UFUG 1504: Honors General Physics II

Chapter 31

Electromagnetic Oscillations and Alternating Current

Summary (1 of 8)

LC Energy Transfer

- In an oscillating LC circuit, instantaneous values of the two forms of energy are

$$U_E = \frac{q^2}{2C} \quad \text{and} \quad U_B = \frac{Li^2}{2} \qquad \textbf{Equation (31-1\&2)}$$

LC Charge and Current Oscillations

- The principle of conservation of energy leads to

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \qquad \textbf{Equation (31-11)}$$

Summary (2 of 8)

- The solution of Equation 31-11 is

$$q = Q\cos(\omega t + \phi)$$

Equation (31-12)

- the angular frequency *v* of the oscillations is

$$\omega = \frac{1}{\sqrt{LC}}.$$

Equation (31-4)

Summary (3 of 8)

Damped Oscillations

- Oscillations in an *LC* circuit are damped when a dissipative element R is also present in the circuit. Then

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

Equation (31-24)

- The solution of this differential equation is

$$q = Qe^{\frac{-Rt}{2L}}\cos(\omega' t + \phi),$$

Equation (31-25)

Summary (4 of 8)

Alternating Currents; Forced Oscillations

- A series *RLC* circuit may be set into forced oscillation at a driving angular frequency by an external alternating emf

$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t.$$

Equation (31-28)

- The current driven in the circuit is

$$i = I \sin\left(\omega_d t - \phi\right)$$

Equation (31-29)

Summary (5 of 8)

Series RLC Circuits

- For a series *RLC* circuit with an alternating external emf and a resulting alternating current,

$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{\mathscr{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d} C\right)^2}}$$

Equation (31-60&63)

- and the phase constant is,

$$\tan\phi = \frac{X_L - X_c}{R}$$

Equation (31-65)

Summary (6 of 8)

- The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$ **Equation (31-61)**

Power

- In a series *RLC* circuit, the average power of the generator is,

$$P_{\text{avg}} = I_{\text{rms}}^2 R = \mathscr{E}_{\text{rms}} I_{\text{rms}} \cos\phi.$$ **Equation (31-71&76)**

Summary (7 of 8)

Transformers

- Primary and secondary voltage in a transformer is related by

$$V_s = V_p \frac{N_s}{N_p}$$

Equation (31-79)

- The currents through the coils,

$$I_s = I_p \frac{N_p}{N_s}$$

Equation (31-80)

Summary (8 of 8)

- The equivalent resistance of the secondary circuit, as seen by the generator, is

$$R_{\mathrm{ep}} = \left(\frac{N_p}{N_s}\right)^2 R, \qquad \textbf{Equation (31-82)}$$

31-1 Electromagnetic Oscillations (5 of 16)

LC Oscillations, Qualitatively

Eight stages in a single cycle of oscillation of a resistance less *LC* circuit. The bar graphs by each figure show the stored magnetic and electrical energies. The magnetic field lines of the inductor and the electric field lines of the capacitor are shown.

(a) Capacitor with maximum charge, no current.

(b) Capacitor discharging, current increasing.

(c) Capacitor fully discharged, current maximum.

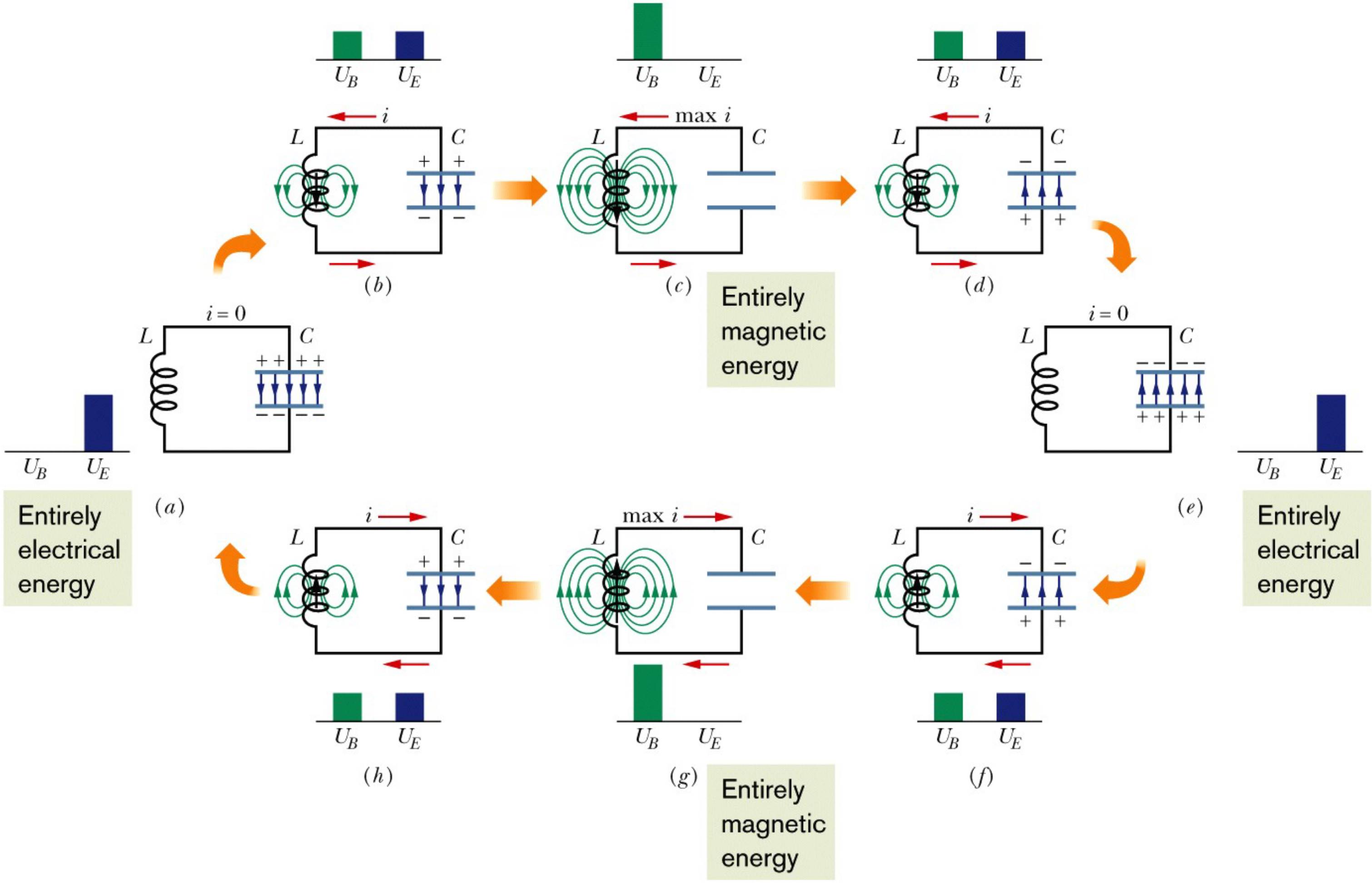
(d) Capacitor charging but with polarity opposite that in (a), current decreasing.

(e) Capacitor with maximum charge having polarity opposite that in (a), no current.

(f) Capacitor discharging, current increasing with direction opposite that in (b).

(g) Capacitor fully discharged, current maximum.

(h) Capacitor charging, current decreasing.



31-1 Electromagnetic Oscillations (7 of 16)

the Figure show succeeding stages of the oscillations in a simple *LC* circuit. The energy stored in the electric field of the **capacitor** at any time is

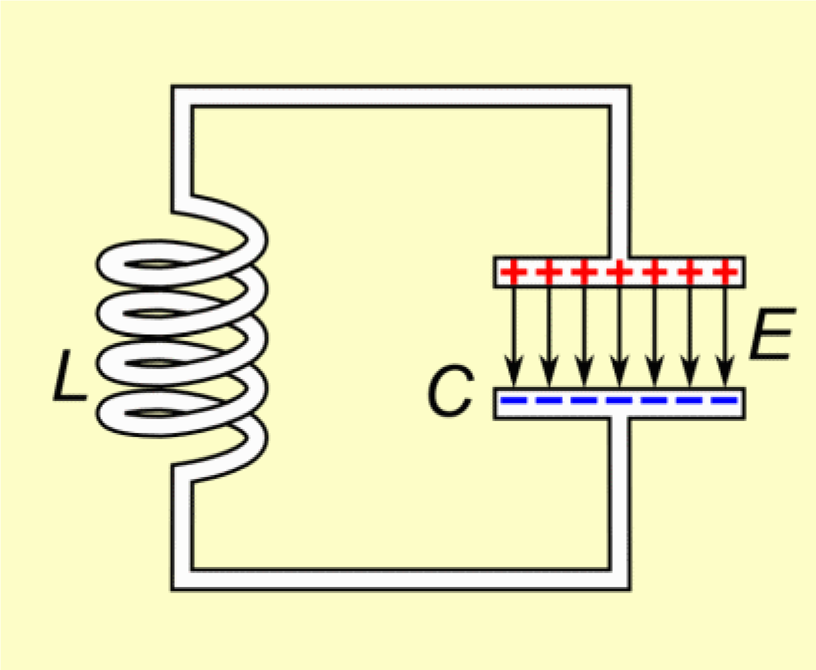
$$U_E = \frac{q^2}{2C}$$

where q is the charge on the capacitor at that time.

The energy stored in the magnetic field of the **inductor** at any time is

$$U_B = \frac{Li^2}{2}$$

where i is the current through the inductor at that time.



The resulting oscillations of the capacitor's electric field and inductor's magnetic field are said to be **electromagnetic oscillations**.(电磁震荡)

31-1 Electromagnetic Oscillations (9 of 16)

Table 31-1 Comparison of the Energy in Two Oscillating Systems

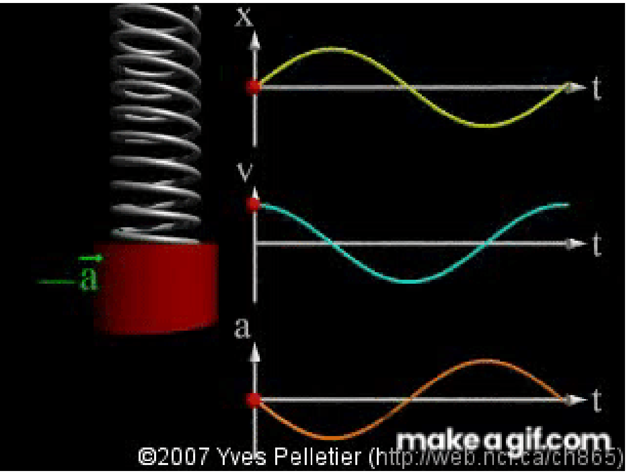


Table 31-1 Comparison of the Energy in Two Oscillating Systems

Block–Spring System		LC Oscillator	
Element	Energy	Element	Energy
Spring	Potential, $\frac{1}{2}kx^2$	Capacitor	Electrical, $\frac{1}{2}(1/C)q^2$
Block	Kinetic, $\frac{1}{2}mv^2$	Inductor	Magnetic, $\frac{1}{2}Li^2$
	$v = dx/dt$		$i = dq/dt$

31-1 Electromagnetic Oscillations (10 of 16)

Table 31-1 Comparison of the Energy in Two Oscillating Systems

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Block	Kinetic, $\frac{1}{2}mv^2$	Inductor	Magnetic, $\frac{1}{2}Li^2$
$v = dx/dt$		$i = dq/dt$	

From the table we can deduce the correspondence between these systems. Thus

$$q \text{ corresponds to } x, \qquad \frac{1}{C} \text{ corresponds to } k,$$

$$i \text{ corresponds to } v, \quad \text{and} \quad L \text{ corresponds to } m.$$

The correspondences listed above suggest that to find the angular frequency of oscillation for an ideal resistanceless) *LC* circuit

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency).}$$

$$\omega = \frac{1}{\sqrt{LC}} \quad (LC \text{ circuit}).$$

15-1 Simple Harmonic Motion (19 of 20)

- **Linear simple harmonic oscillation/linear oscillator** (*F* is proportional to *x* to the first power) gives:

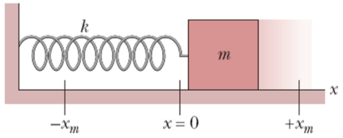
$$F = -(m\omega^2)x. \qquad \vec{F}_s = -k\vec{d}$$

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency).}$$

- **Equation (15-12)**

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period).}$$

- **Equation (15-13)**



31-1 Electromagnetic Oscillations (11 of 16)

LC Oscillator

The total energy U present at any instant in an oscillating LC circuit is given by

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

in which U_B is the energy stored in the magnetic field of the inductor and U_E is the energy stored in the electric field of the capacitor.

Since we have assumed the circuit resistance to be zero, no energy is transferred to thermal energy and U remains constant with time. So $\frac{dU}{dt}=0$

31-1 Electromagnetic Oscillations (12 of 16)

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0.$$

However, $i = \dfrac{dq}{dt}$ and $\dfrac{di}{dt} = \dfrac{d^2q}{dt^2}$. With these substitutions, we get

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

This is the **differential equation (微分方程)** that describes the oscillations of a resistanceless *LC* circuit.

31-1 Electromagnetic Oscillations (13 of 16)

Charge and Current Oscillation

$$L\frac{d^2q}{dt^2}+\frac{1}{C}q=0$$

http://www.drhuang.com/chinese/science/
athematics/handbook/N351/N351.htm

The solution for the differential equation equation that describes the oscillations of a resistanceless *LC* circuit is

$$\boxed{q=Q\cos(\omega t+\phi)}$$

$$\begin{cases}\dfrac{\partial^2 u}{\partial t^2}=a^2\dfrac{\partial^2 u}{\partial x^2}, 0<x<l, t>0\\ u\big|_{t=0}=\varphi(x), \dfrac{\partial u}{\partial t}\Big|_{t=0}=\psi(x), 0\le x\le l\\ u(0,t)=u(l,t)=0,\quad t\ge 0\end{cases}$$

where Q is the amplitude of the charge variation, ω is the angular frequency of the electromagnetic oscillations, and ϕ is the phase constant.

$$X_n(x)=\sin\frac{n\pi}{l}x \; (n=1,2,\cdots)$$

Taking the first derivative of the above Equation with respect to time gives us the current:

$$T_n(t)=A_n\cos\frac{na\pi t}{l}+B_n\sin\frac{na\pi t}{l} \qquad (n=1,2,\cdots)$$

$$\boxed{i=\frac{dq}{dt}=-\omega Q\sin(\omega t+\phi)}$$

31-1 Electromagnetic Oscillations (15 of 16)

Electrical and Magnetic Energy Oscillations

The electrical energy stored in the *LC* circuit at time *t* is

$$q = Q\cos(\omega t + \phi)$$

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi).$$

$$U_E = \frac{q^2}{2C}$$

The magnetic energy is,

$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

$$U_B = \tfrac{1}{2}Li^2 = \tfrac{1}{2}L\omega^2 Q^2 \sin^2(\omega t + \phi).$$

$$\omega = \frac{1}{\sqrt{LC}} \quad (LC \text{ circuit}).$$

31-1 Electromagnetic Oscillations

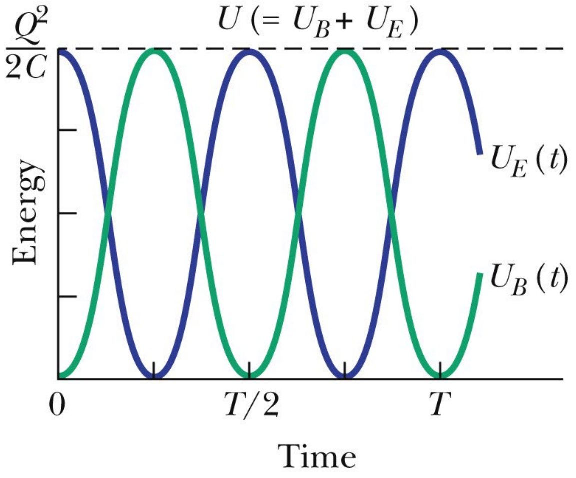
Figure shows plots of $U_E\,(t)$ and $U_B\,(t)$ for the case of $\phi = 0$. Note that

1. The maximum values of U_E and U_B are both $\frac{Q^2}{2C}$.
2. At any instant the sum of U_E and U_B is equal to $\frac{Q^2}{2C}$, a constant.
3. When U_E is maximum, U_B is zero, and conversely.

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi).$$

$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

The electrical and magnetic energies vary but the total is constant.

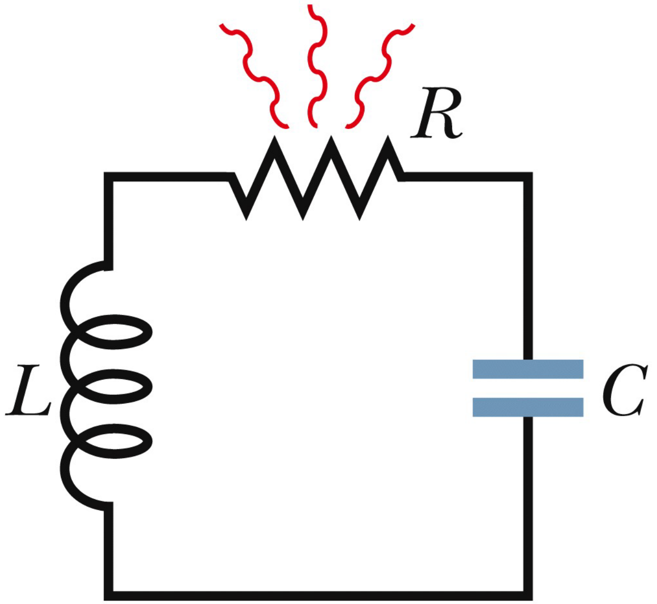


The stored magnetic energy and electrical energy in the RL circuit as a function of time.

31-2 Damped Oscillation in an RLC circuit (3 of 5)

To analyze the oscillations of this circuit, we write an equation for the total electromagnetic energy U in the circuit at any instant. Because the resistance does not store electromagnetic energy, we can write

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}.$$



Now, however, this total energy decreases as energy is transferred to thermal energy. The rate of that transfer is,

$$\frac{dU}{dt} = -i^2 R, \quad \Rightarrow \quad \frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2 R.$$

31-2 Damped Oscillation in an RLC circuit (4 of 5)

where the minus sign indicates that U decreases. By differentiating U with respect to time and then substituting the result we eventually get,

$$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2R.$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

dq/dt for i

d^2q/dt^2t for di/dt

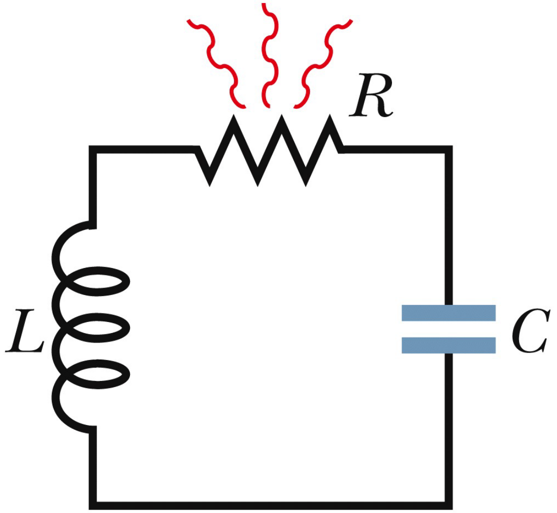
which is the differential equation for **damped oscillations** in an RLC circuit.

31-2 Damped Oscillation in an RLC circuit (5 of 5)

Charge Decay. The solution to above Equation is in which

$$q = Qe^{-\frac{Rt}{2L}}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\omega^2 - \left(\tfrac{R}{2L}\right)^2} \quad \text{and} \quad \omega = \frac{1}{\sqrt{LC}}.$$



A series *RLC* circuit. As the charge contained in the circuit oscillates back and forth through the resistance, electromagnetic energy is dissipated as thermal energy, damping (decreasing the amplitude of) the oscillations.

31-3 Forced Oscillations of Three Simple Circuits (5 of 14)

Why AC (Alternating Current/交流电)?

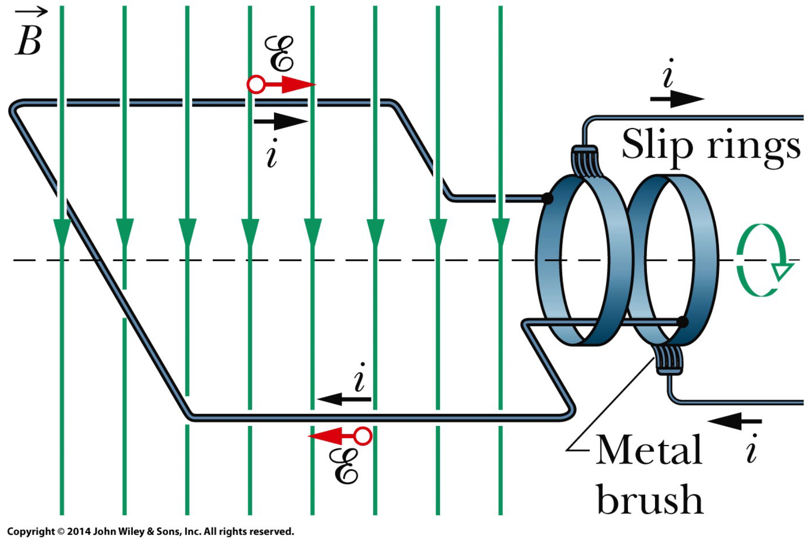
The basic advantage of alternating current is this:

As the current alternates, so does the magnetic field that surrounds the conductor. This makes possible the use of Faraday's law of induction, which, among other things, means that we can step up (increase) or step down (decrease) the magnitude of an alternating potential difference at will, using a device called a transformer, as we shall discuss later.

Moreover, alternating current is more readily adaptable to rotating machinery such as generators and motors than is (nonalternating) direct current.

31-3 Forced Oscillations of Three Simple Circuits (7 of 14)

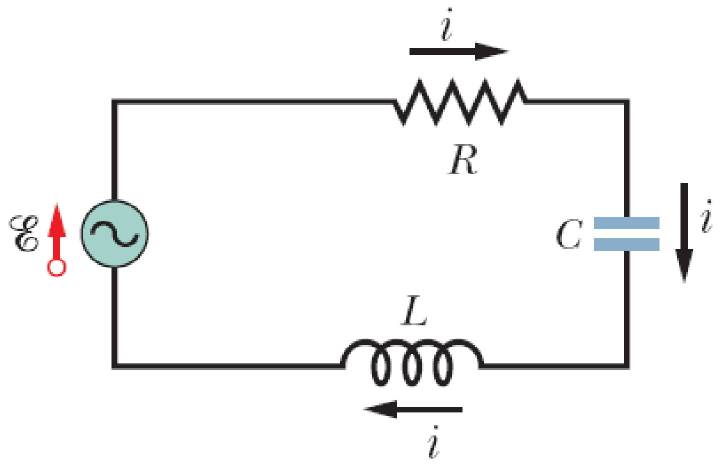
The basic mechanism of an alternating-current generator is a conducting loop rotated in an external magnetic field. In practice, the alternating emf induced in a coil of many turns of wire is made accessible by means of slip rings attached to the rotating loop. Each ring is connected to one end of the loop wire and is electrically connected to the rest of the generator circuit by a conducting brush against which the ring slips as the loop (and ring) rotates.



31-3 Forced Oscillations of Three Simple Circuits (6 of 14)

Forced Oscillations

When the external alternating emf is connected to an *RLC* circuit, the oscillations of charge, potential difference, and current are said to be *driven oscillations* or *forced oscillations* （*强迫震荡*）



Whatever the natural angular frequency ω of a circuit may be, forced oscillations of charge, current, and potential difference in the circuit always occur at the. driving angular frequency ω_d

$$\omega = \frac{1}{\sqrt{LC}}.$$

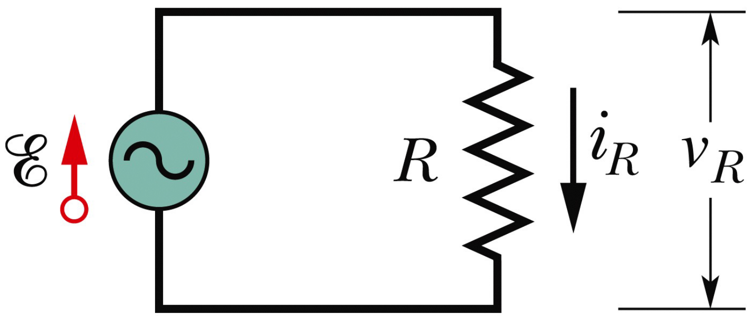
31-3 Forced Oscillations of Three Simple Circuits (8 of 14)

Resistive Load

The alternating potential difference across a resistor has amplitude

$$V_R = I_R R \quad \text{(resistor)}.$$

where V_R and I_R are the amplitudes of alternating current i_R and alternating potential difference v_r across the resistance in the circuit.



A resistor is connected across an alternating-current generator.

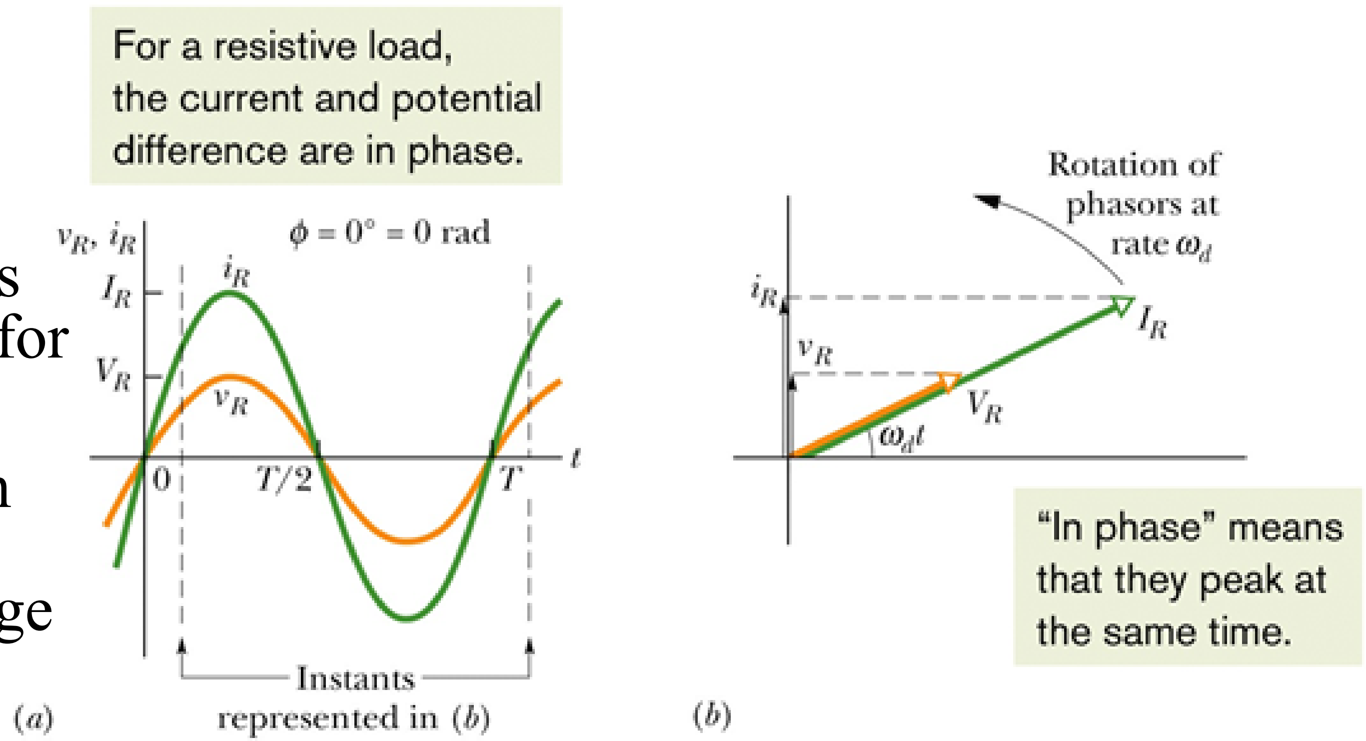
31-3 Forced Oscillations of Three Simple Circuits (9 of 14)

Angular speed: Both current and potential difference phasors rotate counterclockwise about the origin with an angular speed equal to the angular frequency ω_d of v_R and i_R.

Length: The length of each phasor represents the amplitude of the alternating quantity: V_R for the voltage and I_R for the current.

Projection: The projection of each phasor on the vertical axis represents the value of the alternating quantity at time t: v_R for the voltage and i_R for the current.

Rotation angle: The rotation angle of each phasor is equal to the phase of the alternating quantity at time t.



(a) The current i_R and the potential difference v_R across the resistor are plotted on the same graph, both versus time t. They are in phase and complete one cycle in one period T. (b) A phasor diagram shows the same thing as (a).

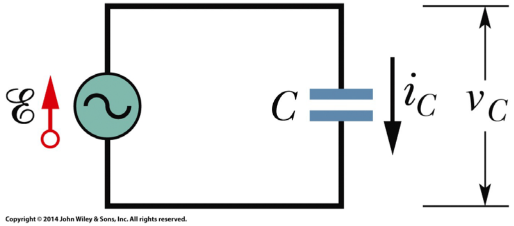
31-3 Forced Oscillations of Three Simple Circuits (13 of 14)

Capacitive Load (电容负载)

The **capacitive reactance (容抗)** of an capacitor is defined as

$$X_C = \frac{1}{\omega_d C}$$

Its value depends not only on the inductance but also on the driving angular frequency ω_d.

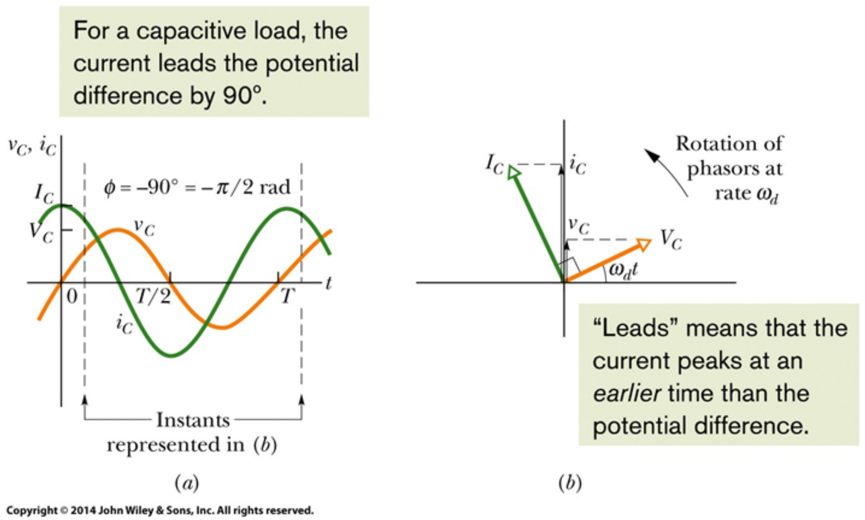


The voltage amplitude and current amplitude are related by

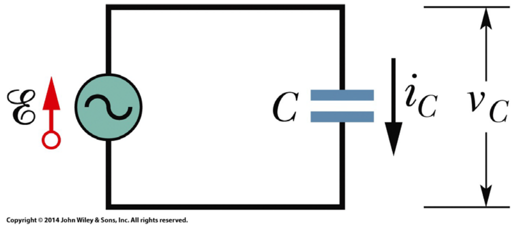
$$V_C = I_C X_C$$

31-3 Forced Oscillations of Three Simple Circuits (12 of 14)

A capacitor is connected across an alternating-current generator.



(a)The current in the capacitor leads the voltage by 90° (= π/2 rad). (b) A phasor diagram shows the same thing.



In the phasor diagram we see that i_C leads v_C, which means that, if you monitored the current i_C and the potential difference v_C in the circuit above, you would find that i_C reaches its maximum before v_C does, by one-quarter cycle.

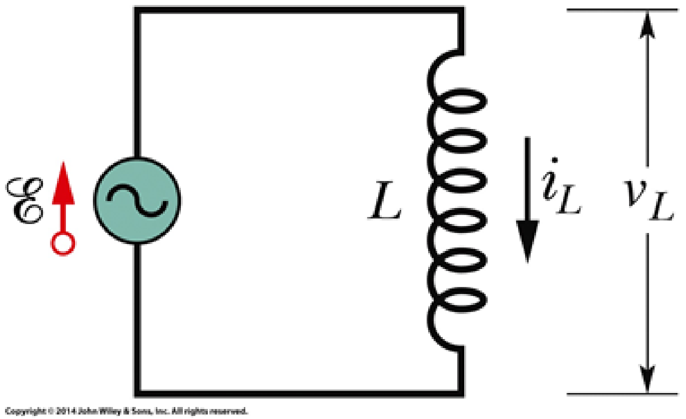
31-3 Forced Oscillations of Three Simple Circuits (11 of 14)

Inductive Load （电感负载）

The **inductive reactance (感性电抗/感抗)**of an inductor is defined as

$$X_L = \omega_d L$$

Its value depends not only on the inductance but also on the driving angular frequency ω_d.

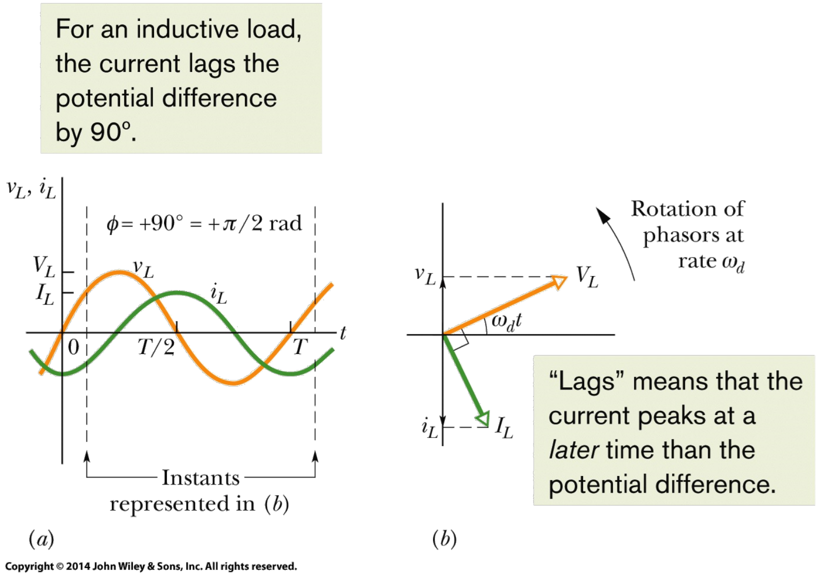


The voltage amplitude and current amplitude are related by

$$V_L = I_L X_L$$

31-3 Forced Oscillations of Three Simple Circuits (14 of 14)

An inductor is connected across an alternating-current generator.



(a)The current in the inductor lags the voltage by 90° (= π/2 rad). (b) A phasor diagram shows the same thing.

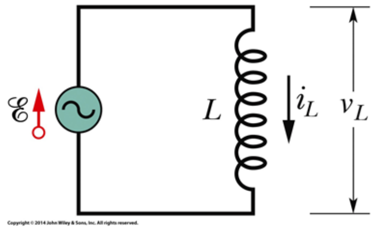


Fig. (left), shows that the quantities i_L and v_L are 90° out of phase. In this case, however, i_L lags v_L; that is, monitoring the current i_L and the potential difference v_L in the circuit of Fig. (top) shows that i_L reaches its maximum value after v_L does, by one-quarter cycle.

31-3 Forced Oscillations of Three Simple Circuits (14 of 14)

Table 31-2 Phase and Amplitude Relations for Alternating Currents and Voltages

Circuit Element	Symbol	Resistance or Reactance	Phase of the Current	Phase Constant (or Angle) ϕ	Amplitude Relation
Resistor	R	R	In phase with v_R	$0°$ (= 0 rad)	$V_R = I_R R$
Capacitor	C	$X_C = 1/\omega_d C$	Leads v_C by $90°$ (= $\pi/2$ rad)	$-90°$ (= $-\pi/2$ rad)	$V_C = I_C X_C$
Inductor	L	$X_L = \omega_d L$	Lags v_L by $90°$ (= $\pi/2$ rad)	$+90°$ (= $+\pi/2$ rad)	$V_L = I_L X_L$

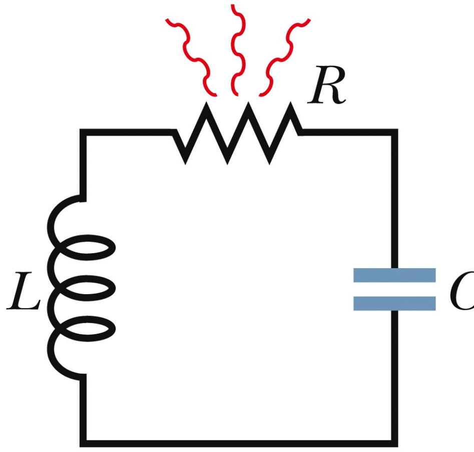
31-4 The Series RLC Circuits (4 of 7)

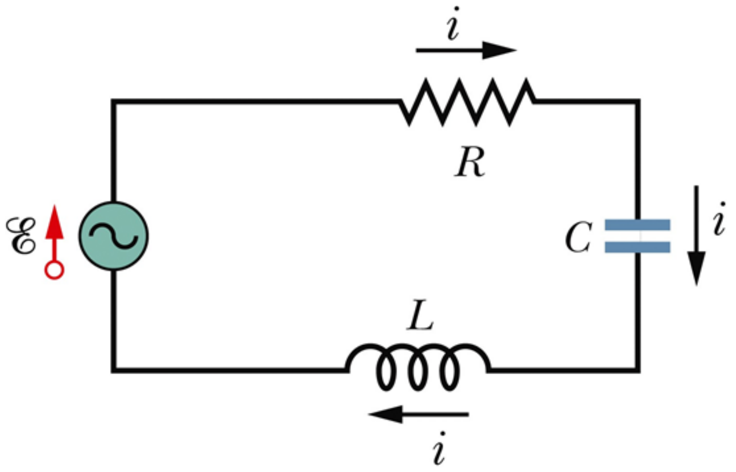
For a series *RLC* circuit with an external emf given by

$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t$$

The current is given by

$$i = I \sin(\omega_d t - \phi)$$





Series RLC circuit with an external emf

31-4 The Series RLC Circuits (5 of 7)

the current amplitude is given by

$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}.$$

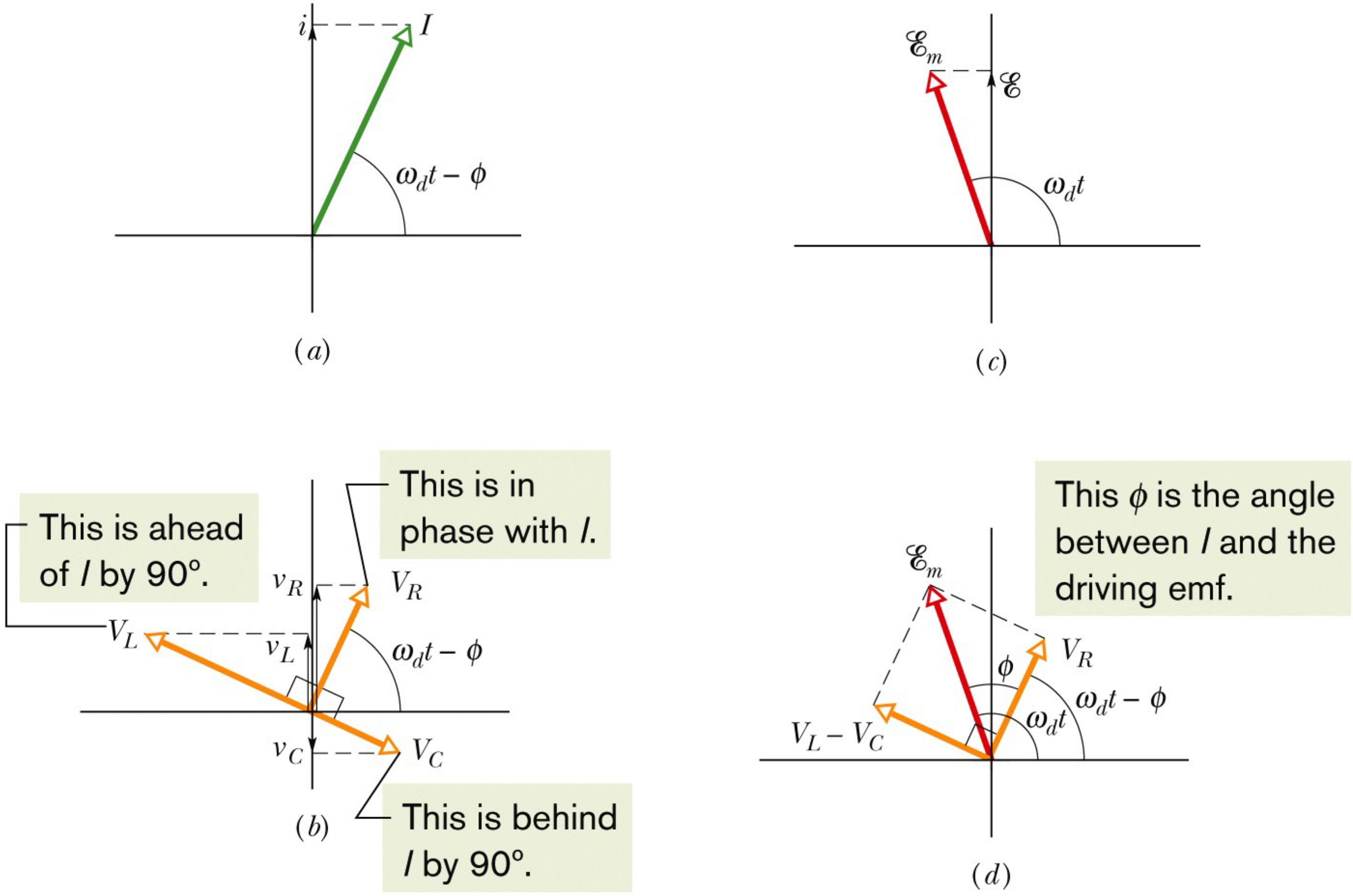
The denominator in the above equation is called the impedance *Z (阻抗)* of the circuit for the driving angular frequency ω_d.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

If we substitute the value of X_L and X_C in the equation for current (I), the equation becomes:

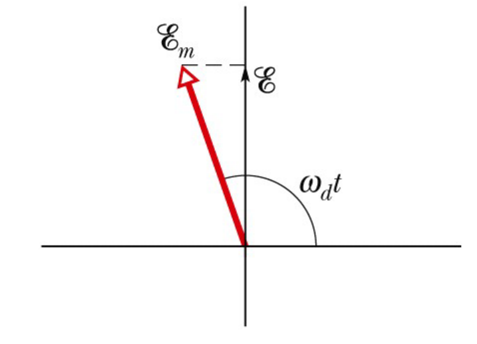
$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$

31-4 The Series RLC Circuits (6 of 7)

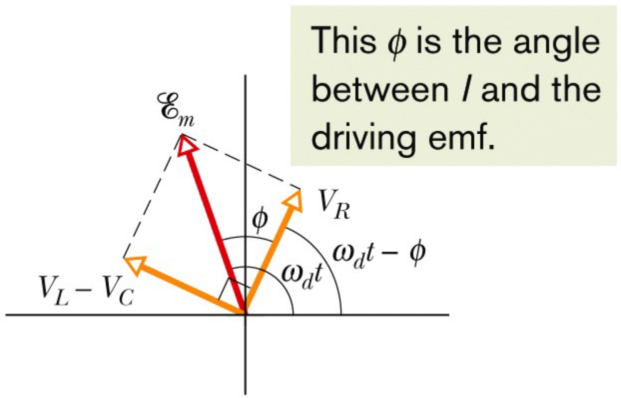


(*a*) A phasor representing the alternating current in the driven *RLC* circuit at time *t*. The amplitude *I*, the instantaneous value *i*, and the phase $(\omega_d t - \phi)$ are shown.
(*b*) Phasors representing the voltages across the inductor, resistor, and capacitor, oriented with respect to the current phasor in (*a*).
(*c*) A phasor representing the alternating emf that drives the current of (*a*).
(*d*) The emf phasor is equal to the vector sum of the three voltage phasors of (*b*). Here, voltage phasors V_L and V_C have been added vectorially to yield their net phasor $(V_L - V_C)$.

31-4 The Series RLC Circuits (7 of 7)



(c)



(d)

From the right-hand phasor triangle in Figure (d) we can write

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}, \quad \Longrightarrow \quad \tan\phi = \frac{X_L - X_C}{R}$$ **Phase Constant**

The current amplitude I is maximum when the driving ω angular frequency ω_d equals the natural angular frequency of the circuit, a condition known as **resonance**. Then $X_C = X_L,$ $\phi = 0,$ and the current is in phase with the emf.

$$\omega_d = \omega = \frac{1}{\sqrt{LC}} \quad (\text{resonance}).$$

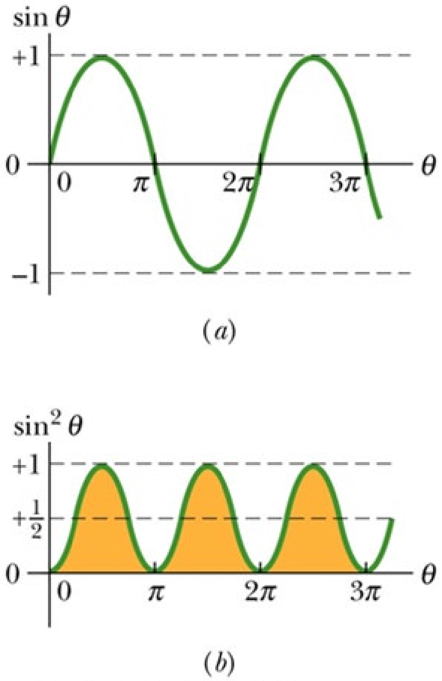
31-5 Power in Alternating-Current Circuits (3 of 5)

The instantaneous rate at which energy is dissipated in the resistor can be written as

$$P = i^2R = \left[I\sin(\omega_d t - \phi)\right]^2 R = I^2R\sin^2(\omega_d t - \phi).$$

Over one complete cycle, the average value of $\sin\theta$, where θ is any variable, is zero (Fig. a) but the average value of $\sin^2\theta$ is $\frac{1}{2}$ (Fig. b). Thus the power

31-5 Power in Alternating-Current Circuits (4 of 5)



$$P = I^2 R \sin^2(\omega_d t - \phi).$$

(a) A plot of $\sin\theta$ versus θ. The average value over one cycle is zero.

(b) A plot of $\sin^2\theta$ versus θ. The average value over one cycle is $\frac{1}{2}$.

31-5 Power in Alternating-Current Circuits (5 of 5)

The quantity $\frac{I}{\sqrt{2}}$ is called the **root-mean-square**, or rms, （均方根） value of the current *i*:

$$I_{\text{rms}} = \frac{I}{\sqrt{2}} \quad \Rightarrow \quad P_{\text{avg}} = I_{\text{rms}}^2 R$$

We can also define rms values of voltages and emfs for alternating-current circuits:

$$V_{\text{rms}} = \frac{V}{\sqrt{2}} \quad \text{and} \quad \mathscr{E}_{\text{rms}} = \frac{\mathscr{E}_m}{\sqrt{2}}$$

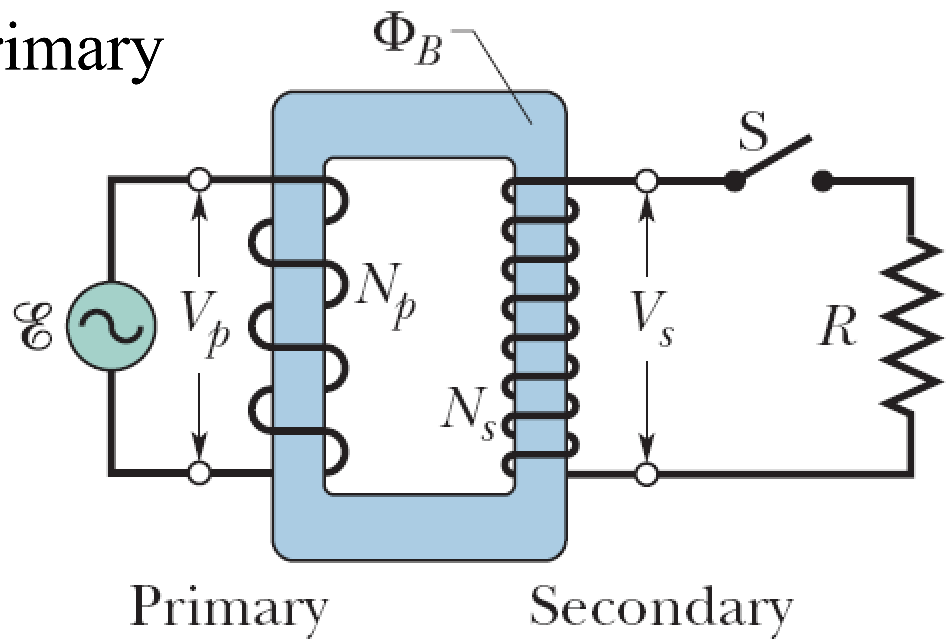
In a series *RLC* circuit, the average power P_{avg} of the generator is equal to the production rate of thermal energy in the resistor:

$$P_{\text{avg}} = \mathscr{E}_{\text{rms}} I_{\text{rms}} \cos\phi$$

31-6 Transformers (4 of 6)

A transformer (assumed to be ideal) is an iron core on which are wound a primary coil (初始线圈)of N_p turns and a secondary coil （二级线圈） of N_s turns. If the primary coil is connected across an alternating-current generator, the primary and secondary voltages are related by

$$V_s = V_p \frac{N_s}{N_p}$$



31-6 Transformers (5 of 6)

Energy Transfers. The rate at which the generator transfers energy to the primary is equal to I_pV_p. The rate at which the primary then transfers energy to the secondary (via the alternating magnetic field linking the two coils) is I_sV_s. Because we assume that no energy is lost along the way, conservation of energy requires that

$$I_pV_p = I_sV_s \quad\Longrightarrow\quad I_s = I_p\frac{N_p}{N_s}$$

The equivalent resistance of the secondary circuit, as seen by the generator, is

$$R_{\text{ep}} = \left(\frac{N_p}{N_s}\right)^2 R.$$

Summary (1 of 8)

LC Energy Transfer

- In an oscillating LC circuit, instantaneous values of the two forms of energy are

$$U_E = \frac{q^2}{2C} \quad \text{and} \quad U_B = \frac{Li^2}{2}$$

Equation (31-1&2)

LC Charge and Current Oscillations

- The principle of conservation of energy leads to

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

Equation (31-11)

Summary (2 of 8)

- The solution of Equation 31-11 is

$$q = Q\cos(\omega t + \phi)$$

Equation (31-12)

- the angular frequency *v* of the oscillations is

$$\omega = \frac{1}{\sqrt{LC}}.$$

Equation (31-4)

Summary (3 of 8)

Damped Oscillations

- Oscillations in an *LC* circuit are damped when a dissipative element R is also present in the circuit. Then

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=0$$

Equation (31-24)

- The solution of this differential equation is

$$q=Qe^{\frac{-Rt}{2L}}\cos(\omega' t+\phi),$$

Equation (31-25)

Summary (4 of 8)

Alternating Currents; Forced Oscillations

- A series *RLC* circuit may be set into forced oscillation at a driving angular frequency by an external alternating emf

$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t.$$

Equation (31-28)

- The current driven in the circuit is

$$i = I \sin\left(\omega_d t - \phi\right)$$

Equation (31-29)

Summary (5 of 8)

Series RLC Circuits

- For a series *RLC* circuit with an alternating external emf and a resulting alternating current,

$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{\mathscr{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d} C\right)^2}}$$

Equation (31-60&63)

- and the phase constant is,

$$\tan\phi = \frac{X_L - X_c}{R}$$

Equation (31-65)

Summary (6 of 8)

- The impedance is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Equation (31-61)

Power

- In a series *RLC* circuit, the average power of the generator is,

$$P_{\text{avg}} = I_{\text{rms}}^2 R = \mathscr{E}_{\text{rms}} I_{\text{rms}} \cos\phi.$$

Equation (31-71&76)

Summary (7 of 8)

Transformers

- Primary and secondary voltage in a transformer is related by

$$V_s = V_p \frac{N_s}{N_p}$$

Equation (31-79)

- The currents through the coils,

$$I_s = I_p \frac{N_p}{N_s}$$

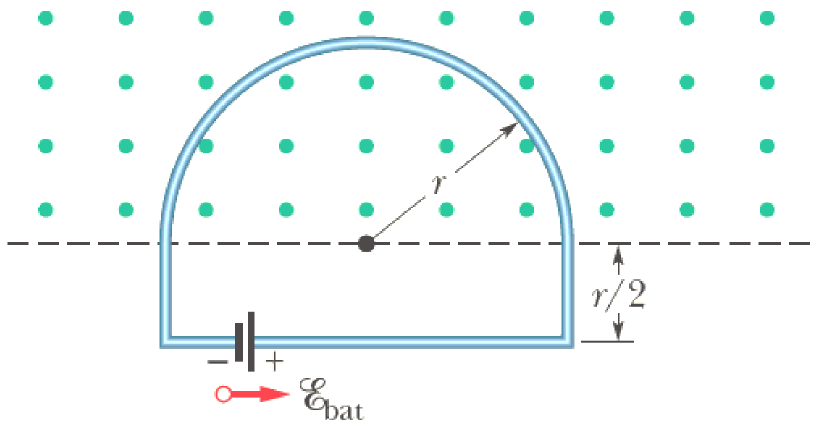
Equation (31-80)

Summary (8 of 8)

- The equivalent resistance of the secondary circuit, as seen by the generator, is

$$R_{\text{ep}} = \left(\frac{N_p}{N_s}\right)^2 R, \qquad \textbf{Equation (31-82)}$$

Figure 30-6 shows a conducting loop consisting of a half-circle of radius $r = 0.20$ m and three straight sections. The half-circle lies in a uniform magnetic field $\vec{B}$ that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf $\mathscr{E}_{bat} = 2.0$ V is connected to the loop. The resistance of the loop is $2.0\ \Omega$.



(a) What are the magnitude and direction of the emf $\mathscr{E}_{ind}$ induced around the loop by field $\vec{B}$ at $t = 10$ s?

(b) What is the current in the loop at $t = 10$ s?

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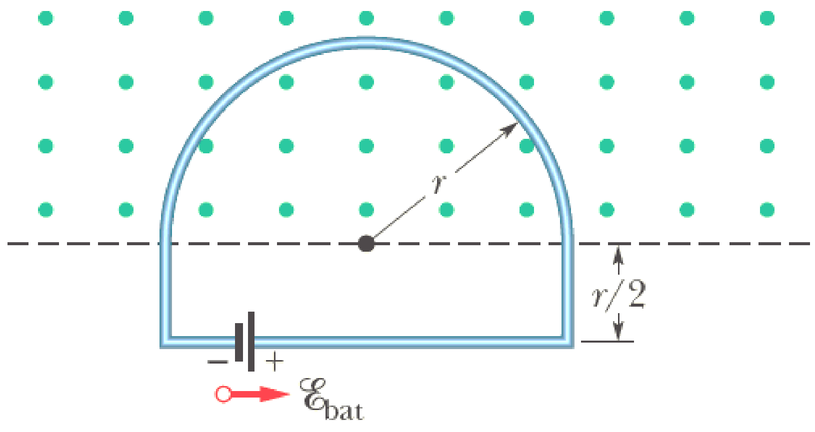


(a) What are the magnitude and direction of the emf $\mathscr{E}_{ind}$ induced around the loop by field $\vec{B}$ at $t = 10$ s?

(b) What is the current in the loop at $t = 10$ s?

$$\mathscr{E}_{ind} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\,\frac{dB}{dt}.$$

$$\mathscr{E}_{ind} = A\frac{dB}{dt} = \frac{\pi r^2}{2}\frac{d}{dt}(4.0t^2 + 2.0t + 3.0)$$

$$= \frac{\pi r^2}{2}(8.0t + 2.0).$$

At $t = 10$ s, then,

$$\mathscr{E}_{ind} = \frac{\pi\,(0.20\text{ m})^2}{2}[8.0(10) + 2.0]$$

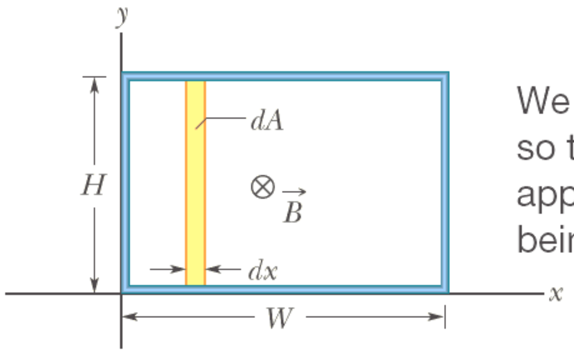
$$= 5.152\text{ V} \approx 5.2\text{ V}.$$

$$i = \frac{\mathscr{E}_{net}}{R} = \frac{\mathscr{E}_{ind} - \mathscr{E}_{bat}}{R}$$

$$= \frac{5.152\text{ V} - 2.0\text{ V}}{2.0\ \Omega} = 1.58\text{ A} \approx 1.6\text{ A}.$$

Figure 30-7 shows a rectangular loop of wire immersed in a nonuniform and varying magnetic field $\vec{B}$ that is perpendicular to and directed into the page. The field's magnitude is given by $B = 4t^2x^2$, with B in teslas, t in seconds, and x in meters. (Note that the function depends on *both* time and position.) The loop has width $W = 3.0$ m and height $H = 2.0$ m. What are the magnitude and direction of the induced emf $\mathscr{E}$ around the loop at $t = 0.10$ s?

If the field varies with position, we must integrate to get the flux through the loop.



We start with a strip so thin that we can approximate the field as being uniform within it.

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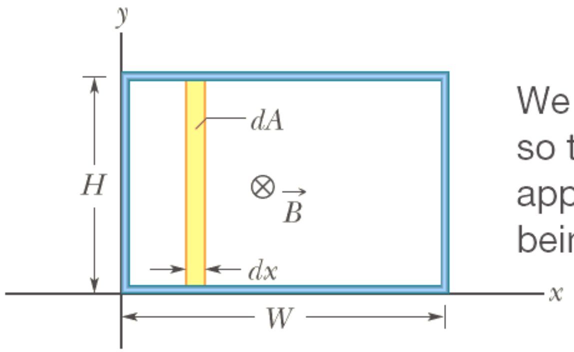


We start with a strip so thin that we can approximate the field as being uniform within it.

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\,dA = \int BH\,dx = \int 4t^2x^2H\,dx.$$

Treating t as a constant for this integration and inserting the integration limits $x = 0$ and $x = 3.0$ m, we obtain

$$\Phi_B = 4t^2H \int_0^{3.0} x^2\,dx = 4t^2H \left[\frac{x^3}{3}\right]_0^{3.0} = 72t^2,$$

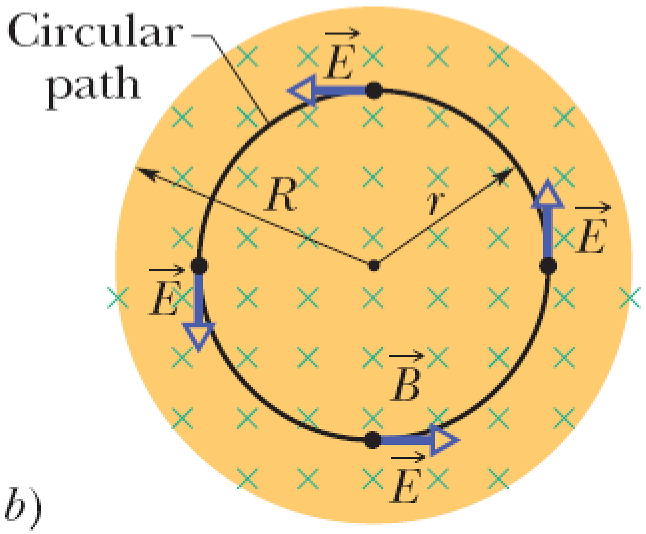
$$\mathscr{E} = \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t,$$

in which $\mathscr{E}$ is in volts. At $t = 0.10$ s,

$$\mathscr{E} = (144\text{ V/s})(0.10\text{ s}) \approx 14\text{ V}.$$

In Fig. 30-11b, take $R = 8.5$ cm and $dB/dt = 0.13$ T/s.

(a) Find an expression for the magnitude E of the induced electric field at points within the magnetic field, at radius r from the center of the magnetic field. Evaluate the expression for $r = 5.2$ cm.



(b)

In Fig. 30-11b, take $R = 8.5$ cm and $dB/dt = 0.13$ T/s.

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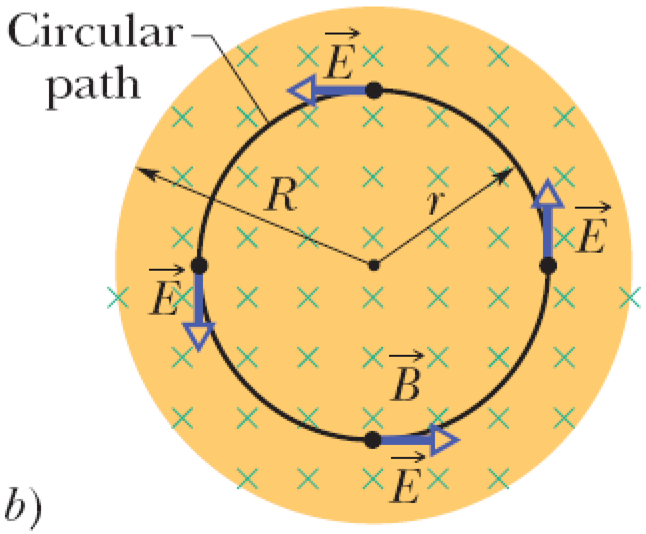


(b)

$$\oint \vec{E} \cdot d\vec{s} = \oint E\, ds = E \oint ds = E(2\pi r).$$

$$\Phi_B = BA = B(\pi r^2).$$

$$E(2\pi r) = (\pi r^2)\frac{dB}{dt}$$

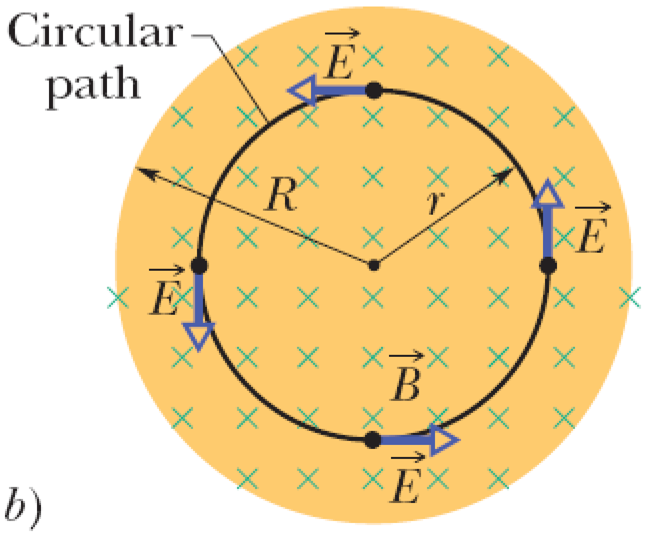
$$E = \frac{r}{2}\frac{dB}{dt}.$$

$$E = \frac{(5.2 \times 10^{-2}\ \text{m})}{2}(0.13\ \text{T/s})$$
$$= 0.0034\ \text{V/m} = 3.4\ \text{mV/m}.$$

In Fig. 30-11b, take $R = 8.5$ cm and $dB/dt = 0.13$ T/s.

(a) Find an expression for the magnitude E of the induced electric field at points within the magnetic field, at radius r from the center of the magnetic field. Evaluate the expression for $r = 5.2$ cm.

(b) Find an expression for the magnitude E of the induced electric field at points that are outside the magnetic field, at radius r from the center of the magnetic field. Evaluate the expression for $r = 12.5$ cm.



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In Fig. 30-11b, take $R = 8.5$ cm and $dB/dt = 0.13$ T/s.

(a) Find an expression for the magnitude E of the induced electric field at points within the magnetic field, at radius r from the center of the magnetic field. Evaluate the expression for $r = 5.2$ cm.

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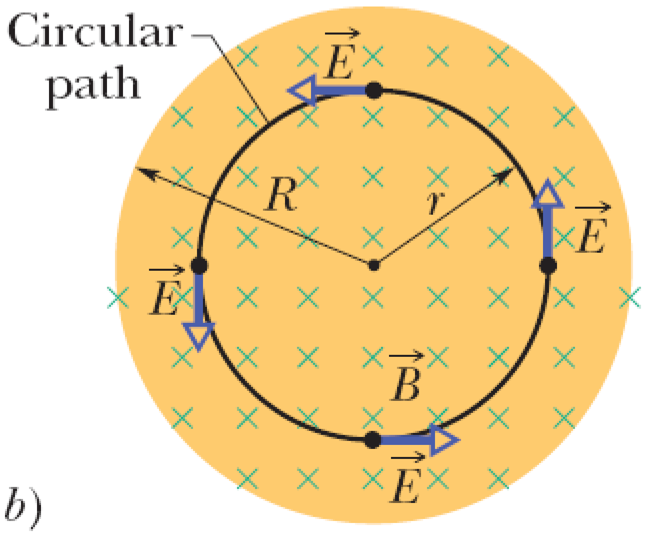


(b)

$$\oint \vec{E} \cdot d\vec{s} = \oint E\, ds = E \oint ds = E(2\pi r).$$

$$\Phi_B = BA = B(\pi R^2)$$

$$E = \frac{R^2}{2r}\frac{dB}{dt}.$$

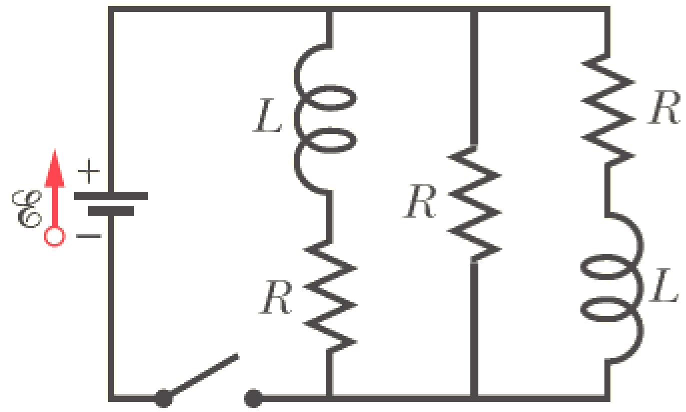
$$E = \frac{(8.5 \times 10^{-2}\ \mathrm{m})^2}{(2)(12.5 \times 10^{-2}\ \mathrm{m})}(0.13\ \mathrm{T/s})$$

$$= 3.8 \times 10^{-3}\ \mathrm{V/m} = 3.8\ \mathrm{mV/m}.$$

Figure 30-18*a* shows a circuit that contains three identical resistors with resistance $R = 9.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH, and an ideal battery with emf $\mathscr{E} = 18$ V.

(a) What is the current i through the battery just after the switch is closed?

(b) What is the current i through the battery long after the switch has been closed?



(*a*)

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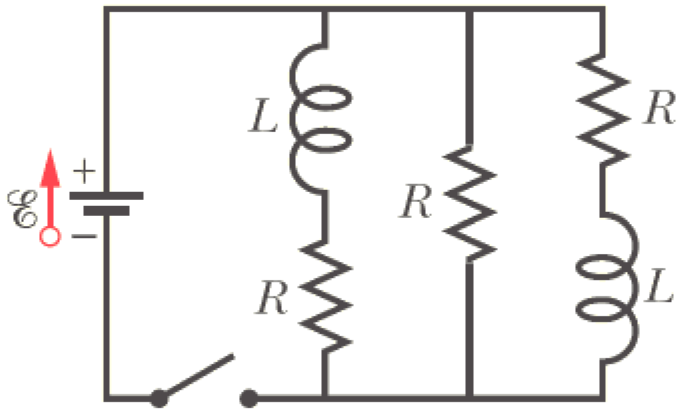
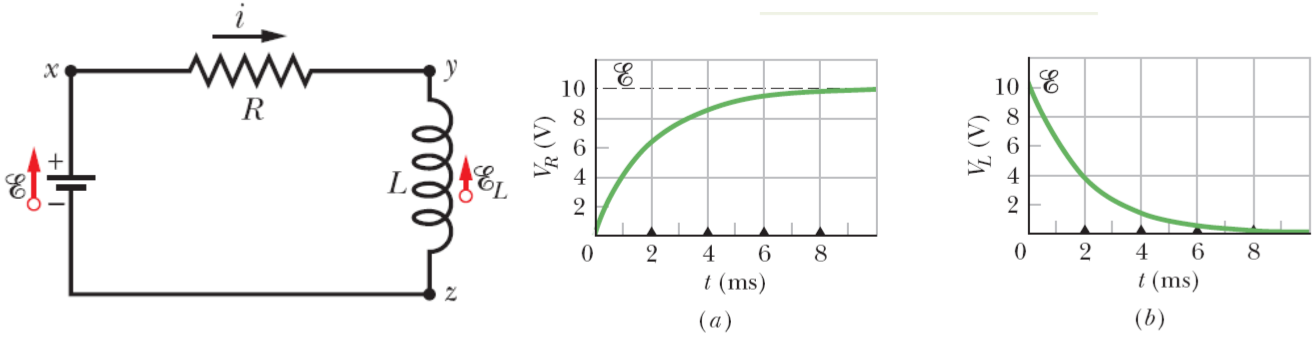
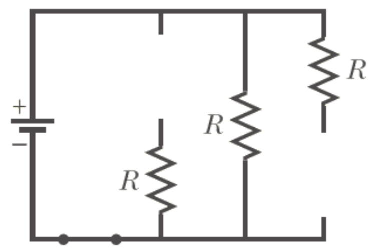
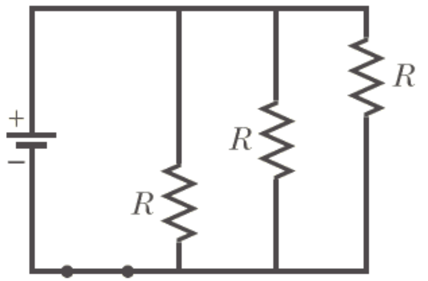


(*a*)





(*b*) Initially, an inductor acts like broken wire.



(*c*) Long later, it acts like ordinary wire.

$$i = \frac{\mathscr{E}}{R} = \frac{18\ \text{V}}{9.0\ \Omega} = 2.0\ \text{A}.$$

$$i = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{18\ \text{V}}{3.0\ \Omega} = 6.0\ \text{A}$$

Figure 30-18*a* shows a circuit that contains three identical resistors with resistance $R = 9.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH, and an ideal battery with emf $\mathscr{E} = 18$ V.

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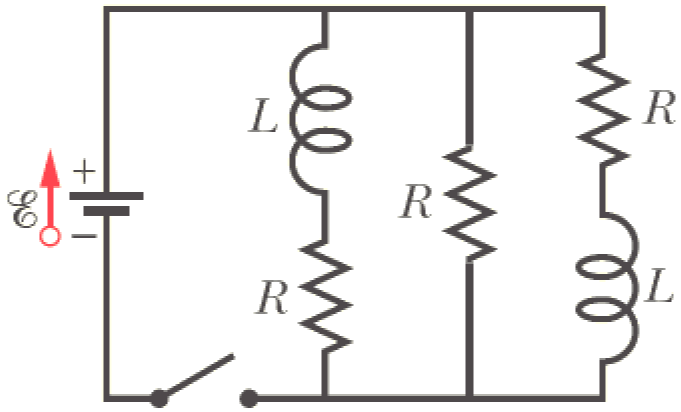
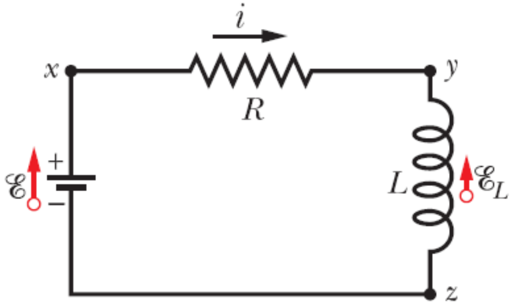
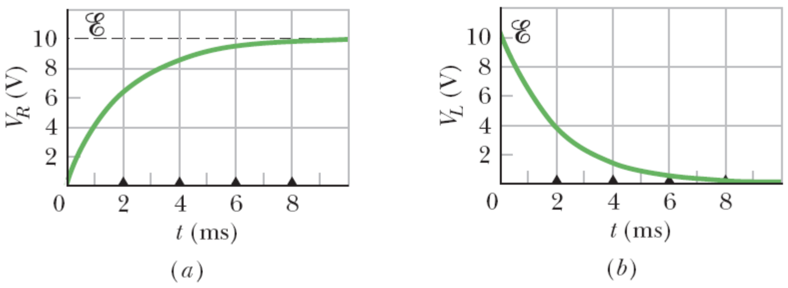
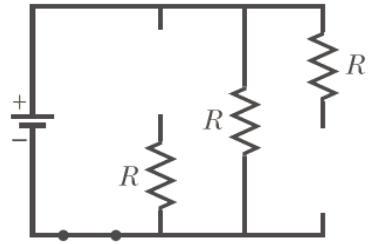


(*a*)







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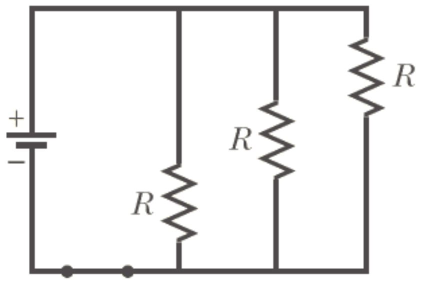
Initially, an inductor acts like broken wire.



(*c*)

Long later, it acts like ordinary wire.

$$i = \frac{\mathscr{E}}{R} = \frac{18\ \text{V}}{9.0\ \Omega} = 2.0\ \text{A}.$$

$$i = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{18\ \text{V}}{3.0\ \Omega} = 6.0\ \text{A}$$

A solenoid has an inductance of 53 mH and a resistance of 0.37 Ω. If the solenoid is connected to a battery, how long will the current take to reach half its final equilibrium value? (This is a *real solenoid* because we are considering its small, but nonzero, internal resistance.)

A solenoid has an inductance of 53 mH and a resistance of 0.37 Ω. If the solenoid is connected to a battery, how long will the current take to reach half its final equilibrium value? (This is a *real solenoid* because we are considering its small, but nonzero, internal resistance.)

$$\frac{1}{2}\frac{\mathscr{E}}{R} = \frac{\mathscr{E}}{R}\left(1 - e^{-t_0/\tau_L}\right)$$

$$t_0 = \tau_L \ln 2 = \frac{L}{R} \ln 2 = \frac{53 \times 10^{-3}\ \mathrm{H}}{0.37\ \Omega} \ln 2$$

$$= 0.10\ \mathrm{s}.$$

A coil has an inductance of 53 mH and a resistance of 0.35 Ω.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

A coil has an inductance of 53 mH and a resistance of 0.35 Ω.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

$$i_\infty = \frac{\mathscr{E}}{R} = \frac{12\ \text{V}}{0.35\ \Omega} = 34.3\ \text{A}.$$

$$\begin{aligned} U_{B\infty} &= \tfrac{1}{2}Li_\infty^2 = (\tfrac{1}{2})(53 \times 10^{-3}\ \text{H})(34.3\ \text{A})^2 \\ &= 31\ \text{J}. \end{aligned}$$

(

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

$$U_B = \tfrac{1}{2} U_{B\infty}$$

$$\tfrac{1}{2} L i^2 = \left(\tfrac{1}{2}\right) \tfrac{1}{2} L i_\infty^2$$

$$i = \left(\frac{1}{\sqrt{2}}\right) i_\infty$$

$$\frac{\mathscr{E}}{R}\left(1 - e^{-t/\tau_L}\right) = \frac{\mathscr{E}}{\sqrt{2}R}.$$

$$e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = 0.293, \qquad t \approx 1.2\tau_L.$$