Università
degli Studi di
Messina
DIPARTIMENTO DI INGEGNERIA

Dependable computing modeling and simulation

Repairable systems

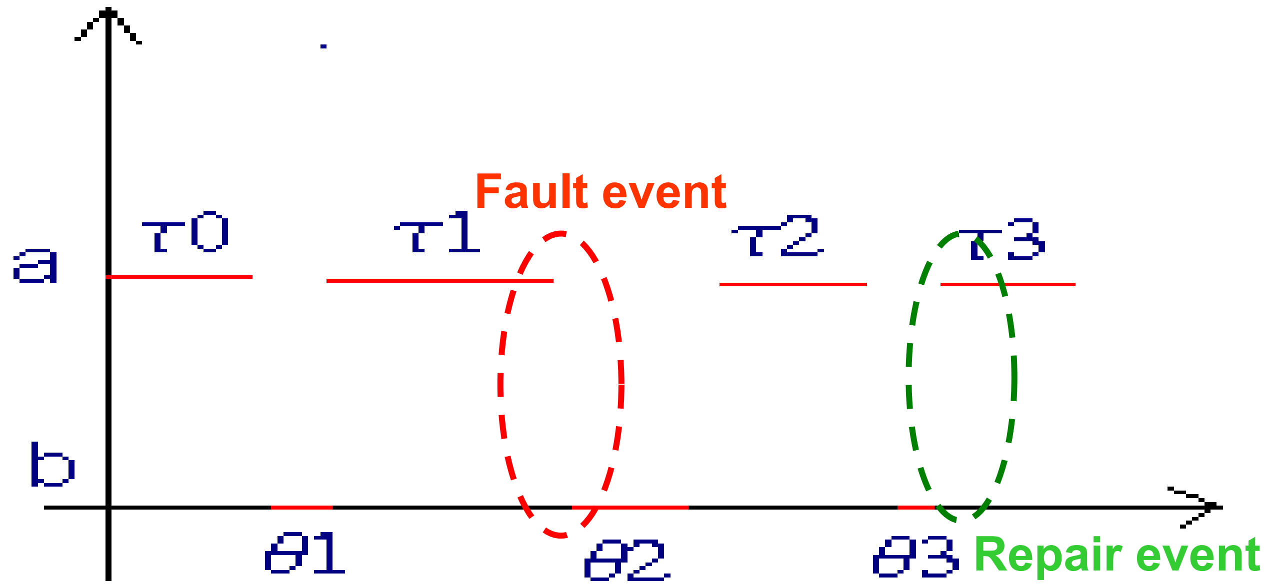
Master degree in

Engineering in Computer Science

Repairable systems

- Maintenance activity is provided in complex system to face faults
- The overall behavior is not related to the faults only
  - *reliability*
- it depends on repair actions also
  - *maintenance*

System dynamic



- **a** = operational state
- **b** = faulty state
- $\boldsymbol{\tau_i}$ = operational time intervals
- $\boldsymbol{\theta_i}$ = NOT operational time interval

System dynamic

- τ_i and θ_i are r.v.
- $F_i(t)$ and $G_i(t)$ are the CDF associated to τ_i and θ_i
- Some simplifying assumptions
  - Each repair is regenerative
  - $F_1(t)=F_2(t)=\ldots=F_i(t)=\ldots=\ldots=F(t)$
  - The repair times have the same distribution
  - $G_1(t)=G_2(t)=\ldots=G_i(t)=\ldots=\ldots=G(t)$
- Are they realistic assumptions?

Distributions

- The CDF of τ is the *unreliability*
  - $F(t)=P\{\tau \le t\}=1-R(t)$
- $G(t)=P\{\theta \le t\}=P\{$*to have a repair within* $[0-t]\}$
- $G(t)$ is called *maintainability*

Some characteristic functions

- Probability distribution function

$$g(t) = \frac{dG(t)}{dt}$$

- Repair rate

$$h_g(t) = \frac{g(t)}{1 - G(t)}$$

- Physical interpretation?

Mean Time to Repair

$$MTTR = E(\Theta) = \int_0^{+\infty} t \cdot g(t)dt = \int_0^{+\infty} t \; dG(t)$$

- Mean time to repair

An example

- Constant repair rate

$$h_g(t) = \mu$$

$$\Downarrow$$

$$G(t) = 1 - e^{-\mu t} \qquad g(t) = \mu e^{-\mu t}$$

$$MTTR = \frac{1}{\mu}$$

- *MTTR* is the inverse ratio of $h_g(t)$

Availability

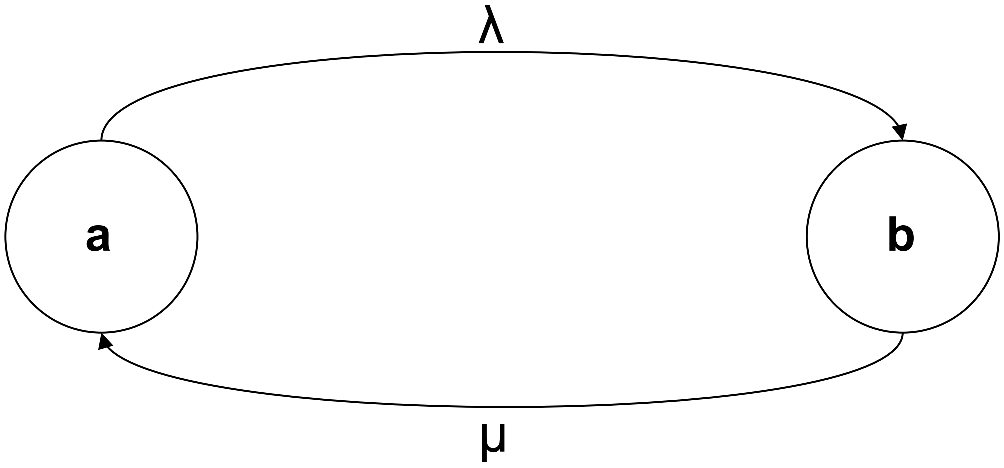
- The *availability* is the quantity that takes into account the faults of an operational system
- $A(t)$ = P{the system is in the state ***a*** at time t}
- When a system is an unrepairable system $A(t)=R(t)$
- $U(t)$ = P{the system is in the state ***b*** at time t} $= 1 - A(t)$

Remarks

- A(t) is meaningful for final users
  - The system operates irrespective of faults
- High availability means that a system have few faults and is quickly repaired

Constant rates

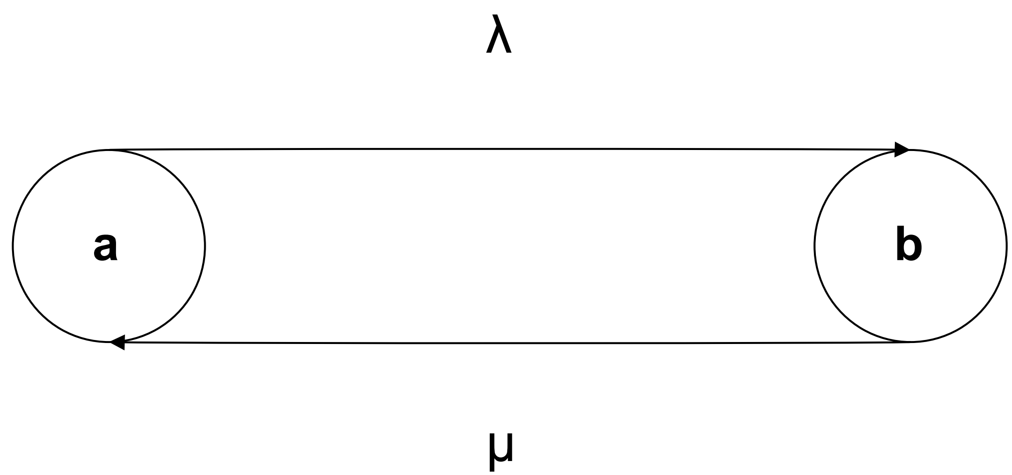
- $h(t)=\lambda$, $h_g(t)=\mu$



Constant rates system

- $P_a(t) = \mathrm{P}\{\text{state} = a \text{ at time } t\}$
- $P_b(t) = \mathrm{P}\{\text{state} = b \text{ at time } t\}$
- $A(t) = P_a(t)$
- $U(t) = P_b(t)$

Analysis



$$\begin{cases} \frac{dP_a(t)}{dt} = -\lambda\ P_a(t) + \mu\ P_b(t) \\ \frac{dP_b(t)}{dt} = \lambda\ P_a(t) - \mu\ P_b(t) \end{cases}$$

$$P_a(0) = 1 \quad \Rightarrow \quad P_b(0) = 0$$

Analysis (cont.)

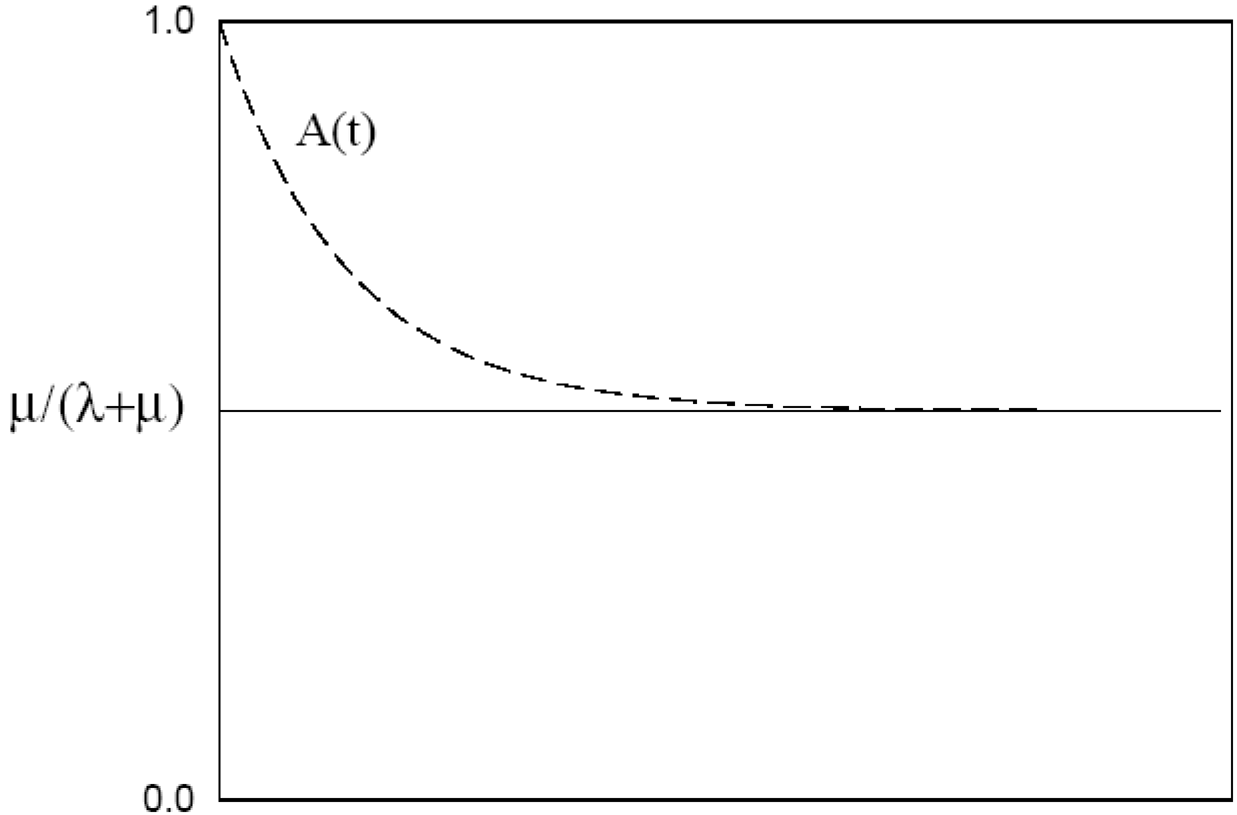
$$A(t) = P_a(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

$$U(t) = P_b(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

$$P_a(t) + P_b(t) = 1, \forall t \in [0, +\infty)$$

$$\lim_{t \to +\infty} A(t) = \frac{\mu}{\lambda + \mu} = A_\infty$$

Availability



Transient availability

- When transient terms is negligible (1%) when $t \approx 4.5/(\lambda+\mu)$
- MTTF>>MTTR => $\lambda \ll \mu$
- E.g.:
  - $\Lambda=10^{-4}h^{-1}$, mean lifetime equal to 10^4h
  - $\mu=10^{-1}h^{-1}$, mean time to repair 10h
  - After t=45h the transitory is negligible

Steady state availability

$$A_{\infty} = \frac{\mu}{\lambda + \mu} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda} + \frac{1}{\mu}} = \frac{MTTF}{MTTF + MTTR}$$

- Percentage of time during which the system correctly operates
- It is a general result

Steady state availability (cont.)

- MUT (*Mean Up Time*)
- MDT (*Mean Down Time*)

$$A_{\infty} = \frac{MUT}{MUT + MDT}$$

$$U_{\infty} = \frac{MDT}{MUT + MDT}$$

Steady state availability (cont.)

- In real systems, MUT>>MDT

$$U_\infty \simeq \frac{MDT}{MUT} \qquad A_\infty \simeq 1 - \frac{MDT}{MUT}$$

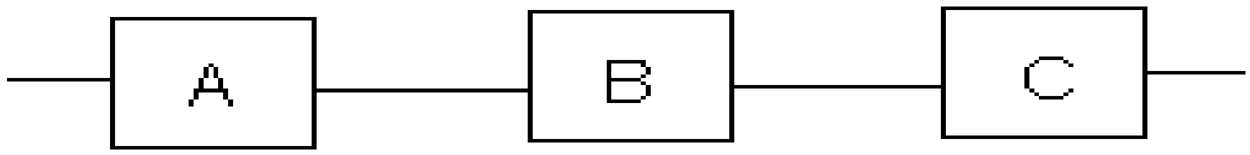
- When faults and repair times are exponentially distributed

$$U_\infty \simeq \frac{\lambda}{\mu} \qquad A_\infty \simeq 1 - \frac{\lambda}{\mu}$$

Complex systems

- As usual, a system could be decomposed into simpler subsystems
- System availability by the knowledge of component availability
- *Statistical independence*

Series system



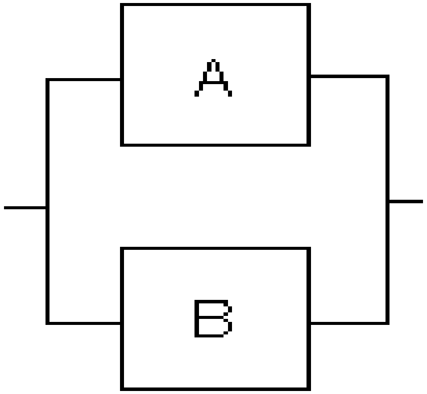
- The system is available iff **all** the subsystems are available
- Statistically independent components

Series system (cont.)

$$A_S = A_1 * A_2 * \ldots * A_n$$

- System availability is less than the availability of each component
- To improve system availability A_S is convenient to improve less available component

Redundant system



- The system is *unavailable* when all the components are unavailable
- $U_S = U_1 * U_2$
- $A_S = (1-U_S) = A_1 + A_2 - A_1 A_2$

Redundant system (cont.)

- System availability of a redundant system is greater than availability of each subsystems
- To improve system availability A_S is convenient to improve the most available component

Example 1

- A plant has a constant production rate of W objects per time unit
- The plant can have faults ($A_\infty = 0.8$)
- Real plant productivity is thus

$$W^* A_\infty$$

- $W = 150$ *units/month*
- Real productivity $= 150^*0.8 = 120$ *units/month*

Example 2

- An organization has N identical terminals; their maintenance is planned
- Each terminal is characterized by the unavailability equal to U_∞
- $N*U_\infty = N_g$ terminals are failed **at the same time**
- $\mu=0.5$ *repairs/hour*
- $U_\infty=0.05$

Example 2 (cont.)

- $N=100 \Rightarrow N_g=5$
  - The time to repair all failed terminal is $N_g/\mu=5/0.5=10$ h
  - One repairman is enough
- $N=150 \Rightarrow N_g=7.5$
  - The time to repair all failed terminal is $N_g/\mu=7.5/0.5=15$ h
  - Two repairmen are necessary

Example 3

- A system design requires R>0.9 at time 1000h and A_∞>0.99
  - Constant failure rate; constant repair rate
  - $R(1000)=\exp(-\lambda*1000)=0.9$
  - $\lambda=-\ln(0.9)/1000=1.05*10^{-5}$ failures/h
  - $\mu/(\mu+\lambda)=0.99 \Rightarrow \mu=1.04*10^{-2}$