

Petri Nets with Time

Fournier-Hotzkin method:

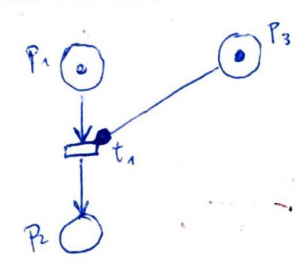
10/74

Merely enabled transition:

- new marking enable the transition
- and - the transition has not just been fired

15/74

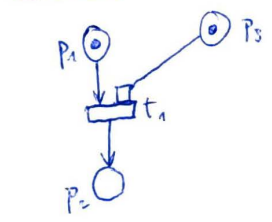
Inhibitor arc:



t_1 is not enabled because there is a token in p_3 that inhibits it

16/74

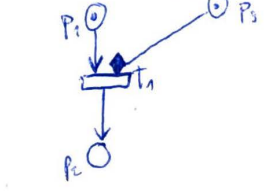
Read arc:



firing t_1 doesn't consume the token in p_2 , but it needs it to be enabled

17/74

Reset arc:



- t_1 reset all the tokens in p_2
- t_1 can be fire if there is no token in p_2

41/74

Exercise:

Computation of state space

t_1

$Z_0: \begin{cases} x_1 = x_2 \\ x_2 \leq 1 \end{cases}$

$Z_0 \cap (x_1 \geq 0) = Z_0 \neq \emptyset$
 $\Rightarrow t_1$ is fireable

t_2

$Z_1: \begin{cases} 0 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 1 \\ 0 \leq x_1 - x_4 \leq 1 \end{cases}$

$Z_1 \cap (1 \leq x_2 \leq 1) \neq \emptyset$
 $\Rightarrow t_2$ is fireable

$t_2 \rightarrow$

t_3

$Z_2: \begin{cases} 0 \leq x_4 \leq 1 \\ 0 \leq x_3 \leq 1 \\ 0 \leq x_1 - x_3 \leq 1 \end{cases}$

$Z_2 \cap (x_3 \geq 1) \neq \emptyset$
 $\Rightarrow t_3$ is fireable

canonical form

$\begin{cases} 0 \leq x_2 \\ 0 \leq 1+x_4 \\ x_4 \leq 1 \\ x_4 \leq 1+x_4 \\ 0 \leq x_4 \leq 1 \end{cases} \Rightarrow \begin{cases} x_4 \geq 0 \\ x_4 \leq 1 \end{cases}$

$t_3 \rightarrow$

t_4

$Z_3: \begin{cases} 1 \leq x_4 \leq 2 \\ 0 \leq x_2 \leq 1 \\ 1 \leq x_1 - x_2 \leq 2 \end{cases}$

$Z_3 \cap (x_2 \geq 1) \neq \emptyset$
 $\Rightarrow t_4$ is fireable

compute the future

$\begin{cases} 1 \leq x_4 \leq 2 \\ 0 \leq x_2 \leq 1 \\ 1 \leq x_1 - x_2 \leq 2 \end{cases}$

add upper bound $x_2 \leq 2$

$\begin{cases} 1 \leq 1 \\ 1 \leq x_4 \\ x_4 - 1 \leq 1 \\ x_4 - 1 \leq x_4 \\ 0 \leq x_4 \leq 2 \end{cases} \Rightarrow \begin{cases} 1 \leq x_4 \leq 2 \end{cases}$

$t_4 \rightarrow$

t_5

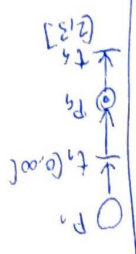
$Z_4: \begin{cases} 2 \leq x_4 \leq 3 \\ 0 \leq x_3 \leq 1 \\ 2 \leq x_1 - x_3 \leq 3 \end{cases}$

$Z_4 \cap (x_3 \geq 1) \neq \emptyset$
 $\Rightarrow t_5$ is fireable

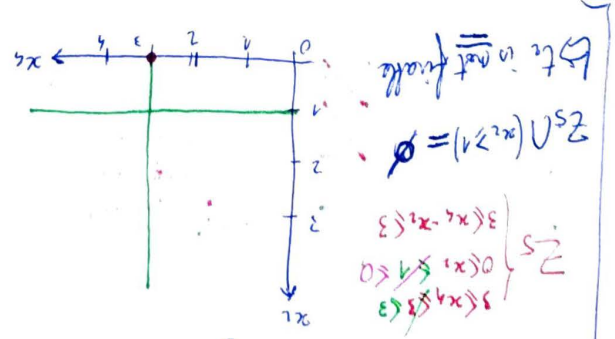
$\begin{cases} 1 \leq x_4 \leq 3 \\ 1 \leq x_2 \\ x_4 - 2 \leq x_2 \\ x_2 \leq x_4 - 1 \end{cases}$

$\begin{cases} 1 \leq 1 \\ 1 \leq x_4 - 1 \\ x_4 - 2 \leq 1 \\ x_4 - 2 \leq x_4 - 1 \\ 1 \leq x_4 \leq 3 \end{cases} \Rightarrow \begin{cases} 2 \leq x_4 \leq 3 \end{cases}$

t_3

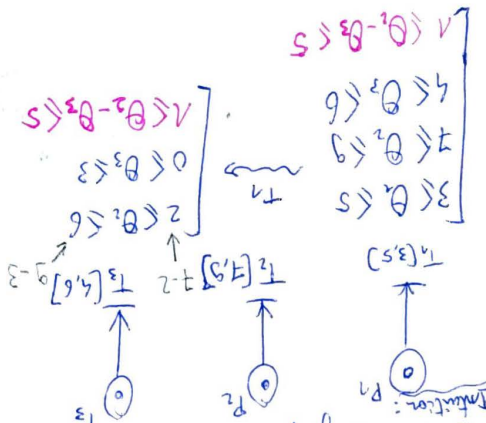


$$\begin{aligned} & \begin{cases} 2 \leq x_4 < 3 \\ 1 \leq x_3 < 2 \\ x_4 - 3 \leq x_3 \\ x_4 - 3 \leq 1 \end{cases} \Rightarrow \begin{cases} x_4 - 3 \leq x_4 - 2 \\ x_4 - 3 \leq 1 \end{cases} \\ & \begin{cases} 1 \leq x_4 < 2 \\ x_4 - 3 \leq x_3 \\ x_4 - 3 \leq 1 \end{cases} \Rightarrow \begin{cases} x_4 - 3 \leq x_4 - 2 \\ x_4 - 3 \leq 1 \end{cases} \end{aligned}$$

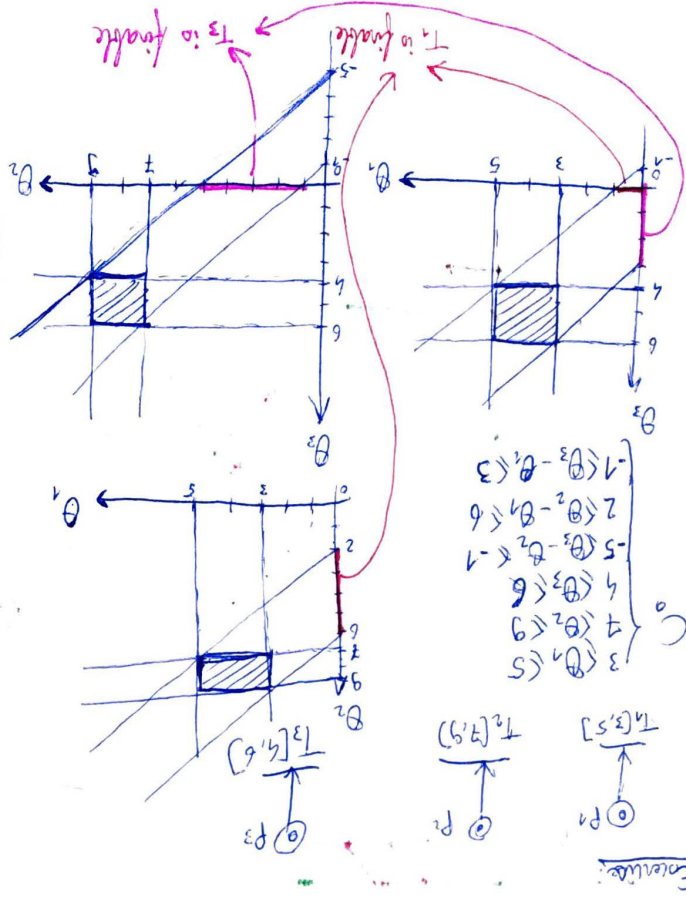


$z^S \cap (x_2 \geq 1) = \emptyset$
 Lst ist nicht lösbar.

State diagram:



$$\begin{aligned} & \begin{cases} 1 \leq \theta_1 - \theta_3 \leq 5 \\ 4 \leq \theta_2 \leq 6 \\ 7 \leq \theta_2 \leq 9 \\ 3 \leq \theta_1 \leq 5 \end{cases} \Rightarrow \begin{cases} 0 \leq \theta_3 \leq 3 \\ 2 \leq \theta_2 \leq 6 \\ 7 \leq \theta_2 \leq 9 \end{cases} \\ & \begin{cases} 1 \leq \theta_2 - \theta_3 \leq 5 \\ 0 \leq \theta_3 \leq 3 \end{cases} \Rightarrow \begin{cases} 2 \leq \theta_2 \leq 6 \\ 7 \leq \theta_2 \leq 9 \end{cases} \end{aligned}$$



(canonical form)

$$\begin{aligned} & \begin{cases} \theta_1 \leq \theta_1 < 1 \\ 2 \leq \theta_2 - \theta_1 < 5 \\ 3 - \theta_1 < \theta_3 \leq 5 - \theta_1 \\ 4 \leq \theta_3 \leq 6 \end{cases} \\ & \begin{cases} \theta_1 \leq \theta_1 < 1 \\ 2 \leq \theta_2 - \theta_1 < 5 \\ 3 - \theta_1 < \theta_3 \leq 5 - \theta_1 \\ 4 \leq \theta_3 \leq 6 \end{cases} \end{aligned}$$

Formal-Methode:

$$\begin{aligned} & \text{Fung T}_3: \\ & \begin{cases} 0 \leq \theta_3 \leq 3 \\ 2 \leq \theta_2 \leq 6 \end{cases} \Rightarrow \begin{cases} \theta_1 = \theta_1 + \theta_3 \\ \theta_2 = \theta_2 + \theta_3 \end{cases} \end{aligned}$$

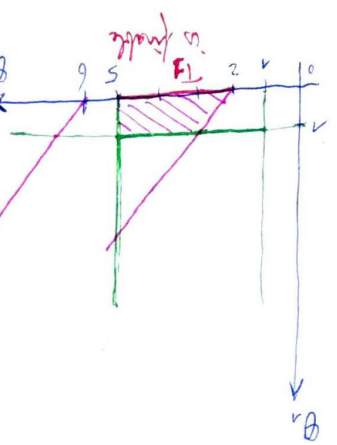
Formal-Methode:

$$\begin{aligned} & \begin{cases} 1 \leq \theta_1 - \theta_3 \leq 5 \\ 4 \leq \theta_2 \leq 6 \\ 7 \leq \theta_2 \leq 9 \\ 3 \leq \theta_1 \leq 5 \end{cases} \Rightarrow \begin{cases} 0 \leq \theta_3 \leq 3 \\ 2 \leq \theta_2 \leq 6 \end{cases} \end{aligned}$$

$$\begin{aligned} & \text{Fung T}_1: \\ & \begin{cases} \theta_2 = \theta_2' + \theta_1 \\ \theta_3 = \theta_3' + \theta_1 \end{cases} \Rightarrow \begin{cases} 1 \leq \theta_2' \leq 4 \\ 2 \leq \theta_3' \leq 5 \end{cases} \end{aligned}$$

$$\begin{aligned} & \begin{cases} 3 \leq \theta_1 < 5 \\ 7 \leq \theta_2' + \theta_1 < 9 \\ 4 \leq \theta_3' + \theta_1 < 6 \\ -5 \leq \theta_3' - \theta_1 < -1 \\ 2 \leq \theta_2' < 6 \\ -1 \leq \theta_3' < 3 \end{cases} \end{aligned}$$

$$\begin{aligned} & \begin{cases} 3 \leq \theta_1 + \theta_3 < 5 \\ 7 \leq \theta_2' + \theta_3 < 9 \\ 4 \leq \theta_3 < 6 \\ 1 \leq \theta_2' < 5 \\ 2 \leq \theta_2' - \theta_1' < 6 \\ -3 \leq \theta_1' < 1 \end{cases} \end{aligned}$$



• Firing T_3 from C_1 :

$$\begin{aligned} \theta_1 &\leftarrow \theta_1 + \theta_3 \\ \theta_2 &\leftarrow \theta_2 + \theta_3 \end{aligned} \Rightarrow \begin{cases} 1 \leq \theta_2 \leq 5 \\ 0 \leq \theta_3 \leq 3 \\ 2 \leq \theta_2 + \theta_3 \leq 6 \end{cases}$$

Fourier-Motzkin:

$$0 \leq \theta_3 \quad \theta_3 \leq 3$$

$$2 - \theta_2 \leq \theta_3 \quad \theta_3 \leq 6 - \theta_2$$

$$C_1 \{ 1 \leq \theta_2 \leq 5$$

• Firing T_1 from C_2 :

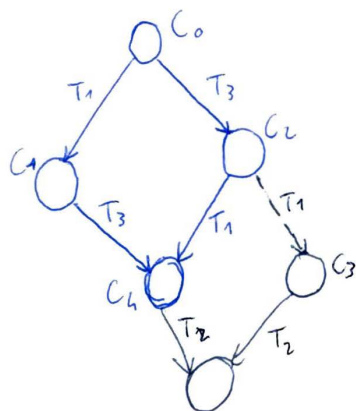
$$\theta_2 \leftarrow \theta_2 + \theta_1 \Rightarrow \begin{cases} 0 \leq \theta_1 \leq 1 \\ 2 \leq \theta_2 + \theta_1 \leq 5 \\ 2 \leq \theta_2 \leq 5 \end{cases}$$

Fourier-Motzkin

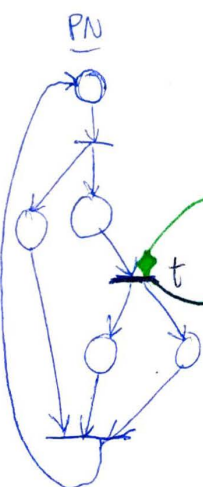
$$0 \leq \theta_1 \quad \theta_1 \leq 1$$

$$2 - \theta_2 \leq \theta_1 \quad \theta_1 \leq 5 - \theta_2$$

$$C_2 \{ 2 \leq \theta_2 \leq 5$$



70/74 | Verification



Observer

Between two firing of t , there is:
• always more than 10 time units

$$\neg EF(H(P_{obs}) > 1)$$

$$\boxed{AG(M(P_{obs}) < 2)}$$

• never more than 10 time units