UFUG 1504: Honors General Physics II

Chapter 40

All About Atoms

Key Ideas

- Atoms have quantized energies and can make quantum jumps between them. If a jump between a higher energy and a lower energy involves the emission or absorption of a photon, the frequency associated with the light is given by

$$hf = E_{\text{high}} - E_{\text{low}}.$$

- States with the same value of quantum number n form a shell.
- States with the same values of quantum numbers n and ℓ form a subshell.
- The magnitude of the orbital angular momentum of an electron trapped in an atom has quantized values given by

$$L = \sqrt{\ell(\ell + 1)}\,\hbar, \quad \text{for } \ell = 0, 1, 2, \ldots, (n - 1),$$

where $\hbar$ is $h/2\pi$, ℓ is the orbital quantum number, and n is the electron's principal quantum number.

Key Ideas

- The component L_z of the orbital angular momentum on a z axis is quantized and given by

$$L_z = m_\ell \hbar, \quad \text{for } m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell,$$

where m_ℓ is the orbital magnetic quantum number.

- The magnitude μ_{orb} of the orbital magnetic moment of the electron is quantized with the values given by

$$\mu_{\text{orb}} = \frac{e}{2m}\sqrt{\ell(\ell+1)}\,\hbar,$$

where m is the electron mass.

- The component $\mu_{\text{orb},z}$ on a z axis is also quantized according to

$$\mu_{\text{orb},z} = -\frac{e}{2m} m_\ell \hbar = -m_\ell \mu_{\text{B}},$$

where μ_{B} is the Bohr magneton:

$$\mu_{\text{B}} = \frac{eh}{4\pi m} = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \text{ J/T}.$$

Key Ideas

- Every electron, whether trapped or free, has an intrinsic spin angular momentum $\vec{S}$ with a magnitude that is quantized as

$$S = \sqrt{s(s+1)}\,\hbar, \quad \text{for } s = \tfrac{1}{2},$$

where s is the spin quantum number. An electron is said to be a spin-$\frac{1}{2}$ particle.

- The component S_z on a z axis is also quantized according to

$$S_z = m_s\hbar, \quad \text{for } m_s = \pm s = \pm\tfrac{1}{2},$$

where m_s is the spin magnetic quantum number.

- Every electron, whether trapped or free, has an intrinsic spin magnetic dipole moment $\vec{\mu}_s$ with a magnitude that is quantized as

$$\mu_s = \frac{e}{m}\sqrt{s(s+1)}\,\hbar, \quad \text{for } s = \tfrac{1}{2}.$$

- The component $\mu_{s,z}$ on a z axis is also quantized according to

$$\mu_{s,z} = -2m_s\mu_B, \quad \text{for } m_s = \pm\tfrac{1}{2}.$$

40.2: Some Properties of Atoms: (1 of 2)

Atoms are stable. Essentially all the atoms that form our tangible world have existed without change for billions of years.

Atoms combine with each other. They stick together to form stable molecules and stack up to form rigid solids.

40.2: Some Properties of Atoms

Atoms Are Put Together Systematically

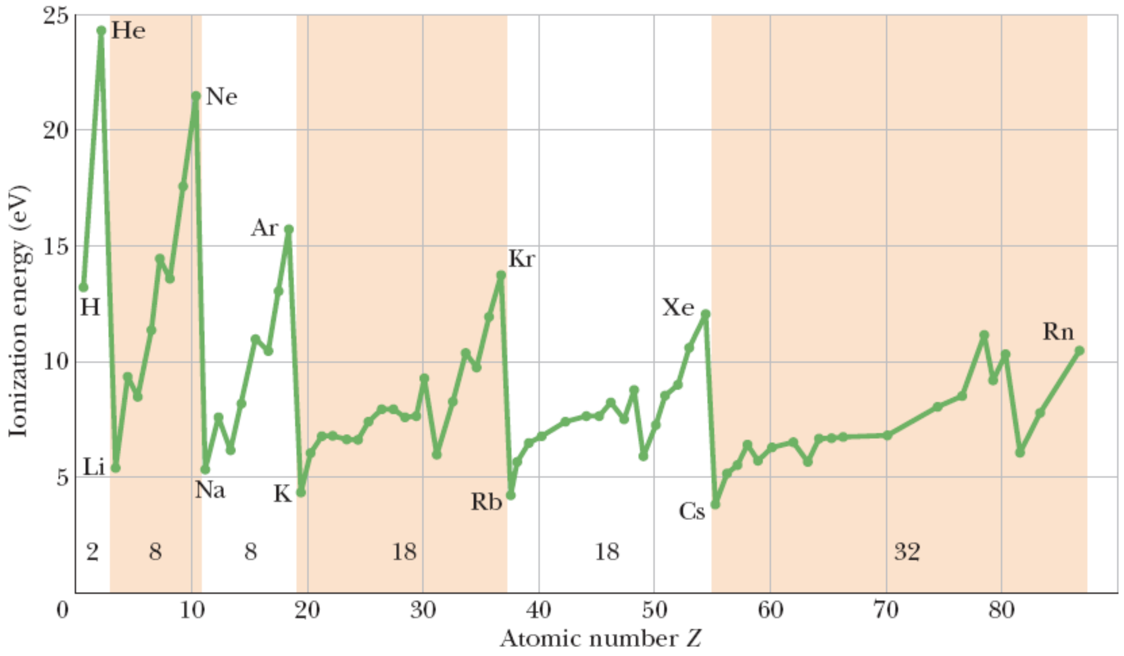


Figure 40-2 A plot of the ionization energies of the elements as a function of atomic number, showing the periodic repetition of properties through the six complete horizontal periods of the periodic table.

ionization energy 电离能: the energy required to **remove the most loosely bound electron from a neutral atom**.

The elements are arranged in the periodic table in six complete horizontal periods

except for the first, each period starts at the left with a highly reactive alkali metal 碱金属 (lithium, sodium, potassium, and so on) and ends at the right with a chemically inert noble gas 惰性气体(neon, argon, krypton, and so on).

The numbers of elements in the six periods are 2, 8, 8, 18, 18, and 32.

Quantum physics predicts these numbers and the chemical properties.

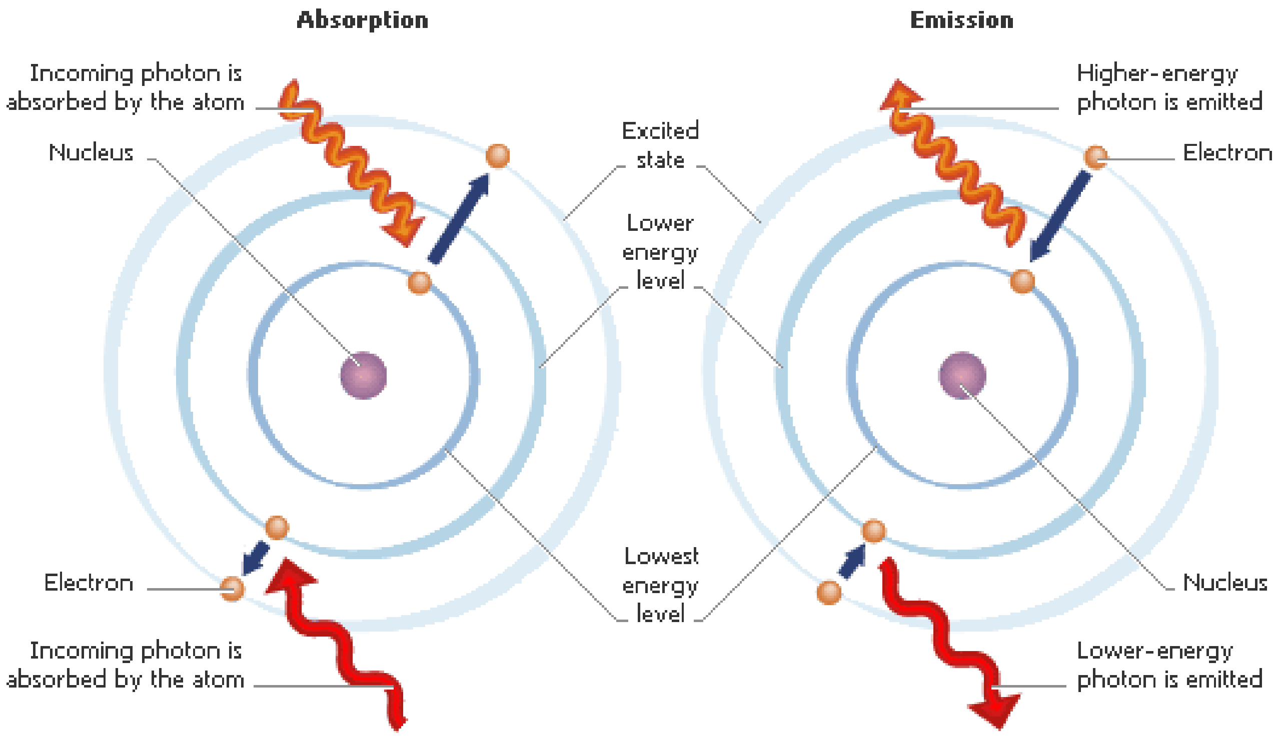
40.2: Some Properties of Atoms

Atoms Emit and Absorb Light:

An atom can make a transition from one state to another by emitting light (to jump to a lower energy level E_{low}) or by absorbing light (to jump to a higher energy level E_{high}).

The light is emitted or absorbed as a photon with energy

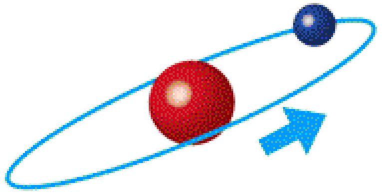
$$hf = E_{high} - E_{low}.$$

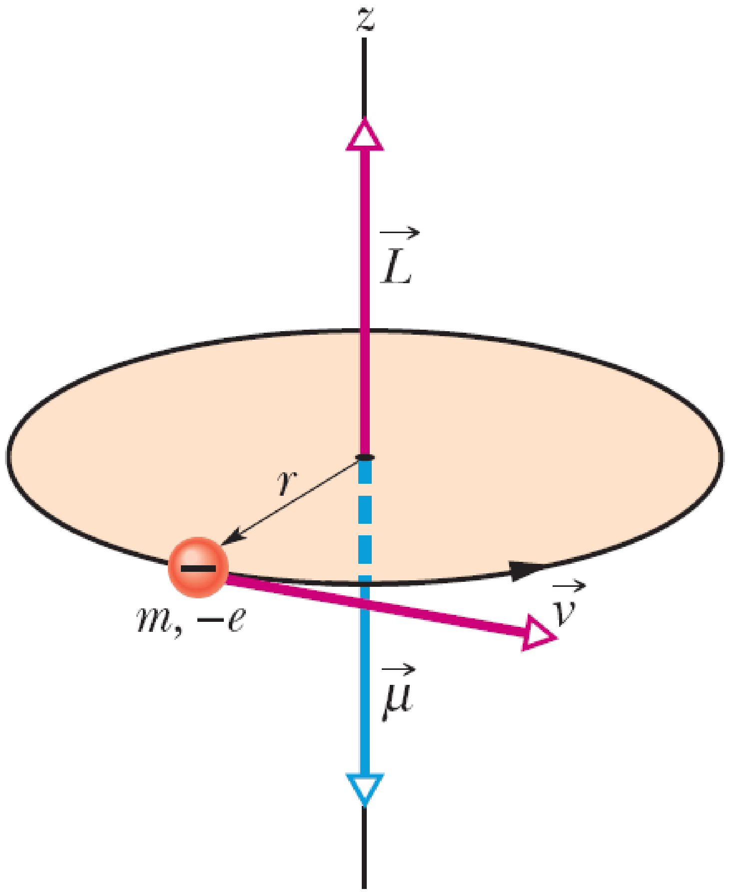


40.2: Some Properties of Atoms:

Atoms have Angular Momentum and Magnetism

A classical model showing a particle of mass m and charge $-e$ moving with speed v in a circle of radius r. The moving particle has an angular momentum $\vec{L}$ given by $\vec{r} \times \vec{p}$, Where $\vec{p}$ is its linear momentum $m\vec{v}$. The particle's motion is equivalent to a current loop that has an associated magnetic dipole moment $\vec{\mu}$ that is directed opposite $\vec{L}$.





40.3: Angular Momenta and Magnetic Dipole Moments: (1 of 9)

Table 40-1 Electron States for an Atom

Quantum Number	Symbol	Allowed Values	Related to
Principal	n	1,2,3, . . .	Distance from the nucleus
Orbital	ℓ	0, 1, 2, . . . , $(n-1)$	Orbital angular momentum
Orbital magnetic	m_ℓ	$0, \pm 1, \pm 2, \ldots, \pm \ell$	Orbital angular momentum (z component)
Spin	s	$\frac{1}{2}$	Spin angular momentum
Spin magnetic	m_s	$\pm \frac{1}{2}$	Spin angular momentum (z component)

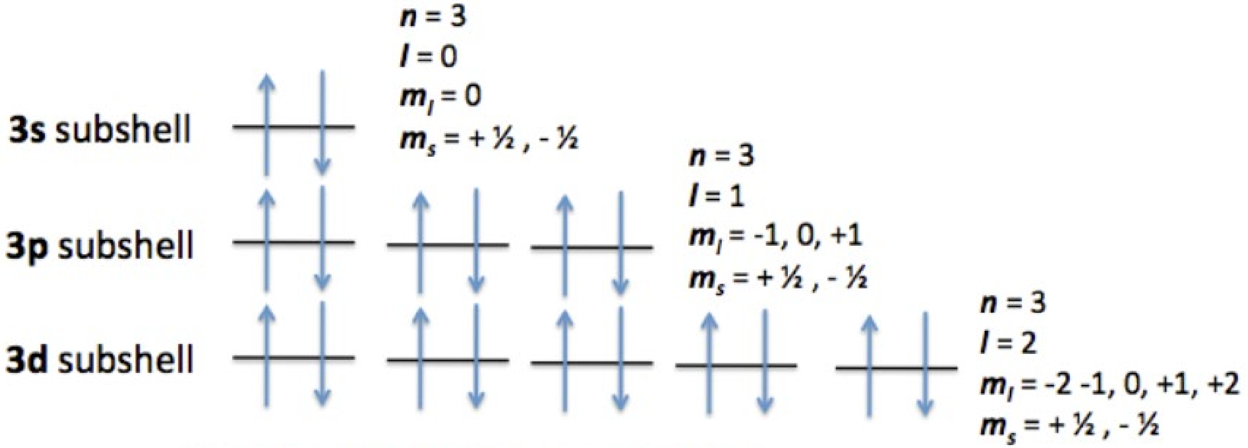
40.3: Angular Momenta and Magnetic Dipole Moments: (1 of 9)

n	$1, 2, 3, \ldots$
ℓ	$0, 1, 2, \ldots, (n-1)$
m_ℓ	$0, \pm 1, \pm 2, \ldots, \pm \ell$
s	$\frac{1}{2}$
m_s	$\pm\frac{1}{2}$

Name of Subshell	Value of *l*
s subshell	0
p subshell	1
d subshell	2
f subshell	3

Example:

Electron Orbital Diagram of subshell/orbital combinations when ***n*** = 3



- The value of *l* indicates s, p, or d subshell.
- Each black line represents one orbital (value count of m_l.)
- The direction of the arrows represent m_s

40.3: Angular Momenta and Magnetic Dipole Moments: (1 of 9)

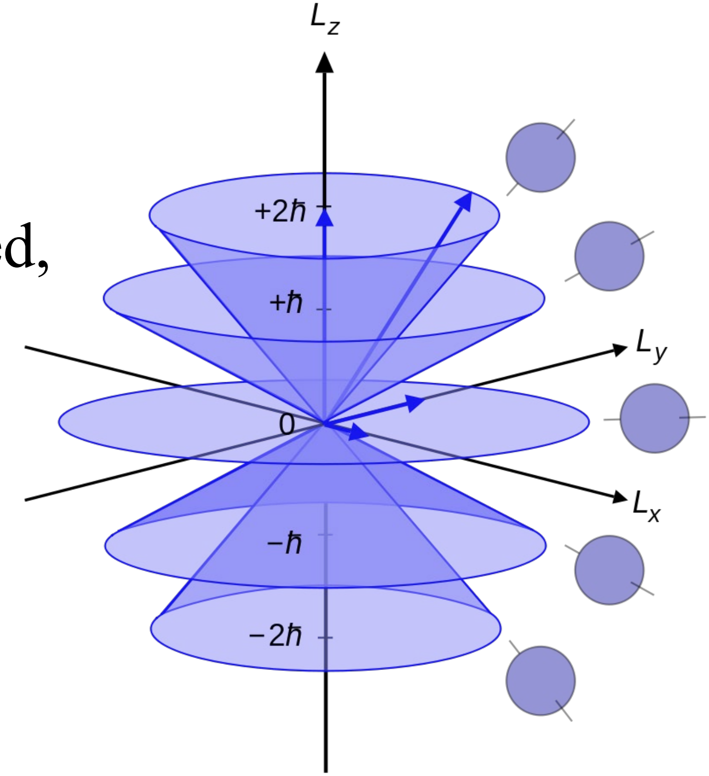
The magnitude $\vec{L}$ of the **orbital angular momentum** $\vec{\mathrm{L}}$ of an of an electron in an atom is quantized; that is, it can have only certain values

$$L = \sqrt{\ell(\ell+1)}\hbar, \qquad \hbar = h/2\pi$$

The components L_z of the angular momentum are also quantized, and they are given by

$$L_z = m_\ell \hbar, \quad \text{for } m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell,$$

m_ℓ is the orbital magnetic quantum number



40.3: Angular Momenta and Magnetic Dipole Moments: (2 of 9)

The magnetic dipole has an **orbital magnetic dipole moment** μ_{orb}, is related to the angular momentum by

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}. = -\frac{e}{2m}\sqrt{\ell(\ell+1)}\hbar.$$

If the atom is located in a magnetic field $\vec{B}$, with a *z* axis extending in the direction of the field lines at the atom's location, we can measure the *z* components of $\vec{\mu}_{\text{orb}}$ and $\vec{L}$ along that axis.

$$\vec{\mu}_{\text{orb},z} = -m_\ell \mu_{\text{B}}.$$

$m_\ell \quad 0, \pm 1, \pm 2, \ldots, \pm \ell$

m_ℓ is the orbital magnetic quantum number

Here μ_{B} is the **Bohr Magneton** 玻尔磁子

$$\mu_{\text{B}} = \frac{eh}{4\pi m} = \frac{e\hbar}{2m} = 9.274 \times 10^{-24}\ \text{J/T}$$

40.3: Angular Momenta and Magnetic Dipole Moments: (2 of 9)

The electron has an intrinsic **spin angular momentum** $\vec{S}$, often called simply spin.

The magnitude of $\vec{S}$ is quantized and depends on a **spin quantum number** s, which is always $\frac{1}{2}$ for electrons (and for protons and neutrons).

$$S = \sqrt{s(s+1)}\hbar$$

$$= \sqrt{\left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)}\hbar = 0.866\hbar,$$

s	$\frac{1}{2}$
m_s	$\pm\frac{1}{2}$

The component of $\vec{S}$ is measured along any axis, is quantized, and depends on a **spin magnetic quantum number** m_s, which can have only the value $+\frac{1}{2}$ or $-\frac{1}{2}$.

$$S_z = m_s\hbar \qquad m_s = \pm s = \pm\frac{1}{2}$$

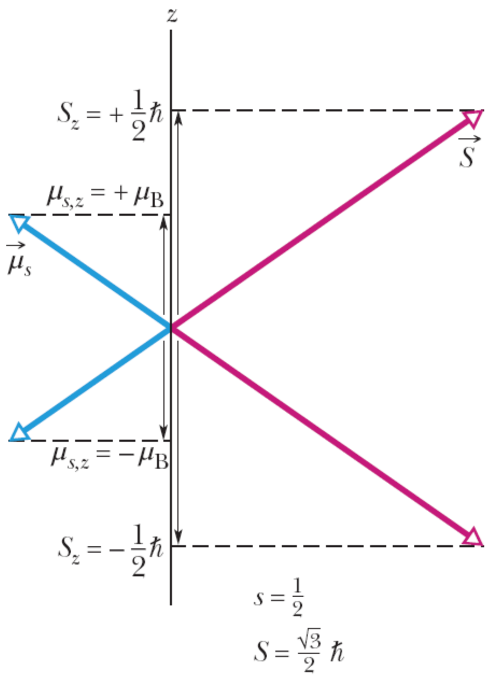


Figure 40-6 The allowed values of S_z and μ_z for an electron.

40.3: Angular Momenta and Magnetic Dipole Moments: (6 of 9)

As with the orbital angular momentum, a magnetic dipole moment is associated with the spin angular momentum:

$$\vec{\mu}_s = -\frac{e}{m}\vec{S}. \qquad \text{(Spin magnetic dipole moment)}$$

$$= \frac{e}{m}\sqrt{s(s+1)}\hbar.$$

The vector can also have a definite component on a *z* axis, given by

$$\mu_{s,z} = -2m_s\mu_B.$$

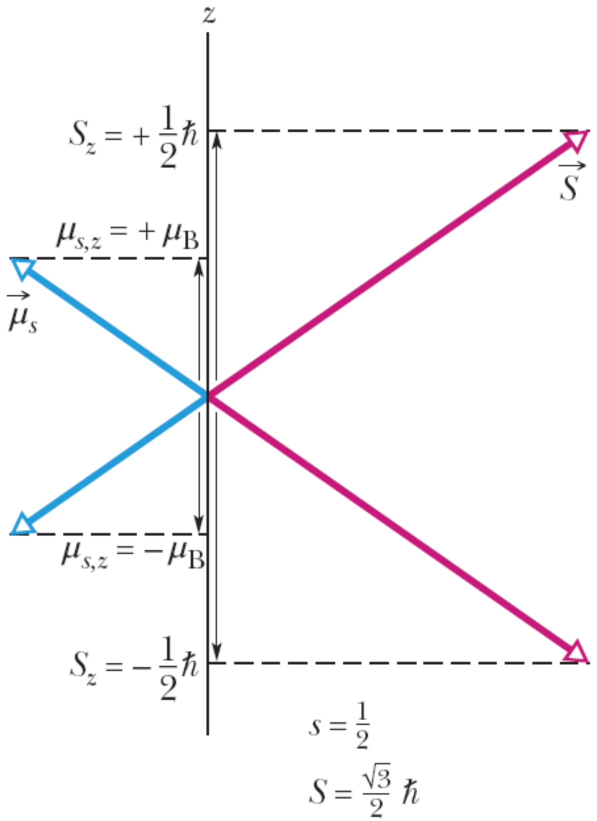


Figure 40-6 The allowed values of S_z and μ_z for an electron.

40.3: Angular Momenta and Magnetic Dipole Moments: (8 of 9)

Orbital and Spin Angular Momenta Combined

For an atom containing more than one electron, we define a total angular momentum, $\vec{J}$, which is the vector sum of orbital and their spin angular momenta of the individual electrons. This number of protons is defined as being the **atomic number** Z of the element.

$$\vec{J} = \left(\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \cdots + \vec{L}_z\right) + \left(\vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \cdots + \vec{S}_z\right).$$

The **effective magnetic dipole moment**, μ_{eff}, for the atom is the component of the vector sum of the individual magnetic dipole moments in the direction of $-\vec{J}$.

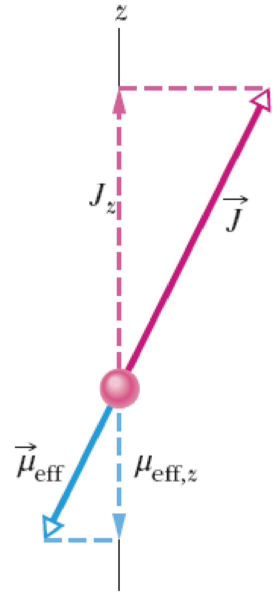


Figure 40-7 A classical model showing the total angular momentum vector $\vec{J}$ and the effective magnetic moment vector $\vec{\mu}_{\text{eff}}$.

40.6: Magnetic Resonance: (1 of 3)

28-8 The Magnetic Dipole Moment (6 of 6)

Summary (5 of 5)

- The dipole has a potential energy U associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}. \qquad \textbf{Equation (22-38)}$$

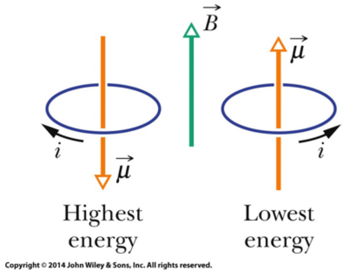
The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

If an external agent rotates a magnetic dipole from an initial orientation θ_i to some other orientation θ_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i.$$

The magnetic moment vector attempts to align with the magnetic field.



A magnetic dipole has its lowest energy (= $-\mu B$ cos 0 = $-\mu B$) when its dipole moment μ is lined up with the magnetic field, It has its highest energy (= $-\mu B$ cos 180° = $+\mu B$) when μ is directed opposite the field

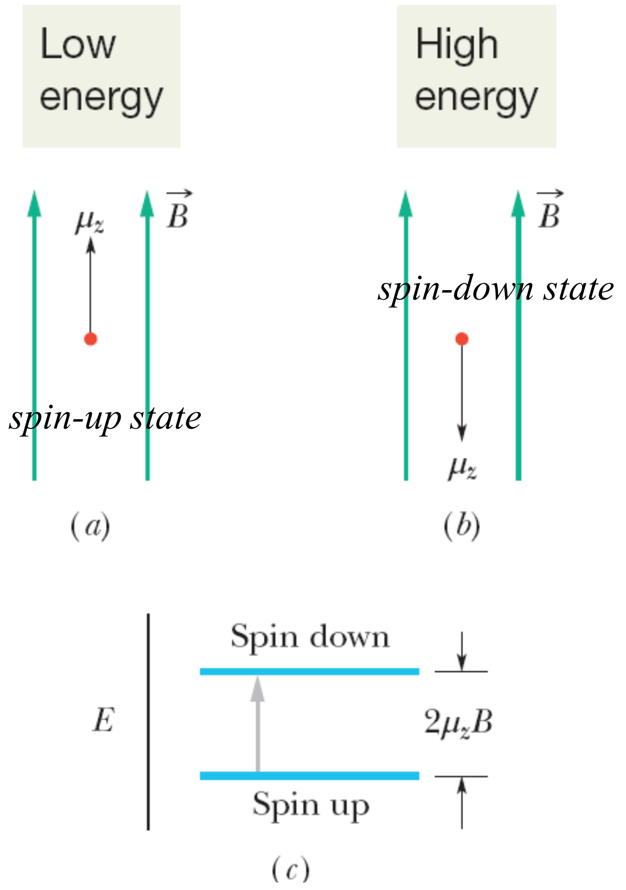


Figure 40-10 The z component of $\vec{\mu}$ for a proton in the (a) lower-energy (spin-up) and (b) higher-energy (spin-down) state. (c) An energy-level diagram for the states, showing the upward quantum jump the proton makes when its spin flips from up to down.

40.6: Magnetic Resonance: (2 of 3)

Thus, the energy difference between these two states is

$$\Delta E = \mu_z B - (-\mu_z B) = 2\mu_z B.$$

Water in a magnetic field B , the protons in the hydrogen portions of each water molecule tend to be in the lower-energy state. Any one of these protons can jump to the higher-energy state by absorbing a photon with an energy hf equal to ΔE.

$$hf = 2\mu_z B. \quad \text{(Energy of absorbing photon)}$$

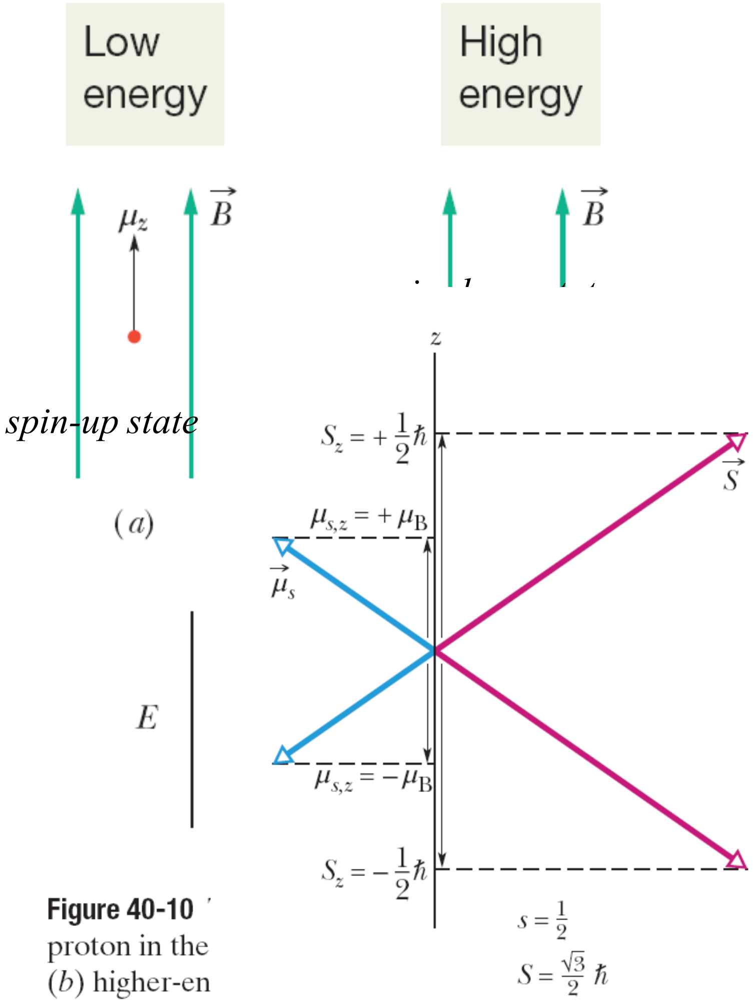
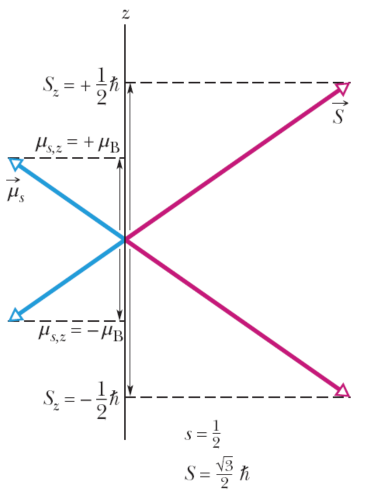


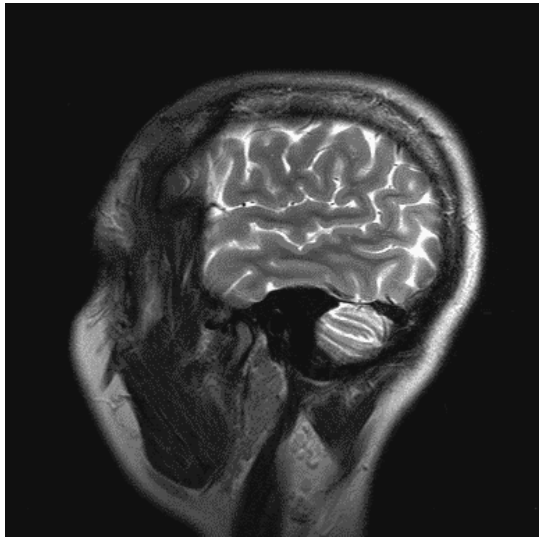
Figure 40-10
proton in the
(b) higher-en
energy-level diagram for the states, showing the upward quantum jump the proton makes when its spin flips from up to down.

40.6: Magnetic Resonance: (2 of 3)

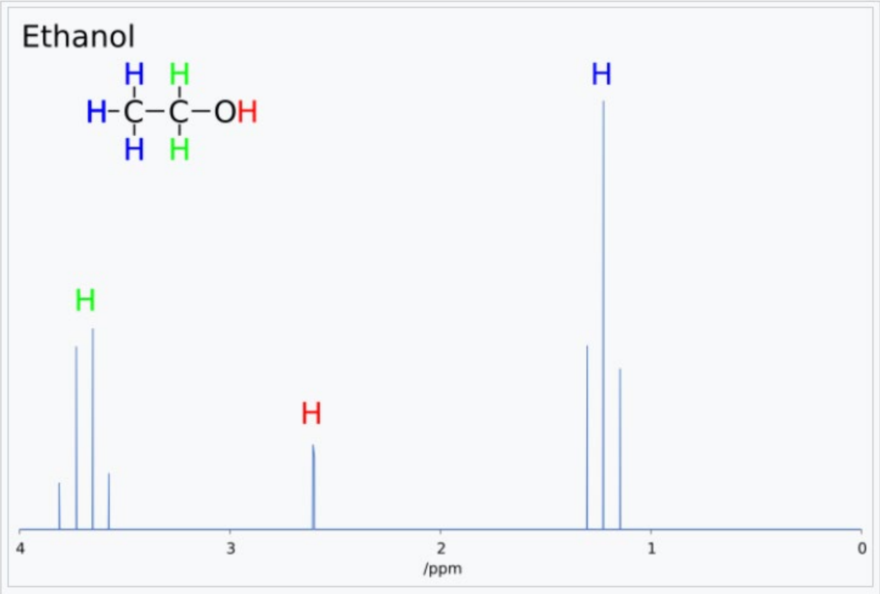
$$hf = 2\mu_z B. \quad (\text{Energy of absorbing photon})$$

Such absorption is called **magnetic resonance** or, **nuclear magnetic resonance (NMR) 核磁共振**, and the consequent reversal of S_z is called **spin-flipping**.





NMR image



NMR spectrum

There are three different types of H atoms in ethanol regarding NMR: the hydrogen (H) on the −OH group is not coupling with the other H atoms and appears as a singlet, but the CH_3− and the −CH_2− hydrogens are coupling with each other, resulting in a triplet and quartet respectively.

^{1}H NMR spectrum

40.7: The Pauli Exclusion Principle:

No two electrons confined to the same trap can have the same set of values for their quantum numbers.

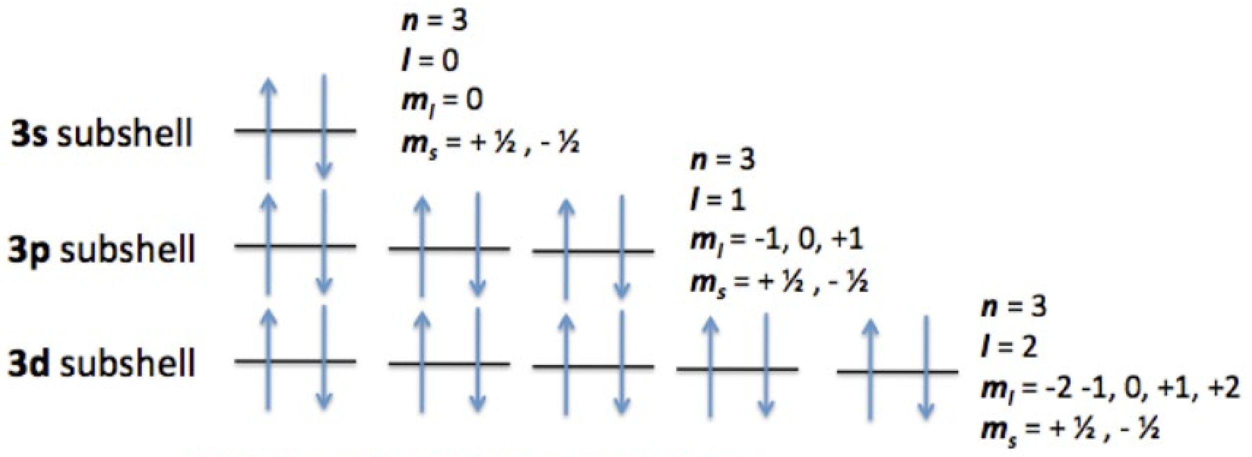
This principle applies not only to electrons but also to protons and neutrons, all of which have $s = \frac{1}{2}$.

The principle is known as the **Pauli exclusion principle** after Wolfgang Pauli, who formulated it in 1925

40.7: The Pauli Exclusion Principle:

Example:

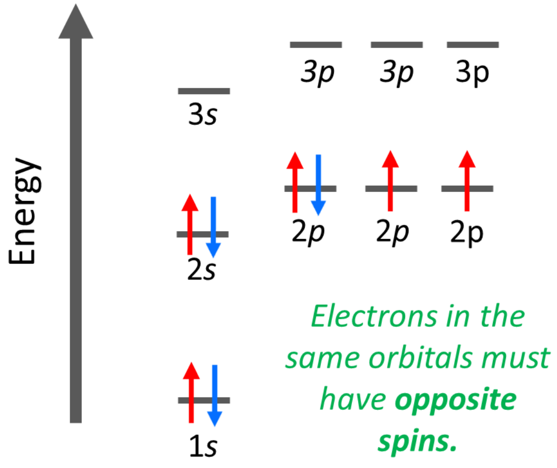
Electron Orbital Diagram of subshell/orbital combinations when $n = 3$



- The value of l indicates s, p, or d subshell.
- Each black line represents one orbital (value count of m_l .)
- The direction of the arrows represent m_s

Pauli's Exclusion Principle

No two electrons in an atom can have the **same *four* quantum numbers.**



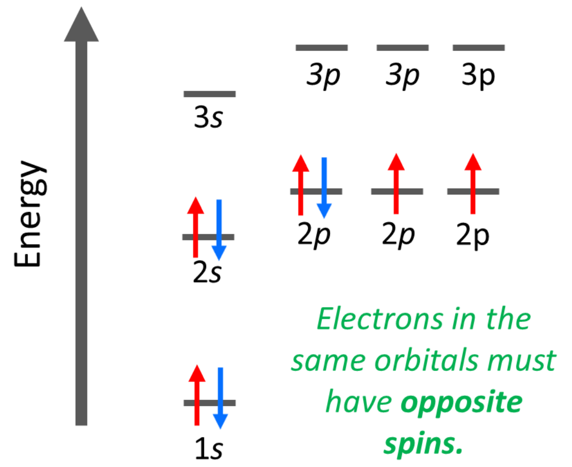
This is because when they are in the **same orbital, the values of n, l, and m_l are the same**. Therefore, they **must have opposite spins (different m_s)** so that they do not have all four quantum numbers the same.

40.8: Multiple Electrons in Rectangular Traps:

- The Pauli exclusion principle disallows any more electrons from occupying that lowest energy level, and the next electron must occupy the next higher level.
- When an energy level cannot be occupied by more electrons because of the Pauli exclusion principle, we say that level is **full or fully occupied**.
- In contrast, a level that is not occupied by any electrons is **empty or unoccupied**.
- For intermediate situations, the level is **partially occupied**.
- The **electron configuration of a system of trapped** electrons is a listing or drawing either of the energy levels the electrons occupy or of the set of the quantum numbers of the electrons.

Pauli's Exclusion Principle

No two electrons in an atom can have the **same *four* quantum numbers.**



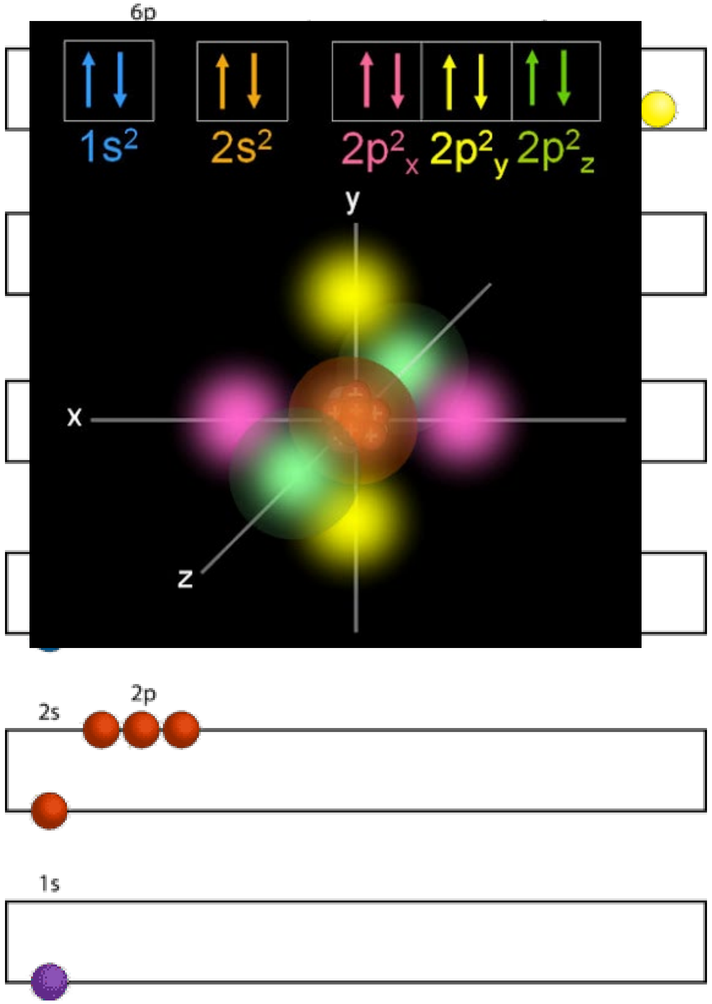
This is because when they are in the **same orbital, the values of n, l, and m_l are the same**. Therefore, they **must have opposite spins (different m_s)** so that they do not have all four quantum numbers the same.

40.9: Building the Periodic Table: (1 of 3)

For the purpose of labeling subshells in atoms of elements, the values of l are represented by letters:

$$\ell = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \ldots$$

$$s \quad p \quad d \quad f \quad g \quad h \quad \ldots.$$



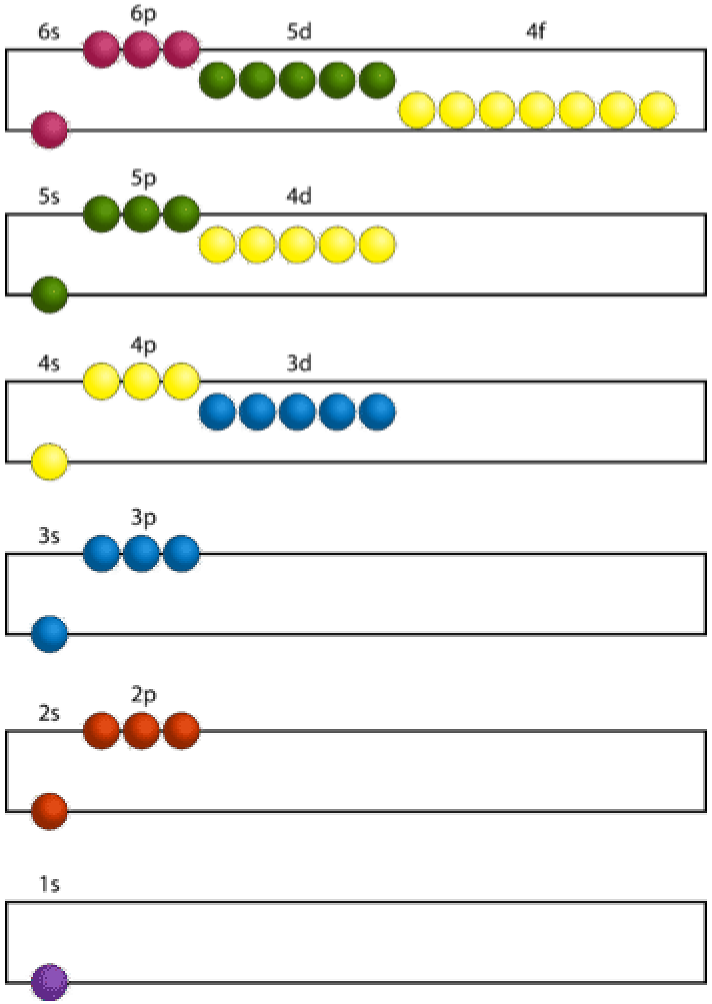
Neon(Ne): The atom has 10 electrons. It has three closed subshells (1*s*, 2*s*, and 2*p*) and, like the other **noble gases** that form the right-hand column of the periodic table, is almost chemically inert.

Quantum Number	Symbol	Allowed Values	Related to
Principal	n	1,2,3, . . .	Distance from the nucleus
Orbital	ℓ	$0, 1, 2, \ldots, (n-1)$	Orbital angular momentum
Orbital magnetic	m_ℓ	$0, \pm 1, \pm 2, \ldots, \pm \ell$	Orbital angular momentum (z component)
Spin	s	$\frac{1}{2}$	Spin angular momentum
Spin magnetic	m_s	$\pm\frac{1}{2}$	Spin angular momentum (z component)

40.9: Building the Periodic Table: (2 of 3)

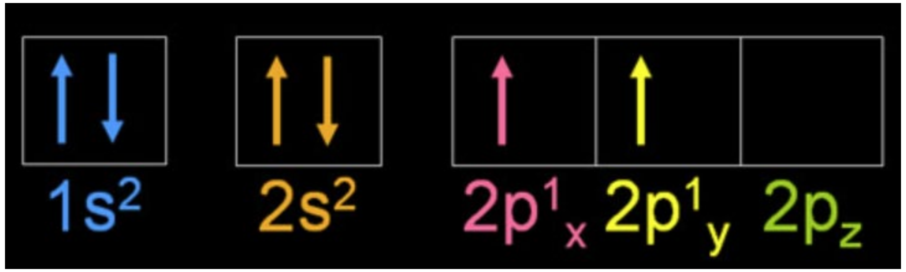
Sodium (Na): The atom has 11 electrons. Ten of them form a closed neon-like core, and has zero angular momentum. The remaining electron is largely outside this inert core, in the 3*s* subshell. This is the **valence electron** of the atom, and the atom's angular momentum and magnetic dipole moment must be due entirely to the spin of this single electron.

Chlorine (Cl): This, with 17 electrons, has the outermost 7 electrons in 3*p* subshell, leaving a "hole" in this state. It is receptive to interacting with other atoms that have a valence electron that might fill this hole. Chlorine, like the other **halogens** that form column VIIA of the periodic table, is chemically active.

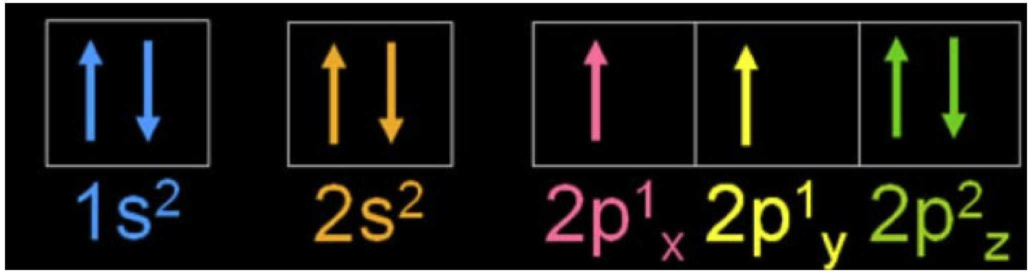


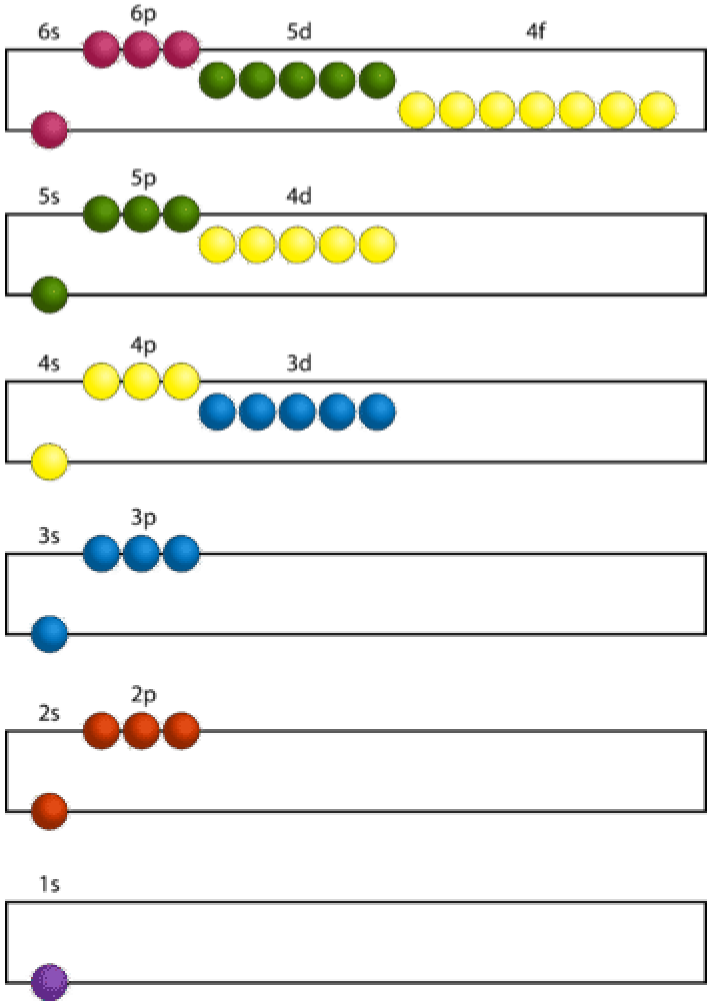
40.9: Building the Periodic Table: (2 of 3)

carbon



oxygen





40.9: Building the Periodic Table: (3 of 3)

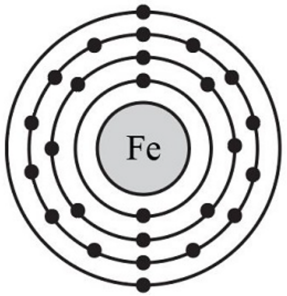
Iron:

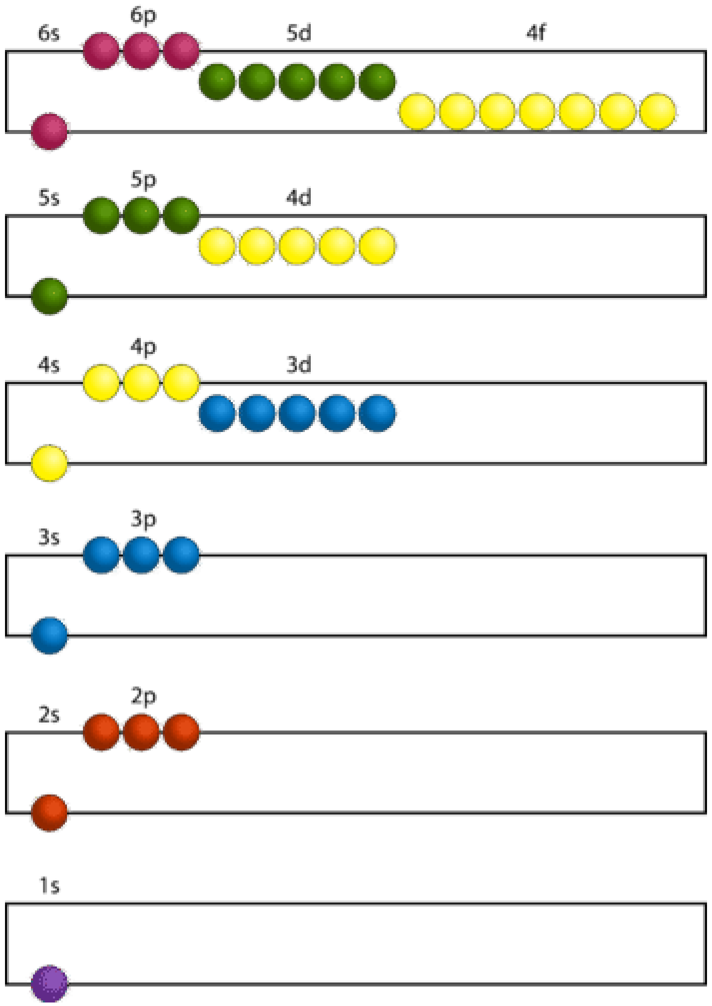
The arrangement of the 26 electrons of the iron atom can be represented as follows:

$$\underbrace{1s^2 \quad 2s^2\, 2p^6 \quad 3s^2 \;\; 3p^6}\; 3d^6 \quad 4s^2.$$

This atom of 26 electrons, has the first 18 electrons form the five filled subshells that are marked off by the bracket. 6 of the remaining 8 electrons go into the $3d$ subshell, and the remaining two go into the $4s$ subshell. The configuration $3d^6 \quad 4s^2$ is of lower energy than $3d^8$.

26: Iron 2,8,14,2





40.10: X-Rays and the Ordering of the Elements: (1 of 2)

The minimum possible x-ray wavelength:

$$K_0 = hf = \frac{hc}{\lambda_{\min}},$$

$$\lambda_{\min} = \frac{hc}{K_0} \quad \text{(cutoff wavelength)}.$$

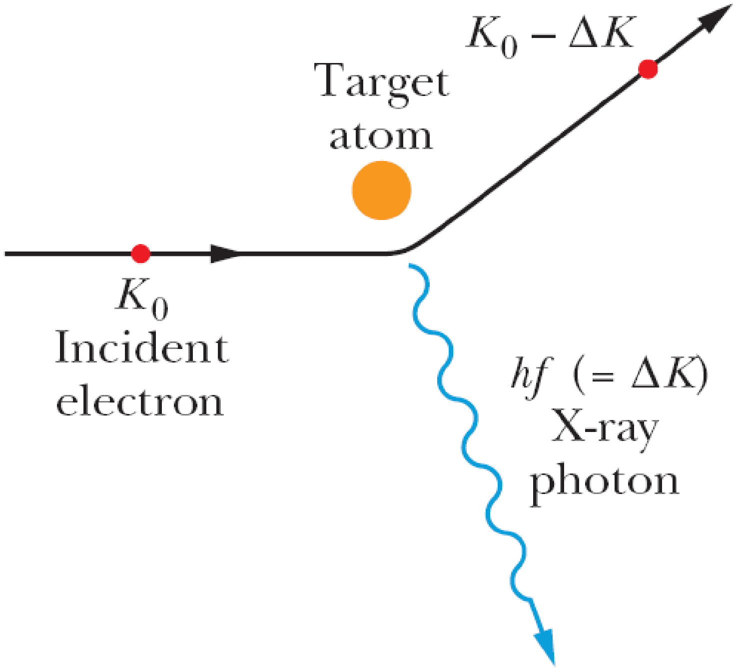


Figure 40-14 An electron of kinetic energy K_0 passing near an atom in the target may generate an x-ray photon, the electron losing part of its energy in the process. The continuous x-ray spectrum arises in this way.

40.10: X Rays and the Ordering of the Elements: The Characteristic X-ray Spectrum (1 of 2)

$$\lambda_{\min} = \frac{hc}{K_0} \quad \text{(cutoff wavelength)}.$$

below λmin the continuous spectrum does not exist

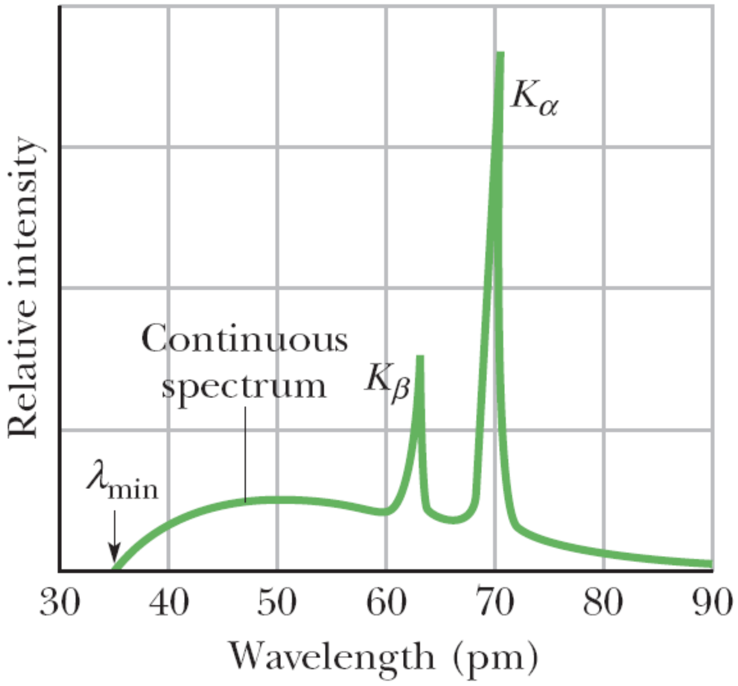


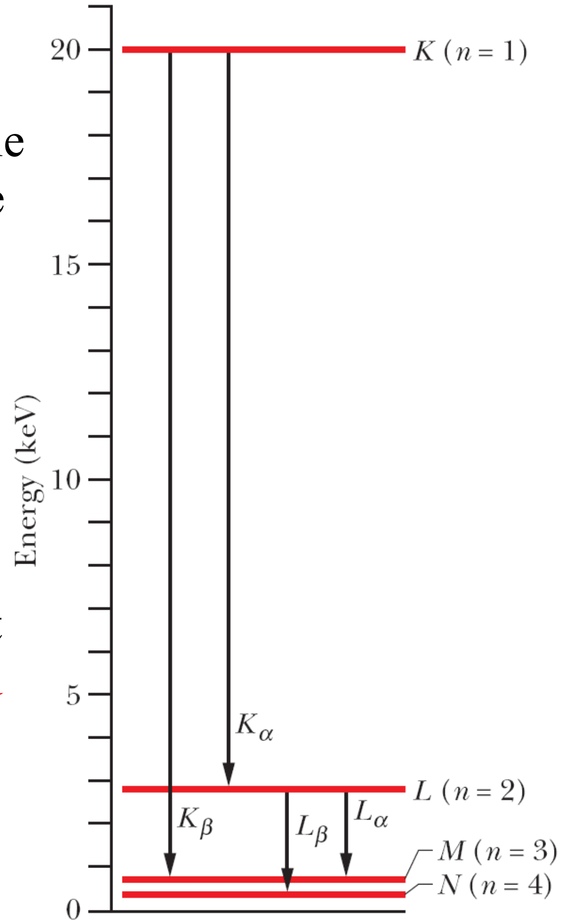
Figure 40-13 The distribution by wavelength of the x rays produced when 35 keV electrons strike a molybdenum target

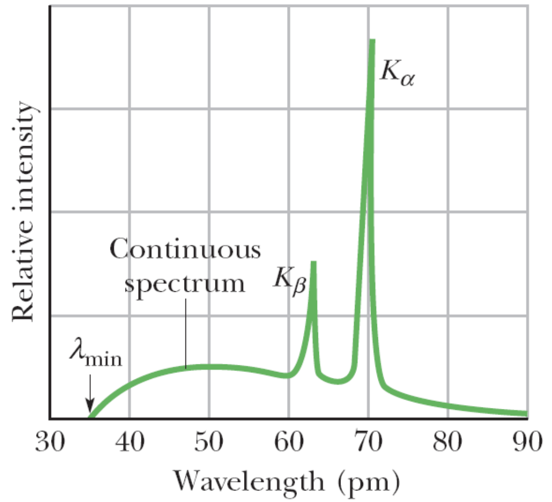
40.10: X Rays and the Ordering of the Elements: The Characteristic X-ray Spectrum (2 of 2)

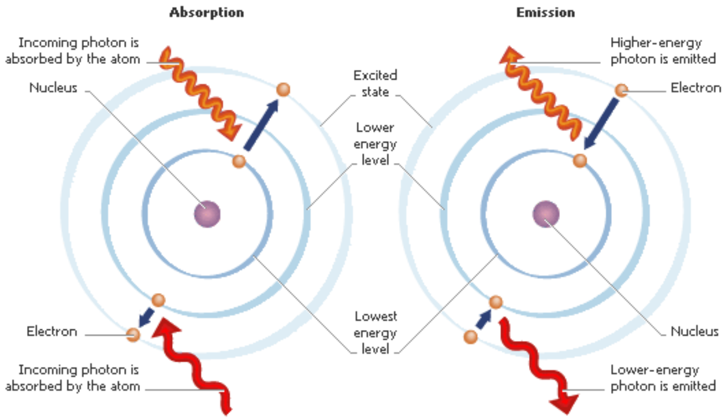
The peaks arise in a two-part process.

(1) An energetic electron strikes an atom in the target and, while it is being scattered, the incident electron knocks out one of the atom's deep-lying (low *n* value) electrons. If the deep-lying electron is in the shell defined by *n* = 1 (called, for historical reasons, the *K* shell), there remains a vacancy, or *hole,* in this shell.

(2) An electron in one of the shells with a higher energy jumps to the *K* shell, filling the hole in this shell. During this jump, the atom emits a characteristic x-ray photon. If the electron that fills the *K*-shell vacancy jumps from the shell with *n* = 2 (called the *L* shell), the emitted radiation is the K_α line; if it jumps from the shell with *n* = 3 (called the *M* shell), it produces the K_β line, and so on. The hole left in either the *L* or *M* shell will be filled by an electron from still farther out in the atom.







40.10: Accounting for Moseley's Plot:

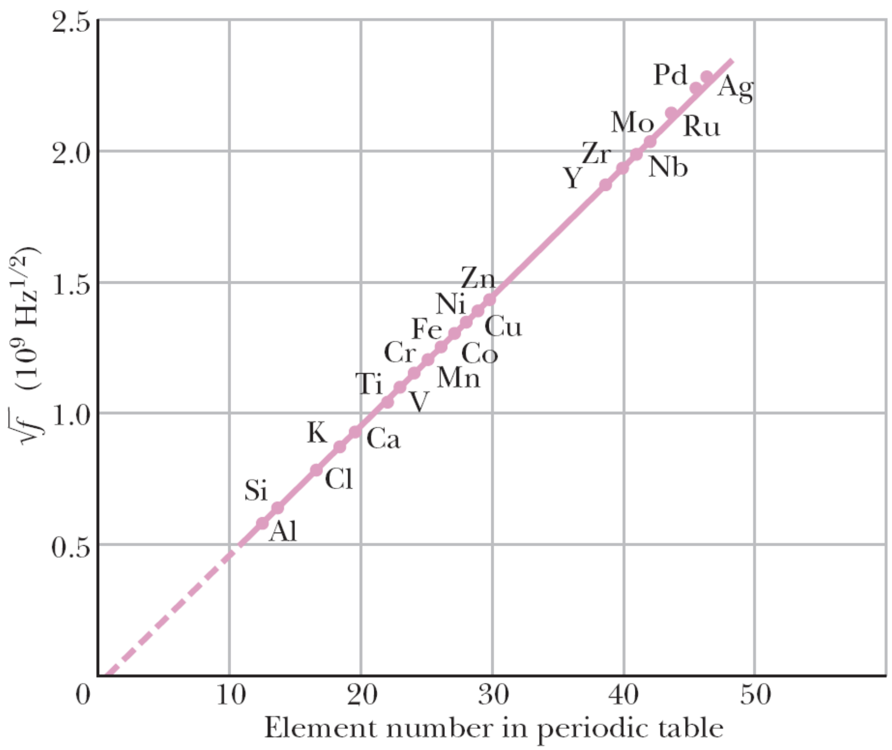


Figure 40-16 A Moseley plot of the K_α line of the characteristic x−ray spectra of 21 elements. The frequency is calculated from the measured wavelength.

40.10: Accounting for Moseley's Plot: (2 of 3)

For hydrogen atom:

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = -\frac{13.60\text{ eV}}{n^2}, \quad \text{for } n = 1, 2, 3, \ldots.$$

$$E_1 = \frac{-13.6}{1^2} eV = -13.6 eV,$$

$$E_2 = \frac{-13.6}{2^2} eV = -3.4 eV,$$

$$E_3 = \frac{-13.6}{3^2} eV = -1.51 eV,$$

$$E_4 = \frac{-13.6}{4^2} eV = -0.85 eV, E_\infty = 0.$$

40.10: Accounting for Moseley's Plot: (2 of 3)

For hydrogen atom:

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = -\frac{13.60\text{ eV}}{n^2},\quad \text{for } n = 1, 2, 3, \ldots.$$

For one of the two innermost electrons in the *K* shell of a multi-electron atom, because of the presence of the other *K*-shell electron, it "sees" an effective nuclear charge of approximately $(Z - 1)e$, where e is the electronic charge. Therefore the effective energy of the atom is:

$$E_n = -\frac{(13.60\text{ eV})(Z-1)^2}{n^2}.$$

40.10: Accounting for Moseley's Plot: (3 of 3)

K_α x-ray photon (of energy hf) arises when an electron makes a transition from the L shell (with n = 2 and energy E_2) to the K shell (with n = 1 and energy E_1).

Therefore,
$$\Delta E = E_2 - E_1 = \frac{-(13.60\ \text{eV})(Z-1)^2}{2^2} - \frac{-(13.60\ \text{eV})(Z-1)^2}{1^2} = (10.2\ \text{eV})(Z-1)^2.$$

And,
$$f = \frac{\Delta E}{h} = \frac{(10.2\ \text{eV})(Z-1)^2}{(4.14\times10^{-15}\ \text{eV}\cdot\text{s})} = (2.46\times10^{15}\ \text{Hz})(Z-1)^2.$$

$$\Rightarrow \sqrt{f} = CZ - C,$$ C is a constant $\left(=4.96\times10^{7}\ \text{Hz}^{\frac{1}{2}}\right)$.

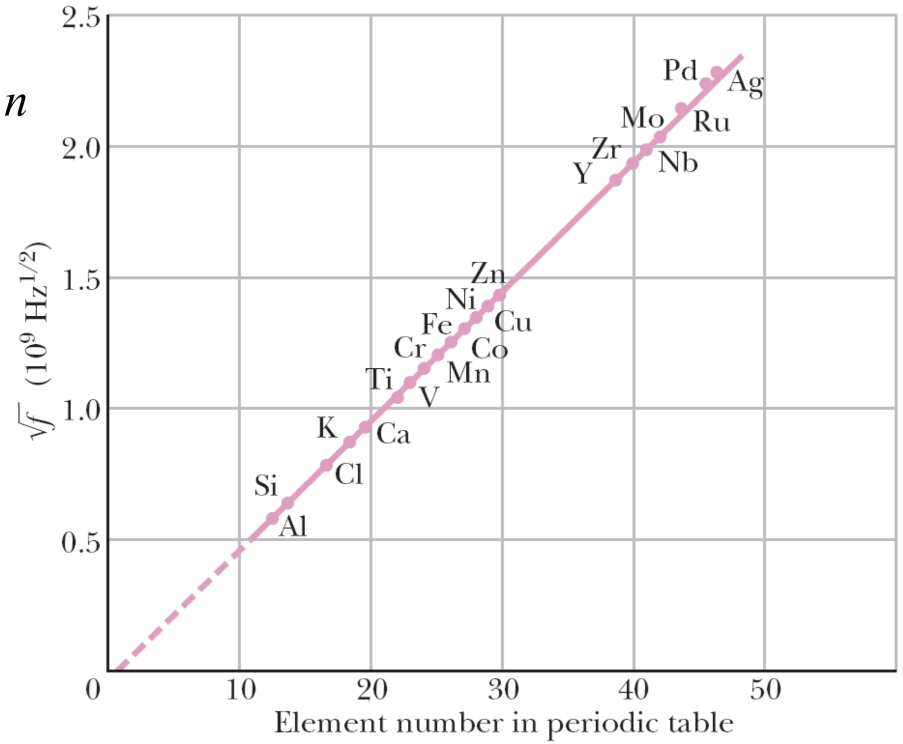
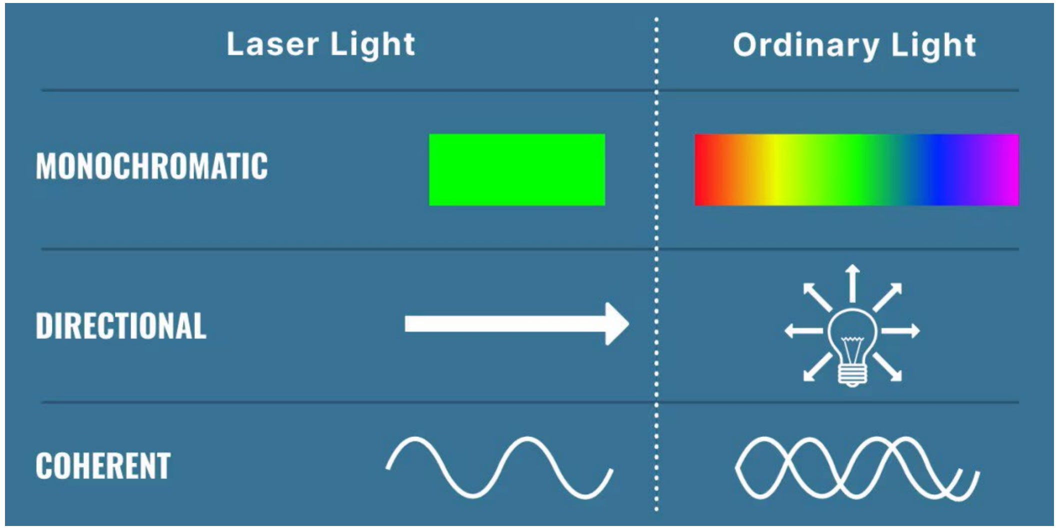


Figure 40-16 A Moseley plot of the K_α line of the characteristic x−ray spectra of 21 elements. The frequency is calculated from the measured wavelength.

40.11: Lasers and Laser Light: (1 of 2)

1. Laser light is highly monochromatic.
2. Laser light is highly coherent.
3. Laser light is highly directional.
4. Laser light can be sharply focused.



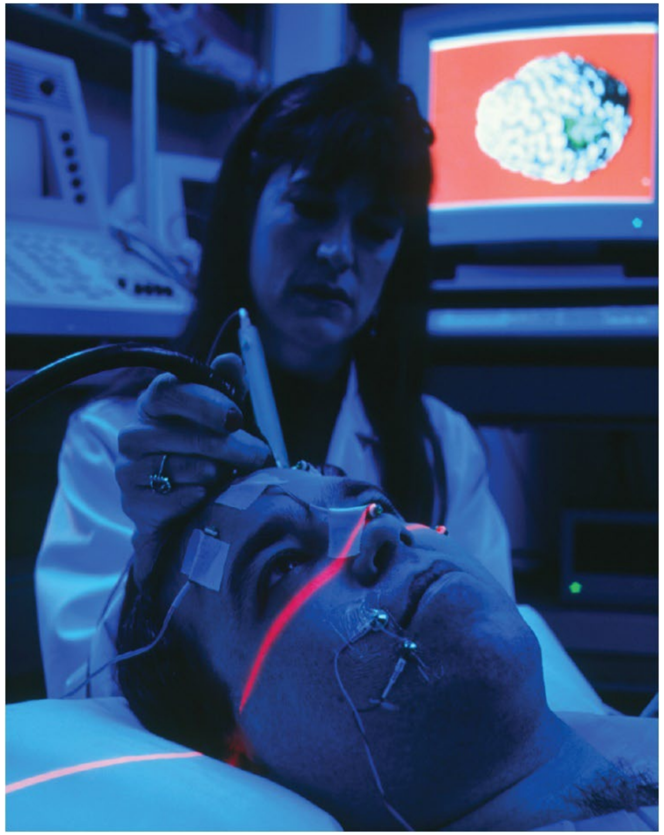


Figure 40-17 A patient's head is scanned and mapped by (red) laser light in preparation for brain surgery. During the surgery, the laser-derived image of the head will be superimposed on the model of the brain shown on the monitor, to guide the surgical team into the region shown in green on the model. (Sam Ogden/Photo Researchers)

40.12: How Lasers Work: (1 of 3)

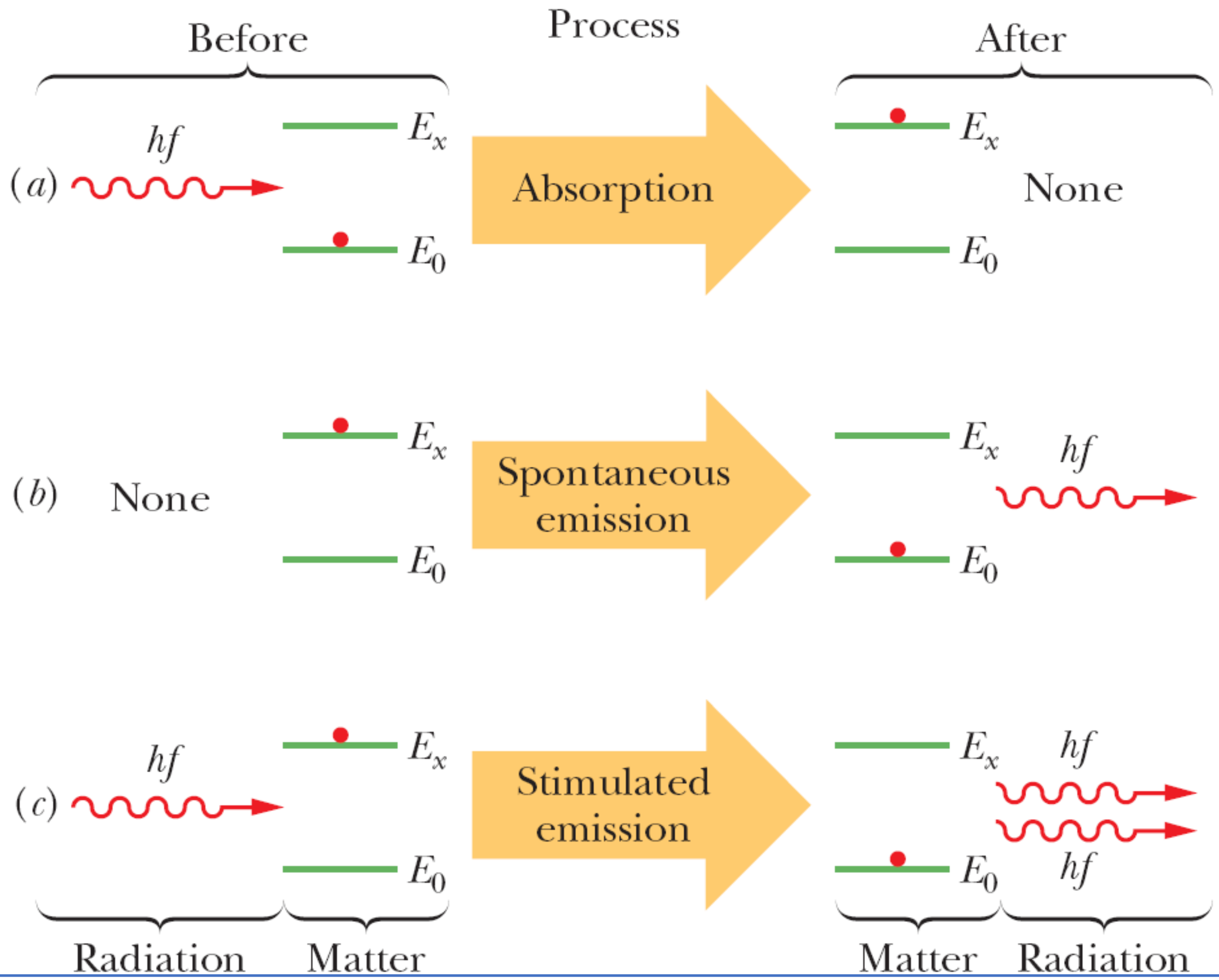


Figure 40-18 The interaction of radiation and matter in the processes of (*a*) absorption. (*b*) spontaneous emission, and (*c*) stimulated emission.

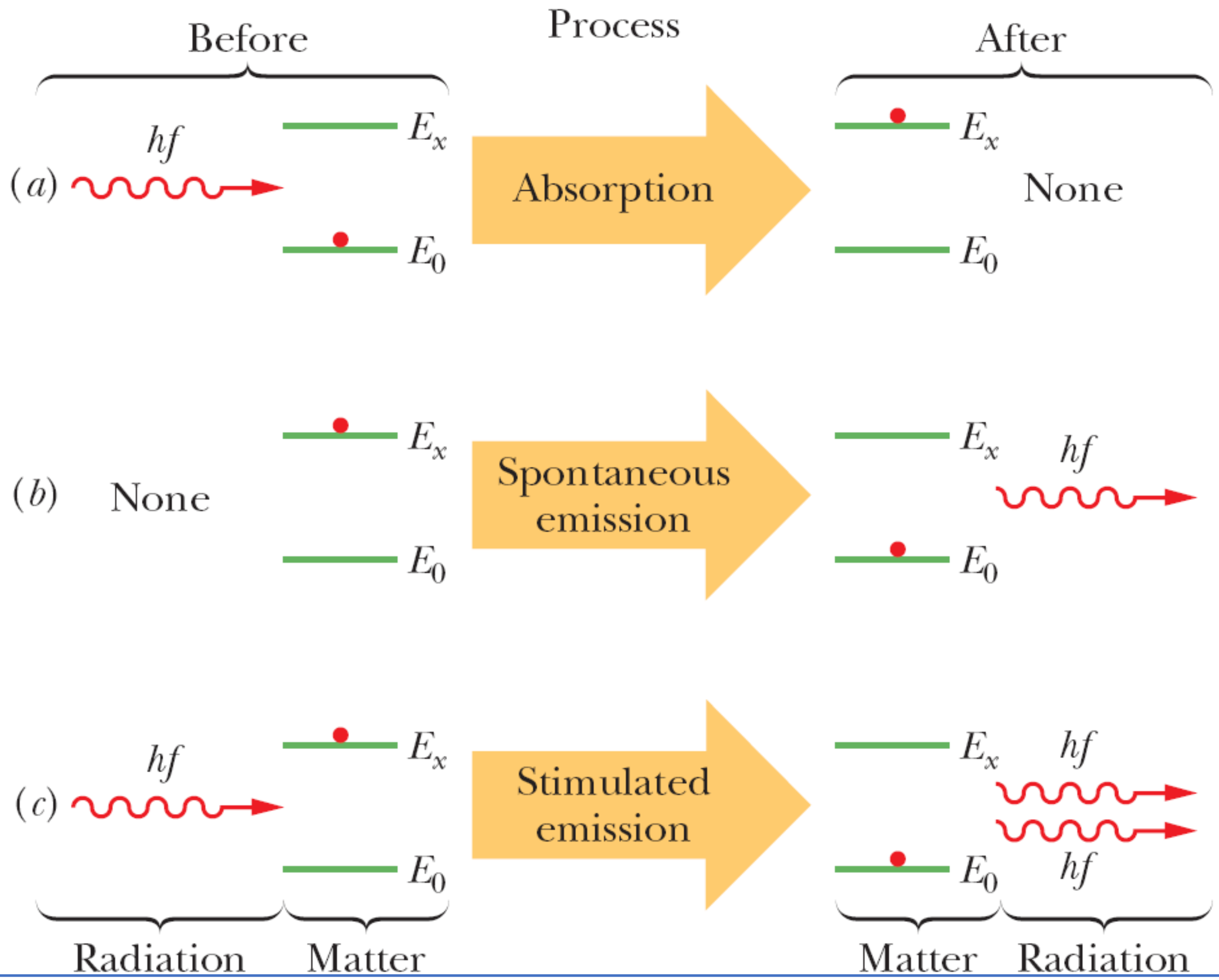
An atom (matter) is represented by the red dot; the atom is in either a lower quantum state with energy E_0 or a higher quantum state with energy E_x.

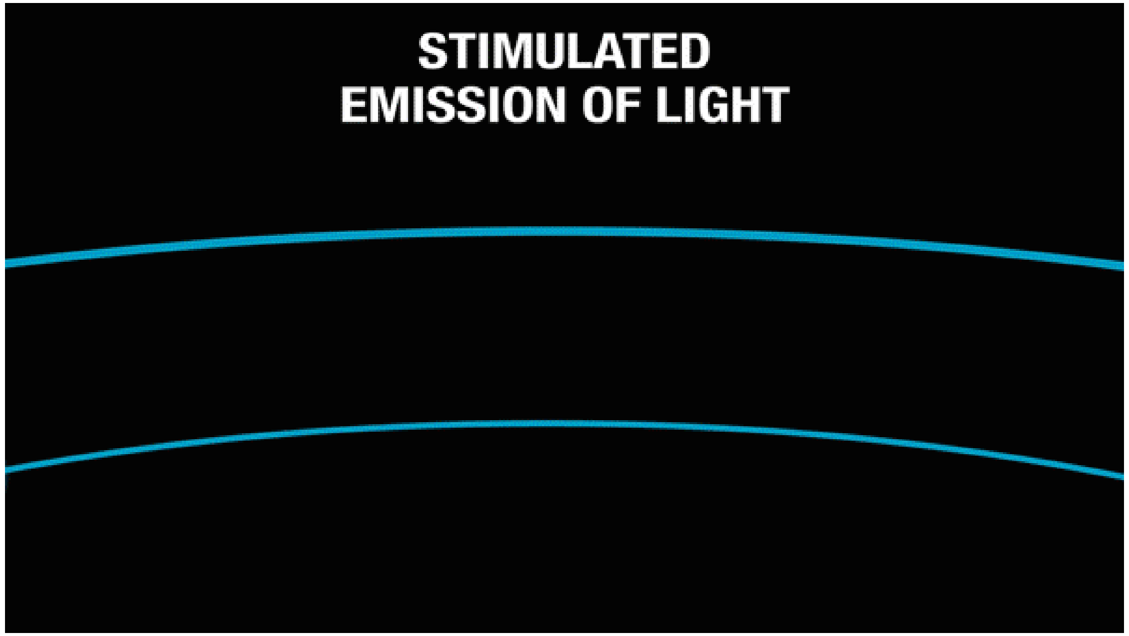
In (*a*) the atom absorbs a photon of energy *hf* from a passing light wave.

In (*b*) it emits a light wave by emitting a photon of energy *hf*.

In (*c*) a passing light wave with photon energy *hf* causes the atom to emit a photon of the same energy, increasing the energy of the light wave.

40.12: How Lasers Work: (1 of 3)





40.12: How Lasers Work: (2 of 3)

If the atoms of Figure 40-19*a* are flooded with photons of energy $(E_x - E_0)$, photons will disappear via absorption by ground-state atoms and photons will be generated largely via stimulated emission of excited-state atoms. Thus, because there are more atoms in the ground state, the net effect will be the absorption of photons.

To produce laser light, one must have more photons emitted than absorbed; that is, one must have a situation in which stimulated emission dominates. Thus, one needs more atoms in the excited state than in the ground state, as in Figure 40-19*b*.

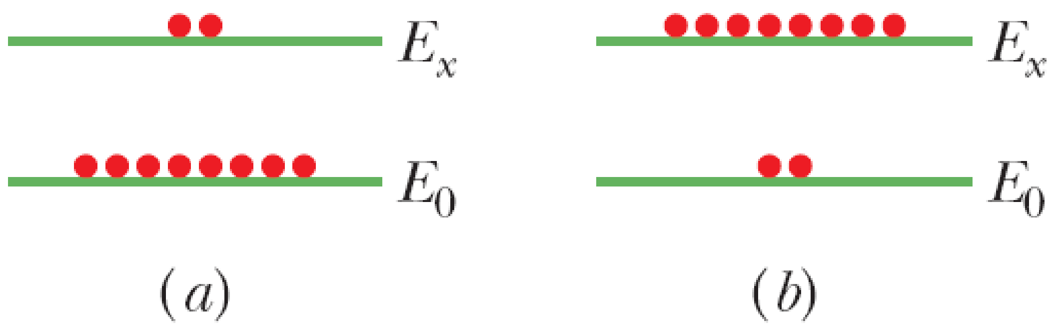
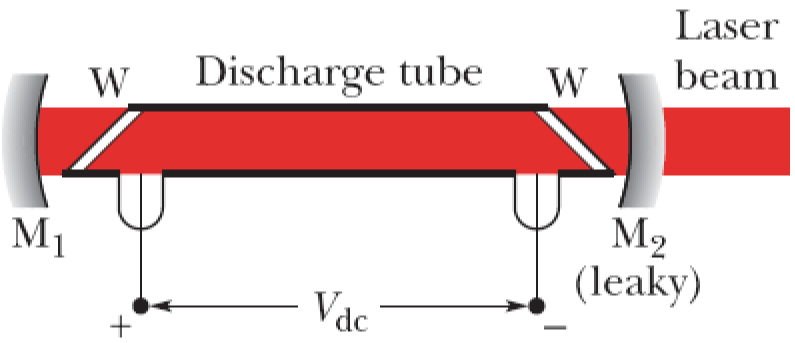


Figure 40-19 (*a*) The equilibrium distribution of atoms between the ground state E_0 and excited state E_x accounted for by thermal agitation, (*b*) An inverted population, obtained by special methods. Such a population inversion is essential for laser action.

40.12: The Helium-Neon Gas Laser: (2 of 2)



An applied potential V_{dc} sends electrons through a discharge tube containing a mixture of helium gas and neon gas. Electrons collide with helium atoms, which then collide with neon atoms, which emit light along the length of the tube. The light passes through transparent windows W and reflects back and forth through the tube from mirrors M_1 and M_2 to cause more neon atom emissions. Some of the light leaks through mirror M_2 to form the laser beam.

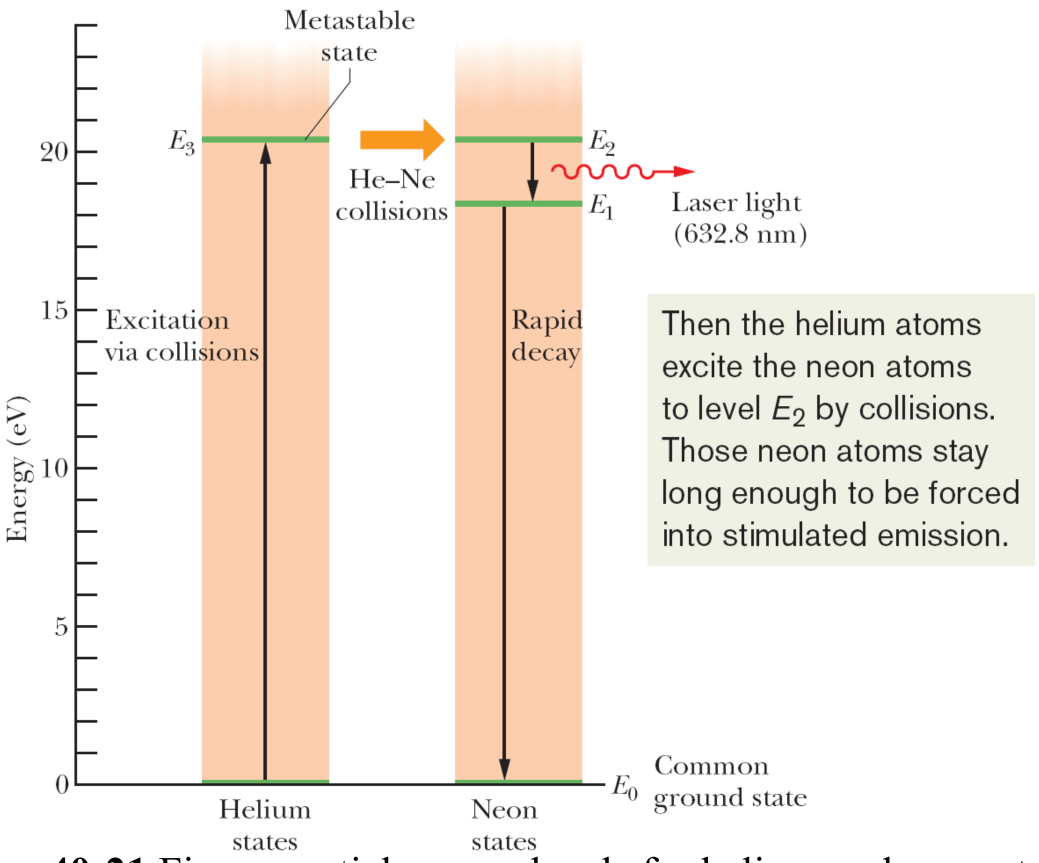


Figure 40-21 Five essential energy levels for helium and neon atoms in a helium-neon gas laser. Laser action occurs between levels E_2 and E_1 of neon when more atoms are at the E_2 level than at the E_1 level.

Review & Summary

Some Properties of Atoms Atoms have quantized energies and can make quantum jumps between them. If a jump between a higher energy and a lower energy involves the emission or absorption of a photon, the frequency associated with the light is given by

$$hf = E_{\text{high}} - E_{\text{low}}. \tag{40-1}$$

States with the same value of quantum number n form a shell. States with the same values of quantum numbers n and ℓ form a subshell.

Review & Summary

Orbital Angular Momentum and Magnetic Dipole Moments The magnitude of the orbital angular momentum of an electron trapped in an atom has quantized values given by

$$L = \sqrt{\ell(\ell+1)}\,\hbar, \qquad \text{for } \ell = 0, 1, 2, \ldots, (n-1), \tag{40-2}$$

where $\hbar$ is $h/2\pi$, ℓ is the orbital magnetic quantum number, and n is the electron's principal quantum number. The component L_z of the orbital angular momentum on a z axis is quantized and given by

$$L_z = m_\ell \hbar, \quad \text{for } m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell, \tag{40-3}$$

where m_ℓ is the orbital magnetic quantum number. The magnitude μ_{orb} of the orbital magnetic moment of the electron is quantized with the values given by

$$\mu_{orb} = \frac{e}{2m}\sqrt{\ell(\ell+1)}\,\hbar, \tag{40-6}$$

where m is the electron mass. The component $\mu_{orb,z}$ on a z axis is also quantized according to

$$\mu_{orb,z} = -\frac{e}{2m} m_\ell \hbar = -m_\ell \mu_B, \tag{40-7}$$

where μ_B is the Bohr magneton:

$$\mu_B = \frac{eh}{4\pi m} = \frac{e\hbar}{2m} = 9.274 \times 10^{-24}\ \text{J/T}. \tag{40-8}$$

Review & Summary

Spin Angular Momentum and Magnetic Dipole Moment Every electron, whether trapped or free, has an intrinsic spin angular momentum $\vec{S}$ with a magnitude that is quantized as

$$S = \sqrt{s(s+1)}\,\hbar, \quad \text{for } s = \tfrac{1}{2}, \tag{40-9}$$

where s is the spin quantum number. An electron is said to be a spin-$\frac{1}{2}$ particle. The component S_z on a z axis is also quantized according to

$$S_z = m_s\hbar, \quad \text{for } m_s = \pm s = \pm\tfrac{1}{2}, \tag{40-10}$$

where m_s is the spin magnetic quantum number. Every electron, whether trapped or free, has an intrinsic spin magnetic dipole moment $\vec{\mu}_s$ with a magnitude that is quantized as

$$\mu_s = \frac{e}{m}\sqrt{s(s+1)}\,\hbar, \quad \text{for } s = \tfrac{1}{2}. \tag{40-12}$$

The component $\mu_{s,z}$ on a z axis is also quantized according to

$$\mu_{s,z} = -2m_s\mu_B, \quad \text{for } m_s = \pm\tfrac{1}{2}. \tag{40-13}$$

Review & Summary

Magnetic Resonance The magnetic dipole moment of a proton in a magnetic field $\vec{B}$ along a z axis has two quantized components on that axis: spin up (μ_z is in the direction $\vec{B}$) and spin down (μ_z is in the opposite direction). Contrary to the situation with an electron, spin up is the lower energy orientation; the difference between the two orientations is $2\mu_z B$. The energy required of a photon to spin-flip the proton between the two orientations is

$$hf = 2\mu_z B. \tag{40-22}$$

The field is the vector sum of an external field set up by equipment and an internal field set up by the atoms and nuclei surrounding the proton. Detection of spin-flips can lead to nuclear magnetic resonance spectra by which specific substances can be identified.

Pauli Exclusion Principle Electrons in atoms and other traps obey the Pauli exclusion principle, which requires that no two electrons in a trap can have the same set of quantum numbers.

Building the Periodic Table In the periodic table, the elements are listed in order of increasing atomic number Z, where Z is the number of protons in the nucleus. For a neutral atom, Z is also the number of electrons. States with the same value of quantum number n form a shell. States with the same values of quantum numbers n and ℓ form a subshell. A closed shell and a closed subshell contain the maximum number of electrons as allowed by the Pauli exclusion principle. The net angular momentum and net magnetic moment of such closed structures is zero.

Review & Summary

X Rays and the Numbering of the Elements When a beam of high-energy electrons impacts a target, the electrons can lose their energy by emitting x rays when they scatter from atoms in the target. The emission is over a range of wavelengths, said to be a continuous spectrum. The shortest wavelength in the spectrum is the cutoff wavelength $\lambda_{\min}$, which is emitted when an incident electron loses its full kinetic energy K_0 in a single scattering event, with a single x-ray emission:

$$\lambda_{\min} = \frac{hc}{K_0}.$$

The characteristic x-ray spectrum is produced when incident electrons eject low-lying electrons in the target atoms and electrons from upper levels jump down to the resulting holes, emitting light. A Moseley plot is a graph of the square root of the characteristic-emission frequencies $\sqrt{f}$ versus atomic number Z of the target atoms. The straight-line plot reveals that the position of an element in the periodic table is set by Z and not by the atomic weight.

Lasers In stimulated emission, an atom in an excited state can be induced to de-excite to a lower energy state by emitting a photon if an identical photon passes the atom. The light emitted in stimulated emission is in phase with and travels in the direction of the light causing the emission.

A laser can emit light via stimulated emission provided that its atoms are in population inversion. That is, for the pair of levels involved in the stimulated emission, more atoms must be in the upper level than the lower level so that there is more stimulated emission than just absorption.