UFUG 1504: Honors General Physics II

Chapter 38

Photons and Matter Waves

38.1: The Photon, the Quantum of Light

In 1905, Einstein proposed that electromagnetic radiation (or simply light) is quantized and exists in elementary amounts (quanta) that we now call **photons.**

The quantum of a light wave of frequency f has the energy

$$E = hf \text{ (photon energy).}$$

$$f = \frac{c}{\lambda}$$

Here h is the **Planck constant**,

$$h = 6.63 \times 10^{-34}\,\mathrm{J \cdot s} = 4.14 \times 10^{-15}\,\mathrm{eV \cdot s}.$$

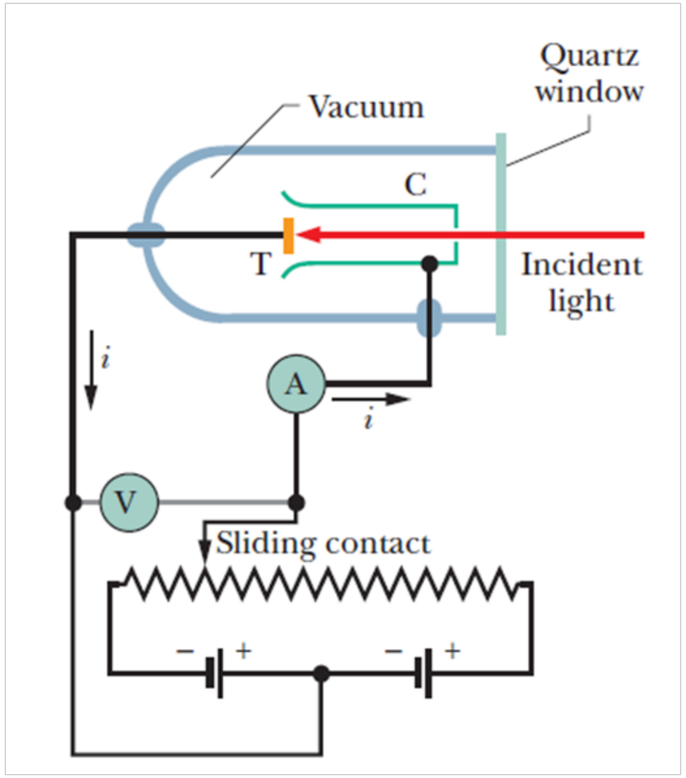
more commonly used

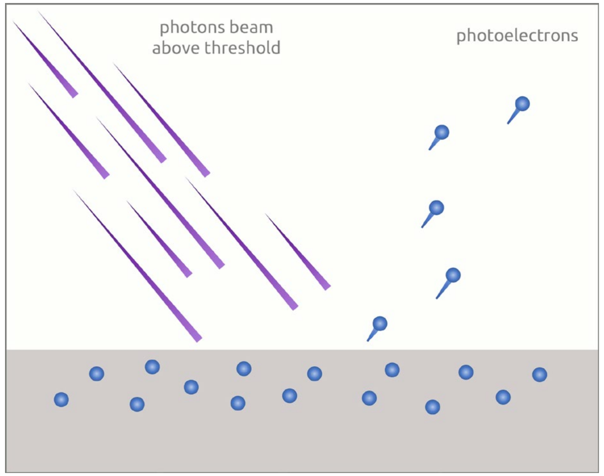
$$E = \frac{hc}{\lambda} \rightarrow E(eV) = \frac{1.24}{\lambda(\mu m)}$$

38.3: The Photoelectric Effect: (1 of 2)

Figure 38-1 An apparatus used to study the photoelectric effect. The incident light shines on target T, ejecting electrons, which are collected by collector cup C.

The batteries and the variable resistor are used to produce and adjust the electric potential difference between T and C.

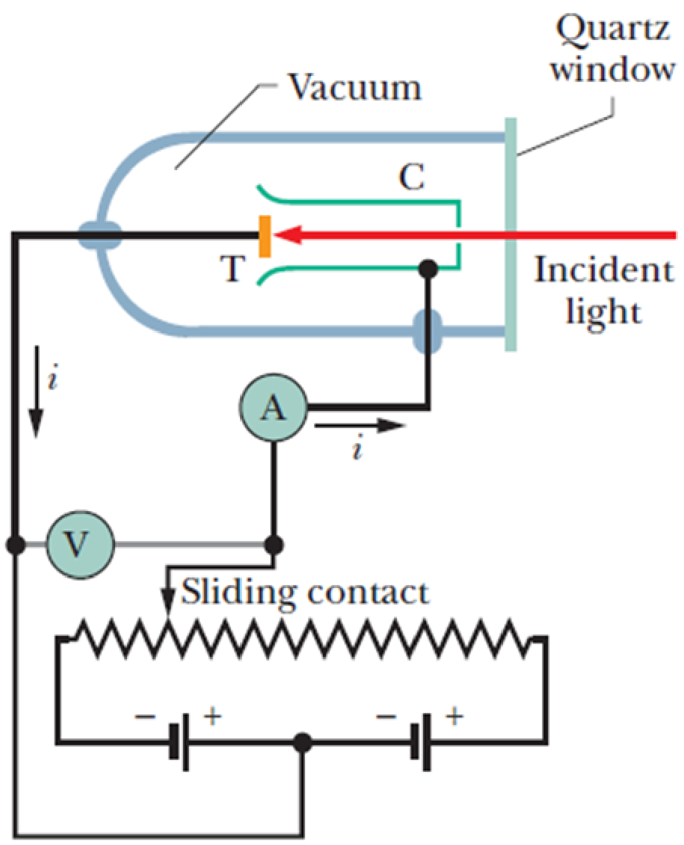




38.2: The Photoelectric Effect 光电效应

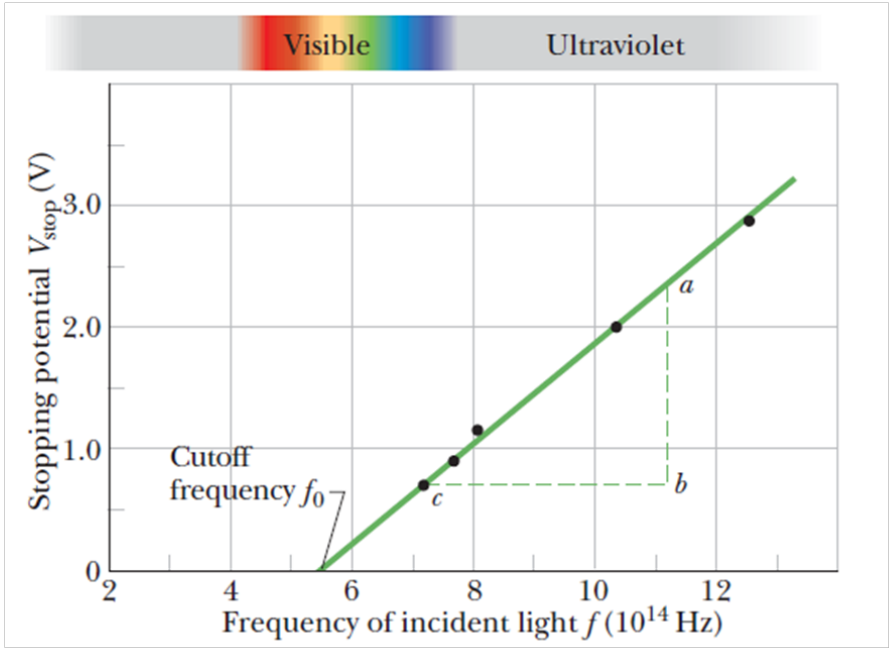
We then vary V until it reaches a certain value, called the **stopping potential V_{stop}**, at which point the reading of meter A has just dropped to zero. When $V = V_{stop}$, the most energetic ejected electrons are turned back just before reaching the collector. Then K_{max}, the kinetic energy of these most energetic electrons, is

$$K_{max} = eV_{stop}.$$



38.3: The Photoelectric Effect: Second Photoelectric Experiment (1 of 2)

If the frequency f of the incident light is varied and the associated stopping potential V_{stop} is measured, then the plot of V_{stop} versus f as shown in the figure is obtained. The photoelectric effect does not occur if the frequency is below a certain **cutoff frequency** f_0 or, if the wavelength is greater than the corresponding **cutoff wavelength** $\lambda_0 = \frac{c}{f_0}$. This is so no matter how intense the incident light is.



38.3: The Photoelectric Effect: Second Photoelectric Experiment (2 of 2)

The electrons within the target are held there by electric forces. To just escape from the target, an electron must pick up a certain minimum energy ϕ, where ϕ is a property of the target material called its **work function.** If the energy *hf* transferred to an electron by a photon exceeds the work function of the material (if $hf > \phi$), the electron can escape the target.

The Photoelectric Equation

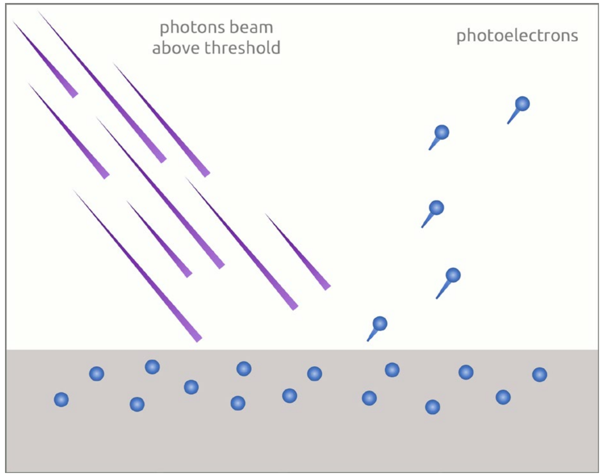
Einstein summed up the results of such photoelectric experiments in the equation

$$hf = K_{\max} + \Phi \quad \text{(photoelectric equation).}$$

K_{max} is the kinetic energy of these most energetic electrons

ϕ **is work function** (a property of the target material)

hf is the energy of light $(hf = hc/\lambda = 1.24/\lambda(\mu m)$ or $1240/\lambda(nm))$



38.4: Photons Have Momentum Compton Effect 康普顿效应

$$p = \frac{hf}{c} = \frac{h}{\lambda} \quad \text{(photon momentum)}$$

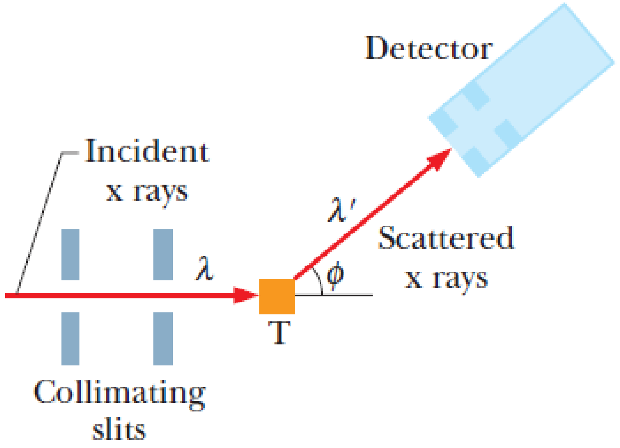
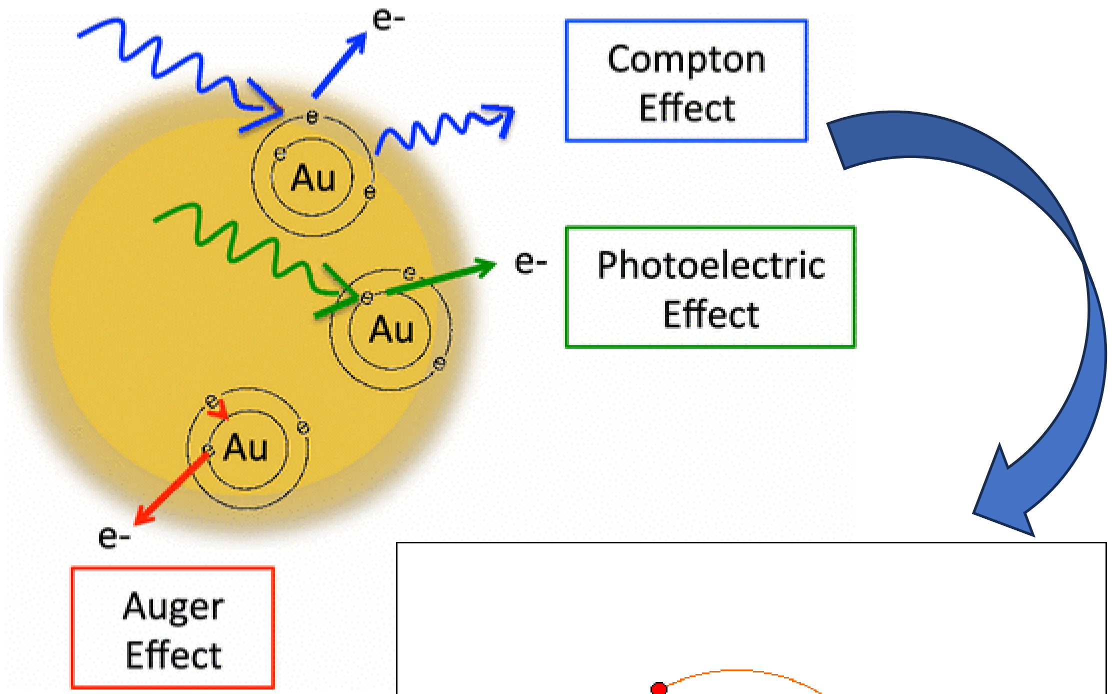
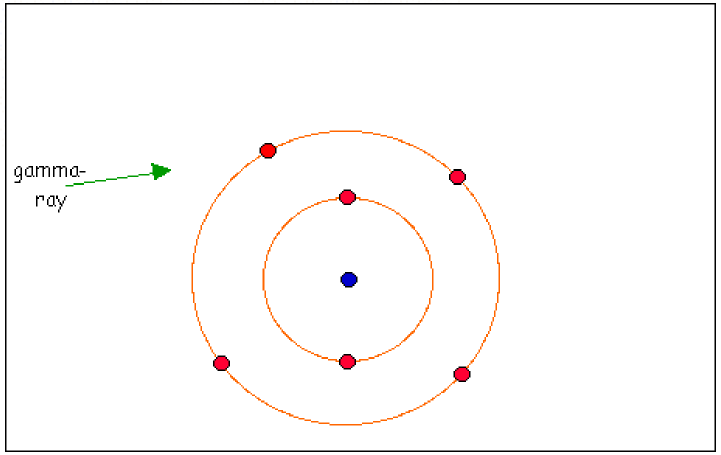


Figure 38-3 Compton's apparatus. A beam of x rays of wavelength $\lambda = 71.1$ pm is directed onto a carbon target T. The x rays scattered from the target are observed at various angles ϕ to the direction of the incident beam. The detector measures both the intensity of the scattered x rays and their wavelength.



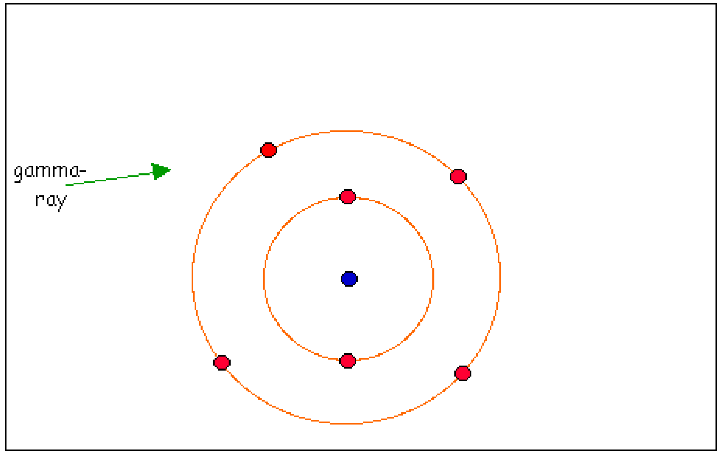


38.4: Photons Have Momentum, Compton Effect: (2 of 4)

As a result of the collision, an x ray of wavelength λ' moves off at an angle ϕ and the electron moves off at an angle θ, as shown. Conservation of energy then gives us

$$hf = hf' + K$$

Here hf is the energy of the incident x-ray photon, hf' is the energy of the scattered x-ray photon, and K is the kinetic energy of the recoiling electron 反冲电子 .



Since the electron may recoil with a speed comparable to that of light,

$$K = mc^2(\gamma - 1), \qquad \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}.$$

38.4: Photons Have Momentum, Compton Effect: (1 of 4)

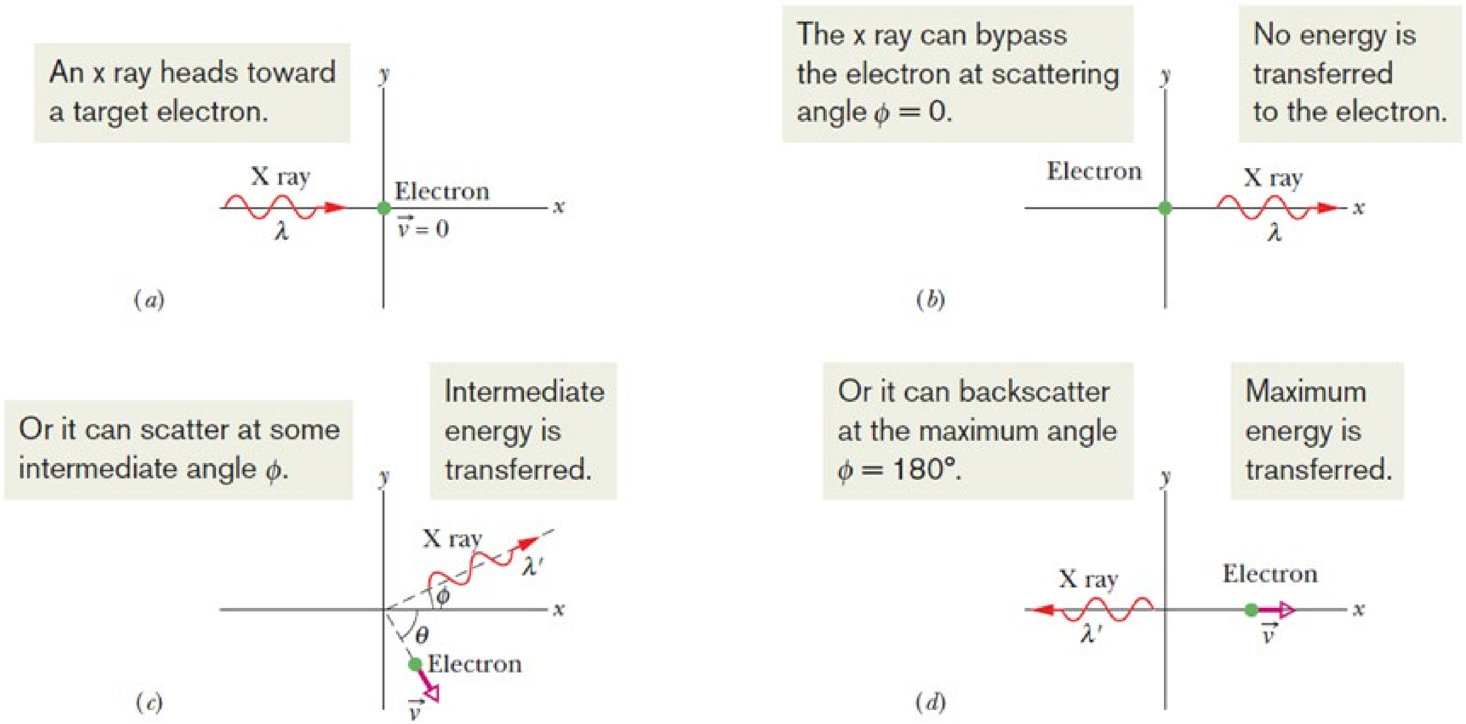


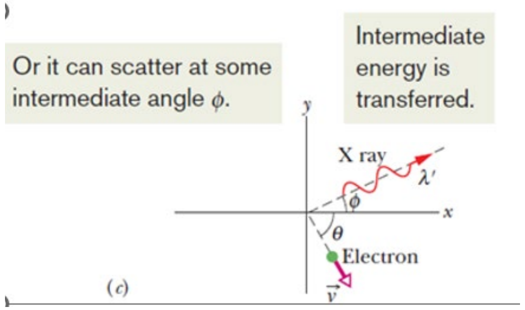
Figure 38-5 (*a*) An x ray approaches a stationary electron. The x ray can (*b*) bypass the electron (forward scatter) with no energy or momentum transfer, (*c*) scatter at some intermediate angle with an intermediate energy and momentum transfer, or (*d*) backscatter with the maximum energy and momentum transfer.

38.4: Photons Have Momentum, Compton Effect: (4 of 4)

$$hf = hf' + mc(\gamma - 1).$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'} + mc(\gamma - 1).$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}.$$



$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\phi + \gamma mv\cos\theta \quad (x \text{ axis})$$

$$0 = \frac{h}{\lambda'}\sin\phi - \gamma mv\sin\theta \quad (y \text{ axis}).$$

$$\boxed{\Delta\lambda = \frac{h}{mc}(1-\cos\phi) \quad (\text{Compton shift}).}$$

康普顿位移 $\Delta\lambda \; (= \lambda' - \lambda)$

The quantity h/mc is a constant called the **Compton wavelength.**

(the electron case value 2.4263×10^{-12} m)

38.4: Photons Have Momentum, Compton Effect: (4 of 4)

$$\boxed{\Delta\lambda = \frac{h}{mc}(1-\cos\phi) \quad \text{(Compton shift).}}$$ 康普顿位移

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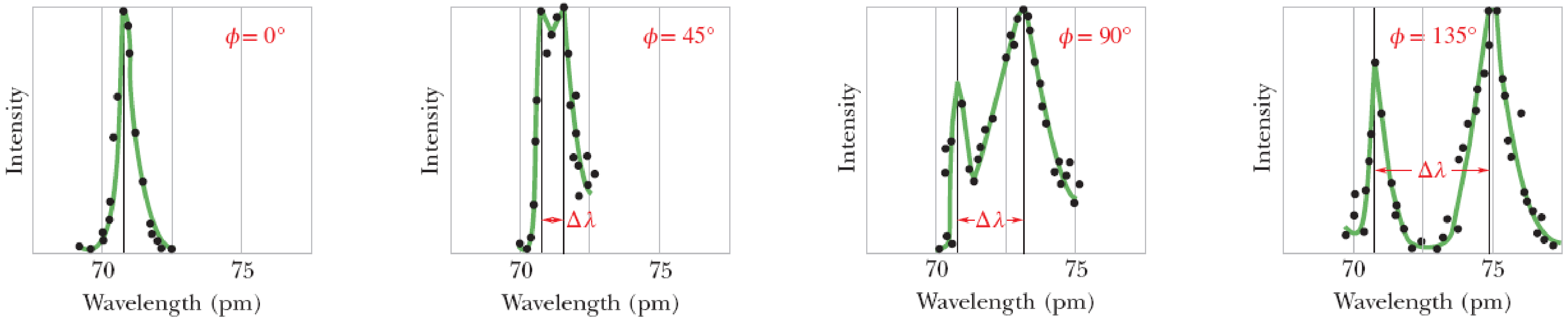
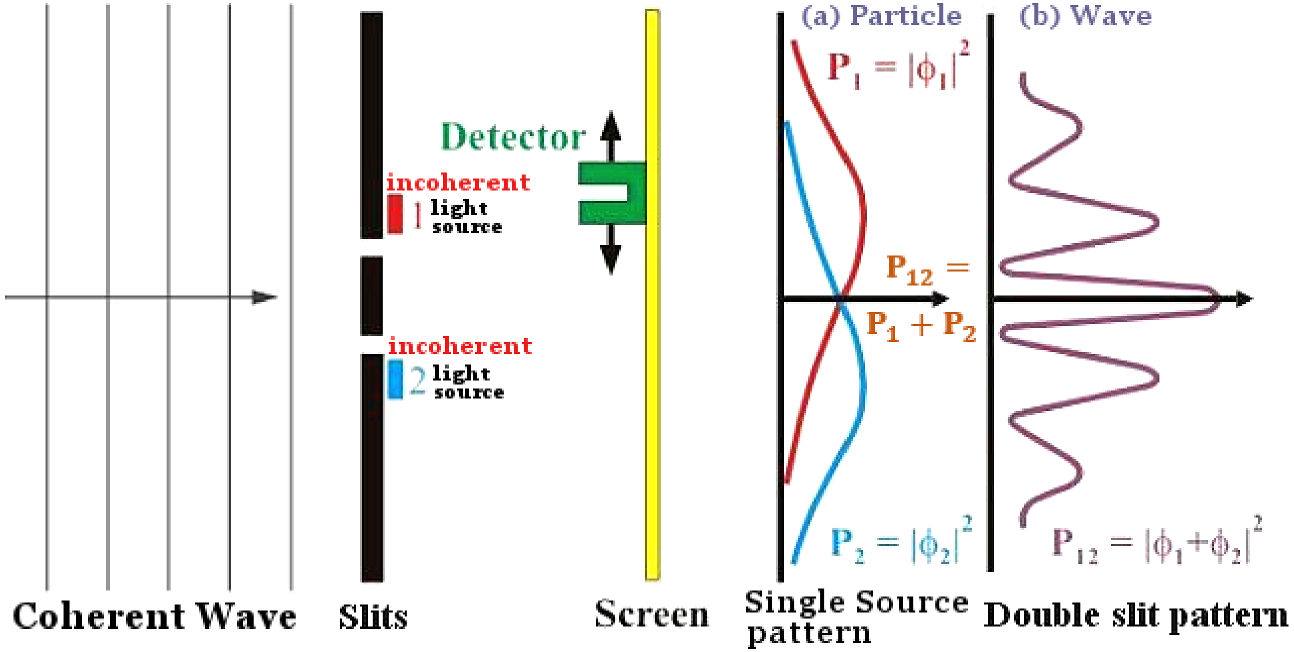
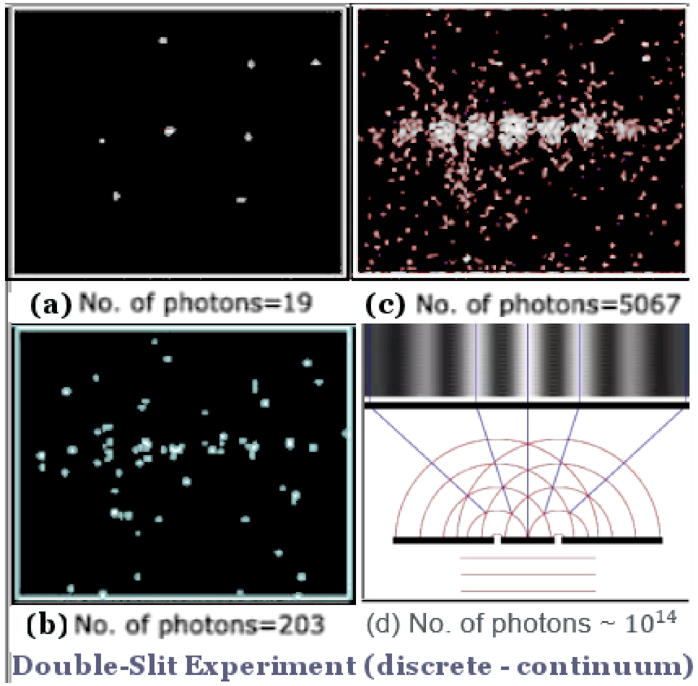


Figure 38-4 Compton's results for four values of the scattering angle ϕ. Note that the Compton shift $\Delta\lambda$ increases as the scattering angle increases.

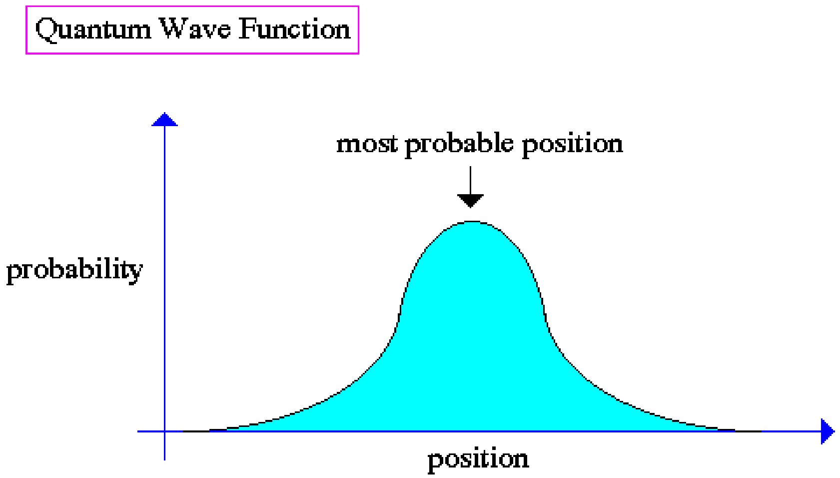
38.5: Light as a Probability Wave 概率波





The probability that a photon will be detected in any small volume centered on a given point in a light wave is proportional to the square of the amplitude of the wave's electric field vector at that point.

38.5: Light as a Probability Wave: (1 of 2)



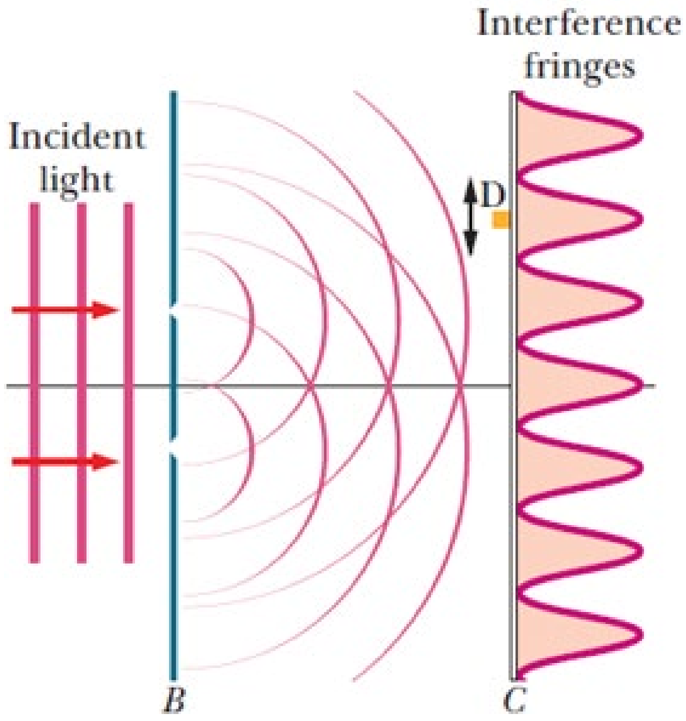


Figure 38-6 Light is directed onto screen *B*, which contains two parallel slits. Light emerging from these slits spreads out by diffraction. The two diffracted waves over lap at screen *C* and form a pattern of interference fringes.

The probabilistic description of a light wave is another way to view light. Light wave is not only an electromagnetic wave but also a **probability wave 概率波.**

That is, to every point in a light wave we can attach a numerical probability (per unit time interval) that a photon can be detected in any small volume centered on that point.

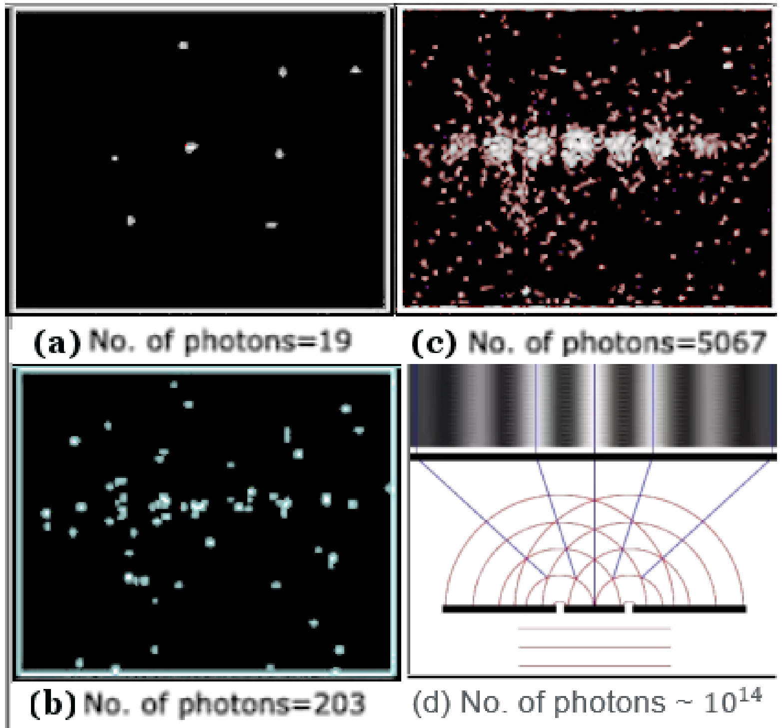
38.5: Light as a Probability Wave, The Single Photon Version: (1 of 4)

Consider the double-slit experiment again.

Since an interference pattern eventually builds up on the screen, we can speculate that each photon travels from source to screen as a wave that fills up the space between source and screen and then vanishes in a photon absorption at some point on the screen, with a transfer of energy and momentum to the screen at that point.

We cannot predict where this transfer will occur (where a photon will be detected) for any given photon originating at the source.

However, we can predict the probability that a transfer will occur at any given point on the screen.



Double-Slit Experiment (discrete - continuum)

https://phet.colorado.edu/sims/html/plinko-probability/latest/plinko-probability_all.html

38.6: Electrons and Matter Waves: (1 of 2)

$$\lambda = \frac{h}{p} \quad (\text{de Brolie wavelength})$$ 德布罗意波长

De Broglie suggested $p = \frac{h}{\lambda}$ might apply not only to photons but also to electrons

$$p = \frac{hf}{c} = \frac{h}{\lambda} \quad (\text{photon momentum})$$

Electrons momentum

38.6: Electrons and Matter Waves: (2 of 2)

De Broglie suggested $p = \frac{h}{\lambda}$ might apply not only to photons but also to electrons

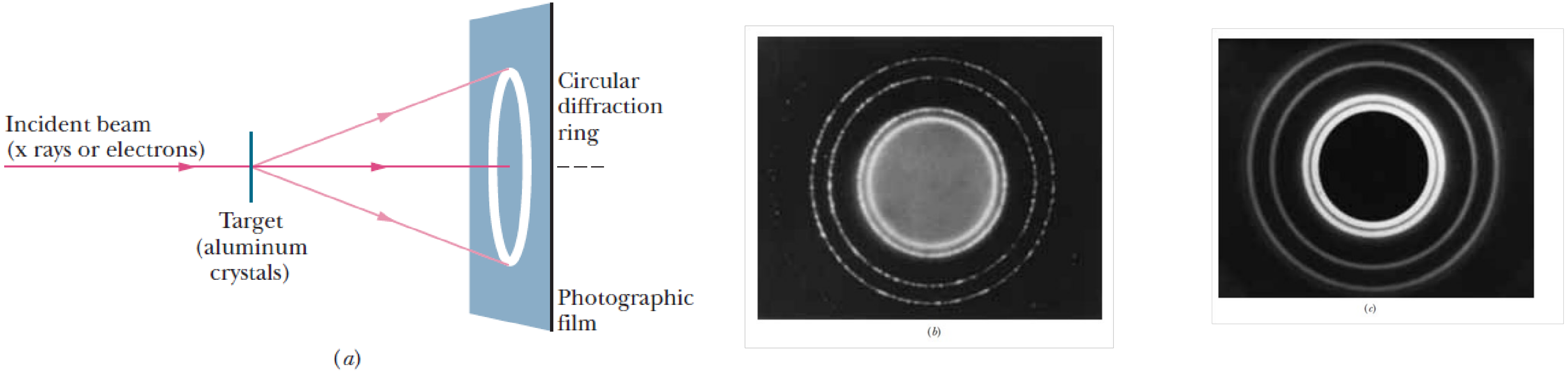


Figure 38-10 (*a*) An experimental arrangement used to demonstrate, by diffraction techniques, the wave-like character of the incident beam. Photographs of the diffraction patterns when the incident beam is (*b*) an x- ray beam (light wave) and (*c*) an electron beam (matter wave). Note that the two patterns are geometrically identical to each other

38.7: Schrödinger's Equation: (1 of 4)

If a **wave function,** $\psi(x, y, z, t)$, can be used to describe matter waves, then its space and time variables can be grouped separately and can be written in the form

$$\psi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$$

Where $\omega = (2\pi f)$ is the angular frequency of the matter wave

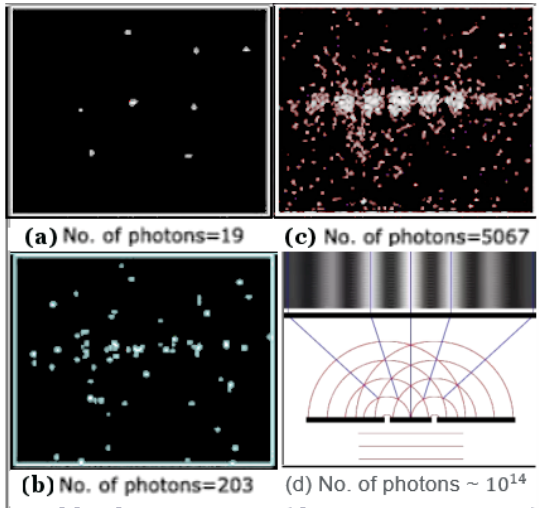
38.7: Schrödinger's Equation: (2 of 4)

$$\psi(x,y,z,t)=\psi(x,y,z)e^{-i\omega t}$$

Suppose that a matter wave reaches a particle detector; then the probability that a particle will be detected in a specified time interval is proportional to $|\psi|^2$, where $|\psi|$ is the absolute value of the wave function at the location of the detector.

- $|\psi|^2$ is always both real and positive, and it is called the **probability density概率密度,**

The probability of detecting a particle in a small volume centered on a given point in a matter wave is proportional to the value of $|\psi|^2$ at that point.



Double-Slit Experiment (discrete - continuum)

38.7: Schrödinger's Equation: (3 of 4)

Matter waves are described by Schrödinger's Equation.

Suppose a particle traveling in the x direction through a region in which forces acting on the particle cause it to have a potential energy $U(x)$. In this special case, Schrödinger's equation can be written as:

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}\left[E-U(x)\right]\psi=0 \quad \text{(Schrödinger' one-dimensional motion)},$$

$$\frac{d^2\psi}{dx^2}+k^2\psi=0 \quad \text{(Schrödinger's equation, uniform } U\text{)},$$

$$k=\frac{2\pi\sqrt{2m(E-U)}}{h} \quad \text{(angular wave number)}.$$

38.7: Schrödinger’s Equation: (4 of 4)

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left[E - U(x)\right]\psi = 0 \quad \text{(Schrödinger' one-dimensional motion)},$$

For a free particle, $U(x)$ is zero, that equation describes a free particle where a moving particle on which no net force acting on it. The particle’s total energy in this case is all kinetic, and the equation becomes:

$$\vec{P} = M\vec{v}_{\text{com}}$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(\frac{mv^2}{2}\right)\psi = 0. \longrightarrow \frac{d^2\psi}{dx^2} + \left(2\pi\frac{p}{h}\right)^2\psi = 0.$$

38.7: Schrödinger's Equation: (4 of 4)

$$\frac{d^2\psi}{dx^2} + \left(2\pi\frac{p}{h}\right)^2\psi = 0. \qquad p = \frac{hf}{c} = \frac{h}{\lambda} \quad (\text{photon momentum}) \qquad k = 2\pi/\lambda:$$

Using the concept of de Broglie wavelength and the definition of wave number,

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad (\text{scrödinger's equation, free particle}),$$

The solution to this is: $\psi(x) = Ae^{ikx} + Be^{-ikx}$, Here A and B are constants.

$$\begin{aligned}\Psi(x,t) &= \psi(x)e^{-i\omega t} = (Ae^{ikx} + Be^{-ikx})\,e^{-i\omega t}\\ &= Ae^{i(kx-\omega t)} + Be^{-i(kx+\omega t)}.\end{aligned}$$

38.7: Schrödinger's Equation, Finding the Probability Density:

Finding the Probability Density | ψ |2

Consider a free particle that travels only in the positive direction of x. Let the arbitrary constant B be zero. At the same time, let us relabel the constant A as ψ_0.

We eliminate the negative motion by setting B to zero, and then the solution at $t = 0$ becomes

$$\Psi(x, t) = Ae^{i(kx-\omega t)} + Be^{-i(kx+\omega t)}.$$

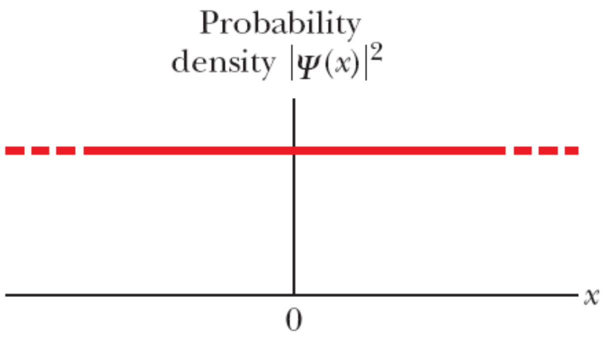
$$\psi(x) = Ae^{ikx}.$$

$$\psi(x) = \psi_0 e^{ikx}.$$

$$|\psi|^2 = |\psi_0 e^{ikx}|^2 = (\psi_0^2)|e^{ikx}|^2.$$

$$|e^{ikx}|^2 = (e^{ikx})(e^{ikx})^* = e^{ikx}e^{-ikx} = e^{ikx-ikx} = e^0 = 1.$$

$$|\psi|^2 = (\psi_0^2)(1)^2 = \psi_0^2 \quad (\text{a constant}).$$



That means that if we make a measurement to locate the particle, the location could turn out to be at any x value. Thus, we cannot say that the particle is moving along the axis in a classical way as a car moves along a street. *In fact, the particle does not have a location until we measure it.*

38.8: Heisenberg's Uncertainty Principle:

$$\Delta x \cdot \Delta p_x \geq \hbar$$

海森堡测不准原理

$$\Delta y \cdot \Delta p_y \geq \hbar \quad (\text{Heisenberg's uncertainty principle})$$

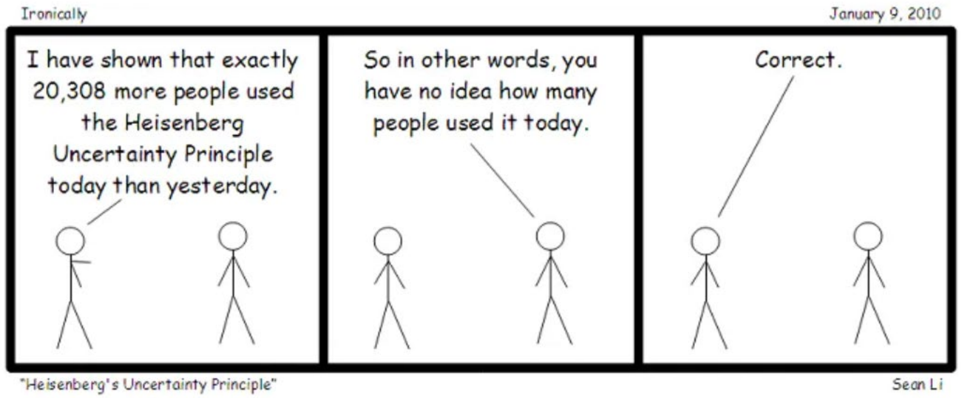
$$\Delta z \cdot \Delta p_z \geq \hbar$$

Heisenberg's Uncertainty Principle states that measured values cannot be assigned to the position and the momentum of a particle simultaneously with unlimited precision.

Here Δx and Δp_x represent the intrinsic uncertainties in the measurements of the x components of r and p, with parallel meanings for the y and z terms. Even with the best measuring instruments, each product of a position uncertainty and a momentum uncertainty will be greater than $\hbar$, never less.

38.8: Heisenberg’s Uncertainty Principle:

The Heisenberg Uncertainty Principle 海生堡测不准原理



uncertainty in momentum

$$\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

uncertainty in position

The more accurately you know the position (i.e., the smaller Δx is), the less accurately you know the momentum (i.e., the larger Δp is); and vice versa

38.9: Barrier Tunneling 势垒隧穿

As the puck climbs the hill, kinetic energy K is transformed into gravitational potential energy U. If the puck reaches the top, its potential energy is U_b. Thus, the puck can pass over the top only if its initial mechanical energy $E > U_b$.

The hill acts as a **potential energy barrier 势垒** (or, for short, a **potential barrier**).

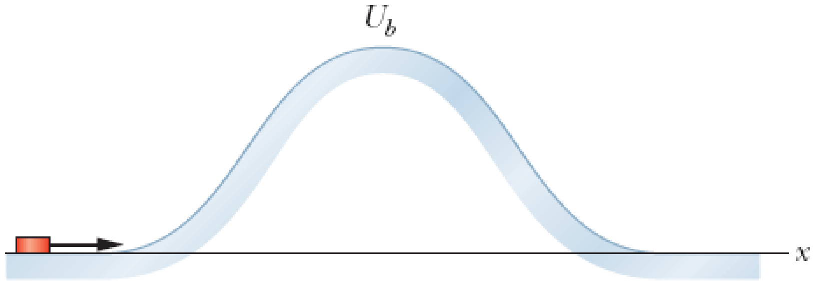


Figure 38-13 A puck slides over friclionless ice toward a hill. The puck's gravitational potential at the top of the hill will be U_b.

38.9: Barrier Tunneling 势垒隧穿

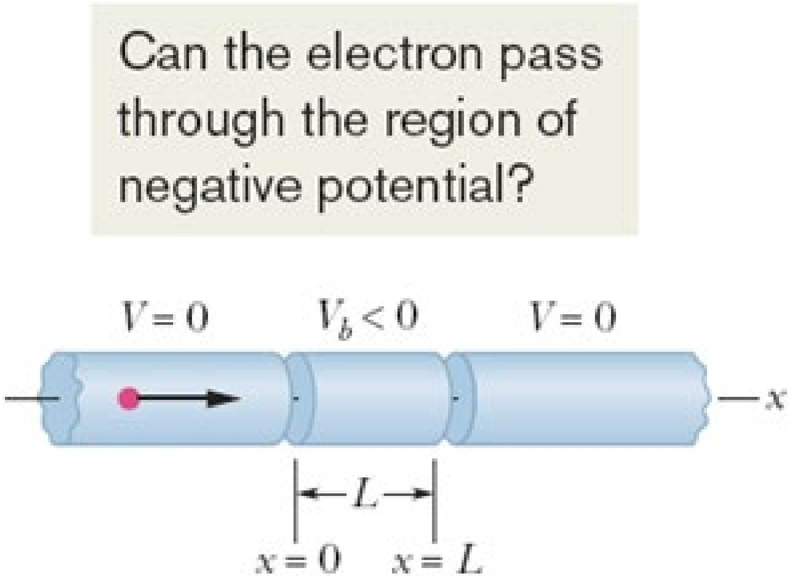


Figure 38-14 The elements of an idealized thin wire in which an election (the dot) approaches a negative electric potential V_b in the region $x = 0$ to $x = L$.

There is a potential barrier for a nonrelativistic electron traveling along an idealized wire of negligible thickness (Figure 38-14). The electron, with mechanical energy *E*, approaches a region (the barrier) in which the electric potential V_b is negative.

38.9: Barrier Tunneling 势垒隧穿

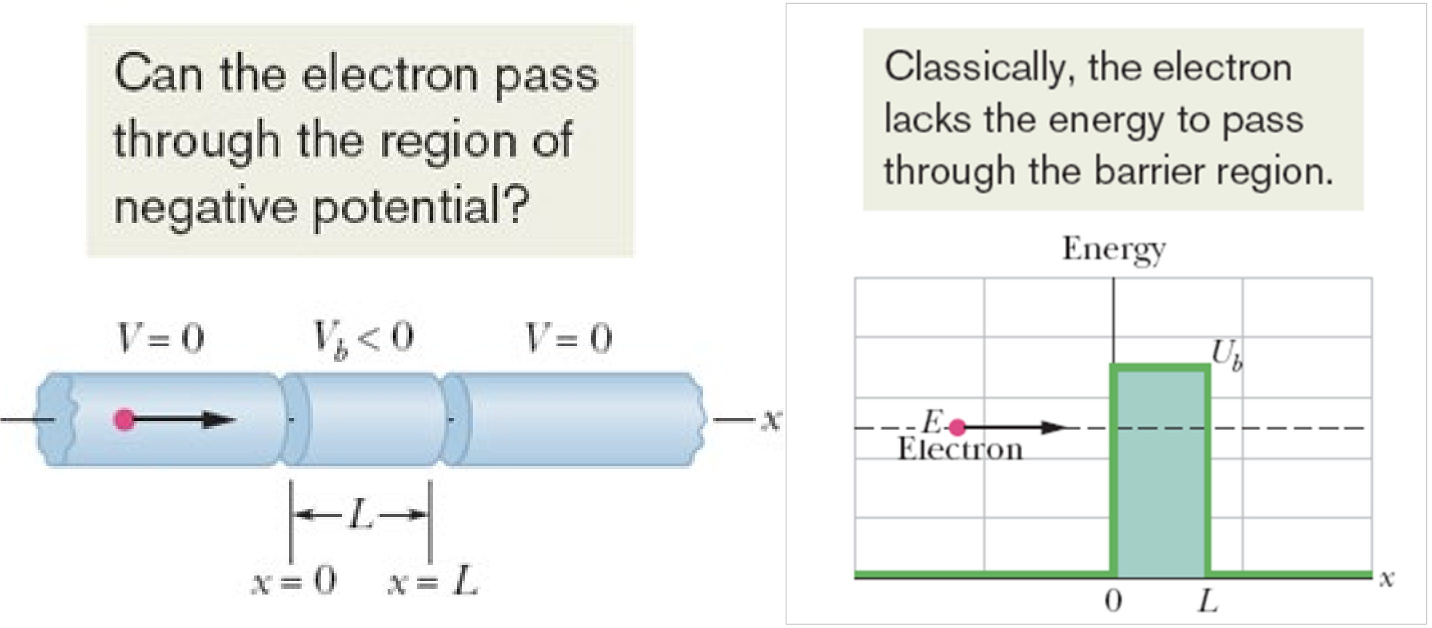


Figure 38-15 An electron's mechanical energy E is plotted when the electron is at any coordinate $x < 0$.

The electron's electric potential energy U is plotted as a function of the electron's position x, assuming that the electron can reach any value of x. The nonzero part of the plot (the potential barrier) has height U_b and thickness L.

mechanical energy E

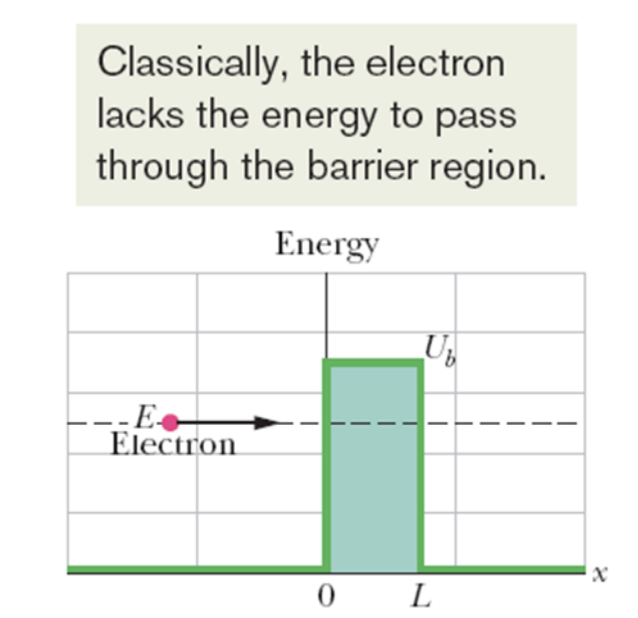
The electron, being negatively charged, will have a positive potential energy U_b ($= qV_b$) in that region (Figure 38-15). If $E > U_b$, we expect the electron to pass through the barrier region and come out to the right of $x = L$ in Figure 38-14.

If $E < U_b$, we expect the electron to be unable to pass through the barrier region.

38.9: Barrier Tunneling: (6 of 9)

Something astounding can happen to the electron when $E < U_b$.

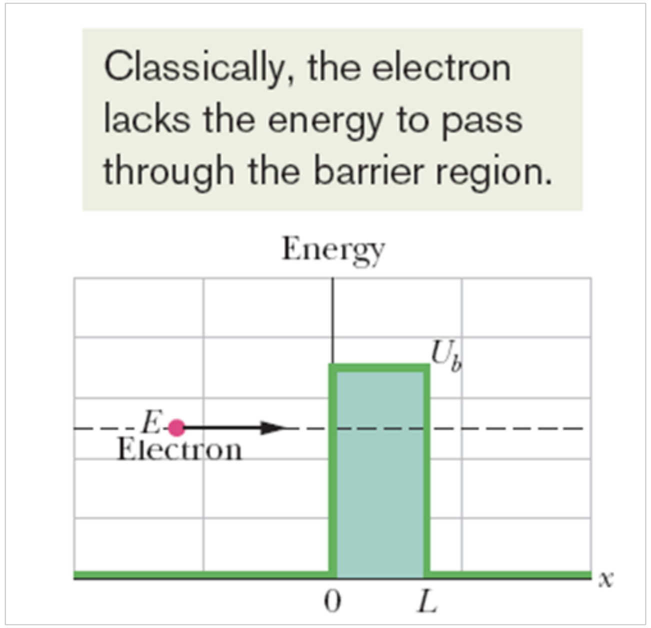
Since it is a matter wave, the electron has a finite probability of leaking (or, tunneling) through the barrier and materializing on the other side, moving rightward with energy E as though nothing had happened in the region of $0 \leq x \leq L$.



The wave function $\psi(x)$ describing the electron can be found by solving Schrödinger's equation separately for the three regions: (1) to the left of the barrier, (2) within the barrier, and (3) to the right of the barrier.

38.9: Barrier Tunneling: (7 of 9)

The arbitrary constants that appear in the solutions can then be chosen so that the values of $\psi(x)$ and its derivative with respect to x join smoothly at $x = 0$ and $x = L$. Squaring the absolute value of $\psi(x)$ then yields the probability density.



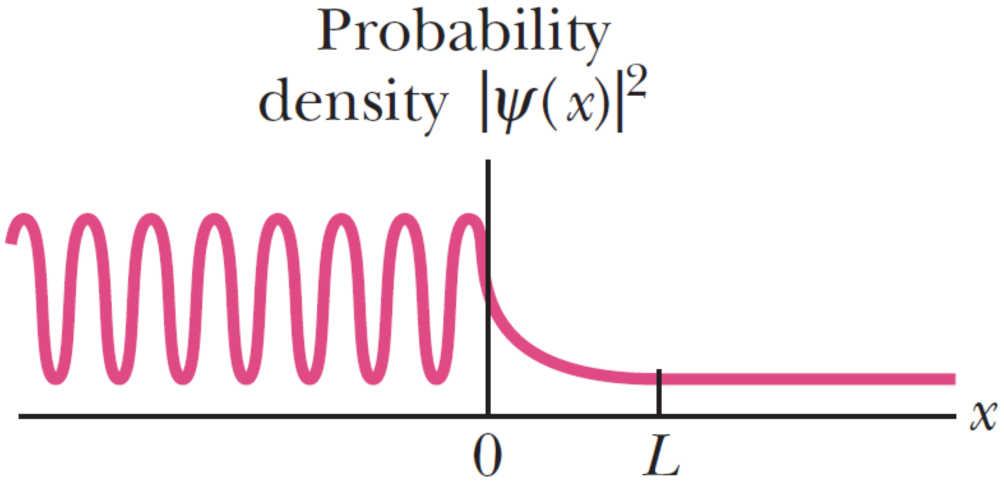


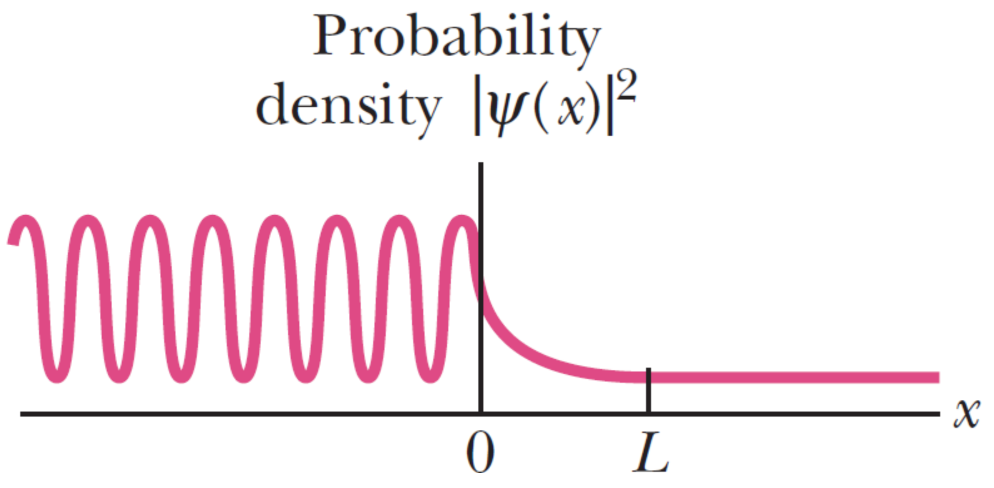
Figure 38-16 A plot of the probability density $|\psi|^2$ of the electron on matter wave for the situation of Figure, 38-15. The value of $|\psi|^2$ is nonzero to the right of the potential barrier.

38.9: Barrier Tunneling: (8 of 9)

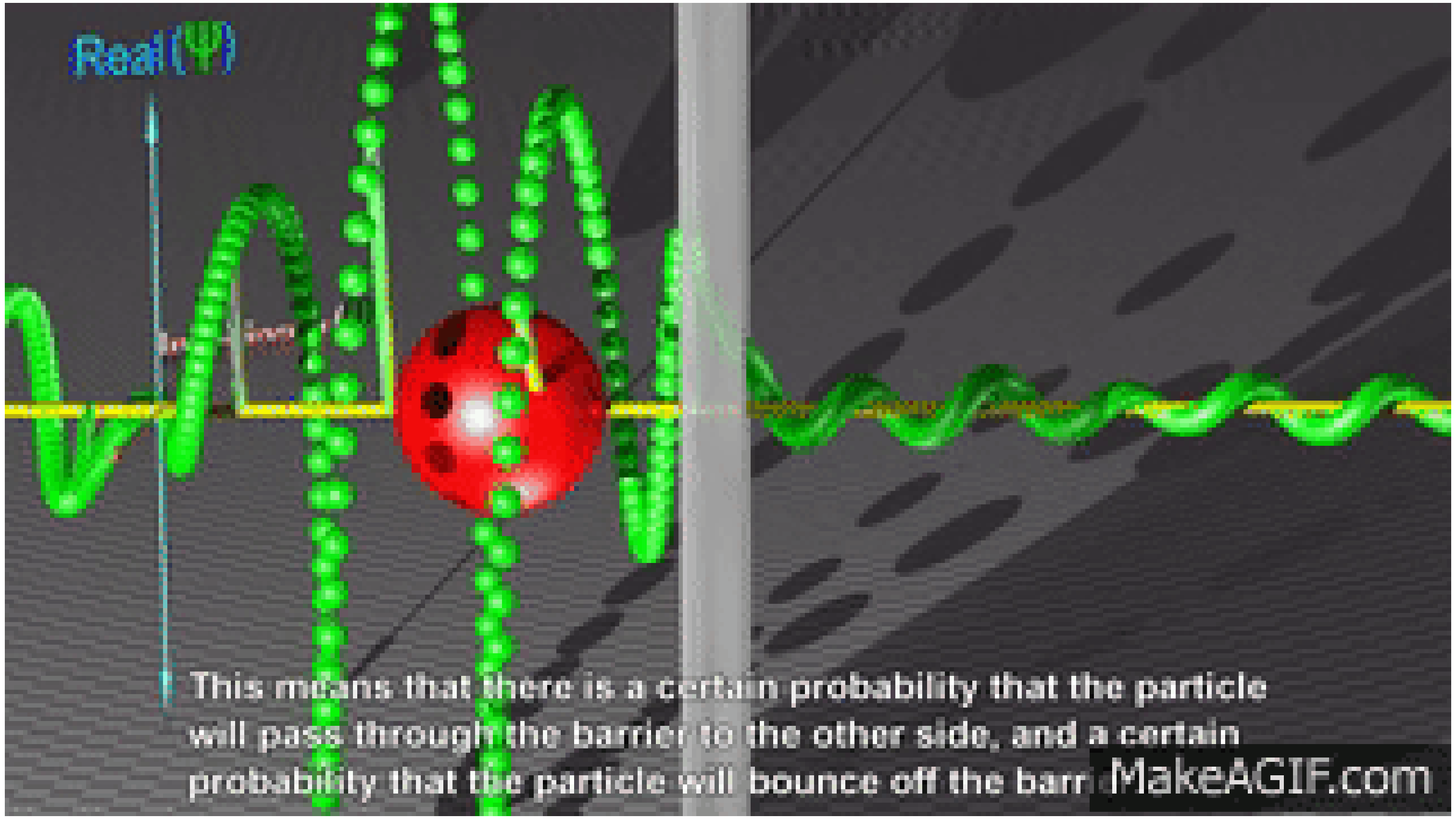
- Within the barrier the probability density decreases exponentially with x.
- To the right of the barrier, the probability density plot describes a transmitted (through the barrier) wave with low but constant amplitude.
- We can assign a **transmission coefficient *T*** to the incident matter wave and the barrier. This coefficient gives the probability with which an approaching electron will be transmitted through the barrier—that is, that tunneling will occur.

Approximately, $T \approx e^{-2bL}$,

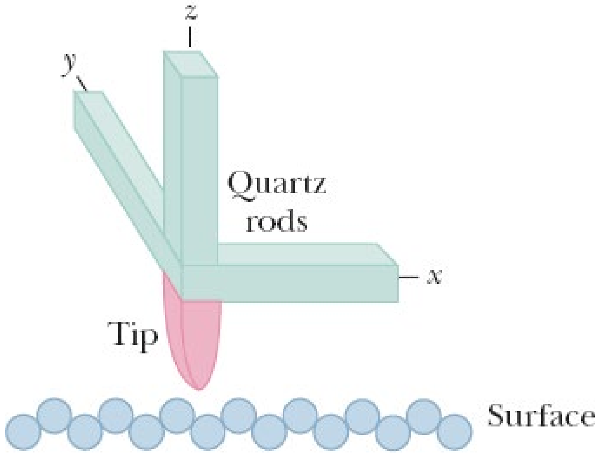
$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}},$$



38.9: Barrier Tunneling: (8 of 9)



38.9: Barrier Tunneling, The Scanning Tunneling Microscope (STM):



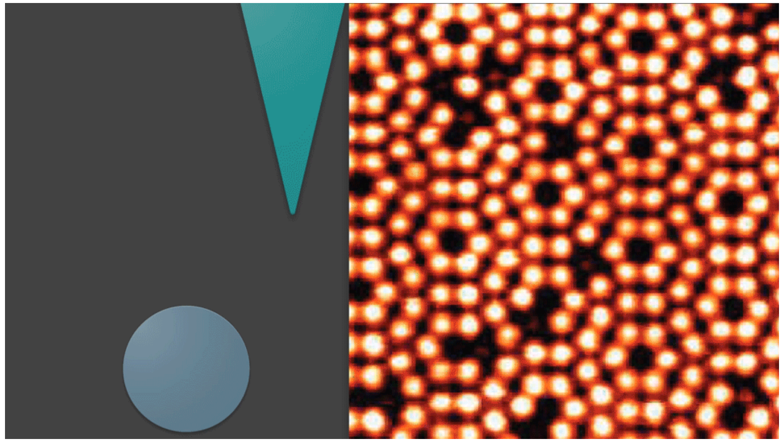


Figure 38-17 The essence of a scanning tunneling microscope (STM). Three quartz rods are used to scan a sharply pointed conducting tip across the surface of interest and to maintain a constant separation between tip and surface. The tip thus moves up and down to match the contours of the surface, and a record of its movement provides information for a computer to create an image of the surface.

38.2.4. Upon which one of the following parameters does the energy of a photon depend?

a) the mass of the photon

b) the amplitude of the electric field

c) the direction of the electric field

d) the relative phase of the electromagnetic wave relative to the source that produced it

e) the frequency of the photon

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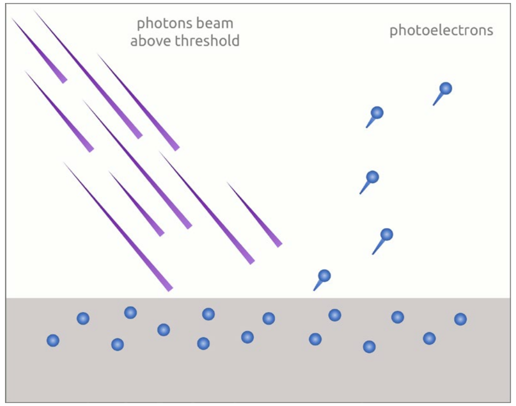
e) the frequency of the photon

38.3.5. In the photoelectric effect experiment, what type of energy process is occurring?



a) Kinetic energy is transformed into thermal energy.

b) Thermal energy is transformed into electromagnetic energy.

c) Radiant energy is transformed into kinetic energy.

d) Electromagnetic energy is transformed into thermal energy.

e) Radiant energy is transformed into potential energy.

$$hf = K_{max} + \Phi \quad \text{(photoelectric equation)}.$$

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38.4.1. X-rays with a wavelength of 0.10 nm are scattered from an argon atom. The scattered x-rays are detected at an angle of 85° relative to the incident beam. What is the Compton shift for the scattered x-rays?

a) 0.0022 nm

b) 0.011 nm

c) 0.022 nm

d) 0.041 nm

e) 0.12 nm

$$\Delta\lambda = \frac{h}{mc}(1-\cos\phi) \quad (\text{Compton shift}).$$

(the electron case value 2.4263×10^{-12} m)

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38.4.3. An x-ray photon with an initial wavelength λ strikes an electron that is initially at rest. Which one of the following statements best describes the wavelength of the photon after the collision?

- $hf = hf' + mc(\gamma - 1).$

a) No photon remains after the collision.

$$\frac{h}{\lambda} = \frac{h}{\lambda'} + mc(\gamma - 1).$$

b) The scattered photon's wavelength will still be λ, but its frequency will decrease.

c) The scattered photon's wavelength will be longer than λ.

d) The scattered photon's wavelength will be λ/2.

e) The scattered photon's wavelength will be between λ/2 and λ.

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uncertainty
in momentum

$$\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

uncertainty
in position

rately you know the position (i.e., the smaller Δx is),
tely you know the momentum (i.e., the larger Δp is)
and vice versa

38.8.2. The position along the *x* axis of an electron is known to be between –0.31 nm and + 0.31 nm. How would the uncertainty in the momentum of the electron change if the electron were allowed to be between –0.62 nm and +0.62 nm?

a) The uncertainty in the momentum would be twice its previous value.

b) The uncertainty in the momentum would be half of its previous value.

c) The uncertainty in the momentum would not be affected by this change.

d) The uncertainty in the momentum would be four times its previous value.

e) The uncertainty in the momentum would be one fourth its previous value.

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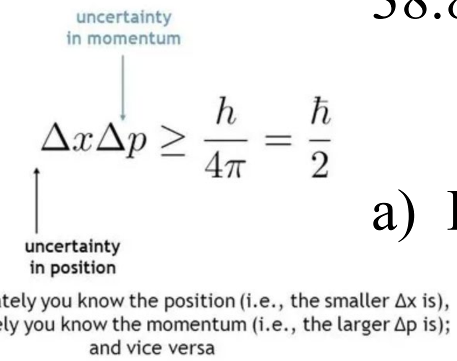

38.8.1. Which one of the following statements provides the best description of the Heisenberg Uncertainty Principle?

a) If a particle is confined to a region Δx, then its momentum is within some range Δp.

b) If the error in measuring the position is Δx, then we can determine the error in measuring the momentum Δp.

c) If one measures the position of a particle, then the value of the momentum will change.

d) It is not possible to be certain of any measurement.

e) Depending on the degree of certainty in measuring the position of a particle, the degree of certainty in measuring the momentum is affected.

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