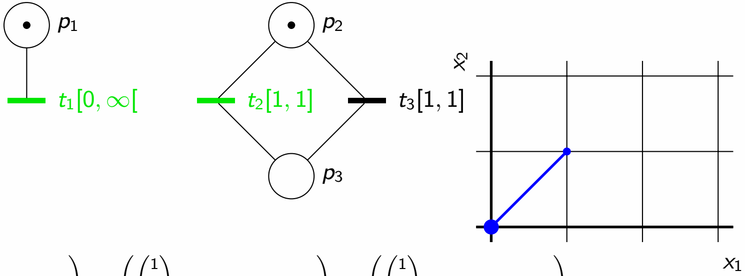

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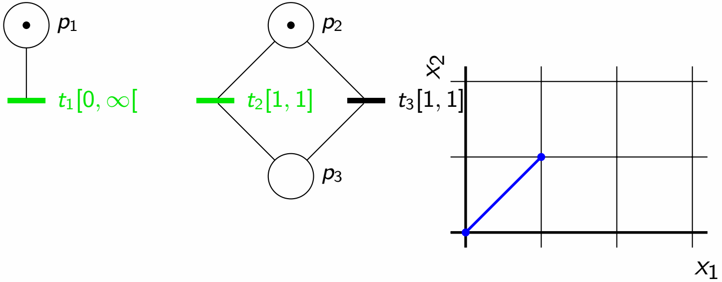
Computation of the state space

Computation of fireable transitions.

Computation of the state space

Computation of fireable transitions.

Example



$$\{t_i \mid (\overrightarrow{Z_0} \cap Inv\,(M_0) \cap x_i \geq \alpha_i) \neq \emptyset\}$$

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Computation of fireable transitions.

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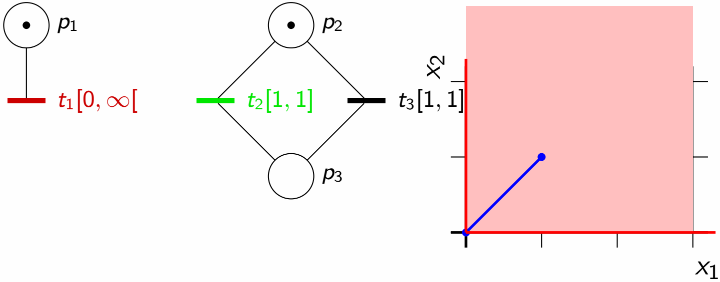

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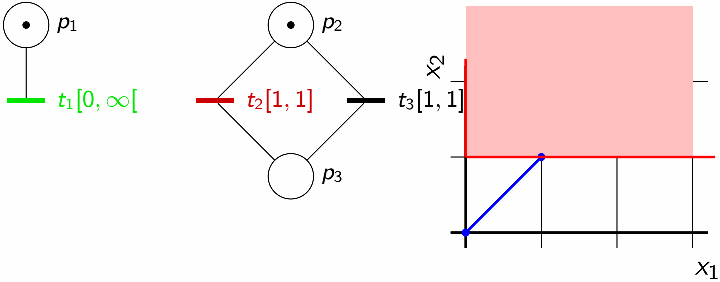

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Computation of the state space

Firing of transitions $\rightarrow (M_i, Z_i)$.

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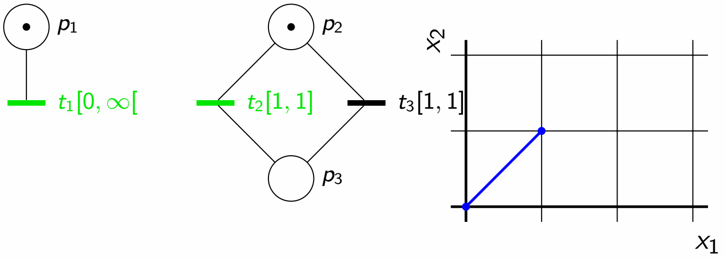

$(M_i, Z_i) = \left(M_0 - {}^\bullet t_i + t_i^\bullet, \left(\overrightarrow{Z_0} \cap Inv\,(M_0) \cap x_i \geq \alpha_i \right) [X_{ne} \leftarrow 0] \right)$ X_{ne} : ensemble des transitions nouvellement sensibilisées

Computation of the state space

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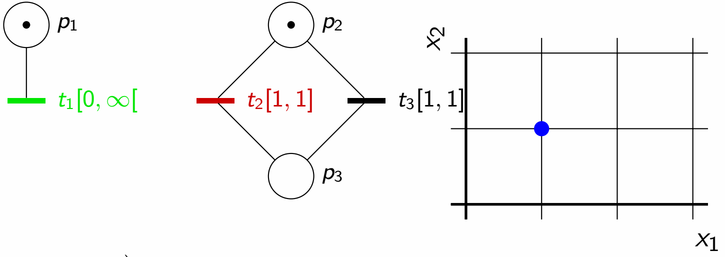

$$\left(\begin{pmatrix}1\\1\\0\end{pmatrix}, x_1 = x_2 \in [1,1]\right)$$

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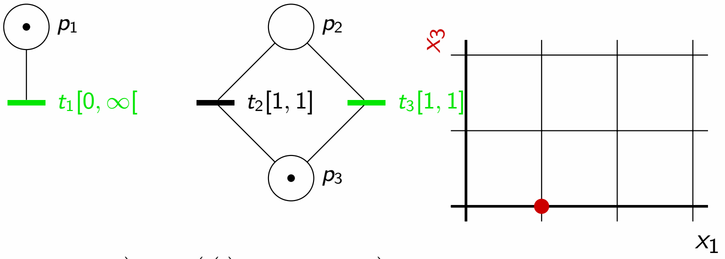

$$\left(\begin{pmatrix}1\\1\\0\end{pmatrix}, x_1 = x_2 \in [1,1]\right) \xrightarrow{t_2} \left(\begin{pmatrix}1\\0\\1\end{pmatrix}, x_1 = 1 \wedge x_3 = 0\right)$$

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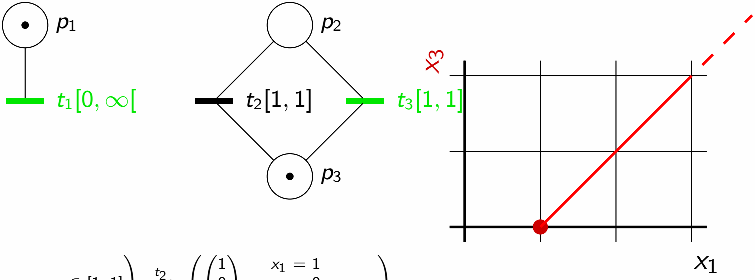

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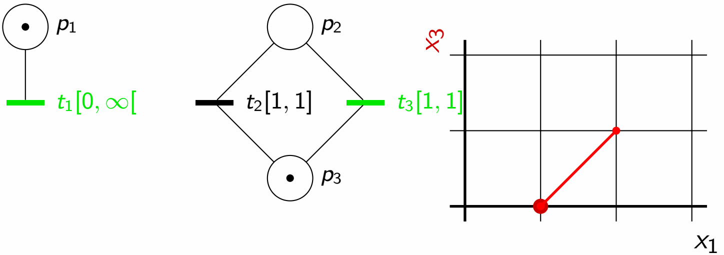

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Next(t)

Let t a transition with the firing interval $[\alpha, \beta]$ fireable from (M_i, Z_i). The computation of the successor of (M_i, Z_i) by the firing of t is $next((M_i, Z_i), t) = (M_j, Z_j)$ where Z_j is computed as follows:

- compute the firing space of the transition t : $Z_i \cap (x_t \geq \alpha)$ where x_t is the clock associated with t
- eliminate x_t (for example by using Fourier-Motzkin method)
- add (or reset) the clocks of the newly enabled transitions: x_{new} and for all other clocks x_{old} (with $min \leq x_{old} \leq max$), add the new diagonal constraints $min \leq x_{old} - x_{new} \leq max$)
- compute the futur (time elapsing)
- add the constraints $x \leq \beta$
- compute the canonical form

Computation of the state space

Exercise

Go back to the previous example and add $t_1 \to P_4 \to t_4[2,3]$ to the net.

- Compute the initial zone Z_0 and its successor Z_1 by the firing of t_1 and give the detail of the method (Conjonction with the guard $x_1 \geq 0$. Elimination of x_1 by Fourier Motzkin. Add x_4. Futur. Conjonction with invariants. Canonical form).
- Compute (literal expression and graphical representation) all the zones of the following sequence: $Z_0 \xrightarrow{t_1} Z_1 \xrightarrow{t_2} Z_2 \xrightarrow{t_3} Z_3 \xrightarrow{t_2} Z_4 \xrightarrow{t_3} Z_5$ and give for each zone, the list of fireable transitions.
  Give the transition fireable from Z_5 ?
- Simulate the TPN with the tool Roméo on example: Ex1-a-Master.xml

Computation of the state space

Theorem (Convergence)

The algorithm converges for time Petri Nets:

- *bounded*
- $\beta : T \to \mathbb{Q}^+$

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Implementation with DBM :

- previous example with $t_1[4, 4]$ with Romeo

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Computation of the state space

Theorem

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Computation of the state space

Theorem

The algorithm converges for time Petri Nets:

- *bounded*
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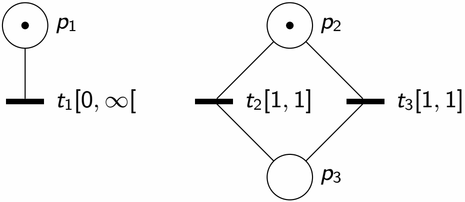
Problem

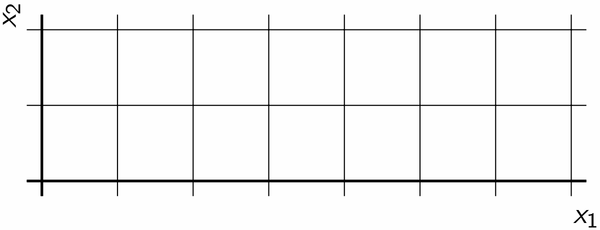
Termination for $\beta : T \to \mathbb{Q}^+ \cup \{\infty\}$?

Computation of the state space

Example

with $\beta = \infty$

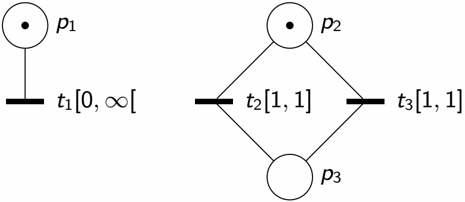


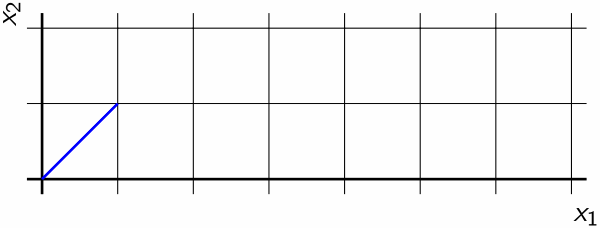


Computation of the state space

Example

with $\beta = \infty$

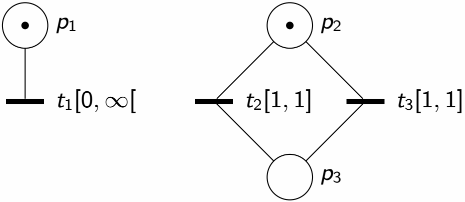


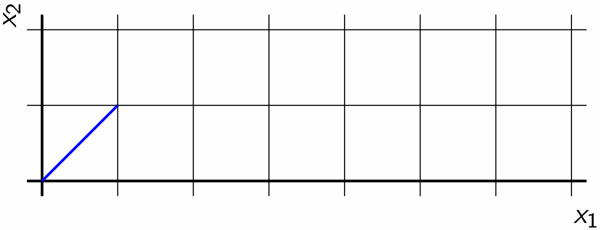


Computation of the state space

Example

with $\beta = \infty$

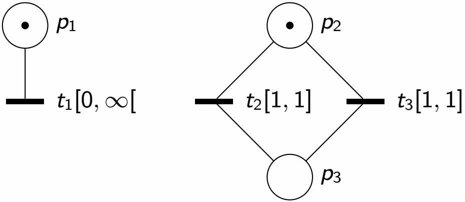


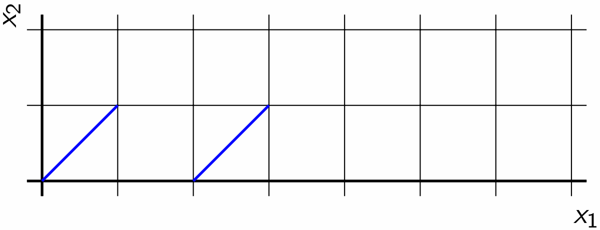


Computation of the state space

Example

with $\beta = \infty$

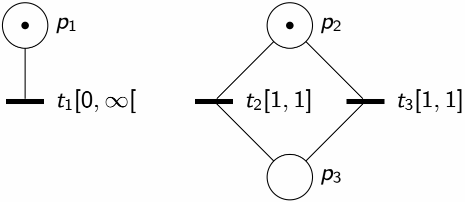


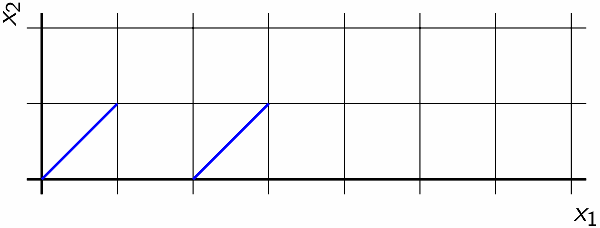


Computation of the state space

Example

with $\beta = \infty$





Computation of the state space

Example

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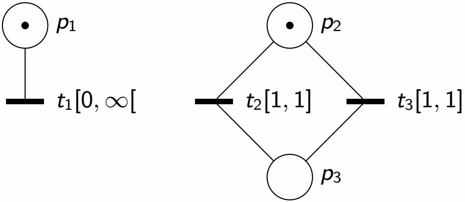
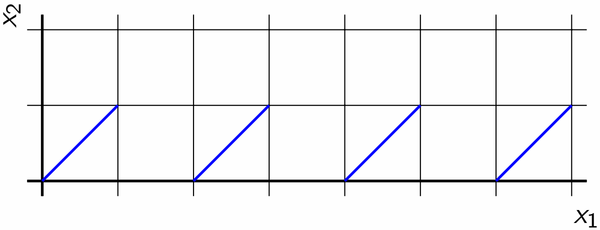




Computation of the state space

Transition with interval $[a, \infty[$:

- Information importante : $x \geq a$

$\rightarrow$ Utilisation d'un opérateur d'approximation *k-approx*

- Choix de k :

$$k = max_{t \in T \mid \beta(t) \neq \infty} (\alpha(t), \beta(t))$$

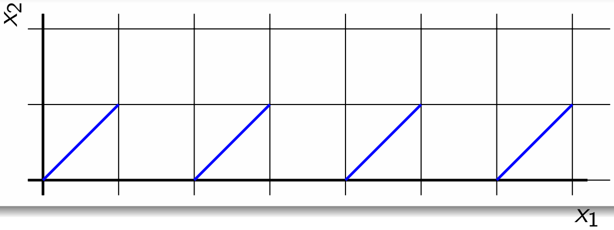
Computation of the state space

Apply *approx* in the computation of the successor

Computation of the state space

Apply *approx* in the computation of the successor

Example

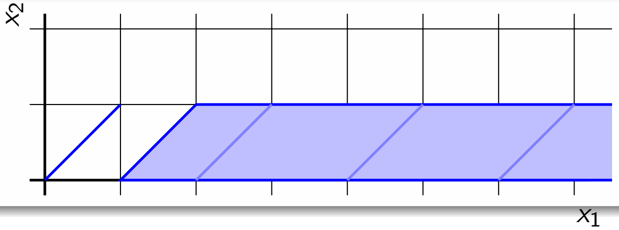


$$(M_i, Z_i) = \left(M_0 -^{\bullet} t_i + t_i^{\bullet}, k - approx\left(\left(\overrightarrow{Z_0} \cap Inv\left(M_0\right) \cap x_i \geq \alpha_i\right)\left[X_{ne} := 0\right]\right)\right)$$

Computation of the state space

Apply *approx* in the computation of the successor

Example



$$(M_i, Z_i) =$$
$$\left(M_0 -^{\bullet} t_i + t_i^{\bullet}, k - approx\left(\left(\overrightarrow{Z_0} \cap Inv\,(M_0) \cap x_i \geq \alpha_i\right)[X_{ne} := 0]\right)\right)$$

Computation of the state space

Theorem

The zone graph algorithm with k-approximation is exact with respect to marking reachability for bounded TPN with latest firing time in the set $\mathbb{Q}^+ \cup \{\infty\}$.

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Implementation with DBM :

- previous example with $t_1[4, \infty]$

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$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \leq x_2 \leq 1 \\ x_1 = x_2 \end{pmatrix} \right)$$

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$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \leq x_2 \leq 1 \\ x_1 = x_2 \end{pmatrix} \right) \xrightarrow{t_2} \left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \leq x_1 \leq 2 \\ 0 \leq x_3 \leq 1 \\ x_1 - x_3 = 1 \end{pmatrix} \right) \xrightarrow{t_3} \left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \leq x_1 \leq 3 \\ 0 \leq x_2 \leq 1 \\ x_1 - x_2 = 2 \end{pmatrix} \right) \ldots$$

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Computation of the state space - State class graph

Definition (State class [BD91])

A State class C is a pair (M, D) where M is a marking and D is a set of inequalities (a polyhedron) called the firing domain.

$$D = \begin{cases} a_i \leq \theta_i \leq b_i, \ \forall i \text{ s.t. } t_i \text{ is enabled}, \\ -c_{kj} \leq \theta_j - \theta_k \leq c_{jk}, \ \forall j, k \text{ s.t. } \begin{cases} j \neq k \\ t_j, t_k \in enabled(M) \end{cases} \end{cases}$$

(θ_i represents the firing time of t_i relatively to the time when the class C was entered in)

$\Rightarrow$ Difference Bound Matrix (DBM)

Computation of the state space - State class graph

Intuition

Let the following Petri net: $P1 \rightarrow T1$, $P2 \rightarrow T2$, $P3 \rightarrow T3$
avec $M_o = \{P1, P2, P3\}$ $T1[3,5]$, $T2[7,9]$, $T3[4,6]$

- Give the firing intervals of the transitions T_1, T_2 and T_3 with the variables θ_1, θ_2 and θ_3
- Deduce the fireable transitions.
- Deduce intuitively (without diagonal constraints)the firing intervals of the transitions T_2 and T_3 after the firing of T_1.

Computation of the state space - State class graph

$\mathcal{C}_0 = (M_0, D_0)$

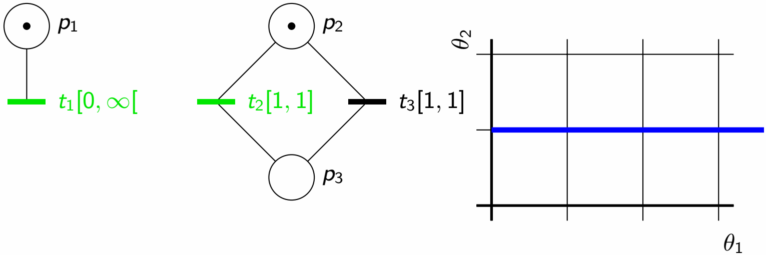
Computation of the fireable transitions

Computation of the state space - State class graph

$\mathcal{C}_0 = (M_0, D_0)$

Computation of the fireable transitions

Example



$$\left(\begin{pmatrix}1\\1\\0\end{pmatrix}, \left\{ \begin{array}{l} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{array} \right. \right)$$

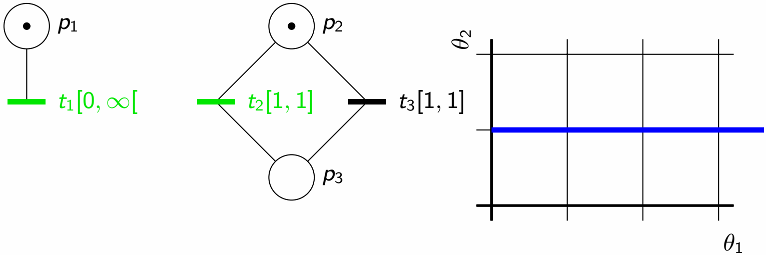
$LSE = min(b_i)$, Fireable transitions: $\{t_j \mid a_j \leq LSE\}$

Computation of the state space - State class graph

$\mathcal{C}_0 = (M_0, D_0)$

Computation of the fireable transitions

Example



$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \left\{ \begin{array}{l} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{array} \right. \right) \rightarrow LSE = b_2 = 1 \text{(From the initial class)}$$

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Computation of the state space - State class graph

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Computation of the fireable transitions

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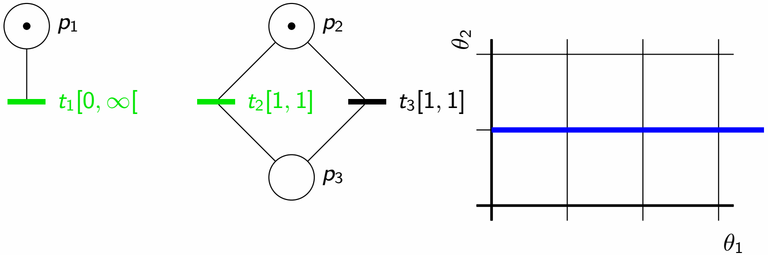

$\left(\begin{pmatrix}1\\1\\0\end{pmatrix}, \left\{\begin{array}{l} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{array}\right.\right) \rightarrow LSE = b_2 = 1$(From the initial class) $\rightarrow t_1$ and t_2 are fireable

$LSE = min(b_i)$, Fireable transitions: $\{t_j \mid a_j \leq LSE\}$

Computation of the state space - State class graph

$\mathcal{C}_0 = (M_0, D_0)$

Computation of the fireable transitions

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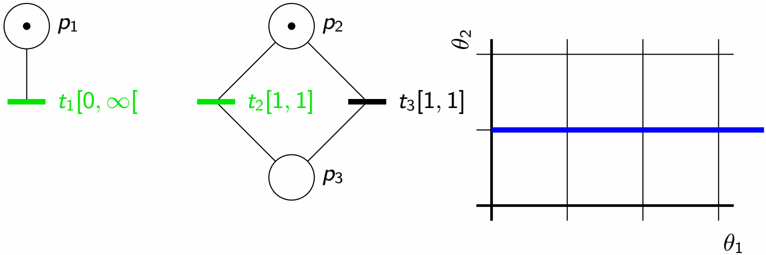

$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \left\{ \begin{array}{l} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{array} \right. \right) \rightarrow t_1 \text{ and } t_2 \text{ are fireable}$$

A transition t_i is fireable if, by elapsing time, one can reach $\theta_i = 0$

Computation of the state space - State class graph

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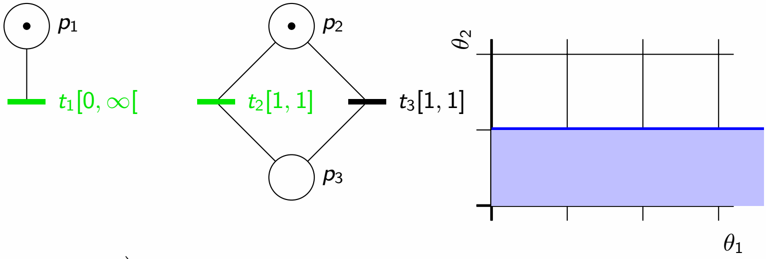
Computation of the fireable transitions

Example



$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \left\{ \begin{array}{l} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{array} \right. \right) \rightarrow t_1 \text{ and } t_2 \text{ are fireable}$$

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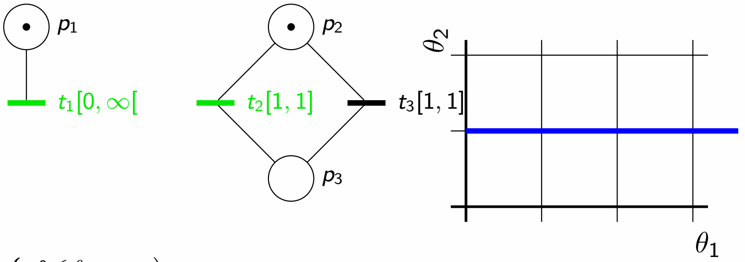
Computation of the state space - State class graph

Firing of transitions $\rightarrow (M_i, D_i)$.

Computation of the state space - State class graph

Firing of transitions $\rightarrow (M_i, D_i)$.

Example



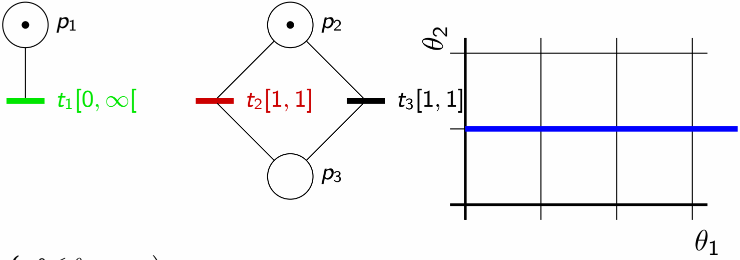
$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{cases} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{cases} \right)$$

$$(M_i, D_i) = (M_0 - {}^\bullet t_i + t_i^\bullet, (D_i = next(D_0, t_i)))$$

Computation of the state space - State class graph

Firing of transitions $\rightarrow (M_i, D_i)$.

Example



$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \left\{ \begin{array}{l} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{array} \right. \right)$$

$$(M_i, D_i) = (M_0 - {}^{\bullet}t_i + t_i^{\bullet}, (D_i = next(D_0, t_i)))$$

Computation of the state space - State class graph

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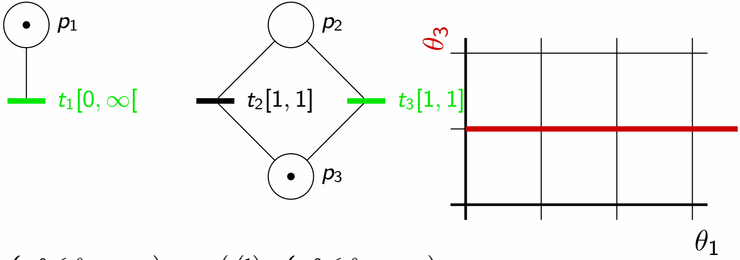

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$$(M_i, D_i) = (M_0 - {}^\bullet t_i + t_i^\bullet, (D_i = next(D_0, t_i)))$$

State class graph - algorithm

Let $C = (M, D)$, a class and t_f, a fireable transition. The class $C' = (M', D')$, successor of C by the firing of t_f is:

- $M' = M - {}^\bullet t_f + t_f^\bullet$
- $D' = next(D, t_f)$ is computed as follows:
  1. variable change: $\forall j, \theta_j = \theta_f + \theta'_j$;
  2. $\forall t_j \neq t_f$, adding the constraints $\theta'_j \geq 0$;
  3. elimination of the variables associated with the transitions disabled by the firing of t_f (including t_f), by using the Fourier-Motzkin method;
  4. adding the news inequalities of the newly enabled transitions t_k:

     $$\forall t_k \in \uparrow enabled(M, t_f), \alpha(t_k) \leq \theta'_k \leq \beta(t_k).$$

  5. Computation of the canonical form D'^* of D'.

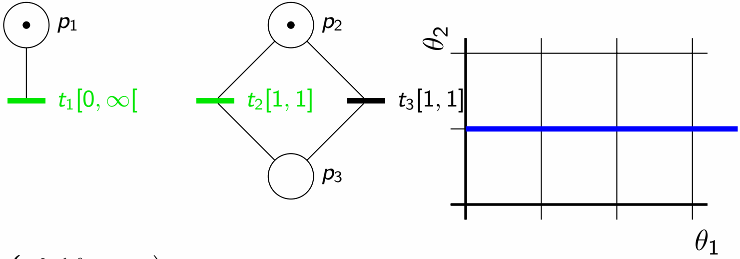
Computation of the state space

Firing of transitions $\rightarrow (M_i, D_i)$.

Computation of the state space

Firing of transitions $\rightarrow (M_i, D_i)$.

Example (Firing of t_1)



$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \left\{ \begin{array}{l} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{array} \right. \right)$$

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Computation of the state space

Firing of transitions $\rightarrow (M_i, D_i)$.

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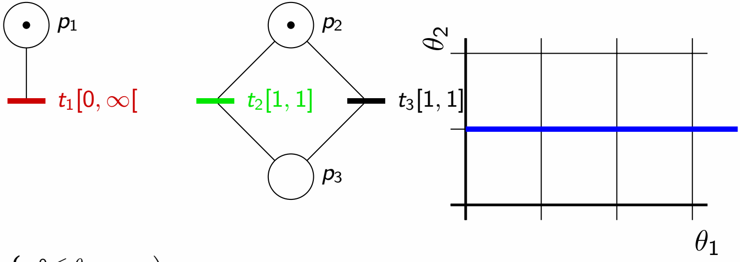

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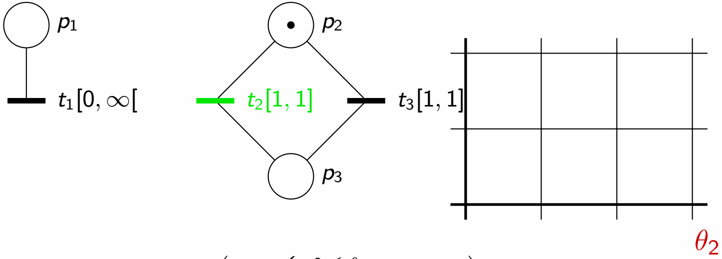

$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \left\{ \begin{array}{l} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{array} \right. \right) \xrightarrow{t_1} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \left\{ \begin{array}{l} 0 \leq \theta_1 \\ \theta_2' \geq 0 \\ 1 \leq \theta_2' + \theta_1 \leq 1 \\ \theta_2' + \theta_1 - \theta_1 \leq 1 \end{array} \right. \right)$$

$$(M_i, D_i) = (M_0 -^\bullet t_i + t_i^\bullet, (D_i = next(D_0, t_i)))$$

Computation of the state space

Firing of transitions $\rightarrow (M_i, D_i)$.

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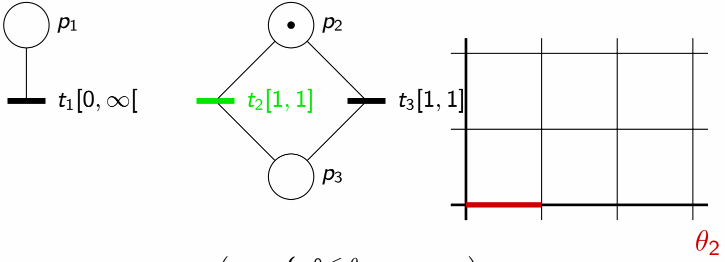


$$\left(\begin{pmatrix}1\\1\\0\end{pmatrix}, \left\{\begin{array}{l} 0 \leq \theta_1 \\ 1 \leq \theta_2 \leq 1 \\ \theta_2 - \theta_1 \leq 1 \end{array}\right.\right) \xrightarrow{t_1} \left(\begin{pmatrix}0\\1\\0\end{pmatrix}, \left\{\begin{array}{l} 0 \leq \theta_1 \\ \theta_2' \geq 0 \\ 1 \leq \theta_2' + \theta_1 \leq 1 \\ \theta_2' + \theta_1 - \theta_1 \leq 1 \end{array}\right.\right) = \left(\begin{pmatrix}0\\1\\0\end{pmatrix}, 0 \leq \theta_2' \leq 1\right)$$

$$(M_i, D_i) = (M_0 - {}^\bullet t_i + t_i^\bullet, (D_i = next(D_0, t_i)))$$

Theorem ([BD91])

The State Class Graph algorithm converges for bounded time Petri Nets:

Theorem ([BD91])

The State Class Graph algorithm converges for bounded time Petri Nets:

Implementations in tools:

- Tina
- Roméo

Computation of the state class graph

Exercise : Ex2-Master.xml

Let the TPN: $P1 \rightarrow T1$, $P2 \rightarrow T2$, $P3 \rightarrow T3$
with $M_o = \{P1, P2, P3\}$ $T1[3,5]$, $T2[7,9]$, $T3[4,6]$

- Compute the initial class C_0
  - Draw the projections of C_0 other 3 planes (θ_1, θ_2), (θ_1, θ_3) and (θ_2, θ_3).
  - Deduce the fireable transitions (Does the elapsing of time makes possible to reach $\theta = 0$?).
- Compute the successor of C_0 by the firing of T_1 (Fourier Motskin) : $C_0 \xrightarrow{T_1} C_1$
  - Draw the polyhedron in the plane (θ_2, θ_3).
  - Is the transition T_2 fireable from C_1 ? (Does the elapsing of time makes possible to reach $\theta_2 = 0$?).
- Same question with T_3 : $C_0 \xrightarrow{T_3} C_2$ (canonical form).

Computation of the state class graph - Convergence

In the state space computation algorithm given in page 34, the convergence criterion is given by $next(\mathcal{Q}, t) \in (Waiting \cup Visited)$

Convergence

For the computation of the state class graph, the convergence can be:

- by equality of classes (the graph preserves the markings and the language)
- by inclusion of the domain of classes (the graph preserves only reachability of markings).

Go back to the previous example. Compute the state class graph obtain:

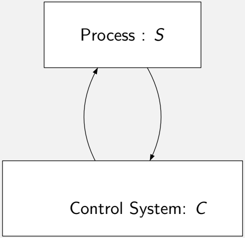
- with convergence by equality
- with convergence by inclusion

Plan

- **Petri nets with time**
- **T-time Petri Nets**
- **Other Semantics**
  - P-time Petri Nets
  - A-time Petri Nets
  - Strong vs Weak Semantics
- **Expressiveness and properties of TPN**
- **State Space of Time Petri Nets**
- **Model-checking of Time Petri Nets**
  - Temporal logics
  - Checking TPN with observers
- **Stopwatch Petri Nets**

Introduction

Design of a real time system



- Closed system : $S \parallel C$
- Model of the process S
- Specification of control properties: φ

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$$\text{Does } S \parallel C \models \varphi \quad ? \tag{3}$$

Control problem:

$$\text{Is there } C \text{ such that } S \parallel C \models \varphi \quad ? \tag{4}$$

- If yes : synthesise this controller.

For real time systems:

- Functional specification
- Timed specification

$\Rightarrow$ logical time is not sufficient.

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Formal model

Timed extensions of

- Process algebra
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Specification

- observers
- temporal logics (LTL, CTL)
- timed temporal logics (TCTL)

Properties

During the execution of a system,

Safety

nothing bad happens

Properties

During the execution of a system,

Safety

nothing bad happens

Liveness

something good eventually happens

Temporal logic CTL*

The temporal operators

Quantifiers over paths

A : *for all* means 'along All paths' (Inevitably)

E : *for some* means 'along at least (there Exists) one path' (possibly)

Path-specific quantifiers

X : next

U : Until

F : (Finally) eventually

G : (Globally) always

Computation tree logic CTL * = State formulae

State formulae (sf):

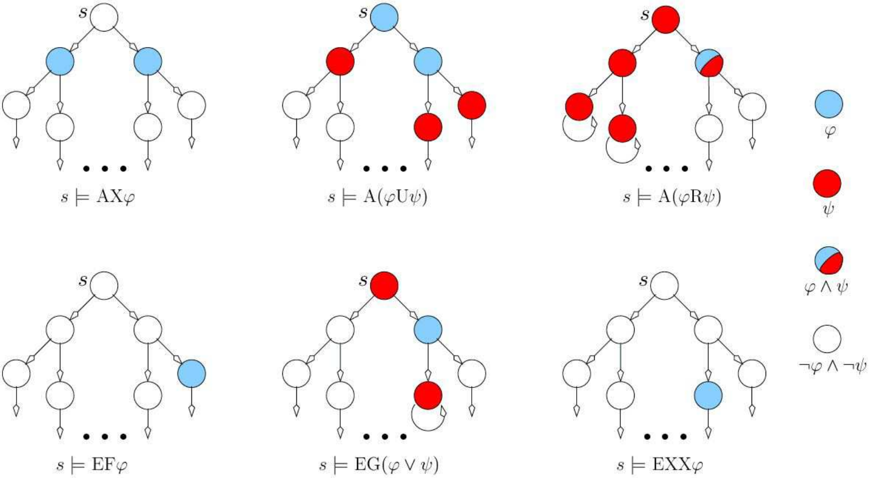
$sf ::= p \mid \neg p \mid \ true \mid false \mid sf \vee sf \mid sf \wedge sf \mid A\ pf \mid E\ pf$

where p ranges over a set of atomic formulas (markings).

Path formulae (pf):

$pf ::= sf \mid sf \vee sf \mid sf \wedge sf \mid pf\ \ U\ pf \mid X\ sf \mid F\ sf \mid G\ sf$

Temporal logic CTL*



Temporal logic CTL*

Other notations:

$$\exists\Diamond\varphi = EF\varphi,$$
$$\forall\Diamond\varphi = AF\varphi,$$
$$\exists\Box\varphi = EG\varphi,$$
$$\forall\Box\varphi = AG\varphi;$$

Deadlock free : $\forall\Box\ \exists X\ \varphi$ with $\varphi = \mathit{true}$ that is to say $AG\ EX\ \mathit{true}$

Temporal logic CTL*

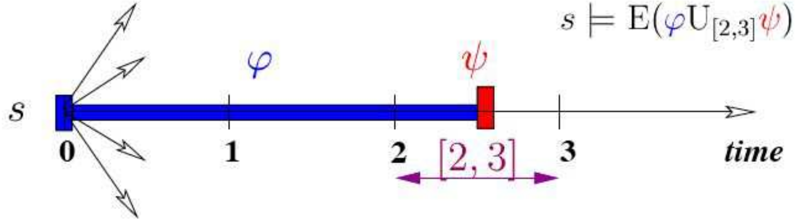
Duality:

$$AF\ \varphi = \neg EG \neg \varphi$$
$$AG\ \varphi = \neg EF \neg \varphi$$
$$AX\ \varphi = \neg EX \neg \varphi$$

F and G quantifiers can be defined from *Until*:

$$AF\ \varphi = A(true U \varphi)$$
$$EF\ \varphi = E(true U \varphi)$$
$$AG\ \varphi = \neg E(true U \neg \varphi)$$
$$EG\ \varphi = \neg A(true U \neg \varphi)$$

Timed Temporal logic TCTL



$\textsc{Gmec}$ = set of legal markings defined as the integer solutions of a convex set

$[\![\mathcal{N}]\!]$ = set of runs of a TPN $\mathcal{N}$

I = time interval

$$
\begin{array}{lll}
(M,v) \models \textsc{Gmec} & \text{iff} & M \models \textsc{Gmec} \\
(M,v) \not\models \textbf{false} & & \\
(M,v) \models \neg\varphi & \text{iff} & (M,v) \not\models \varphi \\
(M,v) \models \varphi \Rightarrow \psi & \text{iff} & (M,v) \not\models \varphi \text{ or } (M,v) \models \psi \\
(M,v) \models \exists\varphi\,\mathcal{U}_I\psi & \text{iff} & \exists\sigma \in [\![\mathcal{N}]\!] \ \textit{such that} \\
& & \begin{cases} (s_0,v_0) = (M,v) \\ \forall i \in [s1..n], \forall d \in [0,d_i), (s_i, v_i + d) \models \varphi \\ \left(\sum_{i=1}^{n} d_i\right) \in I \textit{ and } (s_n,v_n) \models \psi \end{cases} \\
(M,v) \models \forall\varphi\,\mathcal{U}_I\psi & \text{iff} & \forall\sigma \in [\![\mathcal{N}]\!] \ \textit{we have} \\
& & \begin{cases} (s_0,v_0) = (M,v) \\ \forall i \in [1..n], \forall d \in [0,d_i), (s_i, v_i + d) \models \varphi \\ \left(\sum_{i=1}^{n} d_i\right) \in I \textit{ and } (s_n,v_n) \models \psi \end{cases}
\end{array}
$$

Timed Temporal logic TCTL

Notations and shorthands :

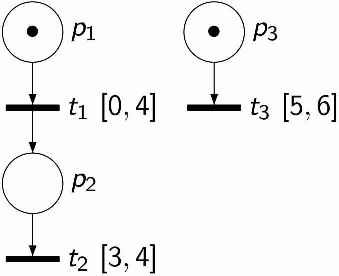
$$\exists\Diamond_I\phi = \exists \textbf{true}\, \mathcal{U}_I \phi,$$
$$\forall\Diamond_I\phi = \forall \textbf{true}\, \mathcal{U}_I \phi,$$
$$\exists\Box_I\phi = \neg\forall\Diamond_I\neg\phi,$$
$$\forall\Box_I\phi = \neg\exists\Diamond_I\neg\phi;$$

$$(\varphi \leadsto_I \psi) = \forall\Box(\varphi \Rightarrow \forall\Diamond_I\psi),$$

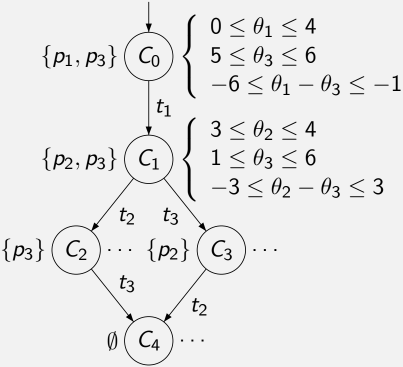
Abstraction and verification...

Example : state class graph [BD91]

Example (Net of L. Gallon)



State class graph



Problème

Find an abstraction for a given property

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Non quantitative properties :

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- Observer and reachability
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Quantitative Properties

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- For a subset of TCTL (on the fly computation) : state class graph or zone graph.

Checking TPN with observers

Observer

- non intrusive
  - Arc (post) from a transition of the net to a place of the observer
  - Read arc or inhibitor arc from a place of the net to a transition of the observer
  - Reset Arc from a place of the observer to a transition of the net
- turns the verification of a particular property into a marking reachability problem

Example : let a Petri Net with a transition t. Write an observer for the followings properties : Between two successive firings of the transition t, there is:

- always more than 10 time units
- never more than 10 time units

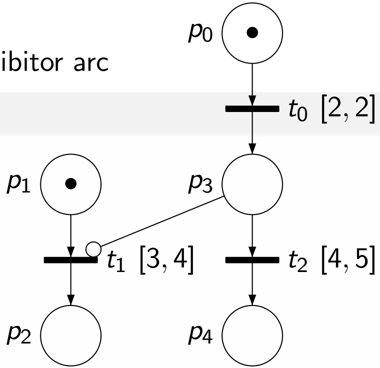
Exercise : ex1.xml

- $EF[0,40](M(P5) - M(P2) > 0)$
- $EF[0,20](M(P5) - M(P2) > 0)$
- $AG[0,20](M(P1) + M(P2) > 0)$
- $AG[0,30](M(P1) + M(P2) > 0)$
- $EG[0,30](M(P1) + M(P2) > 0)$
- $(M(P2) > 0) \rightarrow [0,40](M(P5) > 0)$
- $(M(P2) > 0) \rightarrow [0,35](M(P5) > 0)$
- $(M(P2) > 0) \rightarrow [0,60](M(P3) - M(P4) > 0)$
- Write an observer to check the properties :
  - $(M(P2) > 0) \rightarrow [0,45[(M(P6) > 0)$ or more precisely (firing of T0)$\rightarrow$ $[0,45[$(firing of T3)
  - (firing of T0)$\rightarrow]10,45[$(firing of T3)
  - idem considering that there may be several firings of T0 before a firing of T3 (we consider for the property only the last firing of T0)

Stopwatch Petri Nets

Petri net with timed inhibitor arc

Example (SwPN)



Is it possible to fire t_1 before t_2 ?

Stopwatch Petri Nets

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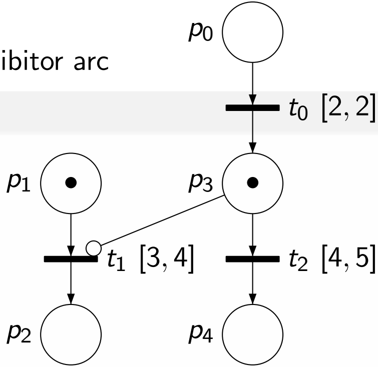

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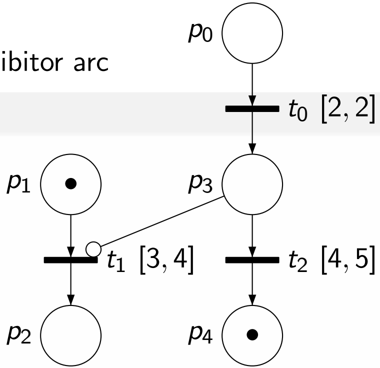

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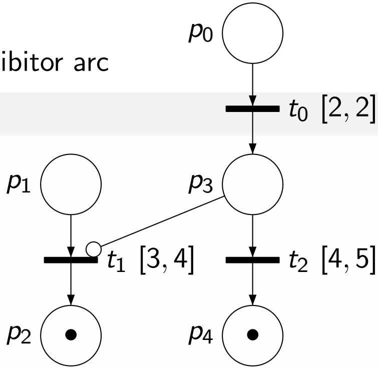

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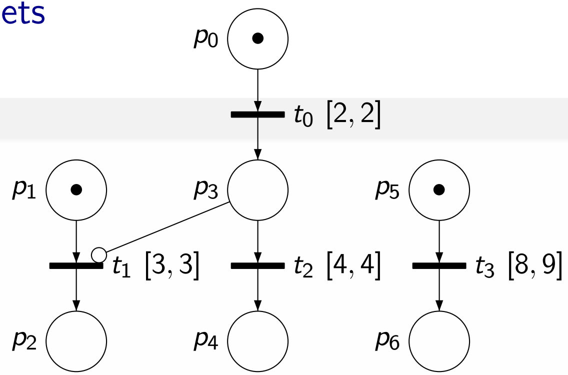
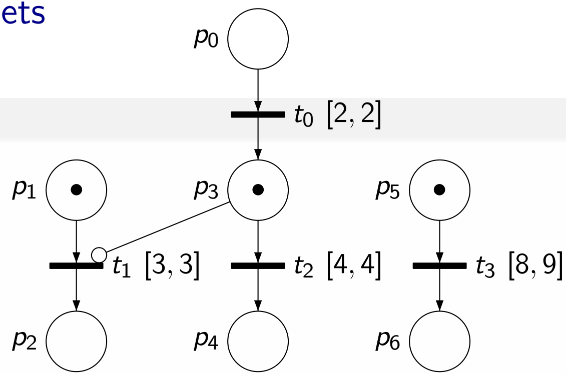


Is it possible to fire t_3 before t_1 ?

Stopwatch Petri Nets

Example (SwPN)



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Stopwatch Petri Nets

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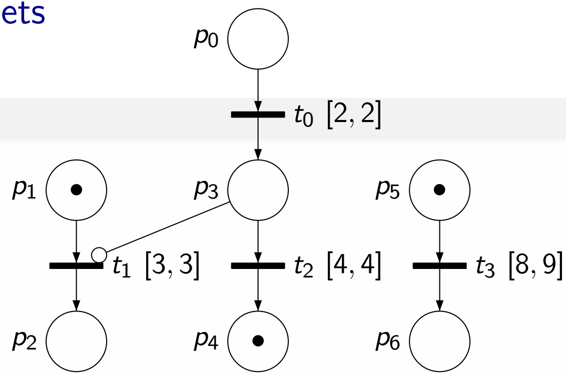

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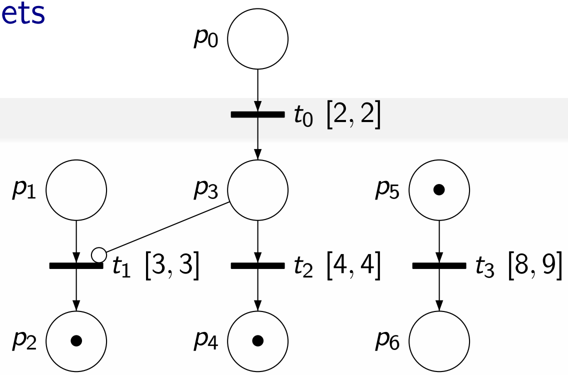

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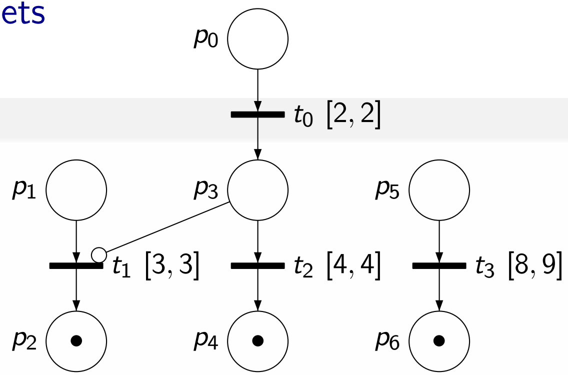
Example (SwPN)



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