

Capacitance

25-1 CAPACITANCE

Learning Objectives

After reading this module, you should be able to . . .

25.01 Sketch a schematic diagram of a circuit with a parallel-plate capacitor, a battery, and an open or closed switch.

25.02 In a circuit with a battery, an open switch, and an uncharged capacitor, explain what happens to the conduction electrons when the switch is closed.

25.03 For a capacitor, apply the relationship between the magnitude of charge q on either plate (“the charge on the capacitor”), the potential difference V between the plates (“the potential across the capacitor”), and the capacitance C of the capacitor.

Key Ideas

- A capacitor consists of two isolated conductors (the plates) with charges $+q$ and $-q$. Its capacitance C is defined from

$$q = CV,$$

where V is the potential difference between the plates.

- When a circuit with a battery, an open switch, and an uncharged capacitor is completed by closing the switch, conduction electrons shift, leaving the capacitor plates with opposite charges.

What Is Physics?

One goal of physics is to provide the basic science for practical devices designed by engineers. The focus of this chapter is on one extremely common example—the capacitor, a device in which electrical energy can be stored. For example, the batteries in a camera store energy in the photoflash unit by charging a capacitor. The batteries can supply energy at only a modest rate, too slowly for the photoflash unit to emit a flash of light. However, once the capacitor is charged, it can supply energy at a much greater rate when the photoflash unit is triggered—enough energy to allow the unit to emit a burst of bright light.

The physics of capacitors can be generalized to other devices and to any situation involving electric fields. For example, Earth’s atmospheric electric field is modeled by meteorologists as being produced by a huge spherical capacitor that partially discharges via lightning. The charge that skis collect as they slide along snow can be modeled as being stored in a capacitor that frequently discharges as sparks (which can be seen by nighttime skiers on dry snow).

The first step in our discussion of capacitors is to determine how much charge can be stored. This “how much” is called capacitance.

Capacitance

Figure 25-1 shows some of the many sizes and shapes of capacitors. Figure 25-2 shows the basic elements of *any* capacitor — two isolated conductors of any



Paul Silvermann/Fundamental Photographs

Figure 25-1 An assortment of capacitors.

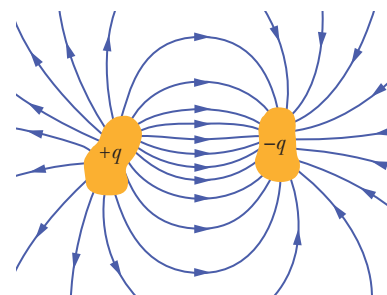
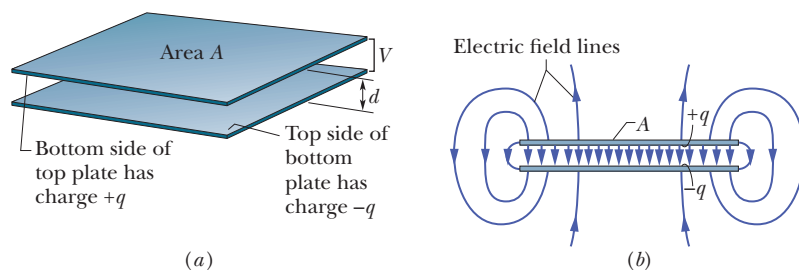


Figure 25-2 Two conductors, isolated electrically from each other and from their surroundings, form a *capacitor*. When the capacitor is charged, the charges on the conductors, or *plates* as they are called, have the same magnitude q but opposite signs.

Figure 25-3 (a) A parallel-plate capacitor, made up of two plates of area A separated by a distance d . The charges on the facing plate surfaces have the same magnitude q but opposite signs. (b) As the field lines show, the electric field due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the “fringing” of the field lines there.



shape. No matter what their geometry, flat or not, we call these conductors *plates*.

Figure 25-3a shows a less general but more conventional arrangement, called a *parallel-plate capacitor*, consisting of two parallel conducting plates of area A separated by a distance d . The symbol we use to represent a capacitor ($\text{--}\text{||}\text{--}$) is based on the structure of a parallel-plate capacitor but is used for capacitors of all geometries. We assume for the time being that no material medium (such as glass or plastic) is present in the region between the plates. In Module 25-5, we shall remove this restriction.

When a capacitor is *charged*, its plates have charges of equal magnitudes but opposite signs: $+q$ and $-q$. However, we refer to the *charge of a capacitor* as being q , the absolute value of these charges on the plates. (Note that q is not the net charge on the capacitor, which is zero.)

Because the plates are conductors, they are equipotential surfaces; all points on a plate are at the same electric potential. Moreover, there is a potential difference between the two plates. For historical reasons, we represent the absolute value of this potential difference with V rather than with the ΔV we used in previous notation.

The charge q and the potential difference V for a capacitor are proportional to each other; that is,

$$q = CV. \quad (25-1)$$

The proportionality constant C is called the **capacitance** of the capacitor. Its value depends only on the geometry of the plates and *not* on their charge or potential difference. The capacitance is a measure of how much charge must be put on the plates to produce a certain potential difference between them: *The greater the capacitance, the more charge is required.*

The SI unit of capacitance that follows from Eq. 25-1 is the coulomb per volt. This unit occurs so often that it is given a special name, the *farad* (F):

$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C/V}. \quad (25-2)$$

As you will see, the farad is a very large unit. Submultiples of the farad, such as the microfarad ($1 \mu\text{F} = 10^{-6} \text{ F}$) and the picofarad ($1 \text{ pF} = 10^{-12} \text{ F}$), are more convenient units in practice.

Charging a Capacitor

One way to charge a capacitor is to place it in an electric circuit with a battery. An *electric circuit* is a path through which charge can flow. A *battery* is a device that maintains a certain potential difference between its *terminals* (points at which charge can enter or leave the battery) by means of internal electrochemical reactions in which electric forces can move internal charge.

In Fig. 25-4a, a battery B, a switch S, an uncharged capacitor C, and interconnecting wires form a circuit. The same circuit is shown in the *schematic diagram* of Fig. 25-4b, in which the symbols for a battery, a switch, and a capacitor represent those devices. The battery maintains potential difference V between its terminals. The terminal of higher potential is labeled $+$ and is often called the *positive terminal*; the terminal of lower potential is labeled $-$ and is often called the *negative terminal*.

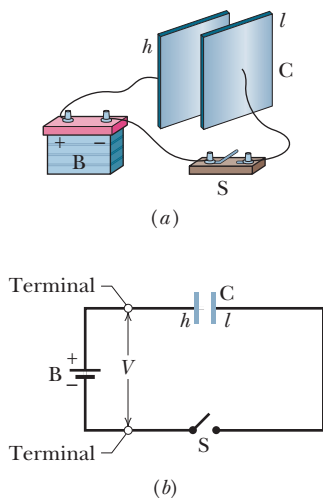


Figure 25-4 (a) Battery B, switch S, and plates h and l of capacitor C, connected in a circuit. (b) A schematic diagram with the *circuit elements* represented by their symbols.

The circuit shown in Figs. 25-4*a* and *b* is said to be *incomplete* because switch *S* is *open*; that is, the switch does not electrically connect the wires attached to it. When the switch is *closed*, electrically connecting those wires, the circuit is complete and charge can then flow through the switch and the wires. As we discussed in Chapter 21, the charge that can flow through a conductor, such as a wire, is that of electrons. When the circuit of Fig. 25-4 is completed, electrons are driven through the wires by an electric field that the battery sets up in the wires. The field drives electrons from capacitor plate *h* to the positive terminal of the battery; thus, plate *h*, losing electrons, becomes positively charged. The field drives just as many electrons from the negative terminal of the battery to capacitor plate *l*; thus, plate *l*, gaining electrons, becomes negatively charged *just as much* as plate *h*, losing electrons, becomes positively charged.

Initially, when the plates are uncharged, the potential difference between them is zero. As the plates become oppositely charged, that potential difference increases until it equals the potential difference *V* between the terminals of the battery. Then plate *h* and the positive terminal of the battery are at the same potential, and there is no longer an electric field in the wire between them. Similarly, plate *l* and the negative terminal reach the same potential, and there is then no electric field in the wire between them. Thus, with the field zero, there is no further drive of electrons. The capacitor is then said to be *fully charged*, with a potential difference *V* and charge *q* that are related by Eq. 25-1.

In this book we assume that during the charging of a capacitor and afterward, charge cannot pass from one plate to the other across the gap separating them. Also, we assume that a capacitor can retain (or *store*) charge indefinitely, until it is put into a circuit where it can be *discharged*.



Checkpoint 1

Does the capacitance *C* of a capacitor increase, decrease, or remain the same (a) when the charge *q* on it is doubled and (b) when the potential difference *V* across it is tripled?

25-2 CALCULATING THE CAPACITANCE

Learning Objectives

After reading this module, you should be able to . . .

25.04 Explain how Gauss' law is used to find the capacitance of a parallel-plate capacitor.

25.05 For a parallel-plate capacitor, a cylindrical capacitor, a spherical capacitor, and an isolated sphere, calculate the capacitance.

Key Ideas

- We generally determine the capacitance of a particular capacitor configuration by (1) assuming a charge *q* to have been placed on the plates, (2) finding the electric field \vec{E} due to this charge, (3) evaluating the potential difference *V* between the plates, and (4) calculating *C* from $q = CV$. Some results are the following:

- A parallel-plate capacitor with flat parallel plates of area *A* and spacing *d* has capacitance

$$C = \frac{\epsilon_0 A}{d}.$$

- A cylindrical capacitor (two long coaxial cylinders) of

length *L* and radii *a* and *b* has capacitance

$$C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}.$$

- A spherical capacitor with concentric spherical plates of radii *a* and *b* has capacitance

$$C = 4\pi\epsilon_0 \frac{ab}{b - a}.$$

- An isolated sphere of radius *R* has capacitance

$$C = 4\pi\epsilon_0 R.$$

Calculating the Capacitance

Our goal here is to calculate the capacitance of a capacitor once we know its geometry. Because we shall consider a number of different geometries, it seems wise to develop a general plan to simplify the work. In brief our plan is as follows: (1) Assume a charge q on the plates; (2) calculate the electric field \vec{E} between the plates in terms of this charge, using Gauss' law; (3) knowing \vec{E} , calculate the potential difference V between the plates from Eq. 24-18; (4) calculate C from Eq. 25-1.

Before we start, we can simplify the calculation of both the electric field and the potential difference by making certain assumptions. We discuss each in turn.

Calculating the Electric Field

To relate the electric field \vec{E} between the plates of a capacitor to the charge q on either plate, we shall use Gauss' law:

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \quad (25-3)$$

Here q is the charge enclosed by a Gaussian surface and $\oint \vec{E} \cdot d\vec{A}$ is the net electric flux through that surface. In all cases that we shall consider, the Gaussian surface will be such that whenever there is an electric flux through it, \vec{E} will have a uniform magnitude E and the vectors \vec{E} and $d\vec{A}$ will be parallel. Equation 25-3 then reduces to

$$q = \epsilon_0 EA \quad (\text{special case of Eq. 25-3}), \quad (25-4)$$

in which A is the area of that part of the Gaussian surface through which there is a flux. For convenience, we shall always draw the Gaussian surface in such a way that it completely encloses the charge on the positive plate; see Fig. 25-5 for an example.

Calculating the Potential Difference

In the notation of Chapter 24 (Eq. 24-18), the potential difference between the plates of a capacitor is related to the field \vec{E} by

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}, \quad (25-5)$$

in which the integral is to be evaluated along any path that starts on one plate and ends on the other. We shall always choose a path that follows an electric field line, from the negative plate to the positive plate. For this path, the vectors \vec{E} and $d\vec{s}$ will have opposite directions; so the dot product $\vec{E} \cdot d\vec{s}$ will be equal to $-E ds$. Thus, the right side of Eq. 25-5 will then be positive. Letting V represent the difference $V_f - V_i$, we can then recast Eq. 25-5 as

$$V = \int_-^+ E ds \quad (\text{special case of Eq. 25-5}), \quad (25-6)$$

in which the $-$ and $+$ remind us that our path of integration starts on the negative plate and ends on the positive plate.

We are now ready to apply Eqs. 25-4 and 25-6 to some particular cases.

A Parallel-Plate Capacitor

We assume, as Fig. 25-5 suggests, that the plates of our parallel-plate capacitor are so large and so close together that we can neglect the fringing of the electric

We use Gauss' law to relate q and E . Then we integrate the E to get the potential difference.

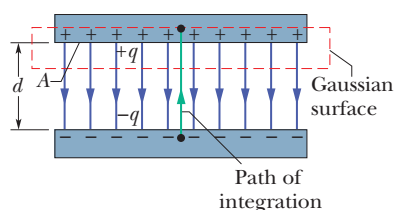


Figure 25-5 A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration of Eq. 25-6 is taken along a path extending directly from the negative plate to the positive plate.

field at the edges of the plates, taking \vec{E} to be constant throughout the region between the plates.

We draw a Gaussian surface that encloses just the charge q on the positive plate, as in Fig. 25-5. From Eq. 25-4 we can then write

$$q = \epsilon_0 EA, \quad (25-7)$$

where A is the area of the plate.

Equation 25-6 yields

$$V = \int_{-}^{+} E \, ds = E \int_0^d ds = Ed. \quad (25-8)$$

In Eq. 25-8, E can be placed outside the integral because it is a constant; the second integral then is simply the plate separation d .

If we now substitute q from Eq. 25-7 and V from Eq. 25-8 into the relation $q = CV$ (Eq. 25-1), we find

$$C = \frac{\epsilon_0 A}{d} \quad (\text{parallel-plate capacitor}). \quad (25-9)$$

Thus, the capacitance does indeed depend only on geometrical factors — namely, the plate area A and the plate separation d . Note that C increases as we increase area A or decrease separation d .

As an aside, we point out that Eq. 25-9 suggests one of our reasons for writing the electrostatic constant in Coulomb's law in the form $1/4\pi\epsilon_0$. If we had not done so, Eq. 25-9 — which is used more often in engineering practice than Coulomb's law — would have been less simple in form. We note further that Eq. 25-9 permits us to express the permittivity constant ϵ_0 in a unit more appropriate for use in problems involving capacitors; namely,

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m} = 8.85 \text{ pF/m}. \quad (25-10)$$

We have previously expressed this constant as

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2. \quad (25-11)$$

A Cylindrical Capacitor

Figure 25-6 shows, in cross section, a cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b . We assume that $L \gg b$ so that we can neglect the fringing of the electric field that occurs at the ends of the cylinders. Each plate contains a charge of magnitude q .

As a Gaussian surface, we choose a cylinder of length L and radius r , closed by end caps and placed as is shown in Fig. 25-6. It is coaxial with the cylinders and encloses the central cylinder and thus also the charge q on that cylinder. Equation 25-4 then relates that charge and the field magnitude E as

$$q = \epsilon_0 EA = \epsilon_0 E(2\pi rL),$$

in which $2\pi rL$ is the area of the curved part of the Gaussian surface. There is no flux through the end caps. Solving for E yields

$$E = \frac{q}{2\pi\epsilon_0 Lr}. \quad (25-12)$$

Substitution of this result into Eq. 25-6 yields

$$V = \int_{-}^{+} E \, ds = -\frac{q}{2\pi\epsilon_0 L} \int_b^a \frac{dr}{r} = \frac{q}{2\pi\epsilon_0 L} \ln\left(\frac{b}{a}\right), \quad (25-13)$$

where we have used the fact that here $ds = -dr$ (we integrated radially inward).

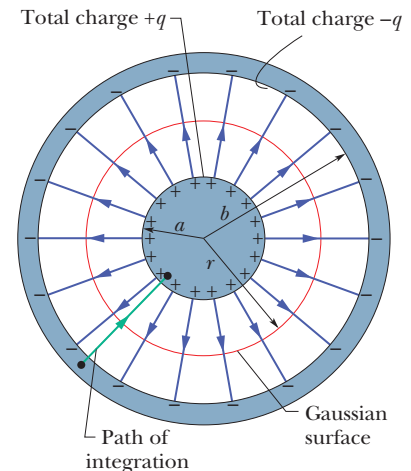


Figure 25-6 A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate) and the radial path of integration along which Eq. 25-6 is to be applied. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

From the relation $C = q/V$, we then have

$$C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)} \quad (\text{cylindrical capacitor}). \quad (25-14)$$

We see that the capacitance of a cylindrical capacitor, like that of a parallel-plate capacitor, depends only on geometrical factors, in this case the length L and the two radii b and a .

A Spherical Capacitor

Figure 25-6 can also serve as a central cross section of a capacitor that consists of two concentric spherical shells, of radii a and b . As a Gaussian surface we draw a sphere of radius r concentric with the two shells; then Eq. 25-4 yields

$$q = \epsilon_0 EA = \epsilon_0 E(4\pi r^2),$$

in which $4\pi r^2$ is the area of the spherical Gaussian surface. We solve this equation for E , obtaining

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}, \quad (25-15)$$

which we recognize as the expression for the electric field due to a uniform spherical charge distribution (Eq. 23-15).

If we substitute this expression into Eq. 25-6, we find

$$V = \int_{-}^{+} E \, ds = -\frac{q}{4\pi\epsilon_0} \int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{q}{4\pi\epsilon_0} \frac{b-a}{ab}, \quad (25-16)$$

where again we have substituted $-dr$ for ds . If we now substitute Eq. 25-16 into Eq. 25-1 and solve for C , we find

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} \quad (\text{spherical capacitor}). \quad (25-17)$$

An Isolated Sphere

We can assign a capacitance to a *single* isolated spherical conductor of radius R by assuming that the “missing plate” is a conducting sphere of infinite radius. After all, the field lines that leave the surface of a positively charged isolated conductor must end somewhere; the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius.

To find the capacitance of the conductor, we first rewrite Eq. 25-17 as

$$C = 4\pi\epsilon_0 \frac{a}{1 - a/b}.$$

If we then let $b \rightarrow \infty$ and substitute R for a , we find

$$C = 4\pi\epsilon_0 R \quad (\text{isolated sphere}). \quad (25-18)$$

Note that this formula and the others we have derived for capacitance (Eqs. 25-9, 25-14, and 25-17) involve the constant ϵ_0 multiplied by a quantity that has the dimensions of a length.



Checkpoint 2

For capacitors charged by the same battery, does the charge stored by the capacitor increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The radius of the inner cylinder of a cylindrical capacitor is increased. (c) The radius of the outer spherical shell of a spherical capacitor is increased.

Sample Problem 25.01 Charging the plates in a parallel-plate capacitor

In Fig. 25-7a, switch S is closed to connect the uncharged capacitor of capacitance $C = 0.25 \mu\text{F}$ to the battery of potential difference $V = 12 \text{ V}$. The lower capacitor plate has thickness $L = 0.50 \text{ cm}$ and face area $A = 2.0 \times 10^{-4} \text{ m}^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 \times 10^{28} \text{ electrons/m}^3$. From what depth d within the plate (Fig. 25-7b) must electrons move to the plate face as the capacitor becomes charged?

KEY IDEA

The charge collected on the plate is related to the capacitance and the potential difference across the capacitor by Eq. 25-1 ($q = CV$).

Calculations: Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. From Eq. 25-1, the total charge

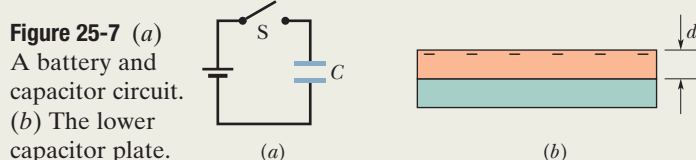


Figure 25-7 (a) A battery and capacitor circuit. (b) The lower capacitor plate.

magnitude that collects there is

$$q = CV = (0.25 \times 10^{-6} \text{ F})(12 \text{ V}) \\ = 3.0 \times 10^{-6} \text{ C}.$$

Dividing this result by e gives us the number N of conduction electrons that come up to the face:

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6} \text{ C}}{1.602 \times 10^{-19} \text{ C}} \\ = 1.873 \times 10^{13} \text{ electrons}.$$

These electrons come from a volume that is the product of the face area A and the depth d we seek. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad},$$

or

$$d = \frac{N}{An} = \frac{1.873 \times 10^{13} \text{ electrons}}{(2.0 \times 10^{-4} \text{ m}^2)(8.49 \times 10^{28} \text{ electrons/m}^3)} \\ = 1.1 \times 10^{-12} \text{ m} = 1.1 \text{ pm}. \quad (\text{Answer})$$

We commonly say that electrons move from the battery to the negative face but, actually, the battery sets up an electric field in the wires and plate such that electrons very close to the plate face move up to the negative face.



Additional examples, video, and practice available at WileyPLUS

25-3 CAPACITORS IN PARALLEL AND IN SERIES

Learning Objectives

After reading this module, you should be able to . . .

- 25.06** Sketch schematic diagrams for a battery and (a) three capacitors in parallel and (b) three capacitors in series.
- 25.07** Identify that capacitors in parallel have the same potential difference, which is the same value that their equivalent capacitor has.
- 25.08** Calculate the equivalent of parallel capacitors.
- 25.09** Identify that the total charge stored on parallel capacitors is the sum of the charges stored on the individual capacitors.
- 25.10** Identify that capacitors in series have the same charge, which is the same value that their equivalent capacitor has.
- 25.11** Calculate the equivalent of series capacitors.
- 25.12** Identify that the potential applied to capacitors in series is equal to the sum of the potentials across the individual capacitors.
- 25.13** For a circuit with a battery and some capacitors in parallel and some in series, simplify the circuit in steps by finding equivalent capacitors, until the charge and potential on the final equivalent capacitor can be determined, and then reverse the steps to find the charge and potential on the individual capacitors.
- 25.14** For a circuit with a battery, an open switch, and one or more uncharged capacitors, determine the amount of charge that moves through a point in the circuit when the switch is closed.
- 25.15** When a charged capacitor is connected in parallel to one or more uncharged capacitors, determine the charge and potential difference on each capacitor when equilibrium is reached.

Key Idea

● The equivalent capacitances C_{eq} of combinations of individual capacitors connected in parallel and in series can be found from

$$C_{\text{eq}} = \sum_{j=1}^n C_j \quad (n \text{ capacitors in parallel})$$

and

$$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^n \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$

Equivalent capacitances can be used to calculate the capacitances of more complicated series–parallel combinations.

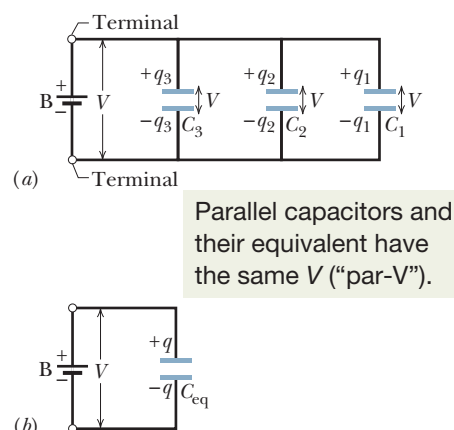


Figure 25-8 (a) Three capacitors connected in parallel to battery B. The battery maintains potential difference V across its terminals and thus across *each* capacitor. (b) The equivalent capacitor, with capacitance C_{eq} , replaces the parallel combination.

Parallel capacitors and their equivalent have the same V (“par- V ”).

Capacitors in Parallel and in Series

When there is a combination of capacitors in a circuit, we can sometimes replace that combination with an **equivalent capacitor**—that is, a single capacitor that has the same capacitance as the actual combination of capacitors. With such a replacement, we can simplify the circuit, affording easier solutions for unknown quantities of the circuit. Here we discuss two basic combinations of capacitors that allow such a replacement.

Capacitors in Parallel

Figure 25-8a shows an electric circuit in which three capacitors are connected *in parallel* to battery B. This description has little to do with how the capacitor plates are drawn. Rather, “in parallel” means that the capacitors are directly wired together at one plate and directly wired together at the other plate, and that the same potential difference V is applied across the two groups of wired-together plates. Thus, each capacitor has the same potential difference V , which produces charge on the capacitor. (In Fig. 25-8a, the applied potential V is maintained by the battery.) In general:



When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.

When we analyze a circuit of capacitors in parallel, we can simplify it with this mental replacement:



Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge q and the same potential difference V as the actual capacitors.

(You might remember this result with the nonsense word “par- V ,” which is close to “party,” to mean “capacitors in parallel have the same V .”) Figure 25-8b shows the equivalent capacitor (with equivalent capacitance C_{eq}) that has replaced the three capacitors (with actual capacitances C_1 , C_2 , and C_3) of Fig. 25-8a.

To derive an expression for C_{eq} in Fig. 25-8b, we first use Eq. 25-1 to find the charge on each actual capacitor:

$$q_1 = C_1 V, \quad q_2 = C_2 V, \quad \text{and} \quad q_3 = C_3 V.$$

The total charge on the parallel combination of Fig. 25-8a is then

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

The equivalent capacitance, with the same total charge q and applied potential difference V as the combination, is then

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

a result that we can easily extend to any number n of capacitors, as

$$C_{eq} = \sum_{j=1}^n C_j \quad (n \text{ capacitors in parallel}). \quad (25-19)$$

Thus, to find the equivalent capacitance of a parallel combination, we simply add the individual capacitances.

Capacitors in Series

Figure 25-9a shows three capacitors connected *in series* to battery B. This description has little to do with how the capacitors are drawn. Rather, “in series” means that the capacitors are wired serially, one after the other, and that a potential difference

V is applied across the two ends of the series. (In Fig. 25-9a, this potential difference V is maintained by battery B.) The potential differences that then exist across the capacitors in the series produce identical charges q on them.



When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q . The sum of the potential differences across all the capacitors is equal to the applied potential difference V .

We can explain how the capacitors end up with identical charge by following a *chain reaction* of events, in which the charging of each capacitor causes the charging of the next capacitor. We start with capacitor 3 and work upward to capacitor 1. When the battery is first connected to the series of capacitors, it produces charge $-q$ on the bottom plate of capacitor 3. That charge then repels negative charge from the top plate of capacitor 3 (leaving it with charge $+q$). The repelled negative charge moves to the bottom plate of capacitor 2 (giving it charge $-q$). That charge on the bottom plate of capacitor 2 then repels negative charge from the top plate of capacitor 2 (leaving it with charge $+q$) to the bottom plate of capacitor 1 (giving it charge $-q$). Finally, the charge on the bottom plate of capacitor 1 helps move negative charge from the top plate of capacitor 1 to the battery, leaving that top plate with charge $+q$.

Here are two important points about capacitors in series:

1. When charge is shifted from one capacitor to another in a series of capacitors, it can move along only one route, such as from capacitor 3 to capacitor 2 in Fig. 25-9a. If there are additional routes, the capacitors are not in series.
2. The battery directly produces charges on only the two plates to which it is connected (the bottom plate of capacitor 3 and the top plate of capacitor 1 in Fig. 25-9a). Charges that are produced on the other plates are due merely to the shifting of charge already there. For example, in Fig. 25-9a, the part of the circuit enclosed by dashed lines is electrically isolated from the rest of the circuit. Thus, its charge can only be redistributed.

When we analyze a circuit of capacitors in series, we can simplify it with this mental replacement:



Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same *total* potential difference V as the actual series capacitors.

(You might remember this with the nonsense word “seri- q ” to mean “capacitors in series have the same q .”) Figure 25-9b shows the equivalent capacitor (with equivalent capacitance C_{eq}) that has replaced the three actual capacitors (with actual capacitances C_1 , C_2 , and C_3) of Fig. 25-9a.

To derive an expression for C_{eq} in Fig. 25-9b, we first use Eq. 25-1 to find the potential difference of each actual capacitor:

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

The total potential difference V due to the battery is the sum

$$V = V_1 + V_2 + V_3 = q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right).$$

The equivalent capacitance is then

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

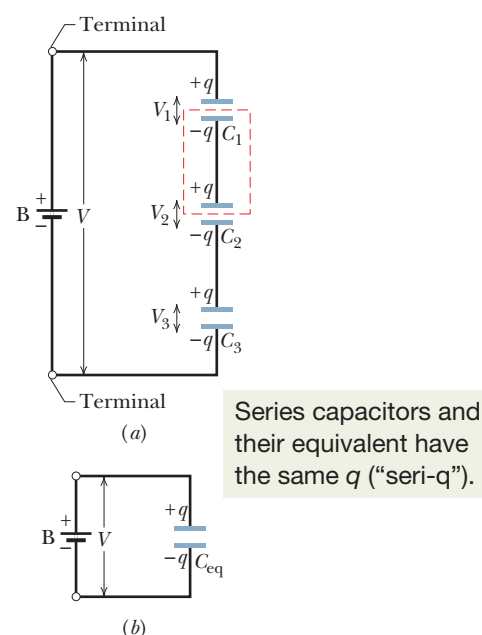


Figure 25-9 (a) Three capacitors connected in series to battery B. The battery maintains potential difference V between the top and bottom plates of the series combination. (b) The equivalent capacitor, with capacitance C_{eq} , replaces the series combination.

We can easily extend this to any number n of capacitors as

$$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^n \frac{1}{C_j} \quad (n \text{ capacitors in series}). \quad (25-20)$$

Using Eq. 25-20 you can show that the equivalent capacitance of a series of capacitances is always *less* than the least capacitance in the series.



Checkpoint 3

A battery of potential V stores charge q on a combination of two identical capacitors. What are the potential difference across and the charge on either capacitor if the capacitors are (a) in parallel and (b) in series?

Sample Problem 25.02 Capacitors in parallel and in series

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10a, across which potential difference V is applied. Assume

$$C_1 = 12.0 \mu\text{F}, \quad C_2 = 5.30 \mu\text{F}, \quad \text{and} \quad C_3 = 4.50 \mu\text{F}.$$

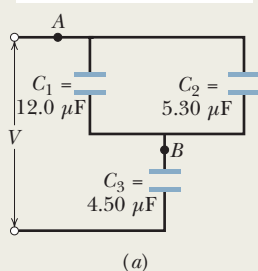
KEY IDEA

Any capacitors connected in series can be replaced with their equivalent capacitor, and any capacitors connected

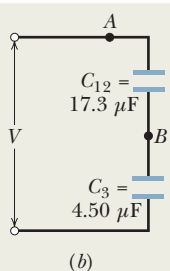
in parallel can be replaced with their equivalent capacitor. Therefore, we should first check whether any of the capacitors in Fig. 25-10a are in parallel or series.

Finding equivalent capacitance: Capacitors 1 and 3 are connected one after the other, but are they in series? No. The potential V that is applied to the capacitors produces charge on the bottom plate of capacitor 3. That charge causes charge to shift from the top plate of capacitor 3. However, note that the shifting charge can move to the

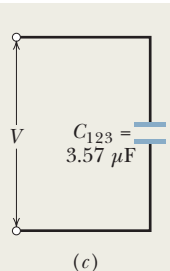
We first reduce the circuit to a single capacitor.



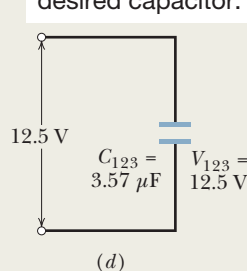
The equivalent of parallel capacitors is larger.



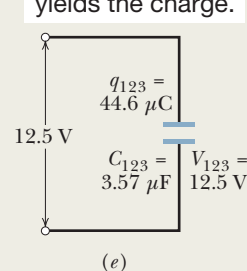
The equivalent of series capacitors is smaller.



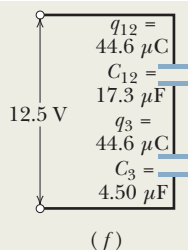
Next, we work backwards to the desired capacitor.



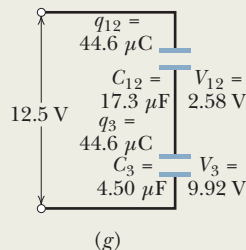
Applying $q = CV$ yields the charge.



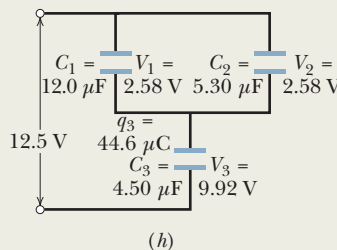
Series capacitors and their equivalent have the same q ("seri- q ").



Applying $V = q/C$ yields the potential difference.



Parallel capacitors and their equivalent have the same V ("par- V ").



Applying $q = CV$ yields the charge.

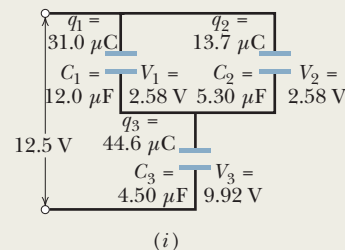


Figure 25-10 (a)–(d) Three capacitors are reduced to one equivalent capacitor. (e)–(i) Working backwards to get the charges.

bottom plates of both capacitor 1 and capacitor 2. Because there is more than one route for the shifting charge, capacitor 3 is not in series with capacitor 1 (or capacitor 2). Any time you think you might have two capacitors in series, apply this check about the shifting charge.

Are capacitor 1 and capacitor 2 in parallel? Yes. Their top plates are directly wired together and their bottom plates are directly wired together, and electric potential is applied between the top-plate pair and the bottom-plate pair. Thus, capacitor 1 and capacitor 2 are in parallel, and Eq. 25-19 tells us that their equivalent capacitance C_{12} is

$$C_{12} = C_1 + C_2 = 12.0 \mu\text{F} + 5.30 \mu\text{F} = 17.3 \mu\text{F}.$$

In Fig. 25-10b, we have replaced capacitors 1 and 2 with their equivalent capacitor, called capacitor 12 (say “one two” and not “twelve”). (The connections at points *A* and *B* are exactly the same in Figs. 25-10a and b.)

Is capacitor 12 in series with capacitor 3? Again applying the test for series capacitances, we see that the charge that shifts from the top plate of capacitor 3 must entirely go to the bottom plate of capacitor 12. Thus, capacitor 12 and capacitor 3 are in series, and we can replace them with their equivalent C_{123} (“one two three”), as shown in Fig. 25-10c. From Eq. 25-20, we have

$$\begin{aligned} \frac{1}{C_{123}} &= \frac{1}{C_{12}} + \frac{1}{C_3} \\ &= \frac{1}{17.3 \mu\text{F}} + \frac{1}{4.50 \mu\text{F}} = 0.280 \mu\text{F}^{-1}, \end{aligned}$$

from which

$$C_{123} = \frac{1}{0.280 \mu\text{F}^{-1}} = 3.57 \mu\text{F}. \quad (\text{Answer})$$

Sample Problem 25.03 One capacitor charging up another capacitor

Capacitor 1, with $C_1 = 3.55 \mu\text{F}$, is charged to a potential difference $V_0 = 6.30 \text{ V}$, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95 \mu\text{F}$. When switch *S* is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

KEY IDEAS

The situation here differs from the previous example because here an applied electric potential is *not* maintained across a combination of capacitors by a battery or some other source. Here, just after switch *S* is closed, the only applied electric potential is that of capacitor 1 on capacitor 2, and that potential is decreasing. Thus, the capacitors in Fig. 25-11 are not connected *in series*; and although they are drawn parallel, in this situation they are not *in parallel*.

(b) The potential difference applied to the input terminals in Fig. 25-10a is $V = 12.5 \text{ V}$. What is the charge on C_1 ?

KEY IDEAS

We now need to work backwards from the equivalent capacitance to get the charge on a particular capacitor. We have two techniques for such “backwards work”: (1) *Seri-q*: Series capacitors have the same charge as their equivalent capacitor. (2) *Par-V*: Parallel capacitors have the same potential difference as their equivalent capacitor.

Working backwards: To get the charge q_1 on capacitor 1, we work backwards to that capacitor, starting with the equivalent capacitor 123. Because the given potential difference $V (= 12.5 \text{ V})$ is applied across the actual combination of three capacitors in Fig. 25-10a, it is also applied across C_{123} in Figs. 25-10d and e. Thus, Eq. 25-1 ($q = CV$) gives us

$$q_{123} = C_{123}V = (3.57 \mu\text{F})(12.5 \text{ V}) = 44.6 \mu\text{C}.$$

The series capacitors 12 and 3 in Fig. 25-10b each have the same charge as their equivalent capacitor 123 (Fig. 25-10f). Thus, capacitor 12 has charge $q_{12} = q_{123} = 44.6 \mu\text{C}$. From Eq. 25-1 and Fig. 25-10g, the potential difference across capacitor 12 must be

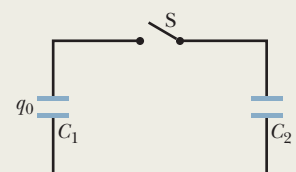
$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6 \mu\text{C}}{17.3 \mu\text{F}} = 2.58 \text{ V}.$$

The parallel capacitors 1 and 2 each have the same potential difference as their equivalent capacitor 12 (Fig. 25-10h). Thus, capacitor 1 has potential difference $V_1 = V_{12} = 2.58 \text{ V}$, and, from Eq. 25-1 and Fig. 25-10i, the charge on capacitor 1 must be

$$\begin{aligned} q_1 &= C_1 V_1 = (12.0 \mu\text{F})(2.58 \text{ V}) \\ &= 31.0 \mu\text{C}. \end{aligned} \quad (\text{Answer})$$

After the switch is closed, charge is transferred until the potential differences match.

Figure 25-11 A potential difference V_0 is applied to capacitor 1 and the charging battery is removed. Switch *S* is then closed so that the charge on capacitor 1 is shared with capacitor 2.



is no electric field within the connecting wires to move conduction electrons. The initial charge on capacitor 1 is then shared between the two capacitors.

Calculations: Initially, when capacitor 1 is connected to the battery, the charge it acquires is, from Eq. 25-1,

$$q_0 = C_1 V_0 = (3.55 \times 10^{-6} \text{ F})(6.30 \text{ V}) \\ = 22.365 \times 10^{-6} \text{ C}.$$

When switch S in Fig. 25-11 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until

$$V_1 = V_2 \quad (\text{equilibrium}).$$

From Eq. 25-1, we can rewrite this as

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \quad (\text{equilibrium}).$$

Because the total charge cannot magically change, the total after the transfer must be

$$q_1 + q_2 = q_0 \quad (\text{charge conservation});$$

thus

$$q_2 = q_0 - q_1.$$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}.$$

Solving this for q_1 and substituting given data, we find

$$q_1 = 6.35 \mu\text{C}. \quad (\text{Answer})$$

The rest of the initial charge ($q_0 = 22.365 \mu\text{C}$) must be on capacitor 2:

$$q_2 = 16.0 \mu\text{C}. \quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS

25-4 ENERGY STORED IN AN ELECTRIC FIELD

Learning Objectives

After reading this module, you should be able to . . .

25.16 Explain how the work required to charge a capacitor results in the potential energy of the capacitor.

25.17 For a capacitor, apply the relationship between the potential energy U , the capacitance C , and the potential difference V .

25.18 For a capacitor, apply the relationship between the

potential energy, the internal volume, and the internal energy density.

25.19 For any electric field, apply the relationship between the potential energy density u in the field and the field's magnitude E .

25.20 Explain the danger of sparks in airborne dust.

Key Ideas

- The electric potential energy U of a charged capacitor,

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2,$$

is equal to the work required to charge the capacitor. This energy can be associated with the capacitor's electric field \vec{E} .

- Every electric field, in a capacitor or from any other source, has an associated stored energy. In vacuum, the energy density u (potential energy per unit volume) in a field of magnitude E is

$$u = \frac{1}{2}\epsilon_0 E^2.$$

Energy Stored in an Electric Field

Work must be done by an external agent to charge a capacitor. We can imagine doing the work ourselves by transferring electrons from one plate to the other, one by one. As the charges build, so does the electric field between the plates, which opposes the continued transfer. So, greater amounts of work are required. Actually, a battery does all this for us, at the expense of its stored chemical energy. We visualize the work as being stored as electric potential energy in the electric field between the plates.

Suppose that, at a given instant, a charge q' has been transferred from one plate of a capacitor to the other. The potential difference V' between the plates at that instant will be q'/C . If an extra increment of charge dq' is then transferred, the increment of work required will be, from Eq. 24-6,

$$dW = V' dq' = \frac{q'}{C} dq'.$$

The work required to bring the total capacitor charge up to a final value q is

$$W = \int dW = \frac{1}{C} \int_0^q q' dq' = \frac{q^2}{2C}.$$

This work is stored as potential energy U in the capacitor, so that

$$U = \frac{q^2}{2C} \quad (\text{potential energy}). \quad (25-21)$$

From Eq. 25-1, we can also write this as

$$U = \frac{1}{2} CV^2 \quad (\text{potential energy}). \quad (25-22)$$

Equations 25-21 and 25-22 hold no matter what the geometry of the capacitor is.

To gain some physical insight into energy storage, consider two parallel-plate capacitors that are identical except that capacitor 1 has twice the plate separation of capacitor 2. Then capacitor 1 has twice the volume between its plates and also, from Eq. 25-9, half the capacitance of capacitor 2. Equation 25-4 tells us that if both capacitors have the same charge q , the electric fields between their plates are identical. And Eq. 25-21 tells us that capacitor 1 has twice the stored potential energy of capacitor 2. Thus, of two otherwise identical capacitors with the same charge and same electric field, the one with twice the volume between its plates has twice the stored potential energy. Arguments like this tend to verify our earlier assumption:



The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Explosions in Airborne Dust


As we discussed in Module 24-8, making contact with certain materials, such as clothing, carpets, and even playground slides, can leave you with a significant electrical potential. You might become painfully aware of that potential if a spark leaps between you and a grounded object, such as a faucet. In many industries involving the production and transport of powder, such as in the cosmetic and food industries, such a spark can be disastrous. Although the powder in bulk may not burn at all, when individual powder grains are airborne and thus surrounded by oxygen, they can burn so fiercely that a cloud of the grains burns as an explosion. Safety engineers cannot eliminate all possible sources of sparks in the powder industries. Instead, they attempt to keep the amount of energy available in the sparks below the threshold value U_t (≈ 150 mJ) typically required to ignite airborne grains.

Suppose a person becomes charged by contact with various surfaces as he walks through an airborne powder. We can roughly model the person as a spherical capacitor of radius $R = 1.8$ m. From Eq. 25-18 ($C = 4\pi\epsilon_0 R$) and Eq. 25-22 ($U = \frac{1}{2} CV^2$), we see that the energy of the capacitor is

$$U = \frac{1}{2}(4\pi\epsilon_0 R)V^2.$$

From this we see that the threshold energy corresponds to a potential of

$$V = \sqrt{\frac{2U_t}{4\pi\epsilon_0 R}} = \sqrt{\frac{2(150 \times 10^{-3} \text{ J})}{4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.8 \text{ m})}} \\ = 3.9 \times 10^4 \text{ V}.$$

Safety engineers attempt to keep the potential of the personnel below this level by “bleeding” off the charge through, say, a conducting floor. 

Energy Density

In a parallel-plate capacitor, neglecting fringing, the electric field has the same value at all points between the plates. Thus, the **energy density** u —that is, the potential energy per unit volume between the plates—should also be uniform. We can find u by dividing the total potential energy by the volume Ad of the space between the plates. Using Eq. 25-22, we obtain

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}. \quad (25-23)$$

With Eq. 25-9 ($C = \epsilon_0 A/d$), this result becomes

$$u = \frac{1}{2} \epsilon_0 \left(\frac{V}{d} \right)^2. \quad (25-24)$$

However, from Eq. 24-42 ($E = -\Delta V/\Delta s$), V/d equals the electric field magnitude E ; so

$$u = \frac{1}{2} \epsilon_0 E^2 \quad (\text{energy density}). \quad (25-25)$$

Although we derived this result for the special case of an electric field of a parallel-plate capacitor, it holds for any electric field. If an electric field \vec{E} exists at any point in space, that site has an electric potential energy with a density (amount per unit volume) given by Eq. 25-25.

Sample Problem 25.04 Potential energy and energy density of an electric field

An isolated conducting sphere whose radius R is 6.85 cm has a charge $q = 1.25 \text{ nC}$.

(a) How much potential energy is stored in the electric field of this charged conductor?

KEY IDEAS

(1) An isolated sphere has capacitance given by Eq. 25-18 ($C = 4\pi\epsilon_0 R$). (2) The energy U stored in a capacitor depends on the capacitor's charge q and capacitance C according to Eq. 25-21 ($U = q^2/2C$).

Calculation: Substituting $C = 4\pi\epsilon_0 R$ into Eq. 25-21 gives us

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\epsilon_0 R} \\ = \frac{(1.25 \times 10^{-9} \text{ C})^2}{(8\pi)(8.85 \times 10^{-12} \text{ F/m})(0.0685 \text{ m})} \\ = 1.03 \times 10^{-7} \text{ J} = 103 \text{ nJ}. \quad (\text{Answer})$$

(b) What is the energy density at the surface of the sphere?

KEY IDEA

The density u of the energy stored in an electric field depends on the magnitude E of the field, according to Eq. 25-25 ($u = \frac{1}{2}\epsilon_0 E^2$).

Calculations: Here we must first find E at the surface of the sphere, as given by Eq. 23-15:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}.$$

The energy density is then

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{q^2}{32\pi^2\epsilon_0 R^4} \\ = \frac{(1.25 \times 10^{-9} \text{ C})^2}{(32\pi^2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0685 \text{ m})^4} \\ = 2.54 \times 10^{-5} \text{ J/m}^3 = 25.4 \mu\text{J/m}^3. \quad (\text{Answer})$$



25-5 CAPACITOR WITH A DIELECTRIC

Learning Objectives

After reading this module, you should be able to . . .

- 25.21** Identify that capacitance is increased if the space between the plates is filled with a dielectric material.
- 25.22** For a capacitor, calculate the capacitance with and without a dielectric.
- 25.23** For a region filled with a dielectric material with a given dielectric constant κ , identify that all electrostatic equations containing the permittivity constant ϵ_0 are modified by multiplying that constant by the dielectric constant to get $\kappa\epsilon_0$.
- 25.24** Name some of the common dielectrics.
- 25.25** In adding a dielectric to a charged capacitor, distinguish the results for a capacitor (a) connected to a battery and (b) not connected to a battery.
- 25.26** Distinguish polar dielectrics from nonpolar dielectrics.
- 25.27** In adding a dielectric to a charged capacitor, explain what happens to the electric field between the plates in terms of what happens to the atoms in the dielectric.

Key Ideas

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C in vacuum (or, effectively, in air) is multiplied by the material's dielectric constant κ , which is a number greater than 1.
- In a region that is completely filled by a dielectric, all electrostatic equations containing the permittivity constant ϵ_0 must be modified by replacing ϵ_0 with $\kappa\epsilon_0$.
- When a dielectric material is placed in an external electric field, it develops an internal electric field that is oriented opposite the external field, thus reducing the magnitude of the electric field inside the material.
- When a dielectric material is placed in a capacitor with a fixed amount of charge on the surface, the net electric field between the plates is decreased.

Capacitor with a Dielectric

If you fill the space between the plates of a capacitor with a *dielectric*, which is an insulating material such as mineral oil or plastic, what happens to the capacitance? Michael Faraday — to whom the whole concept of capacitance is largely due and for whom the SI unit of capacitance is named — first looked into this matter in 1837. Using simple equipment much like that shown in Fig. 25-12, he found that the capacitance *increased* by a numerical factor κ ,



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Figure 25-12 The simple electrostatic apparatus used by Faraday. An assembled apparatus (second from left) forms a spherical capacitor consisting of a central brass ball and a concentric brass shell. Faraday placed dielectric materials in the space between the ball and the shell.

Table 25-1 Some Properties of Dielectrics^a

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, $\kappa = \text{unity}$.

^aMeasured at room temperature, except for the water.

which he called the **dielectric constant** of the insulating material. Table 25-1 shows some dielectric materials and their dielectric constants. The dielectric constant of a vacuum is unity by definition. Because air is mostly empty space, its measured dielectric constant is only slightly greater than unity. Even common paper can significantly increase the capacitance of a capacitor, and some materials, such as strontium titanate, can increase the capacitance by more than two orders of magnitude.

Another effect of the introduction of a dielectric is to limit the potential difference that can be applied between the plates to a certain value V_{max} , called the *breakdown potential*. If this value is substantially exceeded, the dielectric material will break down and form a conducting path between the plates. Every dielectric material has a characteristic *dielectric strength*, which is the maximum value of the electric field that it can tolerate without breakdown. A few such values are listed in Table 25-1.

As we discussed just after Eq. 25-18, the capacitance of any capacitor can be written in the form

$$C = \epsilon_0 \mathcal{L}, \quad (25-26)$$

in which \mathcal{L} has the dimension of length. For example, $\mathcal{L} = A/d$ for a parallel-plate capacitor. Faraday's discovery was that, with a dielectric *completely* filling the space between the plates, Eq. 25-26 becomes

$$C = \kappa \epsilon_0 \mathcal{L} = \kappa C_{\text{air}}, \quad (25-27)$$

where C_{air} is the value of the capacitance with only air between the plates. For example, if we fill a capacitor with strontium titanate, with a dielectric constant of 310, we multiply the capacitance by 310.

Figure 25-13 provides some insight into Faraday's experiments. In Fig. 25-13*a* the battery ensures that the potential difference V between the plates will remain constant. When a dielectric slab is inserted between the plates, the charge q on the plates increases by a factor of κ ; the additional charge is delivered to the capacitor plates by the battery. In Fig. 25-13*b* there is no battery, and therefore the charge q must remain constant when the dielectric slab is inserted; then the potential difference V between the plates decreases by a factor of κ . Both these observations are consistent (through the relation $q = CV$) with the increase in capacitance caused by the dielectric.

Comparison of Eqs. 25-26 and 25-27 suggests that the effect of a dielectric can be summed up in more general terms:



In a region completely filled by a dielectric material of dielectric constant κ , all electrostatic equations containing the permittivity constant ϵ_0 are to be modified by replacing ϵ_0 with $\kappa\epsilon_0$.

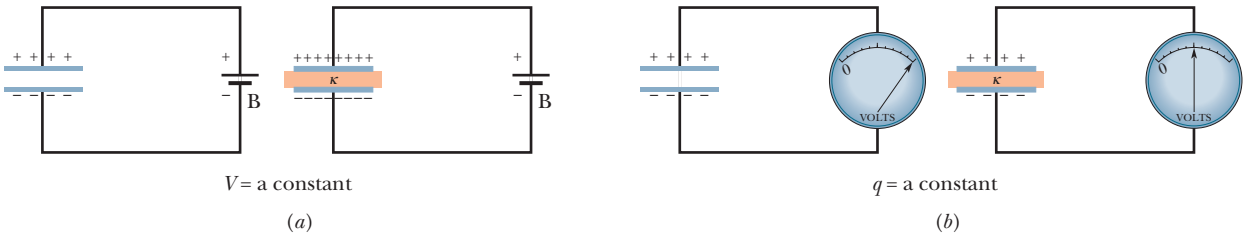


Figure 25-13 (a) If the potential difference between the plates of a capacitor is maintained, as by battery B, the effect of a dielectric is to increase the charge on the plates. (b) If the charge on the capacitor plates is maintained, as in this case, the effect of a dielectric is to reduce the potential difference between the plates. The scale shown is that of a *potentiometer*, a device used to measure potential difference (here, between the plates). A capacitor cannot discharge through a potentiometer.

Thus, the magnitude of the electric field produced by a point charge inside a dielectric is given by this modified form of Eq. 23-15:

$$E = \frac{1}{4\pi\kappa\epsilon_0} \frac{q}{r^2}. \quad (25-28)$$

Also, the expression for the electric field just outside an isolated conductor immersed in a dielectric (see Eq. 23-11) becomes

$$E = \frac{\sigma}{\kappa\epsilon_0}. \quad (25-29)$$

Because κ is always greater than unity, both these equations show that *for a fixed distribution of charges, the effect of a dielectric is to weaken the electric field* that would otherwise be present.

Sample Problem 25.05 Work and energy when a dielectric is inserted into a capacitor

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference $V = 12.5$ V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

KEY IDEA

We can relate the potential energy U_i of the capacitor to the capacitance C and either the potential V (with Eq. 25-22) or the charge q (with Eq. 25-21):

$$U_i = \frac{1}{2}CV^2 = \frac{q^2}{2C}.$$

Calculation: Because we are given the initial potential V ($= 12.5$ V), we use Eq. 25-22 to find the initial stored energy:

$$\begin{aligned} U_i &= \frac{1}{2}CV^2 = \frac{1}{2}(13.5 \times 10^{-12} \text{ F})(12.5 \text{ V})^2 \\ &= 1.055 \times 10^{-9} \text{ J} = 1055 \text{ pJ} \approx 1100 \text{ pJ}. \quad (\text{Answer}) \end{aligned}$$

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

KEY IDEA

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change.

Calculations: Thus, we must now use Eq. 25-21 to write the final potential energy U_f , but now that the slab is within the capacitor, the capacitance is κC . We then have

$$\begin{aligned} U_f &= \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055 \text{ pJ}}{6.50} \\ &= 162 \text{ pJ} \approx 160 \text{ pJ}. \quad (\text{Answer}) \end{aligned}$$

When the slab is introduced, the potential energy decreases by a factor of κ .

The “missing” energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162) \text{ pJ} = 893 \text{ pJ}.$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.



Additional examples, video, and practice available at WileyPLUS

Dielectrics: An Atomic View

What happens, in atomic and molecular terms, when we put a dielectric in an electric field? There are two possibilities, depending on the type of molecule:

1. **Polar dielectrics.** The molecules of some dielectrics, like water, have permanent electric dipole moments. In such materials (called *polar dielectrics*), the

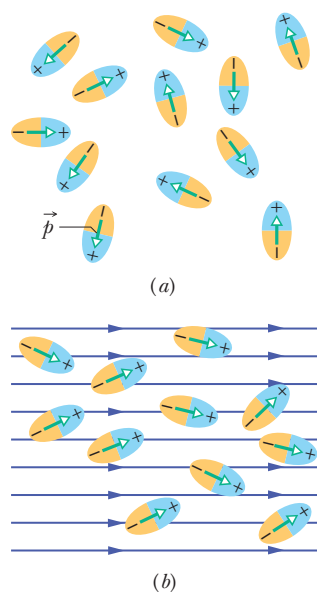


Figure 25-14 (a) Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field. (b) An electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.

electric dipoles tend to line up with an external electric field as in Fig. 25-14. Because the molecules are continuously jostling each other as a result of their random thermal motion, this alignment is not complete, but it becomes more complete as the magnitude of the applied field is increased (or as the temperature, and thus the jostling, are decreased). The alignment of the electric dipoles produces an electric field that is directed opposite the applied field and is smaller in magnitude.

2. Nonpolar dielectrics. Regardless of whether they have permanent electric dipole moments, molecules acquire dipole moments by induction when placed in an external electric field. In Module 24-4 (see Fig. 24-14), we saw that this occurs because the external field tends to “stretch” the molecules, slightly separating the centers of negative and positive charge.

Figure 25-15a shows a nonpolar dielectric slab with no external electric field applied. In Fig. 25-15b, an electric field \vec{E}_0 is applied via a capacitor, whose plates are charged as shown. The result is a slight separation of the centers of the positive and negative charge distributions within the slab, producing positive charge on one face of the slab (due to the positive ends of dipoles there) and negative charge on the opposite face (due to the negative ends of dipoles there). The slab as a whole remains electrically neutral and — within the slab — there is no excess charge in any volume element.

Figure 25-15c shows that the induced surface charges on the faces produce an electric field \vec{E}' in the direction opposite that of the applied electric field \vec{E}_0 . The resultant field \vec{E} inside the dielectric (the vector sum of fields \vec{E}_0 and \vec{E}') has the direction of \vec{E}_0 but is smaller in magnitude.

Both the field \vec{E}' produced by the surface charges in Fig. 25-15c and the electric field produced by the permanent electric dipoles in Fig. 25-14 act in the same way — they oppose the applied field \vec{E} . Thus, the effect of both polar and nonpolar dielectrics is to weaken any applied field within them, as between the plates of a capacitor.

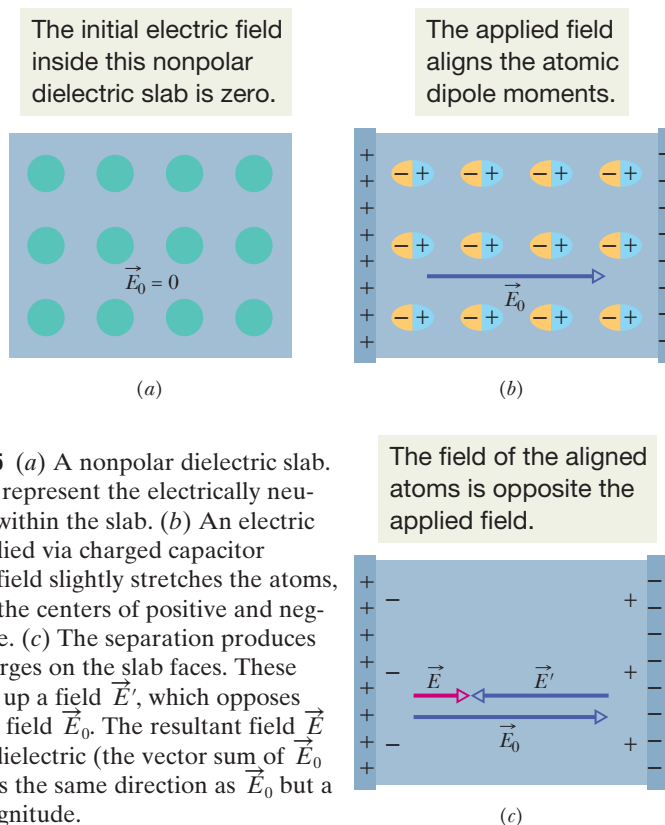


Figure 25-15 (a) A nonpolar dielectric slab. The circles represent the electrically neutral atoms within the slab. (b) An electric field is applied via charged capacitor plates; the field slightly stretches the atoms, separating the centers of positive and negative charge. (c) The separation produces surface charges on the slab faces. These charges set up a field \vec{E}' , which opposes the applied field \vec{E}_0 . The resultant field \vec{E} inside the dielectric (the vector sum of \vec{E}_0 and \vec{E}') has the same direction as \vec{E}_0 but a smaller magnitude.

25-6 DIELECTRICS AND GAUSS' LAW

Learning Objectives

After reading this module, you should be able to . . .

25.28 In a capacitor with a dielectric, distinguish free charge from induced charge.

25.29 When a dielectric partially or fully fills the space in a capacitor, find the free charge, the induced charge, the

electric field between the plates (if there is a gap, there is more than one field value), and the potential between the plates.

Key Ideas

- Inserting a dielectric into a capacitor causes induced charge to appear on the faces of the dielectric and weakens the electric field between the plates.
- The induced charge is less than the free charge on the plates.
- When a dielectric is present, Gauss' law may be

generalized to

$$\epsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q,$$

where q is the free charge. Any induced surface charge is accounted for by including the dielectric constant κ inside the integral.

Dielectrics and Gauss' Law

In our discussion of Gauss' law in Chapter 23, we assumed that the charges existed in a vacuum. Here we shall see how to modify and generalize that law if dielectric materials, such as those listed in Table 25-1, are present. Figure 25-16 shows a parallel-plate capacitor of plate area A , both with and without a dielectric. We assume that the charge q on the plates is the same in both situations. Note that the field between the plates induces charges on the faces of the dielectric by one of the methods described in Module 25-5.

For the situation of Fig. 25-16a, without a dielectric, we can find the electric field \vec{E}_0 between the plates as we did in Fig. 25-5: We enclose the charge $+q$ on the top plate with a Gaussian surface and then apply Gauss' law. Letting E_0 represent the magnitude of the field, we find

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 EA = q, \quad (25-30)$$

or

$$E_0 = \frac{q}{\epsilon_0 A}. \quad (25-31)$$

In Fig. 25-16b, with the dielectric in place, we can find the electric field between the plates (and within the dielectric) by using the same Gaussian surface. However, now the surface encloses two types of charge: It still encloses charge $+q$ on the top plate, but it now also encloses the induced charge $-q'$ on the top face of the dielectric. The charge on the conducting plate is said to be *free charge* because it can move if we change the electric potential of the plate; the induced charge on the surface of the dielectric is not free charge because it cannot move from that surface.

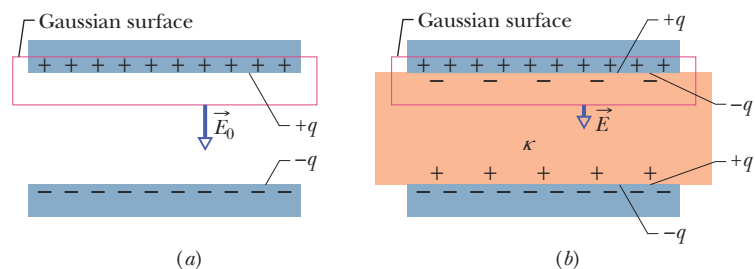


Figure 25-16 A parallel-plate capacitor (a) without and (b) with a dielectric slab inserted. The charge q on the plates is assumed to be the same in both cases.

The net charge enclosed by the Gaussian surface in Fig. 25-16b is $q - q'$, so Gauss' law now gives

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 EA = q - q', \quad (25-32)$$

or
$$E = \frac{q - q'}{\epsilon_0 A}. \quad (25-33)$$

The effect of the dielectric is to weaken the original field E_0 by a factor of κ ; so we may write

$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa \epsilon_0 A}. \quad (25-34)$$

Comparison of Eqs. 25-33 and 25-34 shows that

$$q - q' = \frac{q}{\kappa}. \quad (25-35)$$

Equation 25-35 shows correctly that the magnitude q' of the induced surface charge is less than that of the free charge q and is zero if no dielectric is present (because then $\kappa = 1$ in Eq. 25-35).

By substituting for $q - q'$ from Eq. 25-35 in Eq. 25-32, we can write Gauss' law in the form

$$\epsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad (\text{Gauss' law with dielectric}). \quad (25-36)$$

This equation, although derived for a parallel-plate capacitor, is true generally and is the most general form in which Gauss' law can be written. Note:

1. The flux integral now involves $\kappa \vec{E}$, not just \vec{E} . (The vector $\epsilon_0 \kappa \vec{E}$ is sometimes called the *electric displacement* \vec{D} , so that Eq. 25-36 can be written in the form $\oint \vec{D} \cdot d\vec{A} = q$.)
2. The charge q enclosed by the Gaussian surface is now taken to be the *free charge only*. The induced surface charge is deliberately ignored on the right side of Eq. 25-36, having been taken fully into account by introducing the dielectric constant κ on the left side.
3. Equation 25-36 differs from Eq. 23-7, our original statement of Gauss' law, only in that ϵ_0 in the latter equation has been replaced by $\kappa \epsilon_0$. We keep κ inside the integral of Eq. 25-36 to allow for cases in which κ is not constant over the entire Gaussian surface.

Sample Problem 25.06 Dielectric partially filling the gap in a capacitor

Figure 25-17 shows a parallel-plate capacitor of plate area A and plate separation d . A potential difference V_0 is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume $A = 115 \text{ cm}^2$, $d = 1.24 \text{ cm}$, $V_0 = 85.5 \text{ V}$, $b = 0.780 \text{ cm}$, and $\kappa = 2.61$.

(a) What is the capacitance C_0 before the dielectric slab is inserted?

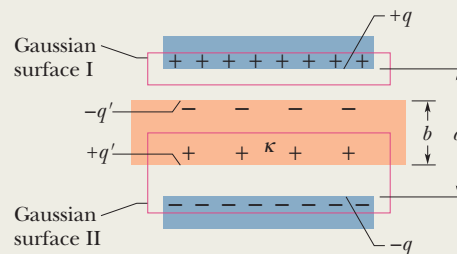


Figure 25-17 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

Calculation: From Eq. 25-9 we have

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ F/m})(115 \times 10^{-4} \text{ m}^2)}{1.24 \times 10^{-2} \text{ m}} \\ = 8.21 \times 10^{-12} \text{ F} = 8.21 \text{ pF.} \quad (\text{Answer})$$

(b) What free charge appears on the plates?

Calculation: From Eq. 25-1,

$$q = C_0 V_0 = (8.21 \times 10^{-12} \text{ F})(85.5 \text{ V}) \\ = 7.02 \times 10^{-10} \text{ C} = 702 \text{ pC.} \quad (\text{Answer})$$

Because the battery was disconnected before the slab was inserted, the free charge is unchanged.

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

KEY IDEA

We need to apply Gauss' law, in the form of Eq. 25-36, to Gaussian surface I in Fig. 25-17.

Calculations: That surface passes through the gap, and so it encloses *only* the free charge on the upper capacitor plate. Electric field pierces only the *bottom* of the Gaussian surface. Because there the area vector $d\vec{A}$ and the field vector \vec{E}_0 are both directed downward, the dot product in Eq. 25-36 becomes

$$\vec{E}_0 \cdot d\vec{A} = E_0 dA \cos 0^\circ = E_0 dA.$$

Equation 25-36 then becomes

$$\epsilon_0 \kappa E_0 \oint dA = q.$$

The integration now simply gives the surface area A of the plate. Thus, we obtain

$$\epsilon_0 \kappa E_0 A = q,$$

or
$$E_0 = \frac{q}{\epsilon_0 \kappa A}.$$

We must put $\kappa = 1$ here because Gaussian surface I does not pass through the dielectric. Thus, we have

$$E_0 = \frac{q}{\epsilon_0 \kappa A} = \frac{7.02 \times 10^{-10} \text{ C}}{(8.85 \times 10^{-12} \text{ F/m})(1)(115 \times 10^{-4} \text{ m}^2)} \\ = 6900 \text{ V/m} = 6.90 \text{ kV/m.} \quad (\text{Answer})$$

Note that the value of E_0 does not change when the slab is introduced because the amount of charge enclosed by Gaussian surface I in Fig. 25-17 does not change.

(d) What is the electric field E_1 in the dielectric slab?

KEY IDEA

Now we apply Gauss' law in the form of Eq. 25-36 to Gaussian surface II in Fig. 25-17.

Calculations: Only the free charge $-q$ is in Eq. 25-36, so

$$\epsilon_0 \oint \kappa \vec{E}_1 \cdot d\vec{A} = -\epsilon_0 \kappa E_1 A = -q. \quad (25-37)$$

The first minus sign in this equation comes from the dot product $\vec{E}_1 \cdot d\vec{A}$ along the *top* of the Gaussian surface because now the field vector \vec{E}_1 is directed downward and the area vector $d\vec{A}$ (which, as always, points outward from the interior of a closed Gaussian surface) is directed upward. With 180° between the vectors, the dot product is negative. Now $\kappa = 2.61$. Thus, Eq. 25-37 gives us

$$E_1 = \frac{q}{\epsilon_0 \kappa A} = \frac{E_0}{\kappa} = \frac{6.90 \text{ kV/m}}{2.61} \\ = 2.64 \text{ kV/m.} \quad (\text{Answer})$$

(e) What is the potential difference V between the plates after the slab has been introduced?

KEY IDEA

We find V by integrating along a straight line directly from the bottom plate to the top plate.

Calculation: Within the dielectric, the path length is b and the electric field is E_1 . Within the two gaps above and below the dielectric, the total path length is $d - b$ and the electric field is E_0 . Equation 25-6 then yields

$$V = \int_{-}^{+} E ds = E_0(d - b) + E_1 b \\ = (6900 \text{ V/m})(0.0124 \text{ m} - 0.00780 \text{ m}) \\ + (2640 \text{ V/m})(0.00780 \text{ m}) \\ = 52.3 \text{ V.} \quad (\text{Answer})$$

This is less than the original potential difference of 85.5 V.

(f) What is the capacitance with the slab in place?

KEY IDEA

The capacitance C is related to q and V via Eq. 25-1.

Calculation: Taking q from (b) and V from (e), we have

$$C = \frac{q}{V} = \frac{7.02 \times 10^{-10} \text{ C}}{52.3 \text{ V}} \\ = 1.34 \times 10^{-11} \text{ F} = 13.4 \text{ pF.} \quad (\text{Answer})$$

This is greater than the original capacitance of 8.21 pF.



Review & Summary

Capacitor; Capacitance A **capacitor** consists of two isolated conductors (the *plates*) with charges $+q$ and $-q$. Its **capacitance** C is defined from

$$q = CV, \quad (25-1)$$

where V is the potential difference between the plates.

Determining Capacitance We generally determine the capacitance of a particular capacitor configuration by (1) assuming a charge q to have been placed on the plates, (2) finding the electric field \vec{E} due to this charge, (3) evaluating the potential difference V , and (4) calculating C from Eq. 25-1. Some specific results are the following:

A *parallel-plate capacitor* with flat parallel plates of area A and spacing d has capacitance

$$C = \frac{\epsilon_0 A}{d}. \quad (25-9)$$

A *cylindrical capacitor* (two long coaxial cylinders) of length L and radii a and b has capacitance

$$C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}. \quad (25-14)$$

A *spherical capacitor* with concentric spherical plates of radii a and b has capacitance

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}. \quad (25-17)$$

An *isolated sphere* of radius R has capacitance

$$C = 4\pi\epsilon_0 R. \quad (25-18)$$

Capacitors in Parallel and in Series The **equivalent capacitances** C_{eq} of combinations of individual capacitors connected in **parallel** and in **series** can be found from

$$C_{\text{eq}} = \sum_{j=1}^n C_j \quad (n \text{ capacitors in parallel}) \quad (25-19)$$

$$\text{and} \quad \frac{1}{C_{\text{eq}}} = \sum_{j=1}^n \frac{1}{C_j} \quad (n \text{ capacitors in series}). \quad (25-20)$$

Equivalent capacitances can be used to calculate the capacitances of more complicated series–parallel combinations.

Potential Energy and Energy Density The **electric potential energy** U of a charged capacitor,

$$U = \frac{q^2}{2C} = \frac{1}{2} CV^2, \quad (25-21, 25-22)$$

is equal to the work required to charge the capacitor. This energy can be associated with the capacitor's electric field \vec{E} . By extension we can associate stored energy with any electric field. In vacuum, the **energy density** u , or potential energy per unit volume, within an electric field of magnitude E is given by

$$u = \frac{1}{2}\epsilon_0 E^2. \quad (25-25)$$

Capacitance with a Dielectric If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ , called the **dielectric constant**, which is characteristic of the material. In a region that is completely filled by a dielectric, all electrostatic equations containing ϵ_0 must be modified by replacing ϵ_0 with $\kappa\epsilon_0$.

The effects of adding a dielectric can be understood physically in terms of the action of an electric field on the permanent or induced electric dipoles in the dielectric slab. The result is the formation of induced charges on the surfaces of the dielectric, which results in a weakening of the field within the dielectric for a given amount of free charge on the plates.

Gauss' Law with a Dielectric When a dielectric is present, Gauss' law may be generalized to

$$\epsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q. \quad (25-36)$$

Here q is the free charge; any induced surface charge is accounted for by including the dielectric constant κ inside the integral.

Questions

1 Figure 25-18 shows plots of charge versus potential difference for three parallel-plate capacitors that have the plate areas and separations given in the table. Which plot goes with which capacitor?

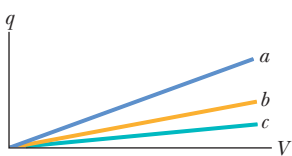


Figure 25-18 Question 1.

Capacitor	Area	Separation
1	A	d
2	$2A$	d
3	A	$2d$

2 What is C_{eq} of three capacitors, each of capacitance C , if they are connected to a battery (a) in series with one another and (b) in parallel? (c) In which arrangement is there more charge on the equivalent capacitance?

3 (a) In Fig. 25-19a, are capacitors 1 and 3 in series? (b) In the same

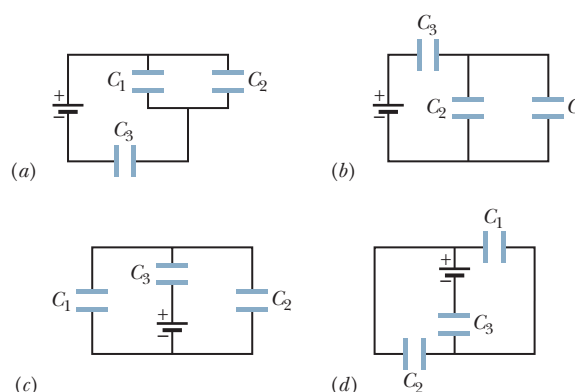


Figure 25-19 Question 3.

figure, are capacitors 1 and 2 in parallel? (c) Rank the equivalent capacitances of the four circuits shown in Fig. 25-19, greatest first.

4 Figure 25-20 shows three circuits, each consisting of a switch and two capacitors, initially charged as indicated (top plate positive). After the switches have been closed, in which circuit (if any) will the charge on the left-hand capacitor (a) increase, (b) decrease, and (c) remain the same?

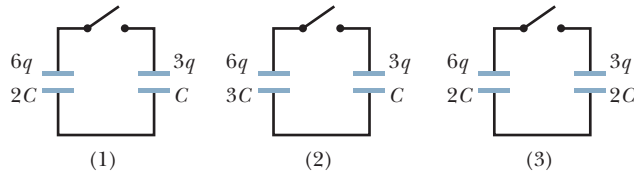


Figure 25-20 Question 4.

5 Initially, a single capacitance C_1 is wired to a battery. Then capacitance C_2 is added in parallel. Are (a) the potential difference across C_1 and (b) the charge q_1 on C_1 now more than, less than, or the same as previously? (c) Is the equivalent capacitance C_{12} of C_1 and C_2 more than, less than, or equal to C_1 ? (d) Is the charge stored on C_1 and C_2 together more than, less than, or equal to the charge stored previously on C_1 ?

6 Repeat Question 5 for C_2 added in series rather than in parallel.

7 For each circuit in Fig. 25-21, are the capacitors connected in series, in parallel, or in neither mode?

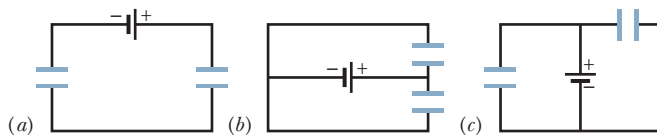


Figure 25-21 Question 7.

8 Figure 25-22 shows an open switch, a battery of potential difference V , a current-measuring meter A , and three identical uncharged capacitors of capacitance C . When the switch is closed and the circuit reaches equilibrium, what are (a) the potential difference across each capacitor and (b) the charge on the left plate of each capacitor? (c) During charging, what net charge passes through the meter?

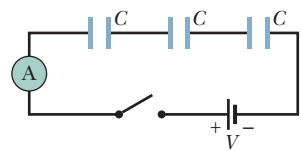


Figure 25-22 Question 8.

9 A parallel-plate capacitor is connected to a battery of electric potential difference V . If the plate separation is decreased, do the following quantities increase, decrease, or remain the same: (a) the capacitor's capacitance, (b) the potential difference across the capacitor, (c) the charge on the capacitor, (d) the energy stored by the capacitor, (e) the magnitude of the electric field between the plates, and (f) the energy density of that electric field?

10 When a dielectric slab is inserted between the plates of one of the two identical capacitors in Fig. 25-23, do the following properties of that capacitor increase, decrease, or remain the same: (a) capacitance, (b) charge, (c) potential difference, and (d) potential energy? (e) How about the same properties of the other capacitor?

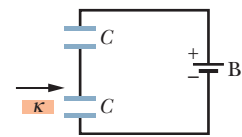


Figure 25-23 Question 10.

11 You are to connect capacitances C_1 and C_2 , with $C_1 > C_2$, to a battery, first individually, then in series, and then in parallel. Rank those arrangements according to the amount of charge stored, greatest first.

Problems

GO Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign

SSM Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at

• • • Number of dots indicates level of problem difficulty

ILW Interactive solution is at

<http://www.wiley.com/college/halliday>

Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Module 25-1 Capacitance

•1 The two metal objects in Fig. 25-24 have net charges of $+70 \text{ pC}$ and -70 pC , which result in a 20 V potential difference between them. (a) What is the capacitance of the system? (b) If the charges are changed to $+200 \text{ pC}$ and -200 pC , what does the capacitance become? (c) What does the potential difference become?

Figure 25-24 Problem 1.



•2 The capacitor in Fig. 25-25 has a capacitance of $25 \mu\text{F}$ and is initially uncharged. The battery provides a potential difference of 120 V . After switch S is closed, how much charge will pass through it?

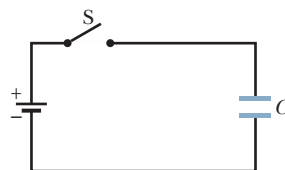


Figure 25-25 Problem 2.

Module 25-2 Calculating the Capacitance

•3 SSM A parallel-plate capacitor has circular plates of 8.20 cm radius and 1.30 mm separation. (a) Calculate the capacitance. (b) Find the charge for a potential difference of 120 V .

•4 The plates of a spherical capacitor have radii 38.0 mm and 40.0 mm . (a) Calculate the capacitance. (b) What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?

•5 What is the capacitance of a drop that results when two mercury spheres, each of radius $R = 2.00 \text{ mm}$, merge?

•6 You have two flat metal plates, each of area 1.00 m^2 , with which to construct a parallel-plate capacitor. (a) If the capacitance of the device is to be 1.00 F , what must be the separation between the plates? (b) Could this capacitor actually be constructed?

•7 If an uncharged parallel-plate capacitor (capacitance C) is connected to a battery, one plate becomes negatively charged

as electrons move to the plate face (area A). In Fig. 25-26, the depth d from which the electrons come in the plate in a particular capacitor is plotted against a range of values for the potential difference V of the battery. The density of conduction electrons in the copper plates is 8.49×10^{28} electrons/m³. The vertical scale is set by $d_s = 1.00$ pm, and the horizontal scale is set by $V_s = 20.0$ V. What is the ratio C/A ?

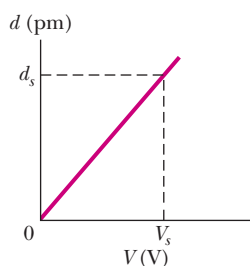


Figure 25-26 Problem 7.

Module 25-3 Capacitors in Parallel and in Series

•8 How many $1.00 \mu\text{F}$ capacitors must be connected in parallel to store a charge of 1.00 C with a potential of 110 V across the capacitors?

•9 Each of the uncharged capacitors in Fig. 25-27 has a capacitance of $25.0 \mu\text{F}$. A potential difference of $V = 4200 \text{ V}$ is established when the switch is closed. How many coulombs of charge then pass through meter A?

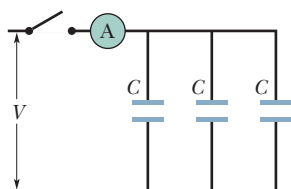


Figure 25-27 Problem 9.

•10 In Fig. 25-28, find the equivalent capacitance of the combination. Assume that C_1 is $10.0 \mu\text{F}$, C_2 is $5.00 \mu\text{F}$, and C_3 is $4.00 \mu\text{F}$.

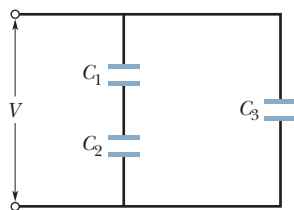


Figure 25-28 Problems 10 and 34.

•11 **ILW** In Fig. 25-29, find the equivalent capacitance of the combination. Assume that $C_1 = 10.0 \mu\text{F}$, $C_2 = 5.00 \mu\text{F}$, and $C_3 = 4.00 \mu\text{F}$.

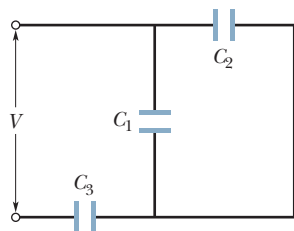


Figure 25-29 Problems 11, 17, and 38.

••12 Two parallel-plate capacitors, $6.0 \mu\text{F}$ each, are connected in parallel to a 10 V battery. One of the capacitors is then squeezed so that its plate separation is 50.0% of its initial value. Because of the squeezing, (a) how much additional charge is transferred to the capacitors by the battery and (b) what is the increase in the total charge stored on the capacitors?

••13 **SSM ILW** A 100 pF capacitor is charged to a potential difference of 50 V , and the charging battery is disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the potential difference across the first

capacitor drops to 35 V , what is the capacitance of this second capacitor?

••14 **GO** In Fig. 25-30, the battery has a potential difference of $V = 10.0 \text{ V}$ and the five capacitors each have a capacitance of $10.0 \mu\text{F}$. What is the charge on (a) capacitor 1 and (b) capacitor 2?

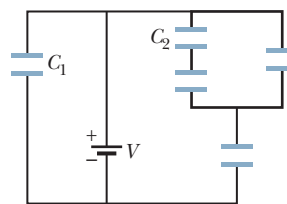


Figure 25-30 Problem 14.

••15 **GO** In Fig. 25-31, a 20.0 V battery is connected across capacitors of capacitances $C_1 = C_6 = 3.00 \mu\text{F}$ and $C_3 = C_5 = 2.00 C_2 = 2.00 C_4 = 4.00 \mu\text{F}$. What are (a) the equivalent capacitance C_{eq} of the capacitors and (b) the charge stored by C_{eq} ? What are (c) V_1 and (d) q_1 of capacitor 1, (e) V_2 and (f) q_2 of capacitor 2, and (g) V_3 and (h) q_3 of capacitor 3?

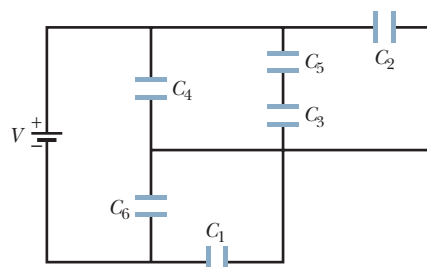


Figure 25-31 Problem 15.

••16 Plot 1 in Fig. 25-32a gives the charge q that can be stored on capacitor 1 versus the electric potential V set up across it. The vertical scale is set by $q_s = 16.0 \mu\text{C}$, and the horizontal scale is set by $V_s = 2.0 \text{ V}$. Plots 2 and 3 are similar plots for capacitors 2 and 3, respectively. Figure 25-32b shows a circuit with those three capacitors and a 6.0 V battery. What is the charge stored on capacitor 2 in that circuit?

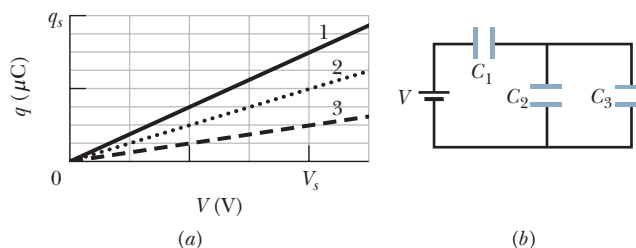


Figure 25-32 Problem 16.

••17 **GO** In Fig. 25-29, a potential difference of $V = 100.0 \text{ V}$ is applied across a capacitor arrangement with capacitances $C_1 = 10.0 \mu\text{F}$, $C_2 = 5.00 \mu\text{F}$, and $C_3 = 4.00 \mu\text{F}$. If capacitor 3 undergoes electrical breakdown so that it becomes equivalent to conducting wire, what is the increase in (a) the charge on capacitor 1 and (b) the potential difference across capacitor 1?

••18 Figure 25-33 shows a circuit section of four air-filled capacitors that is connected to a larger circuit. The graph below the section shows the electric potential $V(x)$ as a function of position x along the lower part of the section, through capacitor 4. Similarly, the graph above the section shows the electric potential $V(x)$ as a function of position x along the upper part of the section, through capacitors 1, 2, and 3.

Capacitor 3 has a capacitance of $0.80\ \mu\text{F}$. What are the capacitances of (a) capacitor 1 and (b) capacitor 2?

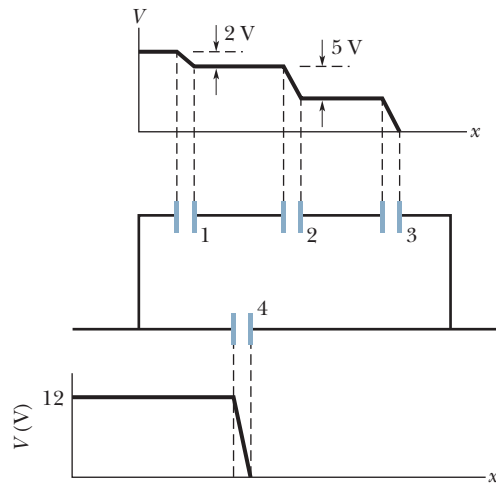


Figure 25-33
Problem 18.

••19 GO In Fig. 25-34, the battery has potential difference $V = 9.0\ \text{V}$, $C_2 = 3.0\ \mu\text{F}$, $C_4 = 4.0\ \mu\text{F}$, and all the capacitors are initially uncharged. When switch S is closed, a total charge of $12\ \mu\text{C}$ passes through point a and a total charge of $8.0\ \mu\text{C}$ passes through point b . What are (a) C_1 and (b) C_3 ?

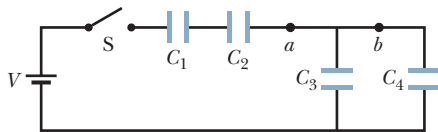


Figure 25-34 Problem 19.

••20 Figure 25-35 shows a variable “air gap” capacitor for manual tuning. Alternate plates are connected together; one group of plates is fixed in position, and the other group is capable of rotation. Consider a capacitor of $n = 8$ plates of alternating polarity, each plate having area $A = 1.25\ \text{cm}^2$ and separated from adjacent plates by distance $d = 3.40\ \text{mm}$. What is the maximum capacitance of the device?

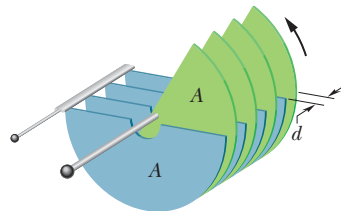


Figure 25-35 Problem 20.

••21 SSM WWW In Fig. 25-36, the capacitances are $C_1 = 1.0\ \mu\text{F}$ and $C_2 = 3.0\ \mu\text{F}$, and both capacitors are charged to a potential difference of $V = 100\ \text{V}$ but with opposite polarity as shown. Switches S_1 and S_2 are now closed. (a) What is now the potential difference between points a and b ? What now is the charge on capacitor (b) 1 and (c) 2?

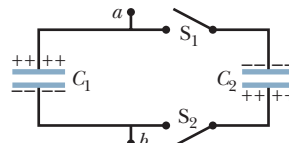


Figure 25-36 Problem 21.

••22 In Fig. 25-37, $V = 10\ \text{V}$, $C_1 = 10\ \mu\text{F}$, and $C_2 = C_3 = 20\ \mu\text{F}$. Switch S is first thrown to the left side until capacitor 1 reaches equilibrium. Then the switch is thrown to the right. When equilibrium is again reached, how much charge is on capacitor 1?

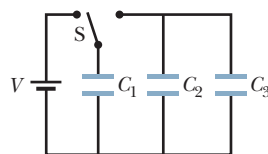


Figure 25-37 Problem 22.

••23 The capacitors in Fig. 25-38 are initially uncharged. The capacitances are $C_1 = 4.0\ \mu\text{F}$, $C_2 = 8.0\ \mu\text{F}$, and $C_3 = 12\ \mu\text{F}$, and the battery's potential difference is $V = 12\ \text{V}$. When switch S is closed, how many electrons travel through (a) point a , (b) point b , (c) point c , and (d) point d ? In the figure, do the electrons travel up or down through (e) point b and (f) point c ?

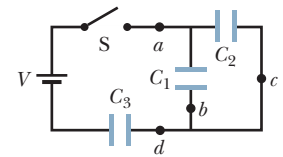


Figure 25-38 Problem 23.

••24 GO Figure 25-39 represents two air-filled cylindrical capacitors connected in series across a battery with potential $V = 10\ \text{V}$. Capacitor 1 has an inner plate radius of $5.0\ \text{mm}$, an outer plate radius of $1.5\ \text{cm}$, and a length of $5.0\ \text{cm}$. Capacitor 2 has an inner plate radius of $2.5\ \text{mm}$, an outer plate radius of $1.0\ \text{cm}$, and a length of $9.0\ \text{cm}$. The outer plate of capacitor 2 is a conducting organic membrane that can be stretched, and the capacitor can be inflated to increase the plate separation. If the outer plate radius is increased to $2.5\ \text{cm}$ by inflation, (a) how many electrons move through point P and (b) do they move toward or away from the battery?

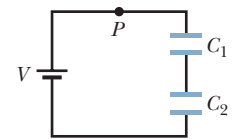


Figure 25-39 Problem 24.

••25 GO In Fig. 25-40, two parallel-plate capacitors (with air between the plates) are connected to a battery. Capacitor 1 has a plate area of $1.5\ \text{cm}^2$ and an electric field (between its plates) of magnitude $2000\ \text{V/m}$. Capacitor 2 has a plate area of $0.70\ \text{cm}^2$ and an electric field of magnitude $1500\ \text{V/m}$. What is the total charge on the two capacitors?

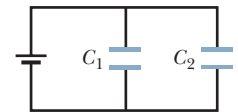


Figure 25-40
Problem 25.

•••26 GO Capacitor 3 in Fig. 25-41a is a variable capacitor (its capacitance C_3 can be varied). Figure 25-41b gives the electric potential V_1 across capacitor 1 versus C_3 . The horizontal scale is set by $C_{3s} = 12.0\ \mu\text{F}$. Electric potential V_1 approaches an asymptote of $10\ \text{V}$ as $C_3 \rightarrow \infty$. What are (a) the electric potential V across the battery, (b) C_1 , and (c) C_2 ?

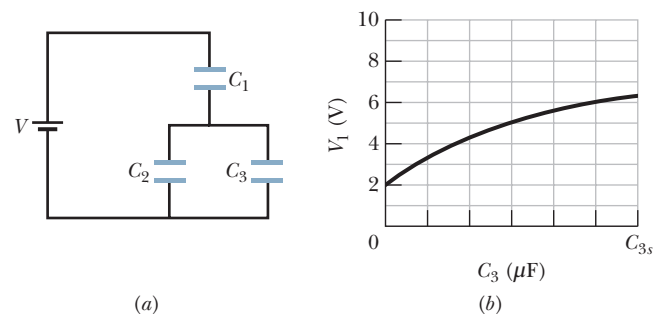


Figure 25-41 Problem 26.

•••27 GO Figure 25-42 shows a $12.0\ \text{V}$ battery and four uncharged capacitors of capacitances $C_1 = 1.00\ \mu\text{F}$, $C_2 = 2.00\ \mu\text{F}$, $C_3 = 3.00\ \mu\text{F}$, and $C_4 = 4.00\ \mu\text{F}$. If only switch S_1 is closed, what is the charge on (a) capacitor 1, (b) capacitor 2, (c) capacitor 3, and (d) capacitor 4? If both switches are closed, what is the charge on (e) capacitor 1, (f) capacitor 2, (g) capacitor 3, and (h) capacitor 4?

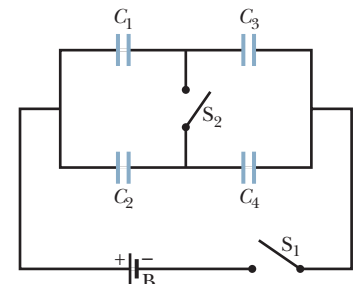



Figure 25-42 Problem 27.

••28  Figure 25-43 displays a 12.0 V battery and 3 uncharged capacitors of capacitances $C_1 = 4.00 \mu\text{F}$, $C_2 = 6.00 \mu\text{F}$, and $C_3 = 3.00 \mu\text{F}$. The switch is thrown to the left side until capacitor 1 is fully charged. Then the switch is thrown to the right. What is the final charge on (a) capacitor 1, (b) capacitor 2, and (c) capacitor 3?

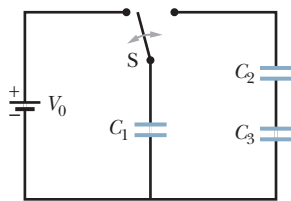



Figure 25-43 Problem 28.

Module 25-4 Energy Stored in an Electric Field

•29 What capacitance is required to store an energy of $10 \text{ kW} \cdot \text{h}$ at a potential difference of 1000 V ?

•30 How much energy is stored in 1.00 m^3 of air due to the “fair weather” electric field of magnitude 150 V/m ?


•31  A $2.0 \mu\text{F}$ capacitor and a $4.0 \mu\text{F}$ capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

•32 A parallel-plate air-filled capacitor having area 40 cm^2 and plate spacing 1.0 mm is charged to a potential difference of 600 V . Find (a) the capacitance, (b) the magnitude of the charge on each plate, (c) the stored energy, (d) the electric field between the plates, and (e) the energy density between the plates.

•33 A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to $V = 0$ at infinity. Calculate the energy density in the electric field near the surface of the sphere.

•34 In Fig. 25-28, a potential difference $V = 100 \text{ V}$ is applied across a capacitor arrangement with capacitances $C_1 = 10.0 \mu\text{F}$, $C_2 = 5.00 \mu\text{F}$, and $C_3 = 4.00 \mu\text{F}$. What are (a) charge q_3 , (b) potential difference V_3 , and (c) stored energy U_3 for capacitor 3, (d) q_1 , (e) V_1 , and (f) U_1 for capacitor 1, and (g) q_2 , (h) V_2 , and (i) U_2 for capacitor 2?

•35 Assume that a stationary electron is a point of charge. What is the energy density u of its electric field at radial distances (a) $r = 1.00 \text{ mm}$, (b) $r = 1.00 \mu\text{m}$, (c) $r = 1.00 \text{ nm}$, and (d) $r = 1.00 \text{ pm}$? (e) What is u in the limit as $r \rightarrow 0$?

•36  As a safety engineer, you must evaluate the practice of storing flammable conducting liquids in nonconducting containers. The company supplying a certain liquid has been using a squat, cylindrical plastic container of radius $r = 0.20 \text{ m}$ and filling it to height $h = 10 \text{ cm}$, which is not the container's full interior height (Fig. 25-44). Your investigation reveals that during handling at the company, the exterior surface of the container commonly acquires a negative charge density of magnitude $2.0 \mu\text{C/m}^2$ (approximately uniform). Because the liquid is a conducting material, the charge on the container induces charge separation within the liquid. (a) How much negative charge is induced in the center of the liquid's bulk? (b) Assume the capacitance of the central portion of the liquid relative to ground is 35 pF . What is the potential energy associated with the negative charge in that effective capacitor? (c) If a spark occurs between the ground and the central portion of the liquid (through the venting port), the potential energy can be fed into the spark. The minimum spark energy needed to ignite the liquid is 10 mJ . In this situation, can a spark ignite the liquid?

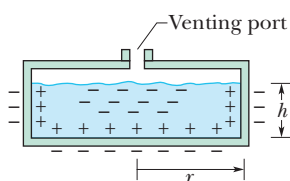






Figure 25-44 Problem 36.

•37    The parallel plates in a capacitor, with a plate area of 8.50 cm^2 and an air-filled separation of 3.00 mm , are charged by a 6.00 V battery. They are then disconnected from the battery and pulled apart (without discharge) to a separation of 8.00 mm . Neglecting fringing, find (a) the potential difference between the plates, (b) the initial stored energy, (c) the final stored energy, and (d) the work required to separate the plates.

•38 In Fig. 25-29, a potential difference $V = 100 \text{ V}$ is applied across a capacitor arrangement with capacitances $C_1 = 10.0 \mu\text{F}$, $C_2 = 5.00 \mu\text{F}$, and $C_3 = 15.0 \mu\text{F}$. What are (a) charge q_3 , (b) potential difference V_3 , and (c) stored energy U_3 for capacitor 3, (d) q_1 , (e) V_1 , and (f) U_1 for capacitor 1, and (g) q_2 , (h) V_2 , and (i) U_2 for capacitor 2?

•39  In Fig. 25-45, $C_1 = 10.0 \mu\text{F}$, $C_2 = 20.0 \mu\text{F}$, and $C_3 = 25.0 \mu\text{F}$. If no capacitor can withstand a potential difference of more than 100 V without failure, what are (a) the magnitude of the maximum potential difference that can exist between points A and B and (b) the maximum energy that can be stored in the three-capacitor arrangement?

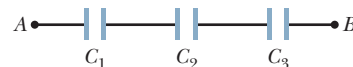



Figure 25-45 Problem 39.

Module 25-5 Capacitor with a Dielectric

•40 An air-filled parallel-plate capacitor has a capacitance of 1.3 pF . The separation of the plates is doubled, and wax is inserted between them. The new capacitance is 2.6 pF . Find the dielectric constant of the wax.

•41  A coaxial cable used in a transmission line has an inner radius of 0.10 mm and an outer radius of 0.60 mm . Calculate the capacitance per meter for the cable. Assume that the space between the conductors is filled with polystyrene.

•42 A parallel-plate air-filled capacitor has a capacitance of 50 pF . (a) If each of its plates has an area of 0.35 m^2 , what is the separation? (b) If the region between the plates is now filled with material having $\kappa = 5.6$, what is the capacitance?

•43 Given a 7.4 pF air-filled capacitor, you are asked to convert it to a capacitor that can store up to $7.4 \mu\text{J}$ with a maximum potential difference of 652 V . Which dielectric in Table 25-1 should you use to fill the gap in the capacitor if you do not allow for a margin of error?

•44 You are asked to construct a capacitor having a capacitance near 1 nF and a breakdown potential in excess of $10\,000 \text{ V}$. You think of using the sides of a tall Pyrex drinking glass as a dielectric, lining the inside and outside curved surfaces with aluminum foil to act as the plates. The glass is 15 cm tall with an inner radius of 3.6 cm and an outer radius of 3.8 cm . What are the (a) capacitance and (b) breakdown potential of this capacitor?

•45 A certain parallel-plate capacitor is filled with a dielectric for which $\kappa = 5.5$. The area of each plate is 0.034 m^2 , and the plates are separated by 2.0 mm . The capacitor will fail (short out and burn up) if the electric field between the plates exceeds 200 kN/C . What is the maximum energy that can be stored in the capacitor?

•46 In Fig. 25-46, how much charge is stored on the parallel-plate capacitors by the 12.0 V battery? One is filled with air, and the other is filled with a dielectric for which $\kappa = 3.00$; both capacitors have a plate area of $5.00 \times 10^{-3} \text{ m}^2$ and a plate separation of 2.00 mm .

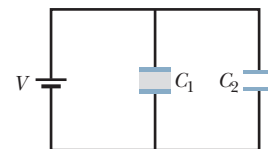


Figure 25-46 Problem 46.

••47 **SSM ILW** A certain substance has a dielectric constant of 2.8 and a dielectric strength of 18 MV/m. If it is used as the dielectric material in a parallel-plate capacitor, what minimum area should the plates of the capacitor have to obtain a capacitance of $7.0 \times 10^{-2} \mu\text{F}$ and to ensure that the capacitor will be able to withstand a potential difference of 4.0 kV?

••48 Figure 25-47 shows a parallel-plate capacitor with a plate area $A = 5.56 \text{ cm}^2$ and separation $d = 5.56 \text{ mm}$. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 7.00$; the right half is filled with material of dielectric constant $\kappa_2 = 12.0$. What is the capacitance?

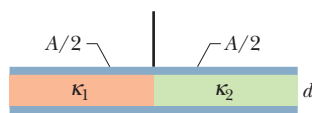


Figure 25-47 Problem 48.

••49 Figure 25-48 shows a parallel-plate capacitor with a plate area $A = 7.89 \text{ cm}^2$ and plate separation $d = 4.62 \text{ mm}$. The top half of the gap is filled with material of dielectric constant $\kappa_1 = 11.0$; the bottom half is filled with material of dielectric constant $\kappa_2 = 12.0$. What is the capacitance?

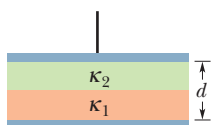


Figure 25-48 Problem 49.

••50 **GO** Figure 25-49 shows a parallel-plate capacitor of plate area $A = 10.5 \text{ cm}^2$ and plate separation $2d = 7.12 \text{ mm}$. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 21.0$; the top of the right half is filled with material of dielectric constant $\kappa_2 = 42.0$; the bottom of the right half is filled with material of dielectric constant $\kappa_3 = 58.0$. What is the capacitance?

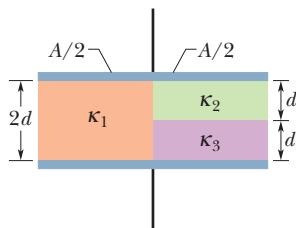


Figure 25-49 Problem 50.

Module 25-6 Dielectrics and Gauss' Law

•51 **SSM WWW** A parallel-plate capacitor has a capacitance of 100 pF, a plate area of 100 cm^2 , and a mica dielectric ($\kappa = 5.4$) completely filling the space between the plates. At 50 V potential difference, calculate (a) the electric field magnitude E in the mica, (b) the magnitude of the free charge on the plates, and (c) the magnitude of the induced surface charge on the mica.

•52 For the arrangement of Fig. 25-17, suppose that the battery remains connected while the dielectric slab is being introduced. Calculate (a) the capacitance, (b) the charge on the capacitor plates, (c) the electric field in the gap, and (d) the electric field in the slab, after the slab is in place.

••53 A parallel-plate capacitor has plates of area 0.12 m^2 and a separation of 1.2 cm. A battery charges the plates to a potential difference of 120 V and is then disconnected. A dielectric slab of thickness 4.0 mm and dielectric constant 4.8 is then placed symmetrically between the plates. (a) What is the capacitance before the slab is inserted? (b) What is the capacitance with the slab in place? What is the free charge q (c) before and (d) after the slab is inserted? What is the magnitude of the electric field (e) in the space between the plates and dielectric and (f) in the dielectric itself? (g) With the slab in place, what is the potential difference across the plates? (h) How much external work is involved in inserting the slab?

••54 Two parallel plates of area 100 cm^2 are given charges of equal magnitudes $8.9 \times 10^{-7} \text{ C}$ but opposite signs. The electric field within the dielectric material filling the space between the plates is $1.4 \times 10^6 \text{ V/m}$. (a) Calculate the dielectric constant of the

material. (b) Determine the magnitude of the charge induced on each dielectric surface.

••55 The space between two concentric conducting spherical shells of radii $b = 1.70 \text{ cm}$ and $a = 1.20 \text{ cm}$ is filled with a substance of dielectric constant $\kappa = 23.5$. A potential difference $V = 73.0 \text{ V}$ is applied across the inner and outer shells. Determine (a) the capacitance of the device, (b) the free charge q on the inner shell, and (c) the charge q' induced along the surface of the inner shell.

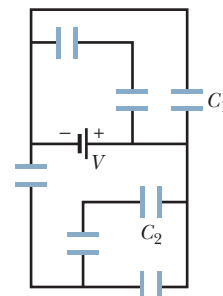


Figure 25-50 Problem 56.

Additional Problems

56 In Fig. 25-50, the battery potential difference V is 10.0 V and each of the seven capacitors has capacitance $10.0 \mu\text{F}$. What is the charge on (a) capacitor 1 and (b) capacitor 2?

57 **SSM** In Fig. 25-51, $V = 9.0 \text{ V}$, $C_1 = C_2 = 30 \mu\text{F}$, and $C_3 = C_4 = 15 \mu\text{F}$. What is the charge on capacitor 4?

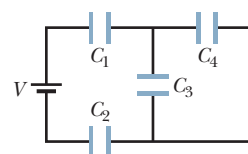


Figure 25-51 Problem 57.

58 (a) If $C = 50 \mu\text{F}$ in Fig. 25-52, what is the equivalent capacitance between points A and B? (Hint: First imagine that a battery is connected between those two points.) (b) Repeat for points A and D.

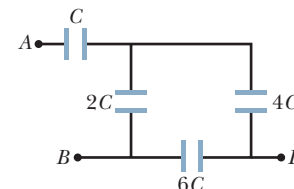


Figure 25-52 Problem 58.

59 In Fig. 25-53, $V = 12 \text{ V}$, $C_1 = C_4 = 2.0 \mu\text{F}$, $C_2 = 4.0 \mu\text{F}$, and $C_3 = 1.0 \mu\text{F}$. What is the charge on capacitor 4?

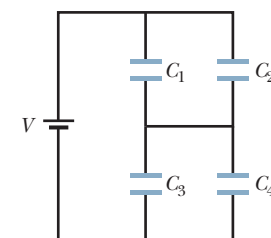


Figure 25-53 Problem 59.

60 **The chocolate crumb mystery.** This story begins with Problem 60 in Chapter 23. As part of the investigation of the biscuit factory explosion, the electric potentials of the workers were measured as they emptied sacks of chocolate crumb powder into the loading bin, stirring up a cloud of the powder around themselves. Each worker had an electric potential of about 7.0 kV relative to the ground, which was taken as zero potential. (a) Assuming that each worker was effectively a capacitor with a typical capacitance of 200 pF, find the energy stored in that effective capacitor. If a single spark between the worker and any conducting object connected to the ground neutralized the worker, that energy would be transferred to the spark. According to measurements, a spark that could ignite a cloud of chocolate crumb powder, and thus set off an explosion, had to have an energy of at least 150 mJ. (b) Could a spark from a worker have set off an explosion in the cloud of powder in the loading bin? (The story continues with Problem 60 in Chapter 26.)

61 Figure 25-54 shows capacitor 1 ($C_1 = 8.00 \mu\text{F}$), capacitor 2 ($C_2 = 6.00 \mu\text{F}$), and capacitor 3 ($C_3 = 8.00 \mu\text{F}$)

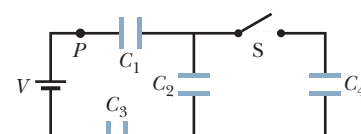



Figure 25-54 Problem 61.

connected to a 12.0 V battery. When switch S is closed so as to connect uncharged capacitor 4 ($C_4 = 6.00 \mu\text{F}$), (a) how much charge passes through point P from the battery and (b) how much charge shows up on capacitor 4? (c) Explain the discrepancy in those two results.

62 Two air-filled, parallel-plate capacitors are to be connected to a 10 V battery, first individually, then in series, and then in parallel. In those arrangements, the energy stored in the capacitors turns out to be, listed least to greatest: $75 \mu\text{J}$, $100 \mu\text{J}$, $300 \mu\text{J}$, and $400 \mu\text{J}$. Of the two capacitors, what is the (a) smaller and (b) greater capacitance?

63 Two parallel-plate capacitors, $6.0 \mu\text{F}$ each, are connected in series to a 10 V battery. One of the capacitors is then squeezed so that its plate separation is halved. Because of the squeezing, (a) how much additional charge is transferred to the capacitors by the battery and (b) what is the increase in the total charge stored on the capacitors (the charge on the positive plate of one capacitor plus the charge on the positive plate of the other capacitor)?

64  In Fig. 25-55, $V = 12 \text{ V}$, $C_1 = C_5 = C_6 = 6.0 \mu\text{F}$, and $C_2 = C_3 = C_4 = 4.0 \mu\text{F}$. What are (a) the net charge stored on the capacitors and (b) the charge on capacitor 4?

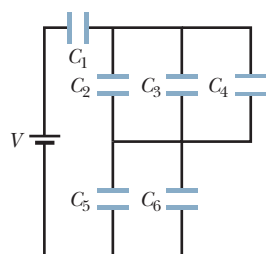



Figure 25-55 Problem 64.

65  In Fig. 25-56, the parallel-plate capacitor of plate area $2.00 \times 10^{-2} \text{ m}^2$ is filled with two dielectric slabs, each with thickness 2.00 mm. One slab has dielectric constant 3.00, and the other, 4.00. How much charge does the 7.00 V battery store on the capacitor?

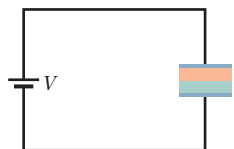


Figure 25-56 Problem 65.

66 A cylindrical capacitor has radii a and b as in Fig. 25-6. Show that half the stored electric potential energy lies within a cylinder whose radius is $r = \sqrt{ab}$.

67 A capacitor of capacitance $C_1 = 6.00 \mu\text{F}$ is connected in series with a capacitor of capacitance $C_2 = 4.00 \mu\text{F}$, and a potential difference of 200 V is applied across the pair. (a) Calculate the equivalent capacitance. What are (b) charge q_1 and (c) potential difference V_1 on capacitor 1 and (d) q_2 and (e) V_2 on capacitor 2?

68 Repeat Problem 67 for the same two capacitors but with them now connected in parallel.

69 A certain capacitor is charged to a potential difference V . If you wish to increase its stored energy by 10%, by what percentage should you increase V ?

70 A slab of copper of thickness $b = 2.00 \text{ mm}$ is thrust into a parallel-plate capacitor of plate area $A = 2.40 \text{ cm}^2$ and plate separation $d = 5.00 \text{ mm}$, as shown in Fig. 25-57; the slab is exactly halfway between the plates. (a) What is the capacitance after the slab is introduced? (b) If a charge $q = 3.40 \mu\text{C}$ is maintained on the plates, what is the ratio of the stored energy before to that after the slab is inserted? (c) How much work is done on the slab as it is inserted? (d) Is the slab sucked in or must it be pushed in?

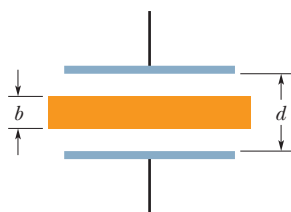


Figure 25-57 Problems 70 and 71.

71 Repeat Problem 70, assuming that a potential difference $V = 85.0 \text{ V}$, rather than the charge, is held constant.

72 A potential difference of 300 V is applied to a series connection of two capacitors of capacitances $C_1 = 2.00 \mu\text{F}$ and $C_2 = 8.00 \mu\text{F}$. What are (a) charge q_1 and (b) potential difference V_1 on capacitor 1 and (c) q_2 and (d) V_2 on capacitor 2? The charged capacitors are then disconnected from each other and from the battery. Then the capacitors are reconnected with plates of the same signs wired together (the battery is not used). What now are (e) q_1 , (f) V_1 , (g) q_2 , and (h) V_2 ? Suppose, instead, the capacitors charged in part (a) are reconnected with plates of opposite signs wired together. What now are (i) q_1 , (j) V_1 , (k) q_2 , and (l) V_2 ?

73 Figure 25-58 shows a four-capacitor arrangement that is connected to a larger circuit at points A and B . The capacitances are $C_1 = 10 \mu\text{F}$ and $C_2 = C_3 = C_4 = 20 \mu\text{F}$. The charge on capacitor 1 is $30 \mu\text{C}$. What is the magnitude of the potential difference $V_A - V_B$?

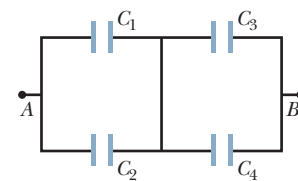



Figure 25-58 Problem 73.

74 You have two plates of copper, a sheet of mica (thickness = 0.10 mm, $\kappa = 5.4$), a sheet of glass (thickness = 2.0 mm, $\kappa = 7.0$), and a slab of paraffin (thickness = 1.0 cm, $\kappa = 2.0$). To make a parallel-plate capacitor with the largest C , which sheet should you place between the copper plates?

75 A capacitor of unknown capacitance C is charged to 100 V and connected across an initially uncharged $60 \mu\text{F}$ capacitor. If the final potential difference across the $60 \mu\text{F}$ capacitor is 40 V, what is C ?

76 A 10 V battery is connected to a series of n capacitors, each of capacitance $2.0 \mu\text{F}$. If the total stored energy is $25 \mu\text{J}$, what is n ?

77  In Fig. 25-59, two parallel-plate capacitors A and B are connected in parallel across a 600 V battery. Each plate has area 80.0 cm^2 ; the plate separations are 3.00 mm. Capacitor A is filled with air; capacitor B is filled with a dielectric of dielectric constant $\kappa = 2.60$. Find the magnitude of the electric field within (a) the dielectric of capacitor B and (b) the air of capacitor A .

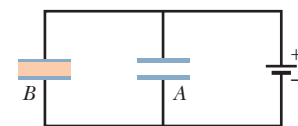


Figure 25-59 Problem 77.

What are the free charge densities σ on the higher-potential plate of (c) capacitor A and (d) capacitor B ? (e) What is the induced charge density σ' on the top surface of the dielectric?

78 You have many $2.0 \mu\text{F}$ capacitors, each capable of withstanding 200 V without undergoing electrical breakdown (in which they conduct charge instead of storing it). How would you assemble a combination having an equivalent capacitance of (a) $0.40 \mu\text{F}$ and (b) $1.2 \mu\text{F}$, each combination capable of withstanding 1000 V?

79 A parallel-plate capacitor has charge q and plate area A . (a) By finding the work needed to increase the plate separation from x to $x + dx$, determine the force between the plates. (Hint: See Eq. 8-22.) (b) Then show that the force per unit area (the electrostatic stress) acting on either plate is equal to the energy density $\epsilon_0 E^2/2$ between the plates.

80 A capacitor is charged until its stored energy is 4.00 J. A second capacitor is then connected to it in parallel. (a) If the charge distributes equally, what is the total energy stored in the electric fields? (b) Where did the missing energy go?

Current and Resistance

26-1 ELECTRIC CURRENT

Learning Objectives

After reading this module, you should be able to . . .

26.01 Apply the definition of current as the rate at which charge moves through a point, including solving for the amount of charge that passes the point in a given time interval.

26.02 Identify that current is normally due to the motion of conduction electrons that are driven by electric fields (such as those set up in a wire by a battery).

26.03 Identify a junction in a circuit and apply the fact that (due to conservation of charge) the total current into a junction must equal the total current out of the junction.

26.04 Explain how current arrows are drawn in a schematic diagram of a circuit, and identify that the arrows are not vectors.

Key Ideas

- An electric current i in a conductor is defined by

$$i = \frac{dq}{dt},$$

where dq is the amount of positive charge that passes in time dt .

- By convention, the direction of electric current is taken as the direction in which positive charge carriers would move even though (normally) only conduction electrons can move.

What Is Physics?

In the last five chapters we discussed electrostatics — the physics of stationary charges. In this and the next chapter, we discuss the physics of **electric currents** — that is, charges in motion.

Examples of electric currents abound and involve many professions. Meteorologists are concerned with lightning and with the less dramatic slow flow of charge through the atmosphere. Biologists, physiologists, and engineers working in medical technology are concerned with the nerve currents that control muscles and especially with how those currents can be reestablished after spinal cord injuries. Electrical engineers are concerned with countless electrical systems, such as power systems, lightning protection systems, information storage systems, and music systems. Space engineers monitor and study the flow of charged particles from our Sun because that flow can wipe out telecommunication systems in orbit and even power transmission systems on the ground. In addition to such scholarly work, almost every aspect of daily life now depends on information carried by electric currents, from stock trades to ATM transfers and from video entertainment to social networking.

In this chapter we discuss the basic physics of electric currents and why they can be established in some materials but not in others. We begin with the meaning of electric current.

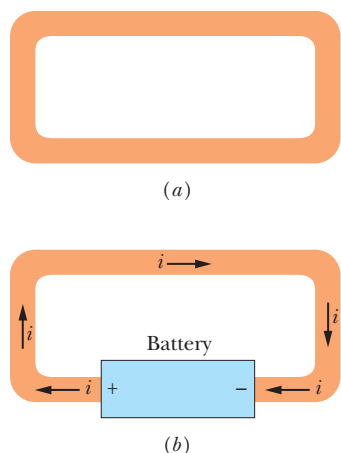


Figure 26-1 (a) A loop of copper in electrostatic equilibrium. The entire loop is at a single potential, and the electric field is zero at all points inside the copper. (b) Adding a battery imposes an electric potential difference between the ends of the loop that are connected to the terminals of the battery. The battery thus produces an electric field within the loop, from terminal to terminal, and the field causes charges to move around the loop. This movement of charges is a current i .

Electric Current

Although an electric current is a stream of moving charges, not all moving charges constitute an electric current. If there is to be an electric current through a given surface, there must be a net flow of charge through that surface. Two examples clarify our meaning.

1. The free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of 10^6 m/s. If you pass a hypothetical plane through such a wire, conduction electrons pass through it *in both directions* at the rate of many billions per second — but there is *no net transport* of charge and thus *no current* through the wire. However, if you connect the ends of the wire to a battery, you slightly bias the flow in one direction, with the result that there now is a net transport of charge and thus an electric current through the wire.
2. The flow of water through a garden hose represents the directed flow of positive charge (the protons in the water molecules) at a rate of perhaps several million coulombs per second. There is no net transport of charge, however, because there is a parallel flow of negative charge (the electrons in the water molecules) of exactly the same amount moving in exactly the same direction.

In this chapter we restrict ourselves largely to the study — within the framework of classical physics — of *steady* currents of *conduction electrons* moving through *metallic conductors* such as copper wires.

As Fig. 26-1a reminds us, any isolated conducting loop — regardless of whether it has an excess charge — is all at the same potential. No electric field can exist within it or along its surface. Although conduction electrons are available, no net electric force acts on them and thus there is no current.

If, as in Fig. 26-1b, we insert a battery in the loop, the conducting loop is no longer at a single potential. Electric fields act inside the material making up the loop, exerting forces on the conduction electrons, causing them to move and thus establishing a current. After a very short time, the electron flow reaches a constant value and the current is in its *steady state* (it does not vary with time).

Figure 26-2 shows a section of a conductor, part of a conducting loop in which current has been established. If charge dq passes through a hypothetical plane (such as aa') in time dt , then the current i through that plane is defined as

$$i = \frac{dq}{dt} \quad (\text{definition of current}). \quad (26-1)$$

We can find the charge that passes through the plane in a time interval extending from 0 to t by integration:

$$q = \int dq = \int_0^t i \, dt, \quad (26-2)$$

in which the current i may vary with time.

Under steady-state conditions, the current is the same for planes aa' , bb' , and cc' and indeed for all planes that pass completely through the conductor, no matter what their location or orientation. This follows from the fact that charge is conserved. Under the steady-state conditions assumed here, an electron must pass through plane aa' for every electron that passes through plane cc' . In the same way, if we have a steady flow of water through a garden hose, a drop of water must leave the nozzle for every drop that enters the hose at the other end. The amount of water in the hose is a conserved quantity.

The SI unit for current is the coulomb per second, or the ampere (A), which is an SI base unit:

$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}.$$

The formal definition of the ampere is discussed in Chapter 29.

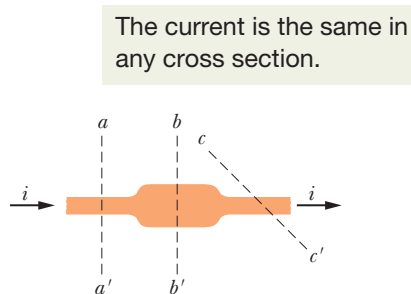


Figure 26-2 The current i through the conductor has the same value at planes aa' , bb' , and cc' .

Current, as defined by Eq. 26-1, is a scalar because both charge and time in that equation are scalars. Yet, as in Fig. 26-1*b*, we often represent a current with an arrow to indicate that charge is moving. Such arrows are not vectors, however, and they do not require vector addition. Figure 26-3*a* shows a conductor with current i_0 splitting at a junction into two branches. Because charge is conserved, the magnitudes of the currents in the branches must add to yield the magnitude of the current in the original conductor, so that

$$i_0 = i_1 + i_2. \quad (26-3)$$

As Fig. 26-3*b* suggests, bending or reorienting the wires in space does not change the validity of Eq. 26-3. Current arrows show only a direction (or sense) of flow along a conductor, not a direction in space.

The Directions of Currents

In Fig. 26-1*b* we drew the current arrows in the direction in which positively charged particles would be forced to move through the loop by the electric field. Such positive *charge carriers*, as they are often called, would move away from the positive battery terminal and toward the negative terminal. Actually, the charge carriers in the copper loop of Fig. 26-1*b* are electrons and thus are negatively charged. The electric field forces them to move in the direction opposite the current arrows, from the negative terminal to the positive terminal. For historical reasons, however, we use the following convention:



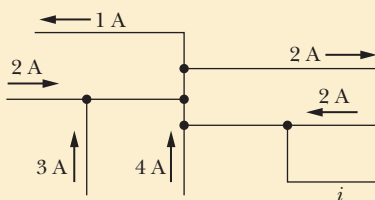
A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

We can use this convention because in *most* situations, the assumed motion of positive charge carriers in one direction has the same effect as the actual motion of negative charge carriers in the opposite direction. (When the effect is not the same, we shall drop the convention and describe the actual motion.)



Checkpoint 1

The figure here shows a portion of a circuit. What are the magnitude and direction of the current i in the lower right-hand wire?



The current into the junction must equal the current out (charge is conserved).

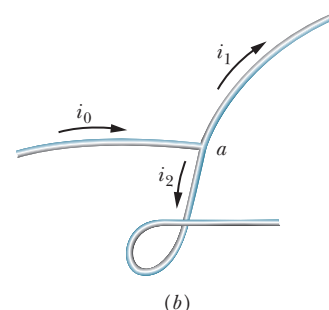
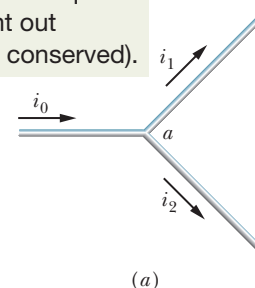


Figure 26-3 The relation $i_0 = i_1 + i_2$ is true at junction a no matter what the orientation in space of the three wires. Currents are scalars, not vectors.

Sample Problem 26.01 Current is the rate at which charge passes a point

Water flows through a garden hose at a volume flow rate dV/dt of $450 \text{ cm}^3/\text{s}$. What is the current of negative charge?

KEY IDEAS

The current i of negative charge is due to the electrons in the water molecules moving through the hose. The current is the rate at which that negative charge passes through any plane that cuts completely across the hose.

Calculations: We can write the current in terms of the number of molecules that pass through such a plane per second as

$$i = \left(\frac{\text{charge}}{\text{electron}} \right) \left(\frac{\text{electrons}}{\text{molecule}} \right) \left(\frac{\text{molecules}}{\text{second}} \right)$$

or
$$i = (e)(10) \frac{dN}{dt}.$$



We substitute 10 electrons per molecule because a water (H_2O) molecule contains 8 electrons in the single oxygen atom and 1 electron in each of the two hydrogen atoms.

We can express the rate dN/dt in terms of the given volume flow rate dV/dt by first writing

$$\left(\begin{array}{c} \text{molecules} \\ \text{per} \\ \text{second} \end{array} \right) = \left(\begin{array}{c} \text{molecules} \\ \text{per} \\ \text{mole} \end{array} \right) \left(\begin{array}{c} \text{moles} \\ \text{per unit} \\ \text{mass} \end{array} \right) \times \left(\begin{array}{c} \text{mass} \\ \text{per unit} \\ \text{volume} \end{array} \right) \left(\begin{array}{c} \text{volume} \\ \text{per} \\ \text{second} \end{array} \right).$$

“Molecules per mole” is Avogadro’s number N_A . “Moles per unit mass” is the inverse of the mass per mole, which is the molar mass M of water. “Mass per unit volume” is the (mass) density ρ_{mass} of water. The volume per second is the volume flow rate dV/dt . Thus, we have

$$\frac{dN}{dt} = N_A \left(\frac{1}{M} \right) \rho_{\text{mass}} \left(\frac{dV}{dt} \right) = \frac{N_A \rho_{\text{mass}}}{M} \frac{dV}{dt}.$$

Substituting this into the equation for i , we find

$$i = 10eN_A M^{-1} \rho_{\text{mass}} \frac{dV}{dt}.$$

We know that Avogadro’s number N_A is 6.02×10^{23} molecules/mol, or $6.02 \times 10^{23} \text{ mol}^{-1}$, and from Table 14-1 we know that the density of water ρ_{mass} under normal conditions is 1000 kg/m^3 . We can get the molar mass of water from the molar masses listed in Appendix F (in grams per mole): We add the molar mass of oxygen (16 g/mol) to twice the molar mass of hydrogen (1 g/mol), obtaining $18 \text{ g/mol} = 0.018 \text{ kg/mol}$. So, the current of negative charge due to the electrons in the water is

$$\begin{aligned} i &= (10)(1.6 \times 10^{-19} \text{ C})(6.02 \times 10^{23} \text{ mol}^{-1}) \\ &\quad \times (0.018 \text{ kg/mol})^{-1}(1000 \text{ kg/m}^3)(450 \times 10^{-6} \text{ m}^3/\text{s}) \\ &= 2.41 \times 10^7 \text{ C/s} = 2.41 \times 10^7 \text{ A} \\ &= 24.1 \text{ MA}. \end{aligned} \quad (\text{Answer})$$

This current of negative charge is exactly compensated by a current of positive charge associated with the nuclei of the three atoms that make up the water molecule. Thus, there is no net flow of charge through the hose.



Additional examples, video, and practice available at WileyPLUS

26-2 CURRENT DENSITY

Learning Objectives

After reading this module, you should be able to . . .

26.05 Identify a current density and a current density vector.

26.06 For current through an area element on a cross section through a conductor (such as a wire), identify the element’s area vector $d\vec{A}$.

26.07 Find the current through a cross section of a conductor by integrating the dot product of the current density vector \vec{J} and the element area vector $d\vec{A}$ over the full cross section.

26.08 For the case where current is uniformly spread over a cross section in a conductor, apply the relationship

between the current i , the current density magnitude J , and the area A .

26.09 Identify streamlines.

26.10 Explain the motion of conduction electrons in terms of their drift speed.

26.11 Distinguish the drift speeds of conduction electrons from their random-motion speeds, including relative magnitudes.

26.12 Identify charge carrier density n .

26.13 Apply the relationship between current density J , charge carrier density n , and charge carrier drift speed v_d .

Key Ideas

● Current i (a scalar quantity) is related to current density \vec{J} (a vector quantity) by

$$i = \int \vec{J} \cdot d\vec{A},$$

where $d\vec{A}$ is a vector perpendicular to a surface element of area dA and the integral is taken over any surface cutting across the conductor. The current density \vec{J} has the same direction as the velocity of the moving

charges if they are positive and the opposite direction if they are negative.

● When an electric field \vec{E} is established in a conductor, the charge carriers (assumed positive) acquire a drift speed v_d in the direction of \vec{E} .

● The drift velocity \vec{v}_d is related to the current density by

$$\vec{J} = (ne)\vec{v}_d,$$

where ne is the carrier charge density.

Current Density

Sometimes we are interested in the current i in a particular conductor. At other times we take a localized view and study the flow of charge through a cross section of the conductor at a particular point. To describe this flow, we can use the **current density** \vec{J} , which has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative. For each element of the cross section, the magnitude J is equal to the current per unit area through that element. We can write the amount of current through the element as $\vec{J} \cdot d\vec{A}$, where $d\vec{A}$ is the area vector of the element, perpendicular to the element. The total current through the surface is then

$$i = \int \vec{J} \cdot d\vec{A}. \quad (26-4)$$

If the current is uniform across the surface and parallel to $d\vec{A}$, then \vec{J} is also uniform and parallel to $d\vec{A}$. Then Eq. 26-4 becomes

$$i = \int J dA = J \int dA = JA, \quad (26-5)$$

so $J = \frac{i}{A},$

where A is the total area of the surface. From Eq. 26-4 or 26-5 we see that the SI unit for current density is the ampere per square meter (A/m^2).

In Chapter 22 we saw that we can represent an electric field with electric field lines. Figure 26-4 shows how current density can be represented with a similar set of lines, which we can call *streamlines*. The current, which is toward the right in Fig. 26-4, makes a transition from the wider conductor at the left to the narrower conductor at the right. Because charge is conserved during the transition, the amount of charge and thus the amount of current cannot change. However, the current density does change—it is greater in the narrower conductor. The spacing of the streamlines suggests this increase in current density; streamlines that are closer together imply greater current density.

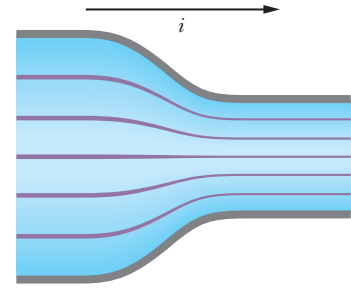


Figure 26-4 Streamlines representing current density in the flow of charge through a constricted conductor.

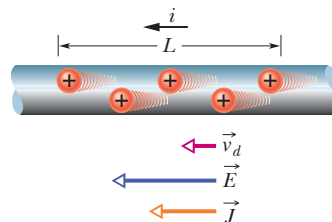
Drift Speed

When a conductor does not have a current through it, its conduction electrons move randomly, with no net motion in any direction. When the conductor does have a current through it, these electrons actually still move randomly, but now they tend to *drift* with a **drift speed** v_d in the direction opposite that of the applied electric field that causes the current. The drift speed is tiny compared with the speeds in the random motion. For example, in the copper conductors of household wiring, electron drift speeds are perhaps 10^{-5} or 10^{-4} m/s, whereas the random-motion speeds are around 10^6 m/s.

We can use Fig. 26-5 to relate the drift speed v_d of the conduction electrons in a current through a wire to the magnitude J of the current density in the wire. For

Figure 26-5 Positive charge carriers drift at speed v_d in the direction of the applied electric field \vec{E} . By convention, the direction of the current density \vec{J} and the sense of the current arrow are drawn in that same direction.

Current is said to be due to positive charges that are propelled by the electric field.



convenience, Fig. 26-5 shows the equivalent drift of *positive* charge carriers in the direction of the applied electric field \vec{E} . Let us assume that these charge carriers all move with the same drift speed v_d and that the current density J is uniform across the wire's cross-sectional area A . The number of charge carriers in a length L of the wire is nAL , where n is the number of carriers per unit volume. The total charge of the carriers in the length L , each with charge e , is then

$$q = (nAL)e.$$

Because the carriers all move along the wire with speed v_d , this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}.$$

Equation 26-1 tells us that the current i is the time rate of transfer of charge across a cross section, so here we have

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d. \quad (26-6)$$

Solving for v_d and recalling Eq. 26-5 ($J = i/A$), we obtain

$$v_d = \frac{i}{nAe} = \frac{J}{ne}$$

or, extended to vector form,

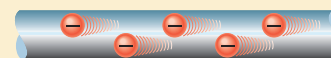
$$\vec{J} = (ne)\vec{v}_d. \quad (26-7)$$

Here the product ne , whose SI unit is the coulomb per cubic meter (C/m^3), is the *carrier charge density*. For positive carriers, ne is positive and Eq. 26-7 predicts that \vec{J} and \vec{v}_d have the same direction. For negative carriers, ne is negative and \vec{J} and \vec{v}_d have opposite directions.



Checkpoint 2

The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward: (a) the current i , (b) the current density \vec{J} , (c) the electric field \vec{E} in the wire?



Sample Problem 26.02 Current density, uniform and nonuniform

(a) The current density in a cylindrical wire of radius $R = 2.0 \text{ mm}$ is uniform across a cross section of the wire and is $J = 2.0 \times 10^5 \text{ A/m}^2$. What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig. 26-6a)?

KEY IDEA

Because the current density is uniform across the cross section, the current density J , the current i , and the cross-sectional area A are related by Eq. 26-5 ($J = i/A$).

Calculations: We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire

area), where

$$\begin{aligned} A' &= \pi R^2 - \pi \left(\frac{R}{2} \right)^2 = \pi \left(\frac{3R^2}{4} \right) \\ &= \frac{3\pi}{4} (0.0020 \text{ m})^2 = 9.424 \times 10^{-6} \text{ m}^2. \end{aligned}$$

So, we rewrite Eq. 26-5 as

$$i = JA'$$

and then substitute the data to find

$$\begin{aligned} i &= (2.0 \times 10^5 \text{ A/m}^2)(9.424 \times 10^{-6} \text{ m}^2) \\ &= 1.9 \text{ A}. \end{aligned} \quad (\text{Answer})$$

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11} \text{ A/m}^4$ and r is in meters. What now is the current through the same outer portion of the wire?

KEY IDEA

Because the current density is not uniform across a cross section of the wire, we must resort to Eq. 26-4 ($i = \int \vec{J} \cdot d\vec{A}$) and integrate the current density over the portion of the wire from $r = R/2$ to $r = R$.

Calculations: The current density vector \vec{J} (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J dA \cos 0 = J dA.$$

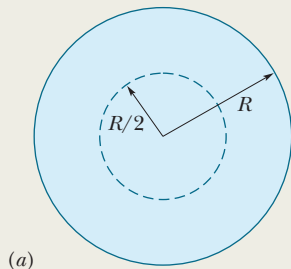
We need to replace the differential area dA with something we can actually integrate between the limits $r = R/2$ and $r = R$. The simplest replacement (because J is given as a function of r) is the area $2\pi r dr$ of a thin ring of circumference $2\pi r$ and width dr (Fig. 26-6b). We can then integrate with r as the variable of integration. Equation 26-4 then gives us

$$\begin{aligned} i &= \int \vec{J} \cdot d\vec{A} = \int J dA \\ &= \int_{R/2}^R ar^2 2\pi r dr = 2\pi a \int_{R/2}^R r^3 dr \\ &= 2\pi a \left[\frac{r^4}{4} \right]_{R/2}^R = \frac{\pi a}{2} \left[R^4 - \frac{R^4}{16} \right] = \frac{15}{32} \pi a R^4 \\ &= \frac{15}{32} \pi (3.0 \times 10^{11} \text{ A/m}^4) (0.0020 \text{ m})^4 = 7.1 \text{ A}. \end{aligned}$$

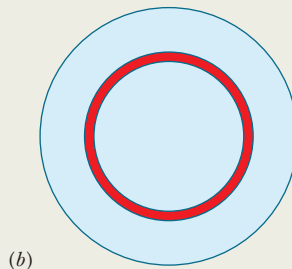
(Answer)



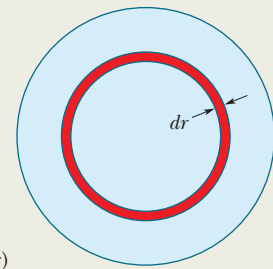
We want the current in the area between these two radii.



If the current is nonuniform, we start with a ring that is so thin that we can approximate the current density as being uniform within it.

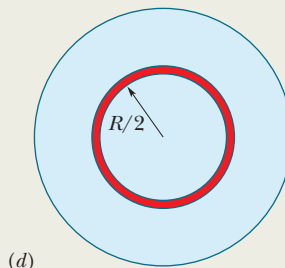


Its area is the product of the circumference and the width.



The current within the ring is the product of the current density and the ring's area.

Our job is to sum the current in all rings from this smallest one ...



... to this largest one.

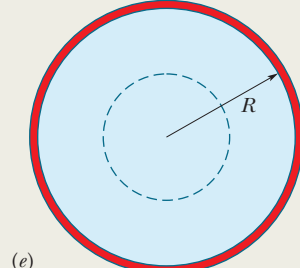


Figure 26-6 (a) Cross section of a wire of radius R . If the current density is uniform, the current is just the product of the current density and the area. (b)–(e) If the current is nonuniform, we must first find the current through a thin ring and then sum (via integration) the currents in all such rings in the given area.





Sample Problem 26.03 In a current, the conduction electrons move very slowly

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900 \mu\text{m}$ when it has a uniform current $i = 17 \text{ mA}$? Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

KEY IDEAS

1. The drift speed v_d is related to the current density \vec{J} and the number n of conduction electrons per unit volume according to Eq. 26-7, which we can write as $J = nev_d$.
2. Because the current density is uniform, its magnitude J is related to the given current i and wire size by Eq. 26-5 ($J = i/A$, where A is the cross-sectional area of the wire).
3. Because we assume one conduction electron per atom, the number n of conduction electrons per unit volume is the same as the number of atoms per unit volume.

Calculations: Let us start with the third idea by writing

$$n = \left(\frac{\text{atoms}}{\text{per unit volume}} \right) = \left(\frac{\text{atoms}}{\text{per mole}} \right) \left(\frac{\text{moles}}{\text{per unit mass}} \right) \left(\frac{\text{mass}}{\text{per unit volume}} \right).$$

The number of atoms per mole is just Avogadro's number N_A ($= 6.02 \times 10^{23} \text{ mol}^{-1}$). Moles per unit mass is the inverse of the mass per mole, which here is the molar mass M of copper. The mass per unit volume is the (mass) density ρ_{mass} of copper. Thus,

$$n = N_A \left(\frac{1}{M} \right) \rho_{\text{mass}} = \frac{N_A \rho_{\text{mass}}}{M}.$$

Taking copper's molar mass M and density ρ_{mass} from Appendix F, we then have (with some conversions of units)

$$\begin{aligned} n &= \frac{(6.02 \times 10^{23} \text{ mol}^{-1})(8.96 \times 10^3 \text{ kg/m}^3)}{63.54 \times 10^{-3} \text{ kg/mol}} \\ &= 8.49 \times 10^{28} \text{ electrons/m}^3 \end{aligned}$$

or $n = 8.49 \times 10^{28} \text{ m}^{-3}.$

Next let us combine the first two key ideas by writing

$$\frac{i}{A} = nev_d.$$

Substituting for A with πr^2 ($= 2.54 \times 10^{-6} \text{ m}^2$) and solving for v_d , we then find

$$\begin{aligned} v_d &= \frac{i}{ne(\pi r^2)} \\ &= \frac{17 \times 10^{-3} \text{ A}}{(8.49 \times 10^{28} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})(2.54 \times 10^{-6} \text{ m}^2)} \\ &= 4.9 \times 10^{-7} \text{ m/s}, \quad (\text{Answer}) \end{aligned}$$

which is only 1.8 mm/h, slower than a sluggish snail.

Lights are fast: You may well ask: “If the electrons drift so slowly, why do the room lights turn on so quickly when I throw the switch?” Confusion on this point results from not distinguishing between the drift speed of the electrons and the speed at which *changes* in the electric field configuration travel along wires. This latter speed is nearly that of light; electrons everywhere in the wire begin drifting almost at once, including into the lightbulbs. Similarly, when you open the valve on your garden hose with the hose full of water, a pressure wave travels along the hose at the speed of sound in water. The speed at which the water itself moves through the hose — measured perhaps with a dye marker — is much slower.



Additional examples, video, and practice available at WileyPLUS



26-3 RESISTANCE AND RESISTIVITY

Learning Objectives

After reading this module, you should be able to . . .

- 26.14 Apply the relationship between the potential difference V applied across an object, the object's resistance R , and the resulting current i through the object, between the application points.
- 26.15 Identify a resistor.
- 26.16 Apply the relationship between the electric field magnitude E set up at a point in a given material, the material's resistivity ρ , and the resulting current density magnitude J at that point.
- 26.17 For a uniform electric field set up in a wire, apply the relationship between the electric field magnitude E ,

the potential difference V between the two ends, and the wire's length L .

- 26.18 Apply the relationship between resistivity ρ and conductivity σ .

- 26.19 Apply the relationship between an object's resistance R , the resistivity of its material ρ , its length L , and its cross-sectional area A .

- 26.20 Apply the equation that approximately gives a conductor's resistivity ρ as a function of temperature T .

- 26.21 Sketch a graph of resistivity ρ versus temperature T for a metal.

Key Ideas

- The resistance R of a conductor is defined as

$$R = \frac{V}{i},$$

where V is the potential difference across the conductor and i is the current.

- The resistivity ρ and conductivity σ of a material are related by

$$\rho = \frac{1}{\sigma} = \frac{E}{J},$$

where E is the magnitude of the applied electric field and J is the magnitude of the current density.

- The electric field and current density are related to the resistivity by

$$\vec{E} = \rho \vec{J}.$$

- The resistance R of a conducting wire of length L and uniform cross section is

$$R = \rho \frac{L}{A},$$

where A is the cross-sectional area.

- The resistivity ρ for most materials changes with temperature. For many materials, including metals, the relation between ρ and temperature T is approximated by the equation

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$

Here T_0 is a reference temperature, ρ_0 is the resistivity at T_0 , and α is the temperature coefficient of resistivity for the material.

Resistance and Resistivity

If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its electrical **resistance**. We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

$$R = \frac{V}{i} \quad (\text{definition of } R). \quad (26-8)$$

The SI unit for resistance that follows from Eq. 26-8 is the volt per ampere. This combination occurs so often that we give it a special name, the **ohm** (symbol Ω); that is,

$$\begin{aligned} 1 \text{ ohm} &= 1 \Omega = 1 \text{ volt per ampere} \\ &= 1 \text{ V/A}. \end{aligned} \quad (26-9)$$

A conductor whose function in a circuit is to provide a specified resistance is called a **resistor** (see Fig. 26-7). In a circuit diagram, we represent a resistor and a resistance with the symbol $\sim\sim\sim$. If we write Eq. 26-8 as

$$i = \frac{V}{R},$$

we see that, for a given V , the greater the resistance, the smaller the current.

The resistance of a conductor depends on the manner in which the potential difference is applied to it. Figure 26-8, for example, shows a given potential difference applied in two different ways to the same conductor. As the current density streamlines suggest, the currents in the two cases — hence the measured resistances — will be different. Unless otherwise stated, we shall assume that any given potential difference is applied as in Fig. 26-8b.



Figure 26-8 Two ways of applying a potential difference to a conducting rod. The gray connectors are assumed to have negligible resistance. When they are arranged as in (a) in a small region at each rod end, the measured resistance is larger than when they are arranged as in (b) to cover the entire rod end.

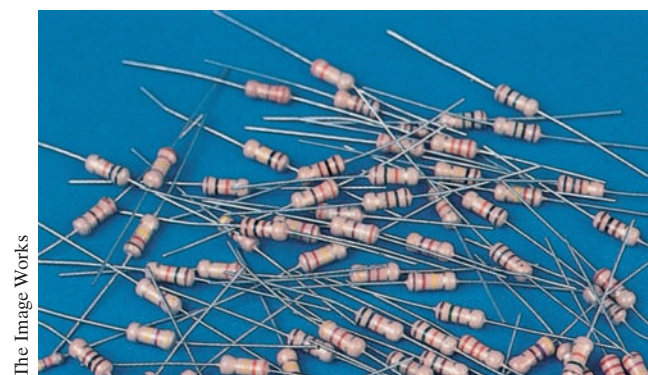


Figure 26-7 An assortment of resistors. The circular bands are color-coding marks that identify the value of the resistance.

Table 26-1 Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot \text{m}$)	Temperature Coefficient of Resistivity, α (K^{-1})
<i>Typical Metals</i>		
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin ^a	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
<i>Typical Semiconductors</i>		
Silicon, pure	2.5×10^3	-70×10^{-3}
Silicon, <i>n</i> -type ^b	8.7×10^{-4}	
Silicon, <i>p</i> -type ^c	2.8×10^{-3}	
<i>Typical Insulators</i>		
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

^aAn alloy specifically designed to have a small value of α .

^bPure silicon doped with phosphorus impurities to a charge carrier density of 10^{23} m^{-3} .

^cPure silicon doped with aluminum impurities to a charge carrier density of 10^{23} m^{-3} .

As we have done several times in other connections, we often wish to take a general view and deal not with particular objects but with materials. Here we do so by focusing not on the potential difference V across a particular resistor but on the electric field \vec{E} at a point in a resistive material. Instead of dealing with the current i through the resistor, we deal with the current density \vec{J} at the point in question. Instead of the resistance R of an object, we deal with the **resistivity** ρ of the *material*:

$$\rho = \frac{E}{J} \quad (\text{definition of } \rho). \quad (26-10)$$

(Compare this equation with Eq. 26-8.)

If we combine the SI units of E and J according to Eq. 26-10, we get, for the unit of ρ , the ohm-meter ($\Omega \cdot \text{m}$):

$$\frac{\text{unit}(E)}{\text{unit}(J)} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}} \text{m} = \Omega \cdot \text{m}.$$

(Do not confuse the *ohm-meter*, the unit of resistivity, with the *ohmmeter*, which is an instrument that measures resistance.) Table 26-1 lists the resistivities of some materials.

We can write Eq. 26-10 in vector form as

$$\vec{E} = \rho \vec{J}. \quad (26-11)$$

Equations 26-10 and 26-11 hold only for *isotropic* materials — materials whose electrical properties are the same in all directions.

We often speak of the **conductivity** σ of a material. This is simply the reciprocal of its resistivity, so

$$\sigma = \frac{1}{\rho} \quad (\text{definition of } \sigma). \quad (26-12)$$

The SI unit of conductivity is the reciprocal ohm-meter, $(\Omega \cdot \text{m})^{-1}$. The unit name mhos per meter is sometimes used (mho is ohm backwards). The definition of σ allows us to write Eq. 26-11 in the alternative form

$$\vec{J} = \sigma \vec{E}. \quad (26-13)$$

Calculating Resistance from Resistivity

We have just made an important distinction:



Resistance is a property of an object. Resistivity is a property of a material.

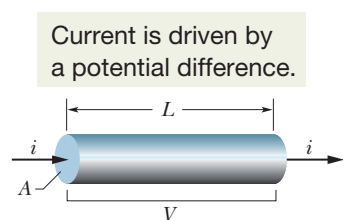


Figure 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A , establishing a current i .

If we know the resistivity of a substance such as copper, we can calculate the resistance of a length of wire made of that substance. Let A be the cross-sectional area of the wire, let L be its length, and let a potential difference V exist between its ends (Fig. 26-9). If the streamlines representing the current density are uniform throughout the wire, the electric field and the current density will be constant for all points within the wire and, from Eqs. 24-42 and 26-5, will have the values

$$E = V/L \quad \text{and} \quad J = i/A. \quad (26-14)$$

We can then combine Eqs. 26-10 and 26-14 to write

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}. \quad (26-15)$$

However, V/i is the resistance R , which allows us to recast Eq. 26-15 as

$$R = \rho \frac{L}{A}. \quad (26-16)$$

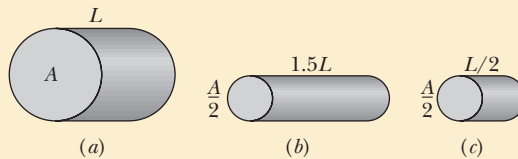
Equation 26-16 can be applied only to a homogeneous isotropic conductor of uniform cross section, with the potential difference applied as in Fig. 26-8b.

The macroscopic quantities V , i , and R are of greatest interest when we are making electrical measurements on specific conductors. They are the quantities that we read directly on meters. We turn to the microscopic quantities E , J , and ρ when we are interested in the fundamental electrical properties of materials.



Checkpoint 3

The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference V is placed across their lengths.



Variation with Temperature

The values of most physical properties vary with temperature, and resistivity is no exception. Figure 26-10, for example, shows the variation of this property for copper over a wide temperature range. The relation between temperature and resistivity for copper — and for metals in general — is fairly linear over a rather broad temperature range. For such linear relations we can write an empirical approximation that is good enough for most engineering purposes:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0). \quad (26-17)$$

Here T_0 is a selected reference temperature and ρ_0 is the resistivity at that temperature. Usually $T_0 = 293 \text{ K}$ (room temperature), for which $\rho_0 = 1.69 \times 10^{-8} \Omega \cdot \text{m}$ for copper.

Because temperature enters Eq. 26-17 only as a difference, it does not matter whether you use the Celsius or Kelvin scale in that equation because the sizes of degrees on these scales are identical. The quantity α in Eq. 26-17, called the *temperature coefficient of resistivity*, is chosen so that the equation gives good agreement with experiment for temperatures in the chosen range. Some values of α for metals are listed in Table 26-1.

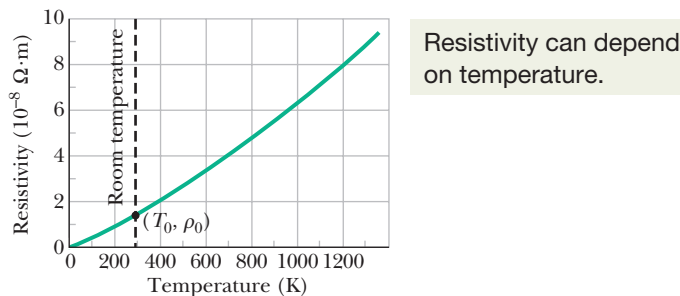


Figure 26-10 The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature $T_0 = 293 \text{ K}$ and resistivity $\rho_0 = 1.69 \times 10^{-8} \Omega \cdot \text{m}$.



Sample Problem 26.04 A material has resistivity, a block of the material has resistance

A rectangular block of iron has dimensions $1.2\text{ cm} \times 1.2\text{ cm} \times 15\text{ cm}$. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8b). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions $1.2\text{ cm} \times 1.2\text{ cm}$) and (2) two rectangular sides (with dimensions $1.2\text{ cm} \times 15\text{ cm}$)?

KEY IDEA

The resistance R of an object depends on how the electric potential is applied to the object. In particular, it depends on the ratio L/A , according to Eq. 26-16 ($R = \rho L/A$), where A is the area of the surfaces to which the potential difference is applied and L is the distance between those surfaces.

Calculations: For arrangement 1, we have $L = 15\text{ cm} = 0.15\text{ m}$ and

$$A = (1.2\text{ cm})^2 = 1.44 \times 10^{-4}\text{ m}^2.$$

Substituting into Eq. 26-16 with the resistivity ρ from Table 26-1, we then find that for arrangement 1,

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega \cdot \text{m})(0.15\text{ m})}{1.44 \times 10^{-4}\text{ m}^2} \\ = 1.0 \times 10^{-4}\ \Omega = 100\ \mu\Omega. \quad (\text{Answer})$$

Similarly, for arrangement 2, with distance $L = 1.2\text{ cm}$ and area $A = (1.2\text{ cm})(15\text{ cm})$, we obtain

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega \cdot \text{m})(1.2 \times 10^{-2}\text{ m})}{1.80 \times 10^{-3}\text{ m}^2} \\ = 6.5 \times 10^{-7}\ \Omega = 0.65\ \mu\Omega. \quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS



26-4 OHM'S LAW

Learning Objectives

After reading this module, you should be able to . . .

- 26.22** Distinguish between an *object* that obeys Ohm's law and one that does not.
- 26.23** Distinguish between a *material* that obeys Ohm's law and one that does not.
- 26.24** Describe the general motion of a conduction electron in a current.

Key Ideas

- A given device (conductor, resistor, or any other electrical device) obeys Ohm's law if its resistance $R (= V/i)$ is independent of the applied potential difference V .
- A given material obeys Ohm's law if its resistivity $\rho (= E/J)$ is independent of the magnitude and direction of the applied electric field \vec{E} .
- The assumption that the conduction electrons in a metal are free to move like the molecules in a gas leads

- 26.25** For the conduction electrons in a conductor, explain the relationship between the mean free time τ , the effective speed, and the thermal (random) motion.

- 26.26** Apply the relationship between resistivity ρ , number density n of conduction electrons, and the mean free time τ of the electrons.

to an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}.$$

Here n is the number of free electrons per unit volume and τ is the mean time between the collisions of an electron with the atoms of the metal.

- Metals obey Ohm's law because the mean free time τ is approximately independent of the magnitude E of any electric field applied to a metal.

Ohm's Law

As we just discussed, a resistor is a conductor with a specified resistance. It has that same resistance no matter what the magnitude and direction (*polarity*) of the applied potential difference are. Other conducting devices, however, might have resistances that change with the applied potential difference.

Figure 26-11a shows how to distinguish such devices. A potential difference V is applied across the device being tested, and the resulting current i through the device is measured as V is varied in both magnitude and polarity. The polarity of V is arbitrarily taken to be positive when the left terminal of the device is at a higher potential than the right terminal. The direction of the resulting current (from left to right) is arbitrarily assigned a plus sign. The reverse polarity of V (with the right terminal at a higher potential) is then negative; the current it causes is assigned a minus sign.

Figure 26-11b is a plot of i versus V for one device. This plot is a straight line passing through the origin, so the ratio i/V (which is the slope of the straight line) is the same for all values of V . This means that the resistance $R = V/i$ of the device is independent of the magnitude and polarity of the applied potential difference V .

Figure 26-11c is a plot for another conducting device. Current can exist in this device only when the polarity of V is positive and the applied potential difference is more than about 1.5 V. When current does exist, the relation between i and V is not linear; it depends on the value of the applied potential difference V .

We distinguish between the two types of device by saying that one obeys Ohm’s law and the other does not.



Ohm’s law is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

(This assertion is correct only in certain situations; still, for historical reasons, the term “law” is used.) The device of Fig. 26-11b — which turns out to be a 1000 Ω resistor — obeys Ohm’s law. The device of Fig. 26-11c — which is called a *pn* junction diode — does not.



A conducting device obeys Ohm’s law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

It is often contended that $V = iR$ is a statement of Ohm’s law. That is not true! This equation is the defining equation for resistance, and it applies to all conducting devices, whether they obey Ohm’s law or not. If we measure the potential difference V across, and the current i through, any device, even a *pn* junction diode, we can find its resistance *at that value of V* as $R = V/i$. The essence of Ohm’s law, however, is that a plot of i versus V is linear; that is, R is independent of V . We can generalize this for conducting materials by using Eq. 26-11 ($\vec{E} = \rho \vec{J}$):



A conducting material obeys Ohm’s law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

All homogeneous materials, whether they are conductors like copper or semiconductors like pure silicon or silicon containing special impurities, obey Ohm’s law within some range of values of the electric field. If the field is too strong, however, there are departures from Ohm’s law in all cases.



Checkpoint 4

The following table gives the current i (in amperes) through two devices for several values of potential difference V (in volts). From these data, determine which device does not obey Ohm’s law.

Device 1		Device 2	
V	i	V	i
2.00	4.50	2.00	1.50
3.00	6.75	3.00	2.20
4.00	9.00	4.00	2.80

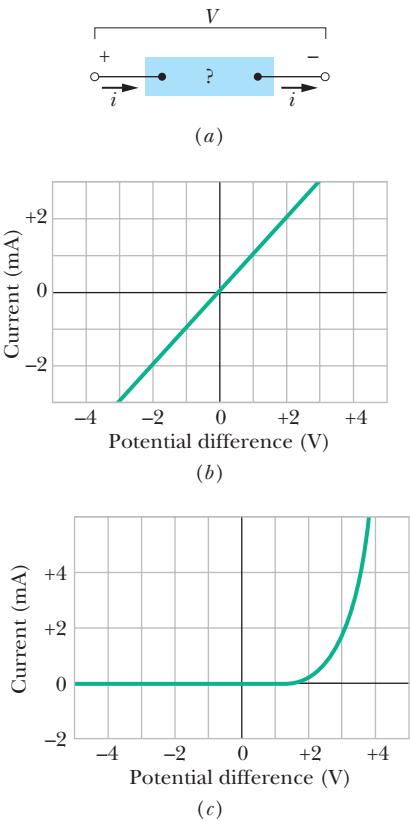


Figure 26-11 (a) A potential difference V is applied to the terminals of a device, establishing a current i . (b) A plot of current i versus applied potential difference V when the device is a 1000 Ω resistor. (c) A plot when the device is a semiconducting *pn* junction diode.

A Microscopic View of Ohm's Law

To find out *why* particular materials obey Ohm's law, we must look into the details of the conduction process at the atomic level. Here we consider only conduction in metals, such as copper. We base our analysis on the *free-electron model*, in which we assume that the conduction electrons in the metal are free to move throughout the volume of a sample, like the molecules of a gas in a closed container. We also assume that the electrons collide not with one another but only with atoms of the metal.

According to classical physics, the electrons should have a Maxwellian speed distribution somewhat like that of the molecules in a gas (Module 19-6), and thus the average electron speed should depend on the temperature. The motions of electrons are, however, governed not by the laws of classical physics but by those of quantum physics. As it turns out, an assumption that is much closer to the quantum reality is that conduction electrons in a metal move with a single effective speed v_{eff} , and this speed is essentially independent of the temperature. For copper, $v_{\text{eff}} \approx 1.6 \times 10^6$ m/s.

When we apply an electric field to a metal sample, the electrons modify their random motions slightly and drift very slowly — in a direction opposite that of the field — with an average drift speed v_d . The drift speed in a typical metallic conductor is about 5×10^{-7} m/s, less than the effective speed (1.6×10^6 m/s) by many orders of magnitude. Figure 26-12 suggests the relation between these two speeds. The gray lines show a possible random path for an electron in the absence of an applied field; the electron proceeds from A to B , making six collisions along the way. The green lines show how the same events *might* occur when an electric field \vec{E} is applied. We see that the electron drifts steadily to the right, ending at B' rather than at B . Figure 26-12 was drawn with the assumption that $v_d \approx 0.02v_{\text{eff}}$. However, because the actual value is more like $v_d \approx (10^{-13})v_{\text{eff}}$, the drift displayed in the figure is greatly exaggerated.

The motion of conduction electrons in an electric field \vec{E} is thus a combination of the motion due to random collisions and that due to \vec{E} . When we consider all the free electrons, their random motions average to zero and make no contribution to the drift speed. Thus, the drift speed is due only to the effect of the electric field on the electrons.

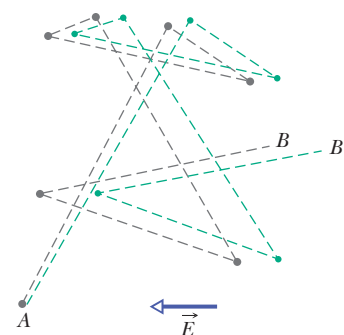
If an electron of mass m is placed in an electric field of magnitude E , the electron will experience an acceleration given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m}. \quad (26-18)$$

After a typical collision, each electron will — so to speak — completely lose its memory of its previous drift velocity, starting fresh and moving off in a random direction. In the average time τ between collisions, the average electron will acquire a drift speed of $v_d = a\tau$. Moreover, if we measure the drift speeds of all the electrons at any instant, we will find that their average drift speed is also $a\tau$. Thus, at any instant, on average, the electrons will have drift speed $v_d = a\tau$. Then Eq. 26-18 gives us

$$v_d = a\tau = \frac{eE\tau}{m}. \quad (26-19)$$

Figure 26-12 The gray lines show an electron moving from A to B , making six collisions en route. The green lines show what the electron's path *might* be in the presence of an applied electric field \vec{E} . Note the steady drift in the direction of $-\vec{E}$. (Actually, the green lines should be slightly curved, to represent the parabolic paths followed by the electrons between collisions, under the influence of an electric field.)



Combining this result with Eq. 26-7 ($\vec{J} = ne\vec{v}_d$), in magnitude form, yields

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m}, \quad (26-20)$$

which we can write as

$$E = \left(\frac{m}{e^2 n \tau} \right) J. \quad (26-21)$$

Comparing this with Eq. 26-11 ($\vec{E} = \rho \vec{J}$), in magnitude form, leads to

$$\rho = \frac{m}{e^2 n \tau}. \quad (26-22)$$

Equation 26-22 may be taken as a statement that metals obey Ohm's law if we can show that, for metals, their resistivity ρ is a constant, independent of the strength of the applied electric field \vec{E} . Let's consider the quantities in Eq. 26-22. We can reasonably assume that n , the number of conduction electrons per volume, is independent of the field, and m and e are constants. Thus, we only need to convince ourselves that τ , the average time (or *mean free time*) between collisions, is a constant, independent of the strength of the applied electric field. Indeed, τ can be considered to be a constant because the drift speed v_d caused by the field is so much smaller than the effective speed v_{eff} that the electron speed — and thus τ — is hardly affected by the field. Thus, because the right side of Eq. 26-22 is independent of the field magnitude, metals obey Ohm's law.

Sample Problem 26.05 Mean free time and mean free distance

(a) What is the mean free time τ between collisions for the conduction electrons in copper?

KEY IDEAS

The mean free time τ of copper is approximately constant, and in particular does not depend on any electric field that might be applied to a sample of the copper. Thus, we need not consider any particular value of applied electric field. However, because the resistivity ρ displayed by copper under an electric field depends on τ , we can find the mean free time τ from Eq. 26-22 ($\rho = m/e^2 n \tau$).

Calculations: That equation gives us

$$\tau = \frac{m}{ne^2 \rho}. \quad (26-23)$$

The number of conduction electrons per unit volume in copper is $8.49 \times 10^{28} \text{ m}^{-3}$. We take the value of ρ from Table 26-1. The denominator then becomes

$$\begin{aligned} (8.49 \times 10^{28} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})^2(1.69 \times 10^{-8} \Omega \cdot \text{m}) \\ = 3.67 \times 10^{-17} \text{ C}^2 \cdot \Omega / \text{m}^2 = 3.67 \times 10^{-17} \text{ kg/s}, \end{aligned}$$

where we converted units as

$$\frac{\text{C}^2 \cdot \Omega}{\text{m}^2} = \frac{\text{C}^2 \cdot \text{V}}{\text{m}^2 \cdot \text{A}} = \frac{\text{C}^2 \cdot \text{J/C}}{\text{m}^2 \cdot \text{C/s}} = \frac{\text{kg} \cdot \text{m}^2/\text{s}^2}{\text{m}^2/\text{s}} = \frac{\text{kg}}{\text{s}}.$$

Using these results and substituting for the electron mass m , we then have

$$\tau = \frac{9.1 \times 10^{-31} \text{ kg}}{3.67 \times 10^{-17} \text{ kg/s}} = 2.5 \times 10^{-14} \text{ s. (Answer)}$$

(b) The mean free path λ of the conduction electrons in a conductor is the average distance traveled by an electron between collisions. (This definition parallels that in Module 19-5 for the mean free path of molecules in a gas.) What is λ for the conduction electrons in copper, assuming that their effective speed v_{eff} is $1.6 \times 10^6 \text{ m/s}$?

KEY IDEA

The distance d any particle travels in a certain time t at a constant speed v is $d = vt$.

Calculation: For the electrons in copper, this gives us

$$\begin{aligned} \lambda &= v_{\text{eff}} \tau \\ &= (1.6 \times 10^6 \text{ m/s})(2.5 \times 10^{-14} \text{ s}) \\ &= 4.0 \times 10^{-8} \text{ m} = 40 \text{ nm. (Answer)} \end{aligned} \quad (26-24)$$

This is about 150 times the distance between nearest-neighbor atoms in a copper lattice. Thus, on the average, each conduction electron passes many copper atoms before finally hitting one.



26-5 POWER, SEMICONDUCTORS, SUPERCONDUCTORS

Learning Objectives

After reading this module, you should be able to . . .

- 26.27** Explain how conduction electrons in a circuit lose energy in a resistive device.
- 26.28** Identify that power is the rate at which energy is transferred from one type to another.
- 26.29** For a resistive device, apply the relationships between power P , current i , voltage V , and resistance R .

26.30 For a battery, apply the relationship between power P , current i , and potential difference V .

26.31 Apply the conservation of energy to a circuit with a battery and a resistive device to relate the energy transfers in the circuit.

26.32 Distinguish conductors, semiconductors, and superconductors.

Key Ideas

- The power P , or rate of energy transfer, in an electrical device across which a potential difference V is maintained is

$$P = iV.$$

- If the device is a resistor, the power can also be written as

$$P = i^2 R = \frac{V^2}{R}.$$

- In a resistor, electric potential energy is converted to

internal thermal energy via collisions between charge carriers and atoms.

- Semiconductors are materials that have few conduction electrons but can become conductors when they are doped with other atoms that contribute charge carriers.

- Superconductors are materials that lose all electrical resistance. Most such materials require very low temperatures, but some become superconducting at temperatures as high as room temperature.

Power in Electric Circuits

Figure 26-13 shows a circuit consisting of a battery B that is connected by wires, which we assume have negligible resistance, to an unspecified conducting device. The device might be a resistor, a storage battery (a rechargeable battery), a motor, or some other electrical device. The battery maintains a potential difference of magnitude V across its own terminals and thus (because of the wires) across the terminals of the unspecified device, with a greater potential at terminal a of the device than at terminal b .

Because there is an external conducting path between the two terminals of the battery, and because the potential differences set up by the battery are maintained, a steady current i is produced in the circuit, directed from terminal a to terminal b . The amount of charge dq that moves between those terminals in time interval dt is equal to $i dt$. This charge dq moves through a decrease in potential of magnitude V , and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq V = i dt V. \quad (26-25)$$

The principle of conservation of energy tells us that the decrease in electric potential energy from a to b is accompanied by a transfer of energy to some other form. The power P associated with that transfer is the rate of transfer dU/dt , which is given by Eq. 26-25 as

$$P = iV \quad (\text{rate of electrical energy transfer}). \quad (26-26)$$

Moreover, this power P is also the rate at which energy is transferred from the battery to the unspecified device. If that device is a motor connected to a mechanical load, the energy is transferred as work done on the load. If the device is a storage battery that is being charged, the energy is transferred to stored chemical energy in the storage battery. If the device is a resistor, the energy is transferred to internal thermal energy, tending to increase the resistor's temperature.

The battery at the left supplies energy to the conduction electrons that form the current.

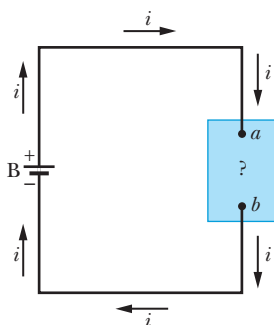


Figure 26-13 A battery B sets up a current i in a circuit containing an unspecified conducting device.

The unit of power that follows from Eq. 26-26 is the volt-ampere ($\text{V} \cdot \text{A}$). We can write it as

$$1 \text{ V} \cdot \text{A} = \left(1 \frac{\text{J}}{\text{C}}\right) \left(1 \frac{\text{C}}{\text{s}}\right) = 1 \frac{\text{J}}{\text{s}} = 1 \text{ W}.$$

As an electron moves through a resistor at constant drift speed, its average kinetic energy remains constant and its lost electric potential energy appears as thermal energy in the resistor and the surroundings. On a microscopic scale this energy transfer is due to collisions between the electron and the molecules of the resistor, which leads to an increase in the temperature of the resistor lattice. The mechanical energy thus transferred to thermal energy is *dissipated* (lost) because the transfer cannot be reversed.

For a resistor or some other device with resistance R , we can combine Eqs. 26-8 ($R = V/i$) and 26-26 to obtain, for the rate of electrical energy dissipation due to a resistance, either

$$P = i^2 R \quad (\text{resistive dissipation}) \quad (26-27)$$

$$\text{or} \quad P = \frac{V^2}{R} \quad (\text{resistive dissipation}). \quad (26-28)$$

Caution: We must be careful to distinguish these two equations from Eq. 26-26: $P = iV$ applies to electrical energy transfers of all kinds; $P = i^2 R$ and $P = V^2/R$ apply only to the transfer of electric potential energy to thermal energy in a device with resistance.



Checkpoint 5

A potential difference V is connected across a device with resistance R , causing current i through the device. Rank the following variations according to the change in the rate at which electrical energy is converted to thermal energy due to the resistance, greatest change first: (a) V is doubled with R unchanged, (b) i is doubled with R unchanged, (c) R is doubled with V unchanged, (d) R is doubled with i unchanged.

Sample Problem 26.06 Rate of energy dissipation in a wire carrying current

You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of $72 \, \Omega$. At what rate is energy dissipated in each of the following situations? (1) A potential difference of 120 V is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of 120 V is applied across the length of each half.

KEY IDEA

Current in a resistive material produces a transfer of mechanical energy to thermal energy; the rate of transfer (dissipation) is given by Eqs. 26-26 to 26-28.

Calculations: Because we know the potential V and resistance R , we use Eq. 26-28, which yields, for situation 1,

$$P = \frac{V^2}{R} = \frac{(120 \text{ V})^2}{72 \, \Omega} = 200 \text{ W}. \quad (\text{Answer})$$

In situation 2, the resistance of each half of the wire is $(72 \, \Omega)/2$, or $36 \, \Omega$. Thus, the dissipation rate for each half is

$$P' = \frac{(120 \text{ V})^2}{36 \, \Omega} = 400 \text{ W},$$

and that for the two halves is

$$P = 2P' = 800 \text{ W}. \quad (\text{Answer})$$

This is four times the dissipation rate of the full length of wire. Thus, you might conclude that you could buy a heating coil, cut it in half, and reconnect it to obtain four times the heat output. Why is this unwise? (What would happen to the amount of current in the coil?)



Semiconductors

Semiconducting devices are at the heart of the microelectronic revolution that ushered in the information age. Table 26-2 compares the properties of silicon — a typical semiconductor — and copper — a typical metallic conductor. We see that silicon has many fewer charge carriers, a much higher resistivity, and a temperature coefficient of resistivity that is both large and negative. Thus, although the resistivity of copper increases with increasing temperature, that of pure silicon decreases.

Pure silicon has such a high resistivity that it is effectively an insulator and thus not of much direct use in microelectronic circuits. However, its resistivity can be greatly reduced in a controlled way by adding minute amounts of specific “impurity” atoms in a process called *doping*. Table 26-1 gives typical values of resistivity for silicon before and after doping with two different impurities.

We can roughly explain the differences in resistivity (and thus in conductivity) between semiconductors, insulators, and metallic conductors in terms of the energies of their electrons. (We need quantum physics to explain in more detail.) In a metallic conductor such as copper wire, most of the electrons are firmly locked in place within the atoms; much energy would be required to free them so they could move and participate in an electric current. However, there are also some electrons that, roughly speaking, are only loosely held in place and that require only little energy to become free. Thermal energy can supply that energy, as can an electric field applied across the conductor. The field would not only free these loosely held electrons but would also propel them along the wire; thus, the field would drive a current through the conductor.

In an insulator, significantly greater energy is required to free electrons so they can move through the material. Thermal energy cannot supply enough energy, and neither can any reasonable electric field applied to the insulator. Thus, no electrons are available to move through the insulator, and hence no current occurs even with an applied electric field.

A semiconductor is like an insulator *except* that the energy required to free some electrons is not quite so great. More important, doping can supply electrons or positive charge carriers that are very loosely held within the material and thus are easy to get moving. Moreover, by controlling the doping of a semiconductor, we can control the density of charge carriers that can participate in a current and thereby can control some of its electrical properties. Most semiconducting devices, such as transistors and junction diodes, are fabricated by the selective doping of different regions of the silicon with impurity atoms of different kinds.

Let us now look again at Eq. 26-22 for the resistivity of a conductor:

$$\rho = \frac{m}{e^2 n \tau}, \quad (26-29)$$

where n is the number of charge carriers per unit volume and τ is the mean time between collisions of the charge carriers. The equation also applies to semiconductors. Let’s consider how n and τ change as the temperature is increased.

In a conductor, n is large but very nearly constant with any change in temperature. The increase of resistivity with temperature for metals (Fig. 26-10) is due to an increase in the collision rate of the charge carriers, which shows up in Eq. 26-29 as a decrease in τ , the mean time between collisions.

Table 26-2 Some Electrical Properties of Copper and Silicon

Property	Copper	Silicon
Type of material	Metal	Semiconductor
Charge carrier density, m^{-3}	8.49×10^{28}	1×10^{16}
Resistivity, $\Omega \cdot \text{m}$	1.69×10^{-8}	2.5×10^3
Temperature coefficient of resistivity, K^{-1}	$+4.3 \times 10^{-3}$	-70×10^{-3}

In a semiconductor, n is small but increases very rapidly with temperature as the increased thermal agitation makes more charge carriers available. This causes a *decrease* of resistivity with increasing temperature, as indicated by the negative temperature coefficient of resistivity for silicon in Table 26-2. The same increase in collision rate that we noted for metals also occurs for semiconductors, but its effect is swamped by the rapid increase in the number of charge carriers.

Superconductors

In 1911, Dutch physicist Kamerlingh Onnes discovered that the resistivity of mercury absolutely disappears at temperatures below about 4 K (Fig. 26-14). This phenomenon of **superconductivity** is of vast potential importance in technology because it means that charge can flow through a superconducting conductor without losing its energy to thermal energy. Currents created in a superconducting ring, for example, have persisted for several years without loss; the electrons making up the current require a force and a source of energy at start-up time but not thereafter.

Prior to 1986, the technological development of superconductivity was throttled by the cost of producing the extremely low temperatures required to achieve the effect. In 1986, however, new ceramic materials were discovered that become superconducting at considerably higher (and thus cheaper to produce) temperatures. Practical application of superconducting devices at room temperature may eventually become commonplace.

Superconductivity is a phenomenon much different from conductivity. In fact, the best of the normal conductors, such as silver and copper, cannot become superconducting at any temperature, and the new ceramic superconductors are actually good insulators when they are not at low enough temperatures to be in a superconducting state.

One explanation for superconductivity is that the electrons that make up the current move in coordinated pairs. One of the electrons in a pair may electrically distort the molecular structure of the superconducting material as it moves through, creating nearby a short-lived concentration of positive charge. The other electron in the pair may then be attracted toward this positive charge. According to the theory, such coordination between electrons would prevent them from colliding with the molecules of the material and thus would eliminate electrical resistance. The theory worked well to explain the pre-1986, lower temperature superconductors, but new theories appear to be needed for the newer, higher temperature superconductors.

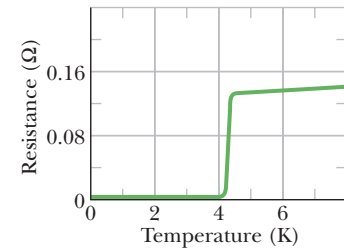
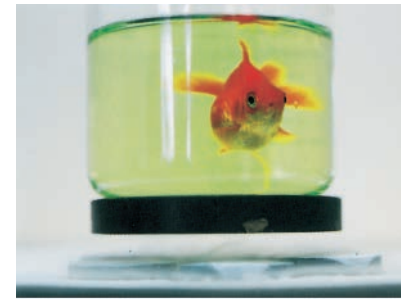


Figure 26-14 The resistance of mercury drops to zero at a temperature of about 4 K.



Courtesy Shoji Tonaka/International Superconductivity Technology Center, Tokyo, Japan

A disk-shaped magnet is levitated above a superconducting material that has been cooled by liquid nitrogen. The goldfish is along for the ride.

Review & Summary

Current An **electric current** i in a conductor is defined by

$$i = \frac{dq}{dt}. \quad (26-1)$$

Here dq is the amount of (positive) charge that passes in time dt through a hypothetical surface that cuts across the conductor. By convention, the direction of electric current is taken as the direction in which positive charge carriers would move. The SI unit of electric current is the **ampere** (A): $1 \text{ A} = 1 \text{ C/s}$.

Current Density Current (a scalar) is related to **current density** \vec{J} (a vector) by

$$i = \int \vec{J} \cdot d\vec{A}, \quad (26-4)$$

where $d\vec{A}$ is a vector perpendicular to a surface element of area dA and the integral is taken over any surface cutting across the conductor. \vec{J} has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

Drift Speed of the Charge Carriers When an electric field \vec{E} is established in a conductor, the charge carriers (assumed positive) acquire a **drift speed** v_d in the direction of \vec{E} ; the velocity \vec{v}_d is related to the current density by

$$\vec{J} = (ne)\vec{v}_d, \quad (26-7)$$

where ne is the *carrier charge density*.

Resistance of a Conductor The **resistance** R of a conductor is defined as

$$R = \frac{V}{i} \quad (\text{definition of } R), \quad (26-8)$$

where V is the potential difference across the conductor and i is the current. The SI unit of resistance is the **ohm** (Ω): $1 \Omega = 1 \text{ V/A}$. Similar equations define the **resistivity** ρ and **conductivity** σ of a material:

$$\rho = \frac{1}{\sigma} = \frac{E}{J} \quad (\text{definitions of } \rho \text{ and } \sigma), \quad (26-12, 26-10)$$

where E is the magnitude of the applied electric field. The SI unit of resistivity is the ohm-meter ($\Omega \cdot \text{m}$). Equation 26-10 corresponds to the vector equation

$$\vec{E} = \rho \vec{J}. \quad (26-11)$$

The resistance R of a conducting wire of length L and uniform cross section is

$$R = \rho \frac{L}{A}, \quad (26-16)$$

where A is the cross-sectional area.

Change of ρ with Temperature The resistivity ρ for most materials changes with temperature. For many materials, including metals, the relation between ρ and temperature T is approximated by the equation

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0). \quad (26-17)$$

Here T_0 is a reference temperature, ρ_0 is the resistivity at T_0 , and α is the temperature coefficient of resistivity for the material.

Ohm's Law A given device (conductor, resistor, or any other electrical device) obeys *Ohm's law* if its resistance R , defined by Eq. 26-8 as V/i , is independent of the applied potential difference V . A given *material* obeys Ohm's law if its resistivity, defined by Eq. 26-10, is independent of the magnitude and direction of the applied electric field \vec{E} .

Resistivity of a Metal By assuming that the conduction electrons in a metal are free to move like the molecules of a gas, it is

possible to derive an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}. \quad (26-22)$$

Here n is the number of free electrons per unit volume and τ is the mean time between the collisions of an electron with the atoms of the metal. We can explain why metals obey Ohm's law by pointing out that τ is essentially independent of the magnitude E of any electric field applied to a metal.

Power The power P , or rate of energy transfer, in an electrical device across which a potential difference V is maintained is

$$P = iV \quad (\text{rate of electrical energy transfer}). \quad (26-26)$$

Resistive Dissipation If the device is a resistor, we can write Eq. 26-26 as

$$P = i^2 R = \frac{V^2}{R} \quad (\text{resistive dissipation}). \quad (26-27, 26-28)$$

In a resistor, electric potential energy is converted to internal thermal energy via collisions between charge carriers and atoms.

Semiconductors *Semiconductors* are materials that have few conduction electrons but can become conductors when they are *doped* with other atoms that contribute charge carriers.

Superconductors *Superconductors* are materials that lose all electrical resistance at low temperatures. Some materials are superconducting at surprisingly high temperatures.

Questions

1 Figure 26-15 shows cross sections through three long conductors of the same length and material, with square cross sections of edge lengths as shown. Conductor B fits snugly within conductor A , and conductor C fits snugly within conductor B . Rank the following according to their end-to-end resistances, greatest first: the individual conductors and the combinations of $A + B$ (B inside A), $B + C$ (C inside B), and $A + B + C$ (B inside A inside C).

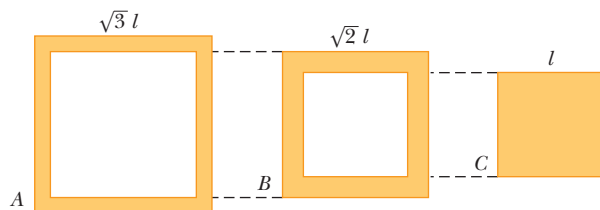


Figure 26-15 Question 1.

2 Figure 26-16 shows cross sections through three wires of identical length and material; the sides are given in millimeters. Rank the wires according to their resistance (measured end to end along each wire's length), greatest first.

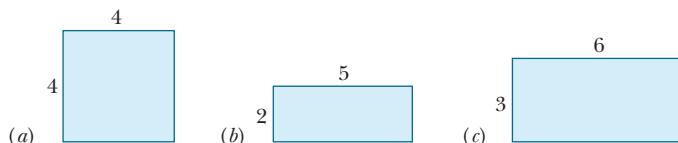


Figure 26-16 Question 2.

3 Figure 26-17 shows a rectangular solid conductor of edge lengths L , $2L$, and $3L$. A potential difference V is to be applied uniformly between pairs of opposite faces of the conductor as in Fig. 26-8b. (The potential difference is applied between the entire face on one side and the entire face on the other side.) First V is applied between the left–right faces, then between the top–bottom faces, and then between the front–back faces. Rank those pairs, greatest first, according to the following (within the conductor): (a) the magnitude of the electric field, (b) the current density, (c) the current, and (d) the drift speed of the electrons.

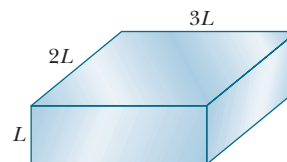


Figure 26-17 Question 3.

4 Figure 26-18 shows plots of the current i through a certain cross section of a wire over four different time periods. Rank the periods according to the net charge that passes through the cross section during the period, greatest first.

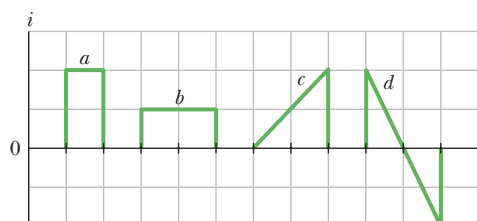


Figure 26-18 Question 4.

5 Figure 26-19 shows four situations in which positive and negative charges move horizontally and gives the rate at which each charge moves. Rank the situations according to the effective current through the regions, greatest first.

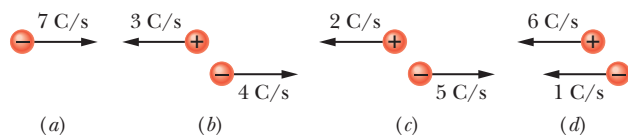


Figure 26-19 Question 5.

6 In Fig. 26-20, a wire that carries a current consists of three sections with different radii. Rank the sections according to the following quantities, greatest first: (a) current, (b) magnitude of current density, and (c) magnitude of electric field.

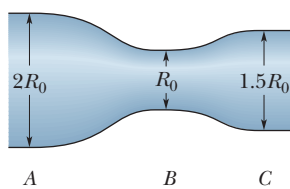


Figure 26-20 Question 6.

7 Figure 26-21 gives the electric potential $V(x)$ versus position x along a copper wire carrying current. The wire consists of three sections that differ in radius. Rank the three sections according to the magnitude of the (a) electric field and (b) current density, greatest first.

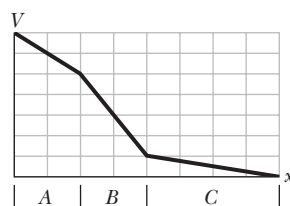


Figure 26-21 Question 7.

8 The following table gives the lengths of three copper rods, their diameters, and the potential differences between their ends. Rank the rods according to (a) the magnitude of the electric field within them, (b) the current density within them, and (c) the drift speed of electrons through them, greatest first.

Rod	Length	Diameter	Potential Difference
1	L	$3d$	V
2	$2L$	d	$2V$
3	$3L$	$2d$	$2V$

9 Figure 26-22 gives the drift speed v_d of conduction electrons in a copper wire versus position x along the wire. The wire consists of three sections that differ in radius. Rank the three sections according to the following quantities, greatest first: (a) radius, (b) number of conduction electrons per cubic meter, (c) magnitude of electric field, (d) conductivity.

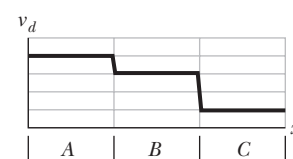


Figure 26-22 Question 9.

10 Three wires, of the same diameter, are connected in turn between two points maintained at a constant potential difference. Their resistivities and lengths are ρ and L (wire A), 1.2ρ and $1.2L$ (wire B), and 0.9ρ and L (wire C). Rank the wires according to the rate at which energy is transferred to thermal energy within them, greatest first.

11 Figure 26-23 gives, for three wires of radius R , the current density $J(r)$ versus radius r , as measured from the center of a circular cross section through the wire. The wires are all made from the same material. Rank the wires according to the magnitude of the electric field (a) at the center, (b) halfway to the surface, and (c) at the surface, greatest first.

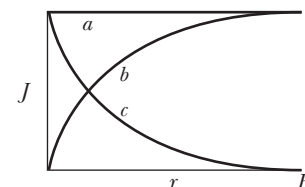


Figure 26-23 Question 11.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at



Number of dots indicates level of problem difficulty

ILW Interactive solution is at



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

<http://www.wiley.com/college/halliday>

Module 26-1 Electric Current

•1 During the 4.0 min a 5.0 A current is set up in a wire, how many (a) coulombs and (b) electrons pass through any cross section across the wire's width?

••2 An isolated conducting sphere has a 10 cm radius. One wire carries a current of 1.000 002 0 A into it. Another wire carries a current of 1.000 000 0 A out of it. How long would it take for the sphere to increase in potential by 1000 V?

••3 A charged belt, 50 cm wide, travels at 30 m/s between a source of charge and a sphere. The belt carries charge into the sphere at a rate corresponding to 100 μ A. Compute the surface charge density on the belt.

Module 26-2 Current Density

•4 The (United States) National Electric Code, which sets maximum safe currents for insulated copper wires of various diameters, is given (in part) in the table. Plot the safe current density as a function of diameter. Which wire gauge has the maximum safe current density? ("Gauge" is a way of identifying wire diameters, and 1 mil = 10^{-3} in.)

Gauge	4	6	8	10	12	14	16	18
Diameter, mils	204	162	129	102	81	64	51	40
Safe current, A	70	50	35	25	20	15	6	3

•5 SSM WWW A beam contains 2.0×10^8 doubly charged positive ions per cubic centimeter, all of which are moving north with a speed of 1.0×10^5 m/s. What are the (a) magnitude and (b) direction of the current density \vec{J} ? (c) What additional quantity do you need to calculate the total current i in this ion beam?

•6 A certain cylindrical wire carries current. We draw a circle of radius r around its central axis in Fig. 26-24a to determine the current i within the circle. Figure 26-24b shows current i as a function of r^2 . The vertical scale is set by $i_s = 4.0$ mA, and the horizontal scale is set

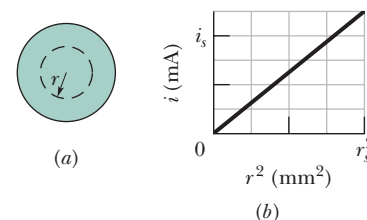


Figure 26-24 Problem 6.

by $r_s^2 = 4.0 \text{ mm}^2$. (a) Is the current density uniform? (b) If so, what is its magnitude?

•7 A fuse in an electric circuit is a wire that is designed to melt, and thereby open the circuit, if the current exceeds a predetermined value. Suppose that the material to be used in a fuse melts when the current density rises to 440 A/cm^2 . What diameter of cylindrical wire should be used to make a fuse that will limit the current to 0.50 A ?

•8 A small but measurable current of $1.2 \times 10^{-10} \text{ A}$ exists in a copper wire whose diameter is 2.5 mm . The number of charge carriers per unit volume is $8.49 \times 10^{28} \text{ m}^{-3}$. Assuming the current is uniform, calculate the (a) current density and (b) electron drift speed.

••9 The magnitude $J(r)$ of the current density in a certain cylindrical wire is given as a function of radial distance from the center of the wire's cross section as $J(r) = Br$, where r is in meters, J is in amperes per square meter, and $B = 2.00 \times 10^5 \text{ A/m}^3$. This function applies out to the wire's radius of 2.00 mm . How much current is contained within the width of a thin ring concentric with the wire if the ring has a radial width of $10.0 \mu\text{m}$ and is at a radial distance of 1.20 mm ?

••10 The magnitude J of the current density in a certain lab wire with a circular cross section of radius $R = 2.00 \text{ mm}$ is given by $J = (3.00 \times 10^8)r^2$, with J in amperes per square meter and radial distance r in meters. What is the current through the outer section bounded by $r = 0.900R$ and $r = R$?

••11 What is the current in a wire of radius $R = 3.40 \text{ mm}$ if the magnitude of the current density is given by (a) $J_a = J_0 r/R$ and (b) $J_b = J_0(1 - r/R)$, in which r is the radial distance and $J_0 = 5.50 \times 10^4 \text{ A/m}^2$? (c) Which function maximizes the current density near the wire's surface?

••12 Near Earth, the density of protons in the solar wind (a stream of particles from the Sun) is 8.70 cm^{-3} , and their speed is 470 km/s . (a) Find the current density of these protons. (b) If Earth's magnetic field did not deflect the protons, what total current would Earth receive?

••13 **GO ILW** How long does it take electrons to get from a car battery to the starting motor? Assume the current is 300 A and the electrons travel through a copper wire with cross-sectional area 0.21 cm^2 and length 0.85 m . The number of charge carriers per unit volume is $8.49 \times 10^{28} \text{ m}^{-3}$.

Module 26-3 Resistance and Resistivity

•14 **ILW** A human being can be electrocuted if a current as small as 50 mA passes near the heart. An electrician working with sweaty hands makes good contact with the two conductors he is holding, one in each hand. If his resistance is 2000Ω , what might the fatal voltage be?

•15 **SSM** A coil is formed by winding 250 turns of insulated 16-gauge copper wire (diameter = 1.3 mm) in a single layer on a cylindrical form of radius 12 cm . What is the resistance of the coil? Neglect the thickness of the insulation. (Use Table 26-1.)

•16 Copper and aluminum are being considered for a high-voltage transmission line that must carry a current of 60.0 A . The resistance per unit length is to be $0.150 \Omega/\text{km}$. The densities of copper and aluminum are 8960 and 2600 kg/m^3 , respectively. Compute (a) the magnitude J of the current density and (b) the mass per unit length λ for a copper cable and (c) J and (d) λ for an aluminum cable.

•17 A wire of Nichrome (a nickel–chromium–iron alloy commonly used in heating elements) is 1.0 m long and 1.0 mm^2 in cross-sectional area. It carries a current of 4.0 A when a 2.0 V potential difference is applied between its ends. Calculate the conductivity σ of Nichrome.

•18 A wire 4.00 m long and 6.00 mm in diameter has a resistance of $15.0 \text{ m}\Omega$. A potential difference of 23.0 V is applied between the ends. (a) What is the current in the wire? (b) What is the magnitude of the current density? (c) Calculate the resistivity of the wire material. (d) Using Table 26-1, identify the material.

•19 **SSM** What is the resistivity of a wire of 1.0 mm diameter, 2.0 m length, and $50 \text{ m}\Omega$ resistance?

•20 A certain wire has a resistance R . What is the resistance of a second wire, made of the same material, that is half as long and has half the diameter?

••21 **ILW** A common flashlight bulb is rated at 0.30 A and 2.9 V (the values of the current and voltage under operating conditions). If the resistance of the tungsten bulb filament at room temperature (20°C) is 1.1Ω , what is the temperature of the filament when the bulb is on?

••22 **ILW** *Kiting during a storm.* The legend that Benjamin Franklin flew a kite as a storm approached is only a legend—he was neither stupid nor suicidal. Suppose a kite string of radius 2.00 mm extends directly upward by 0.800 km and is coated with a 0.500 mm layer of water having resistivity $150 \Omega \cdot \text{m}$. If the potential difference between the two ends of the string is 160 MV , what is the current through the water layer? The danger is not this current but the chance that the string draws a lightning strike, which can have a current as large as $500\,000 \text{ A}$ (way beyond just being lethal).

••23 When 115 V is applied across a wire that is 10 m long and has a 0.30 mm radius, the magnitude of the current density is $1.4 \times 10^8 \text{ A/m}^2$. Find the resistivity of the wire.

••24 **GO** Figure 26-25a gives the magnitude $E(x)$ of the electric fields that have been set up by a battery along a resistive rod of length 9.00 mm (Fig. 26-25b). The vertical scale is set by $E_s = 4.00 \times 10^3 \text{ V/m}$. The rod consists of three sections of the same material but with different radii. (The schematic diagram of Fig. 26-25b does not indicate the different radii.) The radius of section 3 is 2.00 mm . What is the radius of (a) section 1 and (b) section 2?

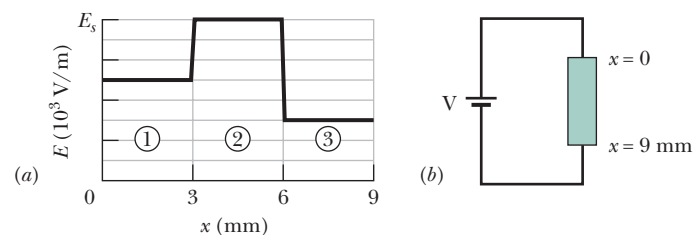


Figure 26-25 Problem 24.

••25 **SSM ILW** A wire with a resistance of 6.0Ω is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire, assuming that the resistivity and density of the material are unchanged.

••26 In Fig. 26-26a, a 9.00 V battery is connected to a resistive strip that consists of three sections with the same cross-sectional areas but different conductivities. Figure 26-26b gives the electric

potential $V(x)$ versus position x along the strip. The horizontal scale is set by $x_s = 8.00$ mm. Section 3 has conductivity 3.00×10^7 ($\Omega \cdot \text{m}$)⁻¹. What is the conductivity of section (a) 1 and (b) 2?

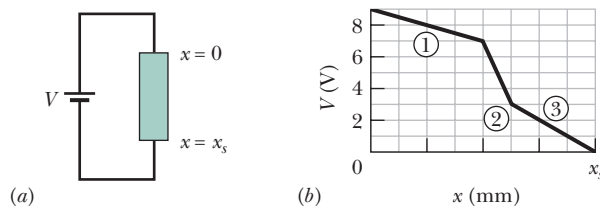


Figure 26-26 Problem 26.

••27 **SSM WWW** Two conductors are made of the same material and have the same length. Conductor *A* is a solid wire of diameter 1.0 mm. Conductor *B* is a hollow tube of outside diameter 2.0 mm and inside diameter 1.0 mm. What is the resistance ratio R_A/R_B , measured between their ends?

••28 **GO** Figure 26-27 gives the electric potential $V(x)$ along a copper wire carrying uniform current, from a point of higher potential $V_s = 12.0$ μV at $x = 0$ to a point of zero potential at $x_s = 3.00$ m. The wire has a radius of 2.00 mm. What is the current in the wire?

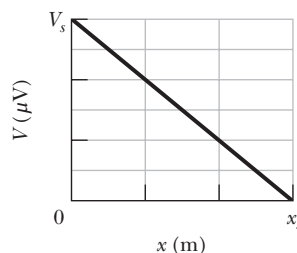


Figure 26-27 Problem 28.

••29 A potential difference of 3.00 nV is set up across a 2.00 cm length of copper wire that has a radius of 2.00 mm. How much charge drifts through a cross section in 3.00 ms?

••30 If the gauge number of a wire is increased by 6, the diameter is halved; if a gauge number is increased by 1, the diameter decreases by the factor $2^{1/6}$ (see the table in Problem 4). Knowing this, and knowing that 1000 ft of 10-gauge copper wire has a resistance of approximately 1.00 Ω , estimate the resistance of 25 ft of 22-gauge copper wire.

••31 An electrical cable consists of 125 strands of fine wire, each having 2.65 $\mu\Omega$ resistance. The same potential difference is applied between the ends of all the strands and results in a total current of 0.750 A. (a) What is the current in each strand? (b) What is the applied potential difference? (c) What is the resistance of the cable?

••32 Earth's lower atmosphere contains negative and positive ions that are produced by radioactive elements in the soil and cosmic rays from space. In a certain region, the atmospheric

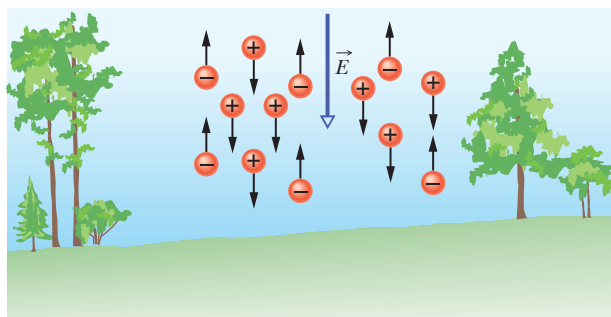


Figure 26-28 Problem 32.

electric field strength is 120 V/m and the field is directed vertically down. This field causes singly charged positive ions, at a density of 620 cm⁻³, to drift downward and singly charged negative ions, at a density of 550 cm⁻³, to drift upward (Fig. 26-28). The measured conductivity of the air in that region is 2.70×10^{-14} ($\Omega \cdot \text{m}$)⁻¹. Calculate (a) the magnitude of the current density and (b) the ion drift speed, assumed to be the same for positive and negative ions.

••33 A block in the shape of a rectangular solid has a cross-sectional area of 3.50 cm² across its width, a front-to-rear length of 15.8 cm, and a resistance of 935 Ω . The block's material contains 5.33×10^{22} conduction electrons/m³. A potential difference of 35.8 V is maintained between its front and rear faces. (a) What is the current in the block? (b) If the current density is uniform, what is its magnitude? What are (c) the drift velocity of the conduction electrons and (d) the magnitude of the electric field in the block?

••34 **GO** Figure 26-29 shows wire section 1 of diameter $D_1 = 4.00R$ and wire section 2 of diameter $D_2 = 2.00R$, connected by a tapered section. The wire is copper and carries a current. Assume that the current is uniformly distributed across any cross-sectional area through the wire's width. The electric potential change V along the length $L = 2.00$ m shown in section 2 is 10.0 μV . The number of charge carriers per unit volume is 8.49×10^{28} m⁻³. What is the drift speed of the conduction electrons in section 1?

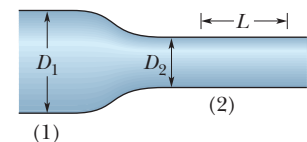


Figure 26-29 Problem 34.

••35 **GO** In Fig. 26-30, current is set up through a truncated right circular cone of resistivity 731 $\Omega \cdot \text{m}$, left radius $a = 2.00$ mm, right radius $b = 2.30$ mm, and length $L = 1.94$ cm. Assume that the current density is uniform across any cross section taken perpendicular to the length. What is the resistance of the cone?

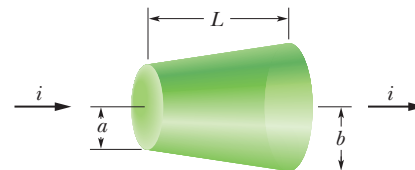


Figure 26-30 Problem 35.

••36 **GO** Swimming during a storm. Figure 26-31 shows a swimmer at distance $D = 35.0$ m from a lightning strike to the water, with current $I = 78$ kA. The water has resistivity 30 $\Omega \cdot \text{m}$, the width of the swimmer along a radial line from the strike is 0.70 m, and his resistance across that width is 4.00 k Ω . Assume that the current spreads through the water over a hemisphere centered on the strike point. What is the current through the swimmer?

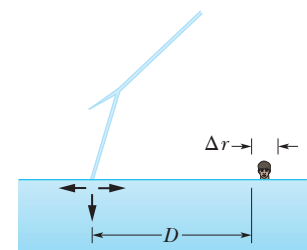


Figure 26-31 Problem 36.

Module 26-4 Ohm's Law

••37 Show that, according to the free-electron model of electrical conduction in metals and classical physics, the resistivity of metals should be proportional to \sqrt{T} , where T is the temperature in kelvins. (See Eq. 19-31.)

Module 26-5 Power, Semiconductors, Superconductors

•38 In Fig. 26-32a, a $20\ \Omega$ resistor is connected to a battery. Figure 26-32b shows the increase of thermal energy E_{th} in the resistor as a function of time t . The vertical scale is set by $E_{th,s} = 2.50\text{ mJ}$, and the horizontal scale is set by $t_s = 4.0\text{ s}$. What is the electric potential across the battery?

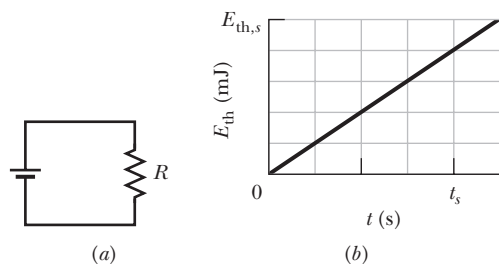


Figure 26-32 Problem 38.

•39 A certain brand of hot-dog cooker works by applying a potential difference of 120 V across opposite ends of a hot dog and allowing it to cook by means of the thermal energy produced. The current is 10.0 A , and the energy required to cook one hot dog is 60.0 kJ . If the rate at which energy is supplied is unchanged, how long will it take to cook three hot dogs simultaneously?

•40 Thermal energy is produced in a resistor at a rate of 100 W when the current is 3.00 A . What is the resistance?

•41 **SSM** A 120 V potential difference is applied to a space heater whose resistance is $14\ \Omega$ when hot. (a) At what rate is electrical energy transferred to thermal energy? (b) What is the cost for 5.0 h at $\text{US}\$0.05/\text{kW}\cdot\text{h}$?

•42 In Fig. 26-33, a battery of potential difference $V = 12\text{ V}$ is connected to a resistive strip of resistance $R = 6.0\ \Omega$. When an electron moves through the strip from one end to the other, (a) in which direction in the figure does the electron move, (b) how much work is done on the electron by the electric field in the strip, and (c) how much energy is transferred to the thermal energy of the strip by the electron?

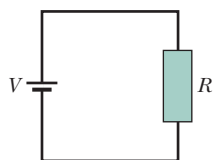


Figure 26-33 Problem 42.

•43 **ILW** An unknown resistor is connected between the terminals of a 3.00 V battery. Energy is dissipated in the resistor at the rate of 0.540 W . The same resistor is then connected between the terminals of a 1.50 V battery. At what rate is energy now dissipated?

•44 A student kept his 9.0 V , 7.0 W radio turned on at full volume from 9:00 P.M. until 2:00 A.M. How much charge went through it?

•45 **SSM ILW** A 1250 W radiant heater is constructed to operate at 115 V . (a) What is the current in the heater when the unit is operating? (b) What is the resistance of the heating coil? (c) How much thermal energy is produced in 1.0 h ?

•46 **GO** A copper wire of cross-sectional area $2.00 \times 10^{-6}\text{ m}^2$ and length 4.00 m has a current of 2.00 A uniformly distributed across that area. (a) What is the magnitude of the electric field along the wire? (b) How much electrical energy is transferred to thermal energy in 30 min ?

•47 A heating element is made by maintaining a potential difference of 75.0 V across the length of a Nichrome wire that

has a $2.60 \times 10^{-6}\text{ m}^2$ cross section. Nichrome has a resistivity of $5.00 \times 10^{-7}\ \Omega\cdot\text{m}$. (a) If the element dissipates 5000 W , what is its length? (b) If 100 V is used to obtain the same dissipation rate, what should the length be?

•48 **Explosion shoes**. The rain-soaked shoes of a person may explode if ground current from nearby lightning vaporizes the water. The sudden conversion of water to water vapor causes a dramatic expansion that can rip apart shoes. Water has density 1000 kg/m^3 and requires 2256 kJ/kg to be vaporized. If horizontal current lasts 2.00 ms and encounters water with resistivity $150\ \Omega\cdot\text{m}$, length 12.0 cm , and vertical cross-sectional area $15 \times 10^{-5}\text{ m}^2$, what average current is required to vaporize the water?

•49 A 100 W lightbulb is plugged into a standard 120 V outlet. (a) How much does it cost per 31-day month to leave the light turned on continuously? Assume electrical energy costs $\text{US}\$0.06/\text{kW}\cdot\text{h}$. (b) What is the resistance of the bulb? (c) What is the current in the bulb?

•50 **GO** The current through the battery and resistors 1 and 2 in Fig. 26-34a is 2.00 A . Energy is transferred from the current to thermal energy E_{th} in both resistors. Curves 1 and 2 in Fig. 26-34b give that thermal energy E_{th} for resistors 1 and 2, respectively, as a function of time t . The vertical scale is set by $E_{th,s} = 40.0\text{ mJ}$, and the horizontal scale is set by $t_s = 5.00\text{ s}$. What is the power of the battery?

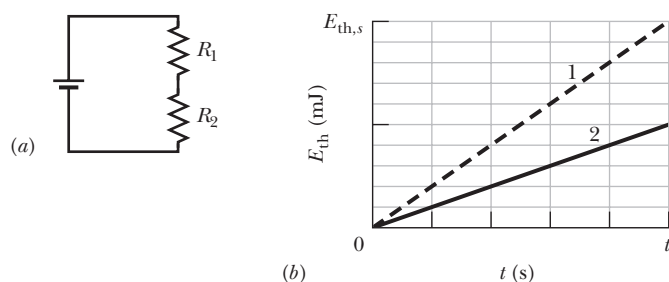


Figure 26-34 Problem 50.

•51 **GO SSM WWW** Wire C and wire D are made from different materials and have length $L_C = L_D = 1.0\text{ m}$. The resistivity and diameter of wire C are $2.0 \times 10^{-6}\ \Omega\cdot\text{m}$ and 1.00 mm , and those of wire D are $1.0 \times 10^{-6}\ \Omega\cdot\text{m}$ and 0.50 mm .

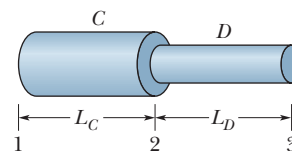



Figure 26-35 Problem 51.

The wires are joined as shown in Fig. 26-35, and a current of 2.0 A is set up in them. What is the electric potential difference between (a) points 1 and 2 and (b) points 2 and 3? What is the rate at which energy is dissipated between (c) points 1 and 2 and (d) points 2 and 3?

•52 **GO** The current-density magnitude in a certain circular wire is $J = (2.75 \times 10^{10}\text{ A/m}^4)r^2$, where r is the radial distance out to the wire's radius of 3.00 mm . The potential applied to the wire (end to end) is 60.0 V . How much energy is converted to thermal energy in 1.00 h ?

•53 A 120 V potential difference is applied to a space heater that dissipates 500 W during operation. (a) What is its resistance during operation? (b) At what rate do electrons flow through any cross section of the heater element?

•••54  Figure 26-36a shows a rod of resistive material. The resistance per unit length of the rod increases in the positive direction of the x axis. At any position x along the rod, the resistance dR of a narrow (differential) section of width dx is given by $dR = 5.00x \, dx$, where dR is in ohms and x is in meters. Figure 26-36b shows such a narrow section. You are to slice off a length of the rod between $x = 0$ and some position $x = L$ and then connect that length to a battery with potential difference $V = 5.0 \, \text{V}$ (Fig. 26-36c). You want the current in the length to transfer energy to thermal energy at the rate of $200 \, \text{W}$. At what position $x = L$ should you cut the rod?

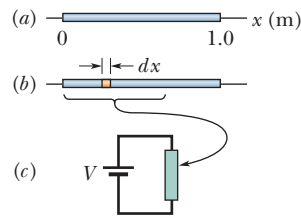



Figure 26-36 Problem 54.

Additional Problems


55  A Nichrome heater dissipates $500 \, \text{W}$ when the applied potential difference is $110 \, \text{V}$ and the wire temperature is 800°C . What would be the dissipation rate if the wire temperature were held at 200°C by immersing the wire in a bath of cooling oil? The applied potential difference remains the same, and α for Nichrome at 800°C is $4.0 \times 10^{-4} \, \text{K}^{-1}$.

56 A potential difference of $1.20 \, \text{V}$ will be applied to a $33.0 \, \text{m}$ length of 18-gauge copper wire (diameter = $0.0400 \, \text{in.}$). Calculate (a) the current, (b) the magnitude of the current density, (c) the magnitude of the electric field within the wire, and (d) the rate at which thermal energy will appear in the wire.

57 An $18.0 \, \text{W}$ device has $9.00 \, \text{V}$ across it. How much charge goes through the device in $4.00 \, \text{h}$?

58 An aluminum rod with a square cross section is $1.3 \, \text{m}$ long and $5.2 \, \text{mm}$ on edge. (a) What is the resistance between its ends? (b) What must be the diameter of a cylindrical copper rod of length $1.3 \, \text{m}$ if its resistance is to be the same as that of the aluminum rod?


59 A cylindrical metal rod is $1.60 \, \text{m}$ long and $5.50 \, \text{mm}$ in diameter. The resistance between its two ends (at 20°C) is $1.09 \times 10^{-3} \, \Omega$. (a) What is the material? (b) A round disk, $2.00 \, \text{cm}$ in diameter and $1.00 \, \text{mm}$ thick, is formed of the same material. What is the resistance between the round faces, assuming that each face is an equipotential surface?

60  *The chocolate crumb mystery.* This story begins with Problem 60 in Chapter 23 and continues through Chapters 24 and 25. The chocolate crumb powder moved to the silo through a pipe of radius R with uniform speed v and uniform charge density ρ . (a) Find an expression for the current i (the rate at which charge on the powder moved) through a perpendicular cross section of the pipe. (b) Evaluate i for the conditions at the factory: pipe radius $R = 5.0 \, \text{cm}$, speed $v = 2.0 \, \text{m/s}$, and charge density $\rho = 1.1 \times 10^{-3} \, \text{C/m}^3$.

If the powder were to flow through a change V in electric potential, its energy could be transferred to a spark at the rate $P = iV$. (c) Could there be such a transfer within the pipe due to the radial potential difference discussed in Problem 70 of Chapter 24?

As the powder flowed from the pipe into the silo, the electric potential of the powder changed. The magnitude of that change was at least equal to the radial potential difference within the pipe (as evaluated in Problem 70 of Chapter 24). (d) Assuming that value for the potential difference and using the current found in (b) above, find the rate at which energy could have been transferred from the powder to a spark as the powder exited the pipe. (e) If a spark did occur at the exit and lasted for $0.20 \, \text{s}$ (a reasonable expectation), how much energy would have been transferred to the spark? Recall

from Problem 60 in Chapter 23 that a minimum energy transfer of $150 \, \text{mJ}$ is needed to cause an explosion. (f) Where did the powder explosion most likely occur: in the powder cloud at the unloading bin (Problem 60 of Chapter 25), within the pipe, or at the exit of the pipe into the silo?

61  A steady beam of alpha particles ($q = +2e$) traveling with constant kinetic energy $20 \, \text{MeV}$ carries a current of $0.25 \, \mu\text{A}$. (a) If the beam is directed perpendicular to a flat surface, how many alpha particles strike the surface in $3.0 \, \text{s}$? (b) At any instant, how many alpha particles are there in a given $20 \, \text{cm}$ length of the beam? (c) Through what potential difference is it necessary to accelerate each alpha particle from rest to bring it to an energy of $20 \, \text{MeV}$?

62 A resistor with a potential difference of $200 \, \text{V}$ across it transfers electrical energy to thermal energy at the rate of $3000 \, \text{W}$. What is the resistance of the resistor?

63 A $2.0 \, \text{kW}$ heater element from a dryer has a length of $80 \, \text{cm}$. If a $10 \, \text{cm}$ section is removed, what power is used by the now shortened element at $120 \, \text{V}$?

64 A cylindrical resistor of radius $5.0 \, \text{mm}$ and length $2.0 \, \text{cm}$ is made of material that has a resistivity of $3.5 \times 10^{-5} \, \Omega \cdot \text{m}$. What are (a) the magnitude of the current density and (b) the potential difference when the energy dissipation rate in the resistor is $1.0 \, \text{W}$?

65 A potential difference V is applied to a wire of cross-sectional area A , length L , and resistivity ρ . You want to change the applied potential difference and stretch the wire so that the energy dissipation rate is multiplied by 30.0 and the current is multiplied by 4.00 . Assuming the wire's density does not change, what are (a) the ratio of the new length to L and (b) the ratio of the new cross-sectional area to A ?

66 The headlights of a moving car require about $10 \, \text{A}$ from the $12 \, \text{V}$ alternator, which is driven by the engine. Assume the alternator is 80% efficient (its output electrical power is 80% of its input mechanical power), and calculate the horsepower the engine must supply to run the lights.

67 A $500 \, \text{W}$ heating unit is designed to operate with an applied potential difference of $115 \, \text{V}$. (a) By what percentage will its heat output drop if the applied potential difference drops to $110 \, \text{V}$? Assume no change in resistance. (b) If you took the variation of resistance with temperature into account, would the actual drop in heat output be larger or smaller than that calculated in (a)?

68 The copper windings of a motor have a resistance of $50 \, \Omega$ at 20°C when the motor is idle. After the motor has run for several hours, the resistance rises to $58 \, \Omega$. What is the temperature of the windings now? Ignore changes in the dimensions of the windings. (Use Table 26-1.)

69 How much electrical energy is transferred to thermal energy in $2.00 \, \text{h}$ by an electrical resistance of $400 \, \Omega$ when the potential applied across it is $90.0 \, \text{V}$?

70 A caterpillar of length $4.0 \, \text{cm}$ crawls in the direction of electron drift along a 5.2-mm -diameter bare copper wire that carries a uniform current of $12 \, \text{A}$. (a) What is the potential difference between the two ends of the caterpillar? (b) Is its tail positive or negative relative to its head? (c) How much time does the caterpillar take to crawl $1.0 \, \text{cm}$ if it crawls at the drift speed of the electrons in the wire? (The number of charge carriers per unit volume is $8.49 \times 10^{28} \, \text{m}^{-3}$.)



71  (a) At what temperature would the resistance of a copper conductor be double its resistance at 20.0°C ? (Use 20.0°C as the reference point in Eq. 26-17; compare your answer with

Fig. 26-10.) (b) Does this same “doubling temperature” hold for all copper conductors, regardless of shape or size?

72 A steel trolley-car rail has a cross-sectional area of 56.0 cm^2 . What is the resistance of 10.0 km of rail? The resistivity of the steel is $3.00 \times 10^{-7} \Omega \cdot \text{m}$.

73 A coil of current-carrying Nichrome wire is immersed in a liquid. (Nichrome is a nickel–chromium–iron alloy commonly used in heating elements.) When the potential difference across the coil is 12 V and the current through the coil is 5.2 A , the liquid evaporates at the steady rate of 21 mg/s . Calculate the heat of vaporization of the liquid (see Module 18-4).

74  The current density in a wire is uniform and has magnitude $2.0 \times 10^6 \text{ A/m}^2$, the wire’s length is 5.0 m , and the density of conduction electrons is $8.49 \times 10^{28} \text{ m}^{-3}$. How long does an electron take (on the average) to travel the length of the wire?

75 A certain x-ray tube operates at a current of 7.00 mA and a potential difference of 80.0 kV . What is its power in watts?

76 A current is established in a gas discharge tube when a sufficiently high potential difference is applied across the two electrodes in the tube. The gas ionizes; electrons move toward the positive terminal and singly charged positive ions toward the negative terminal. (a) What is the current in a hydrogen discharge tube in which 3.1×10^{18} electrons and 1.1×10^{18} protons move past a cross-sectional area of the tube each second? (b) Is the direction of the current density \vec{J} toward or away from the negative terminal?

77 In Fig. 26-37, a resistance coil, wired to an external battery, is placed inside a thermally insulated cylinder fitted with a frictionless piston and containing an ideal gas. A current $i = 240 \text{ mA}$ flows through the coil, which has a resistance $R = 550 \Omega$. If the gas temperature remains constant while the 12 kg piston moves upward, what is the limit of the piston’s speed?

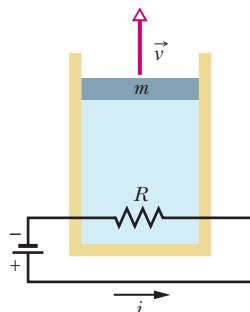


Figure 26-37 Problem 77.

78 An insulating belt moves at speed 30 m/s and has a width of 50 cm . It carries charge into an experimental device at a rate corresponding to $100 \mu\text{A}$. What is the surface charge density on the belt?

79 In a hypothetical fusion research lab, high temperature helium gas is completely ionized and each helium atom is separated into two free electrons and the remaining positively charged nucleus, which is called an alpha particle. An applied electric field causes the alpha particles to drift to the east at 25.0 m/s while the electrons drift to the west at 88.0 m/s . The alpha particle density is $2.80 \times 10^{15} \text{ cm}^{-3}$. What are (a) the net current density and (b) the current direction?

80 When a metal rod is heated, not only its resistance but also its length and cross-sectional area change. The relation $R = \rho L/A$ suggests that all three factors should be taken into account in measuring ρ at various temperatures. If the temperature changes by 1.0°C , what percentage changes in (a) L , (b) A , and (c) R occur for a copper conductor? (d) What conclusion do you draw? The coefficient of linear expansion is $1.70 \times 10^{-5} \text{ K}^{-1}$.

81 A beam of 16 MeV deuterons from a cyclotron strikes a copper block. The beam is equivalent to current of $15 \mu\text{A}$. (a) At what rate do deuterons strike the block? (b) At what rate is thermal energy produced in the block?

82 A linear accelerator produces a pulsed beam of electrons. The pulse current is 0.50 A , and the pulse duration is $0.10 \mu\text{s}$. (a) How many electrons are accelerated per pulse? (b) What is the average current for a machine operating at 500 pulses/s ? If the electrons are accelerated to an energy of 50 MeV , what are the (c) average power and (d) peak power of the accelerator?

83 An electric immersion heater normally takes 100 min to bring cold water in a well-insulated container to a certain temperature, after which a thermostat switches the heater off. One day the line voltage is reduced by 6.00% because of a laboratory overload. How long does heating the water now take? Assume that the resistance of the heating element does not change.

84 A 400 W immersion heater is placed in a pot containing 2.00 L of water at 20°C . (a) How long will the water take to rise to the boiling temperature, assuming that 80% of the available energy is absorbed by the water? (b) How much longer is required to evaporate half of the water?

85 A $30 \mu\text{F}$ capacitor is connected across a programmed power supply. During the interval from $t = 0$ to $t = 3.00 \text{ s}$ the output voltage of the supply is given by $V(t) = 6.00 + 4.00t - 2.00t^2$ volts. At $t = 0.500 \text{ s}$ find (a) the charge on the capacitor, (b) the current into the capacitor, and (c) the power output from the power supply.

Circuits

27-1 SINGLE-LOOP CIRCUITS

Learning Objectives

After reading this module, you should be able to . . .

- 27.01** Identify the action of an emf source in terms of the work it does.
- 27.02** For an ideal battery, apply the relationship between the emf, the current, and the power (rate of energy transfer).
- 27.03** Draw a schematic diagram for a single-loop circuit containing a battery and three resistors.
- 27.04** Apply the loop rule to write a loop equation that relates the potential differences of the circuit elements around a (complete) loop.
- 27.05** Apply the resistance rule in crossing through a resistor.
- 27.06** Apply the emf rule in crossing through an emf.
- 27.07** Identify that resistors in series have the same current, which is the same value that their equivalent resistor has.
- 27.08** Calculate the equivalent of series resistors.
- 27.09** Identify that a potential applied to resistors wired in series is equal to the sum of the potentials across the individual resistors.
- 27.10** Calculate the potential difference between any two points in a circuit.
- 27.11** Distinguish a real battery from an ideal battery and, in a circuit diagram, replace a real battery with an ideal battery and an explicitly shown resistance.
- 27.12** With a real battery in a circuit, calculate the potential difference between its terminals for current in the direction of the emf and in the opposite direction.
- 27.13** Identify what is meant by grounding a circuit, and draw a schematic diagram for such a connection.
- 27.14** Identify that grounding a circuit does not affect the current in a circuit.
- 27.15** Calculate the dissipation rate of energy in a real battery.
- 27.16** Calculate the net rate of energy transfer in a real battery for current in the direction of the emf and in the opposite direction.

Key Ideas

- An emf device does work on charges to maintain a potential difference between its output terminals. If dW is the work the device does to force positive charge dq from the negative to the positive terminal, then the emf (work per unit charge) of the device is

$$\mathcal{E} = \frac{dW}{dq} \quad (\text{definition of } \mathcal{E}).$$

- An ideal emf device is one that lacks any internal resistance. The potential difference between its terminals is equal to the emf.
- A real emf device has internal resistance. The potential difference between its terminals is equal to the emf only if there is no current through the device.
- The change in potential in traversing a resistance R in the direction of the current is $-iR$; in the opposite direction it is $+iR$ (resistance rule).
- The change in potential in traversing an ideal emf device in the direction of the emf arrow is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$ (emf rule).
- Conservation of energy leads to the loop rule:

Loop Rule. The algebraic sum of the changes in potential encountered in a complete traversal of any

loop of a circuit must be zero.

Conservation of charge leads to the junction rule (Chapter 26):
Junction Rule. The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

- When a real battery of emf \mathcal{E} and internal resistance r does work on the charge carriers in a current i through the battery, the rate P of energy transfer to the charge carriers is

$$P = iV,$$

where V is the potential across the terminals of the battery.

- The rate P_r at which energy is dissipated as thermal energy in the battery is

$$P_r = i^2 r.$$

- The rate P_{emf} at which the chemical energy in the battery changes is

$$P_{\text{emf}} = i\mathcal{E}.$$

- When resistances are in series, they have the same current. The equivalent resistance that can replace a series combination of resistances is

$$R_{\text{eq}} = \sum_{j=1}^n R_j \quad (n \text{ resistances in series}).$$

What Is Physics?

You are surrounded by electric circuits. You might take pride in the number of electrical devices you own and might even carry a mental list of the devices you wish you owned. Every one of those devices, as well as the electrical grid that powers your home, depends on modern electrical engineering. We cannot easily estimate the current financial worth of electrical engineering and its products, but we can be certain that the financial worth continues to grow yearly as more and more tasks are handled electrically. Radios are now tuned electronically instead of manually. Messages are now sent by email instead of through the postal system. Research journals are now read on a computer instead of in a library building, and research papers are now copied and filed electronically instead of photocopied and tucked into a filing cabinet. Indeed, you may be reading an electronic version of this book.

The basic science of electrical engineering is physics. In this chapter we cover the physics of electric circuits that are combinations of resistors and batteries (and, in Module 27-4, capacitors). We restrict our discussion to circuits through which charge flows in one direction, which are called either *direct-current circuits* or *DC circuits*. We begin with the question: How can you get charges to flow?

“Pumping” Charges

If you want to make charge carriers flow through a resistor, you must establish a potential difference between the ends of the device. One way to do this is to connect each end of the resistor to one plate of a charged capacitor. The trouble with this scheme is that the flow of charge acts to discharge the capacitor, quickly bringing the plates to the same potential. When that happens, there is no longer an electric field in the resistor, and thus the flow of charge stops.

To produce a steady flow of charge, you need a “charge pump,” a device that — by doing work on the charge carriers — maintains a potential difference between a pair of terminals. We call such a device an **emf device**, and the device is said to provide an **emf** \mathcal{E} , which means that it does work on charge carriers. An emf device is sometimes called a *seat of emf*. The term *emf* comes from the outdated phrase *electromotive force*, which was adopted before scientists clearly understood the function of an emf device.

In Chapter 26, we discussed the motion of charge carriers through a circuit in terms of the electric field set up in the circuit — the field produces forces that move the charge carriers. In this chapter we take a different approach: We discuss the motion of the charge carriers in terms of the required energy — an emf device supplies the energy for the motion via the work it does.

A common emf device is the *battery*, used to power a wide variety of machines from wristwatches to submarines. The emf device that most influences our daily lives, however, is the *electric generator*, which, by means of electrical connections (wires) from a generating plant, creates a potential difference in our homes and workplaces. The emf devices known as *solar cells*, long familiar as the wing-like panels on spacecraft, also dot the countryside for domestic applications. Less familiar emf devices are the *fuel cells* that powered the space shuttles and the *thermopiles* that provide onboard electrical power for some spacecraft and for remote stations in Antarctica and elsewhere. An emf device does not have to be an instrument — living systems, ranging from electric eels and human beings to plants, have physiological emf devices.

Although the devices we have listed differ widely in their modes of operation, they all perform the same basic function — they do work on charge carriers and thus maintain a potential difference between their terminals.

Work, Energy, and Emf

Figure 27-1 shows an emf device (consider it to be a battery) that is part of a simple circuit containing a single resistance R (the symbol for resistance and a resistor is $\text{---}\diagup\diagdown\text{---}$). The emf device keeps one of its terminals (called the positive terminal and often labeled $+$) at a higher electric potential than the other terminal (called the negative terminal and labeled $-$). We can represent the emf of the device with an arrow that points from the negative terminal toward the positive terminal as in Fig. 27-1. A small circle on the tail of the emf arrow distinguishes it from the arrows that indicate current direction.

When an emf device is not connected to a circuit, the internal chemistry of the device does not cause any net flow of charge carriers within it. However, when it is connected to a circuit as in Fig. 27-1, its internal chemistry causes a net flow of positive charge carriers from the negative terminal to the positive terminal, in the direction of the emf arrow. This flow is part of the current that is set up around the circuit in that same direction (clockwise in Fig. 27-1).

Within the emf device, positive charge carriers move from a region of low electric potential and thus low electric potential energy (at the negative terminal) to a region of higher electric potential and higher electric potential energy (at the positive terminal). This motion is just the opposite of what the electric field between the terminals (which is directed from the positive terminal toward the negative terminal) would cause the charge carriers to do.

Thus, there must be some source of energy within the device, enabling it to do work on the charges by forcing them to move as they do. The energy source may be chemical, as in a battery or a fuel cell. It may involve mechanical forces, as in an electric generator. Temperature differences may supply the energy, as in a thermopile; or the Sun may supply it, as in a solar cell.

Let us now analyze the circuit of Fig. 27-1 from the point of view of work and energy transfers. In any time interval dt , a charge dq passes through any cross section of this circuit, such as aa' . This same amount of charge must enter the emf device at its low-potential end and leave at its high-potential end. The device must do an amount of work dW on the charge dq to force it to move in this way. We define the emf of the emf device in terms of this work:

$$\mathcal{E} = \frac{dW}{dq} \quad (\text{definition of } \mathcal{E}). \quad (27-1)$$

In words, the emf of an emf device is the work per unit charge that the device does in moving charge from its low-potential terminal to its high-potential terminal. The SI unit for emf is the joule per coulomb; in Chapter 24 we defined that unit as the *volt*.

An **ideal emf device** is one that lacks any internal resistance to the internal movement of charge from terminal to terminal. The potential difference between the terminals of an ideal emf device is equal to the emf of the device. For example, an ideal battery with an emf of 12.0 V always has a potential difference of 12.0 V between its terminals.

A **real emf device**, such as any real battery, has internal resistance to the internal movement of charge. When a real emf device is not connected to a circuit, and thus does not have current through it, the potential difference between its terminals is equal to its emf. However, when that device has current through it, the potential difference between its terminals differs from its emf. We shall discuss such real batteries near the end of this module.

When an emf device is connected to a circuit, the device transfers energy to the charge carriers passing through it. This energy can then be transferred from the charge carriers to other devices in the circuit, for example, to light a bulb. Figure 27-2a shows a circuit containing two ideal rechargeable (*storage*) batteries A and B, a resistance R , and an electric motor M that can lift an object by using

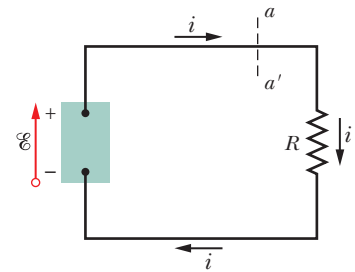
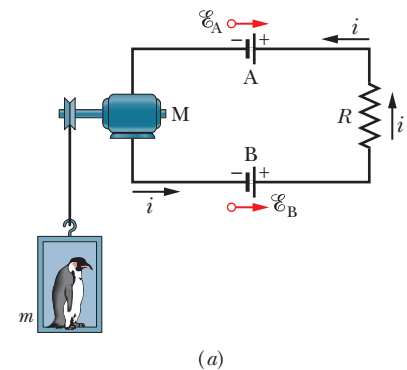
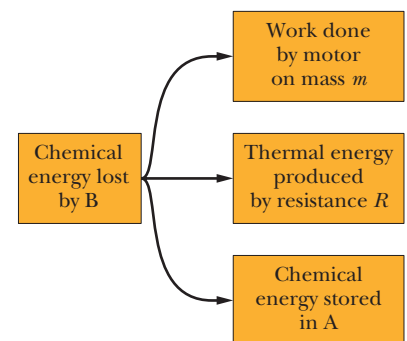


Figure 27-1 A simple electric circuit, in which a device of emf \mathcal{E} does work on the charge carriers and maintains a steady current i in a resistor of resistance R .



(a)



(b)

Figure 27-2 (a) In the circuit, $\mathcal{E}_B > \mathcal{E}_A$; so battery B determines the direction of the current. (b) The energy transfers in the circuit.

energy it obtains from charge carriers in the circuit. Note that the batteries are connected so that they tend to send charges around the circuit in opposite directions. The actual direction of the current in the circuit is determined by the battery with the larger emf, which happens to be battery B, so the chemical energy within battery B is decreasing as energy is transferred to the charge carriers passing through it. However, the chemical energy within battery A is increasing because the current in it is directed from the positive terminal to the negative terminal. Thus, battery B is charging battery A. Battery B is also providing energy to motor M and energy that is being dissipated by resistance R. Figure 27-2b shows all three energy transfers from battery B; each decreases that battery's chemical energy.

Calculating the Current in a Single-Loop Circuit

We discuss here two equivalent ways to calculate the current in the simple *single-loop* circuit of Fig. 27-3; one method is based on energy conservation considerations, and the other on the concept of potential. The circuit consists of an ideal battery B with emf \mathcal{E} , a resistor of resistance R , and two connecting wires. (Unless otherwise indicated, we assume that wires in circuits have negligible resistance. Their function, then, is merely to provide pathways along which charge carriers can move.)

Energy Method

Equation 26-27 ($P = i^2 R$) tells us that in a time interval dt an amount of energy given by $i^2 R dt$ will appear in the resistor of Fig. 27-3 as thermal energy. As noted in Module 26-5, this energy is said to be *dissipated*. (Because we assume the wires to have negligible resistance, no thermal energy will appear in them.) During the same interval, a charge $dq = i dt$ will have moved through battery B, and the work that the battery will have done on this charge, according to Eq. 27-1, is

$$dW = \mathcal{E} dq = \mathcal{E} i dt.$$

From the principle of conservation of energy, the work done by the (ideal) battery must equal the thermal energy that appears in the resistor:

$$\mathcal{E} i dt = i^2 R dt.$$

This gives us

$$\mathcal{E} = iR.$$

The emf \mathcal{E} is the energy per unit charge transferred to the moving charges by the battery. The quantity iR is the energy per unit charge transferred *from* the moving charges to thermal energy within the resistor. Therefore, this equation means that the energy per unit charge transferred to the moving charges is equal to the energy per unit charge transferred from them. Solving for i , we find

$$i = \frac{\mathcal{E}}{R}. \quad (27-2)$$

The battery drives current through the resistor, from high potential to low potential.

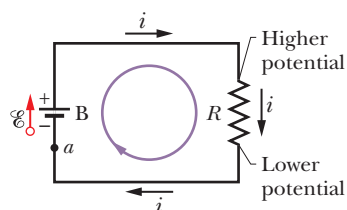


Figure 27-3 A single-loop circuit in which a resistance R is connected across an ideal battery B with emf \mathcal{E} . The resulting current i is the same throughout the circuit.

Potential Method

Suppose we start at any point in the circuit of Fig. 27-3 and mentally proceed around the circuit in either direction, adding algebraically the potential differences that we encounter. Then when we return to our starting point, we must also have returned to our starting potential. Before actually doing so, we shall formalize this idea in a statement that holds not only for single-loop circuits such as that of Fig. 27-3 but also for any complete loop in a *multiloop* circuit, as we shall discuss in Module 27-2:



LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

This is often referred to as *Kirchhoff's loop rule* (or *Kirchhoff's voltage law*), after German physicist Gustav Robert Kirchhoff. This rule is equivalent to saying that each point on a mountain has only one elevation above sea level. If you start from any point and return to it after walking around the mountain, the algebraic sum of the changes in elevation that you encounter must be zero.

In Fig. 27-3, let us start at point a , whose potential is V_a , and mentally walk clockwise around the circuit until we are back at a , keeping track of potential changes as we move. Our starting point is at the low-potential terminal of the battery. Because the battery is ideal, the potential difference between its terminals is equal to \mathcal{E} . When we pass through the battery to the high-potential terminal, the change in potential is $+\mathcal{E}$.

As we walk along the top wire to the top end of the resistor, there is no potential change because the wire has negligible resistance; it is at the same potential as the high-potential terminal of the battery. So too is the top end of the resistor. When we pass through the resistor, however, the potential changes according to Eq. 26-8 (which we can rewrite as $V = iR$). Moreover, the potential must decrease because we are moving from the higher potential side of the resistor. Thus, the change in potential is $-iR$.

We return to point a by moving along the bottom wire. Because this wire also has negligible resistance, we again find no potential change. Back at point a , the potential is again V_a . Because we traversed a complete loop, our initial potential, as modified for potential changes along the way, must be equal to our final potential; that is,

$$V_a + \mathcal{E} - iR = V_a.$$

The value of V_a cancels from this equation, which becomes

$$\mathcal{E} - iR = 0.$$

Solving this equation for i gives us the same result, $i = \mathcal{E}/R$, as the energy method (Eq. 27-2).

If we apply the loop rule to a complete *counterclockwise* walk around the circuit, the rule gives us

$$-\mathcal{E} + iR = 0$$

and we again find that $i = \mathcal{E}/R$. Thus, you may mentally circle a loop in either direction to apply the loop rule.

To prepare for circuits more complex than that of Fig. 27-3, let us set down two rules for finding potential differences as we move around a loop:



RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

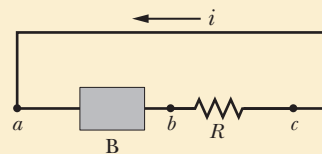


EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$.



Checkpoint 1

The figure shows the current i in a single-loop circuit with a battery B and a resistance R (and wires of negligible resistance). (a) Should the emf arrow at B be drawn pointing leftward or rightward? At points a , b , and c , rank (b) the magnitude of the current, (c) the electric potential, and (d) the electric potential energy of the charge carriers, greatest first.



Other Single-Loop Circuits

Next we extend the simple circuit of Fig. 27-3 in two ways.

Internal Resistance

Figure 27-4a shows a real battery, with internal resistance r , wired to an external resistor of resistance R . The internal resistance of the battery is the electrical resistance of the conducting materials of the battery and thus is an unremovable feature of the battery. In Fig. 27-4a, however, the battery is drawn as if it could be separated into an ideal battery with emf \mathcal{E} and a resistor of resistance r . The order in which the symbols for these separated parts are drawn does not matter.

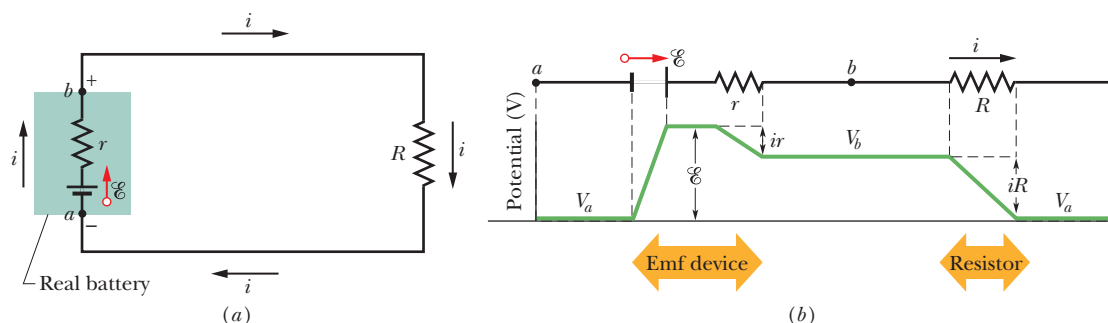
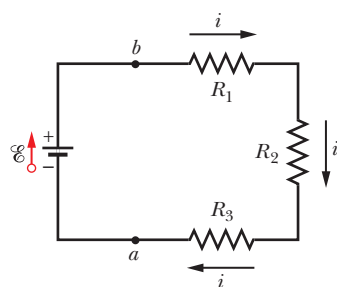
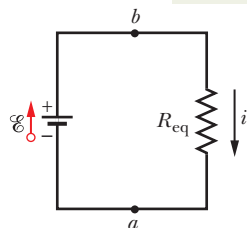


Figure 27-4 (a) A single-loop circuit containing a real battery having internal resistance r and emf \mathcal{E} . (b) The same circuit, now spread out in a line. The potentials encountered in traversing the circuit clockwise from a are also shown. The potential V_a is arbitrarily assigned a value of zero, and other potentials in the circuit are graphed relative to V_a .



(a)

Series resistors and their equivalent have the same current (“ser- i ”).



(b)

Figure 27-5 (a) Three resistors are connected in series between points a and b . (b) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq} .

If we apply the loop rule clockwise beginning at point a , the *changes* in potential give us

$$\mathcal{E} - ir - iR = 0. \quad (27-3)$$

Solving for the current, we find

$$i = \frac{\mathcal{E}}{R + r}. \quad (27-4)$$

Note that this equation reduces to Eq. 27-2 if the battery is ideal — that is, if $r = 0$.

Figure 27-4b shows graphically the changes in electric potential around the circuit. (To better link Fig. 27-4b with the *closed circuit* in Fig. 27-4a, imagine curling the graph into a cylinder with point a at the left overlapping point a at the right.) Note how traversing the circuit is like walking around a (potential) mountain back to your starting point — you return to the starting elevation.

In this book, when a battery is not described as real or if no internal resistance is indicated, you can generally assume that it is ideal — but, of course, in the real world batteries are always real and have internal resistance.

Resistances in Series

Figure 27-5a shows three resistances connected **in series** to an ideal battery with emf \mathcal{E} . This description has little to do with how the resistances are drawn. Rather, “in series” means that the resistances are wired one after another and that a potential difference V is applied across the two ends of the series. In Fig. 27-5a, the resistances are connected one after another between a and b , and a potential difference is maintained across a and b by the battery. The potential differences that then exist across the resistances in the series produce identical currents i in them. In general,



When a potential difference V is applied across resistances connected in series, the resistances have identical currents i . The sum of the potential differences across the resistances is equal to the applied potential difference V .

Note that charge moving through the series resistances can move along only a single route. If there are additional routes, so that the currents in different resistances are different, the resistances are not connected in series.



Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same *total* potential difference V as the actual resistances.

You might remember that R_{eq} and all the actual series resistances have the same current i with the nonsense word “ser-i.” Figure 27-5*b* shows the equivalent resistance R_{eq} that can replace the three resistances of Fig. 27-5*a*.

To derive an expression for R_{eq} in Fig. 27-5*b*, we apply the loop rule to both circuits. For Fig. 27-5*a*, starting at a and going clockwise around the circuit, we find

$$\mathcal{E} - iR_1 - iR_2 - iR_3 = 0,$$

or

$$i = \frac{\mathcal{E}}{R_1 + R_2 + R_3}. \quad (27-5)$$

For Fig. 27-5*b*, with the three resistances replaced with a single equivalent resistance R_{eq} , we find

$$\mathcal{E} - iR_{\text{eq}} = 0,$$

or

$$i = \frac{\mathcal{E}}{R_{\text{eq}}}. \quad (27-6)$$

Comparison of Eqs. 27-5 and 27-6 shows that

$$R_{\text{eq}} = R_1 + R_2 + R_3.$$

The extension to n resistances is straightforward and is

$$R_{\text{eq}} = \sum_{j=1}^n R_j \quad (n \text{ resistances in series}). \quad (27-7)$$

Note that when resistances are in series, their equivalent resistance is greater than any of the individual resistances.



Checkpoint 2

In Fig. 27-5*a*, if $R_1 > R_2 > R_3$, rank the three resistances according to (a) the current through them and (b) the potential difference across them, greatest first.

Potential Difference Between Two Points

We often want to find the potential difference between two points in a circuit. For example, in Fig. 27-6, what is the potential difference $V_b - V_a$ between points a and b ? To find out, let's start at point a (at potential V_a) and move through the battery to point b (at potential V_b) while keeping track of the potential changes we encounter. When we pass through the battery's emf, the potential increases by \mathcal{E} . When we pass through the battery's internal resistance r , we move in the direction of the current and thus the potential decreases by ir . We are then at the

The internal resistance reduces the potential difference between the terminals.

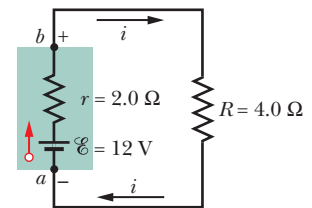


Figure 27-6 Points a and b , which are at the terminals of a real battery, differ in potential.

potential of point b and we have

$$V_a + \mathcal{E} - ir = V_b,$$

or

$$V_b - V_a = \mathcal{E} - ir. \quad (27-8)$$

To evaluate this expression, we need the current i . Note that the circuit is the same as in Fig. 27-4a, for which Eq. 27-4 gives the current as

$$i = \frac{\mathcal{E}}{R + r}. \quad (27-9)$$

Substituting this equation into Eq. 27-8 gives us

$$\begin{aligned} V_b - V_a &= \mathcal{E} - \frac{\mathcal{E}}{R + r} r \\ &= \frac{\mathcal{E}}{R + r} R. \end{aligned} \quad (27-10)$$

Now substituting the data given in Fig. 27-6, we have

$$V_b - V_a = \frac{12 \text{ V}}{4.0 \, \Omega + 2.0 \, \Omega} 4.0 \, \Omega = 8.0 \text{ V}. \quad (27-11)$$

Suppose, instead, we move from a to b counterclockwise, passing through resistor R rather than through the battery. Because we move opposite the current, the potential increases by iR . Thus,

$$V_a + iR = V_b$$

or

$$V_b - V_a = iR. \quad (27-12)$$

Substituting for i from Eq. 27-9, we again find Eq. 27-10. Hence, substitution of the data in Fig. 27-6 yields the same result, $V_b - V_a = 8.0 \text{ V}$. In general,



To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.

Potential Difference Across a Real Battery

In Fig. 27-6, points a and b are located at the terminals of the battery. Thus, the potential difference $V_b - V_a$ is the terminal-to-terminal potential difference V across the battery. From Eq. 27-8, we see that

$$V = \mathcal{E} - ir. \quad (27-13)$$

If the internal resistance r of the battery in Fig. 27-6 were zero, Eq. 27-13 tells us that V would be equal to the emf \mathcal{E} of the battery — namely, 12 V. However, because $r = 2.0 \, \Omega$, Eq. 27-13 tells us that V is less than \mathcal{E} . From Eq. 27-11, we know that V is only 8.0 V. Note that the result depends on the value of the current through the battery. If the same battery were in a different circuit and had a different current through it, V would have some other value.

Grounding a Circuit

Figure 27-7a shows the same circuit as Fig. 27-6 except that here point a is directly connected to *ground*, as indicated by the common symbol \perp . *Grounding a circuit* usually means connecting the circuit to a conducting path to Earth's surface (actually to the electrically conducting moist dirt and rock below ground). Here, such a connection means only that the potential is defined to be zero at the grounding point in the circuit. Thus in Fig. 27-7a, the potential at a is defined to be $V_a = 0$. Equation 27-11 then tells us that the potential at b is $V_b = 8.0 \text{ V}$.

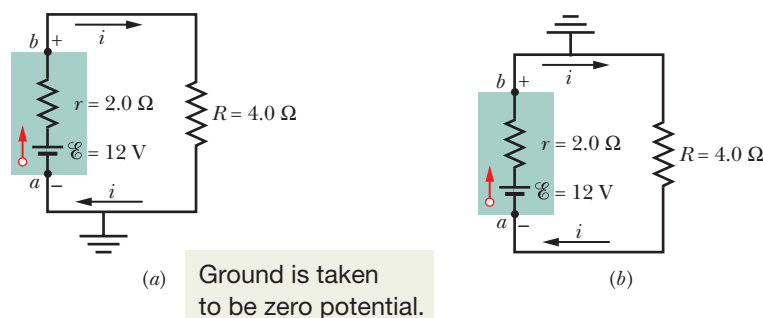


Figure 27-7 (a) Point a is directly connected to ground. (b) Point b is directly connected to ground.

Figure 27-7b is the same circuit except that point b is now directly connected to ground. Thus, the potential there is defined to be $V_b = 0$. Equation 27-11 now tells us that the potential at a is $V_a = -8.0$ V.

Power, Potential, and Emf

When a battery or some other type of emf device does work on the charge carriers to establish a current i , the device transfers energy from its source of energy (such as the chemical source in a battery) to the charge carriers. Because a real emf device has an internal resistance r , it also transfers energy to internal thermal energy via resistive dissipation (Module 26-5). Let us relate these transfers.

The net rate P of energy transfer from the emf device to the charge carriers is given by Eq. 26-26:

$$P = iV, \quad (27-14)$$

where V is the potential across the terminals of the emf device. From Eq. 27-13, we can substitute $V = \mathcal{E} - ir$ into Eq. 27-14 to find

$$P = i(\mathcal{E} - ir) = i\mathcal{E} - i^2r. \quad (27-15)$$

From Eq. 26-27, we recognize the term i^2r in Eq. 27-15 as the rate P_r of energy transfer to thermal energy within the emf device:

$$P_r = i^2r \quad (\text{internal dissipation rate}). \quad (27-16)$$

Then the term $i\mathcal{E}$ in Eq. 27-15 must be the rate P_{emf} at which the emf device transfers energy *both* to the charge carriers and to internal thermal energy. Thus,

$$P_{\text{emf}} = i\mathcal{E} \quad (\text{power of emf device}). \quad (27-17)$$

If a battery is being *recharged*, with a “wrong way” current through it, the energy transfer is then *from* the charge carriers *to* the battery — both to the battery’s chemical energy and to the energy dissipated in the internal resistance r . The rate of change of the chemical energy is given by Eq. 27-17, the rate of dissipation is given by Eq. 27-16, and the rate at which the carriers supply energy is given by Eq. 27-14.



Checkpoint 3

A battery has an emf of 12 V and an internal resistance of 2 Ω . Is the terminal-to-terminal potential difference greater than, less than, or equal to 12 V if the current in the battery is (a) from the negative to the positive terminal, (b) from the positive to the negative terminal, and (c) zero?

Sample Problem 27.01 Single-loop circuit with two real batteries

The emfs and resistances in the circuit of Fig. 27-8a have the following values:

$$\mathcal{E}_1 = 4.4 \text{ V}, \quad \mathcal{E}_2 = 2.1 \text{ V}, \\ r_1 = 2.3 \, \Omega, \quad r_2 = 1.8 \, \Omega, \quad R = 5.5 \, \Omega.$$

(a) What is the current i in the circuit?

KEY IDEA

We can get an expression involving the current i in this single-loop circuit by applying the loop rule, in which we sum the potential changes around the full loop.

Calculations: Although knowing the direction of i is not necessary, we can easily determine it from the emfs of the

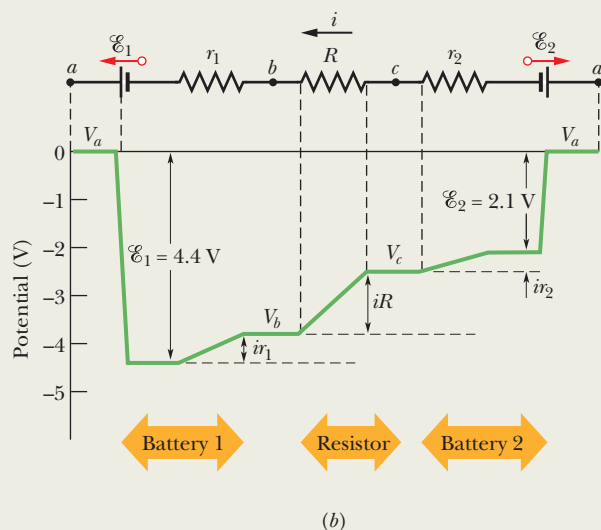
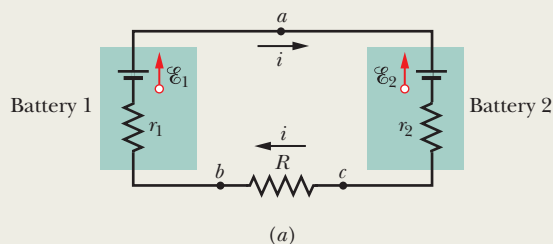


Figure 27-8 (a) A single-loop circuit containing two real batteries and a resistor. The batteries oppose each other; that is, they tend to send current in opposite directions through the resistor. (b) A graph of the potentials, counterclockwise from point a , with the potential at a arbitrarily taken to be zero. (To better link the circuit with the graph, mentally cut the circuit at a and then unfold the left side of the circuit toward the left and the right side of the circuit toward the right.)

two batteries. Because \mathcal{E}_1 is greater than \mathcal{E}_2 , battery 1 controls the direction of i , so the direction is clockwise. Let us then apply the loop rule by going counterclockwise — against the current — and starting at point a . (These decisions about where to start and which way you go are arbitrary but, once made, you must be consistent with decisions about the plus and minus signs.) We find

$$-\mathcal{E}_1 + ir_1 + iR + ir_2 + \mathcal{E}_2 = 0.$$

Check that this equation also results if we apply the loop rule clockwise or start at some point other than a . Also, take the time to compare this equation term by term with Fig. 27-8b, which shows the potential changes graphically (with the potential at point a arbitrarily taken to be zero).

Solving the above loop equation for the current i , we obtain

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R + r_1 + r_2} = \frac{4.4 \text{ V} - 2.1 \text{ V}}{5.5 \, \Omega + 2.3 \, \Omega + 1.8 \, \Omega} \\ = 0.2396 \text{ A} \approx 240 \text{ mA}. \quad (\text{Answer})$$

(b) What is the potential difference between the terminals of battery 1 in Fig. 27-8a?

KEY IDEA

We need to sum the potential differences between points a and b .

Calculations: Let us start at point b (effectively the negative terminal of battery 1) and travel clockwise through battery 1 to point a (effectively the positive terminal), keeping track of potential changes. We find that

$$V_b - ir_1 + \mathcal{E}_1 = V_a,$$

which gives us

$$V_a - V_b = -ir_1 + \mathcal{E}_1 \\ = -(0.2396 \text{ A})(2.3 \, \Omega) + 4.4 \text{ V} \\ = +3.84 \text{ V} \approx 3.8 \text{ V}, \quad (\text{Answer})$$

which is less than the emf of the battery. You can verify this result by starting at point b in Fig. 27-8a and traversing the circuit counterclockwise to point a . We learn two points here. (1) The potential difference between two points in a circuit is independent of the path we choose to go from one to the other. (2) When the current in the battery is in the “proper” direction, the terminal-to-terminal potential difference is low, that is, lower than the stated emf for the battery that you might find printed on the battery.

27-2 MULTILoop CIRCUITS

Learning Objectives

After reading this module, you should be able to . . .

27.17 Apply the junction rule.

27.18 Draw a schematic diagram for a battery and three parallel resistors and distinguish it from a diagram with a battery and three series resistors.

27.19 Identify that resistors in parallel have the same potential difference, which is the same value that their equivalent resistor has.

27.20 Calculate the resistance of the equivalent resistor of several resistors in parallel.

27.21 Identify that the total current through parallel resistors is the sum of the currents through the individual resistors.

27.22 For a circuit with a battery and some resistors in parallel and some in series, simplify the circuit in steps

by finding equivalent resistors, until the current through the battery can be determined, and then reverse the steps to find the currents and potential differences of the individual resistors.

27.23 If a circuit cannot be simplified by using equivalent resistors, identify the several loops in the circuit, choose names and directions for the currents in the branches, set up loop equations for the various loops, and solve these simultaneous equations for the unknown currents.

27.24 In a circuit with identical real batteries in series, replace them with a single ideal battery and a single resistor.

27.25 In a circuit with identical real batteries in parallel, replace them with a single ideal battery and a single resistor.

Key Idea

● When resistances are in parallel, they have the same potential difference. The equivalent resistance that can replace a parallel combination of resistances is given by

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^n \frac{1}{R_j} \quad (n \text{ resistances in parallel}).$$

Multiloop Circuits

Figure 27-9 shows a circuit containing more than one loop. For simplicity, we assume the batteries are ideal. There are two *junctions* in this circuit, at *b* and *d*, and there are three *branches* connecting these junctions. The branches are the left branch (*bad*), the right branch (*bcd*), and the central branch (*bd*). What are the currents in the three branches?

We arbitrarily label the currents, using a different subscript for each branch. Current i_1 has the same value everywhere in branch *bad*, i_2 has the same value everywhere in branch *bcd*, and i_3 is the current through branch *bd*. The directions of the currents are assumed arbitrarily.

Consider junction *d* for a moment: Charge comes into that junction via incoming currents i_1 and i_3 , and it leaves via outgoing current i_2 . Because there is no variation in the charge at the junction, the total incoming current must equal the total outgoing current:

$$i_1 + i_3 = i_2. \quad (27-18)$$

You can easily check that applying this condition to junction *b* leads to exactly the same equation. Equation 27-18 thus suggests a general principle:



JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

This rule is often called *Kirchhoff's junction rule* (or *Kirchhoff's current law*). It is simply a statement of the conservation of charge for a steady flow of charge — there is neither a buildup nor a depletion of charge at a junction. Thus, our basic tools for solving complex circuits are the *loop rule* (based on the conservation of energy) and the *junction rule* (based on the conservation of charge).

The current into the junction must equal the current out (charge is conserved).

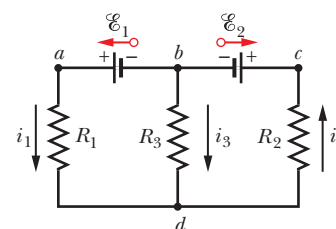


Figure 27-9 A multiloop circuit consisting of three branches: left-hand branch *bad*, right-hand branch *bcd*, and central branch *bd*. The circuit also consists of three loops: left-hand loop *badb*, right-hand loop *bcd b*, and big loop *badcb*.

Equation 27-18 is a single equation involving three unknowns. To solve the circuit completely (that is, to find all three currents), we need two more equations involving those same unknowns. We obtain them by applying the loop rule twice. In the circuit of Fig. 27-9, we have three loops from which to choose: the left-hand loop ($badb$), the right-hand loop ($bcd b$), and the big loop ($badcb$). Which two loops we choose does not matter — let's choose the left-hand loop and the right-hand loop.

If we traverse the left-hand loop in a counterclockwise direction from point b , the loop rule gives us

$$\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0. \quad (27-19)$$

If we traverse the right-hand loop in a counterclockwise direction from point b , the loop rule gives us

$$-i_3 R_3 - i_2 R_2 - \mathcal{E}_2 = 0. \quad (27-20)$$

We now have three equations (Eqs. 27-18, 27-19, and 27-20) in the three unknown currents, and they can be solved by a variety of techniques.

If we had applied the loop rule to the big loop, we would have obtained (moving counterclockwise from b) the equation

$$\mathcal{E}_1 - i_1 R_1 - i_2 R_2 - \mathcal{E}_2 = 0.$$

However, this is merely the sum of Eqs. 27-19 and 27-20.

Resistances in Parallel

Figure 27-10a shows three resistances connected *in parallel* to an ideal battery of emf \mathcal{E} . The term “in parallel” means that the resistances are directly wired together on one side and directly wired together on the other side, and that a potential difference V is applied across the pair of connected sides. Thus, all three resistances have the same potential difference V across them, producing a current through each. In general,



When a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V .

In Fig. 27-10a, the applied potential difference V is maintained by the battery. In Fig. 27-10b, the three parallel resistances have been replaced with an equivalent resistance R_{eq} .

Parallel resistors and their equivalent have the same potential difference (“par- V ”).

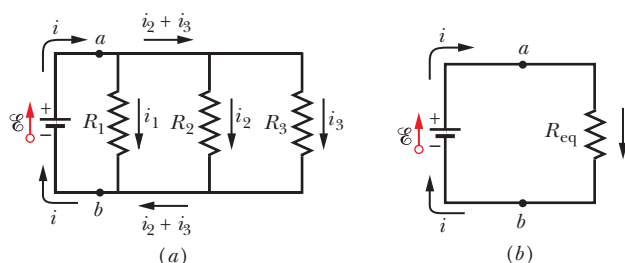


Figure 27-10 (a) Three resistors connected in parallel across points a and b . (b) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq} .



Resistances connected in parallel can be replaced with an equivalent resistance R_{eq} that has the same potential difference V and the same *total* current i as the actual resistances.

You might remember that R_{eq} and all the actual parallel resistances have the same potential difference V with the nonsense word “par- V .”

To derive an expression for R_{eq} in Fig. 27-10*b*, we first write the current in each actual resistance in Fig. 27-10*a* as

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

where V is the potential difference between a and b . If we apply the junction rule at point a in Fig. 27-10*a* and then substitute these values, we find

$$i = i_1 + i_2 + i_3 = V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right). \tag{27-21}$$

If we replaced the parallel combination with the equivalent resistance R_{eq} (Fig. 27-10*b*), we would have

$$i = \frac{V}{R_{\text{eq}}}. \tag{27-22}$$

Comparing Eqs. 27-21 and 27-22 leads to

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \tag{27-23}$$

Extending this result to the case of n resistances, we have

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^n \frac{1}{R_j} \quad (n \text{ resistances in parallel}). \tag{27-24}$$

For the case of two resistances, the equivalent resistance is their product divided by their sum; that is,

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}. \tag{27-25}$$

Note that when two or more resistances are connected in parallel, the equivalent resistance is smaller than any of the combining resistances. Table 27-1 summarizes the equivalence relations for resistors and capacitors in series and in parallel.

Table 27-1 Series and Parallel Resistors and Capacitors

Series	Parallel	Series	Parallel
<u>Resistors</u>		<u>Capacitors</u>	
$R_{\text{eq}} = \sum_{j=1}^n R_j$ Eq. 27-7	$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^n \frac{1}{R_j}$ Eq. 27-24	$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^n \frac{1}{C_j}$ Eq. 25-20	$C_{\text{eq}} = \sum_{j=1}^n C_j$ Eq. 25-19
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors



Checkpoint 4

A battery, with potential V across it, is connected to a combination of two identical resistors and then has current i through it. What are the potential difference across and the current through either resistor if the resistors are (a) in series and (b) in parallel?



Sample Problem 27.02 Resistors in parallel and in series

Figure 27-11a shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$$R_1 = 20\ \Omega, \quad R_2 = 20\ \Omega, \quad \mathcal{E} = 12\ \text{V},$$

$$R_3 = 30\ \Omega, \quad R_4 = 8.0\ \Omega.$$

(a) What is the current through the battery?

KEY IDEA

Noting that the current through the battery must also be the current through R_1 , we see that we might find the current by applying the loop rule to a loop that includes R_1 because the current would be included in the potential difference across R_1 .

Incorrect method: Either the left-hand loop or the big loop should do. Noting that the emf arrow of the battery points upward, so the current the battery supplies is clockwise, we might apply the loop rule to the left-hand loop, clockwise from point a . With i being the current through the battery, we would get

$$+\mathcal{E} - iR_1 - iR_2 - iR_4 = 0 \quad (\text{incorrect}).$$

However, this equation is incorrect because it assumes that R_1 , R_2 , and R_4 all have the same current i . Resistances R_1 and R_4 do have the same current, because the current passing through R_4 must pass through the battery and then through R_1 with no change in value. However, that current splits at junction point b — only part passes through R_2 , the rest through R_3 .

Dead-end method: To distinguish the several currents in the circuit, we must label them individually as in Fig. 27-11b. Then, circling clockwise from a , we can write the loop rule for the left-hand loop as

$$+\mathcal{E} - i_1R_1 - i_2R_2 - i_1R_4 = 0.$$

Unfortunately, this equation contains two unknowns, i_1 and i_2 ; we would need at least one more equation to find them.

Successful method: A much easier option is to simplify the circuit of Fig. 27-11b by finding equivalent resistances. Note carefully that R_1 and R_2 are *not* in series and thus cannot be replaced with an equivalent resistance. However, R_2 and R_3 are in parallel, so we can use either Eq. 27-24 or Eq. 27-25 to find their equivalent resistance R_{23} . From the latter,

$$R_{23} = \frac{R_2R_3}{R_2 + R_3} = \frac{(20\ \Omega)(30\ \Omega)}{50\ \Omega} = 12\ \Omega.$$

We can now redraw the circuit as in Fig. 27-11c; note that the current through R_{23} must be i_1 because charge that moves through R_1 and R_4 must also move through R_{23} . For this simple one-loop circuit, the loop rule (applied clockwise from point a as in Fig. 27-11d) yields

$$+\mathcal{E} - i_1R_1 - i_1R_{23} - i_1R_4 = 0.$$

Substituting the given data, we find

$$12\ \text{V} - i_1(20\ \Omega) - i_1(12\ \Omega) - i_1(8.0\ \Omega) = 0,$$

which gives us

$$i_1 = \frac{12\ \text{V}}{40\ \Omega} = 0.30\ \text{A}. \quad (\text{Answer})$$

(b) What is the current i_2 through R_2 ?

KEY IDEAS

(1) we must now work backward from the equivalent circuit of Fig. 27-11d, where R_{23} has replaced R_2 and R_3 . (2) Because R_2 and R_3 are in parallel, they both have the same potential difference across them as R_{23} .

Working backward: We know that the current through R_{23} is $i_1 = 0.30\ \text{A}$. Thus, we can use Eq. 26-8 ($R = V/i$) and Fig. 27-11e to find the potential difference V_{23} across R_{23} . Setting $R_{23} = 12\ \Omega$ from (a), we write Eq. 26-8 as

$$V_{23} = i_1R_{23} = (0.30\ \text{A})(12\ \Omega) = 3.6\ \text{V}.$$

The potential difference across R_2 is thus also $3.6\ \text{V}$ (Fig. 27-11f), so the current i_2 in R_2 must be, by Eq. 26-8 and Fig. 27-11g,

$$i_2 = \frac{V_2}{R_2} = \frac{3.6\ \text{V}}{20\ \Omega} = 0.18\ \text{A}. \quad (\text{Answer})$$

(c) What is the current i_3 through R_3 ?

KEY IDEAS

We can answer by using either of two techniques: (1) Apply Eq. 26-8 as we just did. (2) Use the junction rule, which tells us that at point b in Fig. 27-11b, the incoming current i_1 and the outgoing currents i_2 and i_3 are related by

$$i_1 = i_2 + i_3.$$

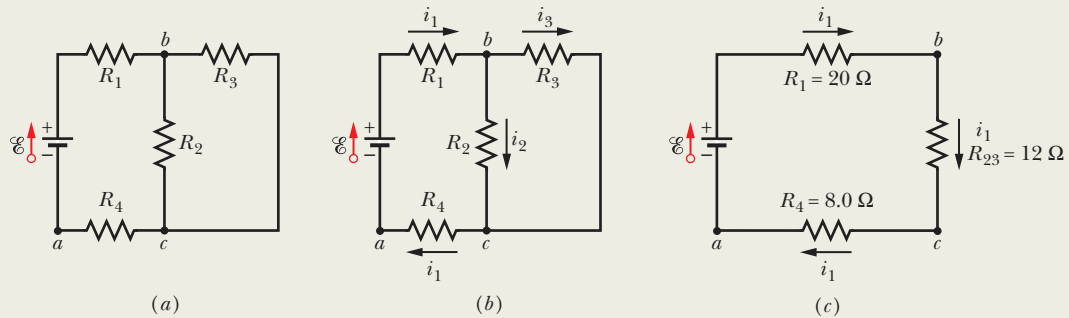
Calculation: Rearranging this junction-rule result yields the result displayed in Fig. 27-11g:

$$\begin{aligned} i_3 &= i_1 - i_2 = 0.30\ \text{A} - 0.18\ \text{A} \\ &= 0.12\ \text{A}. \end{aligned} \quad (\text{Answer})$$

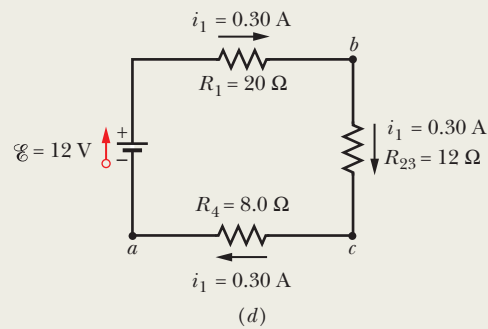




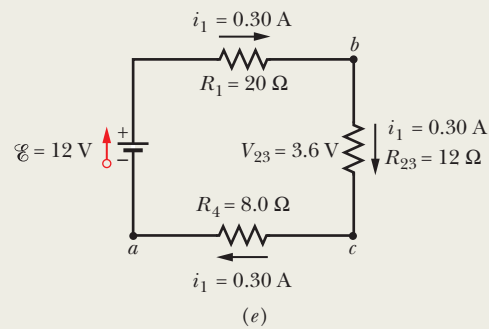
The equivalent of parallel resistors is smaller.



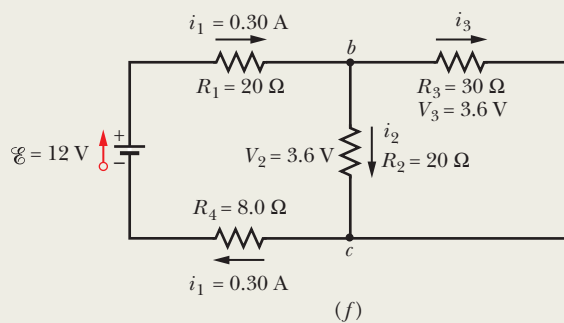
Applying the loop rule yields the current.



Applying $V = iR$ yields the potential difference.



Parallel resistors and their equivalent have the same V ("par-V").



Applying $i = V/R$ yields the current.

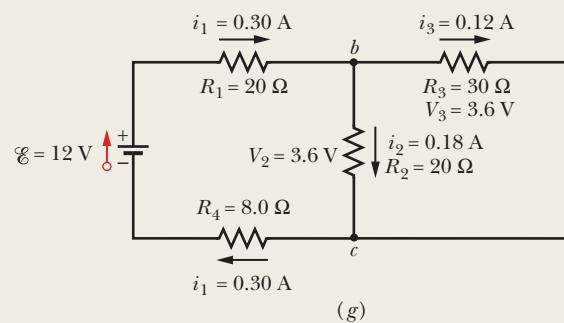


Figure 27-11 (a) A circuit with an ideal battery. (b) Label the currents. (c) Replacing the parallel resistors with their equivalent. (d)–(g) Working backward to find the currents through the parallel resistors.





Sample Problem 27.03 Many real batteries in series and in parallel in an electric fish

Electric fish can generate current with biological emf cells called *electroplaques*. In the South American eel they are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 cells, as suggested by Fig. 27-12a. Each electroplaque has an emf \mathcal{E} of 0.15 V and an internal resistance r of $0.25\ \Omega$. The water surrounding the eel completes a circuit between the two ends of the electroplaque array, one end at the head of the animal and the other near the tail.

(a) If the surrounding water has resistance $R_w = 800\ \Omega$, how much current can the eel produce in the water?

KEY IDEA

We can simplify the circuit of Fig. 27-12a by replacing combinations of emfs and internal resistances with equivalent emfs and resistances.

Calculations: We first consider a single row. The total emf \mathcal{E}_{row} along a row of 5000 electroplaques is the sum of the emfs:

$$\mathcal{E}_{\text{row}} = 5000\mathcal{E} = (5000)(0.15\ \text{V}) = 750\ \text{V}.$$

The total resistance R_{row} along a row is the sum of the internal resistances of the 5000 electroplaques:

$$R_{\text{row}} = 5000r = (5000)(0.25\ \Omega) = 1250\ \Omega.$$

We can now represent each of the 140 identical rows as having a single emf \mathcal{E}_{row} and a single resistance R_{row} (Fig. 27-12b).

In Fig. 27-12b, the emf between point a and point b on any row is $\mathcal{E}_{\text{row}} = 750\ \text{V}$. Because the rows are identical and because they are all connected together at the left in Fig. 27-12b, all points b in that figure are at the same electric potential. Thus, we can consider them to be connected so that there is only a single point b . The emf between point a and this single point b is $\mathcal{E}_{\text{row}} = 750\ \text{V}$, so we can draw the circuit as shown in Fig. 27-12c.

Between points b and c in Fig. 27-12c are 140 resistances $R_{\text{row}} = 1250\ \Omega$, all in parallel. The equivalent resistance R_{eq} of this combination is given by Eq. 27-24 as

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{140} \frac{1}{R_j} = 140 \frac{1}{R_{\text{row}}},$$

$$\text{or} \quad R_{\text{eq}} = \frac{R_{\text{row}}}{140} = \frac{1250\ \Omega}{140} = 8.93\ \Omega.$$

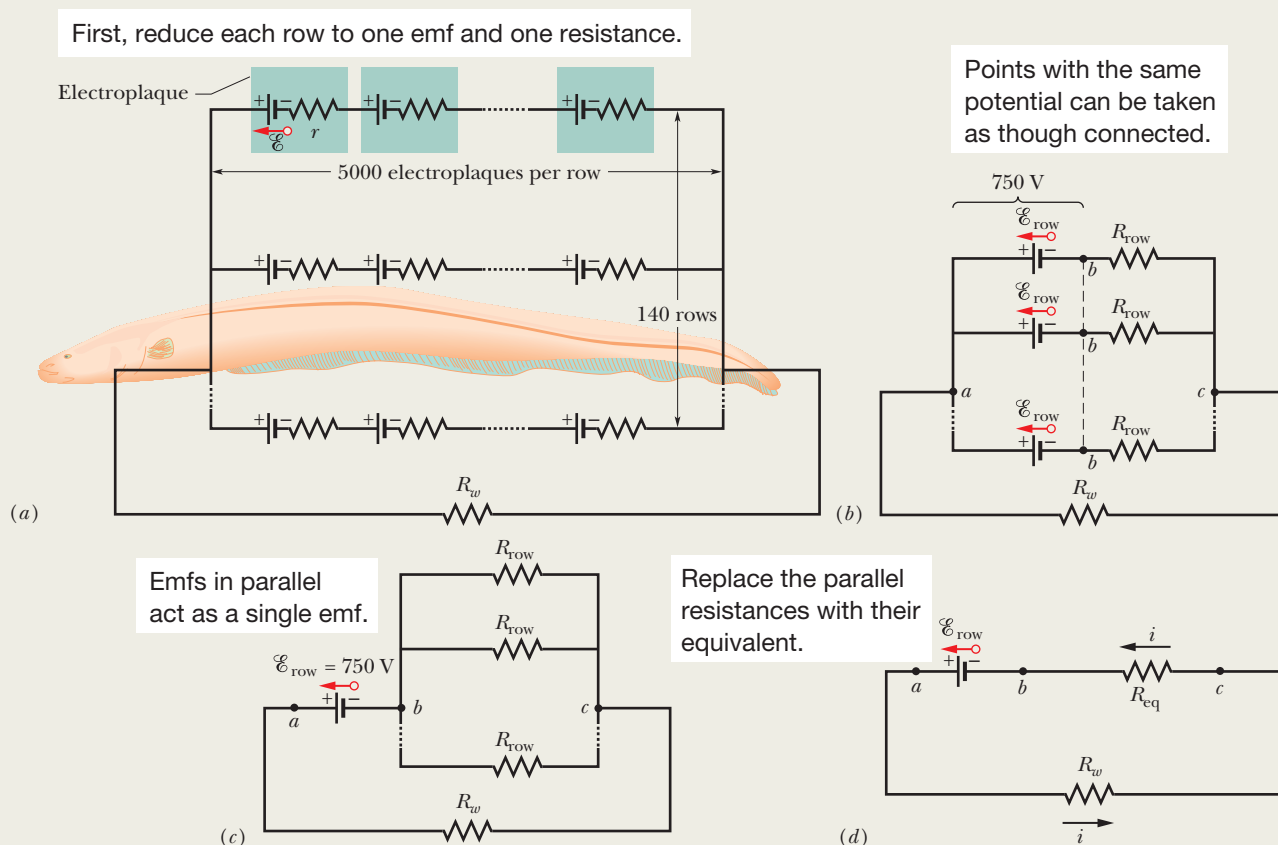


Figure 27-12 (a) A model of the electric circuit of an eel in water. Along each of 140 rows extending from the head to the tail of the eel, there are 5000 electroplaques. The surrounding water has resistance R_w . (b) The emf \mathcal{E}_{row} and resistance R_{row} of each row. (c) The emf between points a and b is \mathcal{E}_{row} . Between points b and c are 140 parallel resistances R_{row} . (d) The simplified circuit.

Replacing the parallel combination with R_{eq} , we obtain the simplified circuit of Fig. 27-12*d*. Applying the loop rule to this circuit counterclockwise from point b , we have

$$\mathcal{E}_{\text{row}} - iR_w - iR_{\text{eq}} = 0.$$

Solving for i and substituting the known data, we find

$$i = \frac{\mathcal{E}_{\text{row}}}{R_w + R_{\text{eq}}} = \frac{750 \text{ V}}{800 \Omega + 8.93 \Omega} \\ = 0.927 \text{ A} \approx 0.93 \text{ A}. \quad (\text{Answer})$$

If the head or tail of the eel is near a fish, some of this current could pass along a narrow path through the fish, stunning or killing it.

(b) How much current i_{row} travels through each row of Fig. 27-12*a*?

KEY IDEA

Because the rows are identical, the current into and out of the eel is evenly divided among them.

Calculation: Thus, we write

$$i_{\text{row}} = \frac{i}{140} = \frac{0.927 \text{ A}}{140} = 6.6 \times 10^{-3} \text{ A}. \quad (\text{Answer})$$

Thus, the current through each row is small, so that the eel need not stun or kill itself when it stuns or kills a fish.

Sample Problem 27.04 Multiloop circuit and simultaneous loop equations

Figure 27-13 shows a circuit whose elements have the following values: $\mathcal{E}_1 = 3.0 \text{ V}$, $\mathcal{E}_2 = 6.0 \text{ V}$, $R_1 = 2.0 \Omega$, $R_2 = 4.0 \Omega$. The three batteries are ideal batteries. Find the magnitude and direction of the current in each of the three branches.

KEY IDEAS

It is not worthwhile to try to simplify this circuit, because no two resistors are in parallel, and the resistors that are in series (those in the right branch or those in the left branch) present no problem. So, our plan is to apply the junction and loop rules.

Junction rule: Using arbitrarily chosen directions for the currents as shown in Fig. 27-13, we apply the junction rule at point a by writing

$$i_3 = i_1 + i_2. \quad (27-26)$$

An application of the junction rule at junction b gives only the same equation, so we next apply the loop rule to any two of the three loops of the circuit.

Left-hand loop: We first arbitrarily choose the left-hand loop, arbitrarily start at point b , and arbitrarily traverse the loop in the clockwise direction, obtaining

$$-i_1 R_1 + \mathcal{E}_1 - i_1 R_1 - (i_1 + i_2) R_2 - \mathcal{E}_2 = 0,$$

where we have used $(i_1 + i_2)$ instead of i_3 in the middle branch. Substituting the given data and simplifying yield

$$i_1(8.0 \Omega) + i_2(4.0 \Omega) = -3.0 \text{ V}. \quad (27-27)$$

Right-hand loop: For our second application of the loop rule, we arbitrarily choose to traverse the right-hand loop counterclockwise from point b , finding

$$-i_2 R_1 + \mathcal{E}_2 - i_2 R_1 - (i_1 + i_2) R_2 - \mathcal{E}_2 = 0.$$

Substituting the given data and simplifying yield

$$i_1(4.0 \Omega) + i_2(8.0 \Omega) = 0. \quad (27-28)$$

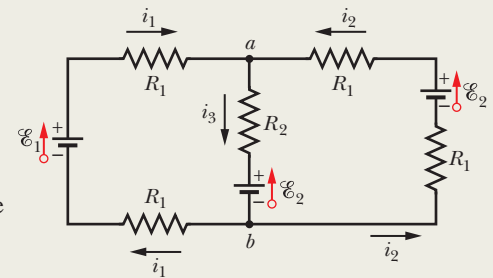


Figure 27-13 A multi-loop circuit with three ideal batteries and five resistances.

Combining equations: We now have a system of two equations (Eqs. 27-27 and 27-28) in two unknowns (i_1 and i_2) to solve either “by hand” (which is easy enough here) or with a “math package.” (One solution technique is Cramer’s rule, given in Appendix E.) We find

$$i_1 = -0.50 \text{ A}. \quad (27-29)$$

(The minus sign signals that our arbitrary choice of direction for i_1 in Fig. 27-13 is wrong, but we must wait to correct it.) Substituting $i_1 = -0.50 \text{ A}$ into Eq. 27-28 and solving for i_2 then give us

$$i_2 = 0.25 \text{ A}. \quad (\text{Answer})$$

With Eq. 27-26 we then find that

$$i_3 = i_1 + i_2 = -0.50 \text{ A} + 0.25 \text{ A} \\ = -0.25 \text{ A}.$$

The positive answer we obtained for i_2 signals that our choice of direction for that current is correct. However, the negative answers for i_1 and i_3 indicate that our choices for those currents are wrong. Thus, as a *last step* here, we correct the answers by reversing the arrows for i_1 and i_3 in Fig. 27-13 and then writing

$$i_1 = 0.50 \text{ A} \quad \text{and} \quad i_3 = 0.25 \text{ A}. \quad (\text{Answer})$$

Caution: Always make any such correction as the last step and not before calculating *all* the currents.



27-3 THE AMMETER AND THE VOLTMETER

Learning Objective

After reading this module, you should be able to . . .

27.26 Explain the use of an ammeter and a voltmeter,

including the resistance required of each in order not to affect the measured quantities.

Key Idea

● Here are three measurement instruments used with circuits: An ammeter measures current. A voltmeter

measures voltage (potential differences). A multimeter can be used to measure current, voltage, or resistance.

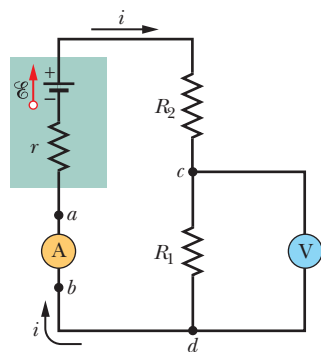


Figure 27-14 A single-loop circuit, showing how to connect an ammeter (A) and a voltmeter (V).

The Ammeter and the Voltmeter

An instrument used to measure currents is called an *ammeter*. To measure the current in a wire, you usually have to break or cut the wire and insert the ammeter so that the current to be measured passes through the meter. (In Fig. 27-14, ammeter A is set up to measure current i .) It is essential that the resistance R_A of the ammeter be very much smaller than other resistances in the circuit. Otherwise, the very presence of the meter will change the current to be measured.

A meter used to measure potential differences is called a *voltmeter*. To find the potential difference between any two points in the circuit, the voltmeter terminals are connected between those points without breaking or cutting the wire. (In Fig. 27-14, voltmeter V is set up to measure the voltage across R_1 .) It is essential that the resistance R_V of a voltmeter be very much larger than the resistance of any circuit element across which the voltmeter is connected. Otherwise, the meter alters the potential difference that is to be measured.

Often a single meter is packaged so that, by means of a switch, it can be made to serve as either an ammeter or a voltmeter — and usually also as an *ohmmeter*, designed to measure the resistance of any element connected between its terminals. Such a versatile unit is called a *multimeter*.

27-4 RC CIRCUITS

Learning Objectives

After reading this module, you should be able to . . .

27.27 Draw schematic diagrams of charging and discharging RC circuits.

27.28 Write the loop equation (a differential equation) for a charging RC circuit.

27.29 Write the loop equation (a differential equation) for a discharging RC circuit.

27.30 For a capacitor in a charging or discharging RC circuit, apply the relationship giving the charge as a function of time.

27.31 From the function giving the charge as a function of time in a charging or discharging RC circuit, find the capacitor's potential difference as a function of time.

27.32 In a charging or discharging RC circuit, find the resistor's current and potential difference as functions of time.

27.33 Calculate the capacitive time constant τ .

27.34 For a charging RC circuit and a discharging RC circuit, determine the capacitor's charge and potential difference at the start of the process and then a long time later.

Key Ideas

● When an emf \mathcal{E} is applied to a resistance R and capacitance C in series, the charge on the capacitor increases according to

$$q = C\mathcal{E}(1 - e^{-t/RC}) \quad (\text{charging a capacitor}),$$

in which $C\mathcal{E} = q_0$ is the equilibrium (final) charge and $RC = \tau$ is the capacitive time constant of the circuit.

● During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R}\right)e^{-t/RC} \quad (\text{charging a capacitor}).$$

● When a capacitor discharges through a resistance R , the charge on the capacitor decays according to

$$q = q_0 e^{-t/RC} \quad (\text{discharging a capacitor}).$$

● During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \quad (\text{discharging a capacitor}).$$

RC Circuits

In preceding modules we dealt only with circuits in which the currents did not vary with time. Here we begin a discussion of time-varying currents.

Charging a Capacitor

The capacitor of capacitance C in Fig. 27-15 is initially uncharged. To charge it, we close switch S on point a . This completes an RC series circuit consisting of the capacitor, an ideal battery of emf \mathcal{E} , and a resistance R .

From Module 25-1, we already know that as soon as the circuit is complete, charge begins to flow (current exists) between a capacitor plate and a battery terminal on each side of the capacitor. This current increases the charge q on the plates and the potential difference $V_C (= q/C)$ across the capacitor. When that potential difference equals the potential difference across the battery (which here is equal to the emf \mathcal{E}), the current is zero. From Eq. 25-1 ($q = CV$), the *equilibrium* (final) charge on the then fully charged capacitor is equal to $C\mathcal{E}$.

Here we want to examine the charging process. In particular we want to know how the charge $q(t)$ on the capacitor plates, the potential difference $V_C(t)$ across the capacitor, and the current $i(t)$ in the circuit vary with time during the charging process. We begin by applying the loop rule to the circuit, traversing it clockwise from the negative terminal of the battery. We find

$$\mathcal{E} - iR - \frac{q}{C} = 0. \quad (27-30)$$

The last term on the left side represents the potential difference across the capacitor. The term is negative because the capacitor's top plate, which is connected to the battery's positive terminal, is at a higher potential than the lower plate. Thus, there is a drop in potential as we move down through the capacitor.

We cannot immediately solve Eq. 27-30 because it contains two variables, i and q . However, those variables are not independent but are related by

$$i = \frac{dq}{dt}. \quad (27-31)$$

Substituting this for i in Eq. 27-30 and rearranging, we find

$$R \frac{dq}{dt} + \frac{q}{C} = \mathcal{E} \quad (\text{charging equation}). \quad (27-32)$$

This differential equation describes the time variation of the charge q on the capacitor in Fig. 27-15. To solve it, we need to find the function $q(t)$ that satisfies this equation and also satisfies the condition that the capacitor be initially uncharged; that is, $q = 0$ at $t = 0$.

We shall soon show that the solution to Eq. 27-32 is

$$q = C\mathcal{E}(1 - e^{-t/RC}) \quad (\text{charging a capacitor}). \quad (27-33)$$

(Here e is the exponential base, 2.718 . . . , and not the elementary charge.) Note that Eq. 27-33 does indeed satisfy our required initial condition, because at $t = 0$ the term $e^{-t/RC}$ is unity; so the equation gives $q = 0$. Note also that as t goes to infinity (that is, a long time later), the term $e^{-t/RC}$ goes to zero; so the equation gives the proper value for the full (equilibrium) charge on the capacitor — namely, $q = C\mathcal{E}$. A plot of $q(t)$ for the charging process is given in Fig. 27-16a.

The derivative of $q(t)$ is the current $i(t)$ charging the capacitor:

$$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R} \right) e^{-t/RC} \quad (\text{charging a capacitor}). \quad (27-34)$$

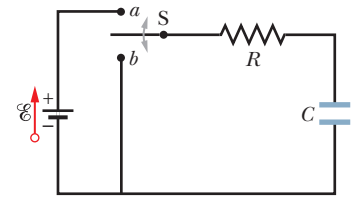


Figure 27-15 When switch S is closed on a , the capacitor is *charged* through the resistor. When the switch is afterward closed on b , the capacitor *discharges* through the resistor.

The capacitor's charge grows as the resistor's current dies out.

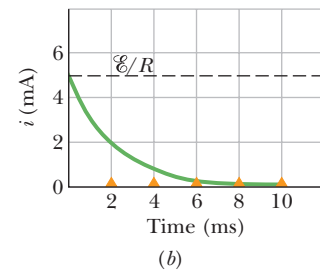
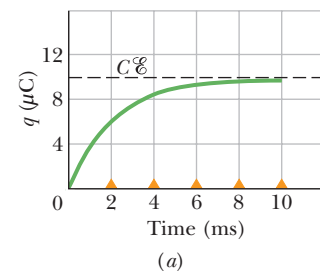


Figure 27-16 (a) A plot of Eq. 27-33, which shows the buildup of charge on the capacitor of Fig. 27-15. (b) A plot of Eq. 27-34, which shows the decline of the charging current in the circuit of Fig. 27-15. The curves are plotted for $R = 2000 \, \Omega$, $C = 1 \, \mu\text{F}$, and $\mathcal{E} = 10 \, \text{V}$; the small triangles represent successive intervals of one time constant τ .

A plot of $i(t)$ for the charging process is given in Fig. 27-16*b*. Note that the current has the initial value \mathcal{E}/R and that it decreases to zero as the capacitor becomes fully charged.



A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

By combining Eq. 25-1 ($q = CV$) and Eq. 27-33, we find that the potential difference $V_C(t)$ across the capacitor during the charging process is

$$V_C = \frac{q}{C} = \mathcal{E}(1 - e^{-t/RC}) \quad (\text{charging a capacitor}). \quad (27-35)$$

This tells us that $V_C = 0$ at $t = 0$ and that $V_C = \mathcal{E}$ when the capacitor becomes fully charged as $t \rightarrow \infty$.

The Time Constant

The product RC that appears in Eqs. 27-33, 27-34, and 27-35 has the dimensions of time (both because the argument of an exponential must be dimensionless and because, in fact, $1.0 \, \Omega \times 1.0 \, \text{F} = 1.0 \, \text{s}$). The product RC is called the **capacitive time constant** of the circuit and is represented with the symbol τ :

$$\tau = RC \quad (\text{time constant}). \quad (27-36)$$

From Eq. 27-33, we can now see that at time $t = \tau (= RC)$, the charge on the initially uncharged capacitor of Fig. 27-15 has increased from zero to

$$q = C\mathcal{E}(1 - e^{-1}) = 0.63C\mathcal{E}. \quad (27-37)$$

In words, during the first time constant τ the charge has increased from zero to 63% of its final value $C\mathcal{E}$. In Fig. 27-16, the small triangles along the time axes mark successive intervals of one time constant during the charging of the capacitor. The charging times for RC circuits are often stated in terms of τ . For example, a circuit with $\tau = 1 \, \mu\text{s}$ charges quickly while one with $\tau = 100 \, \text{s}$ charges much more slowly.

Discharging a Capacitor

Assume now that the capacitor of Fig. 27-15 is fully charged to a potential V_0 equal to the emf \mathcal{E} of the battery. At a new time $t = 0$, switch S is thrown from a to b so that the capacitor can *discharge* through resistance R . How do the charge $q(t)$ on the capacitor and the current $i(t)$ through the discharge loop of capacitor and resistance now vary with time?

The differential equation describing $q(t)$ is like Eq. 27-32 except that now, with no battery in the discharge loop, $\mathcal{E} = 0$. Thus,

$$R \frac{dq}{dt} + \frac{q}{C} = 0 \quad (\text{discharging equation}). \quad (27-38)$$

The solution to this differential equation is

$$q = q_0 e^{-t/RC} \quad (\text{discharging a capacitor}), \quad (27-39)$$

where $q_0 (= CV_0)$ is the initial charge on the capacitor. You can verify by substitution that Eq. 27-39 is indeed a solution of Eq. 27-38.

Equation 27-39 tells us that q decreases exponentially with time, at a rate that is set by the capacitive time constant $\tau = RC$. At time $t = \tau$, the capacitor's charge has been reduced to $q_0 e^{-1}$, or about 37% of the initial value. Note that a greater τ means a greater discharge time.

Differentiating Eq. 27-39 gives us the current $i(t)$:

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \quad (\text{discharging a capacitor}). \quad (27-40)$$

This tells us that the current also decreases exponentially with time, at a rate set by τ . The initial current i_0 is equal to q_0/RC . Note that you can find i_0 by simply applying the loop rule to the circuit at $t = 0$; just then the capacitor's initial potential V_0 is connected across the resistance R , so the current must be $i_0 = V_0/R = (q_0/C)/R = q_0/RC$. The minus sign in Eq. 27-40 can be ignored; it merely means that the capacitor's charge q is decreasing.

Derivation of Eq. 27-33

To solve Eq. 27-32, we first rewrite it as

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{\mathcal{E}}{R}. \quad (27-41)$$

The general solution to this differential equation is of the form

$$q = q_p + Ke^{-at}, \quad (27-42)$$

where q_p is a *particular solution* of the differential equation, K is a constant to be evaluated from the initial conditions, and $a = 1/RC$ is the coefficient of q in Eq. 27-41. To find q_p , we set $dq/dt = 0$ in Eq. 27-41 (corresponding to the final condition of no further charging), let $q = q_p$, and solve, obtaining

$$q_p = C\mathcal{E}. \quad (27-43)$$

To evaluate K , we first substitute this into Eq. 27-42 to get

$$q = C\mathcal{E} + Ke^{-at}.$$

Then substituting the initial conditions $q = 0$ and $t = 0$ yields

$$0 = C\mathcal{E} + K,$$

or $K = -C\mathcal{E}$. Finally, with the values of q_p , a , and K inserted, Eq. 27-42 becomes

$$q = C\mathcal{E} - C\mathcal{E}e^{-t/RC},$$

which, with a slight modification, is Eq. 27-33.



Checkpoint 5

The table gives four sets of values for the circuit elements in Fig. 27-15. Rank the sets according to (a) the initial current (as the switch is closed on a) and (b) the time required for the current to decrease to half its initial value, greatest first.

	1	2	3	4
\mathcal{E} (V)	12	12	10	10
R (Ω)	2	3	10	5
C (μF)	3	2	0.5	2

Sample Problem 27.05 Discharging an RC circuit to avoid a fire in a race car pit stop

As a car rolls along pavement, electrons move from the pavement first onto the tires and then onto the car body. The car stores this excess charge and the associated electric potential energy as if the car body were one plate of a capacitor and the pavement were the other plate (Fig. 27-17a). When the car stops, it discharges its excess charge and energy through the tires, just as a capacitor can discharge through a resistor. If a conducting object comes within a few centimeters of the car before the car is discharged, the remaining energy can be suddenly transferred to a spark between the car and the object. Suppose the conducting object is a fuel dispenser. The spark will not ignite the fuel and cause a fire if the spark energy is less than the critical value $U_{\text{fire}} = 50 \text{ mJ}$.

When the car of Fig. 27-17a stops at time $t = 0$, the car–ground potential difference is $V_0 = 30 \text{ kV}$. The car–ground capacitance is $C = 500 \text{ pF}$, and the resistance of *each* tire is $R_{\text{tire}} = 100 \text{ G}\Omega$. How much time does the car take to discharge through the tires to drop below the critical value U_{fire} ?

KEY IDEAS

(1) At any time t , a capacitor's stored electric potential energy U is related to its stored charge q according to Eq. 25-21 ($U = q^2/2C$). (2) While a capacitor is discharging, the charge decreases with time according to Eq. 27-39 ($q = q_0 e^{-t/RC}$).

Calculations: We can treat the tires as resistors that are connected to one another at their tops via the car body and at their bottoms via the pavement. Figure 27-17b shows how the four resistors are connected in parallel across the car's capacitance, and Fig. 27-17c shows their equivalent resistance R . From Eq. 27-24, R is given by

$$\frac{1}{R} = \frac{1}{R_{\text{tire}}} + \frac{1}{R_{\text{tire}}} + \frac{1}{R_{\text{tire}}} + \frac{1}{R_{\text{tire}}},$$

$$\text{or } R = \frac{R_{\text{tire}}}{4} = \frac{100 \times 10^9 \Omega}{4} = 25 \times 10^9 \Omega. \quad (27-44)$$

When the car stops, it discharges its excess charge and energy through R . We now use our two Key Ideas to analyze the discharge. Substituting Eq. 27-39 into Eq. 25-21 gives

$$\begin{aligned} U &= \frac{q^2}{2C} = \frac{(q_0 e^{-t/RC})^2}{2C} \\ &= \frac{q_0^2}{2C} e^{-2t/RC}. \end{aligned} \quad (27-45)$$

From Eq. 25-1 ($q = CV$), we can relate the initial charge q_0 on the car to the given initial potential difference V_0 : $q_0 = CV_0$. Substituting this equation into Eq. 27-45 brings us to

$$U = \frac{(CV_0)^2}{2C} e^{-2t/RC} = \frac{CV_0^2}{2} e^{-2t/RC},$$

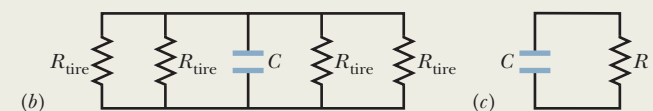
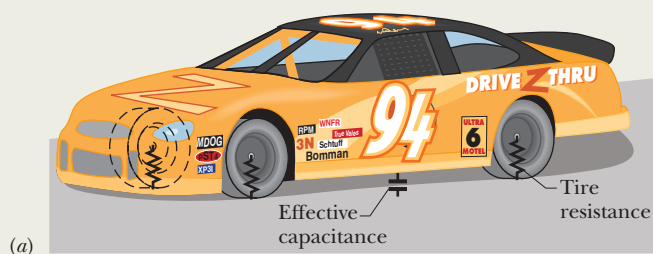
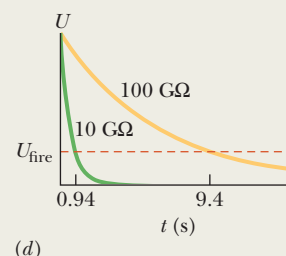


Figure 27-17 (a) A charged car and the pavement acts like a capacitor that can discharge through the tires. (b) The effective circuit of the car–pavement capacitor, with four tire resistances R_{tire} connected in parallel. (c) The equivalent resistance R of the tires. (d) The electric potential energy U in the car–pavement capacitor decreases during discharge.



$$\text{or } e^{-2t/RC} = \frac{2U}{CV_0^2}. \quad (27-46)$$

Taking the natural logarithms of both sides, we obtain

$$-\frac{2t}{RC} = \ln\left(\frac{2U}{CV_0^2}\right),$$

$$\text{or } t = -\frac{RC}{2} \ln\left(\frac{2U}{CV_0^2}\right). \quad (27-47)$$

Substituting the given data, we find that the time the car takes to discharge to the energy level $U_{\text{fire}} = 50 \text{ mJ}$ is

$$\begin{aligned} t &= -\frac{(25 \times 10^9 \Omega)(500 \times 10^{-12} \text{ F})}{2} \\ &\quad \times \ln\left(\frac{2(50 \times 10^{-3} \text{ J})}{(500 \times 10^{-12} \text{ F})(30 \times 10^3 \text{ V})^2}\right) \\ &= 9.4 \text{ s}. \end{aligned} \quad (\text{Answer})$$

Fire or no fire: This car requires at least 9.4 s before fuel can be brought safely near it. A pit crew cannot wait that long. So the tires include some type of conducting material (such as carbon black) to lower the tire resistance and thus increase the discharge rate. Figure 27-17d shows the stored energy U versus time t for tire resistances of $R = 100 \text{ G}\Omega$ (our value) and $R = 10 \text{ G}\Omega$. Note how much more rapidly a car discharges to level U_{fire} with the lower R value.

Review & Summary

Emf An **emf device** does work on charges to maintain a potential difference between its output terminals. If dW is the work the device does to force positive charge dq from the negative to the positive terminal, then the **emf** (work per unit charge) of the device is

$$\mathcal{E} = \frac{dW}{dq} \quad (\text{definition of } \mathcal{E}). \quad (27-1)$$

The volt is the SI unit of emf as well as of potential difference. An **ideal emf device** is one that lacks any internal resistance. The potential difference between its terminals is equal to the emf. A **real emf device** has internal resistance. The potential difference between its terminals is equal to the emf only if there is no current through the device.

Analyzing Circuits The change in potential in traversing a resistance R in the direction of the current is $-iR$; in the opposite direction it is $+iR$ (resistance rule). The change in potential in traversing an ideal emf device in the direction of the emf arrow is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$ (emf rule). Conservation of energy leads to the loop rule:

Loop Rule. *The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.*

Conservation of charge gives us the junction rule:

Junction Rule. *The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.*

Single-Loop Circuits The current in a single-loop circuit containing a single resistance R and an emf device with emf \mathcal{E} and internal resistance r is

$$i = \frac{\mathcal{E}}{R + r}, \quad (27-4)$$

which reduces to $i = \mathcal{E}/R$ for an ideal emf device with $r = 0$.

Power When a real battery of emf \mathcal{E} and internal resistance r does work on the charge carriers in a current i through the battery, the rate P of energy transfer to the charge carriers is

$$P = iV, \quad (27-14)$$

where V is the potential across the terminals of the battery. The rate P_r at which energy is dissipated as thermal energy in the battery is

$$P_r = i^2 r. \quad (27-16)$$

The rate P_{emf} at which the chemical energy in the battery changes is

$$P_{\text{emf}} = i\mathcal{E}. \quad (27-17)$$

Series Resistances When resistances are in **series**, they have the same current. The equivalent resistance that can replace a series combination of resistances is

$$R_{\text{eq}} = \sum_{j=1}^n R_j \quad (n \text{ resistances in series}). \quad (27-7)$$

Parallel Resistances When resistances are in **parallel**, they have the same potential difference. The equivalent resistance that can replace a parallel combination of resistances is given by

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^n \frac{1}{R_j} \quad (n \text{ resistances in parallel}). \quad (27-24)$$

RC Circuits When an emf \mathcal{E} is applied to a resistance R and capacitance C in series, as in Fig. 27-15 with the switch at a , the charge on the capacitor increases according to

$$q = C\mathcal{E}(1 - e^{-t/RC}) \quad (\text{charging a capacitor}), \quad (27-33)$$

in which $C\mathcal{E} = q_0$ is the equilibrium (final) charge and $RC = \tau$ is the **capacitive time constant** of the circuit. During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R} \right) e^{-t/RC} \quad (\text{charging a capacitor}). \quad (27-34)$$

When a capacitor discharges through a resistance R , the charge on the capacitor decays according to

$$q = q_0 e^{-t/RC} \quad (\text{discharging a capacitor}). \quad (27-39)$$

During the discharging, the current is

$$i = \frac{dq}{dt} = - \left(\frac{q_0}{RC} \right) e^{-t/RC} \quad (\text{discharging a capacitor}). \quad (27-40)$$

Questions

1 (a) In Fig. 27-18a, with $R_1 > R_2$, is the potential difference

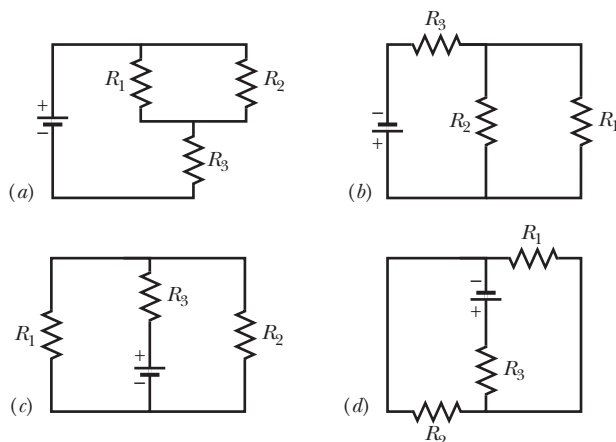


Figure 27-18 Questions 1 and 2.

across R_2 more than, less than, or equal to that across R_1 ? (b) Is the current through resistor R_2 more than, less than, or equal to that through resistor R_1 ?

2 (a) In Fig. 27-18a, are resistors R_1 and R_3 in series? (b) Are resistors R_1 and R_2 in parallel? (c) Rank the equivalent resistances of the four circuits shown in Fig. 27-18, greatest first.

3 You are to connect resistors R_1 and R_2 , with $R_1 > R_2$, to a battery, first individually, then in series, and then in parallel. Rank those arrangements according to the amount of current through the battery, greatest first.

4 In Fig. 27-19, a circuit consists of a battery and two uniform resistors, and the section lying along an x axis is divided into five segments of equal lengths. (a) Assume that $R_1 = R_2$ and rank the segments according to the magnitude of the

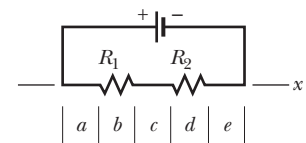


Figure 27-19 Question 4.

average electric field in them, greatest first. (b) Now assume that $R_1 > R_2$ and then again rank the segments. (c) What is the direction of the electric field along the x axis?

5 For each circuit in Fig. 27-20, are the resistors connected in series, in parallel, or neither?

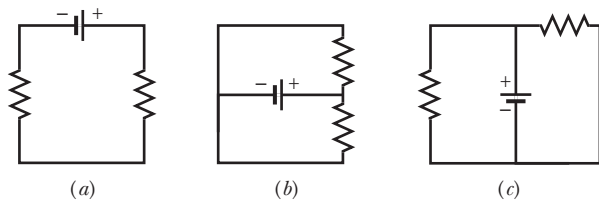


Figure 27-20 Question 5.

6 *Res-monster maze.* In Fig. 27-21, all the resistors have a resistance of $4.0\ \Omega$ and all the (ideal) batteries have an emf of 4.0 V . What is the current through resistor R ? (If you can find the proper loop through this maze, you can answer the question with a few seconds of mental calculation.)

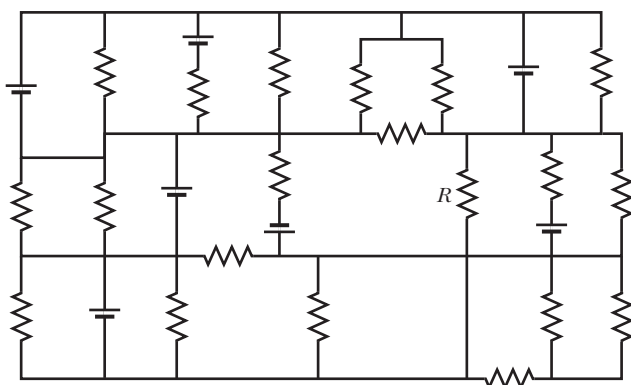


Figure 27-21 Question 6.

7 A resistor R_1 is wired to a battery, then resistor R_2 is added in series. Are (a) the potential difference across R_1 and (b) the current i_1 through R_1 now more than, less than, or the same as previously? (c) Is the equivalent resistance R_{12} of R_1 and R_2 more than, less than, or equal to R_1 ?

8 What is the equivalent resistance of three resistors, each of resistance R , if they are connected to an ideal battery (a) in series with one another and (b) in parallel with one another? (c) Is the potential difference across the series arrangement greater than, less than, or equal to that across the parallel arrangement?

9 Two resistors are wired to a battery. (a) In which arrangement, parallel or series, are the potential differences across each resistor and across the equivalent resistance all equal? (b) In which arrangement are the currents through each resistor and through the equivalent resistance all equal?

10 *Cap-monster maze.* In Fig. 27-22, all the capacitors have a capacitance of $6.0\ \mu\text{F}$, and all the batteries have an emf of 10 V . What is the charge on capacitor C ? (If you can find the proper loop through this maze, you can answer the question with a few seconds of mental calculation.)

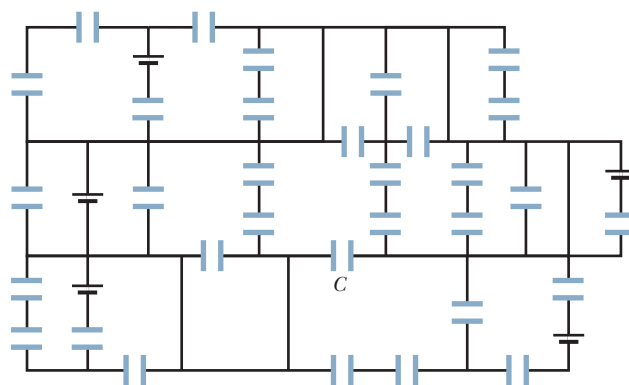


Figure 27-22 Question 10.

11 Initially, a single resistor R_1 is wired to a battery. Then resistor R_2 is added in parallel. Are (a) the potential difference across R_1 and (b) the current i_1 through R_1 now more than, less than, or the same as previously? (c) Is the equivalent resistance R_{12} of R_1 and R_2 more than, less than, or equal to R_1 ? (d) Is the total current through R_1 and R_2 together more than, less than, or equal to the current through R_1 previously?

12 After the switch in Fig. 27-15 is closed on point a , there is current i through resistance R . Figure 27-23 gives that current for four sets of values of R and capacitance C : (1) R_0 and C_0 , (2) $2R_0$ and C_0 , (3) R_0 and $2C_0$, (4) $2R_0$ and $2C_0$. Which set goes with which curve?

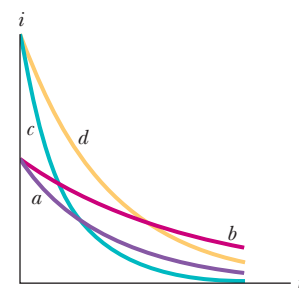


Figure 27-23 Question 12.

13 Figure 27-24 shows three sections of circuit that are to be connected in turn to the same battery via a switch as in Fig. 27-15. The resistors are all identical, as are the capacitors. Rank the sections according to (a) the final (equilibrium) charge on the capacitor and (b) the time required for the capacitor to reach 50% of its final charge, greatest first.

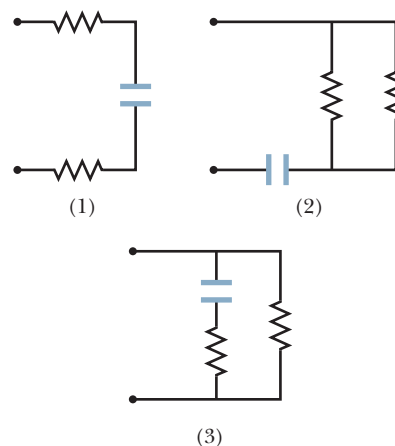


Figure 27-24 Question 13.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Worked-out solution is at



Number of dots indicates level of problem difficulty



Interactive solution is at

<http://www.wiley.com/college/halliday>



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Module 27-1 Single-Loop Circuits

•1 **SSM WWW** In Fig. 27-25, the ideal batteries have emfs $\mathcal{E}_1 = 12 \text{ V}$ and $\mathcal{E}_2 = 6.0 \text{ V}$. What are (a) the current, the dissipation rate in (b) resistor 1 (4.0Ω) and (c) resistor 2 (8.0Ω), and the energy transfer rate in (d) battery 1 and (e) battery 2? Is energy being supplied or absorbed by (f) battery 1 and (g) battery 2?

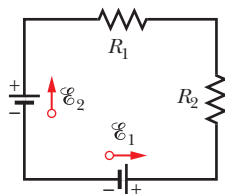


Figure 27-25
Problem 1.

•2 In Fig. 27-26, the ideal batteries have emfs $\mathcal{E}_1 = 150 \text{ V}$ and $\mathcal{E}_2 = 50 \text{ V}$ and the resistances are $R_1 = 3.0 \Omega$ and $R_2 = 2.0 \Omega$. If the potential at P is 100 V , what is it at Q ?

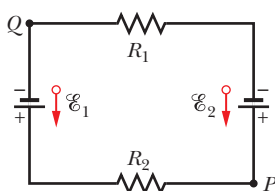


Figure 27-26 Problem 2.

•3 **ILW** A car battery with a 12 V emf and an internal resistance of 0.040Ω is being charged with a current of 50 A . What are (a) the potential difference V across the terminals, (b) the rate P_r of energy dissipation inside the battery, and (c) the rate P_{emf} of energy conversion to chemical form? When the battery is used to supply 50 A to the starter motor, what are (d) V and (e) P_r ?

•4 **GO** Figure 27-27 shows a circuit of four resistors that are connected to a larger circuit. The graph below the circuit shows the electric potential $V(x)$ as a function of position x along the lower branch of the circuit, through resistor 4; the potential V_A is 12.0 V . The graph above the circuit shows the electric potential $V(x)$ versus position x along the upper branch of the circuit, through resistors 1, 2, and 3; the potential differences are $\Delta V_B = 2.00 \text{ V}$ and $\Delta V_C = 5.00 \text{ V}$. Resistor 3 has a resistance of 200Ω . What is the resistance of (a) resistor 1 and (b) resistor 2?

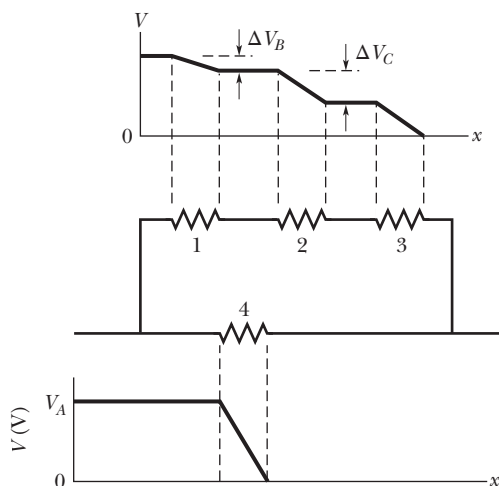


Figure 27-27
Problem 4.

•5 A 5.0 A current is set up in a circuit for 6.0 min by a rechargeable battery with a 6.0 V emf. By how much is the chemical energy of the battery reduced?

•6 A standard flashlight battery can deliver about $2.0 \text{ W} \cdot \text{h}$ of energy before it runs down. (a) If a battery costs US\$0.80, what is the cost of operating a 100 W lamp for 8.0 h using batteries? (b) What is the cost if energy is provided at the rate of US\$0.06 per kilowatt-hour?

•7 A wire of resistance 5.0Ω is connected to a battery whose emf \mathcal{E} is 2.0 V and whose internal resistance is 1.0Ω . In 2.0 min , how much energy is (a) transferred from chemical form in the battery, (b) dissipated as thermal energy in the wire, and (c) dissipated as thermal energy in the battery?

•8 A certain car battery with a 12.0 V emf has an initial charge of $120 \text{ A} \cdot \text{h}$. Assuming that the potential across the terminals stays constant until the battery is completely discharged, for how many hours can it deliver energy at the rate of 100 W ?

•9 (a) In electron-volts, how much work does an ideal battery with a 12.0 V emf do on an electron that passes through the battery from the positive to the negative terminal? (b) If 3.40×10^{18} electrons pass through each second, what is the power of the battery in watts?

••10 (a) In Fig. 27-28, what value must R have if the current in the circuit is to be 1.0 mA ? Take $\mathcal{E}_1 = 2.0 \text{ V}$, $\mathcal{E}_2 = 3.0 \text{ V}$, and $r_1 = r_2 = 3.0 \Omega$. (b) What is the rate at which thermal energy appears in R ?

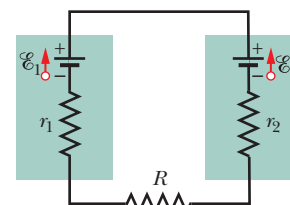


Figure 27-28 Problem 10.

••11 **SSM** In Fig. 27-29, circuit section AB absorbs energy at a rate of 50 W when current $i = 1.0 \text{ A}$ through it is in the indicated direction. Resistance $R = 2.0 \Omega$. (a) What is the potential difference between A and B ? Emf device X lacks internal resistance. (b) What is its emf? (c) Is point B connected to the positive terminal of X or to the negative terminal?

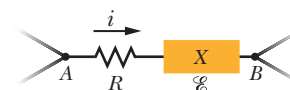


Figure 27-29 Problem 11.

••12 Figure 27-30 shows a resistor of resistance $R = 6.00 \Omega$ connected to an ideal battery of emf $\mathcal{E} = 12.0 \text{ V}$ by means of two copper wires. Each wire has length 20.0 cm and radius 1.00 mm . In dealing with such circuits in this chapter, we generally neglect the potential differences along the wires and the transfer of energy to thermal energy in them. Check the validity of this neglect for the circuit of Fig. 27-30: What is the potential difference across (a) the resistor and (b) each of the two sections of wire? At what rate is energy lost to thermal energy in (c) the resistor and (d) each section of wire?

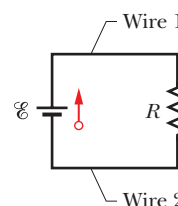


Figure 27-30
Problem 12.

••13 A 10-km -long underground cable extends east to west and consists of two parallel wires, each of which has resistance $13 \Omega/\text{km}$. An electrical short develops at distance x from the west end when

a conducting path of resistance R connects the wires (Fig. 27-31). The resistance of the wires and the short is then $100\ \Omega$ when measured from the east end and $200\ \Omega$ when measured from the west end. What are (a) x and (b) R ?

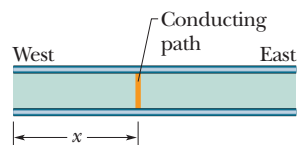


Figure 27-31 Problem 13.

••14 **GO** In Fig. 27-32a, both batteries have emf $\mathcal{E} = 1.20\text{ V}$ and the external resistance R is a variable resistor. Figure 27-32b gives the electric potentials V between the terminals of each battery as functions of R : Curve 1 corresponds to battery 1, and curve 2 corresponds to battery 2. The horizontal scale is set by $R_s = 0.20\ \Omega$. What is the internal resistance of (a) battery 1 and (b) battery 2?

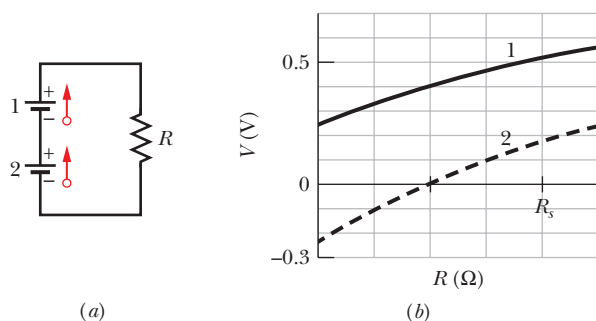


Figure 27-32 Problem 14.

••15 **ILW** The current in a single-loop circuit with one resistance R is 5.0 A . When an additional resistance of $2.0\ \Omega$ is inserted in series with R , the current drops to 4.0 A . What is R ?

•••16 A solar cell generates a potential difference of 0.10 V when a $500\ \Omega$ resistor is connected across it, and a potential difference of 0.15 V when a $1000\ \Omega$ resistor is substituted. What are the (a) internal resistance and (b) emf of the solar cell? (c) The area of the cell is 5.0 cm^2 , and the rate per unit area at which it receives energy from light is 2.0 mW/cm^2 . What is the efficiency of the cell for converting light energy to thermal energy in the $1000\ \Omega$ external resistor?

•••17 **SSM** In Fig. 27-33, battery 1 has emf $\mathcal{E}_1 = 12.0\text{ V}$ and internal resistance $r_1 = 0.016\ \Omega$ and battery 2 has emf $\mathcal{E}_2 = 12.0\text{ V}$ and internal resistance $r_2 = 0.012\ \Omega$. The batteries are connected in series with an external resistance R . (a) What R value makes the terminal-to-terminal potential difference of one of the batteries zero? (b) Which battery is that?

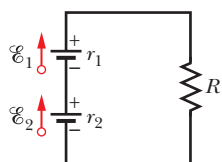


Figure 27-33 Problem 17.

Module 27-2 Multiloop Circuits

•18 In Fig. 27-9, what is the potential difference $V_d - V_c$ between points d and c if $\mathcal{E}_1 = 4.0\text{ V}$, $\mathcal{E}_2 = 1.0\text{ V}$, $R_1 = R_2 = 10\ \Omega$, and $R_3 = 5.0\ \Omega$, and the battery is ideal?

•19 A total resistance of $3.00\ \Omega$ is to be produced by connecting an unknown resistance to a $12.0\ \Omega$ resistance. (a) What must be the value of the unknown resistance, and (b) should it be connected in series or in parallel?

•20 When resistors 1 and 2 are connected in series, the equivalent resistance is $16.0\ \Omega$. When they are connected in parallel, the equivalent resistance is $3.0\ \Omega$. What are (a) the smaller resistance and (b) the larger resistance of these two resistors?

•21 Four $18.0\ \Omega$ resistors are connected in parallel across a 25.0 V ideal battery. What is the current through the battery?

•22 Figure 27-34 shows five $5.00\ \Omega$ resistors. Find the equivalent resistance between points (a) F and H and (b) F and G . (Hint: For each pair of points, imagine that a battery is connected across the pair.)

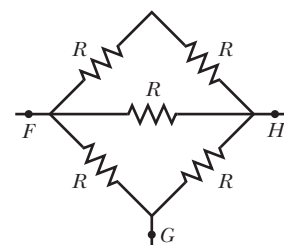


Figure 27-34 Problem 22.

•23 In Fig. 27-35, $R_1 = 100\ \Omega$, $R_2 = 50\ \Omega$, and the ideal batteries have emfs $\mathcal{E}_1 = 6.0\text{ V}$, $\mathcal{E}_2 = 5.0\text{ V}$, and $\mathcal{E}_3 = 4.0\text{ V}$. Find (a) the current in resistor 1, (b) the current in resistor 2, and (c) the potential difference between points a and b .

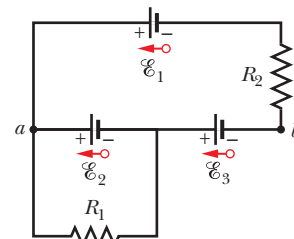


Figure 27-35 Problem 23.

•24 In Fig. 27-36, $R_1 = R_2 = 4.00\ \Omega$ and $R_3 = 2.50\ \Omega$. Find the equivalent resistance between points D and E . (Hint: Imagine that a battery is connected across those points.)

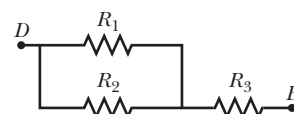


Figure 27-36 Problem 24.

•25 **SSM** Nine copper wires of length l and diameter d are connected in parallel to form a single composite conductor of resistance R . What must be the diameter D of a single copper wire of length l if it is to have the same resistance?

••26 Figure 27-37 shows a battery connected across a uniform resistor R_0 . A sliding contact can move across the resistor from $x = 0$ at the left to $x = 10\text{ cm}$ at the right. Moving the contact changes how much resistance is to the left of the contact and how much is to the right. Find the rate at which energy is dissipated in resistor R as a function of x . Plot the function for $\mathcal{E} = 50\text{ V}$, $R = 2000\ \Omega$, and $R_0 = 100\ \Omega$.

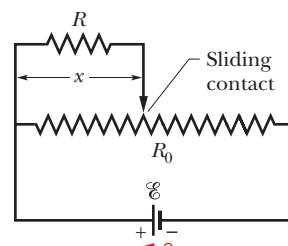


Figure 27-37 Problem 26.

••27 **Side flash.** Figure 27-38 indicates one reason no one should stand under a tree during a lightning storm. If lightning comes down the side of the tree, a portion can jump over to the person, especially if the current on the tree reaches a dry region on the bark and thereafter must travel through air to reach the ground. In the figure, part of the lightning jumps through distance d in air and then travels through the person (who has negligible resistance relative to that of air because of the highly conducting salty fluids within the body). The rest of the current travels through air alongside the tree, for a distance h . If $d/h = 0.400$ and the total current is $I = 5000\text{ A}$, what is the current through the person?

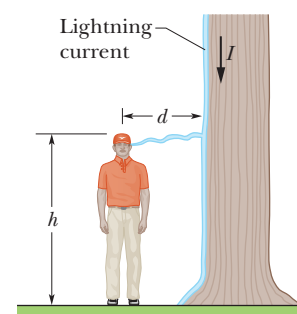


Figure 27-38 Problem 27.

••28 The ideal battery in Fig. 27-39a has emf $\mathcal{E} = 6.0\text{ V}$. Plot 1 in Fig. 27-39b gives the electric potential difference V that can appear across resistor 1 versus the current i in that resistor when the resistor

is individually tested by putting a variable potential across it. The scale of the V axis is set by $V_s = 18.0$ V, and the scale of the i axis is set by $i_s = 3.00$ mA. Plots 2 and 3 are similar plots for resistors 2 and 3, respectively, when they are individually tested by putting a variable potential across them. What is the current in resistor 2 in the circuit of Fig. 27-39a?

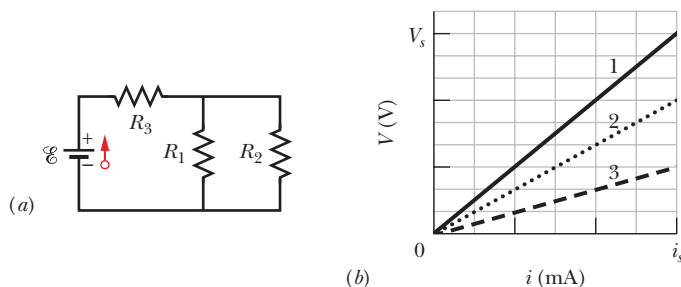


Figure 27-39 Problem 28.

••29 In Fig. 27-40, $R_1 = 6.00 \, \Omega$, $R_2 = 18.0 \, \Omega$, and the ideal battery has emf $\mathcal{E} = 12.0$ V. What are the (a) size and (b) direction (left or right) of current i_1 ? (c) How much energy is dissipated by all four resistors in 1.00 min?

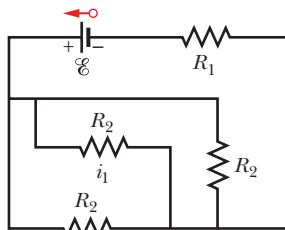


Figure 27-40 Problem 29.

••30 GO In Fig. 27-41, the ideal batteries have emfs $\mathcal{E}_1 = 10.0$ V and $\mathcal{E}_2 = 0.500\mathcal{E}_1$, and the resistances are each $4.00 \, \Omega$. What is the current in (a) resistance 2 and (b) resistance 3?

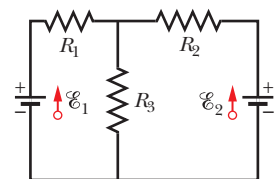


Figure 27-41 Problems 30, 41, and 88.

••31 SSM GO In Fig. 27-42, the ideal batteries have emfs $\mathcal{E}_1 = 5.0$ V and $\mathcal{E}_2 = 12$ V, the resistances are each $2.0 \, \Omega$, and the potential is defined to be zero at the grounded point of the circuit. What are potentials (a) V_1 and (b) V_2 at the indicated points?

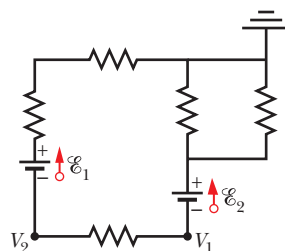


Figure 27-42 Problem 31.

••32 Both batteries in Fig. 27-43a are ideal. Emf \mathcal{E}_1 of battery 1 has a fixed value, but emf \mathcal{E}_2 of battery 2 can be varied between 1.0 V and 10 V. The plots in Fig. 27-43b give the currents through the two batteries as a function of \mathcal{E}_2 . The vertical scale is set by $i_s = 0.20$ A. You must decide which plot corresponds to which battery, but for both plots, a negative current occurs when the direction of the current through the battery is opposite the

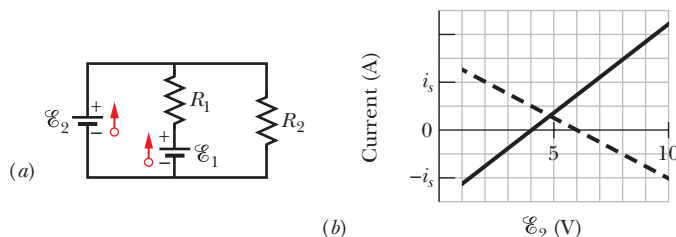


Figure 27-43 Problem 32.

direction of that battery's emf. What are (a) emf \mathcal{E}_1 , (b) resistance R_1 , and (c) resistance R_2 ?

••33 GO In Fig. 27-44, the current in resistance 6 is $i_6 = 1.40$ A and the resistances are $R_1 = R_2 = R_3 = 2.00 \, \Omega$, $R_4 = 16.0 \, \Omega$, $R_5 = 8.00 \, \Omega$, and $R_6 = 4.00 \, \Omega$. What is the emf of the ideal battery?

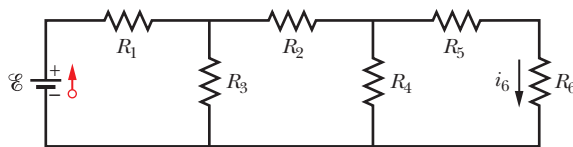


Figure 27-44 Problem 33.

••34 The resistances in Figs. 27-45a and b are all $6.0 \, \Omega$, and the batteries are ideal 12 V batteries. (a) When switch S in Fig. 27-45a is closed, what is the change in the electric potential V_1 across resistor 1, or does V_1 remain the same? (b) When switch S in Fig. 27-45b is closed, what is the change in V_1 across resistor 1, or does V_1 remain the same?

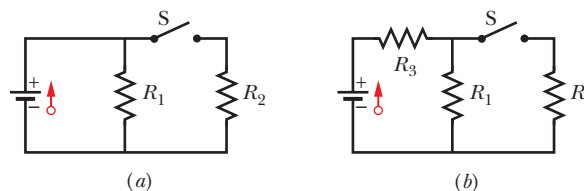


Figure 27-45 Problem 34.

••35 GO In Fig. 27-46, $\mathcal{E} = 12.0$ V, $R_1 = 2000 \, \Omega$, $R_2 = 3000 \, \Omega$, and $R_3 = 4000 \, \Omega$. What are the potential differences (a) $V_A - V_B$, (b) $V_B - V_C$, (c) $V_C - V_D$, and (d) $V_A - V_C$?

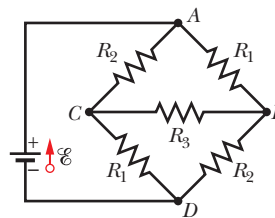


Figure 27-46 Problem 35.

••36 GO In Fig. 27-47, $\mathcal{E}_1 = 6.00$ V, $\mathcal{E}_2 = 12.0$ V, $R_1 = 100 \, \Omega$, $R_2 = 200 \, \Omega$, and $R_3 = 300 \, \Omega$. One point of the circuit is grounded ($V = 0$). What are the (a) size and (b) direction (up or down) of the current through resistance 1, the (c) size and (d) direction (left or right) of the current through resistance 2, and the (e) size and (f) direction of the current through resistance 3? (g) What is the electric potential at point A?

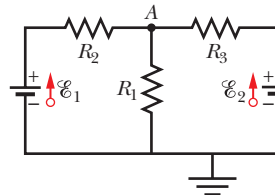


Figure 27-47 Problem 36.

••37 In Fig. 27-48, the resistances are $R_1 = 2.00 \, \Omega$, $R_2 = 5.00 \, \Omega$, and the battery is ideal. What value of R_3 maximizes the dissipation rate in resistance 3?

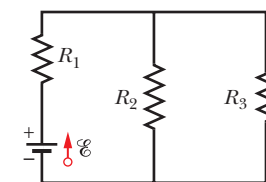


Figure 27-48 Problems 37 and 98.

••38 Figure 27-49 shows a section of a circuit. The resistances are $R_1 = 2.0 \, \Omega$, $R_2 = 4.0 \, \Omega$, and $R_3 = 6.0 \, \Omega$, and the indicated current is $i = 6.0$ A. The electric potential difference between points A and B that connect the section to the rest of the circuit is $V_A - V_B = 78$ V. (a) Is the device represented by "Box" absorbing or providing energy to the circuit, and (b) at what rate?

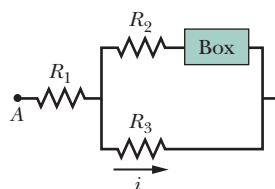


Figure 27-49 Problem 38.

••39 **GO** In Fig. 27-50, two batteries with an emf $\mathcal{E} = 12.0$ V and an internal resistance $r = 0.300$ Ω are connected in parallel across a resistance R . (a) For what value of R is the dissipation rate in the resistor a maximum? (b) What is that maximum?

••40 **GO** Two identical batteries of emf $\mathcal{E} = 12.0$ V and internal resistance $r = 0.200$ Ω are to be connected to an external resistance R , either in parallel (Fig. 27-50) or in series (Fig. 27-51). If $R = 2.00r$, what is the current i in the external resistance in the (a) parallel and (b) series arrangements? (c) For which arrangement is i greater? If $R = r/2.00$, what is i in the external resistance in the (d) parallel arrangement and (e) series arrangement? (f) For which arrangement is i greater now?

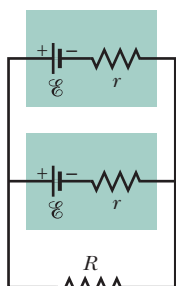


Figure 27-50
Problems 39
and 40.

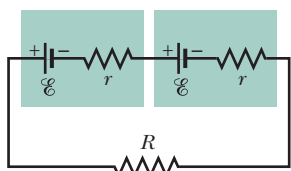


Figure 27-51 Problem 40.

••41 In Fig. 27-41, $\mathcal{E}_1 = 3.00$ V, $\mathcal{E}_2 = 1.00$ V, $R_1 = 4.00$ Ω , $R_2 = 2.00$ Ω , $R_3 = 5.00$ Ω , and both batteries are ideal. What is the rate at which energy is dissipated in (a) R_1 , (b) R_2 , and (c) R_3 ? What is the power of (d) battery 1 and (e) battery 2?

••42 In Fig. 27-52, an array of n parallel resistors is connected in series to a resistor and an ideal battery. All the resistors have the same resistance. If an identical resistor were added in parallel to the parallel array, the current through the battery would change by 1.25%. What is the value of n ?

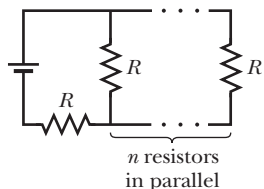


Figure 27-52 Problem 42.

••43 You are given a number of 10 Ω resistors, each capable of dissipating only 1.0 W without being destroyed. What is the minimum number of such resistors that you need to combine in series or in parallel to make a 10 Ω resistance that is capable of dissipating at least 5.0 W?

••44 **GO** In Fig. 27-53, $R_1 = 100$ Ω , $R_2 = R_3 = 50.0$ Ω , $R_4 = 75.0$ Ω , and the ideal battery has emf $\mathcal{E} = 6.00$ V. (a) What is the equivalent resistance? What is i in (b) resistance 1, (c) resistance 2, (d) resistance 3, and (e) resistance 4?

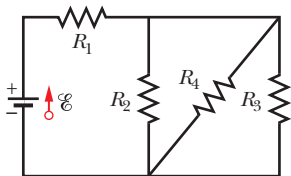


Figure 27-53
Problems 44 and 48.

••45 **ILW** In Fig. 27-54, the resistances are $R_1 = 1.0$ Ω and $R_2 = 2.0$ Ω , and the ideal batteries have emfs $\mathcal{E}_1 = 2.0$ V and $\mathcal{E}_2 = \mathcal{E}_3 = 4.0$ V. What are the (a) size and (b) direction (up or down) of the current in battery 1, the (c) size and (d) direction of the current in battery 2, and the (e) size and (f) direction of the current in battery 3? (g) What is the potential difference $V_a - V_b$?

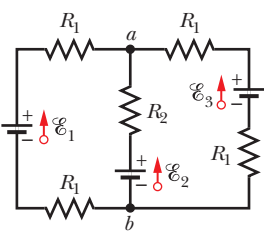
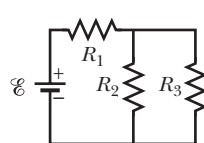


Figure 27-54 Problem 45.

••46 In Fig. 27-55a, resistor 3 is a variable resistor and the ideal battery has emf $\mathcal{E} = 12$ V. Figure 27-55b gives the current i through the battery as a function of R_3 . The horizontal scale

is set by $R_{3s} = 20$ Ω . The curve has an asymptote of 2.0 mA as $R_3 \rightarrow \infty$. What are (a) resistance R_1 and (b) resistance R_2 ?



(a)

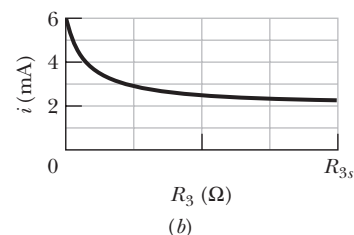


Figure 27-55 Problem 46.

••47 **SSM** A copper wire of radius $a = 0.250$ mm has an aluminum jacket of outer radius $b = 0.380$ mm. There is a current $i = 2.00$ A in the composite wire. Using Table 26-1, calculate the current in (a) the copper and (b) the aluminum. (c) If a potential difference $V = 12.0$ V between the ends maintains the current, what is the length of the composite wire?

••48 **GO** In Fig. 27-53, the resistors have the values $R_1 = 7.00$ Ω , $R_2 = 12.0$ Ω , and $R_3 = 4.00$ Ω , and the ideal battery's emf is $\mathcal{E} = 24.0$ V. For what value of R_4 will the rate at which the battery transfers energy to the resistors equal (a) 60.0 W, (b) the maximum possible rate P_{\max} , and (c) the minimum possible rate P_{\min} ? What are (d) P_{\max} and (e) P_{\min} ?

Module 27-3 The Ammeter and the Voltmeter

••49 **ILW** (a) In Fig. 27-56, what current does the ammeter read if $\mathcal{E} = 5.0$ V (ideal battery), $R_1 = 2.0$ Ω , $R_2 = 4.0$ Ω , and $R_3 = 6.0$ Ω ? (b) The ammeter and battery are now interchanged. Show that the ammeter reading is unchanged.

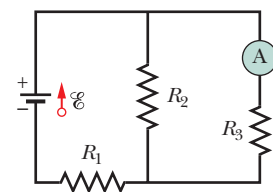


Figure 27-56 Problem 49.

••50 In Fig. 27-57, $R_1 = 2.00R$, the ammeter resistance is zero, and the battery is ideal. What multiple of \mathcal{E}/R gives the current in the ammeter?

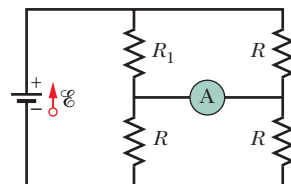


Figure 27-57 Problem 50.

••51 In Fig. 27-58, a voltmeter of resistance $R_V = 300$ Ω and an ammeter of resistance $R_A = 3.00$ Ω are being used to measure a resistance R in a circuit that also contains a resistance $R_0 = 100$ Ω and an ideal battery with an emf of $\mathcal{E} = 12.0$ V. Resistance R is given by $R = V/i$, where V is the potential across R and i is the ammeter reading. The voltmeter reading is V' , which is V plus the potential difference across the ammeter. Thus, the ratio of the two meter readings is not R but only an apparent resistance $R' = V'/i$. If $R = 85.0$ Ω , what are (a) the ammeter reading, (b) the voltmeter reading, and (c) R' ? (d) If R_A is decreased, does the difference between R' and R increase, decrease, or remain the same?

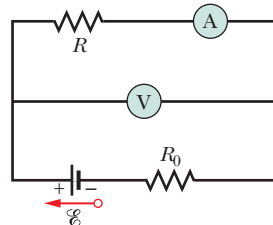


Figure 27-58 Problem 51.

••52 A simple ohmmeter is made by connecting a 1.50 V flashlight battery in series with a resistance R and an ammeter that reads from

0 to 1.00 mA, as shown in Fig. 27-59. Resistance R is adjusted so that when the clip leads are shorted together, the meter deflects to its full-scale value of 1.00 mA. What external resistance across the leads results in a deflection of (a) 10.0%, (b) 50.0%, and (c) 90.0% of full scale? (d) If the ammeter has a resistance of $20.0\ \Omega$ and the internal resistance of the battery is negligible, what is the value of R ?

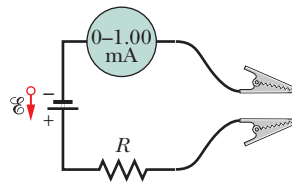


Figure 27-59 Problem 52.

••53 In Fig. 27-14, assume that $\mathcal{E} = 3.0\text{ V}$, $r = 100\ \Omega$, $R_1 = 250\ \Omega$, and $R_2 = 300\ \Omega$. If the voltmeter resistance R_V is $5.0\text{ k}\Omega$, what percent error does it introduce into the measurement of the potential difference across R_1 ? Ignore the presence of the ammeter.

••54 When the lights of a car are switched on, an ammeter in series with them reads 10.0 A and a voltmeter connected across them reads 12.0 V (Fig. 27-60). When the electric starting motor is turned on, the ammeter reading drops to 8.00 A and the lights dim somewhat. If the internal resistance of the battery is $0.0500\ \Omega$ and that of the ammeter is negligible, what are (a) the emf of the battery and (b) the current through the starting motor when the lights are on?

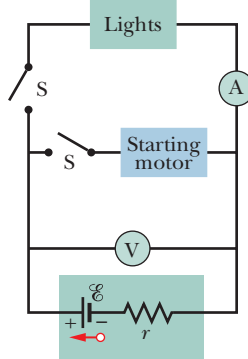


Figure 27-60 Problem 54.

••55 In Fig. 27-61, R_s is to be adjusted in value by moving the sliding contact across it until points a and b are brought to the same potential. (One tests for this condition by momentarily connecting a sensitive ammeter between a and b ; if these points are at the same potential, the ammeter will not deflect.) Show that when this adjustment is made, the following relation holds: $R_x = R_s R_2 / R_1$. An unknown resistance (R_x) can be measured in terms of a standard (R_s) using this device, which is called a Wheatstone bridge.

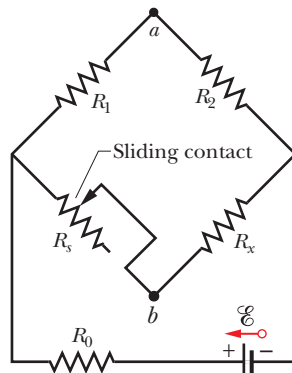


Figure 27-61 Problem 55.

••56 In Fig. 27-62, a voltmeter of resistance $R_V = 300\ \Omega$ and an ammeter of resistance $R_A = 3.00\ \Omega$ are being used to measure a resistance R in a circuit that also contains a resistance $R_0 = 100\ \Omega$ and an ideal battery of emf $\mathcal{E} = 12.0\text{ V}$. Resistance R is given by $R = V/i$, where V is the voltmeter reading and i is the current in resistance R . However, the ammeter reading is not i but rather i' , which is i plus the current through the voltmeter. Thus, the ratio of the two meter readings is not R but only an apparent resistance $R' = V/i'$. If $R = 85.0\ \Omega$, what are (a) the ammeter reading, (b) the voltmeter reading, and (c) R' ? (d) If R_V is increased, does the difference between R' and R increase, decrease, or remain the same?

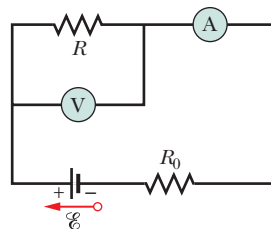


Figure 27-62 Problem 56.

Module 27-4 RC Circuits

•57 Switch S in Fig. 27-63 is closed at time $t = 0$, to begin charging an initially uncharged capacitor of capacitance $C = 15.0\ \mu\text{F}$ through a resistor of resistance $R = 20.0\ \Omega$. At what time is the potential across the capacitor equal to that across the resistor?

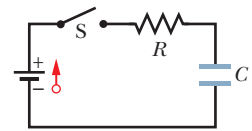


Figure 27-63 Problems 57 and 96.

•58 In an RC series circuit, emf $\mathcal{E} = 12.0\text{ V}$, resistance $R = 1.40\text{ M}\Omega$, and capacitance $C = 1.80\ \mu\text{F}$. (a) Calculate the time constant. (b) Find the maximum charge that will appear on the capacitor during charging. (c) How long does it take for the charge to build up to $16.0\ \mu\text{C}$?

•59 SSM What multiple of the time constant τ gives the time taken by an initially uncharged capacitor in an RC series circuit to be charged to 99.0% of its final charge?

•60 A capacitor with initial charge q_0 is discharged through a resistor. What multiple of the time constant τ gives the time the capacitor takes to lose (a) the first one-third of its charge and (b) two-thirds of its charge?

•61 ILW A $15.0\text{ k}\Omega$ resistor and a capacitor are connected in series, and then a 12.0 V potential difference is suddenly applied across them. The potential difference across the capacitor rises to 5.00 V in $1.30\ \mu\text{s}$. (a) Calculate the time constant of the circuit. (b) Find the capacitance of the capacitor.

••62 Figure 27-64 shows the circuit of a flashing lamp, like those attached to barrels at highway construction sites. The fluorescent lamp L (of negligible capacitance) is connected in parallel across the capacitor C of an RC circuit. There is a current through the lamp only when the potential difference across it reaches the breakdown voltage V_L ; then the capacitor discharges completely through the lamp and the lamp flashes briefly. For a lamp with breakdown voltage $V_L = 72.0\text{ V}$, wired to a 95.0 V ideal battery and a $0.150\ \mu\text{F}$ capacitor, what resistance R is needed for two flashes per second?

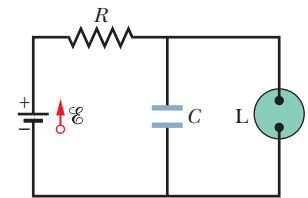


Figure 27-64 Problem 62.

••63 SSM WWW In the circuit of Fig. 27-65, $\mathcal{E} = 1.2\text{ kV}$, $C = 6.5\ \mu\text{F}$, $R_1 = R_2 = R_3 = 0.73\text{ M}\Omega$. With C completely uncharged, switch S is suddenly closed (at $t = 0$). At $t = 0$, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2, and (c) current i_3 in resistor 3? At $t = \infty$ (that is, after many time constants), what are (d) i_1 , (e) i_2 , and (f) i_3 ? What is the potential difference V_2 across resistor 2 at (g) $t = 0$ and (h) $t = \infty$? (i) Sketch V_2 versus t between these two extreme times.

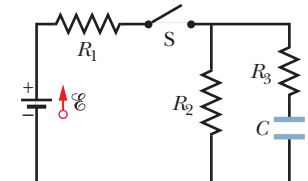


Figure 27-65 Problem 63.

••64 A capacitor with an initial potential difference of 100 V is discharged through a resistor when a switch between them is closed at $t = 0$. At $t = 10.0\text{ s}$, the potential difference across the capacitor is 1.00 V . (a) What is the time constant of the circuit? (b) What is the potential difference across the capacitor at $t = 17.0\text{ s}$?

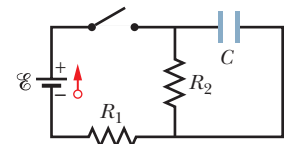


Figure 27-66 Problems 65 and 99.

••65 GO In Fig. 27-66, $R_1 = 10.0\text{ k}\Omega$, $R_2 = 15.0\text{ k}\Omega$, $C = 0.400\ \mu\text{F}$, and the

ideal battery has emf $\mathcal{E} = 20.0$ V. First, the switch is closed a long time so that the steady state is reached. Then the switch is opened at time $t = 0$. What is the current in resistor 2 at $t = 4.00$ ms?

••66 Figure 27-67 displays two circuits with a charged capacitor that is to be discharged through a resistor when a switch is closed. In Fig. 27-67a, $R_1 = 20.0 \, \Omega$ and $C_1 = 5.00 \, \mu\text{F}$. In Fig. 27-67b, $R_2 = 10.0 \, \Omega$ and $C_2 = 8.00 \, \mu\text{F}$. The ratio of the initial charges on the two capacitors is $q_{02}/q_{01} = 1.50$. At time $t = 0$, both switches are closed. At what time t do the two capacitors have the same charge?

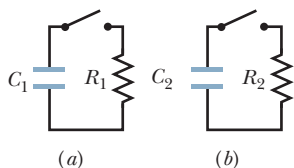


Figure 27-67 Problem 66.

••67 The potential difference between the plates of a leaky (meaning that charge leaks from one plate to the other) $2.0 \, \mu\text{F}$ capacitor drops to one-fourth its initial value in 2.0 s. What is the equivalent resistance between the capacitor plates?

••68 A $1.0 \, \mu\text{F}$ capacitor with an initial stored energy of 0.50 J is discharged through a $1.0 \, \text{M}\Omega$ resistor. (a) What is the initial charge on the capacitor? (b) What is the current through the resistor when the discharge starts? Find an expression that gives, as a function of time t , (c) the potential difference V_C across the capacitor, (d) the potential difference V_R across the resistor, and (e) the rate at which thermal energy is produced in the resistor.

•••69 GO A $3.00 \, \text{M}\Omega$ resistor and a $1.00 \, \mu\text{F}$ capacitor are connected in series with an ideal battery of emf $\mathcal{E} = 4.00$ V. At 1.00 s after the connection is made, what is the rate at which (a) the charge of the capacitor is increasing, (b) energy is being stored in the capacitor, (c) thermal energy is appearing in the resistor, and (d) energy is being delivered by the battery?

Additional Problems

70 GO Each of the six real batteries in Fig. 27-68 has an emf of 20 V and a resistance of $4.0 \, \Omega$. (a) What is the current through the (external) resistance $R = 4.0 \, \Omega$? (b) What is the potential difference across each battery? (c) What is the power of each battery? (d) At what rate does each battery transfer energy to internal thermal energy?

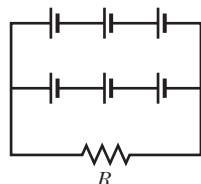


Figure 27-68 Problem 70.

71 In Fig. 27-69, $R_1 = 20.0 \, \Omega$, $R_2 = 10.0 \, \Omega$, and the ideal battery has emf $\mathcal{E} = 120$ V. What is the current at point a if we close (a) only switch S_1 , (b) only switches S_1 and S_2 , and (c) all three switches?

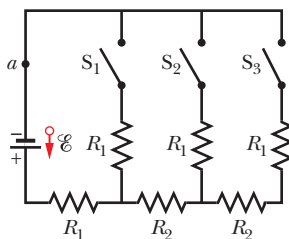


Figure 27-69 Problem 71.

72 In Fig. 27-70, the ideal battery has emf $\mathcal{E} = 30.0$ V, and the resistances are $R_1 = R_2 = 14 \, \Omega$, $R_3 = R_4 = R_5 = 6.0 \, \Omega$, $R_6 = 2.0 \, \Omega$, and $R_7 = 1.5 \, \Omega$. What are currents (a) i_2 , (b) i_4 , (c) i_1 , (d) i_3 , and (e) i_5 ?

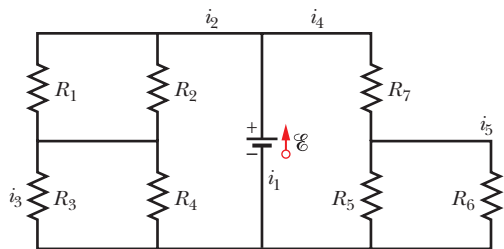


Figure 27-70 Problem 72.

73 SSM Wires A and B , having equal lengths of 40.0 m and equal diameters of 2.60 mm, are connected in series. A potential difference of 60.0 V is applied between the ends of the composite wire. The resistances are $R_A = 0.127 \, \Omega$ and $R_B = 0.729 \, \Omega$. For wire A , what are (a) magnitude J of the current density and (b) potential difference V ? (c) Of what type material is wire A made (see Table 26-1)? For wire B , what are (d) J and (e) V ? (f) Of what type material is B made?

74 What are the (a) size and (b) direction (up or down) of current i in Fig. 27-71, where all resistances are $4.0 \, \Omega$ and all batteries are ideal and have an emf of 10 V? (Hint: This can be answered using only mental calculation.)

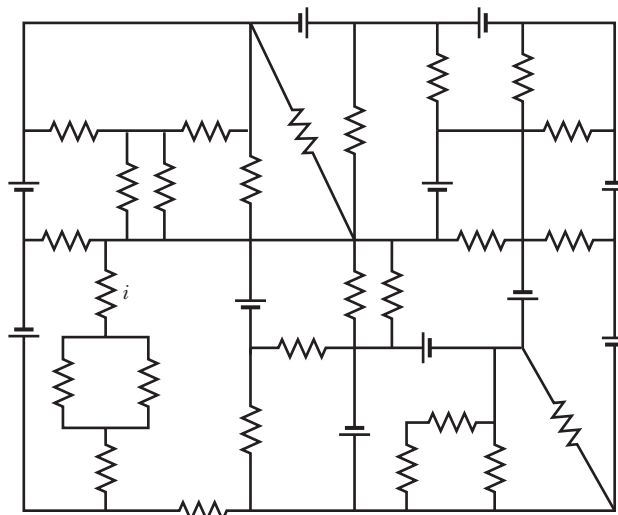


Figure 27-71 Problem 74.

75 Suppose that, while you are sitting in a chair, charge separation between your clothing and the chair puts you at a potential of 200 V, with the capacitance between you and the chair at 150 pF. When you stand up, the increased separation between your body and the chair decreases the capacitance to 10 pF. (a) What then is the potential of your body? That potential is reduced over time, as the charge on you drains through your body and shoes (you are a capacitor discharging through a resistance). Assume that the resistance along that route is $300 \, \text{G}\Omega$. If you touch an electrical component while your potential is greater than 100 V, you could ruin the component. (b) How long must you wait until your potential reaches the safe level of 100 V?

If you wear a conducting wrist strap that is connected to ground, your potential does not increase as much when you stand up; you also discharge more rapidly because the resistance through the grounding connection is much less than through your body and shoes. (c) Suppose that when you stand up, your potential is 1400 V and the chair-to-you capacitance is 10 pF. What resistance in that wrist-strap grounding connection will allow you to discharge to 100 V in 0.30 s, which is less time than you would need to reach for, say, your computer?

76 GO In Fig. 27-72, the ideal batteries have emfs $\mathcal{E}_1 = 20.0$ V, $\mathcal{E}_2 = 10.0$ V, and $\mathcal{E}_3 = 5.00$ V, and the resistances are each $2.00 \, \Omega$. What are the (a) size and (b) direction (left or right) of current i_1 ? (c) Does battery 1 supply or absorb energy, and (d) what is its power? (e) Does battery 2 supply or absorb energy, and

(f) what is its power? (g) Does battery 3 supply or absorb energy, and (h) what is its power?

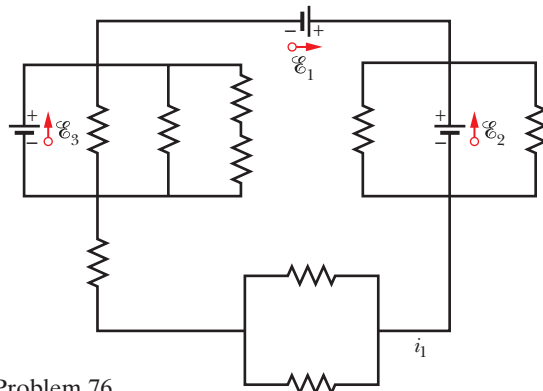


Figure 27-72 Problem 76.

77 SSM A temperature-stable resistor is made by connecting a resistor made of silicon in series with one made of iron. If the required total resistance is $1000\ \Omega$ in a wide temperature range around 20°C , what should be the resistance of the (a) silicon resistor and (b) iron resistor? (See Table 26-1.)

78 In Fig. 27-14, assume that $\mathcal{E} = 5.0\ \text{V}$, $r = 2.0\ \Omega$, $R_1 = 5.0\ \Omega$, and $R_2 = 4.0\ \Omega$. If the ammeter resistance R_A is $0.10\ \Omega$, what percent error does it introduce into the measurement of the current? Assume that the voltmeter is not present.

79 SSM An initially uncharged capacitor C is fully charged by a device of constant emf \mathcal{E} connected in series with a resistor R . (a) Show that the final energy stored in the capacitor is half the energy supplied by the emf device. (b) By direct integration of i^2R over the charging time, show that the thermal energy dissipated by the resistor is also half the energy supplied by the emf device.

80 In Fig. 27-73, $R_1 = 5.00\ \Omega$, $R_2 = 10.0\ \Omega$, $R_3 = 15.0\ \Omega$, $C_1 = 5.00\ \mu\text{F}$, $C_2 = 10.0\ \mu\text{F}$, and the ideal battery has emf $\mathcal{E} = 20.0\ \text{V}$. Assuming that the circuit is in the steady state, what is the total energy stored in the two capacitors?

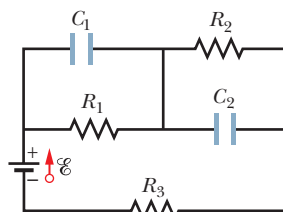


Figure 27-73 Problem 80.

81 In Fig. 27-5a, find the potential difference across R_2 if $\mathcal{E} = 12\ \text{V}$, $R_1 = 3.0\ \Omega$, $R_2 = 4.0\ \Omega$, and $R_3 = 5.0\ \Omega$.

82 In Fig. 27-8a, calculate the potential difference between a and c by considering a path that contains R , r_1 , and \mathcal{E}_1 .

83 SSM A controller on an electronic arcade game consists of a variable resistor connected across the plates of a $0.220\ \mu\text{F}$ capacitor. The capacitor is charged to $5.00\ \text{V}$, then discharged through the resistor. The time for the potential difference across the plates to decrease to $0.800\ \text{V}$ is measured by a clock inside the game. If the range of discharge times that can be handled effectively is from $10.0\ \mu\text{s}$ to $6.00\ \text{ms}$, what should be the (a) lower value and (b) higher value of the resistance range of the resistor?

84 An automobile gasoline gauge is shown schematically in Fig. 27-74. The indicator (on the dashboard) has a resistance of $10\ \Omega$. The tank

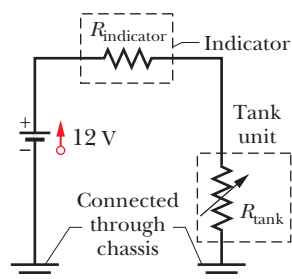


Figure 27-74 Problem 84.

unit is a float connected to a variable resistor whose resistance varies linearly with the volume of gasoline. The resistance is $140\ \Omega$ when the tank is empty and $20\ \Omega$ when the tank is full. Find the current in the circuit when the tank is (a) empty, (b) half-full, and (c) full. Treat the battery as ideal.

85 SSM The starting motor of a car is turning too slowly, and the mechanic has to decide whether to replace the motor, the cable, or the battery. The car's manual says that the $12\ \text{V}$ battery should have no more than $0.020\ \Omega$ internal resistance, the motor no more than $0.200\ \Omega$ resistance, and the cable no more than $0.040\ \Omega$ resistance. The mechanic turns on the motor and measures $11.4\ \text{V}$ across the battery, $3.0\ \text{V}$ across the cable, and a current of $50\ \text{A}$. Which part is defective?

86 Two resistors R_1 and R_2 may be connected either in series or in parallel across an ideal battery with emf \mathcal{E} . We desire the rate of energy dissipation of the parallel combination to be five times that of the series combination. If $R_1 = 100\ \Omega$, what are the (a) smaller and (b) larger of the two values of R_2 that result in that dissipation rate?

87 The circuit of Fig. 27-75 shows a capacitor, two ideal batteries, two resistors, and a switch S . Initially S has been open for a long time. If it is then closed for a long time, what is the change in the charge on the capacitor? Assume $C = 10\ \mu\text{F}$, $\mathcal{E}_1 = 1.0\ \text{V}$, $\mathcal{E}_2 = 3.0\ \text{V}$, $R_1 = 0.20\ \Omega$, and $R_2 = 0.40\ \Omega$.

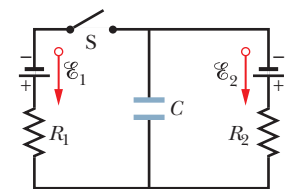


Figure 27-75 Problem 87.

88 In Fig. 27-41, $R_1 = 10.0\ \Omega$, $R_2 = 20.0\ \Omega$, and the ideal batteries have emfs $\mathcal{E}_1 = 20.0\ \text{V}$ and $\mathcal{E}_2 = 50.0\ \text{V}$. What value of R_3 results in no current through battery 1?

89 In Fig. 27-76, $R = 10\ \Omega$. What is the equivalent resistance between points A and B ? (Hint: This circuit section might look simpler if you first assume that points A and B are connected to a battery.)

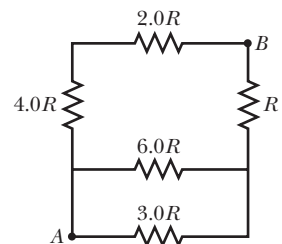


Figure 27-76 Problem 89.

90 (a) In Fig. 27-4a, show that the rate at which energy is dissipated in R as thermal energy is a maximum when $R = r$. (b) Show that this maximum power is $P = \mathcal{E}^2/4r$.

91 In Fig. 27-77, the ideal batteries have emfs $\mathcal{E}_1 = 12.0\ \text{V}$ and $\mathcal{E}_2 = 4.00\ \text{V}$, and the resistances are each $4.00\ \Omega$. What are the (a) size and (b) direction (up or down) of i_1 and the (c) size and (d) direction of i_2 ? (e) Does battery 1 supply or absorb energy, and (f) what is its energy transfer rate? (g) Does battery 2 supply or absorb energy, and (h) what is its energy transfer rate?

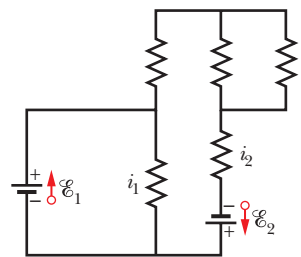


Figure 27-77 Problem 91.

92 Figure 27-78 shows a portion of a circuit through which there is a current $I = 6.00\ \text{A}$. The resistances are $R_1 = R_2 = 2.00R_3 = 2.00R_4 = 4.00\ \Omega$. What is the current i_1 through resistor 1?

93 Thermal energy is to be generated in a $0.10\ \Omega$ resistor at the rate of

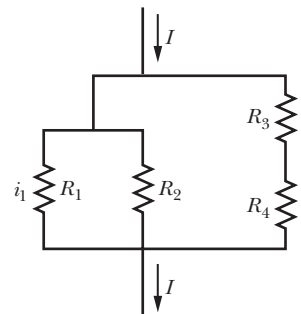


Figure 27-78 Problem 92.

10 W by connecting the resistor to a battery whose emf is 1.5 V. (a) What potential difference must exist across the resistor? (b) What must be the internal resistance of the battery?

94 Figure 27-79 shows three $20.0\ \Omega$ resistors. Find the equivalent resistance between points (a) A and B , (b) A and C , and (c) B and C . (Hint: Imagine that a battery is connected between a given pair of points.)

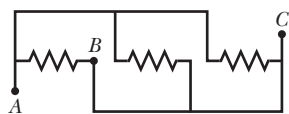


Figure 27-79 Problem 94.

95 A 120 V power line is protected by a 15 A fuse. What is the maximum number of 500 W lamps that can be simultaneously operated in parallel on this line without “blowing” the fuse because of an excess of current?

96 Figure 27-63 shows an ideal battery of emf $\mathcal{E} = 12\text{ V}$, a resistor of resistance $R = 4.0\ \Omega$, and an uncharged capacitor of capacitance $C = 4.0\ \mu\text{F}$. After switch S is closed, what is the current through the resistor when the charge on the capacitor is $8.0\ \mu\text{C}$?

97 SSM A group of N identical batteries of emf \mathcal{E} and internal resistance r may be connected all in series (Fig. 27-80a) or all in parallel (Fig. 27-80b) and then across a resistor R . Show that both arrangements give the same current in R if $R = r$.

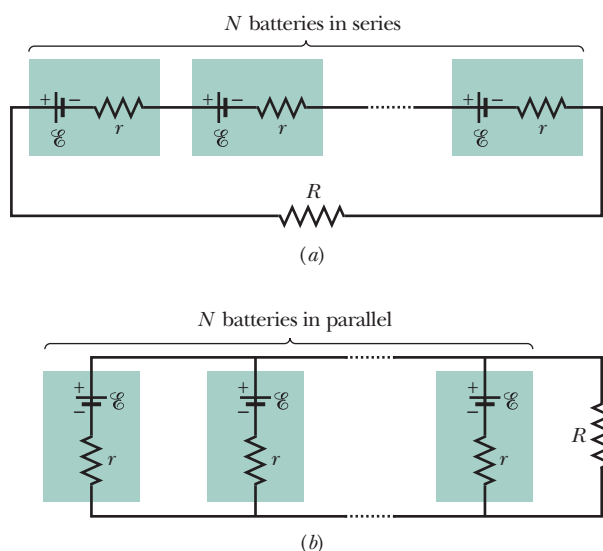


Figure 27-80 Problem 97.

98 SSM In Fig. 27-48, $R_1 = R_2 = 10.0\ \Omega$, and the ideal battery has emf $\mathcal{E} = 12.0\text{ V}$. (a) What value of R_3 maximizes the rate at which the battery supplies energy and (b) what is that maximum rate?

99 SSM In Fig. 27-66, the ideal battery has emf $\mathcal{E} = 30\text{ V}$, the resistances are $R_1 = 20\text{ k}\Omega$ and $R_2 = 10\text{ k}\Omega$, and the capacitor is uncharged. When the switch is closed at time $t = 0$, what is the current in (a) resistance 1 and (b) resistance 2? (c) A long time later, what is the current in resistance 2?

100 In Fig. 27-81, the ideal batteries have emfs $\mathcal{E}_1 = 20.0\text{ V}$, $\mathcal{E}_2 = 10.0\text{ V}$,

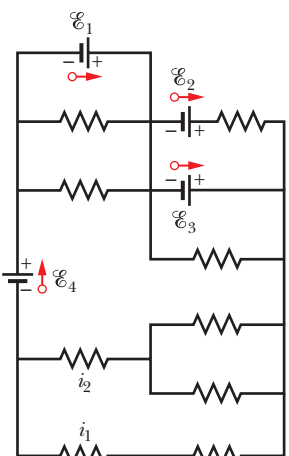


Figure 27-81 Problem 100.

$\mathcal{E}_3 = 5.00\text{ V}$, and $\mathcal{E}_4 = 5.00\text{ V}$, and the resistances are each $2.00\ \Omega$. What are the (a) size and (b) direction (left or right) of current i_1 and the (c) size and (d) direction of current i_2 ? (This can be answered with only mental calculation.) (e) At what rate is energy being transferred in battery 4, and (f) is the energy being supplied or absorbed by the battery?

101 In Fig. 27-82, an ideal battery of emf $\mathcal{E} = 12.0\text{ V}$ is connected to a network of resistances $R_1 = 6.00\ \Omega$, $R_2 = 12.0\ \Omega$, $R_3 = 4.00\ \Omega$, $R_4 = 3.00\ \Omega$, and $R_5 = 5.00\ \Omega$. What is the potential difference across resistance 5?

102 The following table gives the electric potential difference V_T across the terminals of a battery as a function of current i being drawn from the battery. (a) Write an equation that represents the relationship between V_T and i . Enter the data into your graphing calculator and perform a linear regression fit of V_T versus i . From the parameters of the fit, find (b) the battery's emf and (c) its internal resistance.

$i(\text{A})$:	50.0	75.0	100	125	150	175	200
$V_T(\text{V})$:	10.7	9.00	7.70	6.00	4.80	3.00	1.70

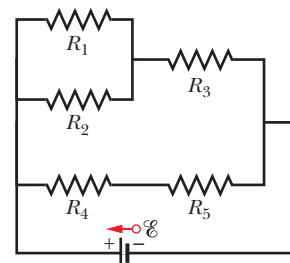


Figure 27-82 Problem 101.

103 In Fig. 27-83, $\mathcal{E}_1 = 6.00\text{ V}$, $\mathcal{E}_2 = 12.0\text{ V}$, $R_1 = 200\ \Omega$, and $R_2 = 100\ \Omega$. What are the (a) size and (b) direction (up or down) of the current through resistance 1, the (c) size and (d) direction of the current through resistance 2, and the (e) size and (f) direction of the current through battery 2?

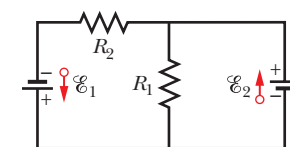


Figure 27-83 Problem 103.

104 A three-way 120 V lamp bulb that contains two filaments is rated for 100-200-300 W. One filament burns out. Afterward, the bulb operates at the same intensity (dissipates energy at the same rate) on its lowest as on its highest switch positions but does not operate at all on the middle position. (a) How are the two filaments wired to the three switch positions? What are the (b) smaller and (c) larger values of the filament resistances?

105 In Fig. 27-84, $R_1 = R_2 = 2.0\ \Omega$, $R_3 = 4.0\ \Omega$, $R_4 = 3.0\ \Omega$, $R_5 = 1.0\ \Omega$, and $R_6 = R_7 = R_8 = 8.0\ \Omega$, and the ideal batteries have emfs $\mathcal{E}_1 = 16\text{ V}$ and $\mathcal{E}_2 = 8.0\text{ V}$. What are the (a) size and (b) direction (up or down) of current i_1 and the (c) size and (d) direction of current i_2 ? What is the energy transfer rate in (e) battery 1 and (f) battery 2? Is energy being supplied or absorbed in (g) battery 1 and (h) battery 2?

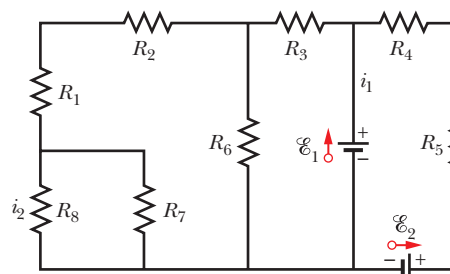


Figure 27-84 Problem 105.