UFUG 1504: Honors General Physics II

Chapter 32

Maxwell Equations; Magnetism of Matter

Summary (1 of 9)

Gauss' Law for Magnetic Fields

- Gauss' law for magnetic fields,

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0,$$

Equation (32-1)

Maxwell's Extension of Ampere's Law

- A changing electric field induces a magnetic field given by,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Equation (32-3)

Summary (2 of 9)

- Maxwell's law and Ampere's law can be written as the single equation

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

Equation (32-5)

Displacement Current

- We define the fictitious displacement current due to a changing electric field as

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}.$$

Equation (32-10)

- Equation 32-5 then becomes

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}}$$

Equation (32-11)

Summary (3 of 9)

Maxwell's Equations

- Four equations are as follows:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_B}{dt} + \mu_0 i_{\text{enc}}$$

Summary (4 of 9)

Spin Magnetic Dipole Moment

- Spin angular momentum of electron is associated with spin magnetic dipole momentum through,

$$\vec{\mu}_s = -\frac{e}{m}\vec{S}.$$

Equation (32-22)

- For a measurement along a z axis, the component S_z can have only the values given by

$$S_z = m_s \frac{h}{2\pi}, \quad \text{for} \quad m_s = \pm\tfrac{1}{2},$$

Equation (32-23)

Summary (5 of 9)

- Similarly,

$$\mu_{s,z} = \pm \frac{eh}{4\pi m} = \pm \mu_{\mathrm{B}},$$ **Equation (32-24 & 26)**

- Where the Bohr magneton is

$$\mu_{\mathrm{B}} = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \mathrm{J/T}.$$ **Equation (32-25)**

- The energy U

$$U = -\vec{\mu}_s \cdot \vec{B}_{\mathrm{ext}} = -\mu_{s,z} B_{\mathrm{ext}}.$$ **Equation (32-27)**

Summary (6 of 9)

Orbital Magnetic Dipole Momentum

- Angular momentum of an electron is associated with orbital magnetic dipole momentum as

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}}.$$

Equation (32-28)

- Orbital angular momentum is quantized,

$$L_{\text{orb},z} = m_\ell \frac{h}{2\pi}, \quad \text{for} \quad m_\ell = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit}).$$

Equation (32-29)

Summary (7 of 9)

- The associated magnetic dipole moment is given by

$$\mu_{\text{orb},z} = -m_\ell \frac{eh}{4\pi m} = m_\ell \mu_{\text{B}}.$$

Equation (32-30 & 31)

- The energy U

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}}.$$

Equation (32-32)

Summary (8 of 9)

Diamagnetism

- Diamagnetic materials exhibit magnetism only when placed in an external magnetic field; there they form magnetic dipoles directed opposite the external field. In a nonuniform field, they are repelled from the region of greater magnetic field.

Paramagnetism

- Paramagnetic materials have atoms with a permanent magnetic dipole moment but the moments are randomly oriented unless the material is in an external magnetic field. The extent of alignment within a volume V is measured as the magnetization M, given by

$$M = \frac{\text{measured magnetic moment}}{V}. \qquad \textbf{Equation (32-28)}$$

Summary (9 of 9)

- Complete alignment (saturation) of all N dipoles in the volume gives a maximum value $M_{\max} = \frac{N\mu}{V}$. At low values of the ratio $\frac{B_{\text{ext}}}{T}$,

$$M = C\frac{B_{\text{ext}}}{T}$$ **Equation (32-39)**

Ferromagnetism

- The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions (domains). Alignment is eliminated at temperatures above a material's Curie temperature. In a nonuniform external field, a ferromagnetic material is attracted to the region of greater magnetic field.

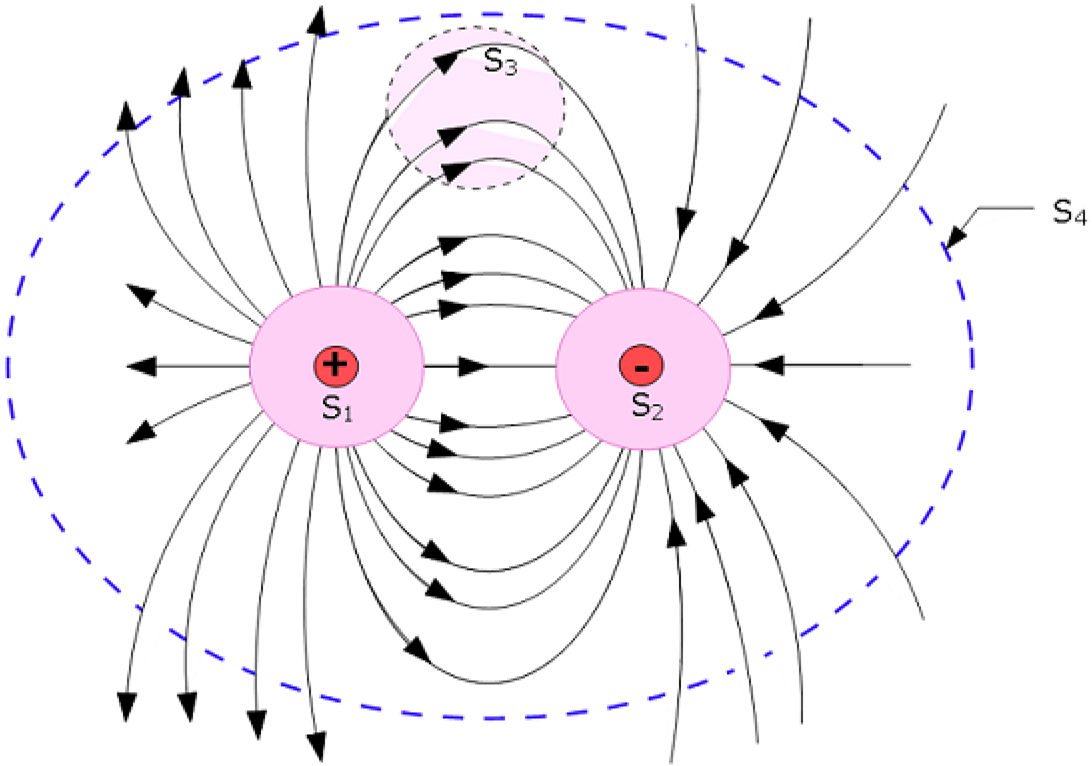
32-1 Gauss' Law for Magnetic Fields (4 of 5)

Surface S_1. The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is positive, and so is the net charge within the surface, as Gauss' law requires

Surface S_2. The electric field is inward for all points on this surface. Thus, the flux of the electric field through this surface is negative and so is the enclosed charge, as Gauss' law requires.

Surface S_3. This surface encloses no charge, and thus $q_{enc} = 0$. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because all the field lines pass entirely through the surface, entering it at the top and leaving at the bottom.

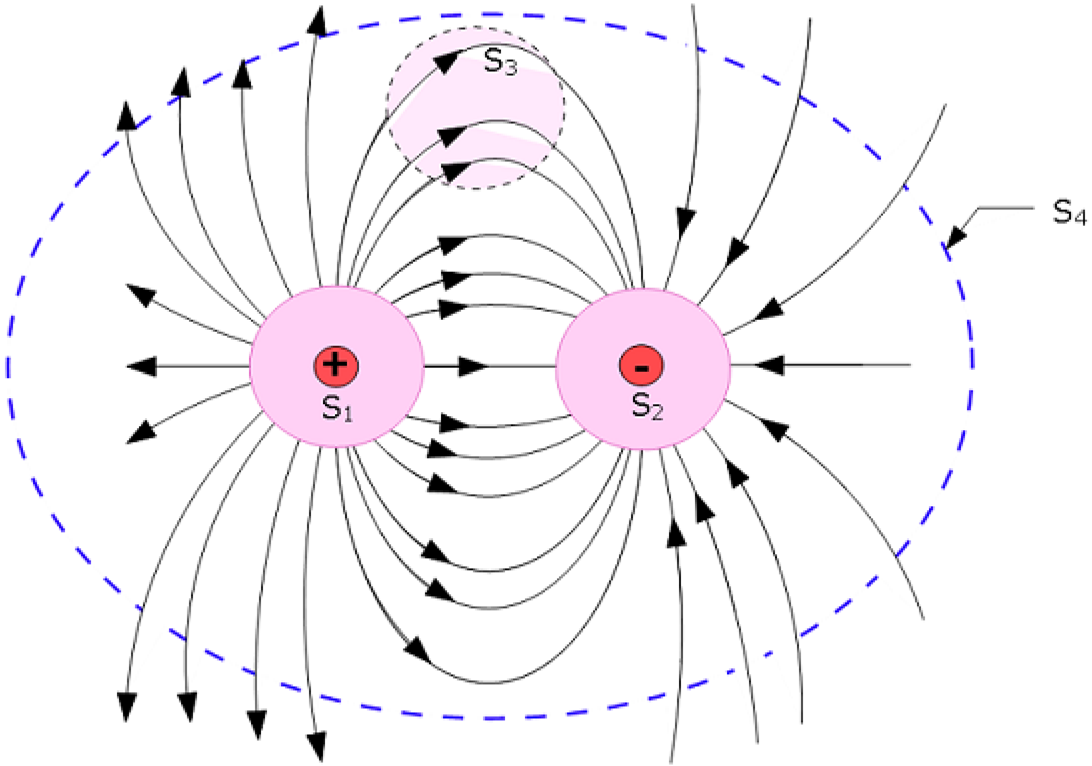
Surface S_4. This surface encloses no net charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because there are as many field lines leaving surface S_4 as entering it.



32-1 Gauss' Law for Magnetic Fields (4 of 5)

Contrast this with Gauss' law for electric fields,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

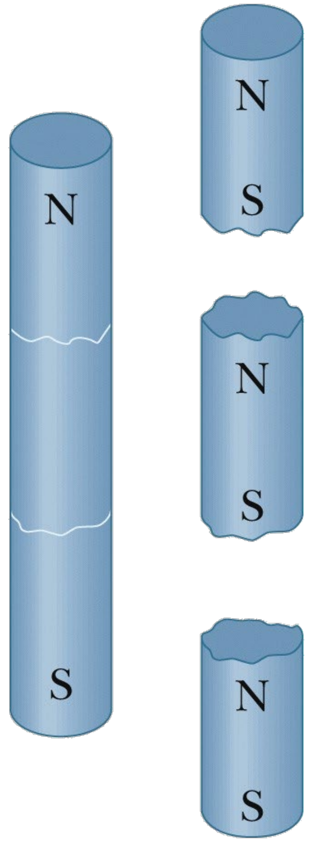


Gauss' law for electric fields says that this integral (the net electric flux through the surface) is proportional to the net electric charge q_{enc} enclosed by the surface

32-1 Gauss’ Law for Magnetic Fields (3 of 5)

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).

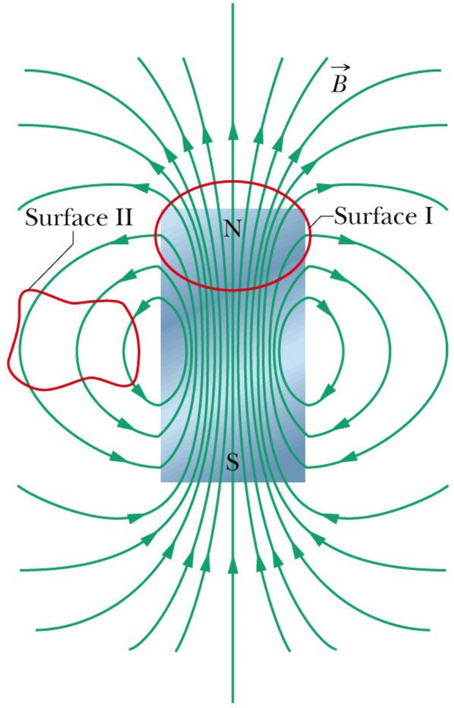
If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.



32-1 Gauss' Law for Magnetic Fields (4 of 5)

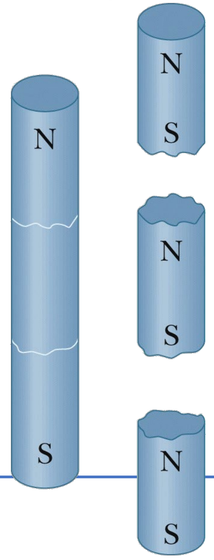
Gauss' law for magnetic fields is a formal way of saying that magnetic monopoles do not exist. The law asserts that the net magnetic flux Φ_B through any closed Gaussian surface is zero:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$



Gaussian surface II near the bar magnet of encloses no poles, and we can easily conclude that the net magnetic flux through it is zero.

Gaussian surface I is more difficult. It may seem to enclose only the north pole of the magnet. However, a south pole must be associated with the lower boundary of the surface because magnetic field lines enter the surface there. (The enclosed section is like one piece of the broken bar magnet) Thus, Gaussian surface I encloses a magnetic dipole, and the net flux through the surface is zero



32-1 Gauss' Law for Magnetic Fields (4 of 5)

Gauss' law for magnetic fields is a formal way of saying that magnetic monopoles do not exist. The law asserts that the net magnetic flux Φ_B through any closed Gaussian surface is zero:

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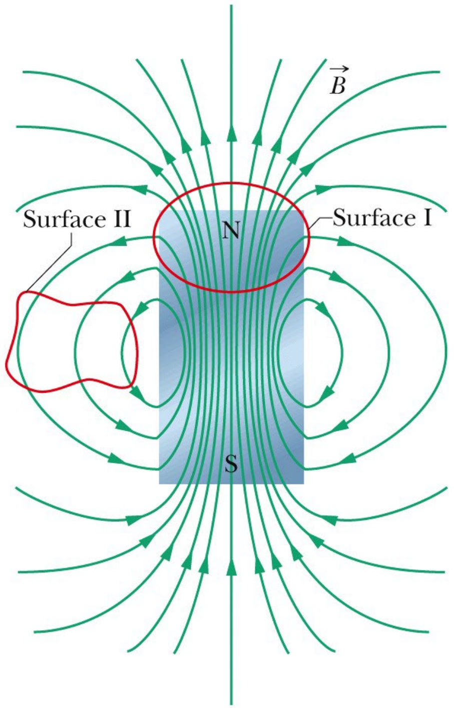


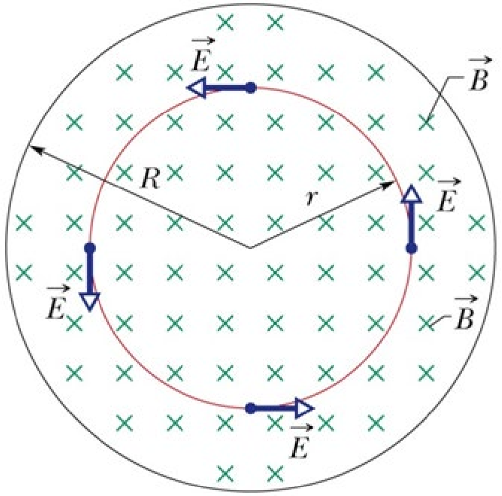
The field lines for the magnetic field $\vec{B}$ of a short bar magnet. The red curves represent cross sections of closed, three-dimensional Gaussian surfaces.

Gauss' law for magnetic fields says that there can be no net magnetic flux through the surface because there can be no net "magnetic charge" (individual magnetic poles) enclosed by the surface.

32-2 Induced Magnetic Fields (8 of 8)

Review Farady law in chapt. 32

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law of induction).}$$



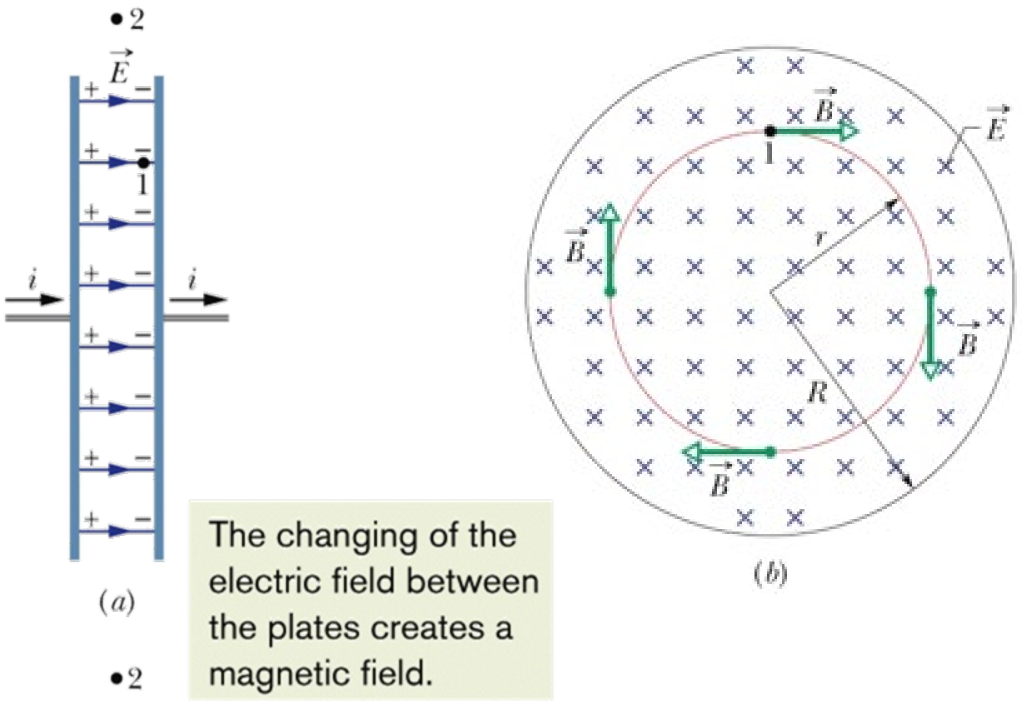
The induced $\vec{E}$ direction here is opposite the induced $\vec{B}$ direction in the preceding figure.

A uniform magnetic field B in a circular region. The field, directed into the page, is increasing in magnitude. The electric field E induced by the changing magnetic field is shown at four points on a circle concentric with the circular region.

32-2 Induced Magnetic Fields (4 of 8)

Charging a Capacitor.

As an example of this sort of induction, we consider the charging of a parallel-plate capacitor with circular plates. The charge on our capacitor is being increased at a steady rate by a constant current i in the connecting wires. Then the electric field magnitude between the plates must also be increasing at a steady rate.

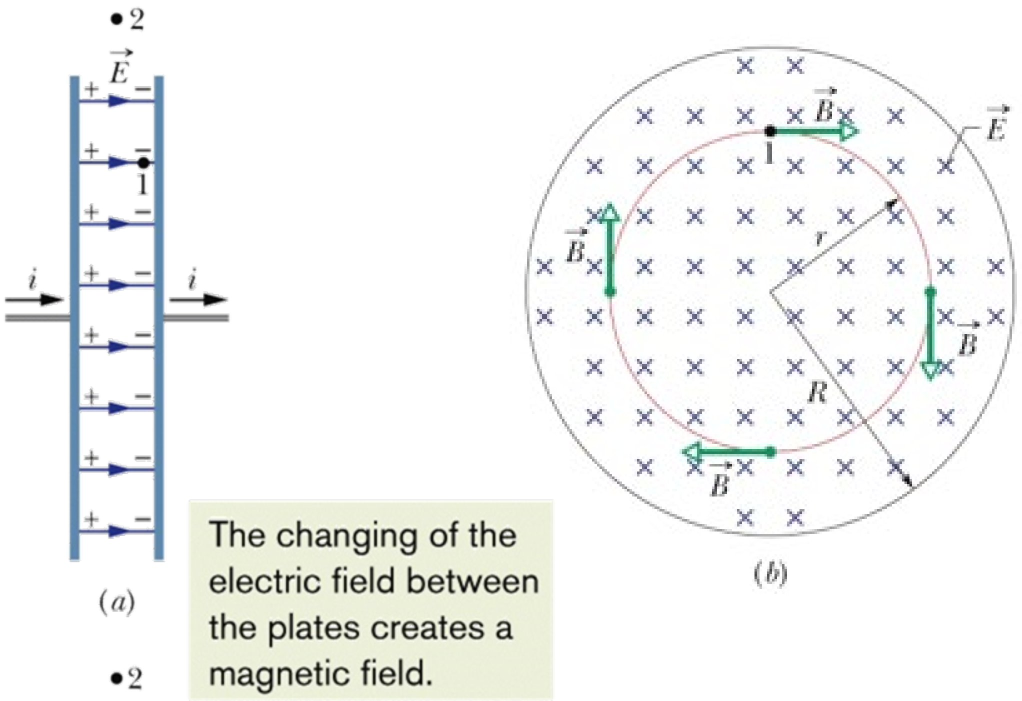


(a) A circular parallel-plate capacitor (side view) is being charged by a constant current i. (b) A view from within the capacitor, looking toward the plate at the right in (a). The electric field E is uniform, is directed into the page (toward the plate), and grows in magnitude as the charge on the capacitor increases. The magnetic field B induced by this changing electric field is shown at four points on a circle with a radius r less than the plate radius R

32-2 Induced Magnetic Fields (6 of 8)

Charging a Capacitor.

Figure (*b*) is a view of the right-hand plate of Fig. (*a*) from between the plates. The electric field is directed into the page. Let us consider a circular loop through point 1 in Figs.(*a*) and (*b*), a loop that is concentric with the capacitor plates and has a radius smaller than that of the plates. Because the electric field through the loop is changing, the electric flux through the loop must also be changing. According to the Maxwell's Law, this changing electric flux induces a magnetic field around the loop.



(*a*) A circular parallel-plate capacitor (side view) is being charged by a constant current *i*. (*b*) A view from within the capacitor, looking toward the plate at the right in (*a*). The electric field *E* is uniform, is directed into the page (toward the plate), and grows in magnitude as the charge on the capacitor increases. The magnetic field *B* induced by this changing electric field is shown at four points on a circle with a radius *r* less than the plate radius *R*

32-2 Induced Magnetic Fields (3 of 8)

A changing electric flux induces a magnetic field $\vec{B}$

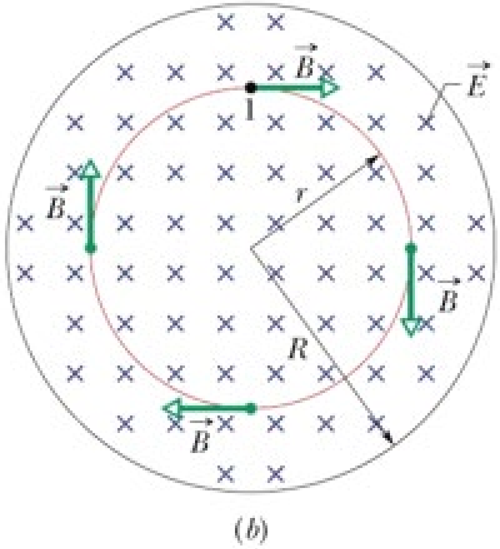
Maxwell's Law,

Not – sign here

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \boxed{\frac{d\Phi_E}{dt}}$$

Change/Increase

B induced by this changing E



Relates the magnetic field induced along a closed loop to the changing electric flux Φ_E through the loop.

– sign here mean opposite direction

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law of induction).}$$

32-2 Induced Magnetic Fields (7 of 8)

Ampere-Maxwell Law

Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

gives the magnetic field generated by a current i_{enc} encircled by a closed loop.

Thus, the two equations (the other being Maxwell's Law) that specify the magnetic field $\vec{B}$ produced by means other than a magnetic material (that is, by a current and by a changing electric field) give the field in exactly the same form. We can combine the two equations into the single equation:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

32-2 Induced Magnetic Fields (8 of 8)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

When there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of Eq. is zero, and so the Eq. reduces to Ampere's law.

When there is a change in electric flux but no current (such as inside or outside the gap of a charging capacitor), the second term on the right side of Eq. is zero, and so Eq. reduces to Maxwell's law of induction.

32-3 Displacement Current （位移电流）

If you compare the two terms on the right side of Eq. (Ampere-Maxwell Law), you will see that the product $\varepsilon_0\left(\frac{d\Phi_E}{dt}\right)$ must have the dimension of a current. In fact, that product has been treated as being a fictitious (虚拟的) current called the **displacement current** i_d:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

Ampere-Maxwell Law then becomes,

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}}$$

where $i_{d,\text{ enc}}$ is the displacement current encircled by the integration loop.

Review

29-3 Ampere's Law (4 of 5)

Magnetic Fields of a long straight wire with current:

$$B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straigth wire)}.$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

B and ds are either parallel or antiparallel at each point of the loop, so at every point the angle θ between ds and B is 0°, so $\cos\theta = \cos 0° = 1$.

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i$$

All of the current is encircled and thus all is used in Ampere's law.

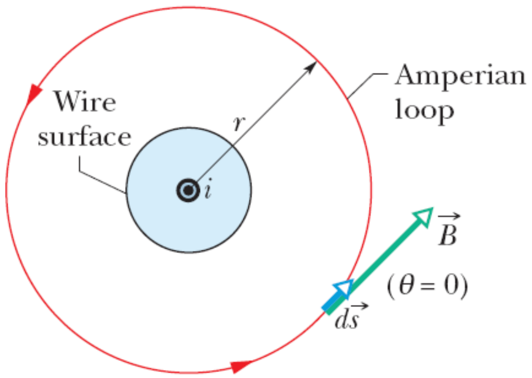


Figure 29-14 Using Ampere's law to find the magnetic field that a current i produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

29-3 Ampere’s Law (4 of 5)

Magnetic Fields of a long straight wire with current:

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \quad \text{(inside straight wire)}.$$

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r).$$

$$i_{\text{enc}} = i \, \frac{\pi r^2}{\pi R^2}.$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

$$B(2\pi r) = \mu_0 i \, \frac{\pi r^2}{\pi R^2}$$

Only the current encircled by the loop is used in Ampere’s law.

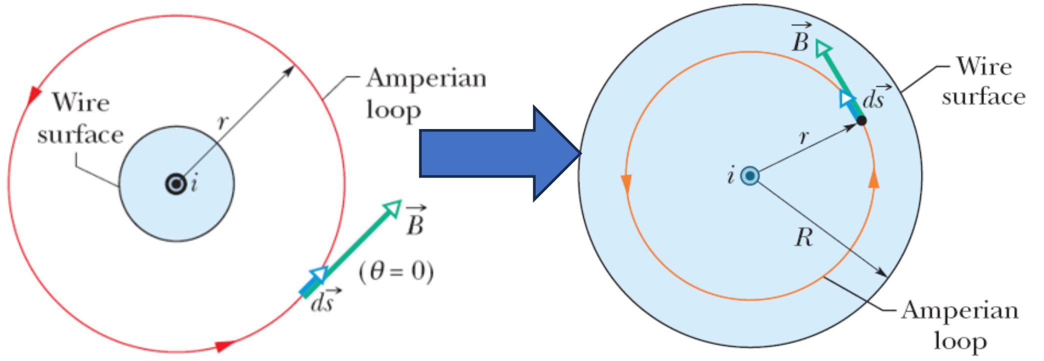


Figure 29-15 Using Ampere’s law to find the magnetic field that a current *i* produces inside a long straight wire of circula cross section. The current is uniformly dis tributed over the cross section of the wire and emerges from the page. An Amperia loop is drawn inside the wire.

32-3 Displacement Current (5 of 8)

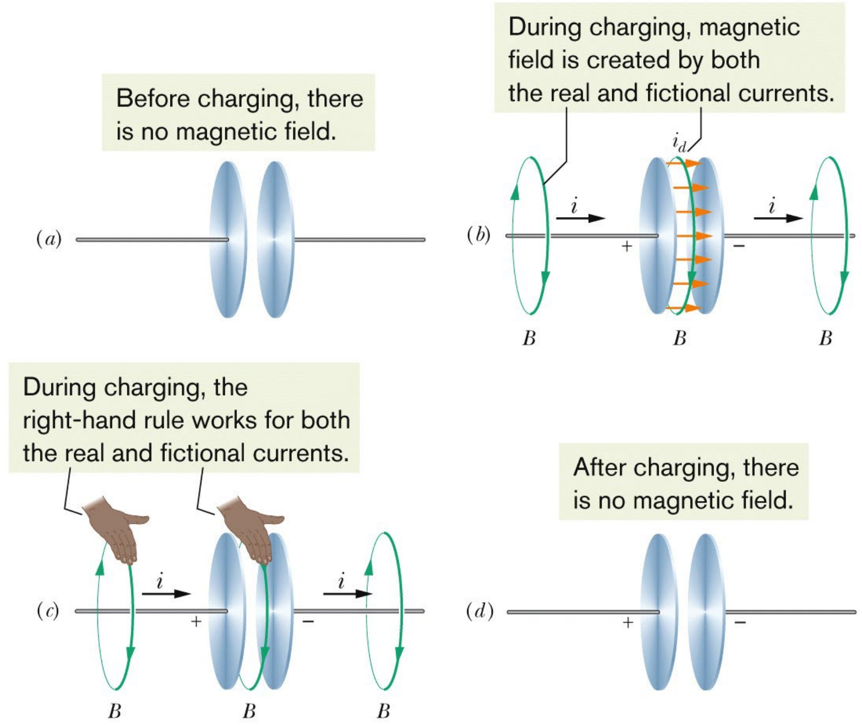
Finding the Induced Magnetic Field: In Chapter 29 we found the direction of the magnetic field produced by a real current *i* by using the right-hand rule. We can apply the same rule to find the direction of an induced magnetic field produced by a fictitious displacement current i_d, as is shown in the center of Fig. (c) for a capacitor.

Then, as done previously, the magnitude of the magnetic field at a point inside the capacitor at radius r from the center is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}} \Rightarrow B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r$$

the magnitude of the magnetic field at a point outside the capacitor at radius r is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}} \Rightarrow B = \frac{\mu_0 i_d}{2\pi r}$$



(*a*) Before and (*d*) after the plates are charged, there is no magnetic field. (*b*) During the charging, magnetic field is created by both the real current and the (fictional) (*c*) displacement current. (*c*) The same right-hand rule works for both currents to give the direction of the magnetic field.

32-3 Displacement Current (7 of 8)

The four fundamental equations of electromagnetism, called Maxwell's equations and are displayed in Table 32-1.

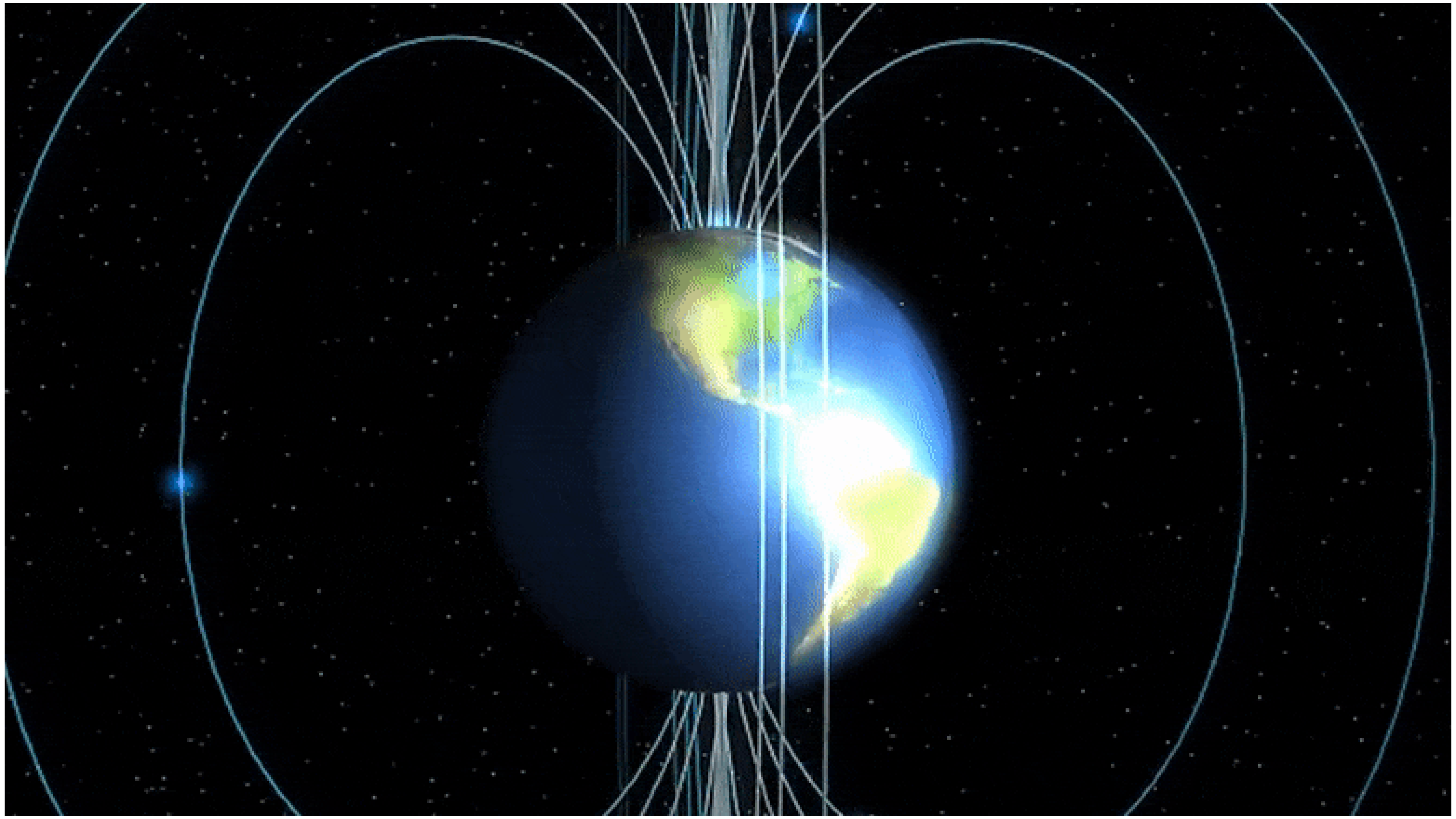
Table 32-1 Maxwell's Equations[a]

Name	Equation	
Gauss' law for electricity	$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere–Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$	Relates induced magnetic field to changing electric flux and to current

[a]Written on the assumption that no dielectric or magnetic materials are present.

These four equations explain a diverse range of phenomena, from why a compass needle points north to why a car starts when you turn the ignition key. They are the basis for the functioning of such electromagnetic devices as electric motors, television transmitters and receivers, telephones, scanners, radar, and microwave ovens.

32-4 Magnets (2 of 3)



32-4 Magnets (2 of 3)

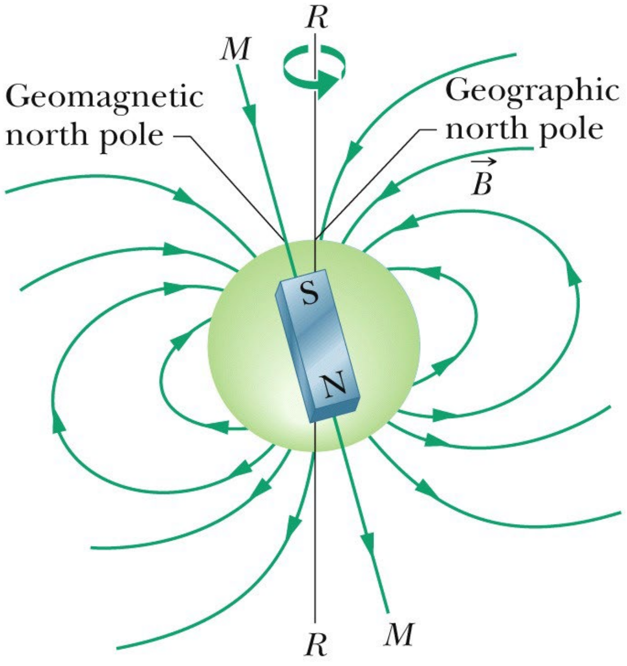
The Magnetism of Earth

Earth is a huge magnet; for points near Earth's surface, its magnetic field can be approximated as the field of a huge bar magnet — a magnetic dipole — that straddles the center of the planet.

Figure shown here is an idealized symmetric depiction of the dipole field, without the distortion caused by passing charged particles from the Sun.

The direction of the magnetic field at any location on Earth's surface is commonly specified in terms of two angles. The **field declination** (场偏差) is the angle (left or right) between geographic north (which is toward 90° latitude) and the horizontal component of the field. The **field inclination** (磁倾角) is the angle (up or down) between a horizontal plane and the field's direction.

For Earth, the south pole of the dipole is actually in the north.



Earth's magnetic field represented as a dipole field. The dipole axis MM makes an angle of 11.5° with Earth's rotational axis RR. The south pole of the dipole is in Earth's Northern Hemisphere.

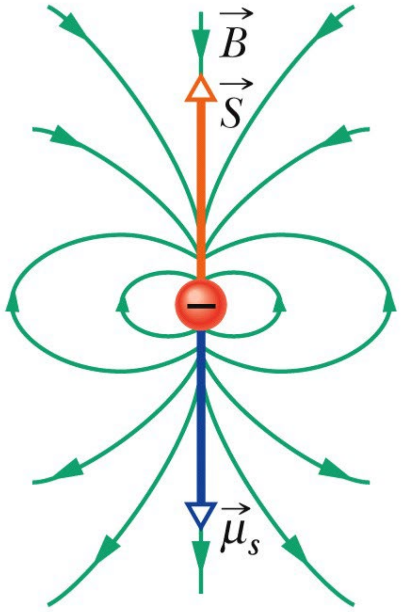
32-5 Magnetism and Electrons (7 of 15)

Spin Magnetic Dipole Moment (自旋磁偶极矩). An electron has an intrinsic angular momentum (固有角动量) called its spin angular momentum (or just spin) $\vec{S}$; associated with this spin is an intrinsic spin magnetic dipole moment $\vec{\mu}_s$. (By intrinsic, we mean that $\vec{S}$ and $\vec{\mu}_s$ are basic characteristics of an electron, like its mass and electric charge.)

Vectors $\vec{S}$ and $\vec{\mu}_s$ are related by

$$\vec{\mu}_s = -\frac{e}{m}\vec{S},$$

For an electron, the spin is opposite the magnetic dipole moment.



32-5 Magnetism and Electrons (8 of 15)

$$\vec{\mu}_s = -\frac{e}{m}\vec{S},$$

in which e is the elementary charge $\left(1.60 \times 10^{-19}\,\mathrm{C}\right)$ and m is the mass of an electron $\left(9.11 \times 10^{-31}\,\mathrm{kg}\right)$. The minus sign means that $\vec{\mu}_s$ and $\vec{S}$ are oppositely directed.

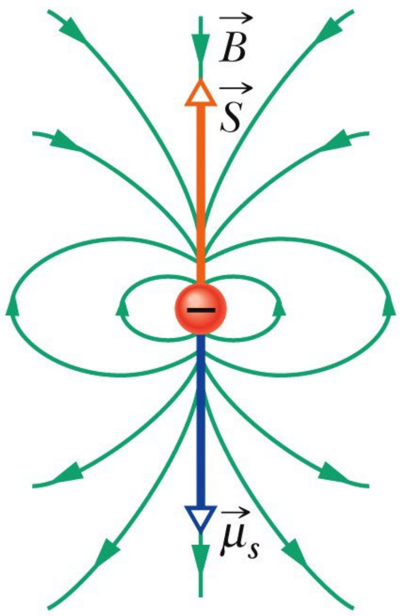
For a measurement along a z axis, the component S_z can have only the values given by

$$S_z = m_s \frac{h}{2\pi}, \qquad \text{for} \qquad m_s = \pm\frac{1}{2}$$

m_s is called the *spin magnetic quantum number* （自旋量子数）

h (= 6.63 ×10 $^{-34}$10 J ・ s) is the Planck constant（普朗克常数）

For an electron, the spin is opposite the magnetic dipole moment.



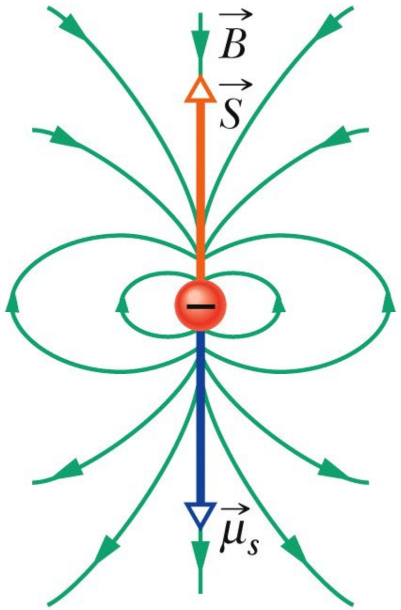
32-5 Magnetism and Electrons (9 of 15)

- Similarly, $\mu_{s,z} = \pm\dfrac{eh}{4\pi m} = \pm\mu_B,$

Where μ_B is the Bohr magneton （玻尔磁子）:

$$\mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}\ \mathrm{J/T}.$$

For an electron, the spin is opposite the magnetic dipole moment.



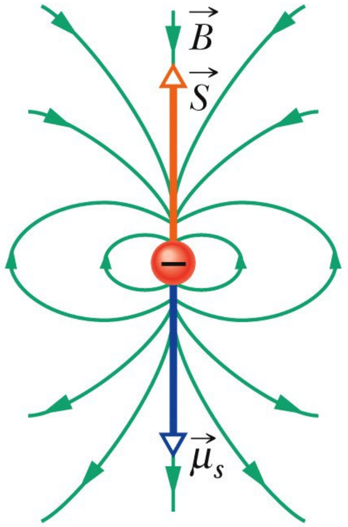
32-5 Magnetism and Electrons

Energy. When an electron is placed in an external magnetic field $\vec{B}_{\text{ext}}$, an energy U can be associated with the orientation of the electron's spin magnetic dipole moment $\vec{\mu}_S$ just as an energy can be associated with the orientation of the magnetic dipole moment $\vec{\mu}$ of a current loop placed in B_{ext}. The orientation energy U for the electron is

$$U = -\vec{\mu}_s \cdot \vec{B}_{\text{ext}} = -\mu_{s,z} B_{\text{ext}},$$

where the z axis is taken to be in the direction of $\vec{B}_{\text{ext}}$.

For an electron, the spin is opposite the magnetic dipole moment.



32-5 Magnetism and Electrons (13 of 15)

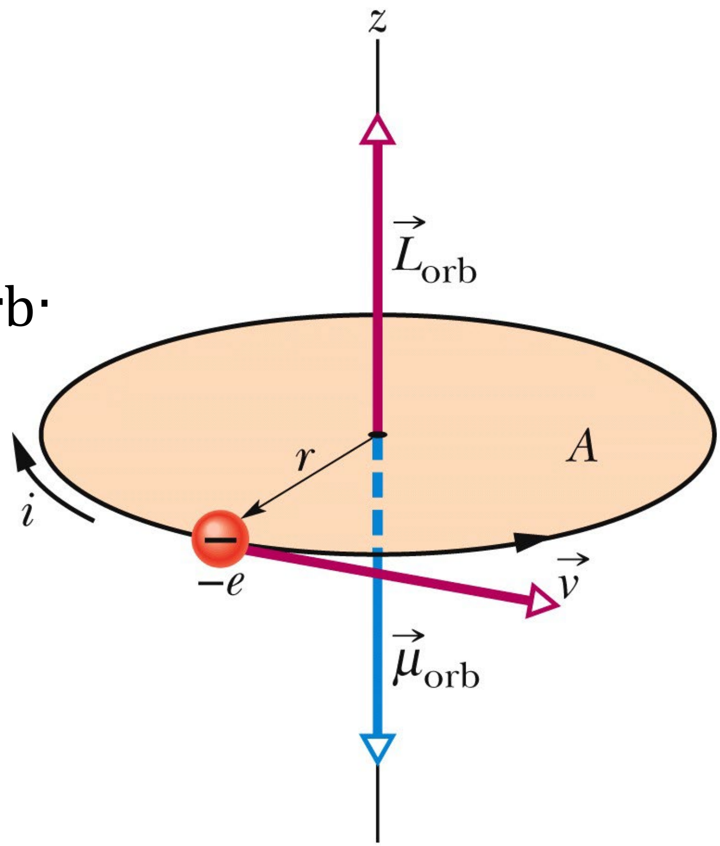
Orbital Magnetic Dipole Moment 轨道磁偶极矩.

When it is in an atom, an electron has an additional angular momentum called its **orbital angular momentum** 轨道角动量 $\vec{L}_{orb}$.

Associated with $\vec{L}_{orb}$ is an orbital magnetic dipole moment $\vec{\mu}_{orb}$; the two are related by

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}.$$

The minus sign means that $\vec{\mu}_{orb}$ and $\vec{L}_{orb}$ have opposite directions.



An electron moving at constant speed *v* in a circular path of radius *r* that encloses an area *A*

32-5 Magnetism and Electrons (14 of 15)

Orbital angular momentum is quantized (量子化的) and can have only measured values given by

$$L_{\mathrm{orb},z} = m_\ell \frac{h}{2\pi}, \qquad \text{for} \qquad m_\ell = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit integer}),$$

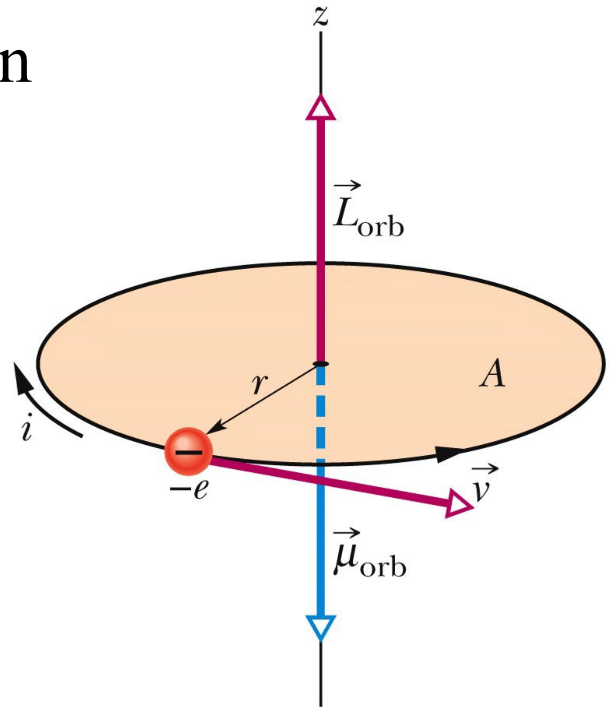
The associated magnetic dipole moment is given by

$$\vec{\mu}_{\mathrm{orb}} = -\frac{e}{2m}\vec{L}_{\mathrm{orb}}. \Rightarrow \qquad \mu_{\mathrm{orb},z} = -m_\ell \frac{eh}{4\pi m} = -m_\ell \mu_B.$$

m_ℓ is called the *orbital magnetic quantum number* （轨道磁量子数）

The energy U associated with the orientation of the orbital magnetic dipole moment in an external magnetic field $\vec{B}_{\mathrm{ext}}$ is

$$U = -\vec{\mu}_{\mathrm{orb}} \cdot \vec{B}_{\mathrm{ext}} = -\mu_{\mathrm{orb},z} B_{\mathrm{ext}}.$$



An electron moving at constant speed *v* in a circular path of radius *r* that encloses an area *A*.

32-6 Diamagnetism (3 of 3)

Magnetic Materials

Each electron in an atom has an orbital magnetic dipole moment and a spin magnetic dipole moment that combine vectorially. The resultant of these two vector quantities combines vectorially with similar resultants for all other electrons in the atom, and the resultant for each atom combines with those for all the other atoms in a sample of a material. If the combination of all these magnetic dipole moments produces a magnetic field, then the material is magnetic.

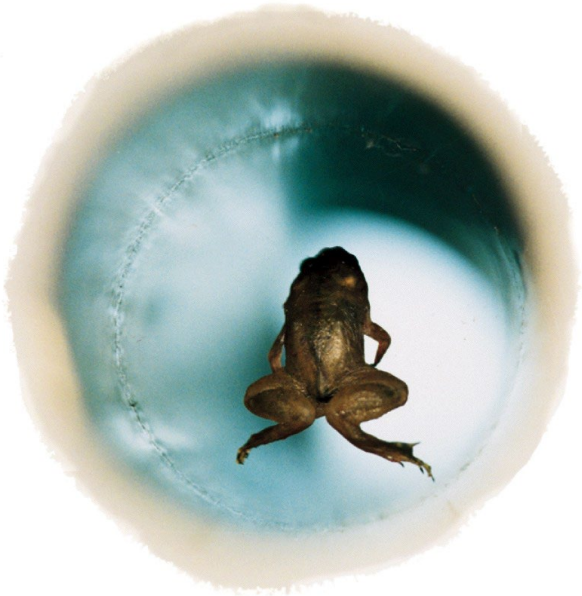
There are three general types of magnetism: diamagnetism (反磁性/抗磁性), paramagnetism (顺磁性/常磁性), and Ferromagnetism (铁磁性).

32-6 Diamagnetism (3 of 3)

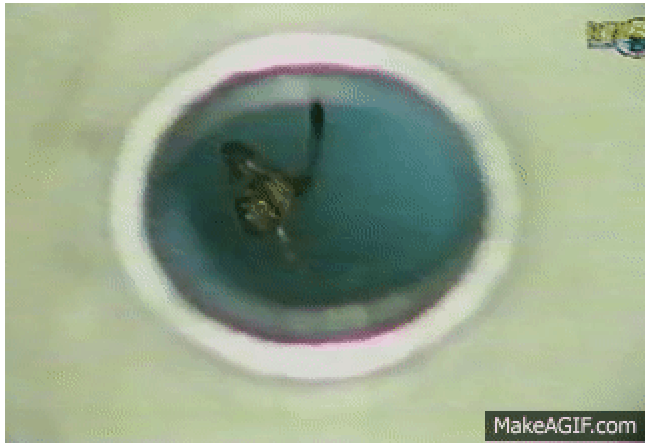
Diamagnetism (反磁性/抗磁性) is exhibited by all common materials but is so feeble that it is masked if the material also exhibits magnetism of either of the other two types. In diamagnetism, weak magnetic dipole moments are produced in the atoms of the material when the material is placed in an external magnetic field B_{ext}; the combination of all those induced dipole moments gives the material as a whole only a feeble （微弱的） net magnetic field. The dipole moments and thus their net field disappear when B_{ext} is removed. The term *diamagnetic material* usually refers to materials that exhibit only diamagnetism.

32-6 Diamagnetism (3 of 3)

Levitating Frog: The frog in the figure is diamagnetic (as is any other animal). When the frog was placed in the diverging magnetic field near the top end of a vertical current-carrying solenoid, every atom in the frog was repelled upward, away from the region of stronger magnetic field at that end of the solenoid. The frog moved upward into weaker and weaker magnetic field until the upward magnetic force balanced the gravitational force on it, and there it hung in midair 悬在半空中. The frog is not in discomfort because every atom is subject to the same forces and thus there is no force variation within the frog.



Courtesy A.K. Geim, University of Manchester, UK



32-7 Paramagnetism (顺磁性/常磁性)

A paramagnetic material placed in an external magnetic field $\vec{B}_{\text{ext}}$ develops a magnetic dipole moment in the direction of $\vec{B}_{\text{ext}}$. If the field is nonuniform, the paramagnetic material is attracted toward a region of greater magnetic field from a region of lesser field.

32-7 Paramagnetism (4 of 5)

Paramagnetic materials have atoms with a permanent magnetic dipole moment 永久磁偶极矩 but the moments are randomly oriented, with no net moment, unless the material is in an external magnetic field $\vec{B}_{\text{ext}}$, where the dipoles tend to align with that field. The extent of alignment within a volume V is measured as the magnetization 磁化 M, given by

$$M = \frac{\text{measured magnetic moment}}{V}.$$

Complete alignment (saturation) of all N dipoles in the volume gives a maximum value

$$M_{\max} = \frac{N\mu}{V}.$$

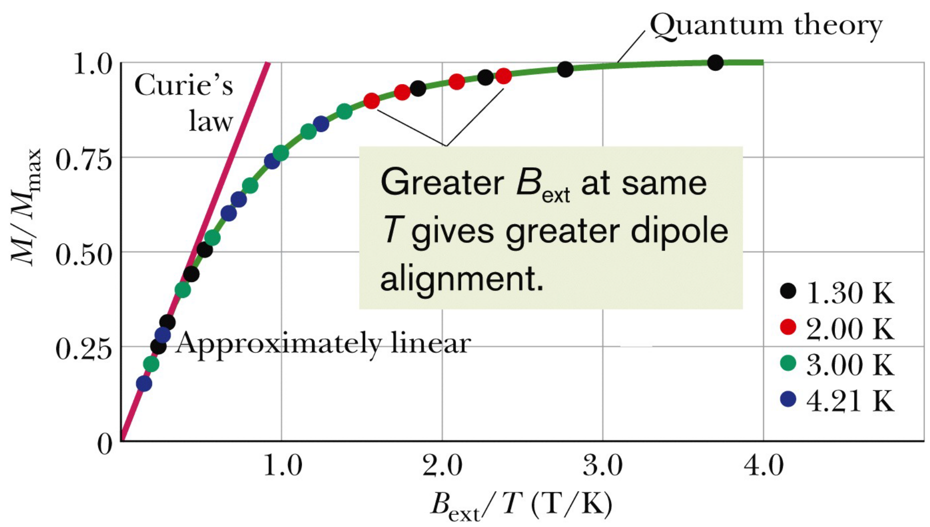
32-7 Paramagnetism (5 of 5)

At low values of the ratio $\frac{B_{ext}}{T}$,

Curie's law $$M = C\frac{B_{ext}}{T}$$

where T is the temperature (in kelvins) and C is a material's Curie constant (居里常数).

In a nonuniform external field, a paramagnetic material is attracted to the region of greater magnetic field.



32-7 Paramagnetism (5 of 5)

Richard Megna/Fundamental Photographs

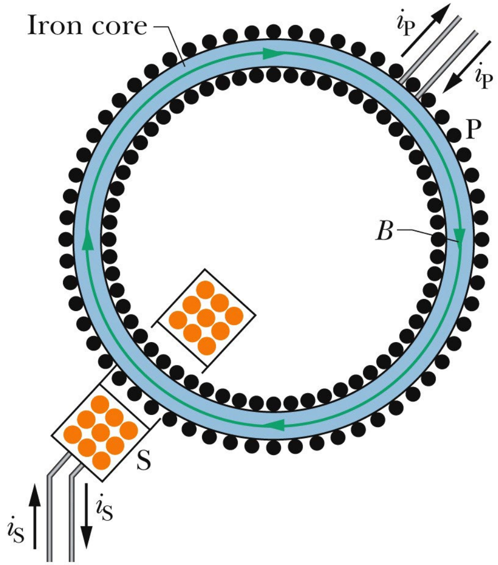
Liquid oxygen is suspended between the two pole faces of a magnet because the Liquid is paramagnetic and is magnetically attracted to the magnet.

32-8 Ferromagnetism铁磁性

A ferromagnetic material placed in an external magnetic field $\vec{B}_{\text{ext}}$ develops a strong magnetic dipole moment in the direction of $\vec{B}_{\text{ext}}$. If the field is nonuniform, the ferromagnetic material is attracted toward a region of greater magnetic field from a region of lesser field.

The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions know as **magnetic domains (磁畴)**.

32-8 Ferromagnetism 铁磁性

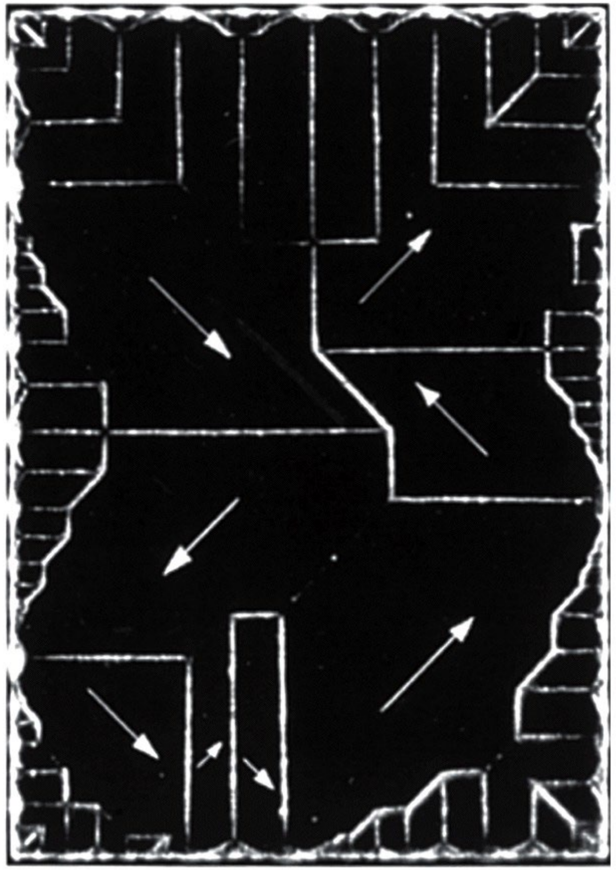


A Rowland ring (罗兰戒指). A primary coil P has a core made of the ferromagnetic material to be studied (here iron). The core is magnetized by a current i_P sent through coil P. (The turns of the coil are represented by dots.) The extent to which the core is magnetized determines the total magnetic field $\vec{B}$ within coil P.

Field $\vec{B}$ can be measured by means of a secondary coil S.

32-8 Ferromagnetism (6 of 8)

A photograph of domain patterns within a single crystal of nickel 镍单晶; white lines reveal the boundaries of the domains. The white arrows superimposed on the photograph show the orientations of the magnetic dipoles within the domains and thus the orientations of the net magnetic dipoles of the domains. The crystal as a whole is unmagnetized if the net magnetic field (the vector sum over all the domains) is zero.

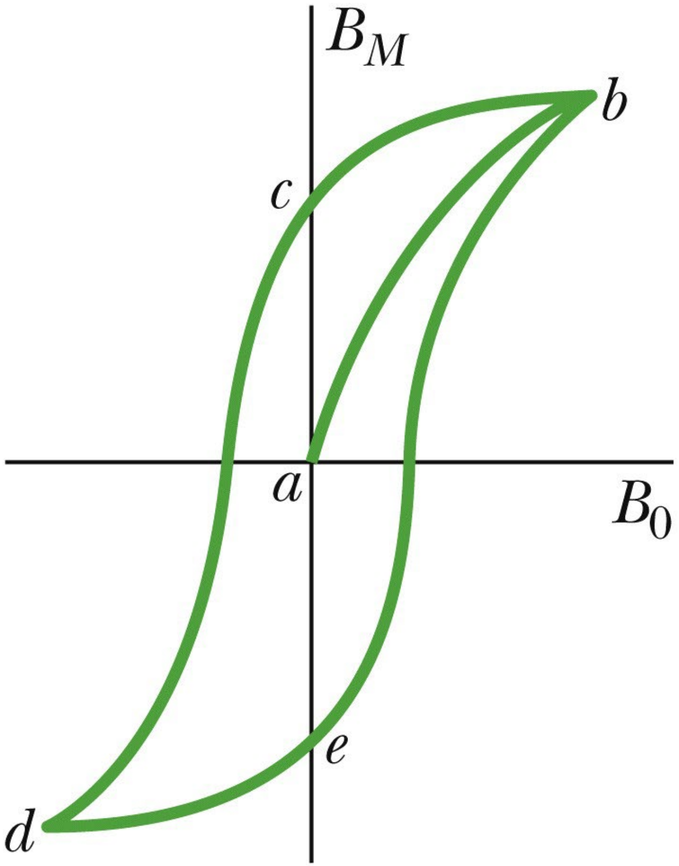


Courtesy Ralph W. DeBlois

32-8 Ferromagnetism (8 of 8)

after the external field is removed, remain partially aligned in regions know as **magnetic domains (磁畴)**.

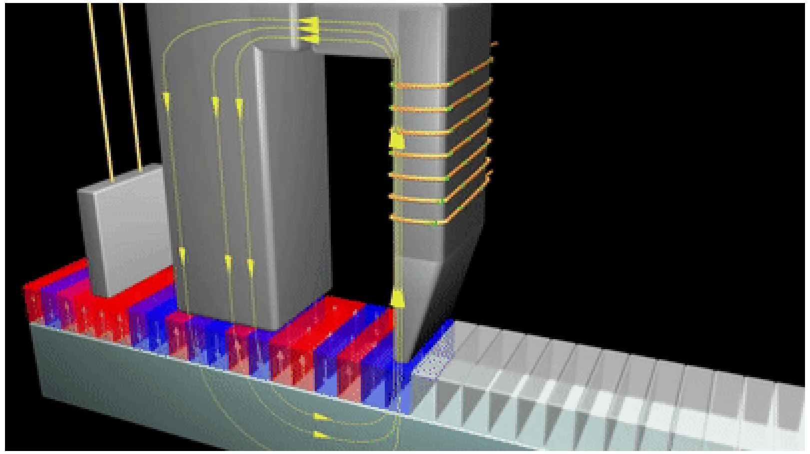
The lack of retraceability (收缩性) shown in the Figure is called **hysteresis** 迟滞现象, and the curve bcdeb is called a **hysteresis** loop迟滞回线. Note that at points c and e the iron core is magnetized, even though there is no current in the toroid windings; this is the familiar phenomenon of permanent magnetism 永磁性.

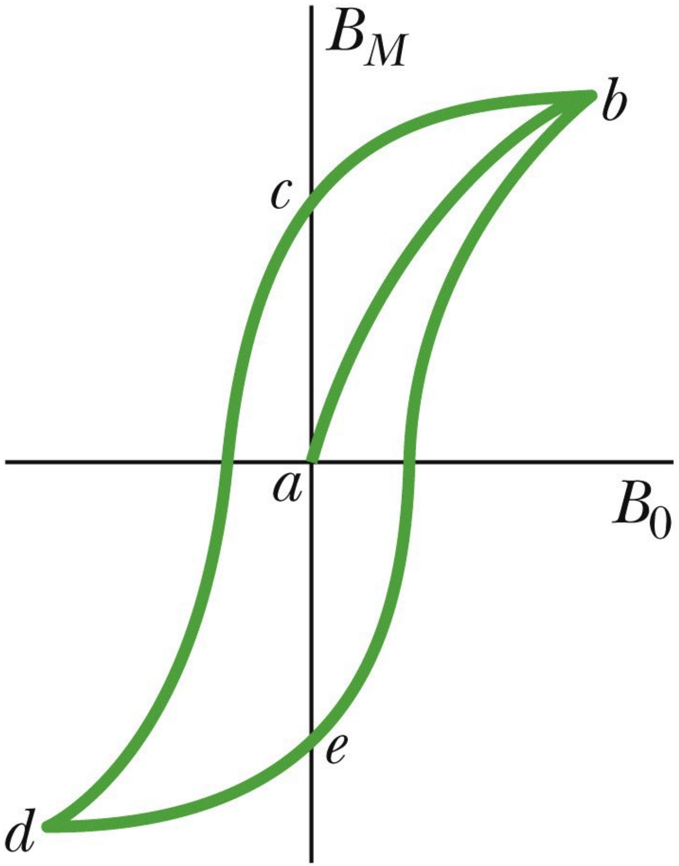


A magnetization curve (*ab*) for a ferromagnetic specimen and an associated hysteresis loop (*bcdeb*).

32-8 Ferromagnetism (8 of 8)

Hysteresis can be understood through the concept of magnetic domains. Evidently the motions of the domain boundaries and the reorientations of the domain directions are not totally reversible. When the applied magnetic field B_0 is increased and then decreased back to its initial value, the domains do not return completely to their original configuration but retain some "memory" of their alignment after the initial increase. This memory of magnetic materials is essential for the magnetic storage of information.





A magnetization curve (ab) for a ferromagnetic specimen and an associated hysteresis loop ($bcdeb$).

Summary (1 of 9)

Gauss' Law for Magnetic Fields

- Gauss' law for magnetic fields,

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0,$$

Equation (32-1)

Maxwell's Extension of Ampere's Law

- A changing electric field induces a magnetic field given by,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Equation (32-3)

Summary (2 of 9)

- Maxwell's law and Ampere's law can be written as the single equation

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$ **Equation (32-5)**

Displacement Current

- We define the fictitious displacement current due to a changing electric field as

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}.$$ **Equation (32-10)**

- Equation 32-5 then becomes

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}}$$ **Equation (32-11)**

Summary (3 of 9)

Maxwell's Equations

- Four equations are as follows:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_B}{dt} + \mu_0 i_{\mathrm{enc}}$$

Summary (4 of 9)

Spin Magnetic Dipole Moment

- Spin angular momentum of electron is associated with spin magnetic dipole momentum through,

$$\vec{\mu}_s = -\frac{e}{m}\vec{S}. \qquad \textbf{Equation (32-22)}$$

- For a measurement along a z axis, the component S_z can have only the values given by

$$S_z = m_s \frac{h}{2\pi}, \quad \text{for} \quad m_s = \pm\tfrac{1}{2}, \qquad \textbf{Equation (32-23)}$$

Summary (5 of 9)

- Similarly,

$$\mu_{s,z} = \pm \frac{eh}{4\pi m} = \pm \mu_{\mathrm{B}},$$ **Equation (32-24 & 26)**

- Where the Bohr magneton is

$$\mu_{\mathrm{B}} = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \mathrm{J/T}.$$ **Equation (32-25)**

- The energy U

$$U = -\vec{\mu}_s \cdot \vec{B}_{\text{ext}} = -\mu_{s,z} B_{\text{ext}}.$$ **Equation (32-27)**

Summary (6 of 9)

Orbital Magnetic Dipole Momentum

- Angular momentum of an electron is associated with orbital magnetic dipole momentum as

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}}.$$

Equation (32-28)

- Orbital angular momentum is quantized,

$$L_{\text{orb},z} = m_\ell \frac{h}{2\pi}, \quad \text{for} \quad m_\ell = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit}).$$

Equation (32-29)

Summary (7 of 9)

- The associated magnetic dipole moment is given by

$$\mu_{\text{orb},z} = -m_\ell \frac{eh}{4\pi m} = m_\ell \mu_\text{B}.$$ **Equation (32-30 & 31)**

- The energy U

$$U = -\vec{\mu}_\text{orb} \cdot \vec{B}_\text{ext} = -\mu_{\text{orb},z} B_\text{ext}.$$ **Equation (32-32)**

Summary (8 of 9)

Diamagnetism

- Diamagnetic materials exhibit magnetism only when placed in an external magnetic field; there they form magnetic dipoles directed opposite the external field. In a nonuniform field, they are repelled from the region of greater magnetic field.

Paramagnetism

- Paramagnetic materials have atoms with a permanent magnetic dipole moment but the moments are randomly oriented unless the material is in an external magnetic field. The extent of alignment within a volume V is measured as the magnetization M, given by

$$M = \frac{\text{measured magnetic moment}}{V}. \qquad \textbf{Equation (32-28)}$$

Summary (9 of 9)

- Complete alignment (saturation) of all N dipoles in the volume gives a maximum value $M_{\max} = \frac{N\mu}{V}$. At low values of the ratio $\frac{B_{\text{ext}}}{T}$,

$$M = C\frac{B_{\text{ext}}}{T}$$

Equation (32-39)

Ferromagnetism

- The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions (domains). Alignment is eliminated at temperatures above a material's Curie temperature. In a nonuniform external field, a ferromagnetic material is attracted to the region of greater magnetic field.

Summary (9 of 9)

- Complete alignment (saturation) of all N dipoles in the volume gives a maximum value $M_{\max} = \frac{N\mu}{V}$. At low values of the ratio $\frac{B_{\text{ext}}}{T}$,

$$M = C\frac{B_{\text{ext}}}{T}$$ **Equation (32-39)**

Ferromagnetism

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32.3.2. When a parallel-plate capacitor is charging, there is both an electric field and an induced magnetic present between the plates. After some time, the charging stops, which of the following statements is true concerning the fields within the capacitor?

a) The magnetic field is zero, but the electric field is constant.

b) The magnetic field is zero; and the electric field slowly decreases to zero over time.

c) Both the electric and magnetic fields are equal to zero.

d) The electric field is zero; and the magnetic field slowly decreases to zero over time.

e) The electric field is zero, but the magnetic field is constant.

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e) The electric field is zero, but the magnetic field is constant.

32.4.1. A circular, parallel-plate capacitor of radius R is being charged. At which of the following locations does the induced magnetic field have its largest magnitude?

a) $r = 0$, the center of the plates

b) $r = R/2$

c) $r = R$

d) $r = 2R$

e) In such a situation, there will be no induced magnetic field.

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c) $r = R$

d) $r = 2R$

e) In such a situation, there will be no induced magnetic field.

32.7.1. Which of the following does not significantly contribute to the magnetism of a given material?

a) the spin magnetic moment of protons

b) the spin magnetic moment of neutrons

c) the spin magnetic moment of electrons

d) the angular magnetic moment of electrons

e) both (a) and (b)

32.7.1. Which of the following does not significantly contribute to the magnetism of a given material?

a) the spin magnetic moment of protons

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c) the spin magnetic moment of electrons

d) the angular magnetic moment of electrons

e) both (a) and (b)

32.9.1. Which of the following will occur when a diamagnetic material is placed in an external magnetic field?

a) Weak magnetic dipole moments will be induced that are generally aligned in opposition to the applied field.

b) Magnetic dipoles within the material do not change in strength or alignment with the external field.

c) Permanent magnetic dipoles within the material increase in strength as they align parallel to the external magnetic field.

d) Permanent magnetic dipoles within the material decrease in strength as they align antiparallel to the external magnetic field.

e) Permanent magnetic dipoles within the material decrease in strength as they align parallel to the external magnetic field.

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