CHAPTER 21

# Coulomb's Law

## 21-1 COULOMB'S LAW

### Learning Objectives

*After reading this module, you should be able to . . .*

**21.01** Distinguish between being electrically neutral, negatively charged, and positively charged and identify excess charge.

**21.02** Distinguish between conductors, nonconductors (insulators), semiconductors, and superconductors.

**21.03** Describe the electrical properties of the particles inside an atom.

**21.04** Identify conduction electrons and explain their role in making a conducting object negatively or positively charged.

**21.05** Identify what is meant by "electrically isolated" and by "grounding."

**21.06** Explain how a charged object can set up induced charge in a second object.

**21.07** Identify that charges with the same electrical sign repel each other and those with opposite electrical signs attract each other.

**21.08** For either of the particles in a pair of charged particles, draw a free-body diagram, showing the electrostatic force (Coulomb force) on it and anchoring the tail of the force vector on that particle.

**21.09** For either of the particles in a pair of charged particles, apply Coulomb's law to relate the magnitude of the electrostatic force, the charge magnitudes of the particles, and the separation between the particles.

**21.10** Identify that Coulomb's law applies only to (point-like) particles and objects that can be treated as particles.

**21.11** If more than one force acts on a particle, find the net force by adding all the forces as vectors, not scalars.

**21.12** Identify that a shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated as a particle at the shell's center.

**21.13** Identify that if a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

**21.14** Identify that if excess charge is put on a spherical conductor, it spreads out uniformly over the external surface area.

**21.15** Identify that if two identical spherical conductors touch or are connected by conducting wire, any excess charge will be shared equally.

**21.16** Identify that a nonconducting object can have any given distribution of charge, including charge at interior points.

**21.17** Identify current as the rate at which charge moves through a point.

**21.18** For current through a point, apply the relationship between the current, a time interval, and the amount of charge that moves through the point in that time interval.

### Key Ideas

- The strength of a particle's electrical interaction with objects around it depends on its electric charge (usually represented as $q$), which can be either positive or negative. Particles with the same sign of charge repel each other, and particles with opposite signs of charge attract each other.

- An object with equal amounts of the two kinds of charge is electrically neutral, whereas one with an imbalance is electrically charged and has an excess charge.

- Conductors are materials in which a significant number of electrons are free to move. The charged particles in nonconductors (insulators) are not free to move.

- Electric current $i$ is the rate $dq/dt$ at which charge passes a point:

$$i = \frac{dq}{dt}.$$

- Coulomb's law describes the electrostatic force (or electric force) between two charged particles. If the particles have charges $q_1$ and $q_2$, are separated by distance $r$, and are at rest (or moving only slowly) relative to each other, then the magnitude of the force acting on each due to the other is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law)},$$

where $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2}$ is the permittivity constant. The ratio $1/4\pi\varepsilon_0$ is often replaced with the electrostatic constant (or Coulomb constant) $k = 8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2}$.

- The electrostatic force vector acting on a charged particle due to a second charged particle is either directly toward the second particle (opposite signs of charge) or directly away from it (same sign of charge).

- If multiple electrostatic forces act on a particle, the net force is the vector sum (not scalar sum) of the individual forces.

- Shell theorem 1: A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.
- Shell theorem 2: A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.
- Charge on a conducting spherical shell spreads uniformly over the (external) surface.

## What Is Physics?

You are surrounded by devices that depend on the physics of electromagnetism, which is the combination of electric and magnetic phenomena. This physics is at the root of computers, television, radio, telecommunications, household lighting, and even the ability of food wrap to cling to a container. This physics is also the basis of the natural world. Not only does it hold together all the atoms and molecules in the world, it also produces lightning, auroras, and rainbows.

The physics of electromagnetism was first studied by the early Greek philosophers, who discovered that if a piece of amber is rubbed and then brought near bits of straw, the straw will jump to the amber. We now know that the attraction between amber and straw is due to an electric force. The Greek philosophers also discovered that if a certain type of stone (a naturally occurring magnet) is brought near bits of iron, the iron will jump to the stone. We now know that the attraction between magnet and iron is due to a magnetic force.

From these modest origins with the Greek philosophers, the sciences of electricity and magnetism developed separately for centuries—until 1820, in fact, when Hans Christian Oersted found a connection between them: an electric current in a wire can deflect a magnetic compass needle. Interestingly enough, Oersted made this discovery, a big surprise, while preparing a lecture demonstration for his physics students.

The new science of electromagnetism was developed further by workers in many countries. One of the best was Michael Faraday, a truly gifted experimenter with a talent for physical intuition and visualization. That talent is attested to by the fact that his collected laboratory notebooks do not contain a single equation. In the mid-nineteenth century, James Clerk Maxwell put Faraday's ideas into mathematical form, introduced many new ideas of his own, and put electromagnetism on a sound theoretical basis.

Our discussion of electromagnetism is spread through the next 16 chapters. We begin with electrical phenomena, and our first step is to discuss the nature of electric charge and electric force.

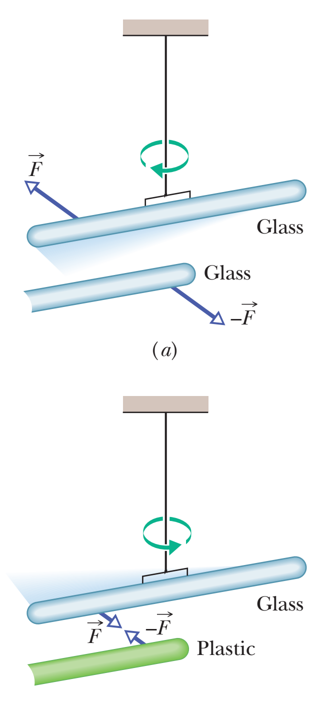



**Figure 21-1** (*a*) The two glass rods were each rubbed with a silk cloth and one was suspended by thread. When they are close to each other, they repel each other. (*b*) The plastic rod was rubbed with fur. When brought close to the glass rod, the rods attract each other.

## Electric Charge

Here are two demonstrations that seem to be magic, but our job here is to make sense of them. After rubbing a glass rod with a silk cloth (on a day when the humidity is low), we hang the rod by means of a thread tied around its center (Fig. 21-1*a*). Then we rub a second glass rod with the silk cloth and bring it near the hanging rod. The hanging rod magically moves away. We can see that a force repels it from the second rod, but how? There is no contact with that rod, no breeze to push on it, and no sound wave to disturb it.

In the second demonstration we replace the second rod with a plastic rod that has been rubbed with fur. This time, the hanging rod moves toward the nearby rod (Fig. 21-1*b*). Like the repulsion, this attraction occurs without any contact or obvious communication between the rods.

In the next chapter we shall discuss how the hanging rod knows of the presence of the other rods, but in this chapter let's focus on just the forces that are involved. In the first demonstration, the force on the hanging rod was *repulsive*, and in the second, *attractive*. After a great many investigations, scientists figured out that the forces in these types of demonstrations are due to the *electric charge* that we set up on the rods when they are in contact with silk or fur. Electric charge is an intrinsic property of the fundamental particles that make up objects such as the rods, silk, and fur. That is, charge is a property that comes automatically with those particles wherever they exist.

***Two Types.*** There are two types of electric charge, named by the American scientist and statesman Benjamin Franklin as positive charge and negative charge. He could have called them anything (such as cherry and walnut), but using algebraic signs as names comes in handy when we add up charges to find the net charge. In most everyday objects, such as a mug, there are about equal numbers of negatively charged particles and positively charged particles, and so the net charge is zero, the charge is said to be *balanced*, and the object is said to be *electrically neutral* (or just *neutral* for short).

***Excess Charge.*** Normally you are approximately neutral. However, if you live in regions where the humidity is low, you know that the charge on your body can become slightly unbalanced when you walk across certain carpets. Either you gain negative charge from the carpet (at the points of contact between your shoes with the carpet) and become negatively charged, or you lose negative charge and become positively charged. Either way, the extra charge is said to be an *excess charge*. You probably don't notice it until you reach for a door handle or another person. Then, if your excess charge is enough, a spark leaps between you and the other object, eliminating your excess charge. Such sparking can be annoying and even somewhat painful. Such *charging* and *discharging* do not happen in humid conditions because the water in the air *neutralizes* your excess charge about as fast as you acquire it.

Two of the grand mysteries in physics are (1) *why* does the universe have particles with electric charge (what is it, really?) and (2) *why* does electric charge come in two types (and not, say, one type or three types). We just do not know. Nevertheless, with lots of experiments similar to our two demonstrations scientists discovered that

> Particles with the same sign of electrical charge repel each other, and particles with opposite signs attract each other.

In a moment we shall put this rule into quantitative form as Coulomb's law of *electrostatic force* (or *electric force*) between charged particles. The term *electrostatic* is used to emphasize that, relative to each other, the charges are either stationary or moving only very slowly.

***Demos.*** Now let's get back to the demonstrations to understand the motions of the rod as being something other than just magic. When we rub the glass rod with a silk cloth, a small amount of negative charge moves from the rod to the silk (a transfer like that between you and a carpet), leaving the rod with a small amount of excess positive charge. (Which way the negative charge moves is not obvious and requires a lot of experimentation.) We *rub* the silk over the rod to increase the number of contact points and thus the amount, still tiny, of transferred charge. We hang the rod from the thread so as to *electrically isolate* it from its surroundings (so that the surroundings cannot neutralize the rod by giving it enough negative charge to rebalance its charge). When we rub the second rod with the silk cloth, it too becomes positively charged. So when we bring it near the first rod, the two rods repel each other (Fig. 21-2*a*).

Next, when we rub the plastic rod with fur, it gains excess negative charge from the fur. (Again, the transfer direction is learned through many experiments.) When we bring the plastic rod (with negative charge) near the hanging glass rod (with positive charge), the rods are attracted to each other (Fig. 21-2*b*). All this is subtle. You cannot see the charge or its transfer, only the results.

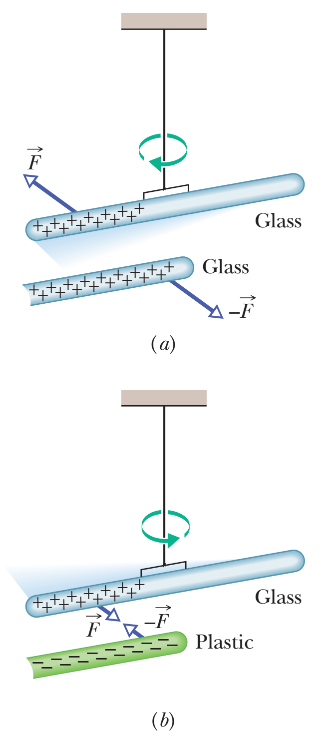



**Figure 21-2** (*a*) Two charged rods of the same sign repel each other. (*b*) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.

## Conductors and Insulators

We can classify materials generally according to the ability of charge to move through them. **Conductors** are materials through which charge can move rather freely; examples include metals (such as copper in common lamp wire), the human body, and tap water. **Nonconductors** — also called **insulators** — are materials through which charge cannot move freely; examples include rubber (such as the insulation on common lamp wire), plastic, glass, and chemically pure water. **Semiconductors** are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips. **Superconductors** are materials that are *perfect* conductors, allowing charge to move without *any* hindrance. In these chapters we discuss only conductors and insulators.

***Conducting Path.*** Here is an example of how conduction can eliminate excess charge on an object. If you rub a copper rod with wool, charge is transferred from the wool to the rod. However, if you are holding the rod while also touching a faucet, you cannot charge the rod in spite of the transfer. The reason is that you, the rod, and the faucet are all conductors connected, via the plumbing, to Earth's surface, which is a huge conductor. Because the excess charges put on the rod by the wool repel one another, they move away from one another by moving first through the rod, then through you, and then through the faucet and plumbing to reach Earth's surface, where they can spread out. The process leaves the rod electrically neutral.

In thus setting up a pathway of conductors between an object and Earth's surface, we are said to *ground* the object, and in neutralizing the object (by eliminating an unbalanced positive or negative charge), we are said to *discharge* the object. If instead of holding the copper rod in your hand, you hold it by an insulating handle, you eliminate the conducting path to Earth, and the rod can then be charged by rubbing (the charge remains on the rod), as long as you do not touch it directly with your hand.

***Charged Particles.*** The properties of conductors and insulators are due to the structure and electrical nature of atoms. Atoms consist of positively charged *protons*, negatively charged *electrons*, and electrically neutral *neutrons.* The protons and neutrons are packed tightly together in a central *nucleus.*

The charge of a single electron and that of a single proton have the same magnitude but are opposite in sign. Hence, an electrically neutral atom contains equal numbers of electrons and protons. Electrons are held near the nucleus because they have the electrical sign opposite that of the protons in the nucleus and thus are attracted to the nucleus. Were this not true, there would be no atoms and thus no you.

When atoms of a conductor like copper come together to form the solid, some of their outermost (and so most loosely held) electrons become free to wander about within the solid, leaving behind positively charged atoms (*positive ions*). We call the mobile electrons *conduction electrons.* There are few (if any) free electrons in a nonconductor.

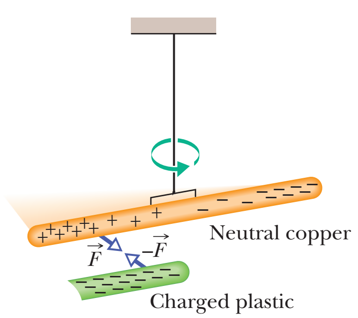



**Figure 21-3** A neutral copper rod is electrically isolated from its surroundings by being suspended on a nonconducting thread. Either end of the copper rod will be attracted by a charged rod. Here, conduction electrons in the copper rod are repelled to the far end of that rod by the negative charge on the plastic rod. Then that negative charge attracts the remaining positive charge on the near end of the copper rod, rotating the copper rod to bring that near end closer to the plastic rod.

***Induced Charge.*** The experiment of Fig. 21-3 demonstrates the mobility of charge in a conductor. A negatively charged plastic rod will attract either end of an isolated neutral copper rod. What happens is that many of the conduction electrons in the closer end of the copper rod are repelled by the negative charge on the plastic rod. Some of the conduction electrons move to the far end of the copper rod, leaving the near end depleted in electrons and thus with an unbalanced positive charge. This positive charge is attracted to the negative charge in the plastic rod. Although the copper rod is still neutral, it is said to have an *induced charge*, which means that some of its positive and negative charges have been separated due to the presence of a nearby charge.

Similarly, if a positively charged glass rod is brought near one end of a neutral copper rod, induced charge is again set up in the neutral copper rod but now the near end gains conduction electrons, becomes negatively charged, and is attracted to the glass rod, while the far end is positively charged.

Note that only conduction electrons, with their negative charges, can move; positive ions are fixed in place. Thus, an object becomes positively charged only through the *removal of negative charges.*

### Blue Flashes from a Wintergreen LifeSaver

Indirect evidence for the attraction of charges with opposite signs can be seen with a wintergreen LifeSaver (the candy shaped in the form of a marine lifesaver). If you adapt your eyes to darkness for about 15 minutes and then have a friend chomp on a piece of the candy in the darkness, you will see a faint blue flash from your friend's mouth with each chomp. Whenever a chomp breaks a sugar crystal into pieces, each piece will probably end up with a different number of electrons. Suppose a crystal breaks into pieces $A$ and $B$, with $A$ ending up with more electrons on its surface than $B$ (Fig. 21-4). This means that $B$ has positive ions (atoms that lost electrons to $A$) on its surface. Because the electrons on $A$ are strongly attracted to the positive ions on $B$, some of those electrons jump across the gap between the pieces.

As $A$ and $B$ move away from each other, air (primarily nitrogen, $N_2$) flows into the gap, and many of the jumping electrons collide with nitrogen molecules in the air, causing the molecules to emit ultraviolet light. You cannot see this type of light. However, the wintergreen molecules on the surfaces of the candy pieces absorb the ultraviolet light and then emit blue light, which you *can* see — it is the blue light coming from your friend's mouth.

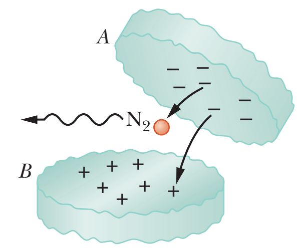
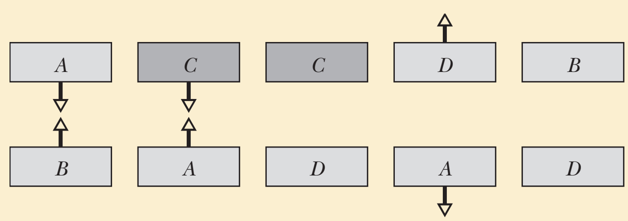



**Figure 21-4** Two pieces of a wintergreen LifeSaver candy as they fall away from each other. Electrons jumping from the negative surface of piece $A$ to the positive surface of piece $B$ collide with nitrogen ($N_2$) molecules in the air.

### Checkpoint 1

The figure shows five pairs of plates: $A$, $B$, and $D$ are charged plastic plates and $C$ is an electrically neutral copper plate. The electrostatic forces between the pairs of plates are shown for three of the pairs. For the remaining two pairs, do the plates repel or attract each other?



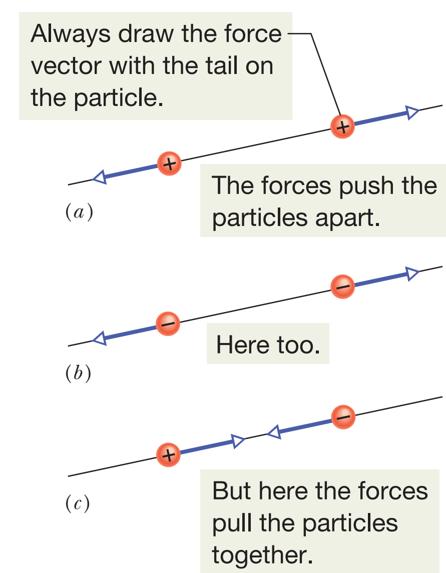




**Figure 21-5** Two charged particles repel each other if they have the same sign of charge, either (*a*) both positive or (*b*) both negative. (*c*) They attract each other if they have opposite signs of charge.

## Coulomb's Law

Now we come to the equation for Coulomb's law, but first a caution. This equation works for only charged particles (and a few other things that can be treated as particles). For extended objects, with charge located in many different places, we need more powerful techniques. So, here we consider just charged particles and not, say, two charged cats.

If two charged particles are brought near each other, they each exert an **electrostatic force** on the other. The direction of the force vectors depends on the signs of the charges. If the particles have the same sign of charge, they repel each other. That means that the force vector on each is directly away from the other particle (Figs. 21-5*a* and *b*). If we release the particles, they accelerate away from each other. If, instead, the particles have opposite signs of charge, they attract each other. That means that the force vector on each is directly toward the other particle (Fig. 21-5*c*). If we release the particles, they accelerate toward each other.

The equation for the electrostatic forces acting on the particles is called **Coulomb's law** after Charles-Augustin de Coulomb, whose experiments in 1785 led him to it. Let's write the equation in vector form and in terms of the particles shown in Fig. 21-6, where particle 1 has charge $q_1$ and particle 2 has charge $q_2$. (These symbols can represent either positive or negative charge.) Let's also focus on particle 1 and write the force acting on it in terms of a unit vector $\hat{r}$ that points along a radial

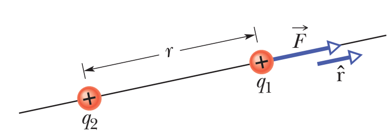



**Figure 21-6** The electrostatic force on particle 1 can be described in terms of a unit vector $\hat{r}$ along an axis through the two particles, radially away from particle 2.

axis extending through the two particles, radially away from particle 2. (As with other unit vectors, $\hat{\mathrm{r}}$ has a magnitude of exactly 1 and no unit; its purpose is to point, like a direction arrow on a street sign.) With these decisions, we write the electrostatic force as

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{\mathrm{r}} \quad \text{(Coulomb's law)}, \tag{21-1}$$

where $r$ is the separation between the particles and $k$ is a positive constant called the *electrostatic constant* or the *Coulomb constant*. (We'll discuss $k$ below.)

Let's first check the direction of the force on particle 1 as given by Eq. 21-1. If $q_1$ and $q_2$ have the same sign, then the product $q_1q_2$ gives us a positive result. So, Eq. 21-1 tells us that the force on particle 1 is in the direction of $\hat{\mathrm{r}}$. That checks, because particle 1 is being repelled from particle 2. Next, if $q_1$ and $q_2$ have opposite signs, the product $q_1q_2$ gives us a negative result. So, now Eq. 21-1 tells us that the force on particle 1 is in the direction opposite $\hat{\mathrm{r}}$. That checks because particle 1 is being attracted toward particle 2.

***An Aside.*** Here is something that is very curious. The form of Eq. 21-1 is the same as that of Newton's equation (Eq. 13-3) for the gravitational force between two particles with masses $m_1$ and $m_2$ and separation $r$:

$$\vec{F} = G\frac{m_1 m_2}{r^2}\,\hat{\mathrm{r}} \quad \text{(Newton's law)}, \tag{21-2}$$

where $G$ is the gravitational constant. Although the two types of forces are wildly different, both equations describe inverse square laws (the $1/r^2$ dependences) that involve a product of a property of the interacting particles—the charge in one case and the mass in the other. However, the laws differ in that gravitational forces are always attractive but electrostatic forces may be either attractive or repulsive, depending on the signs of the charges. This difference arises from the fact that there is only one type of mass but two types of charge.

***Unit.*** The SI unit of charge is the **coulomb**. For practical reasons having to do with the accuracy of measurements, the coulomb unit is derived from the SI unit *ampere* for electric current $i$. We shall discuss current in detail in Chapter 26, but here let's just note that current $i$ is the rate $dq/dt$ at which charge moves past a point or through a region:

$$i = \frac{dq}{dt} \quad \text{(electric current)}. \tag{21-3}$$

Rearranging Eq. 21-3 and replacing the symbols with their units (coulombs C, amperes A, and seconds s) we see that

$$1\ \mathrm{C} = (1\ \mathrm{A})(1\ \mathrm{s}).$$

***Force Magnitude.*** For historical reasons (and because doing so simplifies many other formulas), the electrostatic constant $k$ in Eq. 21-1 is often written as $1/4\pi\varepsilon_0$. Then the magnitude of the electrostatic force in Coulomb's law becomes

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law)}. \tag{21-4}$$

The constants in Eqs. 21-1 and 21-4 have the value

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2}. \tag{21-5}$$

The quantity $\varepsilon_0$, called the **permittivity constant,** sometimes appears separately in equations and is

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2}. \tag{21-6}$$

***Working a Problem.*** Note that the charge magnitudes appear in Eq. 21-4, which gives us the force magnitude. So, in working problems in this chapter, we use Eq. 21-4 to find the magnitude of a force on a chosen particle due to a second particle and we separately determine the direction of the force by considering the charge signs of the two particles.

***Multiple Forces.*** As with all forces in this book, the electrostatic force obeys the principle of superposition. Suppose we have $n$ charged particles near a chosen particle called particle 1; then the net force on particle 1 is given by the vector sum

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \ldots + \vec{F}_{1n}, \tag{21-7}$$

in which, for example, $\vec{F}_{14}$ is the force on particle 1 due to the presence of particle 4.

This equation is the key to many of the homework problems, so let's state it in words. If you want to know the net force acting on a chosen charged particle that is surrounded by other charged particles, first clearly identify that chosen particle and then find the force on it due to each of the other particles. Draw those force vectors in a free-body diagram of the chosen particle, with the tails anchored on the particle. (That may sound trivial, but failing to do so easily leads to errors.) Then add all those forces *as vectors* according to the rules of Chapter 3, not as scalars. (You cannot just willy-nilly add up their magnitudes.) The result is the net force (or resultant force) acting on the particle.

Although the vector nature of the forces makes the homework problems harder than if we simply had scalars, be thankful that Eq. 21-7 works. If two force vectors did not simply add but for some reason amplified each other, the world would be very difficult to understand and manage.

***Shell Theories.*** Analogous to the shell theories for the gravitational force (Module 13-1), we have two shell theories for the electrostatic force:

> Shell theory 1. A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

> Shell theory 2. A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

(In the first theory, we assume that the charge on the shell is much greater than the particle's charge. Thus the presence of the particle has negligible effect on the distribution of charge on the shell.)

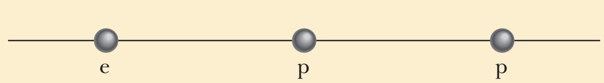
## Spherical Conductors

If excess charge is placed on a spherical shell that is made of conducting material, the excess charge spreads uniformly over the (external) surface. For example, if we place excess electrons on a spherical metal shell, those electrons repel one another and tend to move apart, spreading over the available surface until they are uniformly distributed. That arrangement maximizes the distances between all pairs of the excess electrons. According to the first shell theorem, the shell then will attract or repel an external charge as if all the excess charge on the shell were concentrated at its center.

If we remove negative charge from a spherical metal shell, the resulting positive charge of the shell is also spread uniformly over the surface of the shell. For example, if we remove $n$ electrons, there are then $n$ sites of positive charge (sites missing an electron) that are spread uniformly over the shell. According to the first shell theorem, the shell will again attract or repel an external charge as if all the shell's excess charge were concentrated at its center.

### Checkpoint 2

The figure shows two protons (symbol p) and one electron (symbol e) on an axis. On the central proton, what is the direction of (a) the force due to the electron, (b) the force due to the other proton, and (c) the net force?



### Sample Problem 21.01 Finding the net force due to two other particles

This sample problem actually contains three examples, to build from basic stuff to harder stuff. In each we have the same charged particle 1. First there is a single force acting on it (easy stuff). Then there are two forces, but they are just in opposite directions (not too bad). Then there are again two forces but they are in very different directions (ah, now we have to get serious about the fact that they are vectors). The key to all three examples is to draw the forces correctly *before* you reach for a calculator, otherwise you may be calculating nonsense on the calculator. (Figure 21-7 is available in *WileyPLUS* as an animation with voiceover.)

(a) Figure 21-7*a* shows two positively charged particles fixed in place on an $x$ axis. The charges are $q_1 = 1.60 \times 10^{-19}$ C and $q_2 = 3.20 \times 10^{-19}$ C, and the particle separation is $R = 0.0200$ m. What are the magnitude and direction of the electrostatic force $\vec{F}_{12}$ on particle 1 from particle 2?

#### KEY IDEAS

Because both particles are positively charged, particle 1 is repelled by particle 2, with a force magnitude given by Eq. 21-4. Thus, the direction of force $\vec{F}_{12}$ on particle 1 is *away from* particle 2, in the negative direction of the $x$ axis, as indicated in the free-body diagram of Fig. 21-7*b*.

***Two particles:*** Using Eq. 21-4 with separation $R$ substituted for $r$, we can write the magnitude $F_{12}$ of this force as

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(0.0200\ \text{m})^2}$$
$$= 1.15 \times 10^{-24}\ \text{N}.$$

Thus, force $\vec{F}_{12}$ has the following magnitude and direction (relative to the positive direction of the $x$ axis):

$$1.15 \times 10^{-24}\ \text{N} \quad \text{and} \quad 180°. \qquad \text{(Answer)}$$

We can also write $\vec{F}_{12}$ in unit-vector notation as

$$\vec{F}_{12} = -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}}. \qquad \text{(Answer)}$$

(b) Figure 21-7*c* is identical to Fig. 21-7*a* except that particle 3 now lies on the $x$ axis between particles 1 and 2. Particle 3 has charge $q_3 = -3.20 \times 10^{-19}$ C and is at a distance $\frac{3}{4}R$ from particle 1. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 3?

#### KEY IDEA

The presence of particle 3 does not alter the electrostatic force on particle 1 from particle 2. Thus, force $\vec{F}_{12}$ still acts on particle 1. Similarly, the force $\vec{F}_{13}$ that acts on particle 1 due to particle 3 is not affected by the presence of particle 2. Because

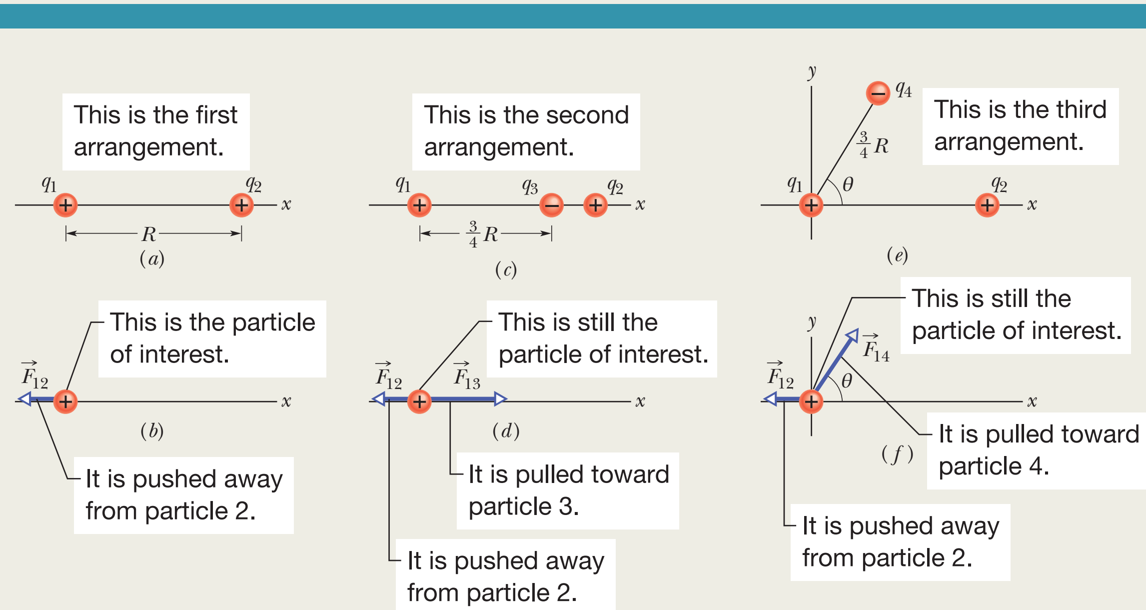



**Figure 21-7** (*a*) Two charged particles of charges $q_1$ and $q_2$ are fixed in place on an $x$ axis. (*b*) The free-body diagram for particle 1, showing the electrostatic force on it from particle 2. (*c*) Particle 3 included. (*d*) Free-body diagram for particle 1. (*e*) Particle 4 included. (*f*) Free-body diagram for particle 1.

particles 1 and 3 have charge of opposite signs, particle 1 is attracted to particle 3. Thus, force $\vec{F}_{13}$ is directed *toward* particle 3, as indicated in the free-body diagram of Fig. 21-7*d*.

***Three particles:*** To find the magnitude of $\vec{F}_{13}$, we can rewrite Eq. 21-4 as

$$\begin{aligned} F_{13} &= \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(\frac{3}{4}R)^2} \\ &= (8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}) \\ &\quad\times \frac{(1.60\times 10^{-19}\ \mathrm{C})(3.20\times 10^{-19}\ \mathrm{C})}{(\frac{3}{4})^2(0.0200\ \mathrm{m})^2} \\ &= 2.05\times 10^{-24}\ \mathrm{N}. \end{aligned}$$

We can also write $\vec{F}_{13}$ in unit-vector notation:

$$\vec{F}_{13} = (2.05\times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}}.$$

The net force $\vec{F}_{1,\mathrm{net}}$ on particle 1 is the vector sum of $\vec{F}_{12}$ and $\vec{F}_{13}$; that is, from Eq. 21-7, we can write the net force $\vec{F}_{1,\mathrm{net}}$ on particle 1 in unit-vector notation as

$$\begin{aligned} \vec{F}_{1,\mathrm{net}} &= \vec{F}_{12} + \vec{F}_{13} \\ &= -(1.15\times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}} + (2.05\times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}} \\ &= (9.00\times 10^{-25}\ \mathrm{N})\hat{\mathrm{i}}. \qquad \text{(Answer)} \end{aligned}$$

Thus, $\vec{F}_{1,\mathrm{net}}$ has the following magnitude and direction (relative to the positive direction of the $x$ axis):

$$9.00\times 10^{-25}\ \mathrm{N} \quad \text{and} \quad 0^\circ. \qquad \text{(Answer)}$$

(c) Figure 21-7*e* is identical to Fig. 21-7*a* except that particle 4 is now included. It has charge $q_4 = -3.20\times 10^{-19}$ C, is at a distance $\frac{3}{4}R$ from particle 1, and lies on a line that makes an angle $\theta = 60^\circ$ with the $x$ axis. What is the net electrostatic force $\vec{F}_{1,\mathrm{net}}$ on particle 1 due to particles 2 and 4?

## KEY IDEA

The net force $\vec{F}_{1,\mathrm{net}}$ is the vector sum of $\vec{F}_{12}$ and a new force $\vec{F}_{14}$ acting on particle 1 due to particle 4. Because particles 1 and 4 have charge of opposite signs, particle 1 is attracted to particle 4. Thus, force $\vec{F}_{14}$ on particle 1 is directed *toward* particle 4, at angle $\theta = 60^\circ$, as indicated in the free-body diagram of Fig. 21-7*f*.

***Four particles:*** We can rewrite Eq. 21-4 as

$$\begin{aligned} F_{14} &= \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(\frac{3}{4}R)^2} \\ &= (8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}) \\ &\quad\times \frac{(1.60\times 10^{-19}\ \mathrm{C})(3.20\times 10^{-19}\ \mathrm{C})}{(\frac{3}{4})^2(0.0200\ \mathrm{m})^2} \\ &= 2.05\times 10^{-24}\ \mathrm{N}. \end{aligned}$$

Then from Eq. 21-7, we can write the net force $\vec{F}_{1,\mathrm{net}}$ on particle 1 as

$$\vec{F}_{1,\mathrm{net}} = \vec{F}_{12} + \vec{F}_{14}.$$

Because the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are not directed along the same axis, we *cannot* sum simply by combining their magnitudes. Instead, we must add them as vectors, using one of the following methods.

***Method 1.*** *Summing directly on a vector-capable calculator.* For $\vec{F}_{12}$, we enter the magnitude $1.15\times 10^{-24}$ and the angle 180°. For $\vec{F}_{14}$, we enter the magnitude $2.05\times 10^{-24}$ and the angle 60°. Then we add the vectors.

***Method 2.*** *Summing in unit-vector notation.* First we rewrite $\vec{F}_{14}$ as

$$\vec{F}_{14} = (F_{14}\cos\theta)\hat{\mathrm{i}} + (F_{14}\sin\theta)\hat{\mathrm{j}}.$$

Substituting $2.05\times 10^{-24}$ N for $F_{14}$ and 60° for $\theta$, this becomes

$$\vec{F}_{14} = (1.025\times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}} + (1.775\times 10^{-24}\mathrm{N})\hat{\mathrm{j}}.$$

Then we sum:

$$\begin{aligned} \vec{F}_{1,\mathrm{net}} &= \vec{F}_{12} + \vec{F}_{14} \\ &= -(1.15\times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}} \\ &\quad + (1.025\times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}} + (1.775\times 10^{-24}\ \mathrm{N})\hat{\mathrm{j}} \\ &\approx (-1.25\times 10^{-25}\ \mathrm{N})\hat{\mathrm{i}} + (1.78\times 10^{-24}\ \mathrm{N})\hat{\mathrm{j}}. \end{aligned}$$

(Answer)

***Method 3.*** *Summing components axis by axis.* The sum of the $x$ components gives us

$$\begin{aligned} F_{1,\mathrm{net},x} &= F_{12,x} + F_{14,x} = F_{12} + F_{14}\cos 60^\circ \\ &= -1.15\times 10^{-24}\ \mathrm{N} + (2.05\times 10^{-24}\ \mathrm{N})(\cos 60^\circ) \\ &= -1.25\times 10^{-25}\ \mathrm{N}. \end{aligned}$$

The sum of the $y$ components gives us

$$\begin{aligned} F_{1,\mathrm{net},y} &= F_{12,y} + F_{14,y} = 0 + F_{14}\sin 60^\circ \\ &= (2.05\times 10^{-24}\ \mathrm{N})(\sin 60^\circ) \\ &= 1.78\times 10^{-24}\ \mathrm{N}. \end{aligned}$$

The net force $\vec{F}_{1,\mathrm{net}}$ has the magnitude

$$F_{1,\mathrm{net}} = \sqrt{F^2_{1,\mathrm{net},x} + F^2_{1,\mathrm{net},y}} = 1.78\times 10^{-24}\ \mathrm{N}. \qquad \text{(Answer)}$$

To find the direction of $\vec{F}_{1,\mathrm{net}}$ we take

$$\theta = \tan^{-1}\frac{F_{1,\mathrm{net},y}}{F_{1,\mathrm{net},x}} = -86.0^\circ.$$

However, this is an unreasonable result because $\vec{F}_{1,\mathrm{net}}$ must have a direction between the directions of $\vec{F}_{12}$ and $\vec{F}_{14}$. To correct $\theta$, we add 180°, obtaining

$$-86.0^\circ + 180^\circ = 94.0^\circ. \qquad \text{(Answer)}$$

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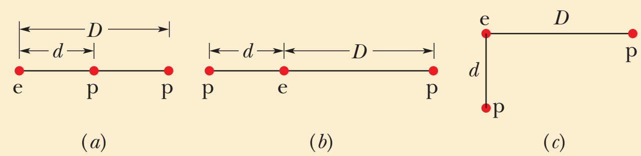
## Checkpoint 3

The figure here shows three arrangements of an electron e and two protons p. (a) Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first. (b) In situation *c*, is the angle between the net force on the electron and the line labeled *d* less than or more than 45°?




## Sample Problem 21.02 Equilibrium of two forces on a particle

Figure 21-8*a* shows two particles fixed in place: a particle of charge $q_1 = +8q$ at the origin and a particle of charge $q_2 = -2q$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in *equilibrium* (the net force on it is zero)? Is that equilibrium *stable* or *unstable*? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it farther away?)

### KEY IDEA

If $\vec{F}_1$ is the force on the proton due to charge $q_1$ and $\vec{F}_2$ is the force on the proton due to charge $q_2$, then the point we seek is where $\vec{F}_1 + \vec{F}_2 = 0$. Thus,

$$\vec{F}_1 = -\vec{F}_2. \tag{21-8}$$

This tells us that at the point we seek, the forces acting on the proton due to the other two particles must be of equal magnitudes,

$$F_1 = F_2, \tag{21-9}$$

and that the forces must have opposite directions.

***Reasoning:*** Because a proton has a positive charge, the proton and the particle of charge $q_1$ are of the same sign, and force $\vec{F}_1$ on the proton must point away from $q_1$. Also, the proton and the particle of charge $q_2$ are of opposite signs, so force $\vec{F}_2$ on the proton must point toward $q_2$. "Away from $q_1$" and "toward $q_2$" can be in opposite directions only if the proton is located on the $x$ axis.

If the proton is on the $x$ axis at any point between $q_1$ and $q_2$, such as point $P$ in Fig. 21-8*b*, then $\vec{F}_1$ and $\vec{F}_2$ are in the same direction and not in opposite directions as required. If the proton is at any point on the $x$ axis to the left of $q_1$, such as point $S$ in Fig. 21-8*c*, then $\vec{F}_1$ and $\vec{F}_2$ are in opposite directions. However, Eq. 21-4 tells us that $\vec{F}_1$ and $\vec{F}_2$ cannot have equal magnitudes there: $F_1$ must be greater than $F_2$, because $F_1$ is produced by a closer charge (with lesser $r$) of greater magnitude ($8q$ versus $2q$).

Finally, if the proton is at any point on the $x$ axis to the right of $q_2$, such as point $R$ in Fig. 21-8*d*, then $\vec{F}_1$ and $\vec{F}_2$ are again in opposite directions. However, because now the charge of greater magnitude ($q_1$) is *farther* away from the proton than the charge of lesser magnitude, there is a point at which $F_1$ is equal to $F_2$. Let $x$ be the coordinate of this point, and let $q_p$ be the charge of the proton.

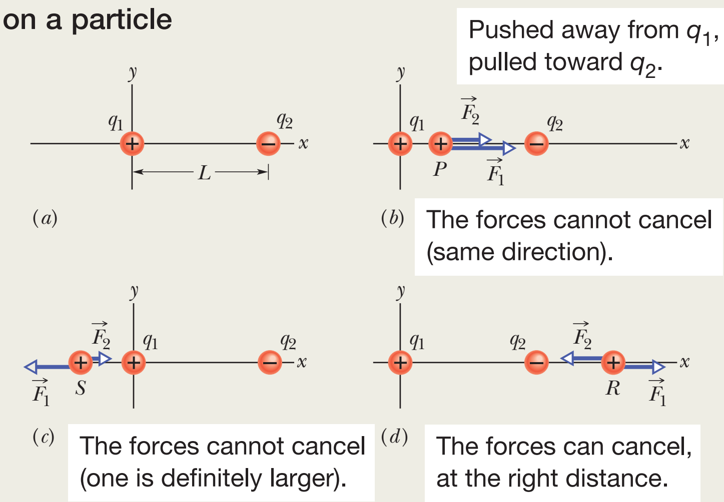



**Figure 21-8** (*a*) Two particles of charges $q_1$ and $q_2$ are fixed in place on an $x$ axis, with separation $L$. (*b*)–(*d*) Three possible locations $P$, $S$, and $R$ for a proton. At each location, $\vec{F}_1$ is the force on the proton from particle 1 and $\vec{F}_2$ is the force on the proton from particle 2.

***Calculations:*** With Eq. 21-4, we can now rewrite Eq. 21-9:

$$\frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(x-L)^2}. \tag{21-10}$$

(Note that only the charge magnitudes appear in Eq. 21-10. We already decided about the directions of the forces in drawing Fig. 21-8*d* and do not want to include any positive or negative signs here.) Rearranging Eq. 21-10 gives us

$$\left(\frac{x-L}{x}\right)^2 = \frac{1}{4}.$$

After taking the square roots of both sides, we find

$$\frac{x-L}{x} = \frac{1}{2}$$

and $$x = 2L. \qquad \text{(Answer)}$$

The equilibrium at $x = 2L$ is unstable; that is, if the proton is displaced leftward from point $R$, then $F_1$ and $F_2$ both increase but $F_2$ increases more (because $q_2$ is closer than $q_1$), and a net force will drive the proton farther leftward. If the proton is displaced rightward, both $F_1$ and $F_2$ decrease but $F_2$ decreases more, and a net force will then drive the proton farther rightward. In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.

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### Sample Problem 21.03 Charge sharing by two identical conducting spheres

In Fig. 21-9*a*, two identical, electrically isolated conducting spheres *A* and *B* are separated by a (center-to-center) distance *a* that is large compared to the spheres. Sphere *A* has a positive charge of $+Q$, and sphere *B* is electrically neutral. Initially, there is no electrostatic force between the spheres. (The large separation means there is no induced charge.)

(a) Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?

#### KEY IDEAS

(1) Because the spheres are identical, connecting them means that they end up with identical charges (same sign and same amount). (2) The initial sum of the charges (including the signs of the charges) must equal the final sum of the charges.

***Reasoning:*** When the spheres are wired together, the (negative) conduction electrons on *B*, which repel one another, have a way to move away from one another (along the wire to positively charged *A*, which attracts them—Fig. 21-9*b*). As *B* loses negative charge, it becomes positively charged, and as *A* gains negative charge, it becomes *less* positively charged. The transfer of charge stops when the charge on *B* has increased to $+Q/2$ and the charge on *A* has decreased to $+Q/2$, which occurs when $-Q/2$ has shifted from *B* to *A*.

After the wire has been removed (Fig. 21-9*c*), we can assume that the charge on either sphere does not disturb the uniformity of the charge distribution on the other sphere, because the spheres are small relative to their separation. Thus, we can apply the first shell theorem to each sphere. By Eq. 21-4 with $q_1 = q_2 = Q/2$ and $r = a$,

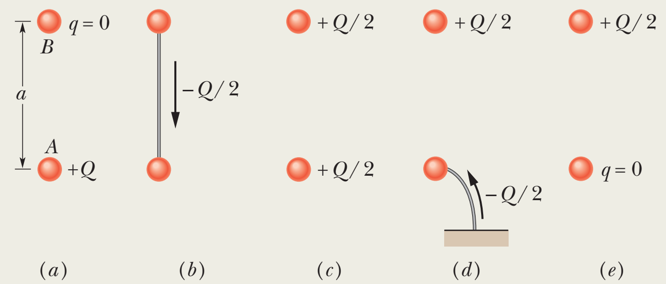


**Figure 21-9** Two small conducting spheres *A* and *B*. (*a*) To start, sphere *A* is charged positively. (*b*) Negative charge is transferred from *B* to *A* through a connecting wire. (*c*) Both spheres are then charged positively. (*d*) Negative charge is transferred through a grounding wire to sphere *A*. (*e*) Sphere *A* is then neutral.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)(Q/2)}{a^2} = \frac{1}{16\pi\varepsilon_0}\left(\frac{Q}{a}\right)^2. \qquad \text{(Answer)}$$

The spheres, now positively charged, repel each other.

(b) Next, suppose sphere *A* is grounded momentarily, and then the ground connection is removed. What now is the electrostatic force between the spheres?

***Reasoning:*** When we provide a conducting path between a charged object and the ground (which is a huge conductor), we neutralize the object. Were sphere *A* negatively charged, the mutual repulsion between the excess electrons would cause them to move from the sphere to the ground. However, because sphere *A* is positively charged, electrons with a total charge of $-Q/2$ move *from* the ground up onto the sphere (Fig. 21-9*d*), leaving the sphere with a charge of 0 (Fig. 21-9*e*). Thus, the electrostatic force is again zero.

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# 21-2 CHARGE IS QUANTIZED

## Learning Objectives

*After reading this module, you should be able to . . .*

**21.19** Identify the elementary charge.

**21.20** Identify that the charge of a particle or object must be a positive or negative integer times the elementary charge.

## Key Ideas

- Electric charge is quantized (restricted to certain values).
- The charge of a particle can be written as $ne$, where $n$ is a positive or negative integer and $e$ is the elementary charge, which is the magnitude of the charge of the electron and proton ($\approx 1.602 \times 10^{-19}$ C).

## Charge Is Quantized

In Benjamin Franklin's day, electric charge was thought to be a continuous fluid — an idea that was useful for many purposes. However, we now know that fluids themselves, such as air and water, are not continuous but are made up of atoms and molecules; matter is discrete. Experiment shows that "electrical fluid" is also not continuous but is made up of multiples of a certain elementary charge. Any positive or negative charge $q$ that can be detected can be written as

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots, \tag{21-11}$$

in which $e$, the **elementary charge,** has the approximate value

$$e = 1.602 \times 10^{-19}\ \mathrm{C}. \tag{21-12}$$

The elementary charge $e$ is one of the important constants of nature. The electron and proton both have a charge of magnitude $e$ (Table 21-1). (Quarks, the constituent particles of protons and neutrons, have charges of $\pm e/3$ or $\pm 2e/3$, but they apparently cannot be detected individually. For this and for historical reasons, we do not take their charges to be the elementary charge.)

You often see phrases — such as "the charge on a sphere," "the amount of charge transferred," and "the charge carried by the electron" — that suggest that charge is a substance. (Indeed, such statements have already appeared in this chapter.) You should, however, keep in mind what is intended: *Particles* are the substance and charge happens to be one of their properties, just as mass is.

When a physical quantity such as charge can have only discrete values rather than any value, we say that the quantity is **quantized.** It is possible, for example, to find a particle that has no charge at all or a charge of $+10e$ or $-6e$, but not a particle with a charge of, say, $3.57e$.

The quantum of charge is small. In an ordinary 100 W lightbulb, for example, about $10^{19}$ elementary charges enter the bulb every second and just as many leave. However, the graininess of electricity does not show up in such large-scale phenomena (the bulb does not flicker with each electron).

**Table 21-1** The Charges of Three Particles

| Particle | Symbol | Charge |
|---|---|---|
| Electron | e or $e^-$ | $-e$ |
| Proton | p | $+e$ |
| Neutron | n | 0 |

### Checkpoint 4

Initially, sphere $A$ has a charge of $-50e$ and sphere $B$ has a charge of $+20e$. The spheres are made of conducting material and are identical in size. If the spheres then touch, what is the resulting charge on sphere $A$?

## Sample Problem 21.04 Mutual electric repulsion in a nucleus

The nucleus in an iron atom has a radius of about $4.0 \times 10^{-15}$ m and contains 26 protons.

(a) What is the magnitude of the repulsive electrostatic force between two of the protons that are separated by $4.0 \times 10^{-15}$ m?

### KEY IDEA

The protons can be treated as charged particles, so the magnitude of the electrostatic force on one from the other is given by Coulomb's law.

***Calculation:*** Table 21-1 tells us that the charge of a proton is $+e$. Thus, Eq. 21-4 gives us

$$\begin{aligned} F &= \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \\ &= \frac{(8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})(1.602 \times 10^{-19}\ \mathrm{C})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2} \\ &= 14\ \mathrm{N}. \end{aligned} \qquad \text{(Answer)}$$

***No explosion:*** This is a small force to be acting on a macroscopic object like a cantaloupe, but an enormous force to be acting on a proton. Such forces should explode the nucleus of any element but hydrogen (which has only one proton in its nucleus). However, they don't, not even in nuclei with a great many protons. Therefore, there must be some enormous attractive force to counter this enormous repulsive electrostatic force.

(b) What is the magnitude of the gravitational force between those same two protons?

### KEY IDEA

Because the protons are particles, the magnitude of the gravitational force on one from the other is given by Newton's equation for the gravitational force (Eq. 21-2).

***Calculation:*** With $m_p$ ($= 1.67 \times 10^{-27}$ kg) representing the mass of a proton, Eq. 21-2 gives us

$$F = G\frac{m_p^2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(1.67 \times 10^{-27}\ \mathrm{kg})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2}$$

$$= 1.2 \times 10^{-35}\ \mathrm{N}. \qquad \text{(Answer)}$$

***Weak versus strong:*** This result tells us that the (attractive) gravitational force is far too weak to counter the repulsive electrostatic forces between protons in a nucleus. Instead, the protons are bound together by an enormous force called (aptly) the *strong nuclear force* — a force that acts between protons (and neutrons) when they are close together, as in a nucleus.

Although the gravitational force is many times weaker than the electrostatic force, it is more important in large-scale situations because it is always attractive. This means that it can collect many small bodies into huge bodies with huge masses, such as planets and stars, that then exert large gravitational forces. The electrostatic force, on the other hand, is repulsive for charges of the same sign, so it is unable to collect either positive charge or negative charge into large concentrations that would then exert large electrostatic forces.



# 21-3 CHARGE IS CONSERVED

## Learning Objectives

*After reading this module, you should be able to . . .*

21.21 Identify that in any isolated physical process, the net charge cannot change (the net charge is always conserved).

21.22 Identify an annihilation process of particles and a pair production of particles.

21.23 Identify mass number and atomic number in terms of the number of protons, neutrons, and electrons.

## Key Ideas

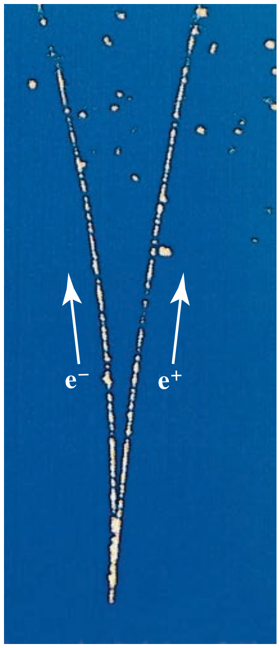
- The net electric charge of any isolated system is always conserved.
- If two charged particles undergo an annihilation process, they have opposite signs of charge.
- If two charged particles appear as a result of a pair production process, they have opposite signs of charge.

## Charge Is Conserved

If you rub a glass rod with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that rubbing does not create charge but only transfers it from one body to another, upsetting the electrical neutrality of each body during the process. This hypothesis of **conservation of charge,** first put forward by Benjamin Franklin, has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles. No exceptions have ever been found. Thus, we add electric charge to our list of quantities — including energy and both linear momentum and angular momentum — that obey a conservation law.

Important examples of the conservation of charge occur in the *radioactive decay* of nuclei, in which a nucleus transforms into (becomes) a different type of nucleus. For example, a uranium-238 nucleus ($^{238}$U) transforms into a thorium-234 nucleus ($^{234}$Th) by emitting an *alpha particle.* Because that particle has the same makeup as a helium-4 nucleus, it has the symbol $^4$He. The number used in the name of a nucleus and as a superscript in the symbol for the nucleus is called the *mass number* and is the total number of the protons and neutrons in the nucleus. For example, the total number in $^{238}$U is 238. The number of protons in a nucleus is the *atomic number Z*, which is listed for all the elements in Appendix F. From that list we find that in the decay

$$^{238}\mathrm{U} \rightarrow {}^{234}\mathrm{Th} + {}^{4}\mathrm{He}, \qquad (21\text{-}13)$$




Courtesy Lawrence Berkeley Laboratory

**Figure 21-10** A photograph of trails of bubbles left in a bubble chamber by an electron and a positron. The pair of particles was produced by a gamma ray that entered the chamber directly from the bottom. Being electrically neutral, the gamma ray did not generate a telltale trail of bubbles along its path, as the electron and positron did.

the *parent* nucleus $^{238}$U contains 92 protons (a charge of $+92e$), the *daughter* nucleus $^{234}$Th contains 90 protons (a charge of $+90e$), and the emitted alpha particle $^{4}$He contains 2 protons (a charge of $+2e$). We see that the total charge is $+92e$ before and after the decay; thus, charge is conserved. (The total number of protons and neutrons is also conserved: 238 before the decay and $234 + 4 = 238$ after the decay.)

Another example of charge conservation occurs when an electron $e^-$ (charge $-e$) and its antiparticle, the *positron* $e^+$ (charge $+e$), undergo an *annihilation process,* transforming into two *gamma rays* (high-energy light):

$$e^- + e^+ \rightarrow \gamma + \gamma \qquad \text{(annihilation)}. \tag{21-14}$$

In applying the conservation-of-charge principle, we must add the charges algebraically, with due regard for their signs. In the annihilation process of Eq. 21-14 then, the net charge of the system is zero both before and after the event. Charge is conserved.

In *pair production*, the converse of annihilation, charge is also conserved. In this process a gamma ray transforms into an electron and a positron:

$$\gamma \rightarrow e^- + e^+ \qquad \text{(pair production)}. \tag{21-15}$$

Figure 21-10 shows such a pair-production event that occurred in a bubble chamber. (This is a device in which a liquid is suddenly made hotter than its boiling point. If a charged particle passes through it, tiny vapor bubbles form along the particle's trail.) A gamma ray entered the chamber from the bottom and at one point transformed into an electron and a positron. Because those new particles were charged and moving, each left a trail of bubbles. (The trails were curved because a magnetic field had been set up in the chamber.) The gamma ray, being electrically neutral, left no trail. Still, you can tell exactly where it underwent pair production — at the tip of the curved V, which is where the trails of the electron and positron begin.

## Review & Summary

**Electric Charge** The strength of a particle's electrical interaction with objects around it depends on its **electric charge** (usually represented as $q$), which can be either positive or negative. Particles with the same sign of charge repel each other, and particles with opposite signs of charge attract each other. An object with equal amounts of the two kinds of charge is electrically neutral, whereas one with an imbalance is electrically charged and has an excess charge.

**Conductors** are materials in which a significant number of electrons are free to move. The charged particles in **nonconductors** (**insulators**) are not free to move.

Electric current $i$ is the rate $dq/dt$ at which charge passes a point:

$$i = \frac{dq}{dt} \qquad \text{(electric current)}. \tag{21-3}$$

**Coulomb's Law** Coulomb's law describes the electrostatic force (or electric force) between two charged particles. If the particles have charges $q_1$ and $q_2$, are separated by distance $r$, and are at rest (or moving only slowly) relative to each other, then the magnitude of the force acting on each due to the other is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \qquad \text{(Coulomb's law)}, \tag{21-4}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$ is the **permittivity constant**. The ratio $1/4\pi\varepsilon_0$ is often replaced with the **electrostatic constant** (or **Coulomb constant**) $k = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

The electrostatic force vector acting on a charged particle due to a second charged particle is either directly toward the second particle (opposite signs of charge) or directly away from it (same sign of charge). As with other types of forces, if multiple electrostatic forces act on a particle, the net force is the vector sum (not scalar sum) of the individual forces.

The two shell theories for electrostatics are

*Shell theorem 1: A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.*

*Shell theorem 2: A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.*

Charge on a conducting spherical shell spreads uniformly over the (external) surface.

**The Elementary Charge** Electric charge is quantized (restricted to certain values). The charge of a particle can be written as $ne$, where $n$ is a positive or negative integer and $e$ is the elementary charge, which is the magnitude of the charge of the electron and proton ($\approx 1.602 \times 10^{-19}$ C).

**Conservation of Charge** The net electric charge of any isolated system is always conserved.

# Questions

**1** Figure 21-11 shows four situations in which five charged particles are evenly spaced along an axis. The charge values are indicated except for the central particle, which has the same charge in all four situations. Rank the situations according to the magnitude of the net electrostatic force on the central particle, greatest first.

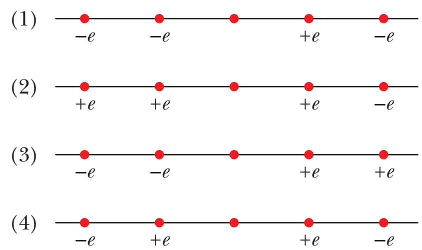



**Figure 21-11** Question 1.

**2** Figure 21-12 shows three pairs of identical spheres that are to be touched together and then separated. The initial charges on them are indicated. Rank the pairs according to (a) the magnitude of the charge transferred during touching and (b) the charge left on the positively charged sphere, greatest first.

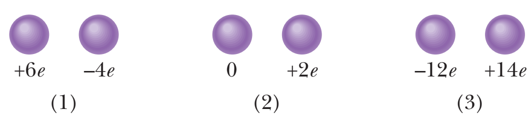



**Figure 21-12** Question 2.

**3** Figure 21-13 shows four situations in which charged particles are fixed in place on an axis. In which situations is there a point to the left of the particles where an electron will be in equilibrium?

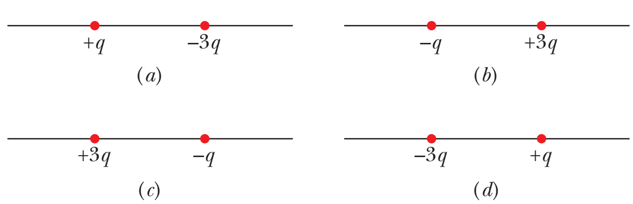



**Figure 21-13** Question 3.

**4** Figure 21-14 shows two charged particles on an axis. The charges are free to move. However, a third charged particle can be placed at a certain point such that all three particles are then in equilibrium. (a) Is that point to the left of the first two particles, to their right, or between them? (b) Should the third particle be positively or negatively charged? (c) Is the equilibrium stable or unstable?

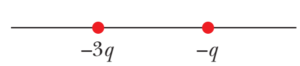



**Figure 21-14** Question 4.

**5** In Fig. 21-15, a central particle of charge $-q$ is surrounded by two circular rings of charged particles. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint:* Consider symmetry.)

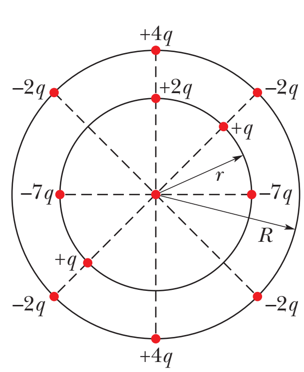



**Figure 21-15** Question 5.

**6** A positively charged ball is brought close to an electrically neutral isolated conductor. The conductor is then grounded while the ball is kept close. Is the conductor charged positively, charged negatively, or neutral if (a) the ball is first taken away and then the ground connection is removed and (b) the ground connection is first removed and then the ball is taken away?

**7** Figure 21-16 shows three situations involving a charged particle and a uniformly charged spherical shell. The charges are given, and the radii of the shells are indicated. Rank the situations according to the magnitude of the force on the particle due to the presence of the shell, greatest first.

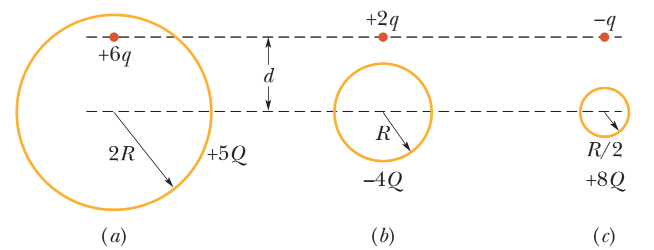



**Figure 21-16** Question 7.

**8** Figure 21-17 shows four arrangements of charged particles. Rank the arrangements according to the magnitude of the net electrostatic force on the particle with charge $+Q$, greatest first.

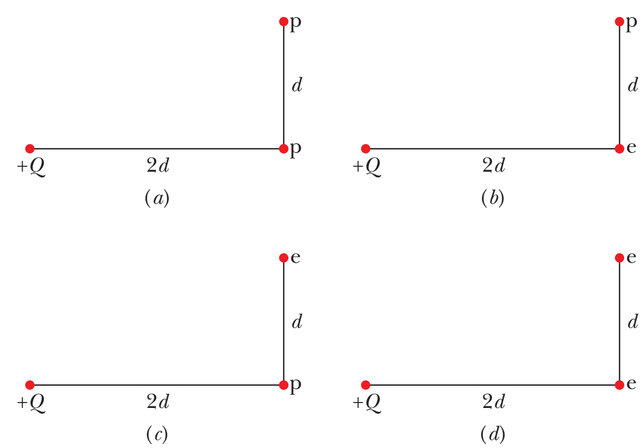



**Figure 21-17** Question 8.

**9** Figure 21-18 shows four situations in which particles of charge $+q$ or $-q$ are fixed in place. In each situation, the particles on the

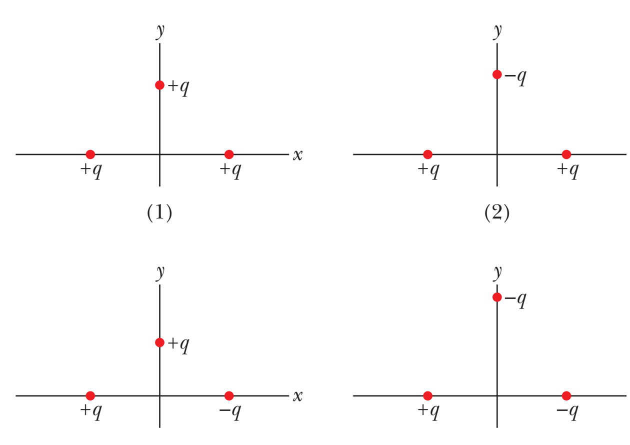



**Figure 21-18** Question 9.

$x$ axis are equidistant from the $y$ axis. First, consider the middle particle in situation 1; the middle particle experiences an electrostatic force from each of the other two particles. (a) Are the magnitudes $F$ of those forces the same or different? (b) Is the magnitude of the net force on the middle particle equal to, greater than, or less than $2F$? (c) Do the $x$ components of the two forces add or cancel? (d) Do their $y$ components add or cancel? (e) Is the direction of the net force on the middle particle that of the canceling components or the adding components? (f) What is the direction of that net force? Now consider the remaining situations: What is the direction of the net force on the middle particle in (g) situation 2, (h) situation 3, and (i) situation 4? (In each situation, consider the symmetry of the charge distribution and determine the canceling components and the adding components.)

**10** In Fig. 21-19, a central particle of charge $-2q$ is surrounded by a square array of charged particles, separated by either distance $d$ or $d/2$ along the perimeter of the square. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint:* Consideration of symmetry can greatly reduce the amount of work required here.)

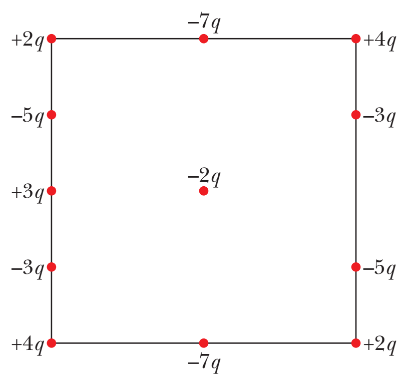



**Figure 21-19** Question 10.

**11** Figure 21-20 shows three identical conducting bubbles $A$, $B$, and $C$ floating in a conducting container that is grounded by a wire. The bubbles initially have the same charge. Bubble $A$ bumps into the container's ceiling and then into bubble $B$. Then bubble $B$ bumps into bubble $C$, which then drifts to the container's floor. When bubble $C$ reaches the floor, a charge of $-3e$ is transferred upward through the wire, from the ground to the container, as indicated. (a) What was the initial charge of each bubble? When (b) bubble $A$ and (c) bubble $B$ reach the floor, what is the charge transfer through the wire? (d) During this whole process, what is the total charge transfer through the wire?

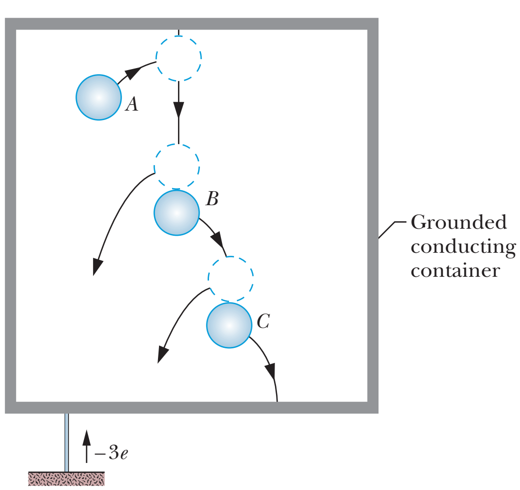



**Figure 21-20** Question 11.

**12** Figure 21-21 shows four situations in which a central proton is partially surrounded by protons or electrons fixed in place along a half-circle. The angles $\theta$ are identical; the angles $\phi$ are also. (a) In each situation, what is the direction of the net force on the central proton due to the other particles? (b) Rank the four situations according to the magnitude of that net force on the central proton, greatest first.

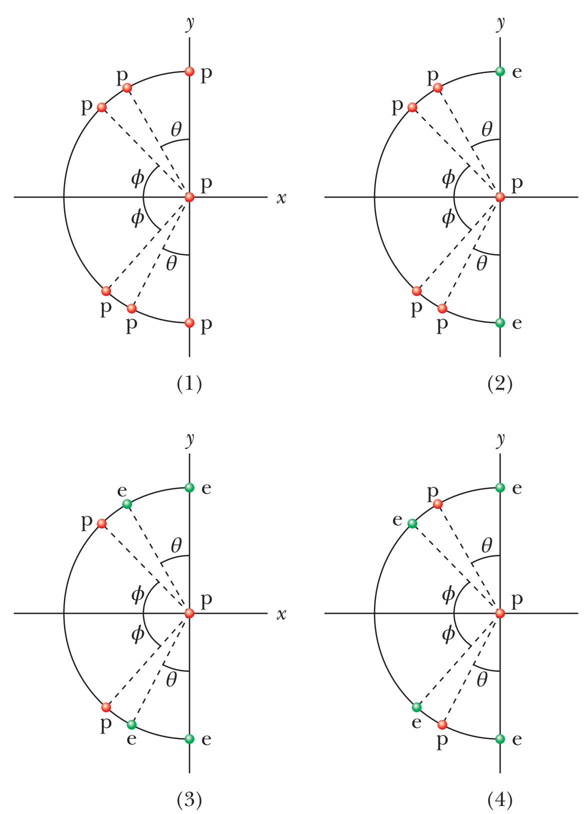



**Figure 21-21** Question 12.

# Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS* and WebAssign
SSM Worked-out solution available in Student Solutions Manual
WWW Worked-out solution is at
• – ••• Number of dots indicates level of problem difficulty
ILW Interactive solution is at
http://www.wiley.com/college/halliday
Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

## Module 21-1 Coulomb's Law

•1 SSM ILW Of the charge $Q$ initially on a tiny sphere, a portion $q$ is to be transferred to a second, nearby sphere. Both spheres can be treated as particles and are fixed with a certain separation. For what value of $q/Q$ will the electrostatic force between the two spheres be maximized?

**•2** Identical isolated conducting spheres 1 and 2 have equal charges and are separated by a distance that is large compared with their diameters (Fig. 21-22*a*). The electrostatic force acting on sphere 2 due to sphere 1 is $\vec{F}$. Suppose now that a third identical sphere 3, having an insulating handle and initially neutral, is touched first to sphere 1 (Fig. 21-22*b*), then to sphere 2 (Fig. 21-22*c*), and finally removed (Fig. 21-22*d*). The electrostatic force that now acts on sphere 2 has magnitude $F'$. What is the ratio $F'/F$?

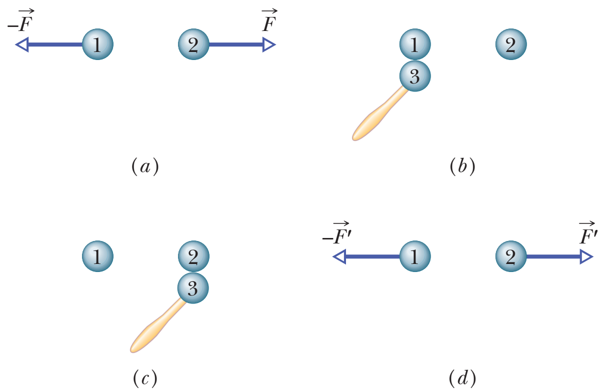



**Figure 21-22** Problem 2.

**•3** **SSM** What must be the distance between point charge $q_1 = 26.0\ \mu C$ and point charge $q_2 = -47.0\ \mu C$ for the electrostatic force between them to have a magnitude of 5.70 N?

**•4** In the return stroke of a typical lightning bolt, a current of $2.5 \times 10^4$ A exists for 20 $\mu s$. How much charge is transferred in this event?

**•5** A particle of charge $+3.00 \times 10^{-6}$ C is 12.0 cm distant from a second particle of charge $-1.50 \times 10^{-6}$ C. Calculate the magnitude of the electrostatic force between the particles.

**•6** **ILW** Two equally charged particles are held $3.2 \times 10^{-3}$ m apart and then released from rest. The initial acceleration of the first particle is observed to be 7.0 m/s² and that of the second to be 9.0 m/s². If the mass of the first particle is $6.3 \times 10^{-7}$ kg, what are (a) the mass of the second particle and (b) the magnitude of the charge of each particle?

**••7** In Fig. 21-23, three charged particles lie on an $x$ axis. Particles 1 and 2 are fixed in place. Particle 3 is free to move, but the net electrostatic force on it from particles 1 and 2 happens to be zero. If $L_{23} = L_{12}$, what is the ratio $q_1/q_2$?

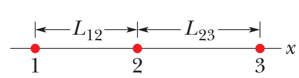



**Figure 21-23** Problems 7 and 40.

**••8** In Fig. 21-24, three identical conducting spheres initially have the following charges: sphere $A$, $4Q$; sphere $B$, $-6Q$; and sphere $C$, 0. Spheres $A$ and $B$ are fixed in place, with a center-to-center separation that is much larger than the spheres. Two experiments are conducted. In experiment 1, sphere $C$ is touched to sphere $A$ and then (separately) to sphere $B$, and then it is removed. In experiment 2, starting with the same initial states, the procedure is reversed: Sphere $C$ is touched to sphere $B$ and then (separately) to sphere $A$, and then it is removed. What is the ratio of the electrostatic force between $A$ and $B$ at the end of experiment 2 to that at the end of experiment 1?

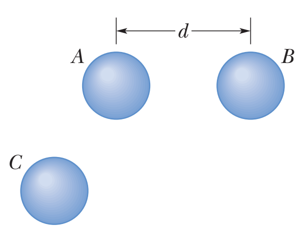



**Figure 21-24** Problems 8 and 65.

**••9** **SSM** **WWW** Two identical conducting spheres, fixed in place, attract each other with an electrostatic force of 0.108 N when their center-to-center separation is 50.0 cm. The spheres are then connected by a thin conducting wire. When the wire is removed, the spheres repel each other with an electrostatic force of 0.0360 N. Of the initial charges on the spheres, with a positive net charge, what was (a) the negative charge on one of them and (b) the positive charge on the other?

**••10** In Fig. 21-25, four particles form a square. The charges are $q_1 = q_4 = Q$ and $q_2 = q_3 = q$. (a) What is $Q/q$ if the net electrostatic force on particles 1 and 4 is zero? (b) Is there any value of $q$ that makes the net electrostatic force on each of the four particles zero? Explain.

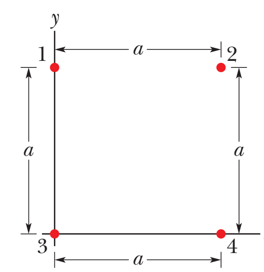



**Figure 21-25** Problems 10, 11, and 70.

**••11** **ILW** In Fig. 21-25, the particles have charges $q_1 = -q_2 = 100$ nC and $q_3 = -q_4 = 200$ nC, and distance $a = 5.0$ cm. What are the (a) $x$ and (b) $y$ components of the net electrostatic force on particle 3?

**••12** Two particles are fixed on an $x$ axis. Particle 1 of charge 40 $\mu C$ is located at $x = -2.0$ cm; particle 2 of charge $Q$ is located at $x = 3.0$ cm. Particle 3 of charge magnitude 20 $\mu C$ is released from rest on the $y$ axis at $y = 2.0$ cm. What is the value of $Q$ if the initial acceleration of particle 3 is in the positive direction of (a) the $x$ axis and (b) the $y$ axis?

**••13** In Fig. 21-26, particle 1 of charge $+1.0\ \mu C$ and particle 2 of charge $-3.0\ \mu C$ are held at separation $L = 10.0$ cm on an $x$ axis. If particle 3 of unknown charge $q_3$ is to be located such that the net electrostatic force on it from particles 1 and 2 is zero, what must be the (a) $x$ and (b) $y$ coordinates of particle 3?

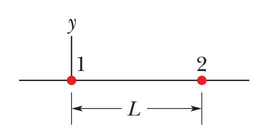



**Figure 21-26** Problems 13, 19, 30, 58, and 67.

**••14** Three particles are fixed on an $x$ axis. Particle 1 of charge $q_1$ is at $x = -a$, and particle 2 of charge $q_2$ is at $x = +a$. If their net electrostatic force on particle 3 of charge $+Q$ is to be zero, what must be the ratio $q_1/q_2$ when particle 3 is at (a) $x = +0.500a$ and (b) $x = +1.50a$?

**••15** The charges and coordinates of two charged particles held fixed in an $xy$ plane are $q_1 = +3.0\ \mu C$, $x_1 = 3.5$ cm, $y_1 = 0.50$ cm, and $q_2 = -4.0\ \mu C$, $x_2 = -2.0$ cm, $y_2 = 1.5$ cm. Find the (a) magnitude and (b) direction of the electrostatic force on particle 2 due to particle 1. At what (c) $x$ and (d) $y$ coordinates should a third particle of charge $q_3 = +4.0\ \mu C$ be placed such that the net electrostatic force on particle 2 due to particles 1 and 3 is zero?

**••16** In Fig. 21-27*a*, particle 1 (of charge $q_1$) and particle 2 (of charge $q_2$) are fixed in place on an $x$ axis, 8.00 cm apart. Particle 3

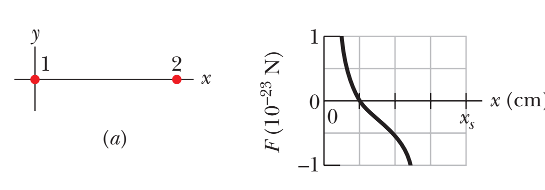



**Figure 21-27** Problem 16.

(of charge $q_3 = +8.00 \times 10^{-19}$ C) is to be placed on the line between particles 1 and 2 so that they produce a net electrostatic force $\vec{F}_{3,\text{net}}$ on it. Figure 21-27*b* gives the *x* component of that force versus the coordinate *x* at which particle 3 is placed. The scale of the *x* axis is set by $x_s = 8.0$ cm. What are (a) the sign of charge $q_1$ and (b) the ratio $q_2/q_1$?

**••17** In Fig. 21-28*a*, particles 1 and 2 have charge 20.0 $\mu$C each and are held at separation distance $d = 1.50$ m. (a) What is the magnitude of the electrostatic force on particle 1 due to particle 2? In Fig. 21-28*b*, particle 3 of charge 20.0 $\mu$C is positioned so as to complete an equilateral triangle. (b) What is the magnitude of the net electrostatic force on particle 1 due to particles 2 and 3?

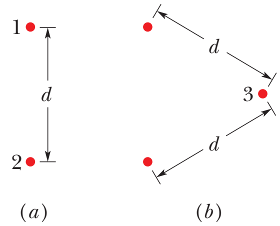



**Figure 21-28** Problem 17.

**••18** In Fig. 21-29*a*, three positively charged particles are fixed on an *x* axis. Particles *B* and *C* are so close to each other that they can be considered to be at the same distance from particle *A*. The net force on particle *A* due to particles *B* and *C* is $2.014 \times 10^{-23}$ N in the negative direction of the *x* axis. In Fig. 21-29*b*, particle *B* has been moved to the opposite side of *A* but is still at the same distance from it. The net force on *A* is now $2.877 \times 10^{-24}$ N in the negative direction of the *x* axis. What is the ratio $q_C/q_B$?

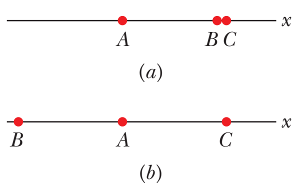



**Figure 21-29** Problem 18.

**••19** SSM WWW In Fig. 21-26, particle 1 of charge $+q$ and particle 2 of charge $+4.00q$ are held at separation $L = 9.00$ cm on an *x* axis. If particle 3 of charge $q_3$ is to be located such that the three particles remain in place when released, what must be the (a) *x* and (b) *y* coordinates of particle 3, and (c) the ratio $q_3/q$?

**•••20** GO Figure 21-30*a* shows an arrangement of three charged particles separated by distance *d*. Particles *A* and *C* are fixed on the *x* axis, but particle *B* can be moved along a circle centered on particle *A*. During the movement, a radial line between *A* and *B* makes an angle $\theta$ relative to the positive direction of the *x* axis (Fig. 21-30*b*). The curves in Fig. 21-30*c* give, for two situations, the magnitude $F_{\text{net}}$ of the net electrostatic force on particle *A* due to the other particles. That net force is given as a function of angle $\theta$ and as a multiple of a basic amount $F_0$. For example on curve 1, at $\theta = 180°$, we see that $F_{\text{net}} = 2F_0$. (a) For the situation corresponding to curve 1, what is the ratio of the charge of particle *C* to that of particle *B* (including sign)? (b) For the situation corresponding to curve 2, what is that ratio?

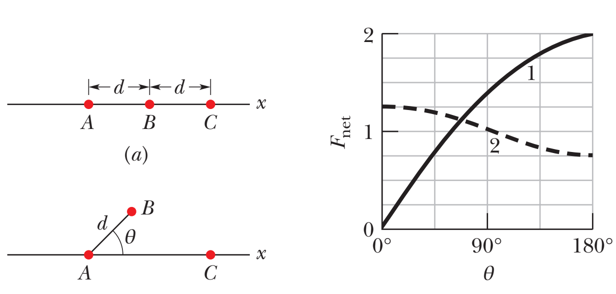



**Figure 21-30** Problem 20.

**•••21** GO A nonconducting spherical shell, with an inner radius of 4.0 cm and an outer radius of 6.0 cm, has charge spread nonuniformly through its volume between its inner and outer surfaces. The *volume charge density* $\rho$ is the charge per unit volume, with the unit coulomb per cubic meter. For this shell $\rho = b/r$, where *r* is the distance in meters from the center of the shell and $b = 3.0\ \mu\text{C/m}^2$. What is the net charge in the shell?

**•••22** GO Figure 21-31 shows an arrangement of four charged particles, with angle $\theta = 30.0°$ and distance $d = 2.00$ cm. Particle 2 has charge $q_2 = +8.00 \times 10^{-19}$ C; particles 3 and 4 have charges $q_3 = q_4 = -1.60 \times 10^{-19}$ C. (a) What is distance *D* between the origin and particle 2 if the net electrostatic force on particle 1 due to the other particles is zero? (b) If particles 3 and 4 were moved closer to the *x* axis but maintained their symmetry about that axis, would the required value of *D* be greater than, less than, or the same as in part (a)?

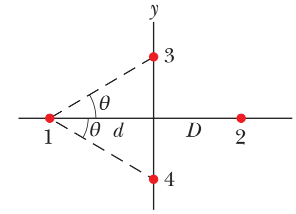



**Figure 21-31** Problem 22.

**•••23** GO In Fig. 21-32, particles 1 and 2 of charge $q_1 = q_2 = +3.20 \times 10^{-19}$ C are on a *y* axis at distance $d = 17.0$ cm from the origin. Particle 3 of charge $q_3 = +6.40 \times 10^{-19}$ C is moved gradually along the *x* axis from $x = 0$ to $x = +5.0$ m. At what values of *x* will the magnitude of the electrostatic force on the third particle from the other two particles be (a) minimum and (b) maximum? What are the (c) minimum and (d) maximum magnitudes?

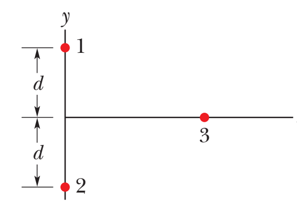



**Figure 21-32** Problem 23.

## Module 21-2 Charge Is Quantized

**•24** Two tiny, spherical water drops, with identical charges of $-1.00 \times 10^{-16}$ C, have a center-to-center separation of 1.00 cm. (a) What is the magnitude of the electrostatic force acting between them? (b) How many excess electrons are on each drop, giving it its charge imbalance?

**•25** ILW How many electrons would have to be removed from a coin to leave it with a charge of $+1.0 \times 10^{-7}$ C?

**•26** What is the magnitude of the electrostatic force between a singly charged sodium ion ($Na^+$, of charge $+e$) and an adjacent singly charged chlorine ion ($Cl^-$, of charge $-e$) in a salt crystal if their separation is $2.82 \times 10^{-10}$ m?

**•27** SSM The magnitude of the electrostatic force between two identical ions that are separated by a distance of $5.0 \times 10^{-10}$ m is $3.7 \times 10^{-9}$ N. (a) What is the charge of each ion? (b) How many electrons are "missing" from each ion (thus giving the ion its charge imbalance)?

**•28** A current of 0.300 A through your chest can send your heart into fibrillation, ruining the normal rhythm of heartbeat and disrupting the flow of blood (and thus oxygen) to your brain. If that current persists for 2.00 min, how many conduction electrons pass through your chest?

**••29** GO In Fig. 21-33, particles 2 and 4, of charge $-e$, are fixed in place on a *y* axis, at $y_2 = -10.0$ cm and

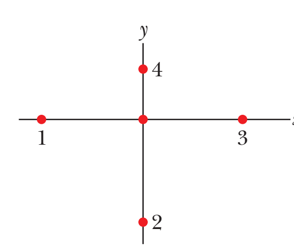



**Figure 21-33** Problem 29.

$y_4 = 5.00$ cm. Particles 1 and 3, of charge $-e$, can be moved along the $x$ axis. Particle 5, of charge $+e$, is fixed at the origin. Initially particle 1 is at $x_1 = -10.0$ cm and particle 3 is at $x_3 = 10.0$ cm. (a) To what $x$ value must particle 1 be moved to rotate the direction of the net electric force $\vec{F}_{net}$ on particle 5 by 30° counterclockwise? (b) With particle 1 fixed at its new position, to what $x$ value must you move particle 3 to rotate $\vec{F}_{net}$ back to its original direction?

**••30** In Fig. 21-26, particles 1 and 2 are fixed in place on an $x$ axis, at a separation of $L = 8.00$ cm. Their charges are $q_1 = +e$ and $q_2 = -27e$. Particle 3 with charge $q_3 = +4e$ is to be placed on the line between particles 1 and 2, so that they produce a net electrostatic force $\vec{F}_{3,net}$ on it. (a) At what coordinate should particle 3 be placed to minimize the magnitude of that force? (b) What is that minimum magnitude?

**••31** ILW Earth's atmosphere is constantly bombarded by *cosmic ray protons* that originate somewhere in space. If the protons all passed through the atmosphere, each square meter of Earth's surface would intercept protons at the average rate of 1500 protons per second. What would be the electric current intercepted by the total surface area of the planet?

**••32** GO Figure 21-34*a* shows charged particles 1 and 2 that are fixed in place on an $x$ axis. Particle 1 has a charge with a magnitude of $|q_1| = 8.00e$. Particle 3 of charge $q_3 = +8.00e$ is initially on the $x$ axis near particle 2. Then particle 3 is gradually moved in the positive direction of the $x$ axis. As a result, the magnitude of the net electrostatic force $\vec{F}_{2,net}$ on particle 2 due to particles 1 and 3 changes. Figure 21-34*b* gives the $x$ component of that net force as a function of the position $x$ of particle 3. The scale of the $x$ axis is set by $x_s = 0.80$ m. The plot has an asymptote of $F_{2,net} = 1.5 \times 10^{-25}$ N as $x \to \infty$. As a multiple of $e$ and including the sign, what is the charge $q_2$ of particle 2?

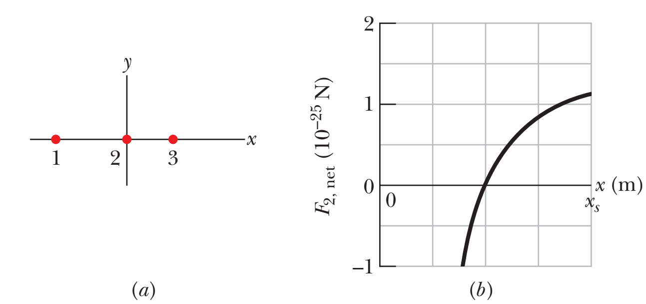



**Figure 21-34** Problem 32.

**••33** Calculate the number of coulombs of positive charge in 250 cm$^3$ of (neutral) water. (*Hint:* A hydrogen atom contains one proton; an oxygen atom contains eight protons.)

**•••34** GO Figure 21-35 shows electrons 1 and 2 on an $x$ axis and charged ions 3 and 4 of identical charge $-q$ and at identical angles $\theta$. Electron 2 is free to move; the other three particles are fixed in place at horizontal distances $R$ from electron 2 and are intended to hold electron 2 in place. For physically possible values of $q \le 5e$, what are the (a) smallest, (b) second smallest, and (c) third smallest values of $\theta$ for which electron 2 is held in place?

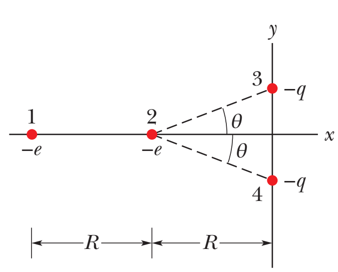



**Figure 21-35** Problem 34.

**•••35** SSM In crystals of the salt cesium chloride, cesium ions $Cs^+$ form the eight corners of a cube and a chlorine ion $Cl^-$ is at the cube's center (Fig. 21-36). The edge length of the cube is 0.40 nm. The $Cs^+$ ions are each deficient by one electron (and thus each has a charge of $+e$), and the $Cl^-$ ion has one excess electron (and thus has a charge of $-e$). (a) What is the magnitude of the net electrostatic force exerted on the $Cl^-$ ion by the eight $Cs^+$ ions at the corners of the cube? (b) If one of the $Cs^+$ ions is missing, the crystal is said to have a *defect*; what is the magnitude of the net electrostatic force exerted on the $Cl^-$ ion by the seven remaining $Cs^+$ ions?

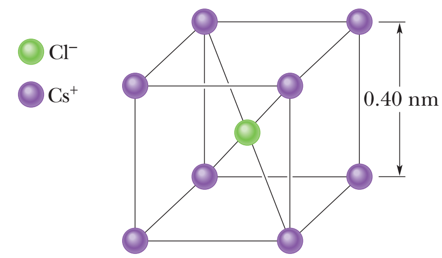



**Figure 21-36** Problem 35.

## Module 21-3 Charge Is Conserved

**•36** Electrons and positrons are produced by the nuclear transformations of protons and neutrons known as *beta decay.* (a) If a proton transforms into a neutron, is an electron or a positron produced? (b) If a neutron transforms into a proton, is an electron or a positron produced?

**•37** SSM Identify X in the following nuclear reactions: (a) $^1H + {}^9Be \to X + n$; (b) $^{12}C + {}^1H \to X$; (c) $^{15}N + {}^1H \to {}^4He + X$. Appendix F will help.

### Additional Problems

**38** GO Figure 21-37 shows four identical conducting spheres that are actually well separated from one another. Sphere $W$ (with an initial charge of zero) is touched to sphere $A$ and then they are separated. Next, sphere $W$ is touched to sphere $B$ (with an initial charge of $-32e$) and then they are separated. Finally, sphere $W$ is touched to sphere $C$ (with an initial charge of $+48e$), and then they are separated. The final charge on sphere $W$ is $+18e$. What was the initial charge on sphere $A$?

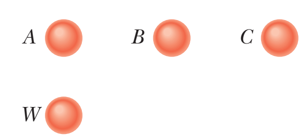



**Figure 21-37** Problem 38.

**39** SSM In Fig. 21-38, particle 1 of charge $+4e$ is above a floor by distance $d_1 = 2.00$ mm and particle 2 of charge $+6e$ is on the floor, at distance $d_2 = 6.00$ mm horizontally from particle 1. What is the $x$ component of the electrostatic force on particle 2 due to particle 1?

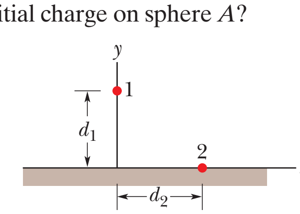



**Figure 21-38** Problem 39.

**40** In Fig. 21-23, particles 1 and 2 are fixed in place, but particle 3 is free to move. If the net electrostatic force on particle 3 due to particles 1 and 2 is zero and $L_{23} = 2.00L_{12}$, what is the ratio $q_1/q_2$?

**41** (a) What equal positive charges would have to be placed on Earth and on the Moon to neutralize their gravitational attraction? (b) Why don't you need to know the lunar distance to solve this problem? (c) How many kilograms of hydrogen ions (that is, protons) would be needed to provide the positive charge calculated in (a)?

**42** In Fig. 21-39, two tiny conducting balls of identical mass $m$ and identical charge $q$ hang from nonconducting threads of length $L$. Assume that $\theta$ is so small that $\tan\theta$ can be replaced by its approximate equal, $\sin\theta$. (a) Show that

$$x = \left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{1/3}$$

gives the equilibrium separation $x$ of the balls. (b) If $L = 120$ cm, $m = 10$ g, and $x = 5.0$ cm, what is $|q|$?

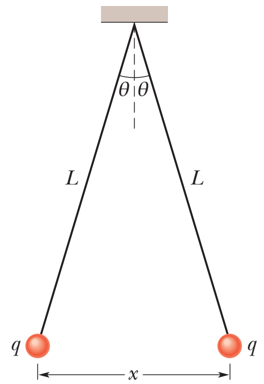



**Figure 21-39** Problems 42 and 43.

**43** (a) Explain what happens to the balls of Problem 42 if one of them is discharged (loses its charge $q$ to, say, the ground). (b) Find the new equilibrium separation $x$, using the given values of $L$ and $m$ and the computed value of $|q|$.

**44** SSM How far apart must two protons be if the magnitude of the electrostatic force acting on either one due to the other is equal to the magnitude of the gravitational force on a proton at Earth's surface?

**45** How many megacoulombs of positive charge are in 1.00 mol of neutral molecular-hydrogen gas ($H_2$)?

**46** In Fig. 21-40, four particles are fixed along an $x$ axis, separated by distances $d = 2.00$ cm. The charges are $q_1 = +2e$, $q_2 = -e$, $q_3 = +e$, and $q_4 = +4e$, with $e = 1.60 \times 10^{-19}$ C. In unit-vector notation, what is the net electrostatic force on (a) particle 1 and (b) particle 2 due to the other particles?

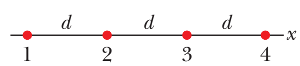



**Figure 21-40** Problem 46.

**47** GO Point charges of $+6.0\,\mu$C and $-4.0\,\mu$C are placed on an $x$ axis, at $x = 8.0$ m and $x = 16$ m, respectively. What charge must be placed at $x = 24$ m so that any charge placed at the origin would experience no electrostatic force?

**48** In Fig. 21-41, three identical conducting spheres form an equilateral triangle of side length $d = 20.0$ cm. The sphere radii are much smaller than $d$, and the sphere charges are $q_A = -2.00$ nC, $q_B = -4.00$ nC, and $q_C = +8.00$ nC. (a) What is the magnitude of the electrostatic force between spheres $A$ and $C$? The following steps are then taken: $A$ and $B$ are connected by a thin wire and then disconnected; $B$ is grounded by the wire, and the wire is then removed; $B$ and $C$ are connected by the wire and then disconnected. What now are the magnitudes of the electrostatic force (b) between spheres $A$ and $C$ and (c) between spheres $B$ and $C$?

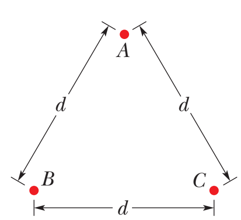



**Figure 21-41** Problem 48.

**49** A neutron consists of one "up" quark of charge $+2e/3$ and two "down" quarks each having charge $-e/3$. If we assume that the down quarks are $2.6 \times 10^{-15}$ m apart inside the neutron, what is the magnitude of the electrostatic force between them?

**50** Figure 21-42 shows a long, nonconducting, massless rod of length $L$, pivoted at its center and balanced with a block of weight $W$ at a distance $x$ from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges $q$ and $2q$, respectively. A distance $h$ directly beneath each of these spheres is a fixed sphere with positive charge $Q$. (a) Find the distance $x$ when the rod is horizontal and balanced. (b) What value should $h$ have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?

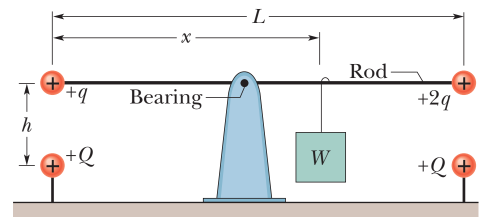



**Figure 21-42** Problem 50.

**51** A charged nonconducting rod, with a length of 2.00 m and a cross-sectional area of 4.00 cm$^2$, lies along the positive side of an $x$ axis with one end at the origin. The *volume charge density* $\rho$ is charge per unit volume in coulombs per cubic meter. How many excess electrons are on the rod if $\rho$ is (a) uniform, with a value of $-4.00\,\mu$C/m$^3$, and (b) nonuniform, with a value given by $\rho = bx^2$, where $b = -2.00\,\mu$C/m$^5$?

**52** A particle of charge $Q$ is fixed at the origin of an $xy$ coordinate system. At $t = 0$ a particle ($m = 0.800$ g, $q = 4.00\,\mu$C) is located on the $x$ axis at $x = 20.0$ cm, moving with a speed of 50.0 m/s in the positive $y$ direction. For what value of $Q$ will the moving particle execute circular motion? (Neglect the gravitational force on the particle.)

**53** What would be the magnitude of the electrostatic force between two 1.00 C point charges separated by a distance of (a) 1.00 m and (b) 1.00 km if such point charges existed (they do not) and this configuration could be set up?

**54** A charge of 6.0 $\mu$C is to be split into two parts that are then separated by 3.0 mm. What is the maximum possible magnitude of the electrostatic force between those two parts?

**55** Of the charge $Q$ on a tiny sphere, a fraction $\alpha$ is to be transferred to a second, nearby sphere. The spheres can be treated as particles. (a) What value of $\alpha$ maximizes the magnitude $F$ of the electrostatic force between the two spheres? What are the (b) smaller and (c) larger values of $\alpha$ that put $F$ at half the maximum magnitude?

**56** If a cat repeatedly rubs against your cotton slacks on a dry day, the charge transfer between the cat hair and the cotton can leave you with an excess charge of $-2.00\,\mu$C. (a) How many electrons are transferred between you and the cat?

You will gradually discharge via the floor, but if instead of waiting, you immediately reach toward a faucet, a painful spark can suddenly appear as your fingers near the faucet. (b) In that spark, do electrons flow from you to the faucet or vice versa? (c) Just before the spark appears, do you induce positive or negative charge in the faucet? (d) If, instead, the cat reaches a paw toward the faucet, which way do electrons flow in the resulting spark? (e) If you stroke a cat with a bare hand on a dry day, you should take care not to bring your fingers near the cat's nose or you will hurt it with a spark. Considering that cat hair is an insulator, explain how the spark can appear.

**57** We know that the negative charge on the electron and the positive charge on the proton are equal. Suppose, however, that these magnitudes differ from each other by 0.00010%. With what force would two copper coins, placed 1.0 m apart, repel each other? Assume that each coin contains $3 \times 10^{22}$ copper atoms. (*Hint:* A neutral copper atom contains 29 protons and 29 electrons.) What do you conclude?

**58** In Fig. 21-26, particle 1 of charge $-80.0\ \mu C$ and particle 2 of charge $+40.0\ \mu C$ are held at separation $L = 20.0$ cm on an $x$ axis. In unit-vector notation, what is the net electrostatic force on particle 3, of charge $q_3 = 20.0\ \mu C$, if particle 3 is placed at (a) $x = 40.0$ cm and (b) $x = 80.0$ cm? What should be the (c) $x$ and (d) $y$ coordinates of particle 3 if the net electrostatic force on it due to particles 1 and 2 is zero?

**59** What is the total charge in coulombs of 75.0 kg of electrons?

**60** GO In Fig. 21-43, six charged particles surround particle 7 at radial distances of either $d = 1.0$ cm or $2d$, as drawn. The charges are $q_1 = +2e$, $q_2 = +4e$, $q_3 = +e$, $q_4 = +4e$, $q_5 = +2e$, $q_6 = +8e$, $q_7 = +6e$, with $e = 1.60 \times 10^{-19}$ C. What is the magnitude of the net electrostatic force on particle 7?

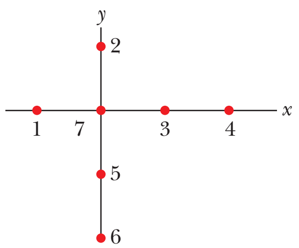


**Figure 21-43** Problem 60.

**61** Three charged particles form a triangle: particle 1 with charge $Q_1 = 80.0$ nC is at $xy$ coordinates (0, 3.00 mm), particle 2 with charge $Q_2$ is at (0, −3.00 mm), and particle 3 with charge $q = 18.0$ nC is at (4.00 mm, 0). In unit-vector notation, what is the electrostatic force on particle 3 due to the other two particles if $Q_2$ is equal to (a) 80.0 nC and (b) −80.0 nC?

**62** SSM In Fig. 21-44, what are the (a) magnitude and (b) direction of the net electrostatic force on particle 4 due to the other three particles? All four particles are fixed in the $xy$ plane, and $q_1 = -3.20 \times 10^{-19}$ C, $q_2 = +3.20 \times 10^{-19}$ C, $q_3 = +6.40 \times 10^{-19}$ C, $q_4 = +3.20 \times 10^{-19}$ C, $\theta_1 = 35.0°$, $d_1 = 3.00$ cm, and $d_2 = d_3 = 2.00$ cm.

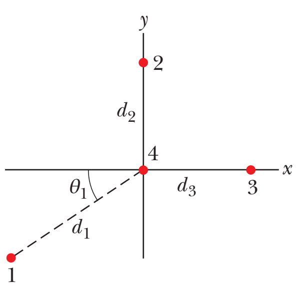


**Figure 21-44** Problem 62.

**63** Two point charges of 30 nC and −40 nC are held fixed on an $x$ axis, at the origin and at $x = 72$ cm, respectively. A particle with a charge of 42 $\mu$C is released from rest at $x = 28$ cm. If the initial acceleration of the particle has a magnitude of 100 km/s$^2$, what is the particle's mass?

**64** Two small, positively charged spheres have a combined charge of $5.0 \times 10^{-5}$ C. If each sphere is repelled from the other by an electrostatic force of 1.0 N when the spheres are 2.0 m apart, what is the charge on the sphere with the smaller charge?

**65** The initial charges on the three identical metal spheres in Fig. 21-24 are the following: sphere $A$, $Q$; sphere $B$, $-Q/4$; and sphere $C$, $Q/2$, where $Q = 2.00 \times 10^{-14}$ C. Spheres $A$ and $B$ are fixed in place, with a center-to-center separation of $d = 1.20$ m, which is much larger than the spheres. Sphere $C$ is touched first to sphere $A$ and then to sphere $B$ and is then removed. What then is the magnitude of the electrostatic force between spheres $A$ and $B$?

**66** An electron is in a vacuum near Earth's surface and located at $y = 0$ on a vertical $y$ axis. At what value of $y$ should a second electron be placed such that its electrostatic force on the first electron balances the gravitational force on the first electron?

**67** SSM In Fig. 21-26, particle 1 of charge $-5.00q$ and particle 2 of charge $+2.00q$ are held at separation $L$ on an $x$ axis. If particle 3 of unknown charge $q_3$ is to be located such that the net electrostatic force on it from particles 1 and 2 is zero, what must be the (a) $x$ and (b) $y$ coordinates of particle 3?

**68** Two engineering students, John with a mass of 90 kg and Mary with a mass of 45 kg, are 30 m apart. Suppose each has a 0.01% imbalance in the amount of positive and negative charge, one student being positive and the other negative. Find the order of magnitude of the electrostatic force of attraction between them by replacing each student with a sphere of water having the same mass as the student.

**69** In the radioactive decay of Eq. 21-13, a $^{238}$U nucleus transforms to $^{234}$Th and an ejected $^{4}$He. (These are nuclei, not atoms, and thus electrons are not involved.) When the separation between $^{234}$Th and $^{4}$He is $9.0 \times 10^{-15}$ m, what are the magnitudes of (a) the electrostatic force between them and (b) the acceleration of the $^{4}$He particle?

**70** In Fig. 21-25, four particles form a square. The charges are $q_1 = +Q$, $q_2 = q_3 = q$, and $q_4 = -2.00Q$. What is $q/Q$ if the net electrostatic force on particle 1 is zero?

**71** In a spherical metal shell of radius $R$, an electron is shot from the center directly toward a tiny hole in the shell, through which it escapes. The shell is negatively charged with a *surface charge density* (charge per unit area) of $6.90 \times 10^{-13}$ C/m$^2$. What is the magnitude of the electron's acceleration when it reaches radial distances (a) $r = 0.500R$ and (b) $2.00R$?

**72** An electron is projected with an initial speed $v_i = 3.2 \times 10^5$ m/s directly toward a very distant proton that is at rest. Because the proton mass is large relative to the electron mass, assume that the proton remains at rest. By calculating the work done on the electron by the electrostatic force, determine the distance between the two particles when the electron instantaneously has speed $2v_i$.

**73** In an early model of the hydrogen atom (the *Bohr model*), the electron orbits the proton in uniformly circular motion. The radius of the circle is restricted (*quantized*) to certain values given by

$$r = n^2 a_0, \quad \text{for } n = 1, 2, 3, \ldots,$$

where $a_0 = 52.92$ pm. What is the speed of the electron if it orbits in (a) the smallest allowed orbit and (b) the second smallest orbit? (c) If the electron moves to larger orbits, does its speed increase, decrease, or stay the same?

**74** A 100 W lamp has a steady current of 0.83 A in its filament. How long is required for 1 mol of electrons to pass through the lamp?

**75** The charges of an electron and a positron are $-e$ and $+e$. The mass of each is $9.11 \times 10^{-31}$ kg. What is the ratio of the electrical force to the gravitational force between an electron and a positron?

CHAPTER 22

# Electric Fields

## 22-1 THE ELECTRIC FIELD

### Learning Objectives

*After reading this module, you should be able to . . .*

**22.01** Identify that at every point in the space surrounding a charged particle, the particle sets up an electric field $\vec{E}$, which is a vector quantity and thus has both magnitude and direction.

**22.02** Identify how an electric field $\vec{E}$ can be used to explain how a charged particle can exert an electrostatic force $\vec{F}$ on a second charged particle even though there is no contact between the particles.

**22.03** Explain how a small positive test charge is used (in principle) to measure the electric field at any given point.

**22.04** Explain electric field lines, including where they originate and terminate and what their spacing represents.

### Key Ideas

- A charged particle sets up an electric field (a vector quantity) in the surrounding space. If a second charged particle is located in that space, an electrostatic force acts on it due to the magnitude and direction of the field at its location.
- The electric field $\vec{E}$ at any point is defined in terms of the electrostatic force $\vec{F}$ that would be exerted on a positive test charge $q_0$ placed there:

$$\vec{E} = \frac{\vec{F}}{q_0}.$$

- Electric field lines help us visualize the direction and magnitude of electric fields. The electric field vector at any point is tangent to the field line through that point. The density of field lines in that region is proportional to the magnitude of the electric field there. Thus, closer field lines represent a stronger field.
- Electric field lines originate on positive charges and terminate on negative charges. So, a field line extending from a positive charge must end on a negative charge.

## What Is Physics?

Figure 22-1 shows two positively charged particles. From the preceding chapter we know that an electrostatic force acts on particle 1 due to the presence of particle 2. We also know the force direction and, given some data, we can calculate the force magnitude. However, here is a leftover nagging question. How does particle 1 "know" of the presence of particle 2? That is, since the particles do not touch, how can particle 2 push on particle 1—how can there be such an *action at a distance*?

One purpose of physics is to record observations about our world, such as the magnitude and direction of the push on particle 1. Another purpose is to provide an explanation of what is recorded. Our purpose in this chapter is to provide such an explanation to this nagging question about electric force at a distance.

The explanation that we shall examine here is this: Particle 2 sets up an **electric field** at all points in the surrounding space, even if the space is a vacuum. If we place particle 1 at any point in that space, particle 1 knows of the presence of particle 2 because it is affected by the electric field particle 2 has already set up at that point. Thus, particle 2 pushes on particle 1 not by touching it as you would push on a coffee mug by making contact. Instead, particle 2 pushes by means of the electric field it has set up.


**Figure 22-1** How does charged particle 2 push on charged particle 1 when they have no contact?

Our goals in this chapter are to (1) define electric field, (2) discuss how to calculate it for various arrangements of charged particles and objects, and (3) discuss how an electric field can affect a charged particle (as in making it move).

## The Electric Field

A lot of different fields are used in science and engineering. For example, a *temperature field* for an auditorium is the distribution of temperatures we would find by measuring the temperature at many points within the auditorium. Similarly, we could define a *pressure field* in a swimming pool. Such fields are examples of *scalar fields* because temperature and pressure are scalar quantities, having only magnitudes and not directions.

In contrast, an electric field is a *vector field* because it is responsible for conveying the information for a force, which involves both magnitude and direction. This field consists of a distribution of electric field vectors $\vec{E}$, one for each point in the space around a charged object. In principle, we can define $\vec{E}$ at some point near the charged object, such as point $P$ in Fig. 22-2*a*, with this procedure: At $P$, we place a particle with a small positive charge $q_0$, called a *test charge* because we use it to test the field. (We want the charge to be small so that it does not disturb the object's charge distribution.) We then measure the electrostatic force $\vec{F}$ that acts on the test charge. The electric field at that point is then

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{(electric field).} \tag{22-1}$$

Because the test charge is positive, the two vectors in Eq. 22-1 are in the same direction, so the direction of $\vec{E}$ is the direction we measure for $\vec{F}$. The magnitude of $\vec{E}$ at point $P$ is $F/q_0$. As shown in Fig. 22-2*b*, we always represent an electric field with an arrow with its tail anchored on the point where the measurement is made. (This may sound trivial, but drawing the vectors any other way usually results in errors. Also, another common error is to mix up the terms *force* and *field* because they both start with the letter f. Electric force is a push or pull. Electric field is an abstract property set up by a charged object.) From Eq. 22-1, we see that the SI unit for the electric field is the newton per coulomb (N/C).

We can shift the test charge around to various other points, to measure the electric fields there, so that we can figure out the distribution of the electric field set up by the charged object. That field exists independent of the test charge. It is something that a charged object sets up in the surrounding space (even vacuum), independent of whether we happen to come along to measure it.

For the next several modules, we determine the field around charged particles and various charged objects. First, however, let's examine a way of visualizing electric fields.

## Electric Field Lines

Look at the space in the room around you. Can you visualize a field of vectors throughout that space—vectors with different magnitudes and directions? As impossible as that seems, Michael Faraday, who introduced the idea of electric fields in the 19th century, found a way. He envisioned lines, now called **electric field lines**, in the space around any given charged particle or object.

Figure 22-3 gives an example in which a sphere is uniformly covered with negative charge. If we place a positive test charge at any point near the sphere (Fig. 22-3*a*), we find that an electrostatic force pulls on it toward the center of the sphere. Thus at every point around the sphere, an electric field vector points radially inward toward the sphere. We can represent this electric field with

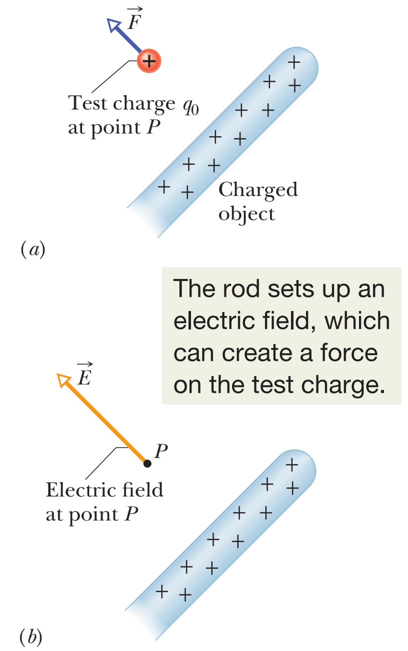



**Figure 22-2** (*a*) A positive test charge $q_0$ placed at point $P$ near a charged object. An electrostatic force $\vec{F}$ acts on the test charge. (*b*) The electric field $\vec{E}$ at point $P$ produced by the charged object.

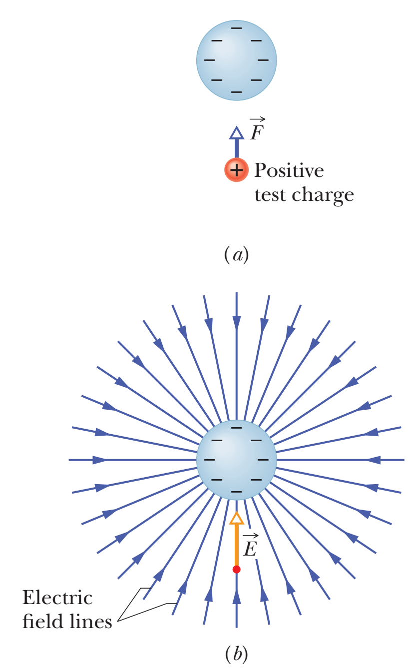



**Figure 22-3** (*a*) The electrostatic force $\vec{F}$ acting on a positive test charge near a sphere of uniform negative charge. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend *toward* the negatively charged sphere. (They originate on distant positive charges.)

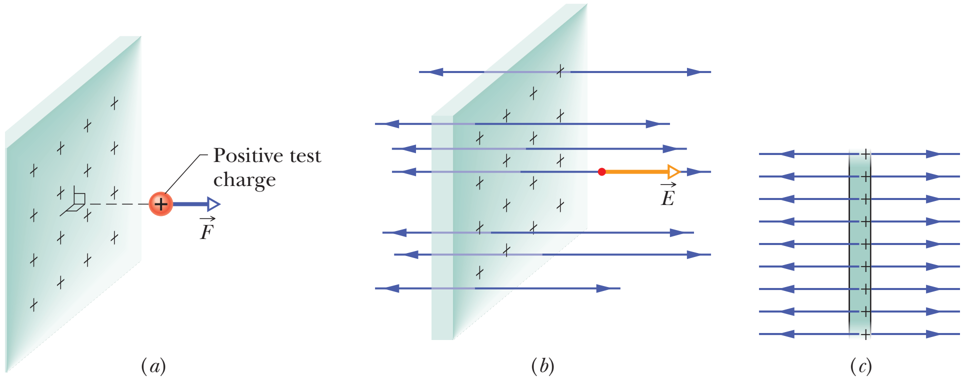



**Figure 22-4** (*a*) The force on a positive test charge near a very large, nonconducting sheet with uniform positive charge on one side. (*b*) The electric field vector $\vec{E}$ at the test charge's location, and the nearby electric field lines, extending away from the sheet. (*c*) Side view.

electric field lines as in Fig. 22-3*b*. At any point, such as the one shown, the direction of the field line through the point matches the direction of the electric vector at that point.

The rules for drawing electric fields lines are these: (1) At any point, the electric field vector must be tangent to the electric field line through that point and in the same direction. (This is easy to see in Fig. 22-3 where the lines are straight, but we'll see some curved lines soon.) (2) In a plane perpendicular to the field lines, the relative density of the lines represents the relative magnitude of the field there, with greater density for greater magnitude.

If the sphere in Fig. 22-3 were uniformly covered with positive charge, the electric field vectors at all points around it would be radially outward and thus so would the electric field lines. So, we have the following rule:

> Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

In Fig. 22-3*b*, they originate on distant positive charges that are not shown.

For another example, Fig. 22-4*a* shows part of an infinitely large, nonconducting *sheet* (or plane) with a uniform distribution of positive charge on one side. If we place a positive test charge at any point near the sheet (on either side), we find that the electrostatic force on the particle is outward and perpendicular to the sheet. The perpendicular orientation is reasonable because any force component that is, say, upward is balanced out by an equal component that is downward. That leaves only outward, and thus the electric field vectors and the electric field lines must also be outward and perpendicular to the sheet, as shown in Figs. 22-4*b* and *c*.

Because the charge on the sheet is uniform, the field vectors and the field lines are also. Such a field is a *uniform electric field*, meaning that the electric field has the same magnitude and direction at every point within the field. (This is a lot easier to work with than a *nonuniform field*, where there is variation from point to point.) Of course, there is no such thing as an infinitely large sheet. That is just a way of saying that we are measuring the field at points close to the sheet relative to the size of the sheet and that we are not near an edge.

Figure 22-5 shows the field lines for two particles with equal positive charges. Now the field lines are curved, but the rules still hold: (1) the electric field vector at any given point must be tangent to the field line at that point and in the same direction, as shown for one vector, and (2) a closer spacing means a larger field magnitude. To imagine the full three-dimensional pattern of field lines around the particles, mentally rotate the pattern in Fig. 22-5 around the *axis of symmetry*, which is a vertical line through both particles.

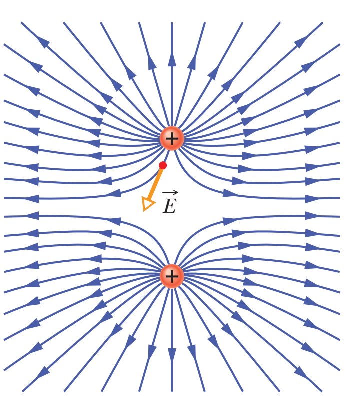



**Figure 22-5** Field lines for two particles with equal positive charge. Doesn't the pattern itself suggest that the particles repel each other?

# 22-2 THE ELECTRIC FIELD DUE TO A CHARGED PARTICLE

## Learning Objectives

*After reading this module, you should be able to . . .*

**22.05** In a sketch, draw a charged particle, indicate its sign, pick a nearby point, and then draw the electric field vector $\vec{E}$ at that point, with its tail anchored on the point.

**22.06** For a given point in the electric field of a charged particle, identify the direction of the field vector $\vec{E}$ when the particle is positively charged and when it is negatively charged.

**22.07** For a given point in the electric field of a charged particle, apply the relationship between the field magnitude $E$, the charge magnitude $|q|$, and the distance $r$ between the point and the particle.

**22.08** Identify that the equation given here for the magnitude of an electric field applies only to a particle, not an extended object.

**22.09** If more than one electric field is set up at a point, draw each electric field vector and then find the net electric field by adding the individual electric fields as vectors (not as scalars).

## Key Ideas

- The magnitude of the electric field $\vec{E}$ set up by a particle with charge $q$ at distance $r$ from the particle is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$

- The electric field vectors set up by a positively charged particle all point directly away from the particle. Those set up by a negatively charged particle all point directly toward the particle.
- If more than one charged particle sets up an electric field at a point, the net electric field is the *vector* sum of the individual electric fields—electric fields obey the superposition principle.

## The Electric Field Due to a Point Charge

To find the electric field due to a charged particle (often called a *point charge*), we place a positive test charge at any point near the particle, at distance $r$. From Coulomb's law (Eq. 21-4), the force on the test charge due to the particle with charge $q$ is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{\mathrm{r}}.$$

As previously, the direction of $\vec{F}$ is directly away from the particle if $q$ is positive (because $q_0$ is positive) and directly toward it if $q$ is negative. From Eq. 22-1, we can now write the electric field set up by the particle (at the location of the test charge) as

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathrm{r}} \quad \text{(charged particle).} \tag{22-2}$$

Let's think through the directions again. The direction of $\vec{E}$ matches that of the force on the positive test charge: directly away from the point charge if $q$ is positive and directly toward it if $q$ is negative.

So, if given another charged particle, we can immediately determine the directions of the electric field vectors near it by just looking at the sign of the charge $q$. We can find the magnitude at any given distance $r$ by converting Eq. 22-2 to a magnitude form:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2} \quad \text{(charged particle).} \tag{22-3}$$

We write $|q|$ to avoid the danger of getting a negative $E$ when $q$ is negative, and then thinking the negative sign has something to do with direction. Equation 22-3 gives magnitude $E$ only. We must think about the direction separately.

Figure 22-6 gives a number of electric field vectors at points around a positively charged particle, but be careful. Each vector represents the vector quantity at the

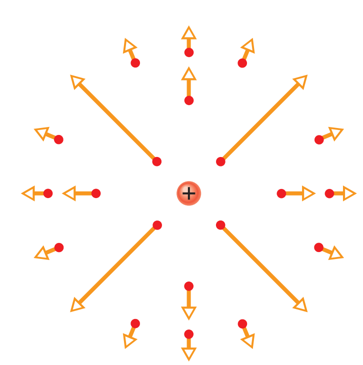



**Figure 22-6** The electric field vectors at various points around a positive point charge.

point where the tail of the arrow is anchored. The vector is not something that stretches from a "here" to a "there" as with a displacement vector.

In general, if several electric fields are set up at a given point by several charged particles, we can find the net field by placing a positive test particle at the point and then writing out the force acting on it due to each particle, such as $\vec{F}_{01}$ due to particle 1. Forces obey the principle of superposition, so we just add the forces as vectors:

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

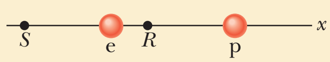
To change over to electric field, we repeatedly use Eq. 22-1 for each of the individual forces:

$$\begin{aligned}\vec{E} &= \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} \\ &= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.\end{aligned} \tag{22-4}$$

This tells us that electric fields also obey the principle of superposition. If you want the net electric field at a given point due to several particles, find the electric field due to each particle (such as $\vec{E}_1$ due to particle 1) and then sum the fields as vectors. (As with electrostatic forces, you cannot just willy-nilly add up the magnitudes.) This addition of fields is the subject of many of the homework problems.

### Checkpoint 1

The figure here shows a proton p and an electron e on an $x$ axis. What is the direction of the electric field due to the electron at (a) point $S$ and (b) point $R$? What is the direction of the net electric field at (c) point $R$ and (d) point $S$?




## Sample Problem 22.01 Net electric field due to three charged particles

Figure 22-7$a$ shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance $d$ from the origin. What net electric field $\vec{E}$ is produced at the origin?

### KEY IDEA

Charges $q_1$, $q_2$, and $q_3$ produce electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, respectively, at the origin, and the net electric field is the vector sum $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. To find this sum, we first must find the magnitudes and orientations of the three field vectors.

***Magnitudes and directions:*** To find the magnitude of $\vec{E}_1$, which is due to $q_1$, we use Eq. 22-3, substituting $d$ for $r$ and $2Q$ for $q$ and obtaining

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

Similarly, we find the magnitudes of $\vec{E}_2$ and $\vec{E}_3$ to be

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

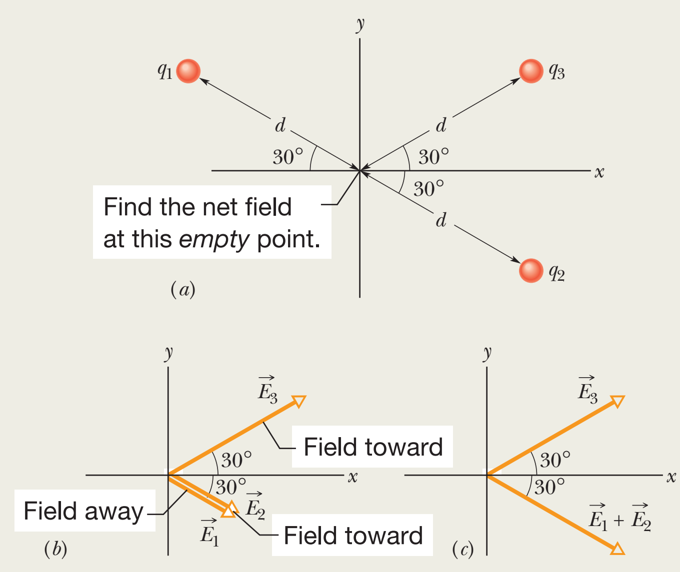



**Figure 22-7** ($a$) Three particles with charges $q_1$, $q_2$, and $q_3$ are at the same distance $d$ from the origin. ($b$) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. ($c$) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

We next must find the orientations of the three electric field vectors at the origin. Because $q_1$ is a positive charge, the field vector it produces points directly *away* from it, and because $q_2$ and $q_3$ are both negative, the field vectors they produce point directly *toward* each of them. Thus, the three electric fields produced at the origin by the three charged particles are oriented as in Fig. 22-7*b*. (*Caution:* Note that we have placed the tails of the vectors at the point where the fields are to be evaluated; doing so decreases the chance of error. Error becomes very probable if the tails of the field vectors are placed on the particles creating the fields.)

*Adding the fields:* We can now add the fields vectorially just as we added force vectors in Chapter 21. However, here we can use symmetry to simplify the procedure. From Fig. 22-7*b*, we see that electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction. Hence, their vector sum has that direction and has the magnitude

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

which happens to equal the magnitude of field $\vec{E}_3$.

We must now combine two vectors, $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$, that have the same magnitude and that are oriented symmetrically about the $x$ axis, as shown in Fig. 22-7*c*. From the symmetry of Fig. 22-7*c*, we realize that the equal $y$ components of our two vectors cancel (one is upward and the other is downward) and the equal $x$ components add (both are rightward). Thus, the net electric field $\vec{E}$ at the origin is in the positive direction of the $x$ axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30^\circ = (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}. \quad \text{(Answer)}$$

WILEY PLUS Additional examples, video, and practice available at *WileyPLUS*

# 22-3 THE ELECTRIC FIELD DUE TO A DIPOLE

## Learning Objectives

*After reading this module, you should be able to . . .*

**22.10** Draw an electric dipole, identifying the charges (sizes and signs), dipole axis, and direction of the electric dipole moment.

**22.11** Identify the direction of the electric field at any given point along the dipole axis, including between the charges.

**22.12** Outline how the equation for the electric field due to an electric dipole is derived from the equations for the electric field due to the individual charged particles that form the dipole.

**22.13** For a single charged particle and an electric dipole, compare the rate at which the electric field magnitude decreases with increase in distance. That is, identify which drops off faster.

**22.14** For an electric dipole, apply the relationship between the magnitude $p$ of the dipole moment, the separation $d$ between the charges, and the magnitude $q$ of either of the charges.

**22.15** For any distant point along a dipole axis, apply the relationship between the electric field magnitude $E$, the distance $z$ from the center of the dipole, and either the dipole moment magnitude $p$ or the product of charge magnitude $q$ and charge separation $d$.

## Key Ideas

- An electric dipole consists of two particles with charges of equal magnitude $q$ but opposite signs, separated by a small distance $d$.
- The electric dipole moment $\vec{p}$ has magnitude $qd$ and points from the negative charge to the positive charge.
- The magnitude of the electric field set up by an electric dipole at a distant point on the dipole axis (which runs through both particles) can be written in terms of either the product $qd$ or the magnitude $p$ of the dipole moment:

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3},$$

where $z$ is the distance between the point and the center of the dipole.

- Because of the $1/z^3$ dependence, the field magnitude of an electric dipole decreases more rapidly with distance than the field magnitude of either of the individual charges forming the dipole, which depends on $1/r^2$.

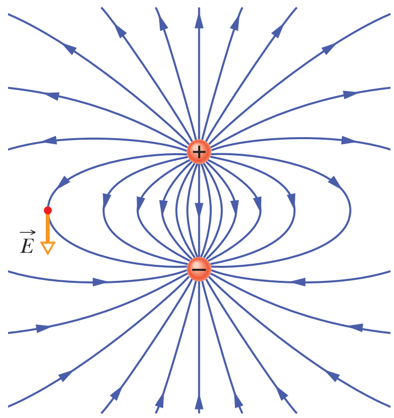



**Figure 22-8** The pattern of electric field lines around an electric dipole, with an electric field vector $\vec{E}$ shown at one point (tangent to the field line through that point).

## The Electric Field Due to an Electric Dipole

Figure 22-8 shows the pattern of electric field lines for two particles that have the same charge magnitude $q$ but opposite signs, a very common and important arrangement known as an **electric dipole**. The particles are separated by distance $d$ and lie along the *dipole axis*, an axis of symmetry around which you can imagine rotating the pattern in Fig. 22-8. Let's label that axis as a $z$ axis. Here we restrict our interest to the magnitude and direction of the electric field $\vec{E}$ at an arbitrary point $P$ along the dipole axis, at distance $z$ from the dipole's midpoint.

Figure 22-9*a* shows the electric fields set up at $P$ by each particle. The nearer particle with charge $+q$ sets up field $E_{(+)}$ in the positive direction of the $z$ axis (directly away from the particle). The farther particle with charge $-q$ sets up a smaller field $E_{(-)}$ in the negative direction (directly toward the particle). We want the net field at $P$, as given by Eq. 22-4. However, because the field vectors are along the same axis, let's simply indicate the vector directions with plus and minus signs, as we commonly do with forces along a single axis. Then we can write the magnitude of the net field at $P$ as

$$\begin{aligned} E &= E_{(+)} - E_{(-)} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2} \\ &= \frac{q}{4\pi\varepsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z+\frac{1}{2}d)^2}. \end{aligned} \tag{22-5}$$

After a little algebra, we can rewrite this equation as

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1-\dfrac{d}{2z}\right)^2} - \frac{1}{\left(1+\dfrac{d}{2z}\right)^2}\right). \tag{22-6}$$

After forming a common denominator and multiplying its terms, we come to

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1-\left(\dfrac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1-\left(\dfrac{d}{2z}\right)^2\right)^2}. \tag{22-7}$$

We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole—that is, at distances such that $z \gg d$. At such large distances, we have $d/2z \ll 1$ in Eq. 22-7. Thus, in our approximation, we can neglect the $d/2z$ term in the denominator, which leaves us with

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}. \tag{22-8}$$

The product $qd$, which involves the two intrinsic properties $q$ and $d$ of the dipole, is the magnitude $p$ of a vector quantity known as the **electric dipole moment** $\vec{p}$ of the dipole. (The unit of $\vec{p}$ is the coulomb-meter.) Thus, we can write Eq. 22-8 as

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole)}. \tag{22-9}$$

The direction of $\vec{p}$ is taken to be from the negative to the positive end of the dipole, as indicated in Fig. 22-9*b*. We can use the direction of $\vec{p}$ to specify the orientation of a dipole.

Equation 22-9 shows that, if we measure the electric field of a dipole only at distant points, we can never find $q$ and $d$ separately; instead, we can find only

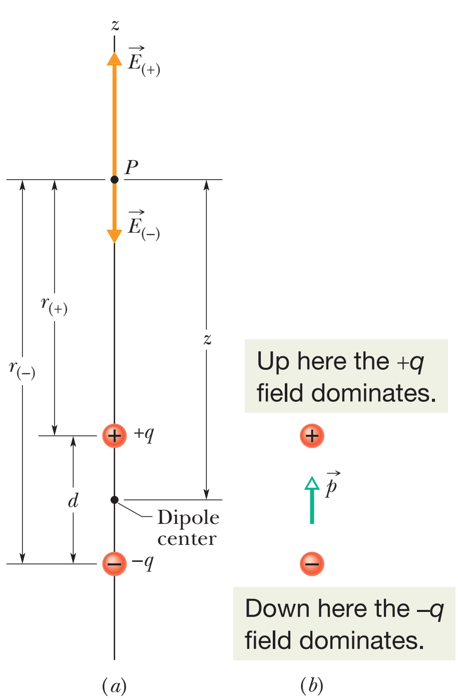



**Figure 22-9** (*a*) An electric dipole. The electric field vectors $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at point $P$ on the dipole axis result from the dipole's two charges. Point $P$ is at distances $r_{(+)}$ and $r_{(-)}$ from the individual charges that make up the dipole. (*b*) The dipole moment $\vec{p}$ of the dipole points from the negative charge to the positive charge.

their product. The field at distant points would be unchanged if, for example, $q$ were doubled and $d$ simultaneously halved. Although Eq. 22-9 holds only for distant points along the dipole axis, it turns out that $E$ for a dipole varies as $1/r^3$ for *all* distant points, regardless of whether they lie on the dipole axis; here $r$ is the distance between the point in question and the dipole center.

Inspection of Fig. 22-9 and of the field lines in Fig. 22-8 shows that the direction of $\vec{E}$ for distant points on the dipole axis is always the direction of the dipole moment vector $\vec{p}$. This is true whether point $P$ in Fig. 22-9*a* is on the upper or the lower part of the dipole axis.

Inspection of Eq. 22-9 shows that if you double the distance of a point from a dipole, the electric field at the point drops by a factor of 8. If you double the distance from a single point charge, however (see Eq. 22-3), the electric field drops only by a factor of 4. Thus the electric field of a dipole decreases more rapidly with distance than does the electric field of a single charge. The physical reason for this rapid decrease in electric field for a dipole is that from distant points a dipole looks like two particles that almost — but not quite — coincide. Thus, because they have charges of equal magnitude but opposite signs, their electric fields at distant points almost — but not quite — cancel each other.

## Sample Problem 22.02 Electric dipole and atmospheric sprites

Sprites (Fig. 22-10*a*) are huge flashes that occur far above a large thunderstorm. They were seen for decades by pilots flying at night, but they were so brief and dim that most pilots figured they were just illusions. Then in the 1990s sprites were captured on video. They are still not well understood but are believed to be produced when especially powerful lightning occurs between the ground and storm clouds, particularly when the lightning transfers a huge amount of negative charge $-q$ from the ground to the base of the clouds (Fig. 22-10*b*).

Just after such a transfer, the ground has a complicated distribution of positive charge. However, we can model the electric field due to the charges in the clouds and the ground by assuming a vertical electric dipole that has charge $-q$ at cloud height $h$ and charge $+q$ at below-ground depth $h$ (Fig. 22-10*c*). If $q = 200$ C and $h = 6.0$ km, what is the magnitude of the dipole's electric field at altitude $z_1 = 30$ km somewhat above the clouds and altitude $z_2 = 60$ km somewhat above the stratosphere?

### KEY IDEA

We can approximate the magnitude $E$ of an electric dipole's electric field on the dipole axis with Eq. 22-8.

***Calculations:*** We write that equation as

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q(2h)}{z^3},$$

where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-10*c*. For the electric field at altitude $z_1 = 30$ km, we find

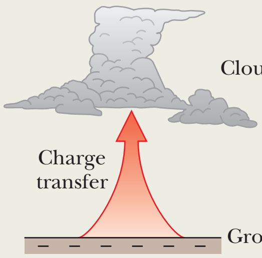
$$E = \frac{1}{2\pi\varepsilon_0}\frac{(200\text{ C})(2)(6.0\times 10^3\text{ m})}{(30\times 10^3\text{ m})^3}$$

$$= 1.6 \times 10^3\text{ N/C}. \qquad \text{(Answer)}$$

Similarly, for altitude $z_2 = 60$ km, we find

$$E = 2.0 \times 10^2\text{ N/C}. \qquad \text{(Answer)}$$

As we discuss in Module 22-6, when the magnitude of

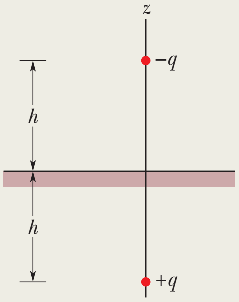




**Figure 22-10** (*a*) Photograph of a sprite. (*b*) Lightning in which a large amount of negative charge is transferred from ground to cloud base. (*c*) The cloud–ground system modeled as a vertical electric dipole.

an electric field exceeds a certain critical value $E_c$, the field can pull electrons out of atoms (ionize the atoms), and then the freed electrons can run into other atoms, causing those atoms to emit light. The value of $E_c$ depends on the density of the air in which the electric field exists. At altitude $z_2 = 60$ km the density of the air is so low that $E = 2.0 \times 10^2$ N/C exceeds $E_c$, and thus light is emitted by the atoms in the air. That light forms sprites. Lower down, just above the clouds at $z_1 = 30$ km, the density of the air is much higher, $E = 1.6 \times 10^3$ N/C does not exceed $E_c$, and no light is emitted. Hence, sprites occur only far above storm clouds.



# 22-4 THE ELECTRIC FIELD DUE TO A LINE OF CHARGE

## Learning Objectives

*After reading this module, you should be able to . . .*

**22.16** For a uniform distribution of charge, find the linear charge density $\lambda$ for charge along a line, the surface charge density $\sigma$ for charge on a surface, and the volume charge density $\rho$ for charge in a volume.

**22.17** For charge that is distributed uniformly along a line, find the net electric field at a given point near the line by splitting the distribution up into charge elements $dq$ and then summing (by integration) the electric field vectors $d\vec{E}$ set up at the point by each element.

**22.18** Explain how symmetry can be used to simplify the calculation of the electric field at a point near a line of uniformly distributed charge.

## Key Ideas

- The equation for the electric field set up by a particle does not apply to an extended object with charge (said to have a continuous charge distribution).
- To find the electric field of an extended object at a point, we first consider the electric field set up by a charge element $dq$ in the object, where the element is small enough for us to apply the equation for a particle. Then we sum, via integration, components of the electric fields $d\vec{E}$ from all the charge elements.
- Because the individual electric fields $d\vec{E}$ have different magnitudes and point in different directions, we first see if symmetry allows us to cancel out any of the components of the fields, to simplify the integration.

## The Electric Field Due to a Line of Charge

So far we have dealt with only charged particles, a single particle or a simple collection of them. We now turn to a much more challenging situation in which a thin (approximately one-dimensional) object such as a rod or ring is charged with a huge number of particles, more than we could ever even count. In the next module, we consider two-dimensional objects, such as a disk with charge spread over a surface. In the next chapter we tackle three-dimensional objects, such as a sphere with charge spread through a volume.

***Heads Up.*** Many students consider this module to be the most difficult in the book for a variety of reasons. There are lots of steps to take, a lot of vector features to keep track of, and after all that, we set up and then solve an integral. The worst part, however, is that the procedure can be different for different arrangements of the charge. Here, as we focus on a particular arrangement (a charged ring), be aware of the general approach, so that you can tackle other arrangements in the homework (such as rods and partial circles).

Figure 22-11 shows a thin ring of radius $R$ with a uniform distribution of positive charge along its circumference. It is made of plastic, which means that the charge is fixed in place. The ring is surrounded by a pattern of electric field lines, but here we restrict our interest to an arbitrary point $P$ on the central axis (the axis through the ring's center and perpendicular to the plane of the ring), at distance $z$ from the center point.

The charge of an extended object is often conveyed in terms of a charge density rather than the total charge. For a line of charge, we use the *linear charge* *density* $\lambda$ (the charge per unit length), with the SI unit of coulomb per meter. Table 22-1 shows the other charge densities that we shall be using for charged surfaces and volumes.

***First Big Problem.*** So far, we have an equation for the electric field of a particle. (We can combine the field of several particles as we did for the electric dipole to generate a special equation, but we are still basically using Eq. 22-3.) Now take a look at the ring in Fig. 22-11. That clearly is not a particle and so Eq. 22-3 does not apply. So what do we do?

The answer is to mentally divide the ring into differential elements of charge that are so small that we can treat them as though they *are* particles. Then we *can* apply Eq. 22-3.

***Second Big Problem.*** We now know to apply Eq. 22-3 to each charge element $dq$ (the front $d$ emphasizes that the charge is very small) and can write an expression for its contribution of electric field $d\vec{E}$ (the front $d$ emphasizes that the contribution is very small). However, each such contributed field vector at $P$ is in its own direction. How can we add them to get the net field at $P$?

The answer is to split the vectors into components and then separately sum one set of components and then the other set. However, first we check to see if one set simply all cancels out. (Canceling out components saves lots of work.)

***Third Big Problem.*** There is a huge number of $dq$ elements in the ring and thus a huge number of $d\vec{E}$ components to add up, even if we can cancel out one set of components. How can we add up more components than we could even count? The answer is to add them by means of integration.

***Do It.*** Let's do all this (but again, be aware of the general procedure, not just the fine details). We arbitrarily pick the charge element shown in Fig. 22-11. Let $ds$ be the arc length of that (or any other) $dq$ element. Then in terms of the linear density $\lambda$ (the charge per unit length), we have

$$dq = \lambda\, ds. \tag{22-10}$$

***An Element's Field.*** This charge element sets up the differential electric field $d\vec{E}$ at $P$, at distance $r$ from the element, as shown in Fig. 22-11. (Yes, we are introducing a new symbol that is not given in the problem statement, but soon we shall replace it with "legal symbols.") Next we rewrite the field equation for a particle (Eq. 22-3) in terms of our new symbols $dE$ and $dq$, but then we replace $dq$ using Eq. 22-10. The field magnitude due to the charge element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}. \tag{22-11}$$

Notice that the illegal symbol $r$ is the hypotenuse of the right triangle displayed in Fig. 22-11. Thus, we can replace $r$ by rewriting Eq. 22-11 as

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}. \tag{22-12}$$

Because every charge element has the same charge and the same distance from point $P$, Eq. 22-12 gives the field magnitude contributed by each of them. Figure 22-11 also tells us that each contributed $d\vec{E}$ leans at angle $\theta$ to the central axis (the $z$ axis) and thus has components perpendicular and parallel to that axis.

***Canceling Components.*** Now comes the neat part, where we eliminate one set of those components. In Fig. 22-11, consider the charge element on the opposite side of the ring. It too contributes the field magnitude $dE$ but the field vector leans at angle $\theta$ in the opposite direction from the vector from our first charge

**Table 22-1** Some Measures of Electric Charge

| Name | Symbol | SI Unit |
|---|---|---|
| Charge | $q$ | C |
| Linear charge density | $\lambda$ | C/m |
| Surface charge density | $\sigma$ | C/m$^2$ |
| Volume charge density | $\rho$ | C/m$^3$ |

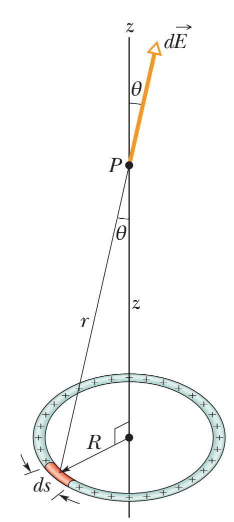



**Figure 22-11** A ring of uniform positive charge. A differential element of charge occupies a length $ds$ (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point $P$.

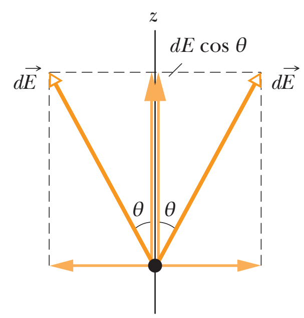



**Figure 22-12** The electric fields set up at $P$ by a charge element and its symmetric partner (on the opposite side of the ring). The components perpendicular to the $z$ axis cancel; the parallel components add.

element, as indicated in the side view of Fig. 22-12. Thus the two perpendicular components cancel. All around the ring, this cancelation occurs for every charge element and its *symmetric partner* on the opposite side of the ring. So we can neglect all the perpendicular components.

***Adding Components.*** We have another big win here. All the remaining components are in the positive direction of the $z$ axis, so we can just add them up as scalars. Thus we can already tell the direction of the net electric field at $P$: directly away from the ring. From Fig. 22-12, we see that the parallel components each have magnitude $dE \cos\theta$, but $\theta$ is another illegal symbol. We can replace $\cos\theta$ with legal symbols by again using the right triangle in Fig. 22-11 to write

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}. \tag{22-13}$$

Multiplying Eq. 22-12 by Eq. 22-13 gives us the parallel field component from each charge element:

$$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{(z^2 + R^2)^{3/2}}\,ds. \tag{22-14}$$

***Integrating.*** Because we must sum a huge number of these components, each small, we set up an integral that moves along the ring, from element to element, from a starting point (call it $s = 0$) through the full circumference ($s = 2\pi R$). Only the quantity $s$ varies as we go through the elements; the other symbols in Eq. 22-14 remain the same, so we move them outside the integral. We find

$$\begin{aligned} E = \int dE\cos\theta &= \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds \\ &= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}. \end{aligned} \tag{22-15}$$

This is a fine answer, but we can also switch to the total charge by using $\lambda = q/(2\pi R)$:

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \quad \text{(charged ring).} \tag{22-16}$$

If the charge on the ring is negative, instead of positive as we have assumed, the magnitude of the field at $P$ is still given by Eq. 22-16. However, the electric field vector then points toward the ring instead of away from it.

Let us check Eq. 22-16 for a point on the central axis that is so far away that $z \gg R$. For such a point, the expression $z^2 + R^2$ in Eq. 22-16 can be approximated as $z^2$, and Eq. 22-16 becomes

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2} \quad \text{(charged ring at large distance).} \tag{22-17}$$

This is a reasonable result because from a large distance, the ring "looks like" a point charge. If we replace $z$ with $r$ in Eq. 22-17, we indeed do have the magnitude of the electric field due to a point charge, as given by Eq. 22-3.

Let us next check Eq. 22-16 for a point at the center of the ring—that is, for $z = 0$. At that point, Eq. 22-16 tells us that $E = 0$. This is a reasonable result because if we were to place a test charge at the center of the ring, there would be no net electrostatic force acting on it; the force due to any element of the ring would be canceled by the force due to the element on the opposite side of the ring. By Eq. 22-1, if the force at the center of the ring were zero, the electric field there would also have to be zero.

## Sample Problem 22.03 Electric field of a charged circular rod

Figure 22-13*a* shows a plastic rod with a uniform charge $-Q$. It is bent in a 120° circular arc of radius $r$ and symmetrically placed across an $x$ axis with the origin at the center of curvature $P$ of the rod. In terms of $Q$ and $r$, what is the electric field $\vec{E}$ due to the rod at point $P$?

### KEY IDEA

Because the rod has a continuous charge distribution, we must find an expression for the electric fields due to differential elements of the rod and then sum those fields via calculus.

***An element:*** Consider a differential element having arc length $ds$ and located at an angle $\theta$ above the $x$ axis (Figs. 22-13*b* and *c*). If we let $\lambda$ represent the linear charge density of the rod, our element $ds$ has a differential charge of magnitude

$$dq = \lambda\, ds. \tag{22-18}$$

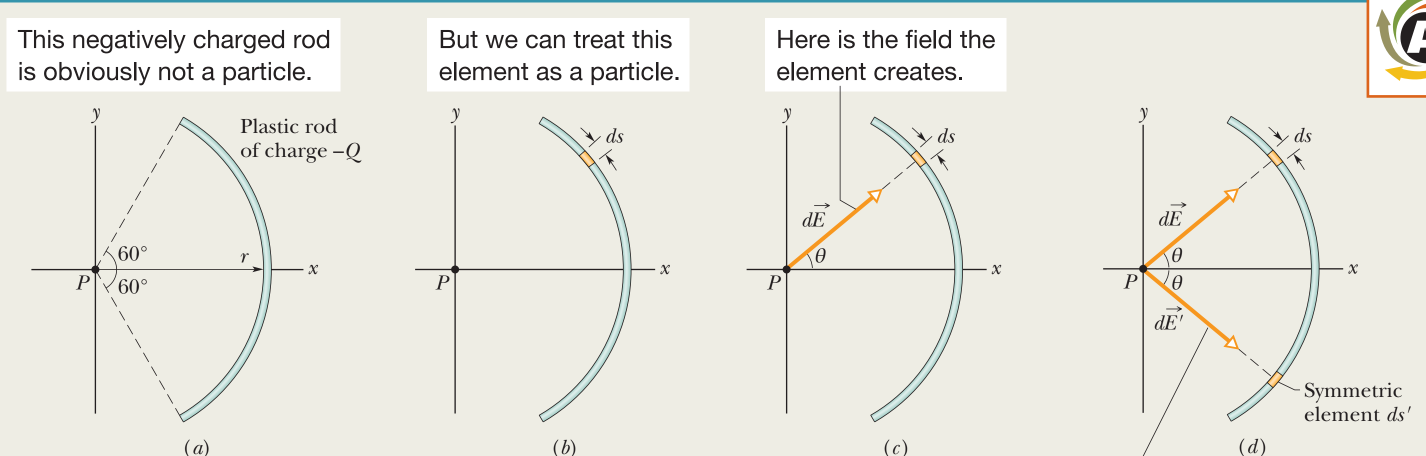
***The element's field:*** Our element produces a differential electric field $d\vec{E}$ at point $P$, which is a distance $r$ from the element. Treating the element as a point charge, we can rewrite Eq. 22-3 to express the magnitude of $d\vec{E}$ as

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}. \tag{22-19}$$

The direction of $d\vec{E}$ is toward $ds$ because charge $dq$ is negative.

***Symmetric partner:*** Our element has a symmetrically located (mirror image) element $ds'$ in the bottom half of the rod. The electric field $d\vec{E}'$ set up at $P$ by $ds'$ also has the magnitude given by Eq. 22-19, but the field vector points toward $ds'$ as shown in Fig. 22-13*d*. If we resolve the electric field vectors of $ds$ and $ds'$ into $x$ and $y$ components as shown in Figs. 22-13*e* and *f*, we see that their $y$ components cancel (because they have equal magnitudes and are in opposite directions). We also see that their $x$ components have equal magnitudes and are in the same direction.

***Summing:*** Thus, to find the electric field set up by the rod, we need sum (via integration) only the $x$ components of the differential electric fields set up by all the differential elements



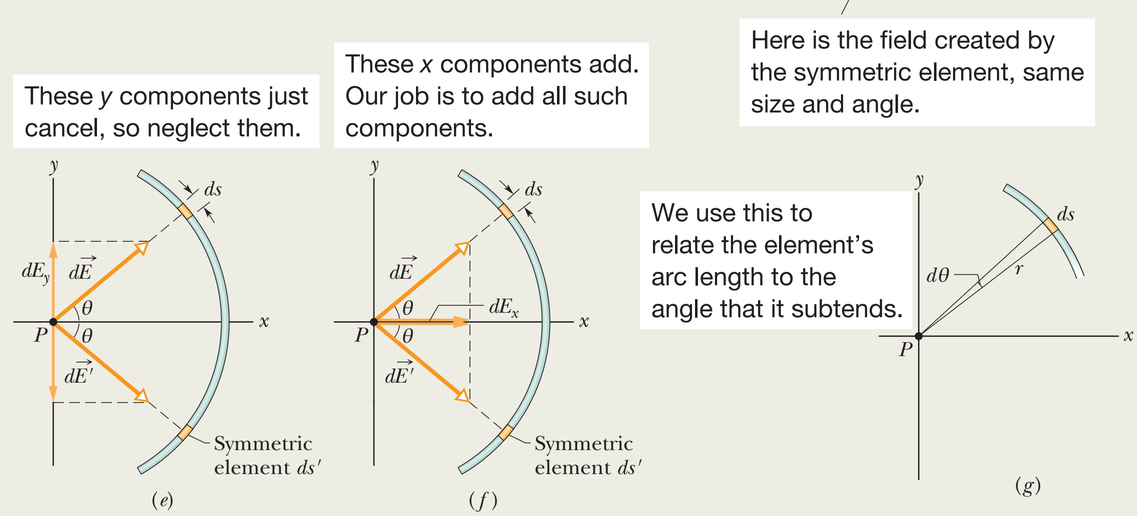




**Figure 22-13** Available in *WileyPLUS* as an animation with voiceover. (*a*) A plastic rod of charge $-Q$ is a circular section of radius $r$ and central angle 120°; point $P$ is the center of curvature of the rod. (*b*)–(*c*) A differential element in the top half of the rod, at an angle $\theta$ to the $x$ axis and of arc length $ds$, sets up a differential electric field $d\vec{E}$ at $P$. (*d*) An element $ds'$, symmetric to $ds$ about the $x$ axis, sets up a field $d\vec{E}'$ at $P$ with the same magnitude. (*e*)–(*f*) The field components. (*g*) Arc length $ds$ makes an angle $d\theta$ about point $P$.

of the rod. From Fig. 22-13*f* and Eq. 22-19, we can write the component $dE_x$ set up by $ds$ as

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, ds. \tag{22-20}$$

Equation 22-20 has two variables, $\theta$ and $s$. Before we can integrate it, we must eliminate one variable. We do so by replacing $ds$, using the relation

$$ds = r\,d\theta,$$

in which $d\theta$ is the angle at $P$ that includes arc length $ds$ (Fig. 22-13*g*). With this replacement, we can integrate Eq. 22-20 over the angle made by the rod at $P$, from $\theta = -60°$ to $\theta = 60°$; that will give us the field magnitude at $P$:

$$\begin{aligned} E &= \int dE_x = \int_{-60°}^{60°} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\,d\theta \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-60°}^{60°}\cos\theta\,d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-60°}^{60°} \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin 60° - \sin(-60°)] \\ &= \frac{1.73\lambda}{4\pi\varepsilon_0 r}. \end{aligned} \tag{22-21}$$

(If we had reversed the limits on the integration, we would have gotten the same result but with a minus sign. Since the integration gives only the magnitude of $\vec{E}$, we would then have discarded the minus sign.)

***Charge density:*** To evaluate $\lambda$, we note that the full rod subtends an angle of 120° and so is one-third of a full circle. Its arc length is then $2\pi r/3$, and its linear charge density must be

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}.$$

Substituting this into Eq. 22-21 and simplifying give us

$$\begin{aligned} E &= \frac{(1.73)(0.477Q)}{4\pi\varepsilon_0 r^2} \\ &= \frac{0.83Q}{4\pi\varepsilon_0 r^2}. \end{aligned} \quad \text{(Answer)}$$

The direction of $\vec{E}$ is toward the rod, along the axis of symmetry of the charge distribution. We can write $\vec{E}$ in unit-vector notation as

$$\vec{E} = \frac{0.83Q}{4\pi\varepsilon_0 r^2}\,\hat{\mathrm{i}}.$$

## Problem-Solving Tactics A Field Guide for Lines of Charge

Here is a generic guide for finding the electric field $\vec{E}$ produced at a point $P$ by a line of uniform charge, either circular or straight. The general strategy is to pick out an element $dq$ of the charge, find $d\vec{E}$ due to that element, and integrate $d\vec{E}$ over the entire line of charge.

**Step 1.** If the line of charge is circular, let $ds$ be the arc length of an element of the distribution. If the line is straight, run an $x$ axis along it and let $dx$ be the length of an element. Mark the element on a sketch.

**Step 2.** Relate the charge $dq$ of the element to the length of the element with either $dq = \lambda\, ds$ or $dq = \lambda\, dx$. Consider $dq$ and $\lambda$ to be positive, even if the charge is actually negative. (The sign of the charge is used in the next step.)

**Step 3.** Express the field $d\vec{E}$ produced at $P$ by $dq$ with Eq. 22-3, replacing $q$ in that equation with either $\lambda\, ds$ or $\lambda\, dx$. If the charge on the line is positive, then at $P$ draw a vector $d\vec{E}$ that points directly away from $dq$. If the charge is negative, draw the vector pointing directly toward $dq$.

**Step 4.** Always look for any symmetry in the situation. If $P$ is on an axis of symmetry of the charge distribution, resolve the field $d\vec{E}$ produced by $dq$ into components that are perpendicular and parallel to the axis of symmetry. Then consider a second element $dq'$ that is located symmetrically to $dq$ about the line of symmetry. At $P$ draw the vector $d\vec{E}'$ that this symmetrical element produces and resolve it into components. One of the components produced by $dq$ is a *canceling component;* it is canceled by the corresponding component produced by $dq'$ and needs no further attention. The other component produced by $dq$ is an *adding component;* it adds to the corresponding component produced by $dq'$. Add the adding components of all the elements via integration.

**Step 5.** Here are four general types of uniform charge distributions, with strategies for the integral of step 4.

*Ring,* with point $P$ on (central) axis of symmetry, as in Fig. 22-11. In the expression for $dE$, replace $r^2$ with $z^2 + R^2$, as in Eq. 22-12. Express the adding component of $d\vec{E}$ in terms of $\theta$. That introduces $\cos\theta$, but $\theta$ is identical for all elements and thus is not a variable. Replace $\cos\theta$ as in Eq. 22-13. Integrate over $s$, around the circumference of the ring.

*Circular arc,* with point $P$ at the center of curvature, as in Fig. 22-13. Express the adding component of $d\vec{E}$ in terms of $\theta$. That introduces either $\sin\theta$ or $\cos\theta$. Reduce the resulting two variables $s$ and $\theta$ to one, $\theta$, by replacing $ds$ with $r\,d\theta$. Integrate over $\theta$ from one end of the arc to the other end.

*Straight line,* with point $P$ on an extension of the line, as in Fig. 22-14*a*. In the expression for $dE$, replace $r$ with $x$. Integrate over $x$, from end to end of the line of charge.

*Straight line,* with point $P$ at perpendicular distance $y$ from the line of charge, as in Fig. 22-14$b$. In the expression for $dE$, replace $r$ with an expression involving $x$ and $y$. If $P$ is on the perpendicular bisector of the line of charge, find an expression for the adding component of $d\vec{E}$. That will introduce either $\sin\theta$ or $\cos\theta$. Reduce the resulting two variables $x$ and $\theta$ to one, $x$, by replacing the trigonometric function with an expression (its definition) involving $x$ and $y$. Integrate over $x$ from end to end of the line of charge. If $P$ is not on a line of symmetry, as in Fig. 22-14$c$, set up an integral to sum the components $dE_x$, and integrate over $x$ to find $E_x$. Also set up an integral to sum the components $dE_y$, and integrate over $x$ again to find $E_y$. Use the components $E_x$ and $E_y$ in the usual way to find the magnitude $E$ and the orientation of $\vec{E}$.

**Step 6.** One arrangement of the integration limits gives a positive result. The reverse gives the same result with a minus sign; discard the minus sign. If the result is to be stated in terms of the total charge $Q$ of the distribution, replace $\lambda$ with $Q/L$, in which $L$ is the length of the distribution.

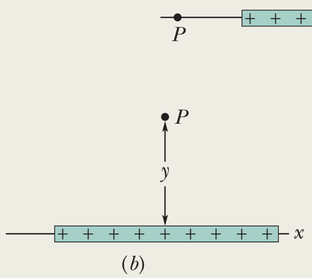
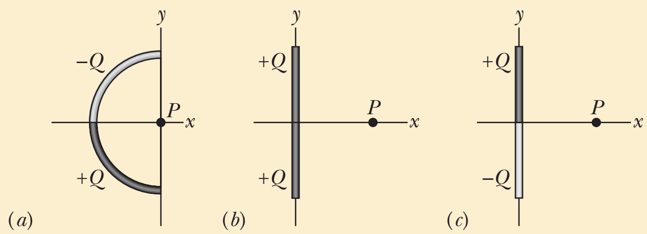



**Figure 22-14** ($a$) Point $P$ is on an extension of the line of charge. ($b$) $P$ is on a line of symmetry of the line of charge, at perpendicular distance $y$ from that line. ($c$) Same as ($b$) except that $P$ is not on a line of symmetry.

Additional examples, video, and practice available at *WileyPLUS*

### Checkpoint 2

The figure here shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude $Q$ along its top half and another along its bottom half. For each rod, what is the direction of the net electric field at point $P$?




# 22-5 THE ELECTRIC FIELD DUE TO A CHARGED DISK

## Learning Objectives

*After reading this module, you should be able to . . .*

**22.19** Sketch a disk with uniform charge and indicate the direction of the electric field at a point on the central axis if the charge is positive and if it is negative.

**22.20** Explain how the equation for the electric field on the central axis of a uniformly charged ring can be used to find the equation for the electric field on the central axis of a uniformly charged disk.

**22.21** For a point on the central axis of a uniformly charged disk, apply the relationship between the surface charge density $\sigma$, the disk radius $R$, and the distance $z$ to that point.

## Key Idea

- On the central axis through a uniformly charged disk,

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

gives the electric field magnitude. Here $z$ is the distance along the axis from the center of the disk, $R$ is the radius of the disk, and $\sigma$ is the surface charge density.

## The Electric Field Due to a Charged Disk

Now we switch from a line of charge to a surface of charge by examining the electric field of a circular plastic disk, with a radius $R$ and a uniform surface charge density $\sigma$ (charge per unit area, Table 22-1) on its top surface. The disk sets up a

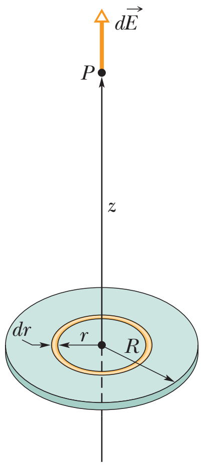



**Figure 22-15** A disk of radius $R$ and uniform positive charge. The ring shown has radius $r$ and radial width $dr$. It sets up a differential electric field $d\vec{E}$ at point $P$ on its central axis.

pattern of electric field lines around it, but here we restrict our attention to the electric field at an arbitrary point $P$ on the central axis, at distance $z$ from the center of the disk, as indicated in Fig. 22-15.

We could proceed as in the preceding module but set up a two-dimensional integral to include all of the field contributions from the two-dimensional distribution of charge on the top surface. However, we can save a lot of work with a neat shortcut using our earlier work with the field on the central axis of a thin ring.

We superimpose a ring on the disk as shown in Fig. 22-15, at an arbitrary radius $r \leq R$. The ring is so thin that we can treat the charge on it as a charge element $dq$. To find its small contribution $dE$ to the electric field at point $P$, we rewrite Eq. 22-16 in terms of the ring's charge $dq$ and radius $r$:

$$dE = \frac{dq\, z}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}}. \tag{22-22}$$

The ring's field points in the positive direction of the $z$ axis.

To find the total field at $P$, we are going to integrate Eq. 22-22 from the center of the disk at $r = 0$ out to the rim at $r = R$ so that we sum all the $dE$ contributions (by sweeping our arbitrary ring over the entire disk surface). However, that means we want to integrate with respect to a variable radius $r$ of the ring.

We get $dr$ into the expression by substituting for $dq$ in Eq. 22-22. Because the ring is so thin, call its thickness $dr$. Then its surface area $dA$ is the product of its circumference $2\pi r$ and thickness $dr$. So, in terms of the surface charge density $\sigma$, we have

$$dq = \sigma\, dA = \sigma\,(2\pi r\, dr). \tag{22-23}$$

After substituting this into Eq. 22-22 and simplifying slightly, we can sum all the $dE$ contributions with

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr, \tag{22-24}$$

where we have pulled the constants (including $z$) out of the integral. To solve this integral, we cast it in the form $\int X^m\, dX$ by setting $X = (z^2 + r^2)$, $m = -\frac{3}{2}$, and $dX = (2r)\, dr$. For the recast integral we have

$$\int X^m\, dX = \frac{X^{m+1}}{m+1},$$

and so Eq. 22-24 becomes

$$E = \frac{\sigma z}{4\varepsilon_0}\left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R. \tag{22-25}$$

Taking the limits in Eq. 22-25 and rearranging, we find

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \quad \text{(charged disk)} \tag{22-26}$$

as the magnitude of the electric field produced by a flat, circular, charged disk at points on its central axis. (In carrying out the integration, we assumed that $z \geq 0$.)

If we let $R \to \infty$ while keeping $z$ finite, the second term in the parentheses in Eq. 22-26 approaches zero, and this equation reduces to

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(infinite sheet).} \tag{22-27}$$

This is the electric field produced by an infinite sheet of uniform charge located on one side of a nonconductor such as plastic. The electric field lines for such a situation are shown in Fig. 22-4.

We also get Eq. 22-27 if we let $z \to 0$ in Eq. 22-26 while keeping $R$ finite. This shows that at points very close to the disk, the electric field set up by the disk is the same as if the disk were infinite in extent.

# 22-6 A POINT CHARGE IN AN ELECTRIC FIELD

## Learning Objectives

*After reading this module, you should be able to . . .*

**22.22** For a charged particle placed in an external electric field (a field due to other charged objects), apply the relationship between the electric field $\vec{E}$ at that point, the particle's charge $q$, and the electrostatic force $\vec{F}$ that acts on the particle, and identify the relative directions of the force and the field when the particle is positively charged and negatively charged.

**22.23** Explain Millikan's procedure of measuring the elementary charge.

**22.24** Explain the general mechanism of ink-jet printing.

## Key Ideas

- If a particle with charge $q$ is placed in an external electric field $\vec{E}$, an electrostatic force $\vec{F}$ acts on the particle:

$$\vec{F} = q\vec{E}.$$

- If charge $q$ is positive, the force vector is in the same direction as the field vector. If charge $q$ is negative, the force vector is in the opposite direction (the minus sign in the equation reverses the force vector from the field vector).

## A Point Charge in an Electric Field

In the preceding four modules we worked at the first of our two tasks: given a charge distribution, to find the electric field it produces in the surrounding space. Here we begin the second task: to determine what happens to a charged particle when it is in an electric field set up by other stationary or slowly moving charges.

What happens is that an electrostatic force acts on the particle, as given by

$$\vec{F} = q\vec{E}, \tag{22-28}$$

in which $q$ is the charge of the particle (including its sign) and $\vec{E}$ is the electric field that other charges have produced at the location of the particle. (The field is *not* the field set up by the particle itself; to distinguish the two fields, the field acting on the particle in Eq. 22-28 is often called the *external field.* A charged particle or object is not affected by its own electric field.) Equation 22-28 tells us

> The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge $q$ of the particle is positive and has the opposite direction if $q$ is negative.

### Measuring the Elementary Charge

Equation 22-28 played a role in the measurement of the elementary charge $e$ by American physicist Robert A. Millikan in 1910–1913. Figure 22-16 is a representation of his apparatus. When tiny oil drops are sprayed into chamber A, some of them become charged, either positively or negatively, in the process. Consider a drop that drifts downward through the small hole in plate $P_1$ and into chamber C. Let us assume that this drop has a negative charge $q$.

If switch S in Fig. 22-16 is open as shown, battery B has no electrical effect on chamber C. If the switch is closed (the connection between chamber C and the positive terminal of the battery is then complete), the battery causes an excess positive charge on conducting plate $P_1$ and an excess negative charge on conducting plate $P_2$. The charged plates set up a downward-directed electric field $\vec{E}$ in chamber C. According to Eq. 22-28, this field exerts an electrostatic force on any charged drop that happens to be in the chamber and affects its motion. In particular, our negatively charged drop will tend to drift upward.

By timing the motion of oil drops with the switch opened and with it closed and thus determining the effect of the charge $q$, Millikan discovered that the

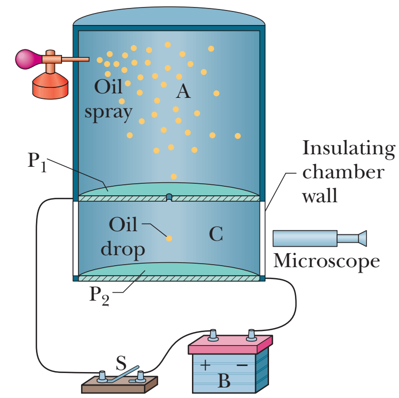



**Figure 22-16** The Millikan oil-drop apparatus for measuring the elementary charge $e$. When a charged oil drop drifted into chamber C through the hole in plate $P_1$, its motion could be controlled by closing and opening switch S and thereby setting up or eliminating an electric field in chamber C. The microscope was used to view the drop, to permit timing of its motion.

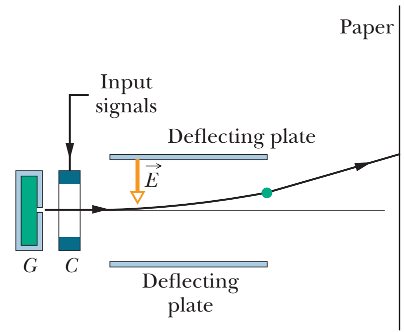



**Figure 22-17** Ink-jet printer. Drops shot from generator $G$ receive a charge in charging unit $C$. An input signal from a computer controls the charge and thus the effect of field $\vec{E}$ on where the drop lands on the paper.

values of $q$ were always given by

$$q = ne, \qquad \text{for } n = 0, \pm 1, \pm 2, \pm 3, \ldots, \tag{22-29}$$

in which $e$ turned out to be the fundamental constant we call the *elementary charge,* $1.60 \times 10^{-19}$ C. Millikan's experiment is convincing proof that charge is quantized, and he earned the 1923 Nobel Prize in physics in part for this work. Modern measurements of the elementary charge rely on a variety of interlocking experiments, all more precise than the pioneering experiment of Millikan.

## Ink-Jet Printing

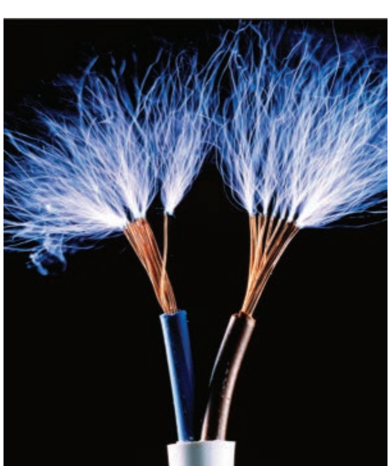
The need for high-quality, high-speed printing has caused a search for an alternative to impact printing, such as occurs in a standard typewriter. Building up letters by squirting tiny drops of ink at the paper is one such alternative.

Figure 22-17 shows a negatively charged drop moving between two conducting deflecting plates, between which a uniform, downward-directed electric field $\vec{E}$ has been set up. The drop is deflected upward according to Eq. 22-28 and then strikes the paper at a position that is determined by the magnitudes of $\vec{E}$ and the charge $q$ of the drop.

In practice, $E$ is held constant and the position of the drop is determined by the charge $q$ delivered to the drop in the charging unit, through which the drop must pass before entering the deflecting system. The charging unit, in turn, is activated by electronic signals that encode the material to be printed.

## Electrical Breakdown and Sparking

If the magnitude of an electric field in air exceeds a certain critical value $E_c$, the air undergoes *electrical breakdown,* a process whereby the field removes electrons from the atoms in the air. The air then begins to conduct electric current because the freed electrons are propelled into motion by the field. As they move, they collide with any atoms in their path, causing those atoms to emit light. We can see the paths, commonly called sparks, taken by the freed electrons because of that emitted light. Figure 22-18 shows sparks above charged metal wires where the electric fields due to the wires cause electrical breakdown of the air.

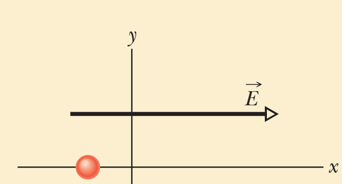


Adam Hart-Davis/Science Source

**Figure 22-18** The metal wires are so charged that the electric fields they produce in the surrounding space cause the air there to undergo electrical breakdown.

### Checkpoint 3

(a) In the figure, what is the direction of the electrostatic force on the electron due to the external electric field shown? (b) In which direction will the electron accelerate if it is moving parallel to the $y$ axis before it encounters the external field? (c) If, instead, the electron is initially moving rightward, will its speed increase, decrease, or remain constant?



### Sample Problem 22.04 Motion of a charged particle in an electric field

Figure 22-19 shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass $m$ of $1.3 \times 10^{-10}$ kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the $x$ axis with speed $v_x = 18$ m/s. The length $L$ of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of $1.4 \times 10^6$ N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)

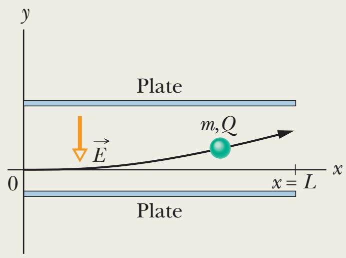



**Figure 22-19** An ink drop of mass $m$ and charge magnitude $Q$ is deflected in the electric field of an ink-jet printer.

#### KEY IDEA

The drop is negatively charged and the electric field is directed *downward.* From Eq. 22-28, a constant electrostatic force of magnitude $QE$ acts *upward* on the charged drop. Thus, as the drop travels parallel to the $x$ axis at constant speed $v_x$, it accelerates upward with some constant acceleration $a_y$.

***Calculations:*** Applying Newton's second law ($F = ma$) for components along the $y$ axis, we find that

$$a_y = \frac{F}{m} = \frac{QE}{m}. \tag{22-30}$$

Let $t$ represent the time required for the drop to pass through the region between the plates. During $t$ the vertical and horizontal displacements of the drop are

$$y = \tfrac{1}{2}a_y t^2 \quad \text{and} \quad L = v_x t, \tag{22-31}$$

respectively. Eliminating $t$ between these two equations and substituting Eq. 22-30 for $a_y$, we find

$$\begin{aligned} y &= \frac{QEL^2}{2mv_x^2} \\ &= \frac{(1.5 \times 10^{-13}\ \text{C})(1.4 \times 10^6\ \text{N/C})(1.6 \times 10^{-2}\ \text{m})^2}{(2)(1.3 \times 10^{-10}\ \text{kg})(18\ \text{m/s})^2} \\ &= 6.4 \times 10^{-4}\ \text{m} \\ &= 0.64\ \text{mm}. \qquad \text{(Answer)} \end{aligned}$$



# 22-7 A DIPOLE IN AN ELECTRIC FIELD

## Learning Objectives

*After reading this module, you should be able to . . .*

**22.25** On a sketch of an electric dipole in an external electric field, indicate the direction of the field, the direction of the dipole moment, the direction of the electrostatic forces on the two ends of the dipole, and the direction in which those forces tend to rotate the dipole, and identify the value of the net force on the dipole.

**22.26** Calculate the torque on an electric dipole in an external electric field by evaluating a cross product of the dipole moment vector and the electric field vector, in magnitude-angle notation and unit-vector notation.

**22.27** For an electric dipole in an external electric field, relate the potential energy of the dipole to the work done by a torque as the dipole rotates in the electric field.

**22.28** For an electric dipole in an external electric field, calculate the potential energy by taking a dot product of the dipole moment vector and the electric field vector, in magnitude-angle notation and unit-vector notation.

**22.29** For an electric dipole in an external electric field, identify the angles for the minimum and maximum potential energies and the angles for the minimum and maximum torque magnitudes.

## Key Ideas

- The torque on an electric dipole of dipole moment $\vec{p}$ when placed in an external electric field $\vec{E}$ is given by a cross product:

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

- A potential energy $U$ is associated with the orientation of the dipole moment in the field, as given by a dot product:

$$U = -\vec{p} \cdot \vec{E}.$$

- If the dipole orientation changes, the work done by the electric field is

$$W = -\Delta U.$$

If the change in orientation is due to an external agent, the work done by the agent is $W_a = -W$.

## A Dipole in an Electric Field

We have defined the electric dipole moment $\vec{p}$ of an electric dipole to be a vector that points from the negative to the positive end of the dipole. As you will see, the behavior of a dipole in a uniform external electric field $\vec{E}$ can be described completely in terms of the two vectors $\vec{E}$ and $\vec{p}$, with no need of any details about the dipole's structure.

A molecule of water ($H_2O$) is an electric dipole; Fig. 22-20 shows why. There the black dots represent the oxygen nucleus (having eight protons) and the two hydrogen nuclei (having one proton each). The colored enclosed areas represent the regions in which electrons can be located around the nuclei.

In a water molecule, the two hydrogen atoms and the oxygen atom do not lie on a straight line but form an angle of about 105°, as shown in Fig. 22-20. As a result, the molecule has a definite "oxygen side" and "hydrogen side." Moreover, the 10 electrons of the molecule tend to remain closer to the oxygen nucleus than to the hydrogen nuclei. This makes the oxygen side of the molecule slightly more negative than the hydrogen side and creates an electric dipole moment $\vec{p}$ that points along the symmetry axis of the molecule as shown. If the water molecule is placed in an external electric field, it behaves as would be expected of the more abstract electric dipole of Fig. 22-9.

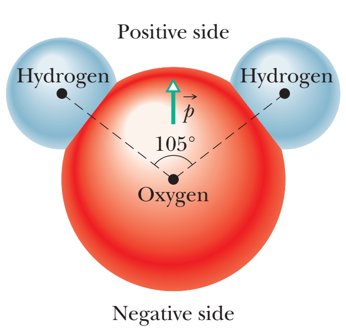



**Figure 22-20** A molecule of $H_2O$, showing the three nuclei (represented by dots) and the regions in which the electrons can be located. The electric dipole moment $\vec{p}$ points from the (negative) oxygen side to the (positive) hydrogen side of the molecule.

To examine this behavior, we now consider such an abstract dipole in a uniform external electric field $\vec{E}$, as shown in Fig. 22-21*a*. We assume that the dipole is a rigid structure that consists of two centers of opposite charge, each of magnitude $q$, separated by a distance $d$. The dipole moment $\vec{p}$ makes an angle $\theta$ with field $\vec{E}$.

Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions (as shown in Fig. 22-21*a*) and with the same magnitude $F = qE$. Thus, *because the field is uniform,* the net force on the dipole from the field is zero and the center of mass of the dipole does not move. However, the forces on the charged ends do produce a net torque $\vec{\tau}$ on the dipole about its center of mass. The center of mass lies on the line connecting the charged ends, at some distance $x$ from one end and thus a distance $d - x$ from the other end. From Eq. 10-39 ($\tau = rF \sin \phi$), we can write the magnitude of the net torque $\vec{\tau}$ as

$$\tau = Fx \sin \theta + F(d - x) \sin \theta = Fd \sin \theta. \qquad (22\text{-}32)$$

We can also write the magnitude of $\vec{\tau}$ in terms of the magnitudes of the electric field $E$ and the dipole moment $p = qd$. To do so, we substitute $qE$ for $F$ and $p/q$ for $d$ in Eq. 22-32, finding that the magnitude of $\vec{\tau}$ is

$$\tau = pE \sin \theta. \qquad (22\text{-}33)$$

We can generalize this equation to vector form as

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).} \qquad (22\text{-}34)$$

Vectors $\vec{p}$ and $\vec{E}$ are shown in Fig. 22-21*b*. The torque acting on a dipole tends to rotate $\vec{p}$ (hence the dipole) into the direction of field $\vec{E}$, thereby reducing $\theta$. In Fig. 22-21, such rotation is clockwise. As we discussed in Chapter 10, we can represent a torque that gives rise to a clockwise rotation by including a minus sign with the magnitude of the torque. With that notation, the torque of Fig. 22-21 is

$$\tau = -pE \sin \theta. \qquad (22\text{-}35)$$

### Potential Energy of an Electric Dipole

Potential energy can be associated with the orientation of an electric dipole in an electric field. The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment $\vec{p}$ is lined up with the field $\vec{E}$ (then $\vec{\tau} = \vec{p} \times \vec{E} = 0$). It has greater potential energy in all other orientations. Thus the dipole is like a pendulum, which has *its* least gravitational potential energy in *its* equilibrium orientation — at its lowest point. To rotate the dipole or the pendulum to any other orientation requires work by some external agent.

In any situation involving potential energy, we are free to define the zero-potential-energy configuration in an arbitrary way because only differences in potential energy have physical meaning. The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle $\theta$ in Fig. 22-21 is 90°. We then can find the potential energy $U$ of the dipole at any other value of $\theta$ with Eq. 8-1 ($\Delta U = -W$) by calculating the work $W$ done by the field on the dipole when the dipole is rotated to that value of $\theta$ from 90°. With the aid of Eq. 10-53 ($W = \int \tau \, d\theta$) and Eq. 22-35, we find that the potential energy $U$ at any angle $\theta$ is

$$U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta. \qquad (22\text{-}36)$$

Evaluating the integral leads to

$$U = -pE\cos\theta. \qquad (22\text{-}37)$$

We can generalize this equation to vector form as

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).} \qquad (22\text{-}38)$$

Equations 22-37 and 22-38 show us that the potential energy of the dipole is least ($U = -pE$) when $\theta = 0$ ($\vec{p}$ and $\vec{E}$ are in the same direction); the potential energy is greatest ($U = pE$) when $\theta = 180°$ ($\vec{p}$ and $\vec{E}$ are in opposite directions).

When a dipole rotates from an initial orientation $\theta_i$ to another orientation $\theta_f$, the work $W$ done on the dipole by the electric field is

$$W = -\Delta U = -(U_f - U_i), \qquad (22\text{-}39)$$

where $U_f$ and $U_i$ are calculated with Eq. 22-38. If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work $W_a$ done on the dipole by the applied torque is the negative of the work done on the dipole by the field; that is,

$$W_a = -W = (U_f - U_i). \qquad (22\text{-}40)$$

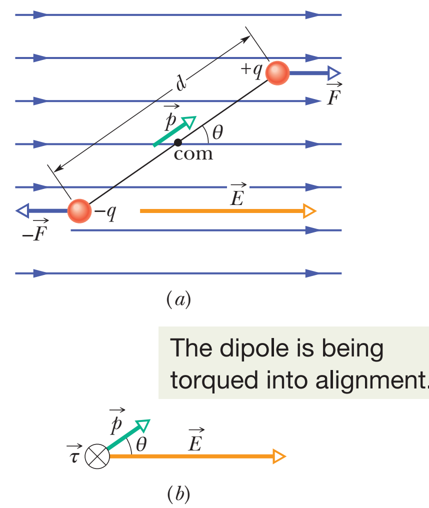



**Figure 22-21** (*a*) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance $d$. The line between them represents their rigid connection. (*b*) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

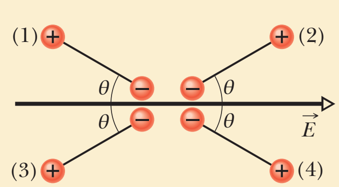
## Microwave Cooking

Food can be warmed and cooked in a microwave oven if the food contains water because water molecules are electric dipoles. When you turn on the oven, the microwave source sets up a rapidly oscillating electric field $\vec{E}$ within the oven and thus also within the food. From Eq. 22-34, we see that any electric field $\vec{E}$ produces a torque on an electric dipole moment $\vec{p}$ to align $\vec{p}$ with $\vec{E}$. Because the oven's $\vec{E}$ oscillates, the water molecules continuously flip-flop in a frustrated attempt to align with $\vec{E}$.

Energy is transferred from the electric field to the thermal energy of the water (and thus of the food) where three water molecules happened to have bonded together to form a group. The flip-flop breaks some of the bonds. When the molecules reform the bonds, energy is transferred to the random motion of the group and then to the surrounding molecules. Soon, the thermal energy of the water is enough to cook the food.

### Checkpoint 4

The figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.



### Sample Problem 22.05 Torque and energy of an electric dipole in an electric field

A neutral water molecule ($H_2O$) in its vapor state has an electric dipole moment of magnitude $6.2 \times 10^{-30}$ C·m.

(a) How far apart are the molecule's centers of positive and negative charge?

#### KEY IDEA

A molecule's dipole moment depends on the magnitude $q$ of the molecule's positive or negative charge and the charge separation $d$.

***Calculations:*** There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which $d$ is the separation we are seeking and $e$ is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \text{C}\cdot\text{m}}{(10)(1.60 \times 10^{-19}\ \text{C})}$$
$$= 3.9 \times 10^{-12}\ \text{m} = 3.9\ \text{pm}. \qquad \text{(Answer)}$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

(b) If the molecule is placed in an electric field of $1.5 \times 10^4$ N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

#### KEY IDEA

The torque on a dipole is maximum when the angle $\theta$ between $\vec{p}$ and $\vec{E}$ is 90°.

***Calculation:*** Substituting $\theta = 90°$ in Eq. 22-33 yields

$$\begin{aligned}\tau &= pE \sin\theta \\ &= (6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})(\sin 90°) \\ &= 9.3 \times 10^{-26}\ \text{N}\cdot\text{m}. \qquad \text{(Answer)}\end{aligned}$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

#### KEY IDEA

The work done by an external agent (by means of a torque applied to the molecule) is equal to the change in the molecule's potential energy due to the change in orientation.

***Calculation:*** From Eq. 22-40, we find

$$\begin{aligned}W_a &= U_{180°} - U_0 \\ &= (-pE\cos 180°) - (-pE\cos 0) \\ &= 2pE = (2)(6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C}) \\ &= 1.9 \times 10^{-25}\ \text{J}. \qquad \text{(Answer)}\end{aligned}$$

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## Review & Summary

**Electric Field** To explain the electrostatic force between two charges, we assume that each charge sets up an electric field in the space around it. The force acting on each charge is then due to the electric field set up at its location by the other charge.

**Definition of Electric Field** The *electric field* $\vec{E}$ at any point is defined in terms of the electrostatic force $\vec{F}$ that would be exerted on a positive test charge $q_0$ placed there:

$$\vec{E} = \frac{\vec{F}}{q_0}. \qquad (22\text{-}1)$$

**Electric Field Lines** *Electric field lines* provide a means for visualizing the direction and magnitude of electric fields. The electric field vector at any point is tangent to a field line through that point. The density of field lines in any region is proportional to the magnitude of the electric field in that region. Field lines originate on positive charges and terminate on negative charges.

**Field Due to a Point Charge** The magnitude of the electric field $\vec{E}$ set up by a point charge $q$ at a distance $r$ from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}. \qquad (22\text{-}3)$$

The direction of $\vec{E}$ is away from the point charge if the charge is positive and toward it if the charge is negative.

**Field Due to an Electric Dipole** An *electric dipole* consists of two particles with charges of equal magnitude $q$ but opposite sign, separated by a small distance $d$. Their **electric dipole moment** $\vec{p}$ has magnitude $qd$ and points from the negative charge to the positive charge. The magnitude of the electric field set up by the dipole at a distant point on the dipole axis (which runs through both charges) is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}, \qquad (22\text{-}9)$$

where $z$ is the distance between the point and the center of the dipole.

**Field Due to a Continuous Charge Distribution** The electric field due to a *continuous charge distribution* is found by treating charge elements as point charges and then summing, via integration, the electric field vectors produced by all the charge elements to find the net vector.

**Field Due to a Charged Disk** The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right), \tag{22-26}$$

where $z$ is the distance along the axis from the center of the disk, $R$ is the radius of the disk, and $\sigma$ is the surface charge density.

**Force on a Point Charge in an Electric Field** When a point charge $q$ is placed in an external electric field $\vec{E}$, the electrostatic force $\vec{F}$ that acts on the point charge is

$$\vec{F} = q\vec{E}. \tag{22-28}$$

Force $\vec{F}$ has the same direction as $\vec{E}$ if $q$ is positive and the opposite direction if $q$ is negative.

**Dipole in an Electric Field** When an electric dipole of dipole moment $\vec{p}$ is placed in an electric field $\vec{E}$, the field exerts a torque $\vec{\tau}$ on the dipole:

$$\vec{\tau} = \vec{p} \times \vec{E}. \tag{22-34}$$

The dipole has a potential energy $U$ associated with its orientation in the field:

$$U = -\vec{p} \cdot \vec{E}. \tag{22-38}$$

This potential energy is defined to be zero when $\vec{p}$ is perpendicular to $\vec{E}$; it is least ($U = -pE$) when $\vec{p}$ is aligned with $\vec{E}$ and greatest ($U = pE$) when $\vec{p}$ is directed opposite $\vec{E}$.

## Questions

**1** Figure 22-22 shows three arrangements of electric field lines. In each arrangement, a proton is released from rest at point $A$ and is then accelerated through point $B$ by the electric field. Points $A$ and $B$ have equal separations in the three arrangements. Rank the arrangements according to the linear momentum of the proton at point $B$, greatest first.

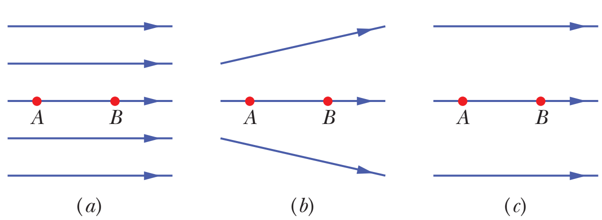


**Figure 22-22** Question 1.

**2** Figure 22-23 shows two square arrays of charged particles. The squares, which are centered on point $P$, are misaligned. The particles are separated by either $d$ or $d/2$ along the perimeters of the squares. What are the magnitude and direction of the net electric field at $P$?

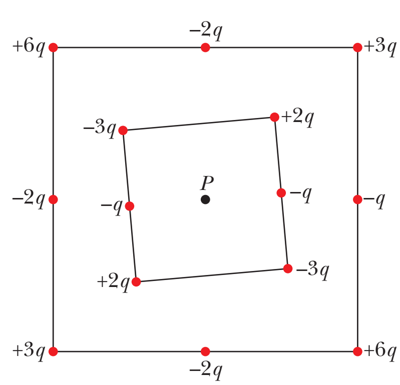


**Figure 22-23** Question 2.

**3** In Fig. 22-24, two particles of charge $-q$ are arranged symmetrically about the $y$ axis; each produces an electric field at point $P$ on that axis. (a) Are the magnitudes of the fields at $P$ equal? (b) Is each electric field directed toward or away from the charge producing it? (c) Is the magnitude of the net electric field at $P$ equal to the sum of the magnitudes $E$ of the two field vectors (is it equal to $2E$)? (d) Do the $x$ components of those two field vectors add or cancel? (e) Do their $y$ components add or cancel? (f) Is the direction of the net field at $P$ that of the canceling components or the adding components? (g) What is the direction of the net field?

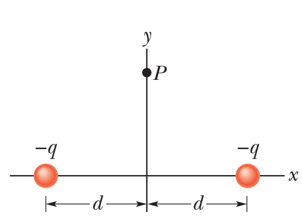


**Figure 22-24** Question 3.

**4** Figure 22-25 shows four situations in which four charged particles are evenly spaced to the left and right of a central point. The charge values are indicated. Rank the situations according to the magnitude of the net electric field at the central point, greatest first.

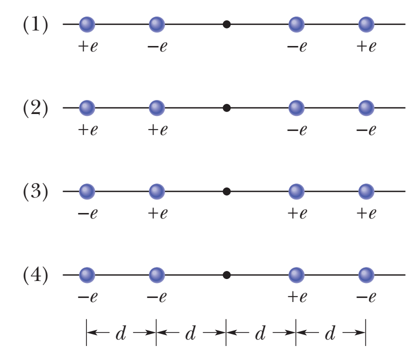


**Figure 22-25** Question 4.

**5** Figure 22-26 shows two charged particles fixed in place on an axis. (a) Where on the axis (other than at an infinite distance) is there a point at which their net electric field is zero: between the charges, to their left, or to their right? (b) Is there a point of zero net electric field anywhere *off* the axis (other than at an infinite distance)?

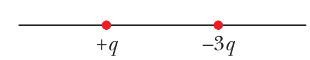


**Figure 22-26** Question 5.

**6** In Fig. 22-27, two identical circular nonconducting rings are centered on the same line with their planes perpendicular to the line. Each ring has charge that is uniformly distributed along its circumference. The rings each produce electric fields at points along the line. For three situations, the charges on rings $A$ and $B$ are, respectively, (1) $q_0$ and $q_0$, (2) $-q_0$ and $-q_0$, and (3) $-q_0$ and $q_0$. Rank the situations according to the magnitude of the net electric field at (a) point $P_1$ midway between the rings, (b) point $P_2$ at the center of ring $B$, and (c) point $P_3$ to the right of ring $B$, greatest first.

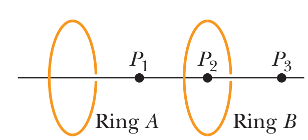


**Figure 22-27** Question 6.

**7** The potential energies associated with four orientations of an electric dipole in an electric field are (1) $-5U_0$, (2) $-7U_0$, (3) $3U_0$, and (4) $5U_0$, where $U_0$ is positive. Rank the orientations according to (a) the angle between the electric dipole moment $\vec{p}$ and the electric field $\vec{E}$ and (b) the magnitude of the torque on the electric dipole, greatest first.

**8** (a) In Checkpoint 4, if the dipole rotates from orientation 1 to orientation 2, is the work done on the dipole by the field positive, negative, or zero? (b) If, instead, the dipole rotates from orientation 1 to orientation 4, is the work done by the field more than, less than, or the same as in (a)?

**9** Figure 22-28 shows two disks and a flat ring, each with the same uniform charge $Q$. Rank the objects according to the magnitude of the electric field they create at points $P$ (which are at the same vertical heights), greatest first.

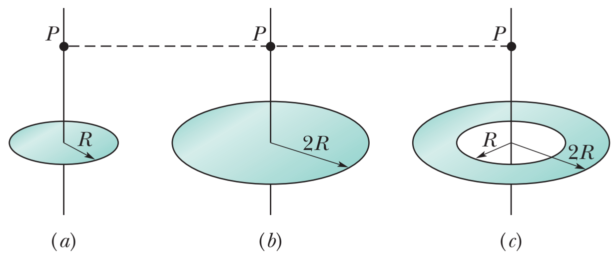


**Figure 22-28** Question 9.

**10** In Fig. 22-29, an electron e travels through a small hole in plate $A$ and then toward plate $B$. A uniform electric field in the region between the plates then slows the electron without deflecting it. (a) What is the direction of the field? (b) Four other particles similarly travel through small holes in either plate $A$ or plate $B$ and then into the region between the plates. Three have charges $+q_1$, $+q_2$, and $-q_3$. The fourth (labeled n) is a neutron, which is electrically neutral. Does the speed of each of those four other particles increase, decrease, or remain the same in the region between the plates?

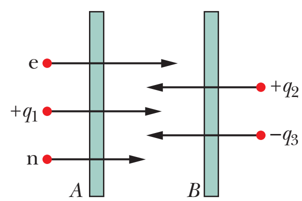


**Figure 22-29** Question 10.

**11** In Fig. 22-30$a$, a circular plastic rod with uniform charge $+Q$ produces an electric field of magnitude $E$ at the center of curvature (at the origin). In Figs. 22-30$b$, $c$, and $d$, more circular rods, each with identical uniform charges $+Q$, are added until the circle is complete. A fifth arrangement (which would be labeled $e$) is like that in $d$ except the rod in the fourth quadrant has charge $-Q$. Rank the five arrangements according to the magnitude of the electric field at the center of curvature, greatest first.

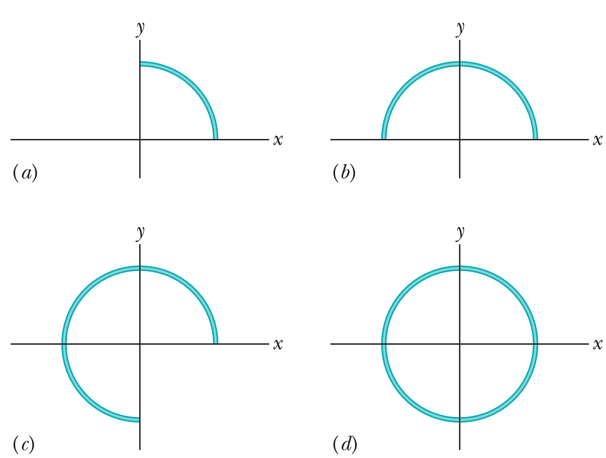


**Figure 22-30** Question 11.

**12** When three electric dipoles are near each other, they each experience the electric field of the other two, and the three-dipole system has a certain potential energy. Figure 22-31 shows two arrangements in which three electric dipoles are side by side. Each dipole has the same magnitude of electric dipole moment, and the spacings between adjacent dipoles are identical. In which arrangement is the potential energy of the three-dipole system greater?

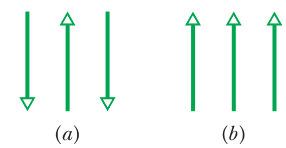


**Figure 22-31** Question 12.

**13** Figure 22-32 shows three rods, each with the same charge $Q$ spread uniformly along its length. Rods $a$ (of length $L$) and $b$ (of length $L/2$) are straight, and points $P$ are aligned with their midpoints. Rod $c$ (of length $L/2$) forms a complete circle about point $P$. Rank the rods according to the magnitude of the electric field they create at points $P$, greatest first.

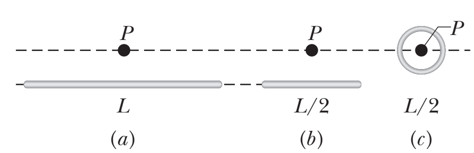


**Figure 22-32** Question 13.

**14** Figure 22-33 shows five protons that are launched in a uniform electric field $\vec{E}$; the magnitude and direction of the launch velocities are indicated. Rank the protons according to the magnitude of their accelerations due to the field, greatest first.

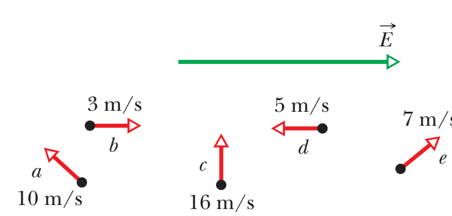


**Figure 22-33** Question 14.

## Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS* and WebAssign
SSM Worked-out solution available in Student Solutions Manual
WWW Worked-out solution is at
• – ••• Number of dots indicates level of problem difficulty
ILW Interactive solution is at
http://www.wiley.com/college/halliday
Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

### Module 22-1 The Electric Field

•**1** Sketch qualitatively the electric field lines both between and outside two concentric conducting spherical shells when a uniform positive charge $q_1$ is on the inner shell and a uniform negative charge $-q_2$ is on the outer. Consider the cases $q_1 > q_2$, $q_1 = q_2$, and $q_1 < q_2$.

•2 In Fig. 22-34 the electric field lines on the left have twice the separation of those on the right. (a) If the magnitude of the field at $A$ is 40 N/C, what is the magnitude of the force on a proton at $A$? (b) What is the magnitude of the field at $B$?

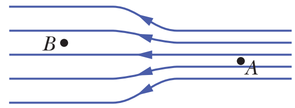


**Figure 22-34** Problem 2.

## Module 22-2 The Electric Field Due to a Charged Particle

•3 SSM The nucleus of a plutonium-239 atom contains 94 protons. Assume that the nucleus is a sphere with radius 6.64 fm and with the charge of the protons uniformly spread through the sphere. At the surface of the nucleus, what are the (a) magnitude and (b) direction (radially inward or outward) of the electric field produced by the protons?

•4 Two charged particles are attached to an $x$ axis: Particle 1 of charge $-2.00 \times 10^{-7}$ C is at position $x = 6.00$ cm and particle 2 of charge $+2.00 \times 10^{-7}$ C is at position $x = 21.0$ cm. Midway between the particles, what is their net electric field in unit-vector notation?

•5 SSM A charged particle produces an electric field with a magnitude of 2.0 N/C at a point that is 50 cm away from the particle. What is the magnitude of the particle's charge?

•6 What is the magnitude of a point charge that would create an electric field of 1.00 N/C at points 1.00 m away?

••7 SSM ILW WWW In Fig. 22-35, the four particles form a square of edge length $a = 5.00$ cm and have charges $q_1 = +10.0$ nC, $q_2 = -20.0$ nC, $q_3 = +20.0$ nC, and $q_4 = -10.0$ nC. In unit-vector notation, what net electric field do the particles produce at the square's center?

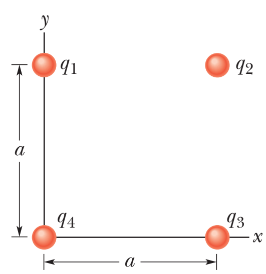


**Figure 22-35** Problem 7.

••8 GO In Fig. 22-36, the four particles are fixed in place and have charges $q_1 = q_2 = +5e$, $q_3 = +3e$, and $q_4 = -12e$. Distance $d = 5.0\ \mu$m. What is the magnitude of the net electric field at point $P$ due to the particles?

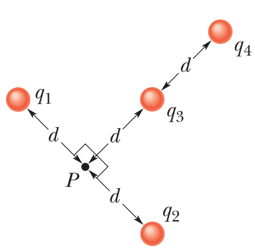


**Figure 22-36** Problem 8.

••9 GO Figure 22-37 shows two charged particles on an $x$ axis: $-q = -3.20 \times 10^{-19}$ C at $x = -3.00$ m and $q = 3.20 \times 10^{-19}$ C at $x = +3.00$ m. What are the (a) magnitude and (b) direction (relative to the positive direction of the $x$ axis) of the net electric field produced at point $P$ at $y = 4.00$ m?

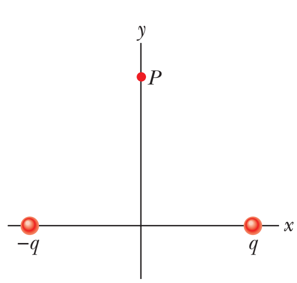


**Figure 22-37** Problem 9.

••10 GO Figure 22-38$a$ shows two charged particles fixed in place on an $x$ axis with separation $L$. The ratio $q_1/q_2$ of their charge magnitudes is 4.00. Figure 22-38$b$ shows the $x$ component $E_{\text{net},x}$ of their net electric field along the $x$ axis just to the right of particle 2. The $x$ axis scale is set by $x_s = 30.0$ cm. (a) At what value of $x > 0$ is $E_{\text{net},x}$ maximum? (b) If particle 2 has charge $-q_2 = -3e$, what is the value of that maximum?

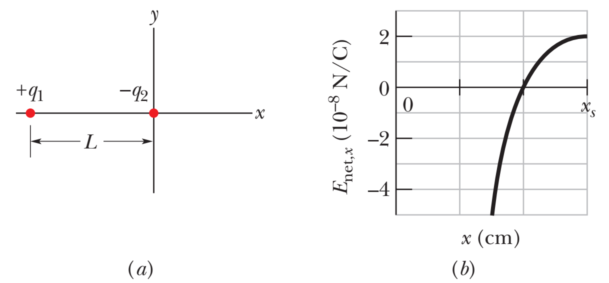


**Figure 22-38** Problem 10.

••11 SSM Two charged particles are fixed to an $x$ axis: Particle 1 of charge $q_1 = 2.1 \times 10^{-8}$ C is at position $x = 20$ cm and particle 2 of charge $q_2 = -4.00q_1$ is at position $x = 70$ cm. At what coordinate on the axis (other than at infinity) is the net electric field produced by the two particles equal to zero?

••12 GO Figure 22-39 shows an uneven arrangement of electrons (e) and protons (p) on a circular arc of radius $r = 2.00$ cm, with angles $\theta_1 = 30.0°$, $\theta_2 = 50.0°$, $\theta_3 = 30.0°$, and $\theta_4 = 20.0°$. What are the (a) magnitude and (b) direction (relative to the positive direction of the $x$ axis) of the net electric field produced at the center of the arc?

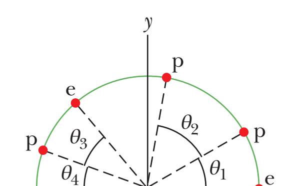


**Figure 22-39** Problem 12.

••13 GO Figure 22-40 shows a proton (p) on the central axis through a disk with a uniform charge density due to excess electrons. The disk is seen from an edge-on view. Three of those electrons are shown: electron $e_c$ at the disk center and electrons $e_s$ at opposite sides of the disk, at radius $R$ from the center. The proton is initially at distance $z = R = 2.00$ cm from the disk. At that location, what are the magnitudes of (a) the electric field $\vec{E}_c$ due to electron $e_c$ and (b) the *net* electric field $\vec{E}_{s,\text{net}}$ due to electrons $e_s$? The proton is then moved to $z = R/10.0$. What then are the magnitudes of (c) $\vec{E}_c$ and (d) $\vec{E}_{s,\text{net}}$ at the proton's location? (e) From (a) and (c) we see that as the proton gets nearer to the disk, the magnitude of $\vec{E}_c$ increases, as expected. Why does the magnitude of $\vec{E}_{s,\text{net}}$ from the two side electrons decrease, as we see from (b) and (d)?

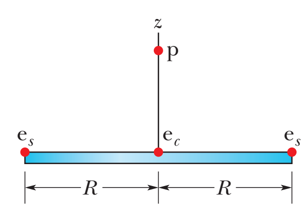


**Figure 22-40** Problem 13.

••14 In Fig. 22-41, particle 1 of charge $q_1 = -5.00q$ and particle 2 of charge $q_2 = +2.00q$ are fixed to an $x$ axis. (a) As a multiple of distance $L$, at what coordinate on the axis is the net electric field of the particles zero? (b) Sketch the net electric field lines between and around the particles.

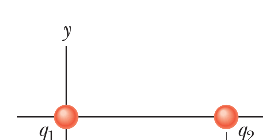


**Figure 22-41** Problem 14.

••**15** In Fig. 22-42, the three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6.00\ \mu$m. What are the (a) magnitude and (b) direction of the net electric field at point $P$ due to the particles?

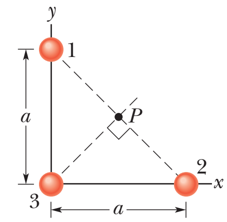

**Figure 22-42** Problem 15.

•••**16** Figure 22-43 shows a plastic ring of radius $R = 50.0$ cm. Two small charged beads are on the ring: Bead 1 of charge $+2.00\ \mu$C is fixed in place at the left side; bead 2 of charge $+6.00\ \mu$C can be moved along the ring. The two beads produce a net electric field of magnitude $E$ at the center of the ring. At what (a) positive and (b) negative value of angle $\theta$ should bead 2 be positioned such that $E = 2.00 \times 10^5$ N/C?

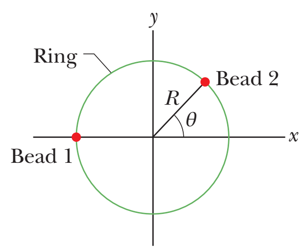


**Figure 22-43** Problem 16.

•••**17** Two charged beads are on the plastic ring in Fig. 22-44*a*. Bead 2, which is not shown, is fixed in place on the ring, which has radius $R = 60.0$ cm. Bead 1, which is not fixed in place, is initially on the $x$ axis at angle $\theta = 0°$. It is then moved to the opposite side, at angle $\theta = 180°$, through the first and second quadrants of the $xy$ coordinate system. Figure 22-44*b* gives the $x$ component of the net electric field produced at the origin by the two beads as a function of $\theta$, and Fig. 22-44*c* gives the $y$ component of that net electric field. The vertical axis scales are set by $E_{xs} = 5.0 \times 10^4$ N/C and $E_{ys} = -9.0 \times 10^4$ N/C. (a) At what angle $\theta$ is bead 2 located? What are the charges of (b) bead 1 and (c) bead 2?

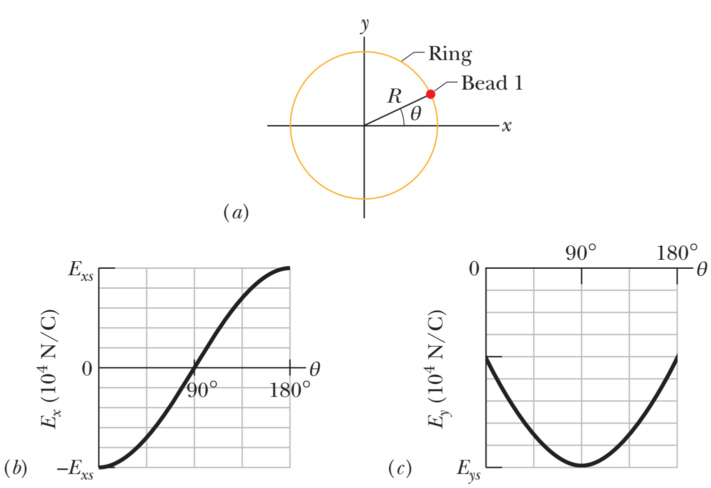


**Figure 22-44** Problem 17.

## Module 22-3 The Electric Field Due to a Dipole

••**18** The electric field of an electric dipole along the dipole axis is approximated by Eqs. 22-8 and 22-9. If a binomial expansion is made of Eq. 22-7, what is the next term in the expression for the dipole's electric field along the dipole axis? That is, what is $E_{next}$ in the expression

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} + E_{next}?$$

••**19** Figure 22-45 shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the $x$ axis) of the dipole's electric field at point $P$, located at distance $r \gg d$?

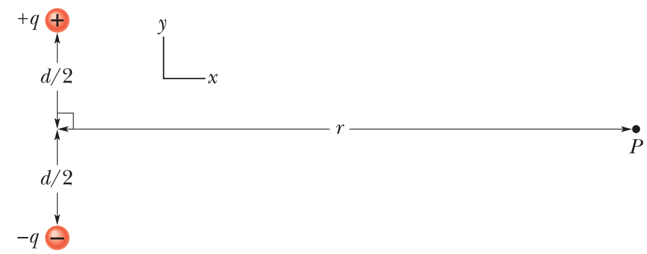

**Figure 22-45** Problem 19.

••**20** Equations 22-8 and 22-9 are approximations of the magnitude of the electric field of an electric dipole, at points along the dipole axis. Consider a point $P$ on that axis at distance $z = 5.00d$ from the dipole center ($d$ is the separation distance between the particles of the dipole). Let $E_{appr}$ be the magnitude of the field at point $P$ as approximated by Eqs. 22-8 and 22-9. Let $E_{act}$ be the actual magnitude. What is the ratio $E_{appr}/E_{act}$?

•••**21** SSM *Electric quadrupole.* Figure 22-46 shows a generic electric quadrupole. It consists of two dipoles with dipole moments that are equal in magnitude but opposite in direction. Show that the value of $E$ on the axis of the quadrupole for a point $P$ a distance $z$ from its center (assume $z \gg d$) is given by

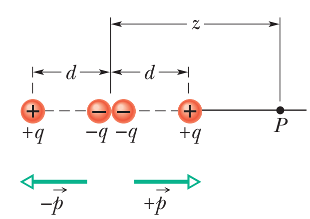

**Figure 22-46** Problem 21.

$$E = \frac{3Q}{4\pi\varepsilon_0 z^4},$$

in which $Q\ (= 2qd^2)$ is known as the *quadrupole moment* of the charge distribution.

## Module 22-4 The Electric Field Due to a Line of Charge

•**22** *Density, density, density.* (a) A charge $-300e$ is uniformly distributed along a circular arc of radius 4.00 cm, which subtends an angle of 40°. What is the linear charge density along the arc? (b) A charge $-300e$ is uniformly distributed over one face of a circular disk of radius 2.00 cm. What is the surface charge density over that face? (c) A charge $-300e$ is uniformly distributed over the surface of a sphere of radius 2.00 cm. What is the surface charge density over that surface? (d) A charge $-300e$ is uniformly spread through the volume of a sphere of radius 2.00 cm. What is the volume charge density in that sphere?

•**23** Figure 22-47 shows two parallel nonconducting rings with their central axes along a common line. Ring 1 has uniform charge $q_1$ and radius $R$; ring 2 has uniform charge $q_2$ and the same radius $R$. The rings are separated by distance $d = 3.00R$. The net electric field at point $P$ on the common line, at distance $R$ from ring 1, is zero. What is the ratio $q_1/q_2$?

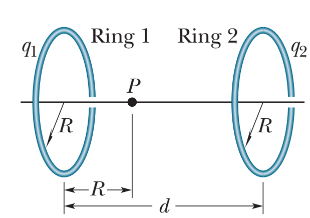


**Figure 22-47** Problem 23.

••**24** A thin nonconducting rod with a uniform distribution of positive charge $Q$ is bent into a complete circle of radius $R$

(Fig. 22-48). The central perpendicular axis through the ring is a $z$ axis, with the origin at the center of the ring. What is the magnitude of the electric field due to the rod at (a) $z = 0$ and (b) $z = \infty$? (c) In terms of $R$, at what positive value of $z$ is that magnitude maximum? (d) If $R = 2.00$ cm and $Q = 4.00\ \mu$C, what is the maximum magnitude?

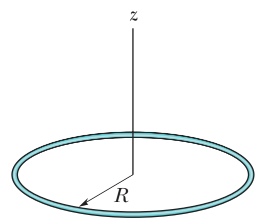


**Figure 22-48** Problem 24.

**••25** Figure 22-49 shows three circular arcs centered on the origin of a coordinate system. On each arc, the uniformly distributed charge is given in terms of $Q = 2.00\ \mu$C. The radii are given in terms of $R = 10.0$ cm. What are the (a) magnitude and (b) direction (relative to the positive $x$ direction) of the net electric field at the origin due to the arcs?

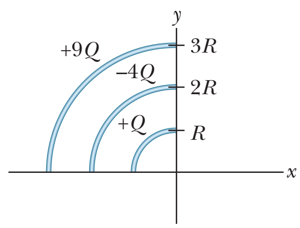


**Figure 22-49** Problem 25.

**••26** GO ILW In Fig. 22-50, a thin glass rod forms a semicircle of radius $r = 5.00$ cm. Charge is uniformly distributed along the rod, with $+q = 4.50$ pC in the upper half and $-q = -4.50$ pC in the lower half. What are the (a) magnitude and (b) direction (relative to the positive direction of the $x$ axis) of the electric field $\vec{E}$ at $P$, the center of the semicircle?

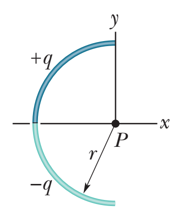


**Figure 22-50** Problem 26.

**••27** GO In Fig. 22-51, two curved plastic rods, one of charge $+q$ and the other of charge $-q$, form a circle of radius $R = 8.50$ cm in an $xy$ plane. The $x$ axis passes through both of the connecting points, and the charge is distributed uniformly on both rods. If $q = 15.0$ pC, what are the (a) magnitude and (b) direction (relative to the positive direction of the $x$ axis) of the electric field $\vec{E}$ produced at $P$, the center of the circle?

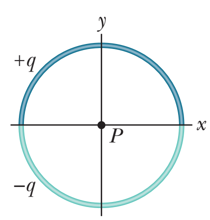


**Figure 22-51** Problem 27.

**••28** Charge is uniformly distributed around a ring of radius $R = 2.40$ cm, and the resulting electric field magnitude $E$ is measured along the ring's central axis (perpendicular to the plane of the ring). At what distance from the ring's center is $E$ maximum?

**••29** GO Figure 22-52*a* shows a nonconducting rod with a uniformly distributed charge $+Q$. The rod forms a half-circle with radius $R$ and produces an electric field of magnitude $E_{arc}$ at its center of curvature $P$. If the arc is collapsed to a point at distance $R$ from $P$ (Fig. 22-52*b*), by what factor is the magnitude of the electric field at $P$ multiplied?

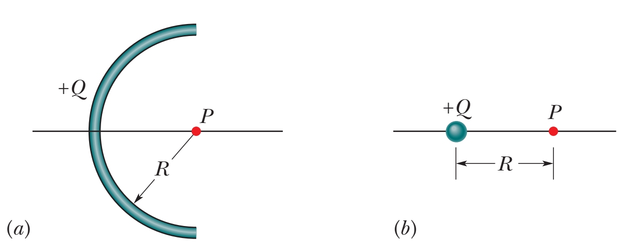


**Figure 22-52** Problem 29.

**••30** GO Figure 22-53 shows two concentric rings, of radii $R$ and $R' = 3.00R$, that lie on the same plane. Point $P$ lies on the central $z$ axis, at distance $D = 2.00R$ from the center of the rings. The smaller ring has uniformly distributed charge $+Q$. In terms of $Q$, what is the uniformly distributed charge on the larger ring if the net electric field at $P$ is zero?

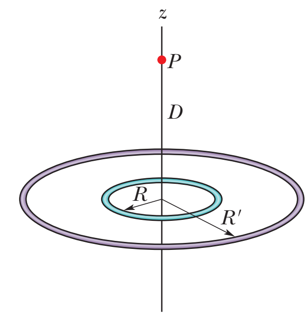


**Figure 22-53** Problem 30.

**••31** SSM ILW WWW In Fig. 22-54, a nonconducting rod of length $L = 8.15$ cm has a charge $-q = -4.23$ fC uniformly distributed along its length. (a) What is the linear charge density of the rod? What are the (b) magnitude and (c) direction (relative to the positive direction of the $x$ axis) of the electric field produced at point $P$, at distance $a = 12.0$ cm from the rod? What is the electric field magnitude produced at distance $a$ =50 m by (d) the rod and (e) a particle of charge $-q = -4.23$ fC that we use to replace the rod? (At that distance, the rod "looks" like a particle.)

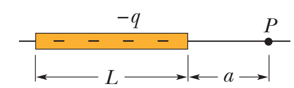


**Figure 22-54** Problem 31.

**•••32** GO In Fig. 22-55, positive charge $q = 7.81$ pC is spread uniformly along a thin nonconducting rod of length $L = 14.5$ cm. What are the (a) magnitude and (b) direction (relative to the positive direction of the $x$ axis) of the electric field produced at point $P$, at distance $R = 6.00$ cm from the rod along its perpendicular bisector?

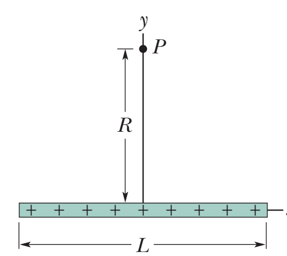


**Figure 22-55** Problem 32.

**•••33** GO In Fig. 22-56, a "semi-infinite" nonconducting rod (that is, infinite in one direction only) has uniform linear charge density $\lambda$. Show that the electric field $\vec{E}_p$ at point $P$ makes an angle of 45° with the rod and that this result is independent of the distance $R$. (*Hint:* Separately find the component of $\vec{E}_p$ parallel to the rod and the component perpendicular to the rod.)

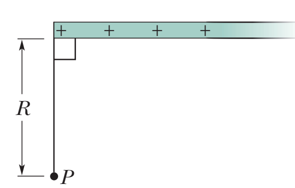


**Figure 22-56** Problem 33.

## Module 22-5 The Electric Field Due to a Charged Disk

**•34** A disk of radius 2.5 cm has a surface charge density of 5.3 $\mu$C/m$^2$ on its upper face. What is the magnitude of the electric field produced by the disk at a point on its central axis at distance $z = 12$ cm from the disk?

**•35** SSM WWW At what distance along the central perpendicular axis of a uniformly charged plastic disk of radius 0.600 m is the magnitude of the electric field equal to one-half the magnitude of the field at the center of the surface of the disk?

**••36** A circular plastic disk with radius $R = 2.00$ cm has a uniformly distributed charge $Q = +(2.00 \times 10^6)e$ on one face. A circular ring of width 30 $\mu$m is centered on that face, with the center of that width at radius $r = 0.50$ cm. In coulombs, what charge is contained within the width of the ring?

**••37** Suppose you design an apparatus in which a uniformly charged disk of radius $R$ is to produce an electric field. The field magnitude is most important along the central perpendicular axis of the disk, at a point $P$ at distance $2.00R$ from the disk (Fig. 22-57$a$). Cost analysis suggests that you switch to a ring of the same outer radius $R$ but with inner radius $R/2.00$ (Fig. 22-57$b$). Assume that the ring will have the same surface charge density as the original disk. If you switch to the ring, by what percentage will you decrease the electric field magnitude at $P$?

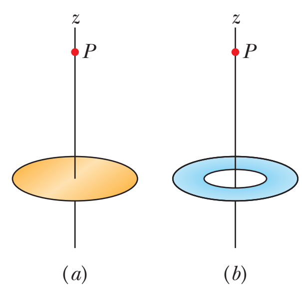


**Figure 22-57** Problem 37.

**••38** Figure 22-58$a$ shows a circular disk that is uniformly charged. The central $z$ axis is perpendicular to the disk face, with the origin at the disk. Figure 22-58$b$ gives the magnitude of the electric field along that axis in terms of the maximum magnitude $E_m$ at the disk surface. The $z$ axis scale is set by $z_s = 8.0$ cm. What is the radius of the disk?

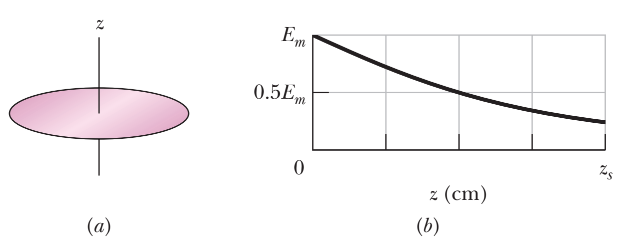


**Figure 22-58** Problem 38.

## Module 22-6 A Point Charge in an Electric Field

**•39** In Millikan's experiment, an oil drop of radius 1.64 $\mu$m and density 0.851 g/cm$^3$ is suspended in chamber C (Fig. 22-16) when a downward electric field of $1.92 \times 10^5$ N/C is applied. Find the charge on the drop, in terms of $e$.

**•40** GO An electron with a speed of $5.00 \times 10^8$ cm/s enters an electric field of magnitude $1.00 \times 10^3$ N/C, traveling along a field line in the direction that retards its motion. (a) How far will the electron travel in the field before stopping momentarily, and (b) how much time will have elapsed? (c) If the region containing the electric field is 8.00 mm long (too short for the electron to stop within it), what fraction of the electron's initial kinetic energy will be lost in that region?

**•41** SSM A charged cloud system produces an electric field in the air near Earth's surface. A particle of charge $-2.0 \times 10^{-9}$ C is acted on by a downward electrostatic force of $3.0 \times 10^{-6}$ N when placed in this field. (a) What is the magnitude of the electric field? What are the (b) magnitude and (c) direction of the electrostatic force $\vec{F}_{el}$ on a proton placed in this field? (d) What is the magnitude of the gravitational force $\vec{F}_g$ on the proton? (e) What is the ratio $F_{el}/F_g$ in this case?

**•42** Humid air breaks down (its molecules become ionized) in an electric field of $3.0 \times 10^6$ N/C. In that field, what is the magnitude of the electrostatic force on (a) an electron and (b) an ion with a single electron missing?

**•43** SSM An electron is released from rest in a uniform electric field of magnitude $2.00 \times 10^4$ N/C. Calculate the acceleration of the electron. (Ignore gravitation.)

**•44** An alpha particle (the nucleus of a helium atom) has a mass of $6.64 \times 10^{-27}$ kg and a charge of $+2e$. What are the (a) magnitude and (b) direction of the electric field that will balance the gravitational force on the particle?

**•45** ILW An electron on the axis of an electric dipole is 25 nm from the center of the dipole. What is the magnitude of the electrostatic force on the electron if the dipole moment is $3.6 \times 10^{-29}$ C·m? Assume that 25 nm is much larger than the separation of the charged particles that form the dipole.

**•46** An electron is accelerated eastward at $1.80 \times 10^9$ m/s$^2$ by an electric field. Determine the field (a) magnitude and (b) direction.

**•47** SSM Beams of high-speed protons can be produced in "guns" using electric fields to accelerate the protons. (a) What acceleration would a proton experience if the gun's electric field were $2.00 \times 10^4$ N/C? (b) What speed would the proton attain if the field accelerated the proton through a distance of 1.00 cm?

**••48** In Fig. 22-59, an electron (e) is to be released from rest on the central axis of a uniformly charged disk of radius $R$. The surface charge density on the disk is $+4.00$ $\mu$C/m$^2$. What is the magnitude of the electron's initial acceleration if it is released at a distance (a) $R$, (b) $R/100$, and (c) $R/1000$ from the center of the disk? (d) Why does the acceleration magnitude increase only slightly as the release point is moved closer to the disk?

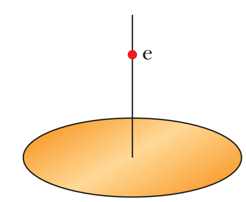


**Figure 22-59** Problem 48.

**••49** A 10.0 g block with a charge of $+8.00 \times 10^{-5}$ C is placed in an electric field $\vec{E} = (3000\hat{i} - 600\hat{j})$ N/C. What are the (a) magnitude and (b) direction (relative to the positive direction of the $x$ axis) of the electrostatic force on the block? If the block is released from rest at the origin at time $t = 0$, what are its (c) $x$ and (d) $y$ coordinates at $t = 3.00$ s?

**••50** At some instant the velocity components of an electron moving between two charged parallel plates are $v_x = 1.5 \times 10^5$ m/s and $v_y = 3.0 \times 10^3$ m/s. Suppose the electric field between the plates is uniform and given by $\vec{E} = (120 \text{ N/C})\hat{j}$. In unit-vector notation, what are (a) the electron's acceleration in that field and (b) the electron's velocity when its $x$ coordinate has changed by 2.0 cm?

**••51** Assume that a honeybee is a sphere of diameter 1.000 cm with a charge of +45.0 pC uniformly spread over its surface. Assume also that a spherical pollen grain of diameter 40.0 $\mu$m is electrically held on the surface of the bee because the bee's charge induces a charge of −1.00 pC on the near side of the grain and a charge of +1.00 pC on the far side. (a) What is the magnitude of the net electrostatic force on the grain due to the bee? Next, assume that the bee brings the grain to a distance of 1.000 mm from the tip of a flower's stigma and that the tip is a particle of charge −45.0 pC. (b) What is the magnitude of the net electrostatic force on the grain due to the stigma? (c) Does the grain remain on the bee or does it move to the stigma?

**••52** An electron enters a region of uniform electric field with an initial velocity of 40 km/s in the same direction as the electric field, which has magnitude $E = 50$ N/C. (a) What is the speed of the electron 1.5 ns after entering this region? (b) How far does the electron travel during the 1.5 ns interval?

••53 GO Two large parallel copper plates are 5.0 cm apart and have a uniform electric field between them as depicted in Fig. 22-60. An electron is released from the negative plate at the same time that a proton is released from the positive plate. Neglect the force of the particles on each other and find their distance from the positive plate when they pass each other. (Does it surprise you that you need not know the electric field to solve this problem?)

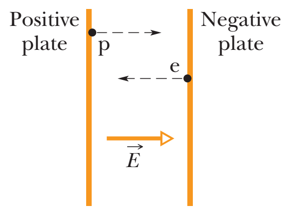


**Figure 22-60** Problem 53.

••54 GO In Fig. 22-61, an electron is shot at an initial speed of $v_0 = 2.00 \times 10^6$ m/s, at angle $\theta_0 = 40.0°$ from an $x$ axis. It moves through a uniform electric field $\vec{E} = (5.00\text{ N/C})\hat{j}$. A screen for detecting electrons is positioned parallel to the $y$ axis, at distance $x = 3.00$ m. In unit-vector notation, what is the velocity of the electron when it hits the screen?

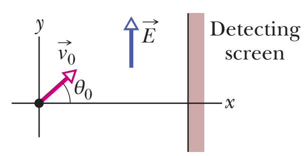


**Figure 22-61** Problem 54.

••55 ILW A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0 cm away, in a time $1.5 \times 10^{-8}$ s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field $\vec{E}$?

## Module 22-7 A Dipole in an Electric Field

•56 An electric dipole consists of charges $+2e$ and $-2e$ separated by 0.78 nm. It is in an electric field of strength $3.4 \times 10^6$ N/C. Calculate the magnitude of the torque on the dipole when the dipole moment is (a) parallel to, (b) perpendicular to, and (c) antiparallel to the electric field.

•57 SSM An electric dipole consisting of charges of magnitude 1.50 nC separated by 6.20 $\mu$m is in an electric field of strength 1100 N/C. What are (a) the magnitude of the electric dipole moment and (b) the difference between the potential energies for dipole orientations parallel and antiparallel to $\vec{E}$?

••58 A certain electric dipole is placed in a uniform electric field $\vec{E}$ of magnitude 20 N/C. Figure 22-62 gives the potential energy $U$ of the dipole versus the angle $\theta$ between $\vec{E}$ and the dipole moment $\vec{p}$. The vertical axis scale is set by $U_s = 100 \times 10^{-28}$ J. What is the magnitude of $\vec{p}$?

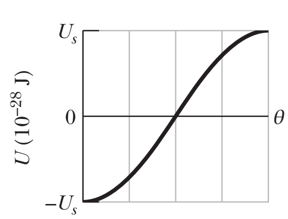


**Figure 22-62** Problem 58.

••59 How much work is required to turn an electric dipole 180° in a uniform electric field of magnitude $E = 46.0$ N/C if the dipole moment has a magnitude of $p = 3.02 \times 10^{-25}$ C·m and the initial angle is 64°?

••60 A certain electric dipole is placed in a uniform electric field $\vec{E}$ of magnitude 40 N/C. Figure 22-63 gives the magnitude $\tau$ of the torque on the dipole versus the angle $\theta$ between field $\vec{E}$ and the dipole moment $\vec{p}$. The vertical axis scale is set by $\tau_s = 100 \times 10^{-28}$ N·m. What is the magnitude of $\vec{p}$?

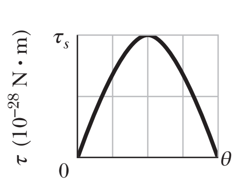


**Figure 22-63** Problem 60.

••61 Find an expression for the oscillation frequency of an electric dipole of dipole moment $\vec{p}$ and rotational inertia $I$ for small amplitudes of oscillation about its equilibrium position in a uniform electric field of magnitude $E$.

### Additional Problems

62 (a) What is the magnitude of an electron's acceleration in a uniform electric field of magnitude $1.40 \times 10^6$ N/C? (b) How long would the electron take, starting from rest, to attain one-tenth the speed of light? (c) How far would it travel in that time?

63 A spherical water drop 1.20 $\mu$m in diameter is suspended in calm air due to a downward-directed atmospheric electric field of magnitude $E = 462$ N/C. (a) What is the magnitude of the gravitational force on the drop? (b) How many excess electrons does it have?

64 Three particles, each with positive charge $Q$, form an equilateral triangle, with each side of length $d$. What is the magnitude of the electric field produced by the particles at the midpoint of any side?

65 In Fig. 22-64$a$, a particle of charge $+Q$ produces an electric field of magnitude $E_{part}$ at point $P$, at distance $R$ from the particle. In Fig. 22-64$b$, that same amount of charge is spread uniformly along a circular arc that has radius $R$ and subtends an angle $\theta$. The charge on the arc produces an electric field of magnitude $E_{arc}$ at its center of curvature $P$. For what value of $\theta$ does $E_{arc} = 0.500E_{part}$? (*Hint:* You will probably resort to a graphical solution.)

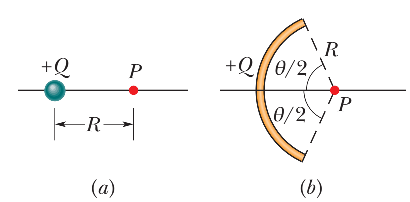


**Figure 22-64** Problem 65.

66 A proton and an electron form two corners of an equilateral triangle of side length $2.0 \times 10^{-6}$ m. What is the magnitude of the net electric field these two particles produce at the third corner?

67 A charge (uniform linear density = 9.0 nC/m) lies on a string that is stretched along an $x$ axis from $x = 0$ to $x = 3.0$ m. Determine the magnitude of the electric field at $x = 4.0$ m on the $x$ axis.

68 In Fig. 22-65, eight particles form a square in which distance $d = 2.0$ cm. The charges are $q_1 = +3e$, $q_2 = +e$, $q_3 = -5e$, $q_4 = -2e$, $q_5 = +3e$, $q_6 = +e$, $q_7 = -5e$, and $q_8 = +e$. In unit-vector notation, what is the net electric field at the square's center?

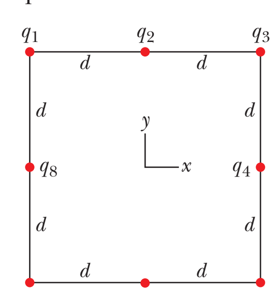


**Figure 22-65** Problem 68.

69 Two particles, each with a charge of magnitude 12 nC, are at two of the vertices of an equilateral triangle with edge length 2.0 m. What is the magnitude of the electric field at the third vertex if (a) both charges are positive and (b) one charge is positive and the other is negative?

70 The following table gives the charge seen by Millikan at different times on a single drop in his experiment. From the data, calculate the elementary charge $e$.

| | | |
|---|---|---|
| $6.563 \times 10^{-19}$ C | $13.13 \times 10^{-19}$ C | $19.71 \times 10^{-19}$ C |
| $8.204 \times 10^{-19}$ C | $16.48 \times 10^{-19}$ C | $22.89 \times 10^{-19}$ C |
| $11.50 \times 10^{-19}$ C | $18.08 \times 10^{-19}$ C | $26.13 \times 10^{-19}$ C |

**71** A charge of 20 nC is uniformly distributed along a straight rod of length 4.0 m that is bent into a circular arc with a radius of 2.0 m. What is the magnitude of the electric field at the center of curvature of the arc?

**72** An electron is constrained to the central axis of the ring of charge of radius $R$ in Fig. 22-11, with $z \ll R$. Show that the electrostatic force on the electron can cause it to oscillate through the ring center with an angular frequency

$$\omega = \sqrt{\frac{eq}{4\pi\varepsilon_0 m R^3}},$$

where $q$ is the ring's charge and $m$ is the electron's mass.

**73** SSM The electric field in an $xy$ plane produced by a positively charged particle is $7.2(4.0\hat{\imath} + 3.0\hat{\jmath})$ N/C at the point (3.0, 3.0) cm and $100\hat{\imath}$ N/C at the point (2.0, 0) cm. What are the (a) $x$ and (b) $y$ coordinates of the particle? (c) What is the charge of the particle?

**74** (a) What total (excess) charge $q$ must the disk in Fig. 22-15 have for the electric field on the surface of the disk at its center to have magnitude $3.0 \times 10^6$ N/C, the $E$ value at which air breaks down electrically, producing sparks? Take the disk radius as 2.5 cm. (b) Suppose each surface atom has an effective cross-sectional area of 0.015 nm$^2$. How many atoms are needed to make up the disk surface? (c) The charge calculated in (a) results from some of the surface atoms having one excess electron. What fraction of these atoms must be so charged?

**75** In Fig. 22-66, particle 1 (of charge $+1.00\ \mu$C), particle 2 (of charge $+1.00\ \mu$C), and particle 3 (of charge $Q$) form an equilateral triangle of edge length $a$. For what value of $Q$ (both sign and magnitude) does the net electric field produced by the particles at the center of the triangle vanish?

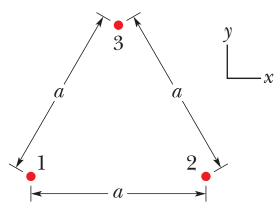

**Figure 22-66** Problems 75 and 86.

**76** In Fig. 22-67, an electric dipole swings from an initial orientation $i$ ($\theta_i = 20.0°$) to a final orientation $f$ ($\theta_f = 20.0°$) in a uniform external electric field $\vec{E}$. The electric dipole moment is $1.60 \times 10^{-27}$ C·m; the field magnitude is $3.00 \times 10^6$ N/C. What is the change in the dipole's potential energy?

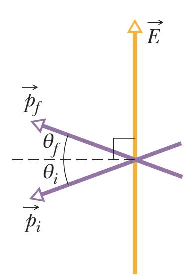

**Figure 22-67** Problem 76.

**77** A particle of charge $-q_1$ is at the origin of an $x$ axis. (a) At what location on the axis should a particle of charge $-4q_1$ be placed so that the net electric field is zero at $x = 2.0$ mm on the axis? (b) If, instead, a particle of charge $+4q_1$ is placed at that location, what is the direction (relative to the positive direction of the $x$ axis) of the net electric field at $x = 2.0$ mm?

**78** Two particles, each of positive charge $q$, are fixed in place on a $y$ axis, one at $y = d$ and the other at $y = -d$. (a) Write an expression that gives the magnitude $E$ of the net electric field at points on the $x$ axis given by $x = \alpha d$. (b) Graph $E$ versus $\alpha$ for the range $0 < \alpha < 4$. From the graph, determine the values of $\alpha$ that give (c) the maximum value of $E$ and (d) half the maximum value of $E$.

**79** A clock face has negative point charges $-q, -2q, -3q, \ldots, -12q$ fixed at the positions of the corresponding numerals. The clock hands do not perturb the net field due to the point charges. At what time does the hour hand point in the same direction as the electric field vector at the center of the dial? (*Hint:* Use symmetry.)

**80** Calculate the electric dipole moment of an electron and a proton 4.30 nm apart.

**81** An electric field $\vec{E}$ with an average magnitude of about 150 N/C points downward in the atmosphere near Earth's surface. We wish to "float" a sulfur sphere weighing 4.4 N in this field by charging the sphere. (a) What charge (both sign and magnitude) must be used? (b) Why is the experiment impractical?

**82** A circular rod has a radius of curvature $R = 9.00$ cm and a uniformly distributed positive charge $Q = 6.25$ pC and subtends an angle $\theta = 2.40$ rad. What is the magnitude of the electric field that $Q$ produces at the center of curvature?

**83** SSM An electric dipole with dipole moment

$$\vec{p} = (3.00\hat{\imath} + 4.00\hat{\jmath})(1.24 \times 10^{-30}\ \text{C}\cdot\text{m})$$

is in an electric field $\vec{E} = (4000\ \text{N/C})\hat{\imath}$. (a) What is the potential energy of the electric dipole? (b) What is the torque acting on it? (c) If an external agent turns the dipole until its electric dipole moment is

$$\vec{p} = (-4.00\hat{\imath} + 3.00\hat{\jmath})(1.24 \times 10^{-30}\ \text{C}\cdot\text{m}),$$

how much work is done by the agent?

**84** In Fig. 22-68, a uniform, upward electric field $\vec{E}$ of magnitude $2.00 \times 10^3$ N/C has been set up between two horizontal plates by charging the lower plate positively and the upper plate negatively. The plates have length $L = 10.0$ cm and separation $d = 2.00$ cm. An electron is then shot between the plates from the left edge of the lower plate. The initial velocity $\vec{v}_0$ of the electron makes an angle $\theta = 45.0°$ with the lower plate and has a magnitude of $6.00 \times 10^6$ m/s. (a) Will the electron strike one of the plates? (b) If so, which plate and how far horizontally from the left edge will the electron strike?

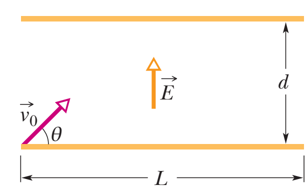

**Figure 22-68** Problem 84.

**85** For the data of Problem 70, assume that the charge $q$ on the drop is given by $q = ne$, where $n$ is an integer and $e$ is the elementary charge. (a) Find $n$ for each given value of $q$. (b) Do a linear regression fit of the values of $q$ versus the values of $n$ and then use that fit to find $e$.

**86** In Fig. 22-66, particle 1 (of charge +2.00 pC), particle 2 (of charge −2.00 pC), and particle 3 (of charge +5.00 pC) form an equilateral triangle of edge length $a = 9.50$ cm. (a) Relative to the positive direction of the $x$ axis, determine the direction of the force $\vec{F}_3$ on particle 3 due to the other particles by sketching electric field lines of the other particles. (b) Calculate the magnitude of $\vec{F}_3$.

**87** In Fig. 22-69, particle 1 of charge $q_1 = 1.00$ pC and particle 2 of charge $q_2 = -2.00$ pC are fixed at a distance $d = 5.00$ cm apart. In unit-vector notation, what is the net electric field at points (a) $A$, (b) $B$, and (c) $C$? (d) Sketch the electric field lines.

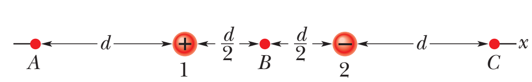

**Figure 22-69** Problem 87.