Olivier H. ROUX

Petri Nets with Time

10/74

Newly enabled transition:

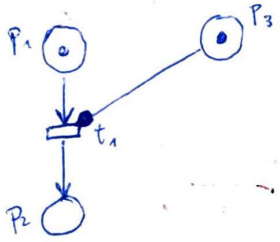
- new marking enable the transition

and

- the transition has not just been fired

15/74

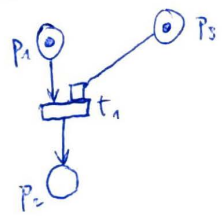
Inhibitor arc:



t_1 is not enabled because there is token in p_3 that inhibite it

16/74

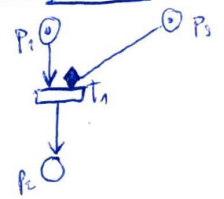
Read arc:



firing t_1 doesn't consume the token in p_3, but it needs it to be enabled

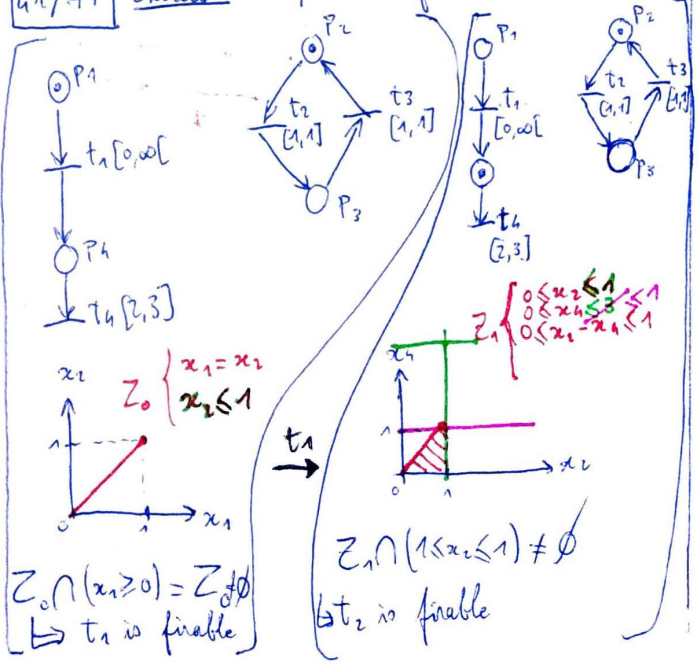
17/74

Reset arc:

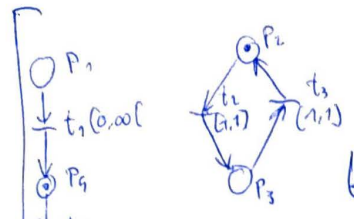


- t_1 reset all the tokens in p_3
- t_1 can be fire if there is no token in p_3

41/74 **Exercise:** Computation of state space



t_3 ⇒



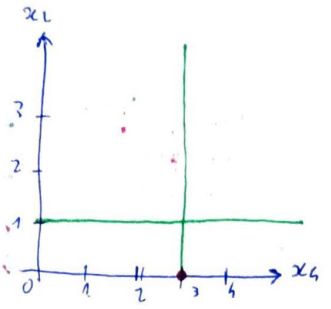
$2 \le x_4 \le 3$
$1 \le x_3 \quad x_3 \le 1$
$x_4 - 3 \le x_3 \quad x_3 \le x_4 - 2$

$$\hookrightarrow \begin{cases} 2 \le x_4 \le 3 \\ 1 \le 1 \\ 1 \le x_4 - 2 \\ x_4 - 3 \le 1 \\ x_4 - 3 \le x_4 - 2 \end{cases} \Rightarrow \boxed{3 \le x_4 \le 3}$$

$$\supseteq_5 \begin{cases} 3 \le x_4 \le 3 \le 3 \\ 0 \le x_1 \le 1 \le a \\ 3 \le x_4 - x_2 \le 3 \end{cases}$$

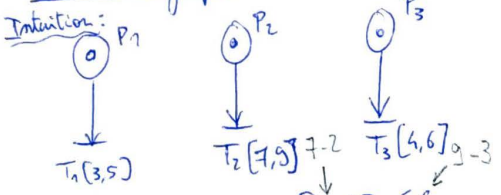
$Z_5 \cap (x_2 \ge 1) = \emptyset$

$\hookrightarrow t_2$ is not firable



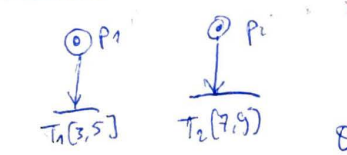
State class graph:

Intuition:

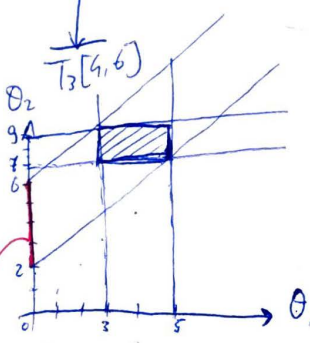


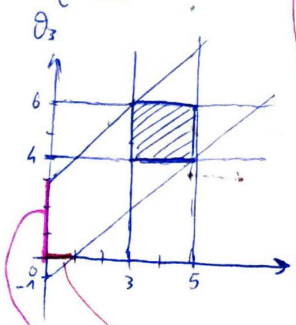
$$\begin{cases} 3 \le \theta_1 \le 5 \\ 7 \le \theta_2 \le 9 \\ 4 \le \theta_3 \le 6 \\ 1 \le \theta_2 - \theta_3 \le 5 \end{cases} \xrightarrow{T_1} \begin{cases} 2 \le \theta_2 \le 6 \\ 0 \le \theta_3 \le 3 \\ 1 \le \theta_2 - \theta_3 \le 5 \end{cases}$$

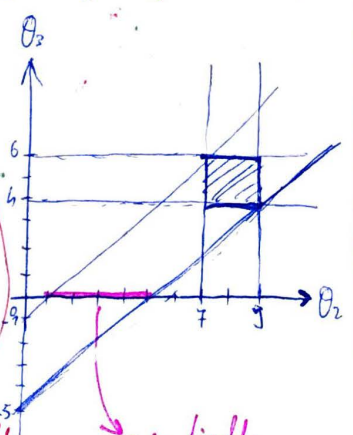
Exercise:



$$C_0 \begin{cases} 3 \le \theta_1 \le 5 \\ 7 \le \theta_2 \le 9 \\ 4 \le \theta_3 \le 6 \\ -5 \le \theta_3 - \theta_2 \le -1 \\ 2 \le \theta_2 - \theta_1 \le 6 \\ -1 \le \theta_3 - \theta_1 \le 3 \end{cases}$$







T_1 is firable

T_3 is firable

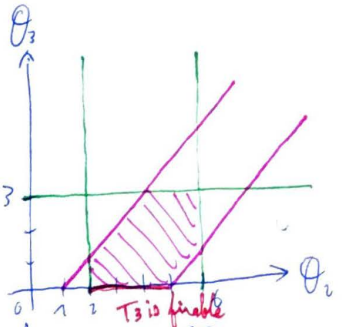
Firing T_1:

$$\theta_2 = \theta_2' + \theta_1 \quad \theta_3 = \theta_3' + \theta_1 \Rightarrow \begin{cases} 3 \le \theta_1 \le 5 \\ 7 \le \theta_2' + \theta_1 \le 9 \\ 4 \le \theta_3' + \theta_1 \le 6 \\ -5 \le \theta_3' - \theta_2' \le -1 \\ 2 \le \theta_2' \le 6 \\ -1 \le \theta_3' \le 3 \end{cases}$$

Fourier-Motzkin:

$$\begin{cases} 3 \le \theta_1 \le 5 \\ 7 - \theta_2' \le \theta_1 \le 9 - \theta_2' \\ 4 - \theta_3' \le \theta_1 \le 6 - \theta_3' \end{cases}$$

$$C_1 \begin{cases} 1 \le \theta_2' - \theta_3' \le 5 \\ 0 \le \theta_3' \le 3 \\ 2 \le \theta_2' \le 6 \end{cases}$$



Firing T_3:

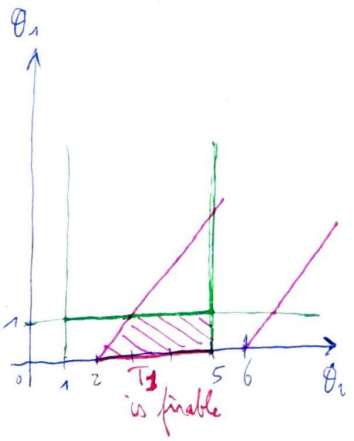
$$\theta_1 = \theta_1' + \theta_3 \quad \theta_2 = \theta_2' + \theta_3 \Rightarrow \begin{cases} 3 \le \theta_1' + \theta_3 \le 5 \\ 7 \le \theta_2' + \theta_3 \le 9 \\ 4 \le \theta_3 \le 6 \\ 1 \le \theta_2' \le 5 \\ 2 \le \theta_2' - \theta_1' \le 6 \\ -3 \le \theta_1' \le 1 \end{cases}$$

Fourier-Motzkin:

$$\begin{cases} 4 \le \theta_3 \le 6 \\ 3 - \theta_1' \le \theta_3 \le 5 - \theta_1' \\ 7 - \theta_2' \le \theta_3 \le 9 - \theta_2' \end{cases}$$

$$C_2 \begin{cases} 2 \le \theta_2' - \theta_1' \le 5 \\ 2 \le \theta_2' \le 5 \\ 0 \le \theta_1' \le 1 \end{cases}$$

(canonical form)



• Firing T_3 from C_1:

$\theta_1 \leftarrow \theta_1 + \theta_3$
$\theta_2 \leftarrow \theta_2 + \theta_3$

$\Rightarrow \begin{cases} 1 \leq \theta_2 \leq 5 \\ 0 \leq \theta_3 \leq 3 \\ 2 \leq \theta_2 + \theta_3 \leq 6 \end{cases}$

Fourier - Motzkin:

$0 \leq \theta_3 \quad \theta_3 \leq 3$
$2 - \theta_2 \leq \theta_3 \quad \theta_3 \leq 6 - \theta_2$

$C_4 \begin{cases} 1 \leq \theta_2 \leq 5 \end{cases}$

• Firing T_1 from C_2:

$\theta_2 \leftarrow \theta_2 + \theta_1$

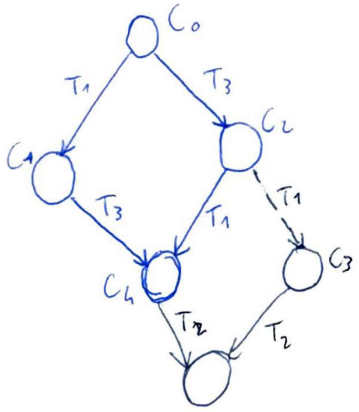
$\Rightarrow \begin{cases} 0 \leq \theta_1 \leq 1 \\ 2 \leq \theta_2 + \theta_1 \leq 5 \\ 2 \leq \theta_2 \leq 5 \end{cases}$

Fourier - Motzkin

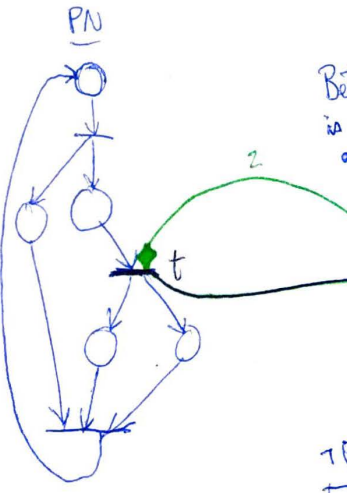
$0 \leq \theta_1 \quad \theta_1 \leq 1$
$2 - \theta_2 \leq \theta_1 \quad \theta_1 \leq 5 - \theta_2$

$C_3 \begin{cases} 2 \leq \theta_2 \leq 5 \end{cases}$

$C_3 \subset C_4$



70/74 | Verification



Observer

Between two firing of **t**, there is:

- always more than 10 time units
- never more than 10 time units

$TEF(M(P_{obs}) > 1)$

$AG(M(P_{obs}) < 2)$