UFUG 1504: Honors General Physics II

Chapter 29

Magnetic Fields due to Currents

Summary (1 of 5)

The Biot-Savart Law

- The magnetic field set up by a current- carrying conductor can be found from the Biot–Savart law.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$$

Equation (29-3)

- The quantity μ_0, called the permeability constant, has the value

$$4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\approx1.26\times10^{-6}\,\mathrm{T\cdot m/A}.$$

Summary (2 of 5)

Magnetic Field of a Long Straight Wire

- For a long straight wire carrying a current i, the Biot–Savart law gives,

$$B = \frac{\mu_0 i}{2\pi R}$$ **Equation (29-4)**

Magnetic Field of a Circular Arc

- The magnitude of the magnetic field at the center of a circular arc,

$$B = \frac{\mu_0 i\phi}{4\pi R}$$ **Equation (29-9)**

Summary (3 of 5)

Force Between Parallel Currents

- The magnitude of the force on a length L of either wire is

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}$$ **Equation (29-13)**

Ampere’s Law

- Ampere’s law states that,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$ **Equation (29-14)**

Summary (4 of 5)

Fields of a Solenoid and a Toroid

- Inside a long solenoid carrying current i, at points not near its ends, the magnitude B of the magnetic field is

$$B = \mu_0 i n$$

Equation (29-23)

- At a point inside a toroid, the magnitude B of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r}$$

Equation (29-24)

Summary (5 of 5)

Field of a Magnetic Dipole

- The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point *P* located a distance *z* along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$$

Equation (29-9)

29-1 Magnetic Field due to a Current (5 of 8)

The magnitude of the field $d\vec{B}$ produced at point P at distance r by a current-length element $d\vec{s}$ turns out to be

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

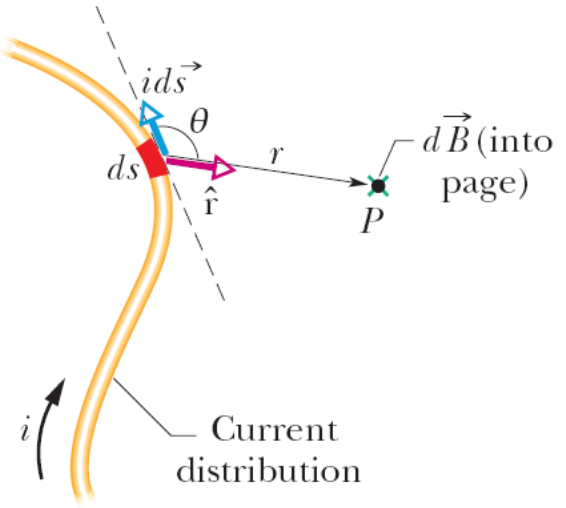
where θ is the angle between the directions of $d\vec{s}$ and $\hat{r}$,

a unit vector that points from $d\vec{s}$ toward P.

μ_0 is a constant, called the permeability constant, whose value is defined to be exactly

$$\mu_0 = 4\pi\times10^{-7}\,\mathrm{T\cdot m/A} \approx 1.26\times10^{-6}\,\mathrm{T\cdot m/A}.$$

This element of current creates a magnetic field at P, into the page.



A current-length element $i\,ds$ produces a differential magnetic field dB at point P. The green × (the tail of an arrow) at the dot for point P indicates that dB is directed *into* the page there.

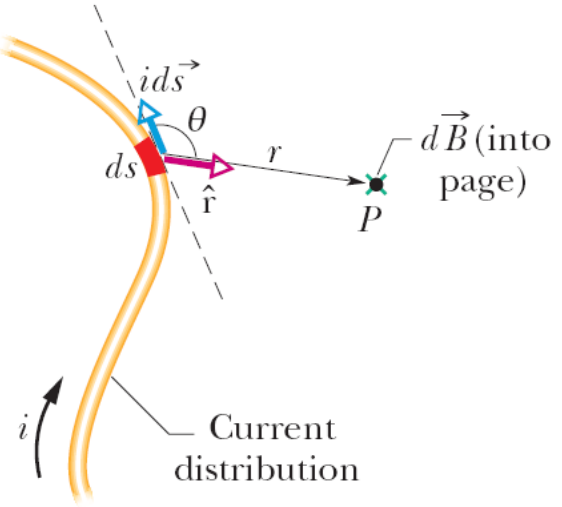
29-1 Magnetic Field due to a Current (6 of 8)

The direction of $d\vec{B}$, shown as being into the page in the figure, is that of the cross product $d\vec{s} \times \hat{r}$. We can therefore write the above equation containing $d\vec{B}$ in vector form as

(Biot–Savart law)
毕奥-萨伐尔定律

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

This element of current creates a magnetic field at *P*, into the page.



A current-length element *i ds* produces a differential magnetic field *dB* at point *P*. The green × (the tail of an arrow) at the dot for point *P* indicates that *dB* is directed *into* the page there.

29-1 Magnetic Field due to a Current

For a **long straight wire** carrying a current i, the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance R from the wire,

$$B = \frac{\mu_0 i}{2\pi R}$$

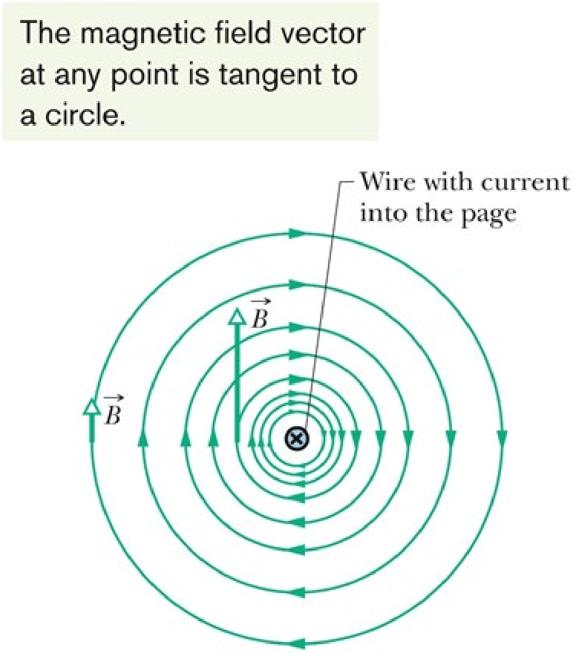
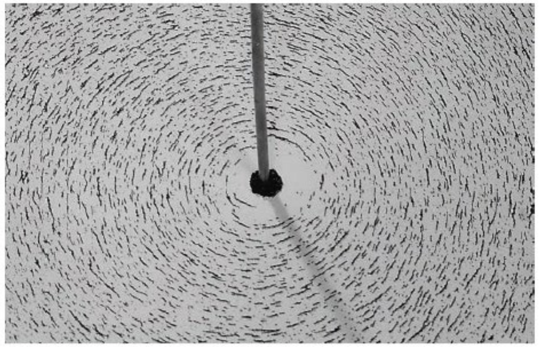
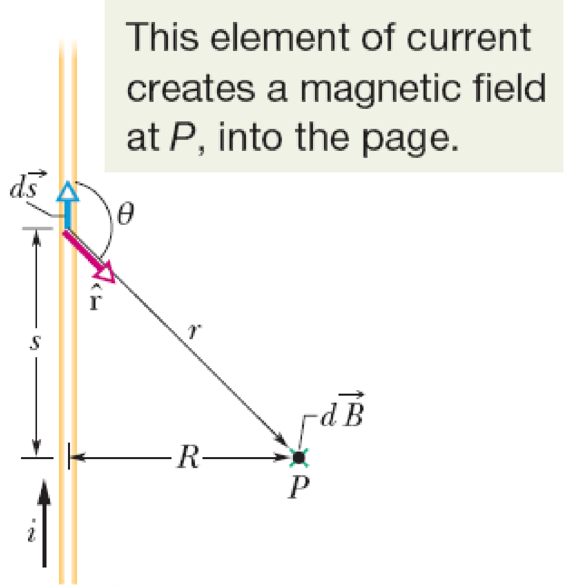


Figure: The magnetic field lines produced by a current in along straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the ×.

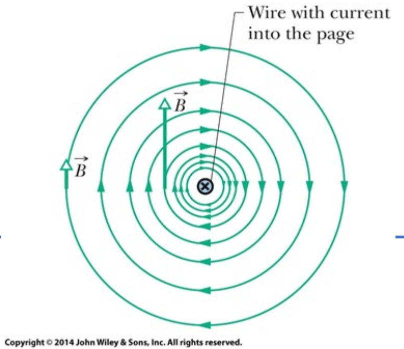


Iron filings that have been sprinkled onto cardboard collect in concentric circles when current is sent through the central wire. The alignment, which is along magnetic field lines, is caused by the magnetic field produced by the current.

29-1 Magnetic Field due to a Current (7 of 8)



The magnetic field vector at any point is tangent to a circle.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}.$$

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta\,ds}{r^2}$$

$$\Rightarrow$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R\,ds}{(s^2+R^2)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi R}\left[\frac{s}{(s^2+R^2)^{1/2}}\right]_0^\infty = \frac{\mu_0 i}{2\pi R},$$

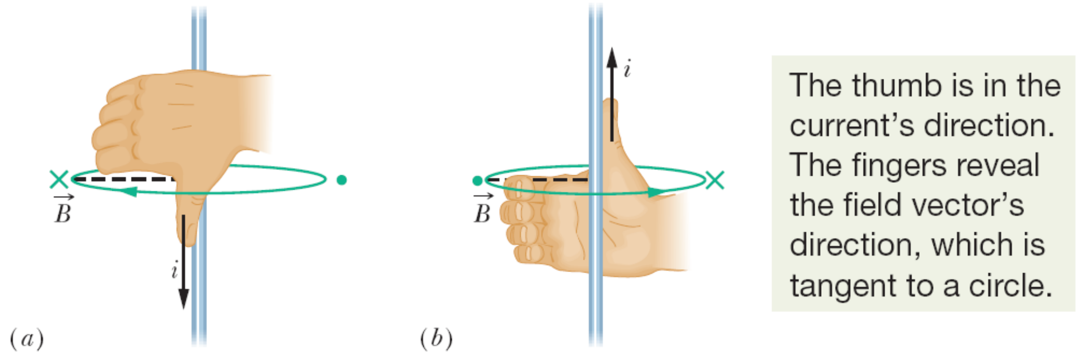
The variables θ, s, and r in this equation are not independent

$$r = \sqrt{s^2+R^2}$$

$$\sin\theta = \sin(\pi-\theta) = \frac{R}{\sqrt{s^2+R^2}}$$

29-1 Magnetic Field due to a Current (8 of 8)

Curled–straight right-hand rule (弯曲右手法则): Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.



A right-hand rule gives the direction of the magnetic field due to a current in a wire.

29-1 Magnetic Field due to a Current (8 of 8)

Magnetic Field Due to a Current in a Circular Arc of Wire (圆弧导线)

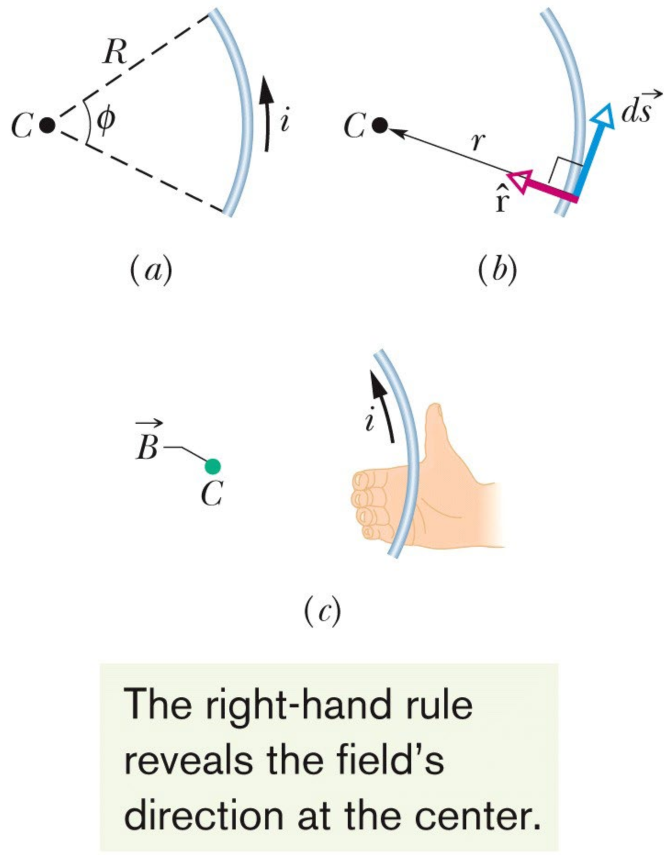
The magnitude of the **magnetic field at the center of a circular arc**

Of radius R and central angle ϕ (in radians), carrying current i, is

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

$$B = \int dB = \int_0^{\phi} \frac{\mu_0}{4\pi} \frac{iR\,d\phi}{R^2} = \frac{\mu_0 i}{4\pi R} \int_0^{\phi} d\phi.$$

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds \sin\theta}{r^2},$$



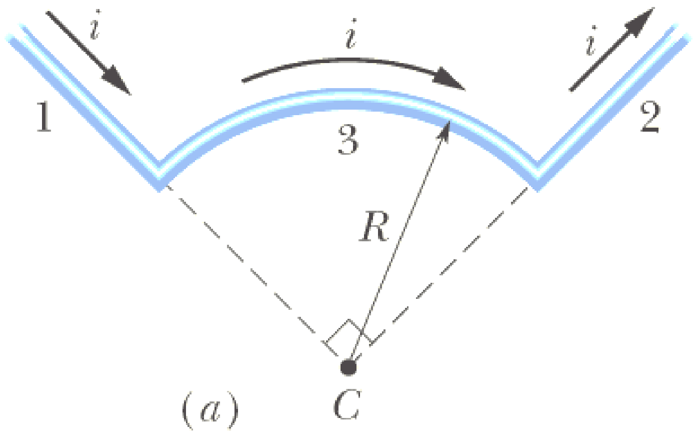
(a) A wire in the shape of a circular arc with center C carries current i. (b) For any element of wire along the arc, the angle between the directions of ds and r̂ is 90°. (c) Determining the direction of the magnetic field at the center C due to the current in the wire; the field is out of the page, in the direction of the fingertips, as indicated by the colored dot at C.

29-1 Magnetic Field due to a Current (8 of 8)

The magnitude of the **magnetic field at the center of a circular arc**

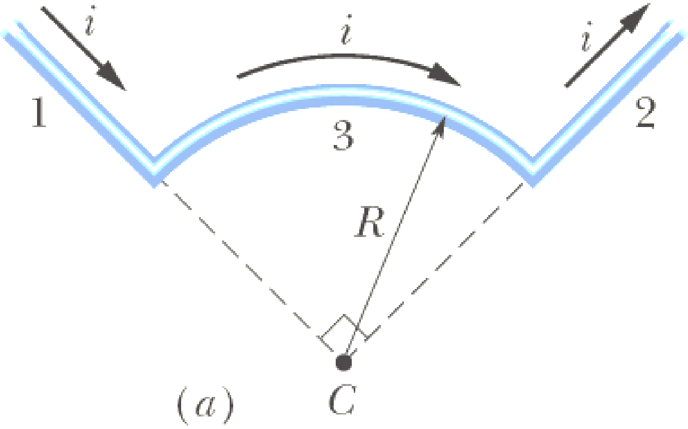
$$B = \frac{\mu_0 i \phi}{4\pi R} \xrightarrow{\text{at center of full circle}} B = \frac{\mu_0 i (2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}$$

29-1 Magnetic Field due to a Current



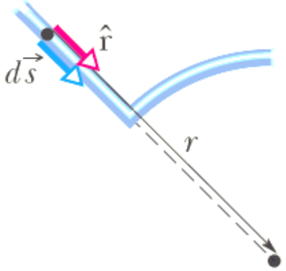
The wire in Figure carries a current i and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field B (magnitude and direction) does the current produce at C?

29-1 Magnetic Field due to a Current (8 of 8)



B separately for the three distinguishable sections of the wire

(1) the straight section at the left,
(2) the straight section at the right
(3) the circular arc.



the angle θ between ds and r̂ is zero

$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0.$$

$$B_1 = B_2 = 0.$$

$$B = \frac{\mu_0 i\phi}{4\pi R} \qquad B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}$$

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}$$

The direction of B is into the plane

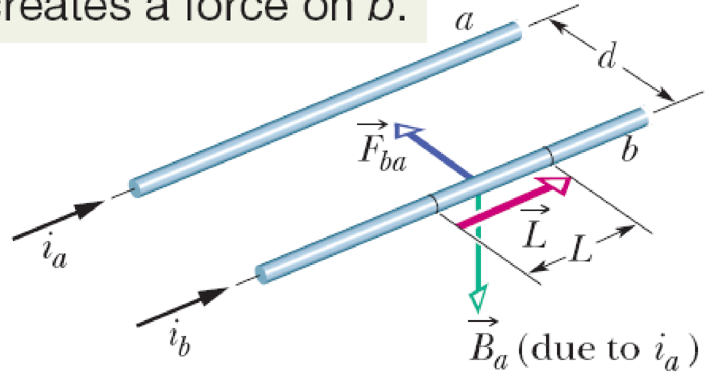
29-2 Force Between Two Parallel Currents (2 of 4)

Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length L of either wire is

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a \qquad F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d},$$

where d is the wire separation, and i_a and i_b are the currents in the wires.

The field due to *a* at the position of *b* creates a force on *b*.

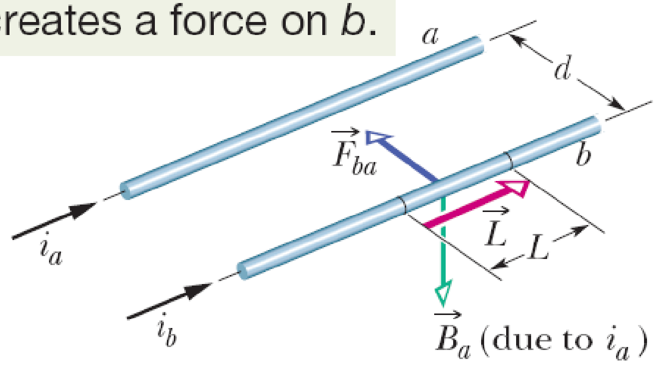


29-2 Force Between Two Parallel Currents (3 of 4)

The general procedure for finding the force on a current-carrying wire is this:

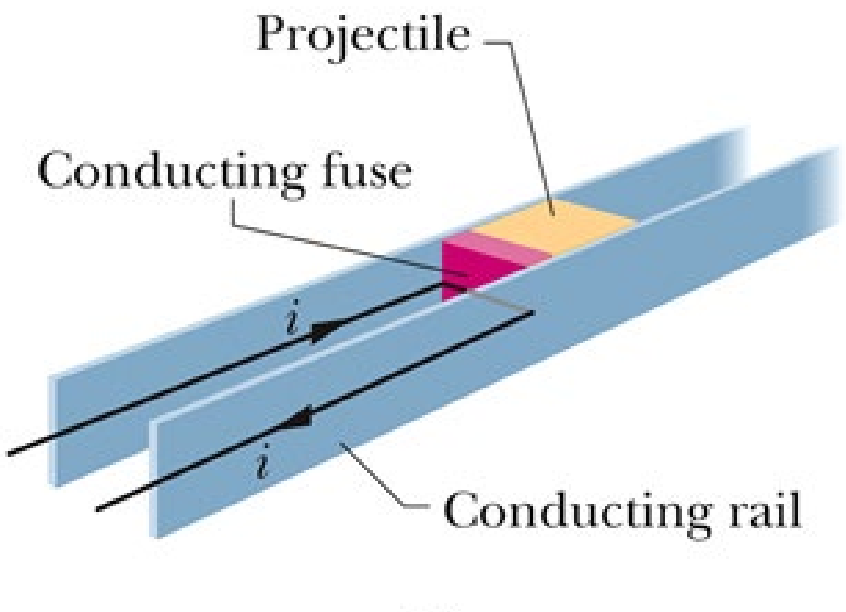
To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

The field due to a at the position of b creates a force on b.

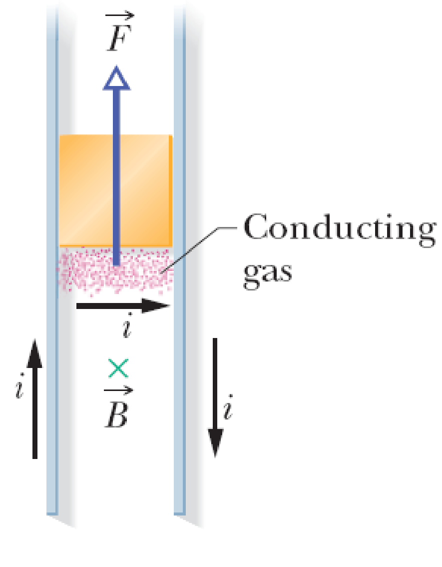


Similarly, if the two currents were anti-parallel (反平行的), we could show that the two wires repel each other.

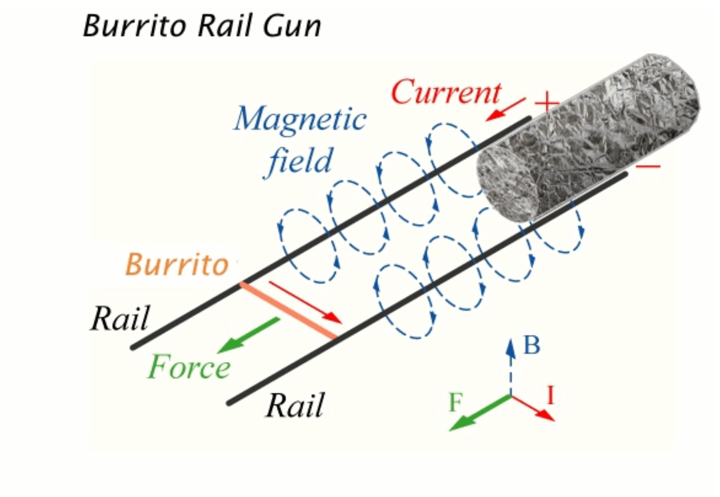
29-2 Force Between Two Parallel Currents (4 of 4)



(a)



(b)



A rail gun, as a current i is set up in it. The current rapidly causes the conducting fuse to vaporize.

Someday rail guns may be used to launch materials into space from mining operations on the Moon or an asteroid.

29-3 Ampere's Law (3 of 5)

if the distribution has some symmetry, we may be able to apply **Ampere's law (安培环路定律)** to find the magnetic field with considerably less effort.

Ampere's law states that

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

The line integral in this equation is evaluated around a closed loop called an Amperian loop. The current *i* on the right side is the net current encircled by the loop

29-3 Ampere's Law (4 of 5)

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

Only the currents encircled by the loop are used in Ampere's law.

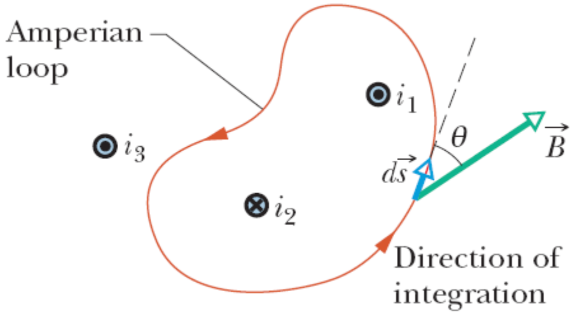
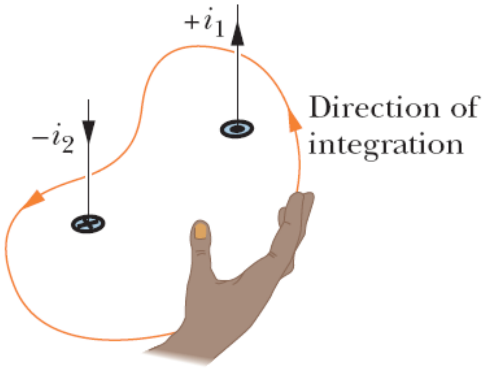


Figure 29-12 Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

This is how to assign a sign to a current used in Ampere's law.



igure 29-13 A right-hand rule for ımpere's law, to determine the signs for urrents encircled by an Amperian loop. 'he situation is that of Fig. 29-12.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \xrightarrow{i_{enc} = i_1 - i_2} \oint B \cos\theta \, ds = \mu_0 (i_1 - i_2).$$

29-3 Ampere's Law (4 of 5)

29-1 Magnetic Field due to a Current

For a **long straight wire** carrying a current i, the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance R from the wire,

$$B = \frac{\mu_0 i}{2\pi R}$$

Lets use Ampere's law to prove it!

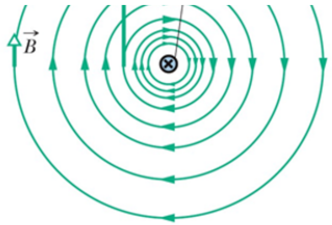


Figure: The magnetic field lines produced by a current in along straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the ×.

Courtesy Education Development Center

Iron filings that have been sprinkled onto cardboard collect in concentric circles when current is sent through the central wire. The alignment, which is along magnetic field lines, is caused by the magnetic field produced by the current.

29-3 Ampere's Law (4 of 5)

Magnetic Fields of a long straight wire with current:

$$B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straigth wire).}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

B and ds are either parallel or antiparallel at each point of the loop, so at every point the angle θ between ds and B is 0°, so $\cos\theta = \cos 0° = 1$.

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i$$

All of the current is encircled and thus all is used in Ampere's law.

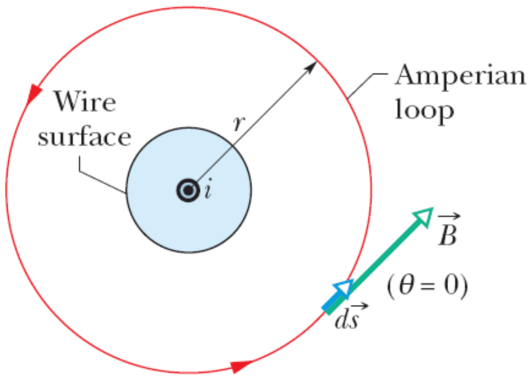


Figure 29-14 Using Ampere's law to find the magnetic field that a current i produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

29-3 Ampere's Law (4 of 5)

Magnetic Fields of a long straight wire with current:

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \quad \text{(inside straight wire).}$$

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r).$$

$$i_{\text{enc}} = i\,\frac{\pi r^2}{\pi R^2}.$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \Rightarrow B(2\pi r) = \mu_0 i\,\frac{\pi r^2}{\pi R^2}$$

Only the current encircled by the loop is used in Ampere's law.

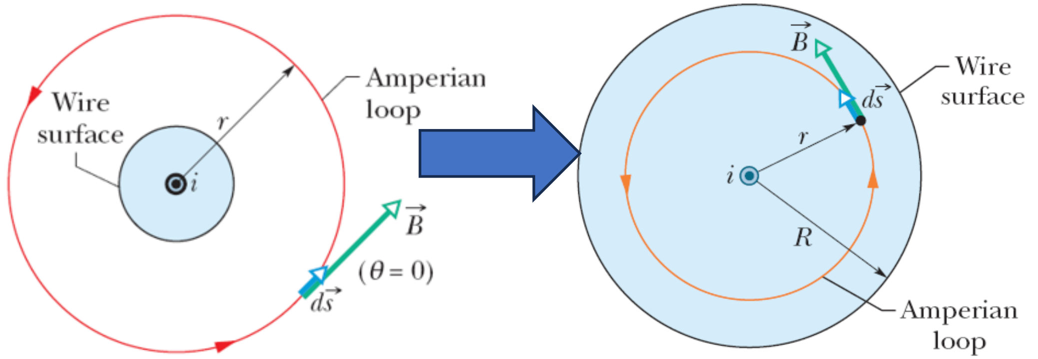
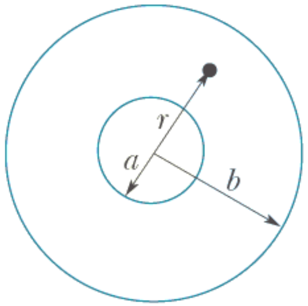


Figure 29-15 Using Ampere's law to find the magnetic field that a current i produces inside a long straight wire of circula[r] cross section. The current is uniformly dis[-]tributed over the cross section of the wire and emerges from the page. An Amperia[n] loop is drawn inside the wire.

29-3 Ampere's Law (4 of 5)

Figure 29-16*a* shows the cross section of a long conducting cylinder with inner radius $a = 2.0$ cm and outer radius $b = 4.0$ cm. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $J = cr^2$, with $c = 3.0 \times 10^6$ A/m^4 and r in meters. What is the magnetic field $\vec{B}$ at the dot in Fig. 29-16*a*, which is at radius $r = 3.0$ cm from the central axis of the cylinder?

We want the magnetic field at the dot at radius *r*.



(*a*)

29-3 Ampere's Law (4 of 5)

Figure 29-16*a* shows the cross section of a long conducting cylinder with inner radius $a = 2.0$ cm and outer radius $b = 4.0$ cm. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $J = cr^2$, with $c = 3.0 \times 10^6$ A/m^4 and r in meters. What is the magnetic field $\vec{B}$ at the dot in Fig. 29-16*a*, which is at radius $r = 3.0$ cm from the central axis of the cylinder?

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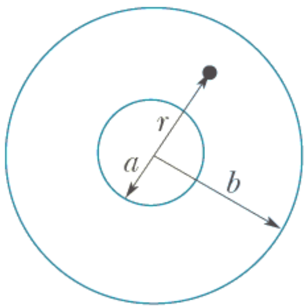
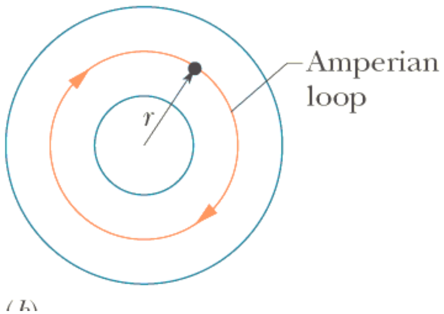


(*a*)

So, we put a concentric Amperian loop through the dot.



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

$$i_{\text{enc}} = \int J\, dA = \int_a^r cr^2 (2\pi r\, dr)$$

$$= 2\pi c \int_a^r r^3\, dr = 2\pi c \left[\frac{r^4}{4}\right]_a^r$$

$$= \frac{\pi c (r^4 - a^4)}{2}.$$

$$B(2\pi r) = -\frac{\mu_0 \pi c}{2}(r^4 - a^4)$$

$$B = -\frac{\mu_0\, c}{4r}(r^4 - a^4)$$

$$= -\frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(3.0 \times 10^6\ \text{A/m}^4)}{4(0.030\ \text{m})}$$

$$\times [(0.030\ \text{m})^4 - (0.020\ \text{m})^4]$$

$$= -2.0 \times 10^{-5}\ \text{T}.$$

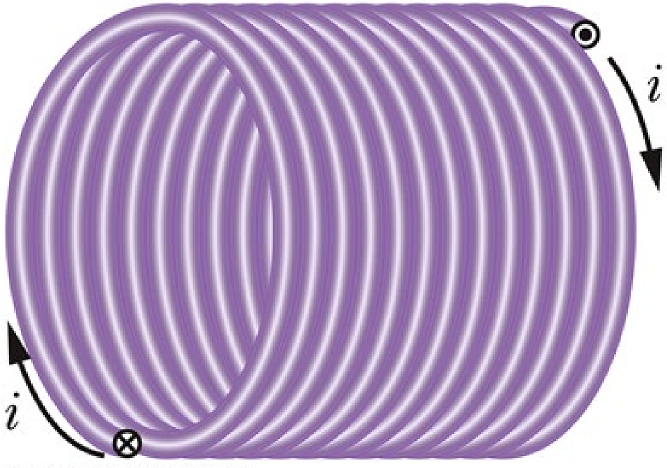
Direction is counterclockwise

29-4 Solenoids and Toroids (螺旋管和环形管)

Magnetic Field of a Solenoid

Figure (a) is a solenoid carrying current *i*.

(a)



(b)

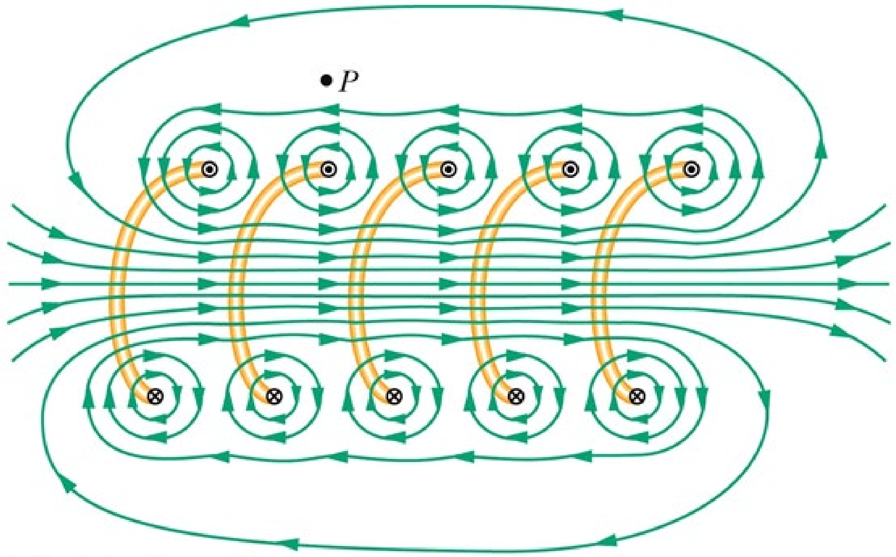


Figure (b) shows a section through a portion of a "stretched-out" solenoid. The solenoid's magnetic field is the vector sum of the fields produced by the individual turns (windings) that make up the solenoid. For points very close to a turn, the wire behaves magnetically almost like a long straight wire, and the lines of B there are almost concentric circles (同心圆). Figure (b) suggests that the field tends to cancel between adjacent turns. It also suggests that, at points inside the solenoid and reasonably far from the wire, B is approximately parallel to the (central) solenoid axis.

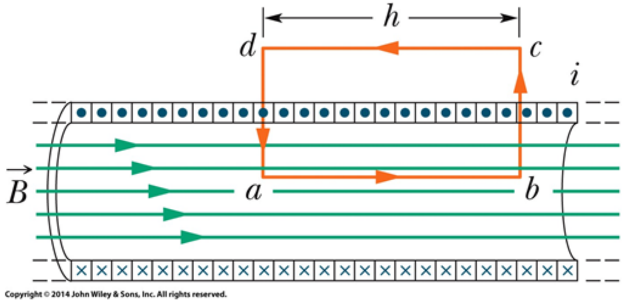
29-4 Solenoids and Toroids (5 of 8)

Magnetic Field of a Solenoid

Let us now apply Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}},$$

to the ideal solenoid of Fig. (a), where B is uniform within the solenoid and zero outside it, using the rectangular Amperian loop abcda



(a)

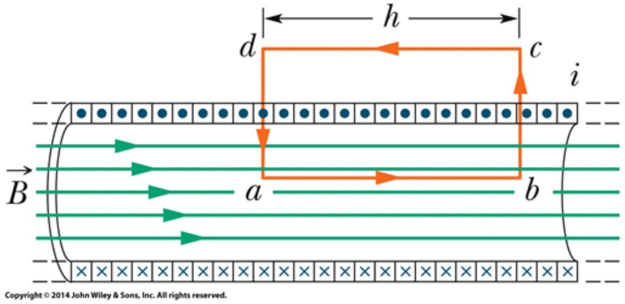
We write $\oint \vec{B} \cdot d\vec{s}$ as the sum of four integrals, one for each loop segment:

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_a^b \vec{B} \cdot d\vec{s}.$$

29-4 Solenoids and Toroids (6 of 8)

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_a^b \vec{B} \cdot d\vec{s}.$$

The first integral on the right of equation is *Bh*, where *B* is the magnitude of the uniform field B inside the solenoid and *h* is the (arbitrary) length of the segment from *a* to *b*. The second and fourth integrals are zero because for every element *ds* of these segments, B either is perpendicular to *ds* or is zero, and thus the product $\vec{B} \cdot d\vec{s}$ is zero. The third integral, which is taken along a segment points. Thus, $\oint \vec{B} \cdot d\vec{s}$ for the entire rectangular loop has the value Bh.



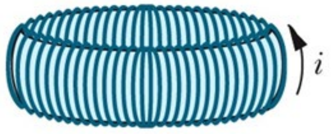
Inside a long solenoid carrying current i, at points not near its ends, the magnitude *B* of the magnetic field is

$$i_{enc} = i(nh)$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad B = \mu_0 in \quad \text{(ideal solenoid)}.$$

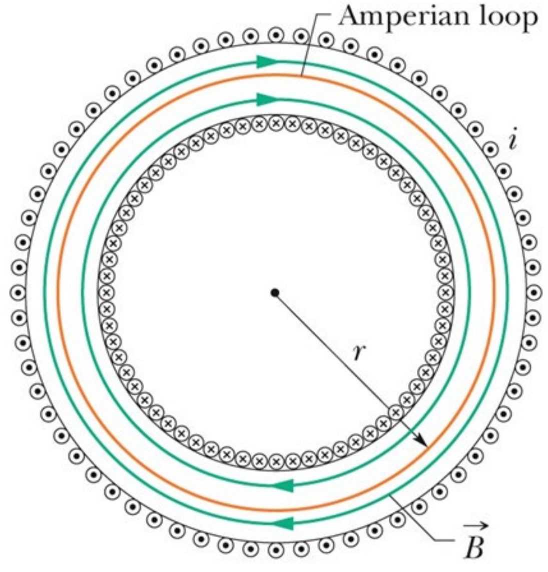
29-4 Solenoids and Toroids (7 of 8)

Magnetic Field of a Toroid (环形管)



(a)

Figure (a) shows a toroid, which we may describe as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow (空心的) bracelet. We can find out from Ampere's law and the symmetry of the bracelet. From the symmetry, we see that the lines of B form concentric circles inside the toroid, directed as shown in Fig. (b). Let us choose a concentric circle of radius *r* as an Amperian loop and traverse it in the clockwise direction. Ampere's law yields



(b)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}, \quad (B)(2\pi r) = \mu_0 i N,$$

29-4 Solenoids and Toroids (8 of 8)

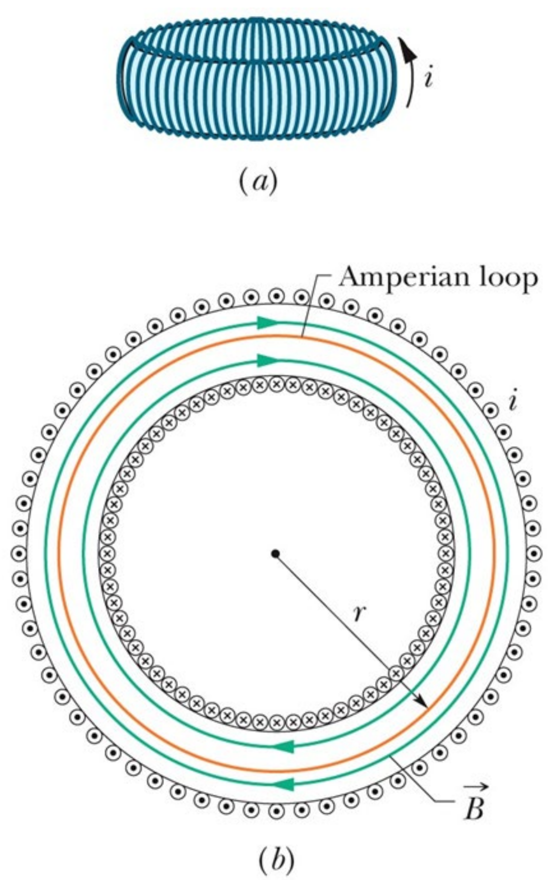
$$(B)(2\pi r) = \mu_0 iN,$$

where i is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and N is the total number of turns. This gives

$$B = \frac{\mu_0 iN}{2\pi}\frac{1}{r} \quad (\text{toroid}).$$

In contrast to the situation for a solenoid, B is **not constant** over the cross section of a toroid.

$$B = \mu_0 in \quad (\text{ideal solenoid}).$$

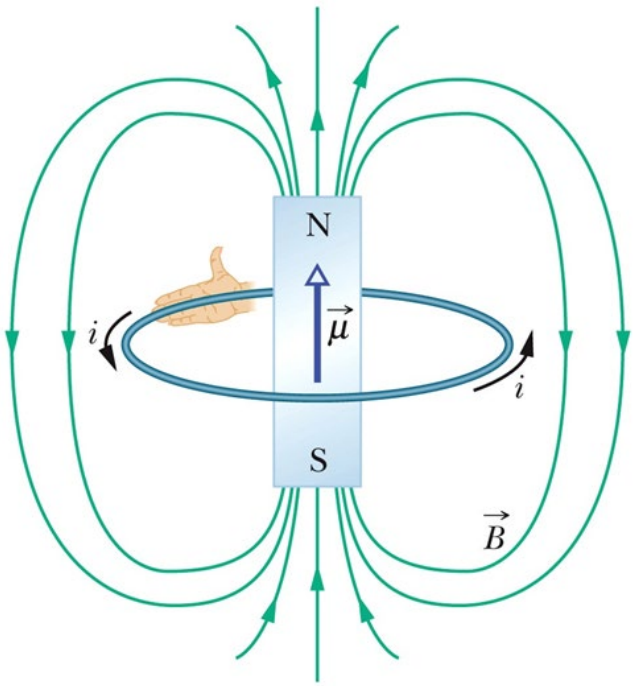


29-5 A Current-Carrying Coil as a Magnetic Dipole (2 of 4)

The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point *P* located a distance *z* along the coil's perpendicular central axis is parallel to the axis and is given by

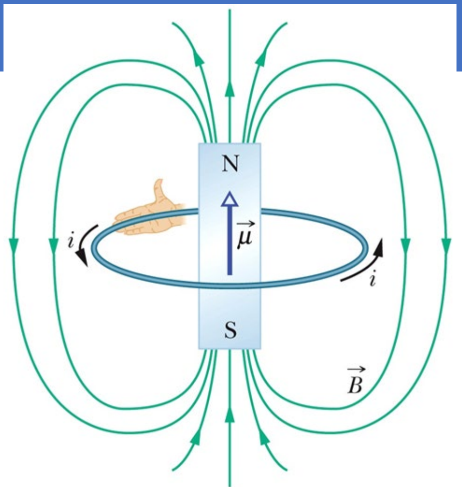
$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \quad \text{(current-carrying coil).}$$

Here μ is the dipole moment of the coil. This equation applies only when *z* is much greater than the dimensions of the coil.



A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles.

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \quad \text{(current-carrying coil).}$$



The problem does not have enough symmetry to make Ampere’s law useful; so we must turn to the law of Biot and Savart.

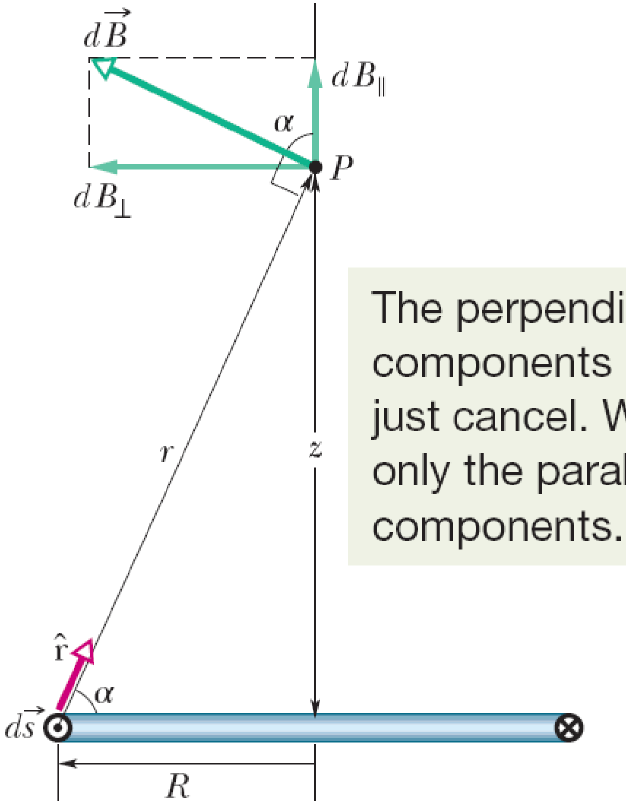
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i\, ds \sin 90^\circ}{r^2}. \qquad dB_{\parallel} = dB \cos\alpha$$

$$dB_{\parallel} = \frac{\mu_0\, i \cos\alpha\, ds}{4\pi r^2}$$

$$\cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}. \qquad r = \sqrt{R^2 + z^2}$$

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}}\, ds$$



The perpendicular components just cancel. We add only the parallel components.

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi(R^2 + z^2)^{3/2}}\, ds$$

$$B = \int dB_{\parallel}$$

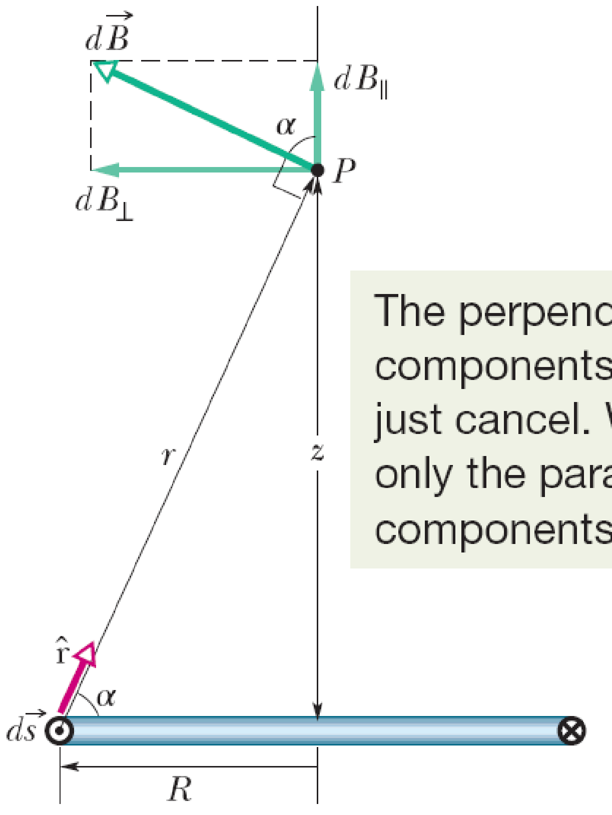
$$= \frac{\mu_0\, iR}{4\pi(R^2 + z^2)^{3/2}} \int ds \qquad B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.$$

$$\text{If } z \gg R \qquad B(z) \approx \frac{\mu_0 i R^2}{2z^3}$$

$$B(z) = \frac{\mu_0}{2\pi}\frac{NiA}{z^3} \qquad (A = 2\pi R)$$

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \quad \text{(current-carrying coil).}$$

$$\mu = NiA:$$



The perpendicular components just cancel. We add only the parallel components.

29-5 A Current-Carrying Coil as a Magnetic Dipole (4 of 4)

We have two ways in which we can regard **a current-carrying coil as a magnetic dipole**:

1. It experiences a torque when we place it in an external magnetic field.
2. It generates its own intrinsic magnetic field, given, for distant points along its axis, by the above equation. Figure shows the magnetic field of a current loop; one side of the loop acts as a north pole (in the direction of μ)

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \quad \text{(current-carrying coil)}.$$

Summary (1 of 5)

The Biot-Savart Law

- The magnetic field set up by a current- carrying conductor can be found from the Biot–Savart law.

$$d\vec{B} = \frac{\mu_0}{4} \frac{i d\vec{s} \times \hat{r}}{r^2}$$ **Equation (29-3)**

- The quantity μ_0, called the permeability constant, has the value

$$4\pi \times 10^{-7}\,\mathrm{T \cdot m/A} \approx 1.26 \times 10^{-6}\,\mathrm{T \cdot m/A}.$$

Summary (2 of 5)

Magnetic Field of a Long Straight Wire

- For a long straight wire carrying a current i, the Biot–Savart law gives,

$$B = \frac{\mu_0 i}{2\pi R}$$

Equation (29-4)

Magnetic Field of a Circular Arc

- The magnitude of the magnetic field at the center of a circular arc,

$$B = \frac{\mu_0 i\phi}{4\pi R}$$

Equation (29-9)

Summary (3 of 5)

Force Between Parallel Currents

- The magnitude of the force on a length L of either wire is

$$F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}$$

Equation (29-13)

Ampere's Law

- Ampere's law states that,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

Equation (29-14)

Summary (4 of 5)

Fields of a Solenoid and a Toroid

- Inside a long solenoid carrying current i, at points not near its ends, the magnitude B of the magnetic field is

$$B = \mu_0 in$$

Equation (29-23)

- At a point inside a toroid, the magnitude B of the magnetic field is

$$B = \frac{\mu_0 iN}{2\pi}\frac{1}{r}$$

Equation (29-24)

Summary (5 of 5)

Field of a Magnetic Dipole

- The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point *P* located a distance *z* along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$

Equation (29-9)

In-class quiz

•8 In Fig. 29-40, two semicircular arcs have radii $R_2 = 7.80$ cm and $R_1 = 3.15$ cm, carry current $i = 0.281$ A, and have the same center of curvature C. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at C?

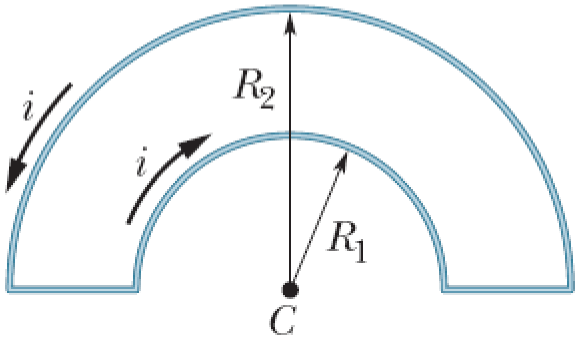


Figure 29-40 Problem 8.

The magnitude of the **magnetic field at the center of a circular arc**

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

In-class quiz

In-class quiz

•8 In Fig. 29-40, two semicircular arcs have radii $R_2 = 7.80$ cm and $R_1 = 3.15$ cm, carry current $i = 0.281$ A, and have the same center of curvature C. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at C?

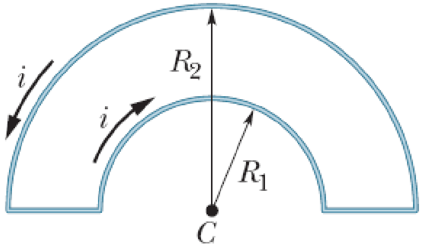


Figure 29-40 Problem 8.

8. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in segments AH and JD do not contribute to the field at point C. Using Eq. 29-9 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc HJ contributes $\mu_0 i/4R_1$ (into the page) to the field at C. Also, arc DA contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at C is

$$B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(4\text{p}\times 10^{-7}\,\text{T}\cdot\text{m/A})(0.281\text{A})}{4}\left(\frac{1}{0.0315\text{m}} - \frac{1}{0.0780\text{m}}\right) = 1.67\times 10^{-6}\,\text{T}.$$

(b) The direction of the field is into the page.

In-class quiz

••**13** In Fig. 29-44, point P_1 is at distance $R = 13.1$ cm on the perpendicular bisector of a straight wire of length $L = 18.0$ cm carrying

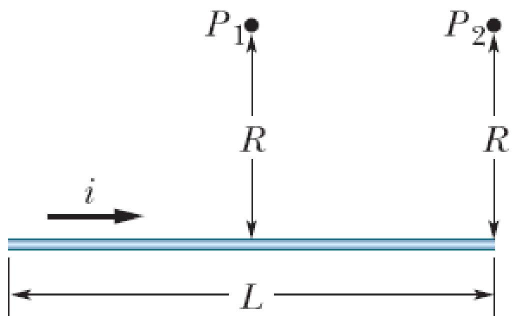


Figure 29-44 Problems 13 and 17.

In-class quiz

•**46** Eight wires cut the page perpendicularly at the points shown in Fig. 29-70. A wire labeled with the integer k ($k = 1, 2, \ldots, 8$) carries the current ki, where $i = 4.50$ mA. For those wires with odd k, the current is out of the page; for those with even k, it is into the page. Evaluate $\oint \vec{B} \cdot d\vec{s}$ along the closed path indicated and in the direction shown.

••**47** ILW The current density $\vec{J}$

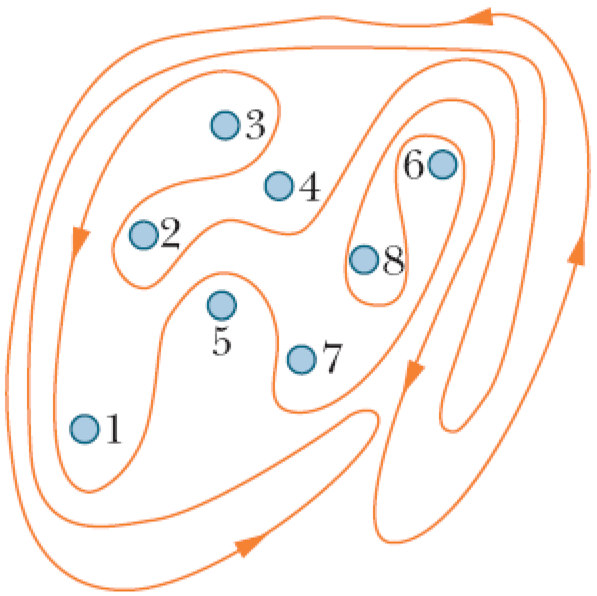


Figure 29-70 Problem 46.