

UFUG 1504: Honors General Physics II

Chapter 30

Induction and Inductance

Summary (1 of 7)

Magnetic Flux

- The magnetic flux through an area A in a magnetic field \vec{B} is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad \text{Equation 30-1}$$

- If \vec{B} is perpendicular to the area and uniform over it, Equation 30-1 becomes

$$\Phi_B = BA \quad (\vec{B} \perp A, \vec{B} \text{ uniform}). \quad \text{Equation 30-2}$$

Summary (2 of 7)

Faraday's Law of Induction

- The induced emf is,

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Equation 30-4

- If the loop is replaced by a closely packed coil of N turns, the induced emf is

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}.$$

Equation 30-5

Summary (3 of 7)

Lenz's Law

- An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current.

emf and the Induced Magnetic Field

- The induced emf is related to \vec{E} by

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s},$$

Equation 30-19

- Faraday's law in its most general form,

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Equation 30-20

Summary (4 of 7)

Inductor

- The inductance L of the inductor is

$$L = \frac{N\Phi_B}{i}$$

Equation 30-28

- The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A$$

Equation 30-31

Summary (5 of 7)

Self-Induction

- This self-induced emf is,

$$\mathcal{E}_L = -L \frac{di}{dt}.$$

Equation 30-35

Series RL Circuit

- Rise of current,

$$i = \frac{\mathcal{E}}{R} \left(1 - e^{\frac{-t}{\tau_L}} \right)$$

Equation 30-41

- Decay of current

$$i = i_0 e^{\frac{-t}{\tau_L}}$$

Equation 30-45

Summary (6 of 7)

Magnetic Energy

- the inductor's magnetic field stores an energy given by

$$U_B = \frac{1}{2}Li^2$$

Equation 30-49

- The density of stored magnetic energy,

$$u_B = \frac{B^2}{2\mu_0}$$

Equation 30-55

Summary (7 of 7)

Mutual Induction

- The mutual induction is described by,

$$\mathcal{E}_2 = -M \frac{di_1}{dt}$$

Equation 30-64

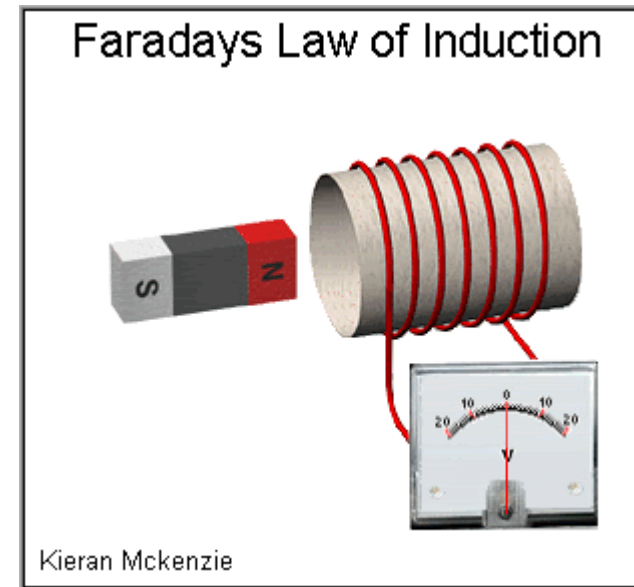
$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

Equation 30-65

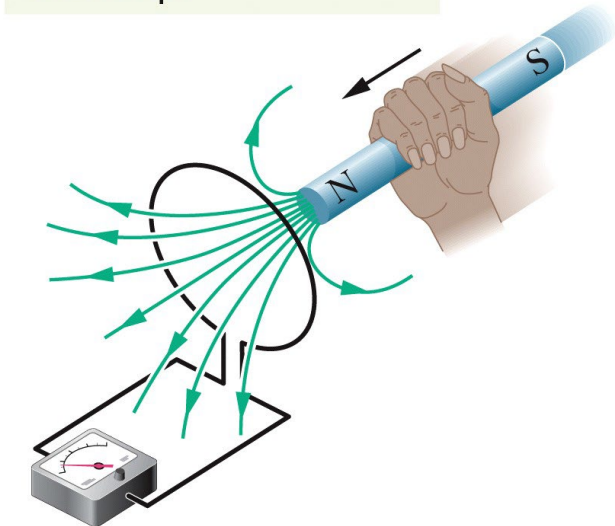
30-1 Faraday's Law and Lenz's Law (4 of 14)

Let us examine two simple experiments to prepare for Faraday's law of induction (法拉第感应定律).

First Experiment. Figure shows a conducting loop connected to a sensitive ammeter. Because there is **no battery or other source of emf** included, there is **no current in the circuit**. However, if we **move a bar magnet** toward the loop, a **current** suddenly **appears** in the circuit. The current disappears when the magnet stops moving. If we then **move the magnet away**, a **current** again suddenly appears, but now in the **opposite direction**. If we experimented for a while, we would discover the following:



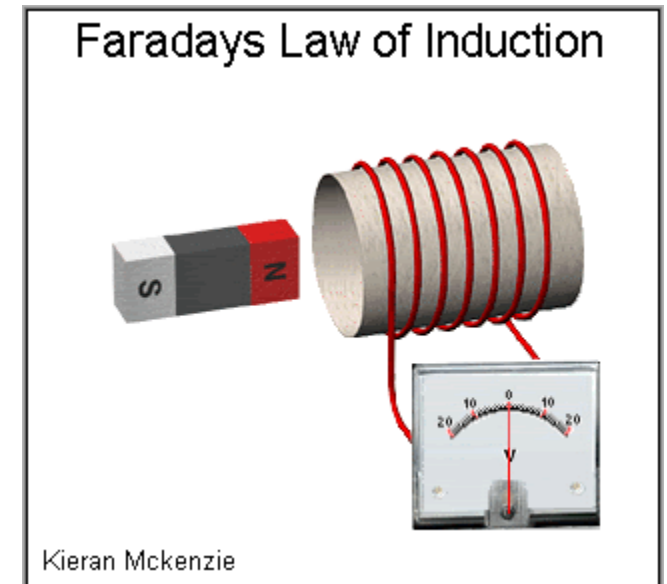
The magnet's motion creates a current in the loop.



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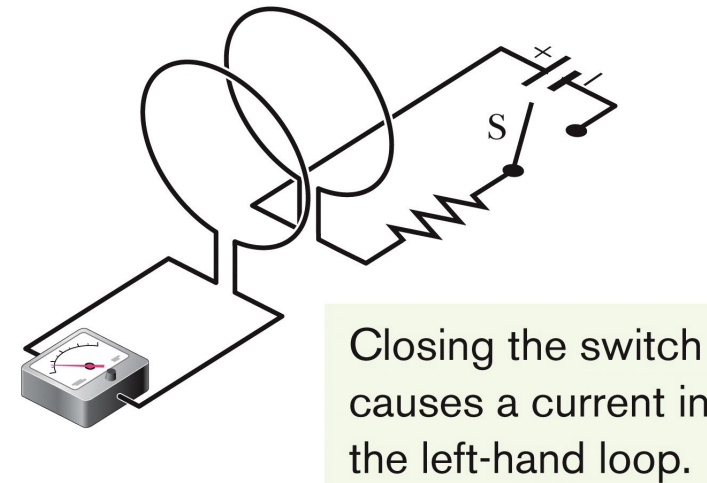
30-1 Faraday's Law and Lenz's Law (5 of 14)

1. A **current appears only** if there is **relative motion** between the **loop and the magnet** (one must move relative to the other); the current disappears when the relative motion between them ceases (停止).
2. **Faster** motion of the magnet produces a **greater** current.
3. If moving the magnet's **north pole toward the loop** causes, say, **clockwise current**, then moving **the north pole away** causes **counterclockwise current**. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions from the north pole effects.



30-1 Faraday's Law and Lenz's Law (6 of 14)

Second Experiment. For this experiment we use the apparatus shown in the figure, with the two conducting loops close to each other but not touching. If **we close switch S** to turn on a current in the **right-hand loop**, the meter suddenly and briefly registers a current—an **induced current (感应电流)**—**in the left-hand loop**. If the switch **remains closed**, **no further current** is observed. If we then **open the switch**, another sudden and brief **induced current** appears in the left-hand loop, but **in the opposite direction**.



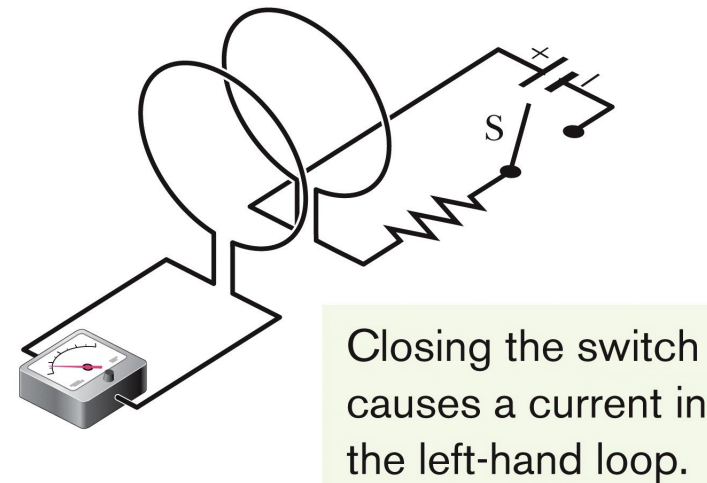
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30-1 Faraday's Law and Lenz's Law (8 of 14)

Faraday's Law of Induction

Faraday realized that an **emf and a current can be induced in a loop**, as in our two experiments, **by changing the amount of magnetic field passing through the loop**.

He further realized that the “amount of magnetic field” can be visualized in terms of the magnetic field lines passing through the loop.



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30-1 Faraday's Law and Lenz's Law (9 of 14)

The magnetic flux Φ_B through an area A in a magnetic field \vec{B} is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

where the integral is taken over the area. The SI unit of magnetic flux is the weber, where $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$.

If B is perpendicular to the area and uniform over it, the flux is

$$\Phi_B = BA \quad \left(\vec{B} \perp \text{area } A, \vec{B} \text{ uniform} \right).$$

30-1 Faraday's Law and Lenz's Law (10 of 14)

Faraday's Law of Induction

The magnitude of the emf \mathcal{E} induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

Faraday's Law. With the notion of magnetic flux, we can state Faraday's law in a more quantitative and useful way:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

the **induced emf** tends to **oppose the flux change** and the minus sign indicates this opposition. This minus sign is referred to as Lenz's Law.

30-1 Faraday's Law and Lenz's Law (12 of 14)

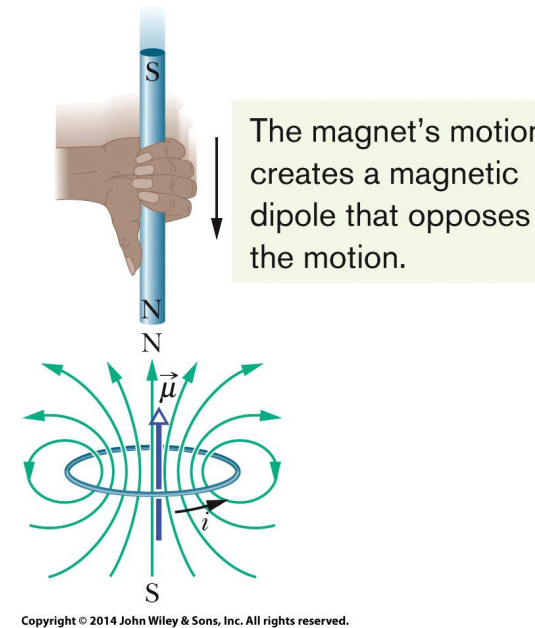
Lenz's Law (楞次定律)

An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current.

感应电流的磁场总要阻碍引起感应电流的磁通量的变化

1. *Opposition to Pole Movement*

2. *Opposition to Flux Change.*



Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

30-1 Faraday's Law and Lenz's Law (13 of 14)

Lenz's Law

Increasing the external field \vec{B} induces a current with a field \vec{B}_{ind} that *opposes the change*.

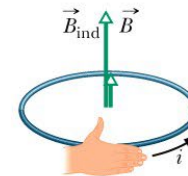
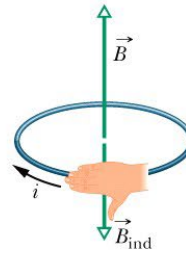
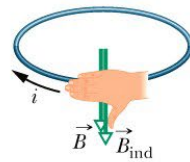
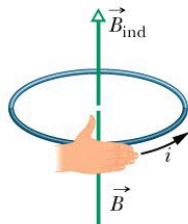
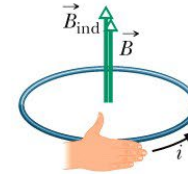
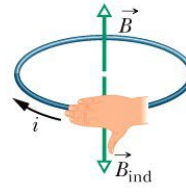
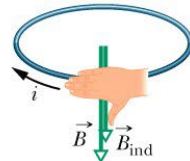
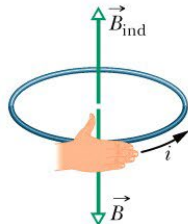
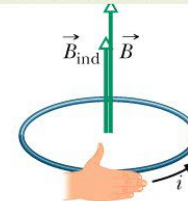
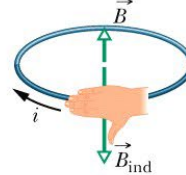
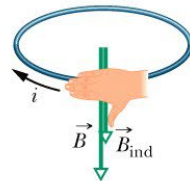
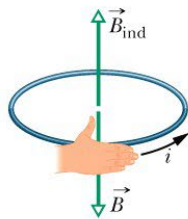
Decreasing the external field \vec{B} induces a current with a field \vec{B}_{ind} that *opposes the change*.

Increasing the external field \vec{B} induces a current with a field \vec{B}_{ind} that *opposes the change*.

Decreasing the external field \vec{B} induces a current with a field \vec{B}_{ind} that *opposes the change*.

The induced current creates this field, trying to offset the change.

The fingers are in the current's direction; the thumb is in the induced field's direction.



(a)

(b)

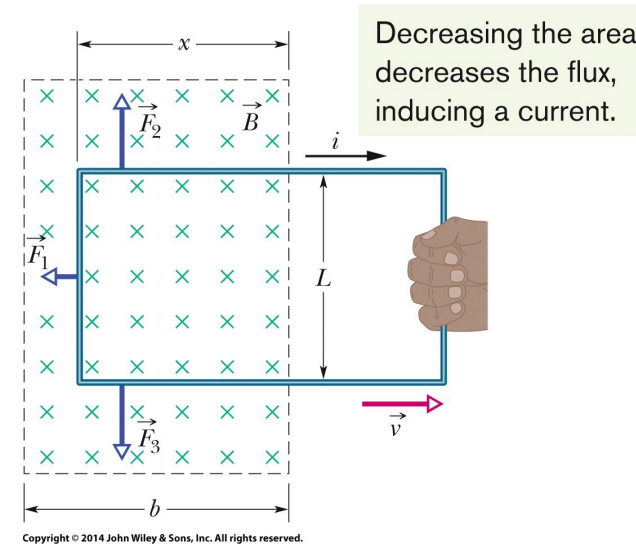
(c)

(d)

The direction of the current i induced in a loop is such that the current's magnetic field \vec{B}_{ind} opposes the change in the opposite an increasing field opposite an increasing field and in the same direction as a decreasing field $\vec{B}(b, d)$. The curled – straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

30-2 Induction and Energy Transfer (2 of 9)

In the figure, a rectangular loop of wire of width L has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in the figure show the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are to pull this loop to the right at a constant velocity v

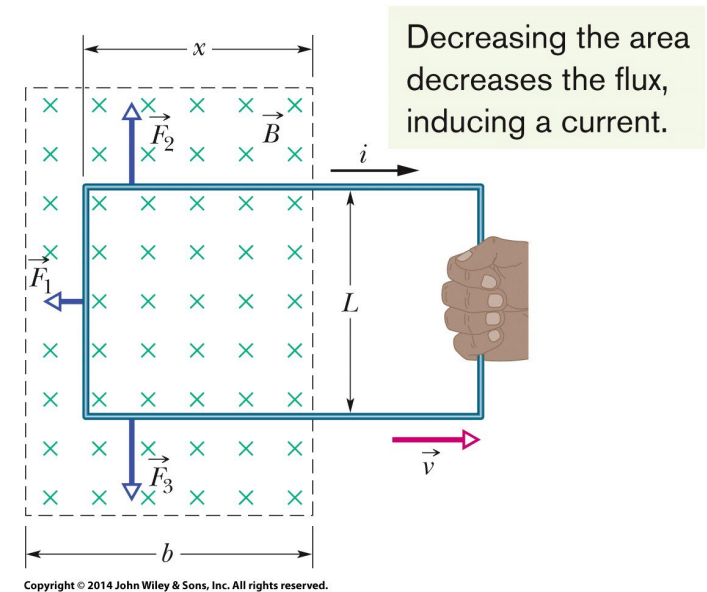


30-2 Induction and Energy Transfer (3 of 9)

Flux change: Therefore, in the figure a magnetic field and a conducting loop are in **relative motion at speed v** and the **flux of the field through the loop is changing with time** (here the flux is changing as the area of the loop still in the magnetic field is changing).

Rate of Work: To pull the loop at a constant velocity \vec{v} , you must apply a constant force \vec{F} to the loop because a magnetic force of equal magnitude but opposite direction acts on the loop to oppose you. The rate at which you do work — that is, the power — is then where F is the magnitude of the force you apply to the loop.

$$P = Fv,$$



In the figure, a rectangular loop of wire of width L has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. The dashed lines in the figure show the assumed limits of the magnetic field; You are to pull this loop to the right at a constant velocity v

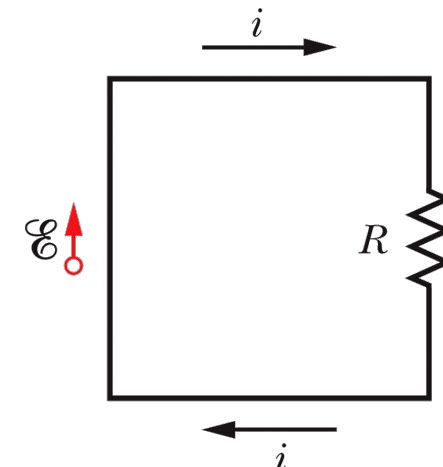
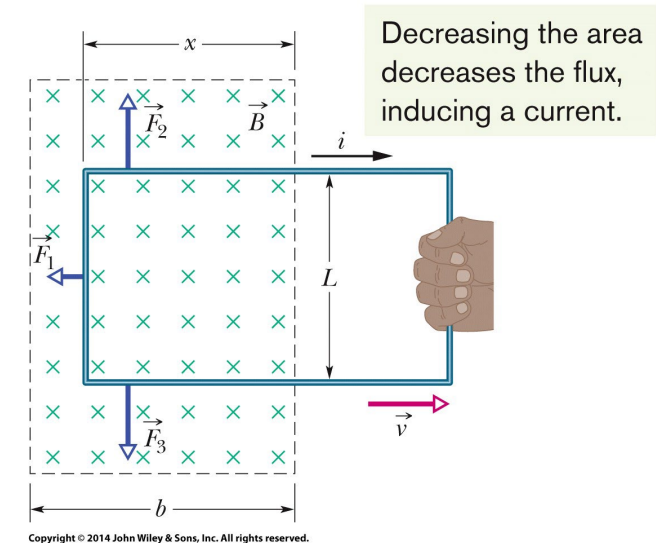
30-2 Induction and Energy Transfer (4 of 9)

Induced emf: To find the current, we first apply Faraday's law. When x is the length of the loop still in the magnetic field, the area of the loop still in the field is Lx . Then, the magnitude of the flux through the loop is

$$\Phi_B = BA = BLx.$$

As x decreases, the flux decreases. Faraday's law tells us that with this flux i decrease, an emf is induced in the loop. We can write the magnitude of this emf as

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv,$$



30-2 Induction and Energy Transfer (6 of 9)

Induced Current: Figure (bottom) shows the loop as a circuit: induced emf is represented on the left, and the collective resistance R of the loop is represented on the right. To find the magnitude of the induced current, we can apply the equation $i = \frac{\mathcal{E}}{R}$ which gives

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv, \quad \longrightarrow \quad i = \frac{BLv}{R}.$$

30-2 Induction and Energy Transfer (8 of 9)

In the Fig. (top), the deflecting forces acting on the three segments of the loop are marked \vec{F}_1 , \vec{F}_2 , and \vec{F}_3 . Note, however, that from the symmetry, forces \vec{F}_2 and \vec{F}_3 are equal in magnitude and cancel. This leaves only force \vec{F}_1 , which is directed opposite your force \vec{F} on the loop and thus is the force opposing you.

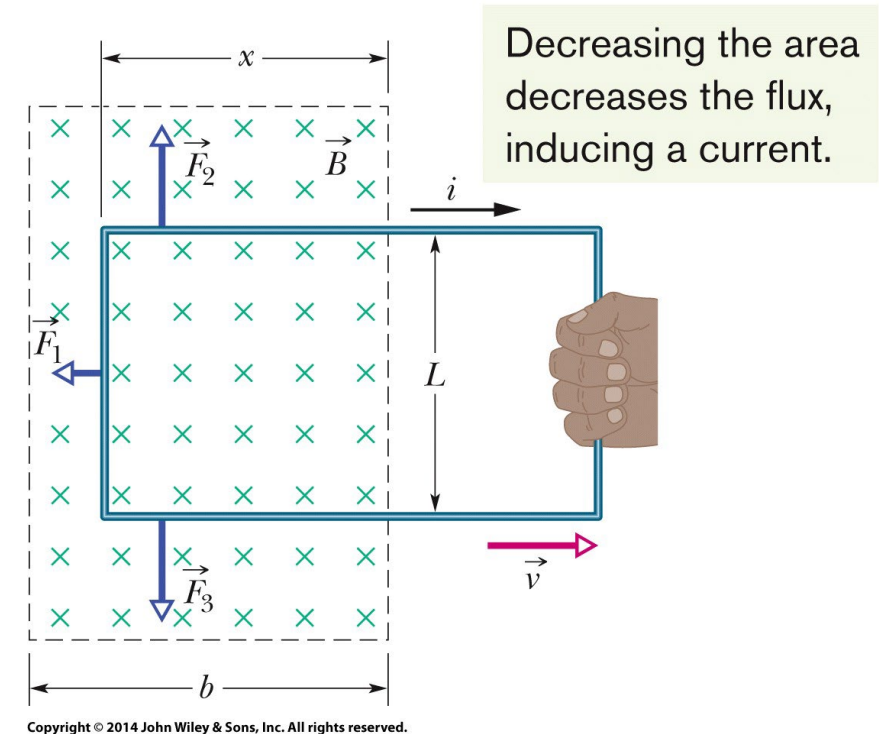
So, $\vec{F} = -\vec{F}_1$, the magnitude of \vec{F}_1 thus

$$F = F_1 = iLB \sin 90^\circ = iLB. \quad \left(\text{from } \vec{F}_d = i\vec{L} \times \vec{B}. \right)$$

where the angle between B and the length vector L for the left segment is 90° . This gives us

$$F = \frac{B^2 L^2 v}{R}. \quad \left(i = \frac{BLv}{R} \right)$$

Because B , L , and R are constants, the speed v at which you move the loop is constant so the force you apply to the loop is also constant.

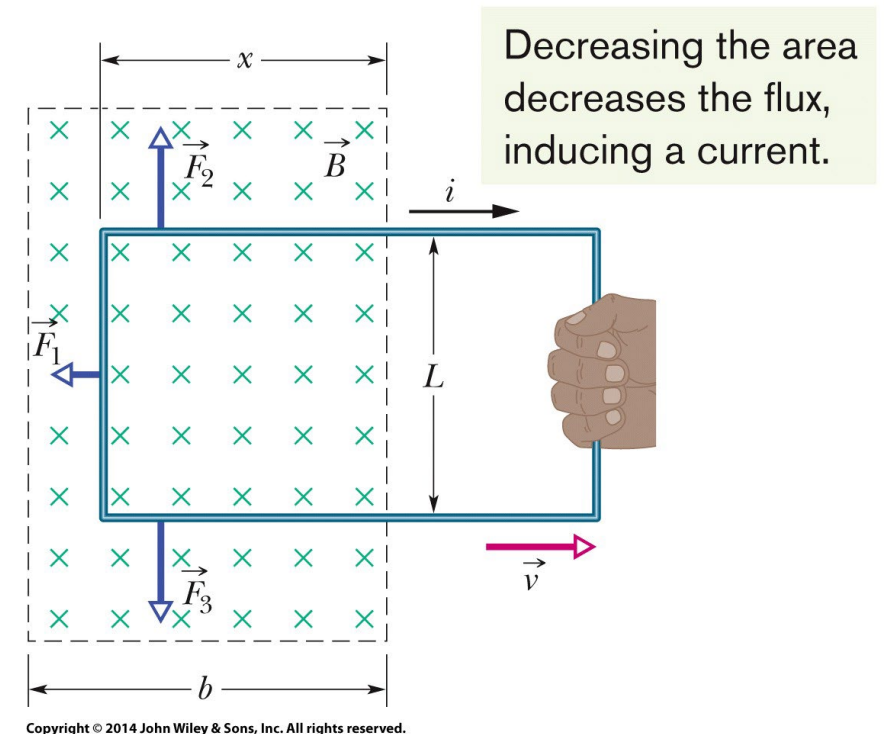


30-2 Induction and Energy Transfer (9 of 9)

Rate of Work: We find the rate at which you do work on the loop as you pull it from the magnetic field:

$$P = Fv = \frac{B^2 L^2 v^2}{R}$$

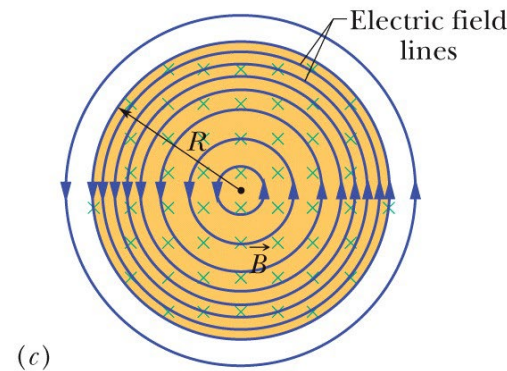
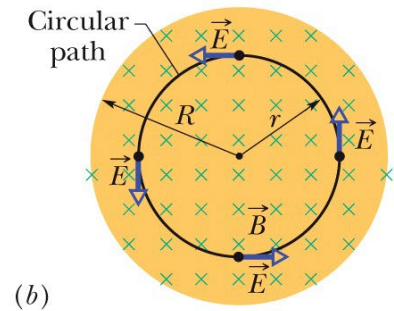
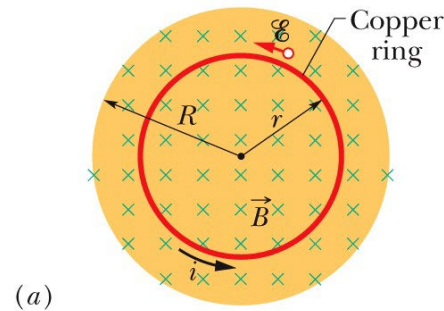
NOTE: The work that you do in pulling the loop through the magnetic field appears as **thermal energy** in the loop.



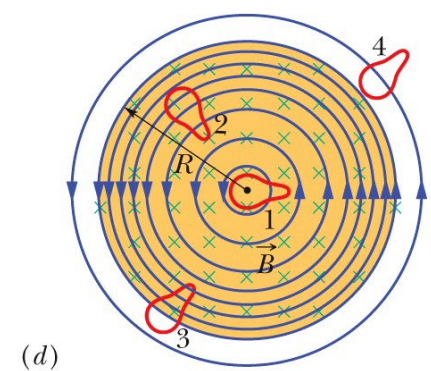
30-3 Induced Electric Field (2 of 5)

A changing magnetic field produces an electric field.

- (a) If the **magnetic field increases at a steady rate**, a constant induced current appears, as shown, in the copper ring of radius r .
- (b) An induced electric field exists even when the ring is removed; the electric field is shown at four points.
- (c) The complete picture of the induced electric field, displayed as field lines.
- (d) Four similar closed paths that enclose identical areas. Equal emfs are induced around paths 1 and 2, which lie entirely within the region of changing magnetic field. A smaller emf is induced around path 3, which only partially lies in that region. No net emf is induced around path 4, which lies entirely outside the magnetic field.



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30-3 Induced Electric Field (3 of 5)

Therefore, an emf is induced by a changing magnetic flux even if the loop through which the flux is changing is not a physical conductor but an imaginary line. The changing magnetic field induces an electric field \vec{E} at every point of such a loop; the induced emf is related to \vec{E} by

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}.$$

Using the induced electric field, we can write Faraday's law in its most general form as

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \Rightarrow \quad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

30-4 Inductors and Inductance (2 of 3)



Air-Core Inductor



Iron-Core Inductor



Ferrite-Core Inductor



Bobbin Based Inductor



Variable Inductor



Multilayer Ceramic Inductor

30-4 Inductors and Inductance (2 of 3)

An **inductor** is a device that can be used to **produce a known magnetic field** in a specified region. If a current i is established through each of the N windings of an inductor, a magnetic flux Φ_B links those windings. The inductance L of the inductor is

$$L = \frac{N\Phi_B}{i}$$

The SI unit of inductance is the henry (H), where $1 \text{ henry} = 1 \text{ H} = 1 \text{ T} \cdot \text{m}^2 / \text{A}$.

30-4 Inductors and Inductance (2 of 3)

The inductance per unit length near the middle of a long solenoid (长螺旋管) of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A$$

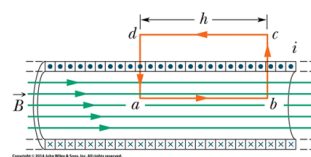
29-4 Solenoids and Toroids (6 of 8)

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}.$$

The first integral on the right of equation is Bh , where B is the magnitude of the uniform field B inside the solenoid and h is the (arbitrary) length of the segment from a to b . The second and fourth integrals are zero because for every element $d\vec{s}$ of these segments, B either is perpendicular to $d\vec{s}$ or is zero, and thus the product $\vec{B} \cdot d\vec{s}$ is zero. The third integral, which is taken along a segment points. Thus, $\oint \vec{B} \cdot d\vec{s}$ for the entire rectangular loop has the value Bh .

Inside a long solenoid carrying current i , at points not near its ends, the magnitude B of the magnetic field is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad B = \mu_0 in \quad (\text{ideal solenoid}).$$



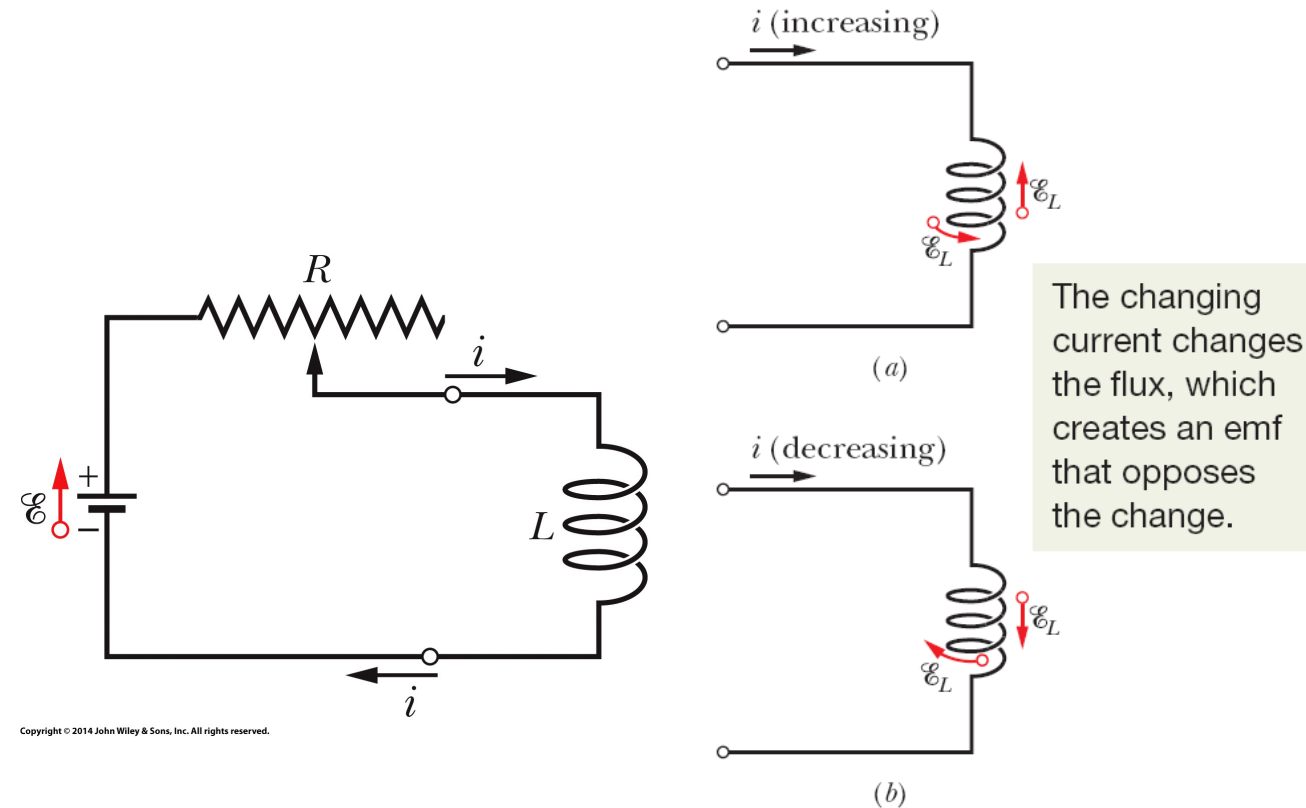
$$L = \frac{N\Phi_B}{i} \quad N\Phi_B = (nl)(BA), \quad B = \mu_0 in.$$



$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 in)(A)}{i} = \mu_0 n^2 l A.$$

30-5 Self-Induction (2 of 3)

If two coils — which we can now call inductors — are near each other, a current i in one coil produces a magnetic flux Φ_B through the second coil. We have seen that if we change this flux by changing the current, an induced emf appears in the second coil according to Faraday's law. An induced emf appears in the first coil as well. This process (see Figure) is called self-induction, and the emf that appears is called a self-induced emf. It obeys Faraday's law of induction just as other induced emfs do.



If the current in a coil is changed by varying the contact position on a variable resistor, a self-induced emf \mathcal{E}_L will appear in the coil *while the current is changing*.

30-5 Self-Induction (3 of 3)

For any inductor, $N\Phi_B = Li$.

Faraday's law tells us that $\mathcal{E}_L = -\frac{d(N\Phi_B)}{dt}$.

By combining these equations, we can write

$$\mathcal{E}_L = -L \frac{di}{dt} \quad (\text{self-induced emf}).$$

An induced emf \mathcal{E}_L appears in any coil in which the current is changing.

Note: Thus, in any inductor (such as a coil, a solenoid, or a toroid) a **self-induced emf** appears whenever **the current changes with time**. The magnitude of the current has no influence on the magnitude of the induced emf; only the rate of change of the current counts.

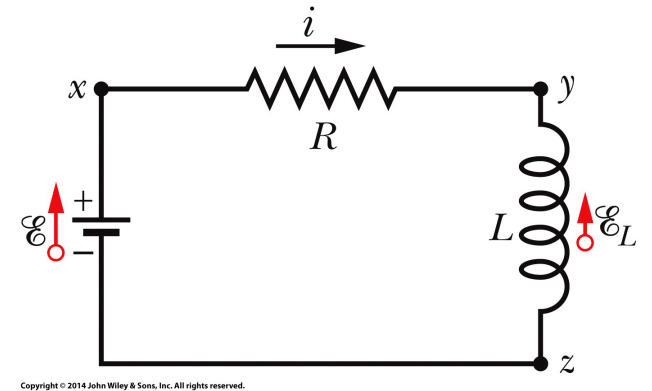
30-6 RL Circuits (4 of 6)

If a constant emf \mathcal{E} is introduced into a single-loop circuit containing a resistance R and an inductance L , the current rises to an equilibrium value of $\frac{\mathcal{E}}{R}$ according to

$$i = \frac{\mathcal{E}}{R} \left(1 - e^{\frac{-t}{\tau_L}} \right)$$

Here τ_L , the **inductive time constant**, is given by

$$\tau_L = \frac{L}{R}$$

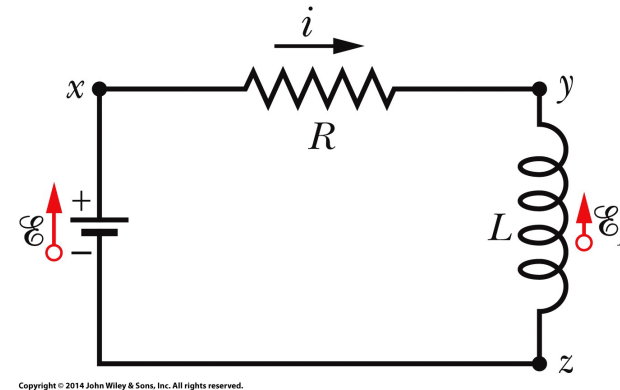


30-6 RL Circuits (4 of 6)

$$i = \frac{\mathcal{E}}{R} \left(1 - e^{-\frac{t}{\tau_L}} \right)$$

$$\mathcal{E}_L = -L \frac{di}{dt} \quad (\text{self-induced emf}).$$

$$-iR - L \frac{di}{dt} + \mathcal{E} = 0$$



五. 线性方程

一阶线性常微分方程的一般形式为

$$\frac{dy}{dx} + f(x)y = g(x)。$$

利用分离变量法，易知齐次线性方程

$$\frac{dy}{dx} + f(x)y = 0$$

的通解为

$$y = C e^{-\int f(x)dx}。$$

Solving firstly $di/dt + iR/L = 0$

$$i_1 = K e^{-tL/R}$$

Solving secondly $i_2 = Q(x)$

$$i_2 = \mathcal{E}/L$$



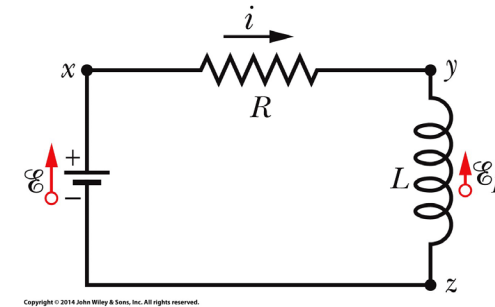
Initial condition $i = 0$ at $t = 0$.

$$i = i_1 + i_2 = K e^{-tL/R} + \mathcal{E}/L$$

$$i = \mathcal{E}/R \left(1 - e^{-\frac{tL}{R}} \right) \quad . \quad \tau_L = \frac{L}{R}$$

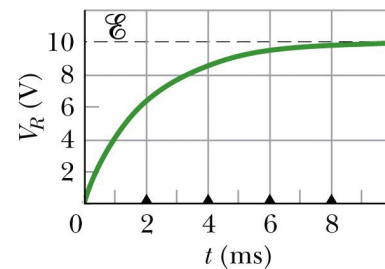
30-6 RL Circuits (5 of 6)

Plot (a) and (b) shows how the potential differences $V_R (= iR)$ across the resistor and $V_L \left(= L \frac{di}{dt} \right)$ across the inductor vary with time for particular values of \mathcal{E} , L , and R .

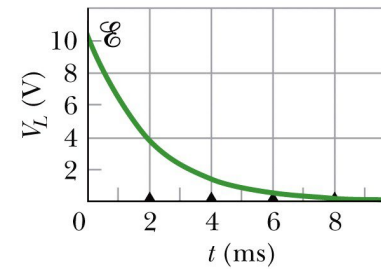


An RL circuit.

The resistor's potential difference turns on.
The inductor's potential difference turns off.



(a)



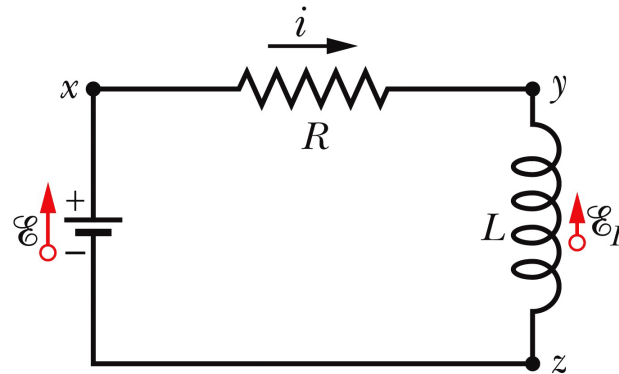
(b)

30-6 RL Circuits (6 of 6)

- When the source of constant emf is removed and replaced by a conductor, the **current decays** from a value i_0 according to

$$i = \frac{\mathcal{E}}{R} e^{\frac{-t}{\tau_L}} = i_0 e^{\frac{-t}{\tau_L}}$$

$$L \frac{di}{dt} + iR = 0.$$



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五. 线性方程

一阶线性常微分方程的一般形式为

$$\frac{dy}{dx} + f(x)y = g(x).$$

利用分离变量法，易知齐次线性方程

$$\frac{dy}{dx} + f(x)y = 0$$

的通解为

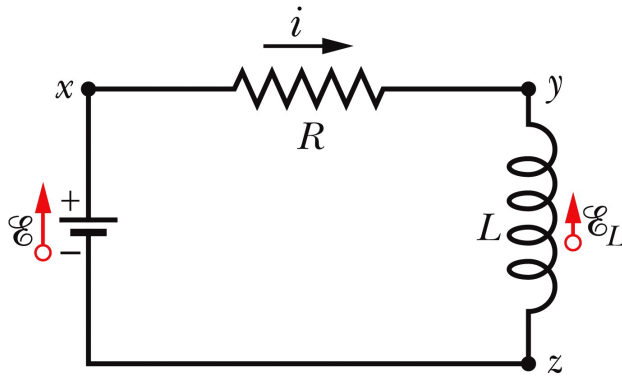
$$y = C e^{-\int f(x) dx}.$$

30-7 Energy Stored in a Magnetic Field (2 of 7)

If an inductor L carries a current i , the inductor's magnetic field stores an energy given by

magnetic potential energy U_B

$$U_B = \frac{1}{2} Li^2$$



An RL circuit.

$$\mathcal{E} = L \frac{di}{dt} + iR,$$



$$\mathcal{E}i = Li \frac{di}{dt} + i^2 R,$$

$$\frac{dU_B}{dt} = Li \frac{di}{dt} \quad \Rightarrow \quad \int_0^{U_B} dU_B = \int_0^i Li \, di$$

30-7 Energy Stored in a Magnetic Field (2 of 7)

A coil has an inductance of 53 mH and a resistance of $0.35\ \Omega$.

- (a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?
- (b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

A coil has an inductance of 53 mH and a resistance of 0.35 Ω .

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

$$i_{\infty} = \frac{\mathcal{E}}{R} = \frac{12 \text{ V}}{0.35 \Omega} = 34.3 \text{ A.}$$

$$U_{B\infty} = \frac{1}{2}Li_{\infty}^2 = \left(\frac{1}{2}\right)(53 \times 10^{-3} \text{ H})(34.3 \text{ A})^2 = 31 \text{ J.}$$

$$U_B = \frac{1}{2}U_{B\infty} \longrightarrow \frac{1}{2}Li^2 = \left(\frac{1}{2}\right)\frac{1}{2}Li_{\infty}^2$$

$$\frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L}) = \frac{\mathcal{E}}{\sqrt{2}R} \quad e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = 0.293,$$

$$\frac{t}{\tau_L} = -\ln 0.293 = 1.23$$

$$t \approx 1.2\tau_L.$$

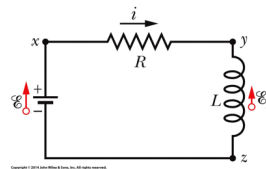
6 RL Circuits (4 of 6)

If a constant emf \mathcal{E} is introduced into a single-loop circuit containing a resistance R and an inductance L , the current rises to an equilibrium value of $\frac{\mathcal{E}}{R}$ according to

$$i = \frac{\mathcal{E}}{R} \left(1 - e^{-\frac{t}{\tau_L}}\right)$$

Here τ_L , the **inductive time constant**, is given by

$$\tau_L = \frac{L}{R}$$



30-8 Energy Density of a Magnetic Field (2 of 3)

Consider a length l near the middle of a long solenoid of cross-sectional area A carrying current i ; the volume associated with this length is Al . The energy U_B stored by the length l of the solenoid must lie entirely within this volume because the magnetic field outside such a solenoid is approximately zero. Moreover, the stored energy must be uniformly distributed within the solenoid because the magnetic field is (approximately) uniform everywhere inside. Thus, the energy stored per unit volume of the field is

magnetic energy density $u_B = \frac{U_B}{Al} \quad \longrightarrow \quad U_B = \frac{1}{2} Li^2$

30-8 Energy Density of a Magnetic Field (3 of 3)

We have,

$$u_B = \frac{Li^2}{2Al} = \frac{L}{l} \frac{i^2}{2A}.$$

here L is the inductance of length l of the solenoid

Substituting for $\frac{L}{l}$ we get

$$u_B = \frac{1}{2} \mu_0 n^2 i^2$$

And we can write the **energy density** as

$$u_B = \frac{B^2}{2\mu_0}$$

30-4 Inductors and Inductance (2 of 3)

The inductance per unit length near the middle of a long solenoid (长螺旋管) of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A$$

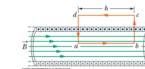
29-4 Solenoids and Toroids (6 of 8)

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}.$$

The first integral on the right of equation is Bh , where B is the magnitude of the uniform field B inside the solenoid and h is the (arbitrary) length of the segment from a to b . The second and fourth integrals are zero because for every element $d\vec{s}$ of these segments, B is either perpendicular to $d\vec{s}$ or is zero, and thus the product $\vec{B} \cdot d\vec{s}$ is zero. The third integral, which is taken along a segment points. Thus, $\oint \vec{B} \cdot d\vec{s}$ for the entire rectangular loop has the value Bh .

Inside a long solenoid carrying current i , at points not near its ends, the magnitude B of the magnetic field is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad B = \mu_0 n i \quad (\text{ideal solenoid}).$$



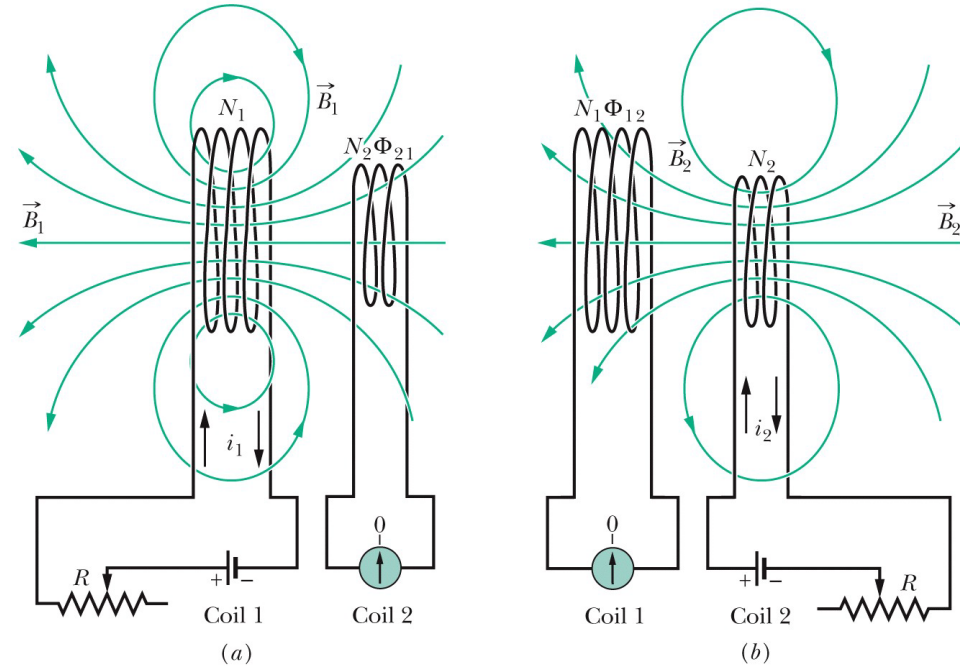
$$L = \frac{N\Phi_B}{i} \quad N\Phi_B = (nl)(BA).$$

$$B = \mu_0 n i$$

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 n i)(A)}{i} = \mu_0 n^2 l A.$$

30-9 Mutual Induction (互感)_(2 of 3)

Mutual induction. (a) The magnetic field B_1 produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an emf is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.



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30-9 Mutual Induction (3 of 3)

If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \qquad \mathcal{E}_1 = -M \frac{di_2}{dt}.$$

the mutual inductance $M_{21} = \frac{N_2 \Phi_{21}}{i_1}$, the mutual inductance M_{21} of coil 2 with respect to coil 1 as

$$M_{21} i_1 = N_2 \Phi_{21}.$$

$$M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt} \qquad \mathcal{E}_2 = -M_{21} \frac{di_1}{dt}$$

Summary (1 of 7)

Magnetic Flux

- The magnetic flux through an area A in a magnetic field \vec{B} is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Equation 30-1

- If \vec{B} is perpendicular to the area and uniform over it, Equation 30-1 becomes

$$\Phi_B = BA \quad (\vec{B} \perp A, \vec{B} \text{ uniform}).$$

Equation 30-2

Summary (2 of 7)

Faraday's Law of Induction

- The induced emf is,

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Equation 30-4

- If the loop is replaced by a closely packed coil of N turns, the induced emf is

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}.$$

Equation 30-5

Summary (3 of 7)

Lenz's Law

- An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current.

emf and the Induced Magnetic Field

- The induced emf is related to \vec{E} by

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s},$$

Equation 30-19

- Faraday's law in its most general form,

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Equation 30-20

Summary (4 of 7)

Inductor

- The inductance L of the inductor is

$$L = \frac{N\Phi_B}{i}$$

Equation 30-28

- The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A$$

Equation 30-31

Summary (5 of 7)

Self-Induction

- This self-induced emf is,

$$\mathcal{E}_L = -L \frac{di}{dt}.$$

Equation 30-35

Series RL Circuit

- Rise of current,

$$i = \frac{\mathcal{E}}{R} \left(1 - e^{\frac{-t}{\tau_L}} \right)$$

Equation 30-41

- Decay of current

$$i = i_0 e^{\frac{-t}{\tau_L}}$$

Equation 30-45

Summary (6 of 7)

Magnetic Energy

- the inductor's magnetic field stores an energy given by

$$U_B = \frac{1}{2}Li^2$$

Equation 30-49

- The density of stored magnetic energy,

$$u_B = \frac{B^2}{2\mu_0}$$

Equation 30-55

Summary (7 of 7)

Mutual Induction

- The mutual induction is described by,

$$\mathcal{E}_2 = -M \frac{di_1}{dt}$$

Equation 30-64

$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

Equation 30-65