UFUG 1504: Honors General Physics II

Chapter 23

Gauss' Law

Summary (1 of 4)

Gauss' Law

- Gauss' law is

$$\varepsilon_0 \Phi = q_{\text{enc}}$$

Equation (23-6)

- the net flux of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

Equation (23-6)

Summary (2 of 4)

Applications of Gauss' Law

- surface of a charged conductor

$$E = \frac{\sigma}{\varepsilon_0}$$

Equation (23-11)

- Within the surface $E = 0$.
- line of charge

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Equation (23-12)

Summary (3 of 4)

- Infinite non-conducting sheet

$$E = \frac{\sigma}{2\varepsilon_0}$$

Equation (23-13)

- Outside a spherical shell of charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Equation (23-15)

Summary (4 of 4)

- Inside a uniform spherical shell

$$E = 0$$

Equation (23-16)

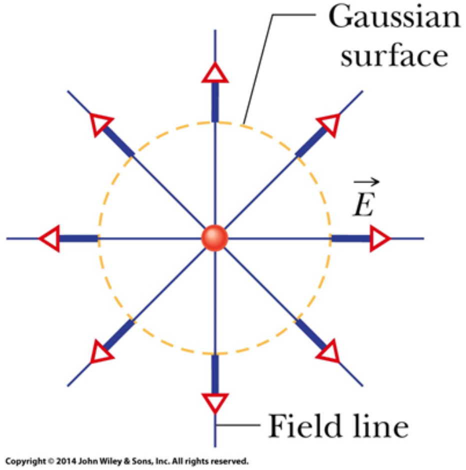
- Inside a uniform sphere of charge

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r.$$

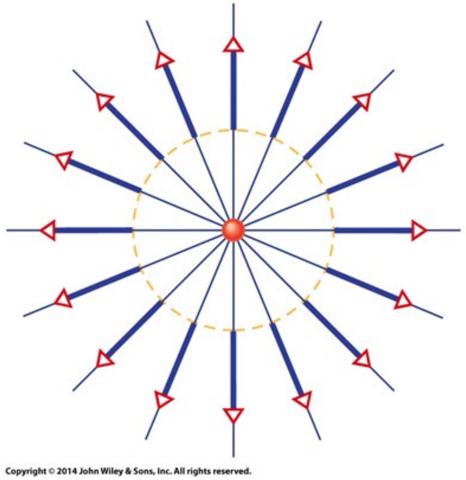
Equation (23-20)

23-1 Electric Flux (电通量) (4 of 13)

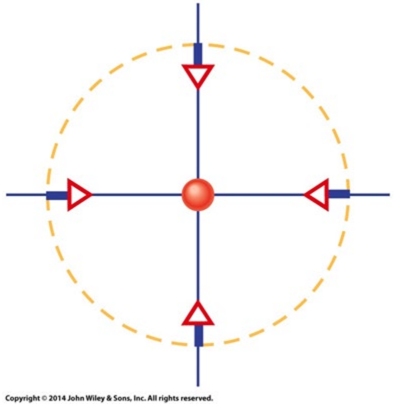
Guass' law relates the electric field at points on a (closed) Gaussian surface to the net charge enclosed by that surface.



Electric field vectors and field lines pierce an imaginary, spherical Gaussian surface that encloses a particle with charge $+Q$. 8 line



Now the enclosed particle has charge $+2Q$.
16 line



Can you tell what the enclosed charge is now? 4 line but toward to the center

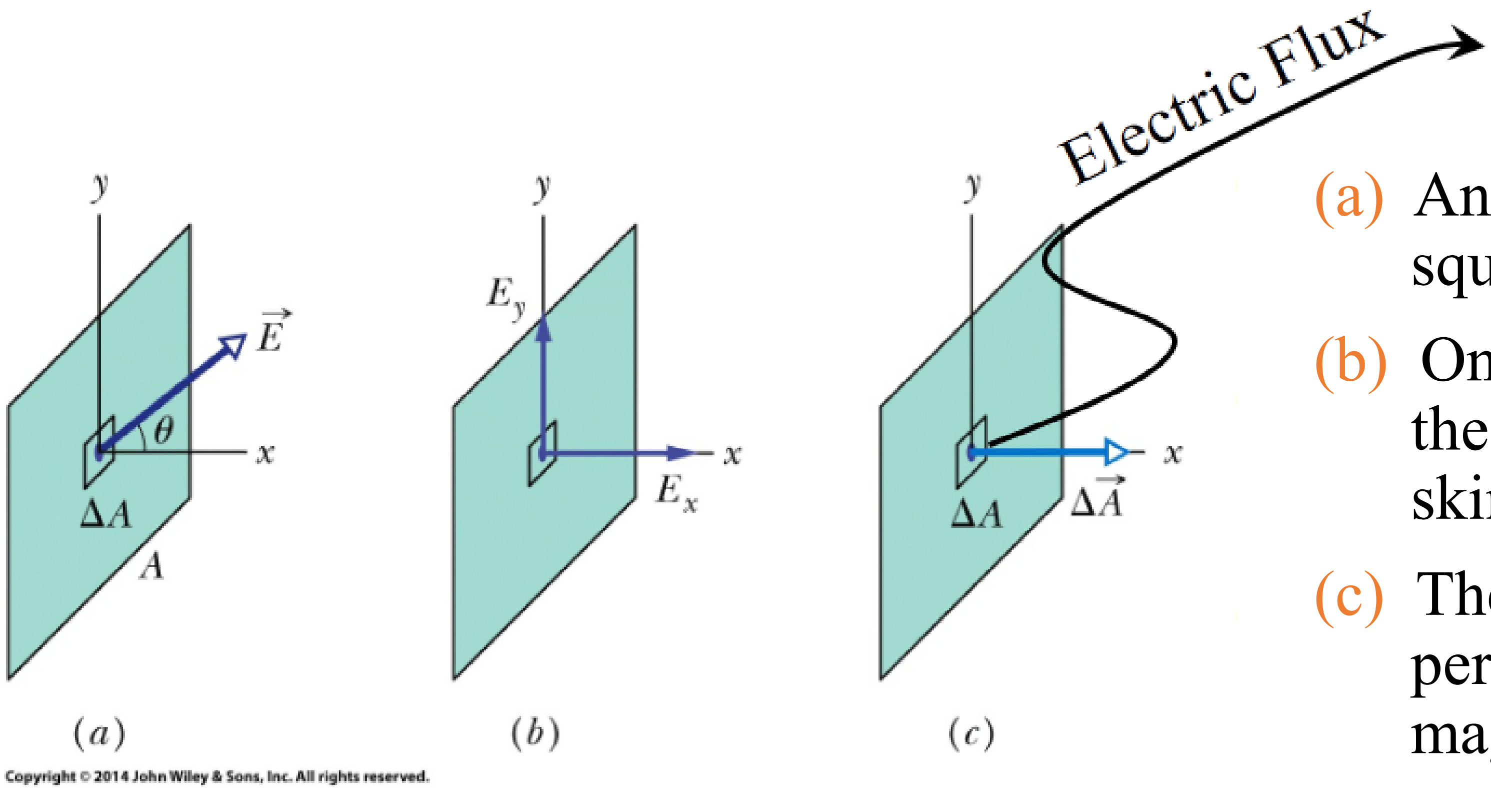
Answer: $-0.5Q$

23-1 Electric Flux (电通量) (4 of 13)

However, we cannot do all this by simply comparing the density of field lines in a drawing as we just did.

We need a quantitative way of determining how much electric field pierces a surface. That measure is called the electric flux.

23-1 Electric Flux (6 of 13)



(a) An electric field vector pierces a small square patch on a flat surface.

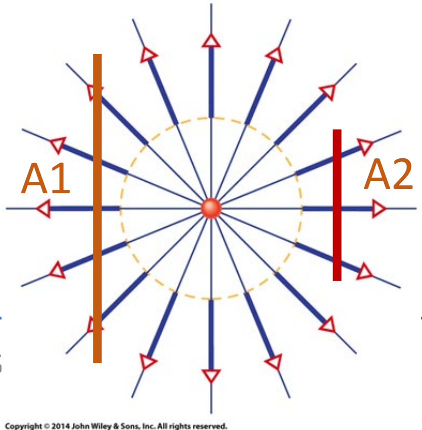
(b) Only the x component actually pierces the patch ($E\cos\theta$); the y component skims across it.

(c) The area vector of the patch is perpendicular to the patch, with a magnitude equal to the patch's area.

$$\Delta\Phi = (E\cos\theta)\Delta A$$

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}.$$

phi



A1>A1, more electric field line

23-1 Electric Flux (5 of 13)

However, because we do not want to sum hundreds (or more) flux values,we transform the summation into an integral

$$\Delta\Phi = (E\cos\theta)\Delta A$$

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}.$$

$$d\Phi = \vec{E} \cdot d\vec{A}.$$

$$\Phi = \int \vec{E} \cdot d\vec{A} \qquad (\text{total flux}).$$

The **area vector** $d\vec{A}$ for an area element (patch element) on a surface is a vector that is perpendicular to the element and has a magnitude equal to the area dA of the element.

The **electric flux** $d\Phi$ through a patch element with area vector

23-1 Electric Flux (8 of 13)

- The **net flux (净通量)** through a closed surface (which is used in Gauss' law) is given by

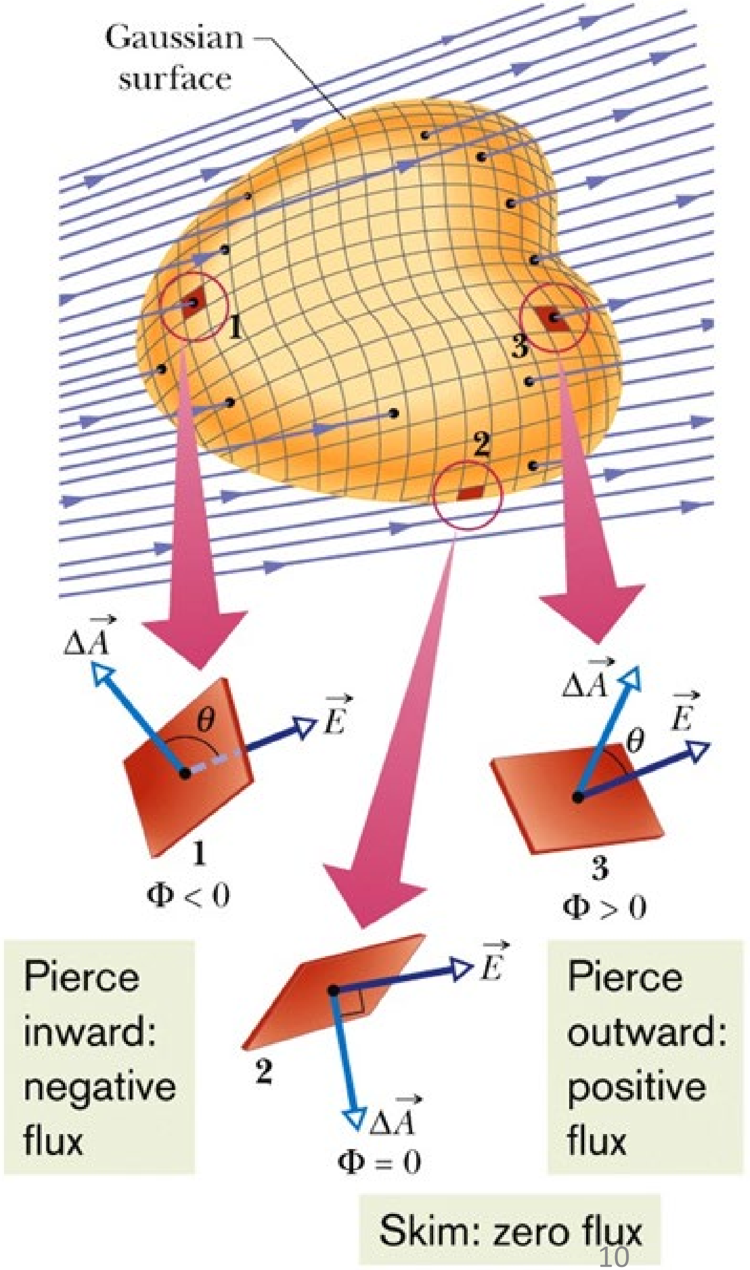
$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad (\text{net flux}).$$

- where the integration is carried out over the entire surface.

An inward piercing field is negative flux.

An outward piercing field is positive flux.

A skimming field is zero flux.



23-1 Electric Flux (7 of 13)

Now we can find the total flux by integrating the dot product over the full surface.

The **total flux** through a surface is given by

$$\Phi = \int \vec{E} \cdot d\vec{A} \qquad (\text{total flux}).$$

The **net flux** through a closed surface (which is used in Gauss' law) is given by

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad (\text{net flux}).$$

where the integration is carried out over the entire surface.

23-1 Electric Flux (9 of 13)

Flux through a closed cylinder, uniform field

Figure 23-6 shows a Gaussian surface in the form of a closed cylinder (a Gaussian cylinder or G-cylinder) of radius *R*. It lies in a uniform electric field E with the cylinder's central axis (along the length of the cylinder) parallel to the field. What is the net flux of the electric field through the cylinder?

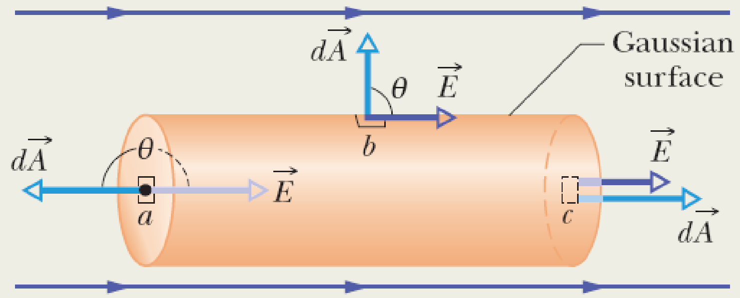


Figure 23-6 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

Calculations: We break the integral of Eq. 23-4 into three terms: integrals over the left cylinder cap *a*, the curved cylindrical surface *b*, and the right cap *c*

$$\Phi = \oint \vec{E}\cdot d\vec{A}$$

$$= \int_a \vec{E}\cdot d\vec{A} + \int_b \vec{E}\cdot d\vec{A} + \int_c \vec{E}\cdot d\vec{A}. \quad =0$$

$$\int_a \vec{E}\cdot d\vec{A} = \int E(\cos 180^\circ)\,dA = -E\int dA = -EA,$$

23-1 Electric Flux (9 of 13)

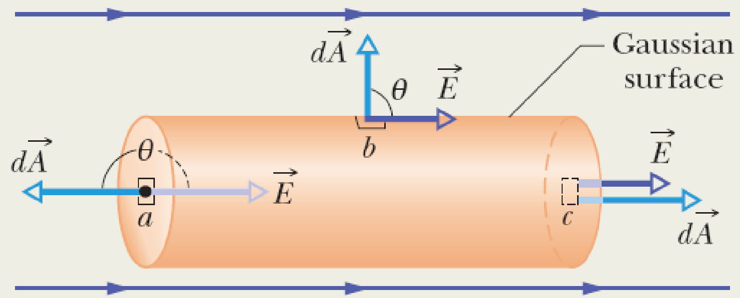


Figure 23-6 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

$$\Phi = \oint \vec{E}\cdot d\vec{A}$$

$$= \int_a \vec{E}\cdot d\vec{A} + \int_b \vec{E}\cdot d\vec{A} + \int_c \vec{E}\cdot d\vec{A}.$$

$$\int_a \vec{E}\cdot d\vec{A} = \int E(\cos 180°)\,dA = -E\int dA = -EA, \quad A(=\pi R^2).$$

$$\int_c \vec{E}\cdot d\vec{A} = \int E(\cos 0)\,dA = EA.$$

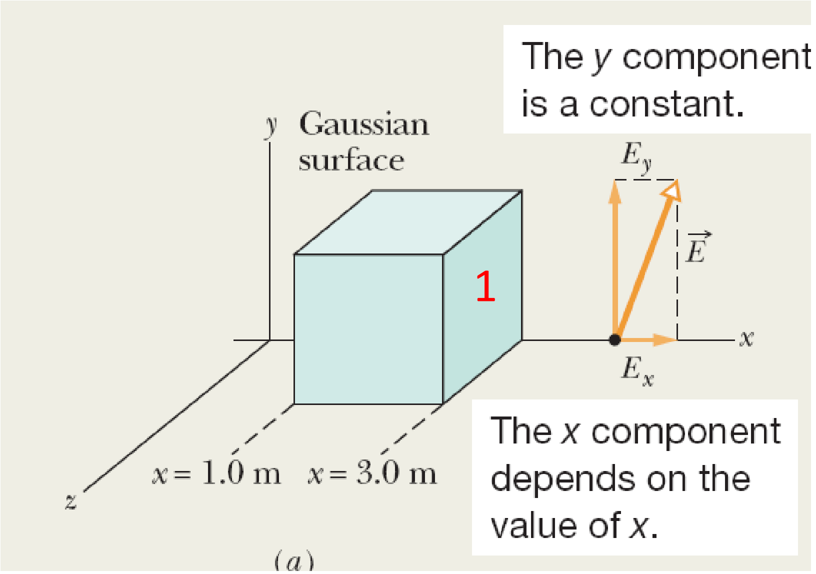
$$\int_b \vec{E}\cdot d\vec{A} = \int E(\cos 90°)\,dA = 0.$$

$$\Phi = -EA + 0 + EA = 0.$$

The net flux is zero that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

23-1 Electric Flux (9 of 13)

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-7*a*. (*E* is in newtons per coulomb and *x* is in meters.) What is the electric flux through the right face, the left face, and the top face?



1

$$\Phi_r = \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i})$$

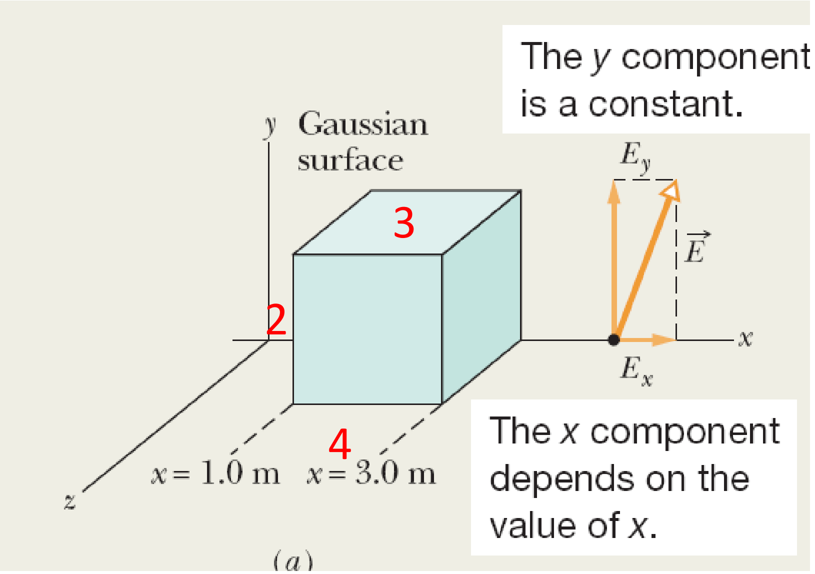
$$= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i}]$$

$$= \int (3.0x\, dA + 0) = 3.0 \int x\, dA.$$

$$= (9.0\ \mathrm{N/C})(4.0\ \mathrm{m^2}) = 36\ \mathrm{N \cdot m^2/C}.$$

23-1 Electric Flux (9 of 13)

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-7*a*. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face?



2 $\Phi_l = -12\ \mathrm{N \cdot m^2/C}.$

3
$$\Phi_t = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{j})$$
$$= \int [(3.0x)(dA)\hat{i} \cdot \hat{j} + (4.0)(dA)\hat{j} \cdot \hat{j}]$$
$$= \int (0 + 4.0\ dA) = 4.0 \int dA$$
$$= 16\ \mathrm{N \cdot m^2/C}.$$

Actually, net Φ of surface 3 and surface 4 are 0, only 1 and 2 contribute Φ.

23-2 Gauss' Law (3 of 5)

Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the net charge q_{enc} that is enclosed by that surface. It tells us that

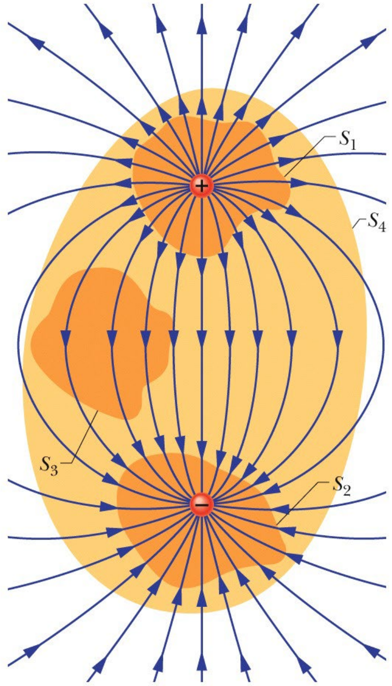
q_{enc} is the algebraic sum of all the *enclosed* positive and negative charges

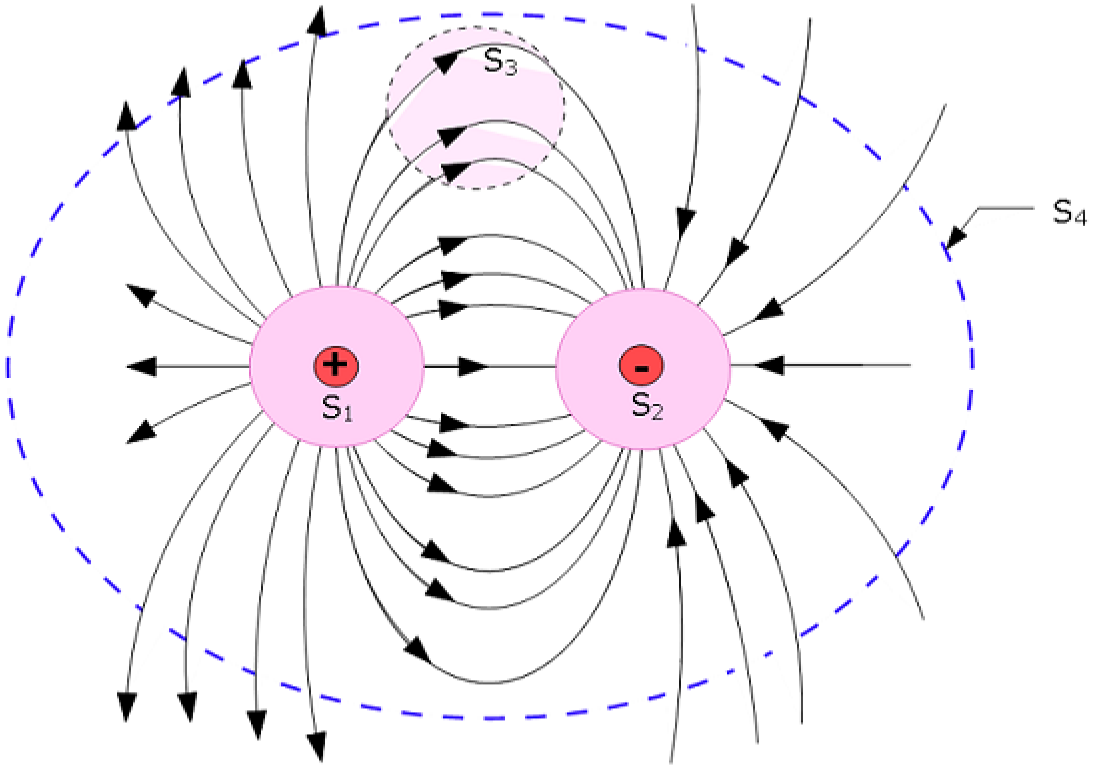
$$\varepsilon_0 \Phi = q_{\text{enc}} \qquad (\text{Gauss' law}).$$

because $\Phi = \oint \vec{E} \cdot d\vec{A} \qquad (\text{net flux}).$

$$\boxed{\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \qquad (\text{Gauss' law}).}$$

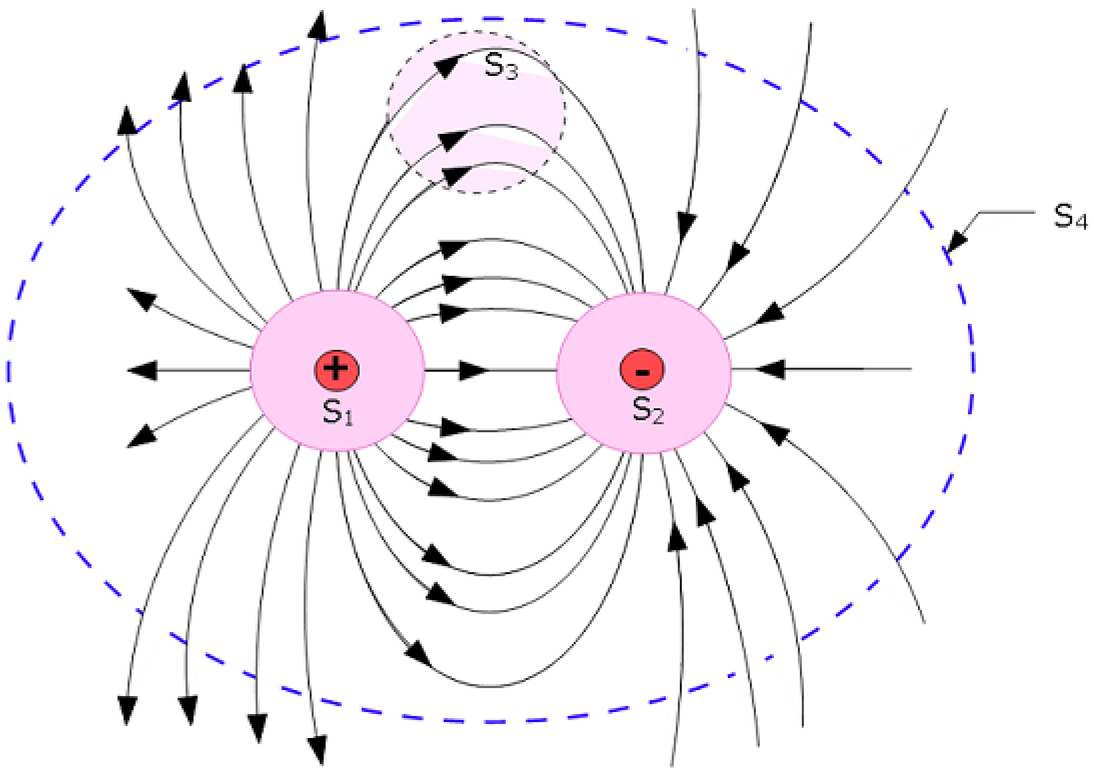
23-2 Gauss' Law (4 of 5)





Two charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section.

23-2 Gauss' Law (4 of 5)



$$\varepsilon_0 \Phi = q_{\text{enc}} \qquad (\text{Gauss' law}).$$

Surface S_1. The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is positive, and so is the net charge within the surface, as Gauss' law requires

Surface S_2. The electric field is inward for all points on this surface. Thus, the flux of the electric field through this surface is negative and so is the enclosed charge, as Gauss' law requires.

Surface S_3. This surface encloses no charge, and thus $q_{\text{enc}} = 0$. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because all the field lines pass entirely through the surface, entering it at the top and leaving at the bottom.

Surface S_4. This surface encloses no net charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because there are as many field lines leaving surface S_4 as entering it.

23-2 Gauss' Law (4 of 5)

Can anyone use Gauss' law to demonstrate Coulomb's Law?

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad \text{(Gauss' law)}. \Rightarrow \quad f = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law)},$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}.$$

$$\varepsilon_0 E \oint dA = q. \qquad A(= 4\pi R^2).$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}. \qquad \vec{F} = q\vec{E}. \Rightarrow \quad f = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law)},$$

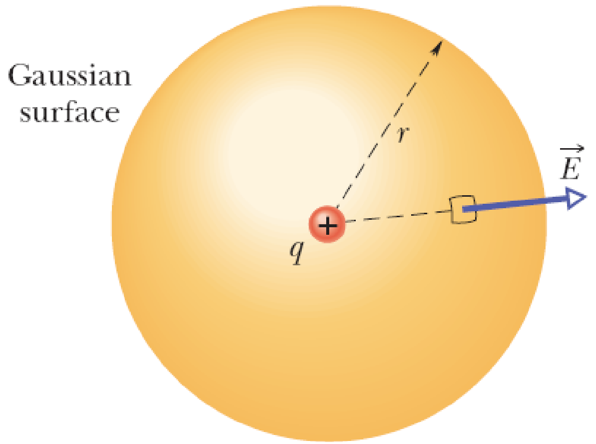


Figure 23-9 A spherical Gaussian surface centered on a particle with charge q.

23-2 Gauss' Law (4 of 5)

Using Gauss' law to find the electric field

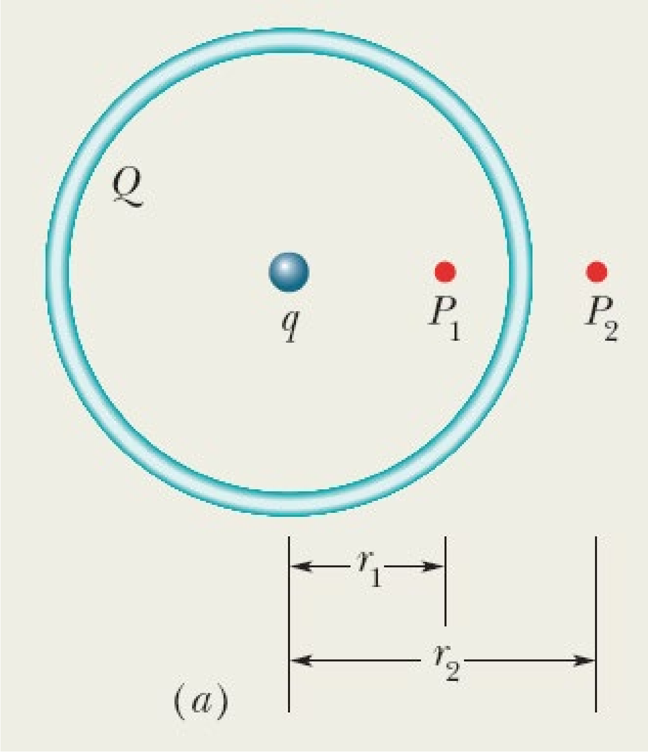


Figure *a* shows, in cross section, a plastic, spherical shell with uniform charge $Q = -16e$ and radius $R = 10$ cm. A particle with charge $q = +5e$ is at the center. What is the electric field (magnitude and direction) at point P_2 at radial distance $r_2 = 12.0$ cm?

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{enc}.$$

$$\varepsilon_0 E \oint dA = q. \qquad A(= 4\pi R^2)$$

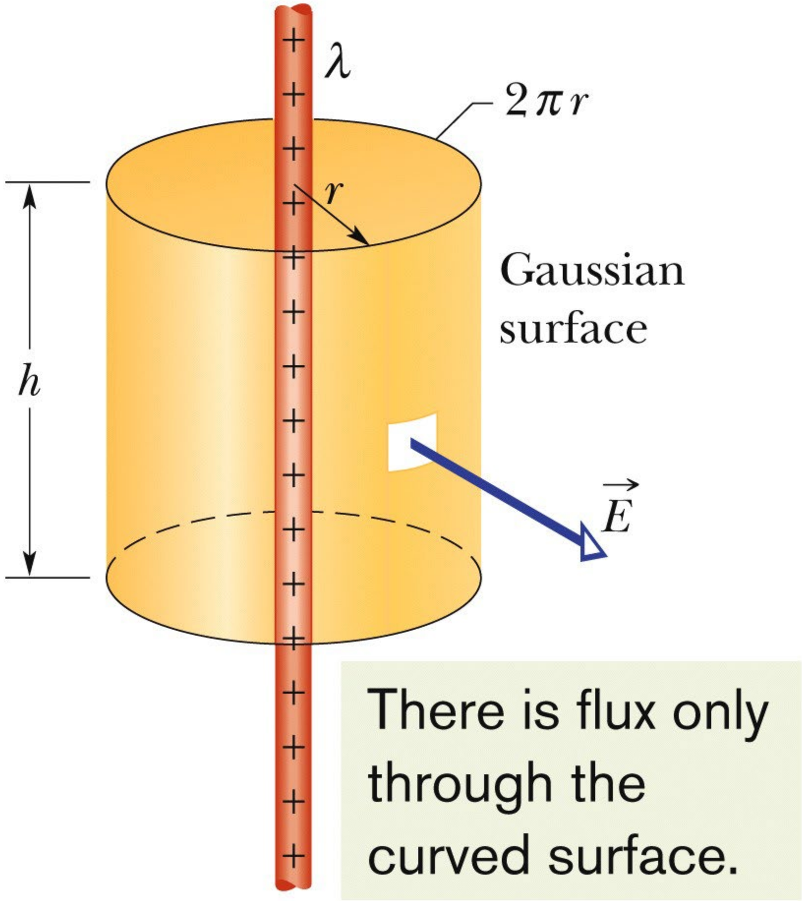
$$q_{enc} = q + Q = 5e + (-16e) = -11e.$$

$$E = \frac{-q_{enc}}{4\pi\varepsilon_0 r^2}$$

$$= \frac{-[-11(1.60 \times 10^{-19}\ \text{C})]}{4\pi(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.120\ \text{m})^2}$$

$$= 1.10 \times 10^{-6}\ \text{N/C}.$$

23-4 Applying Gauss' Law: Cylindrical Symmetry



A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

First, because of the symmetry, the electric field at any point must be radially outward (the charge is positive). So, the flux through each end cap is zero.

Second, To find the flux through the cylinder's curved surface, E and dA is in the same direction

$$\Phi = EA\cos\theta = E(2\pi rh)\cos 0 = E(2\pi rh).$$

Gauss' Law: $\varepsilon_0 \Phi = q_{\text{enc}},$

Replacing: $\varepsilon_0 E(2\pi rh) = \lambda h,$ λ *is line* charge density

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(line of charge)}.$$

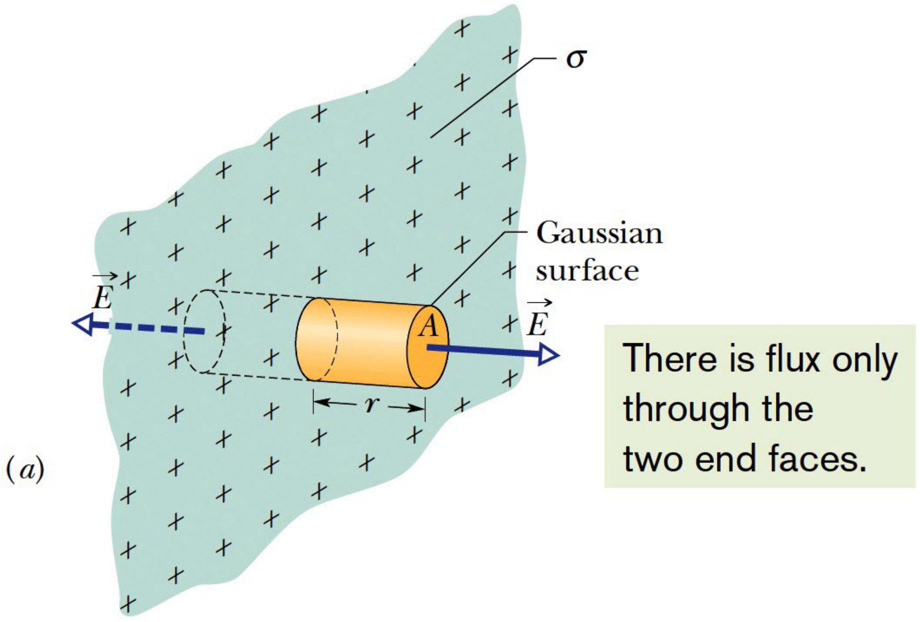
Table 22-1 Some Measures Charge

Name	Symbo
Charge	q
Linear charge density	λ
Surface charge density	σ
Volume charge density	ρ

23-5 Applying Gauss' Law: Planar Symmetry (3 of 7)

Non-conducting Sheet

Let us find the Electric field E a distance r in front of the sheet



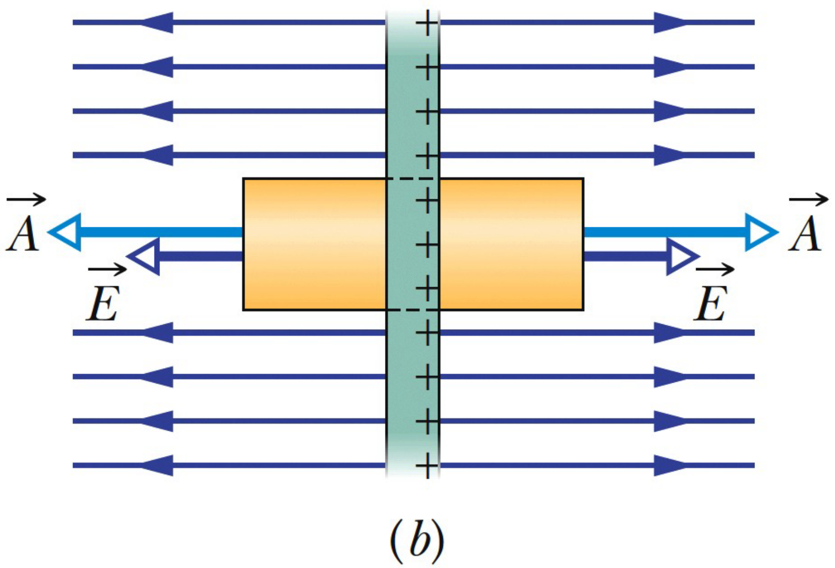
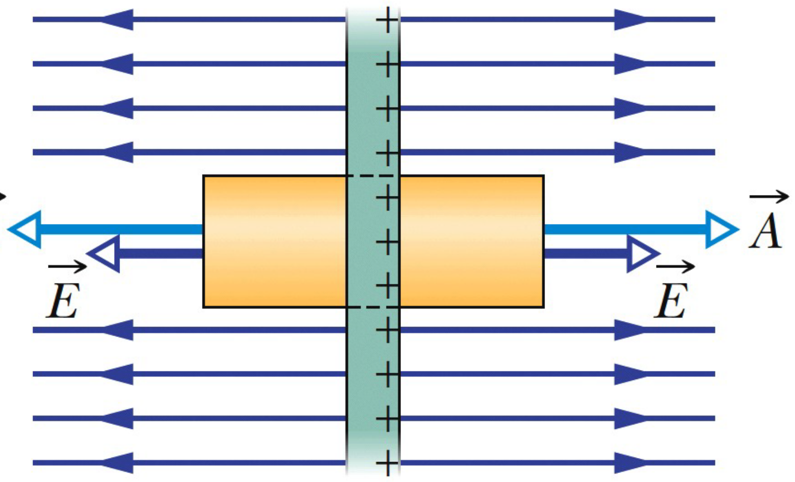


Figure 23-17 shows (*a*) Perspective view and (*b*) side view a portion of a thin, infinite, nonconducting sheet with a uniform (positive) surface charge density σ. A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

23-5 Applying Gauss' Law: Planar Symmetry (4 of 7)



(*b*)

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \qquad (\text{Gauss' law}).$$

becomes $\varepsilon_0 (EA + EA) = \sigma A,$

where σA is the charge enclosed by the Gaussian surface.

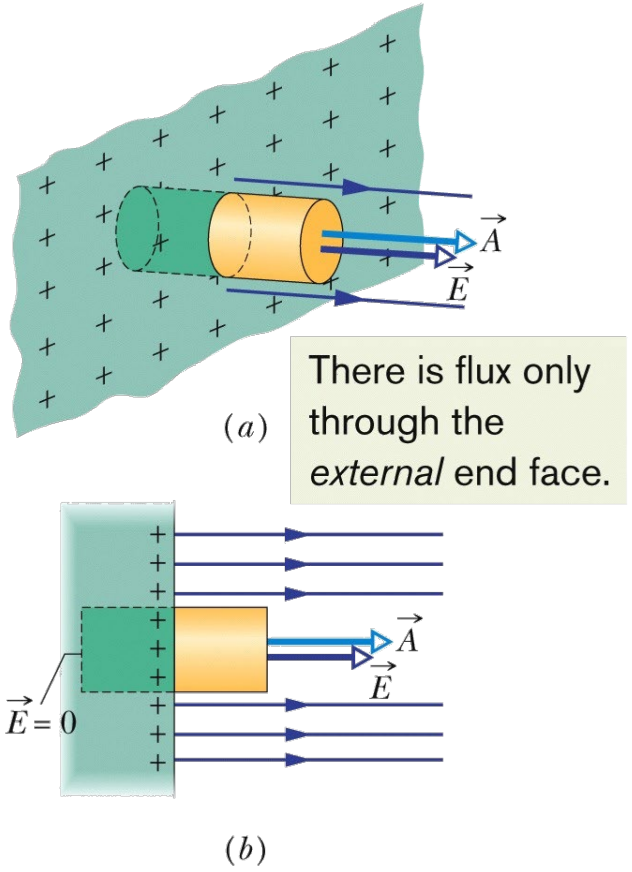
Hence $$E = \frac{\sigma}{2\varepsilon_0} \qquad (\text{sheet of charge}).$$

Non-conducting Sheet

Table 22-1 Some Measure: Charge

Name	Symbo
Charge	q
Linear charge density	λ
Surface charge density	σ
Volume charge density	ρ

23-3 A Charged Isolated Conductor (3 of 4)



One conducting Sheet

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \qquad (\text{Gauss' law}).$$

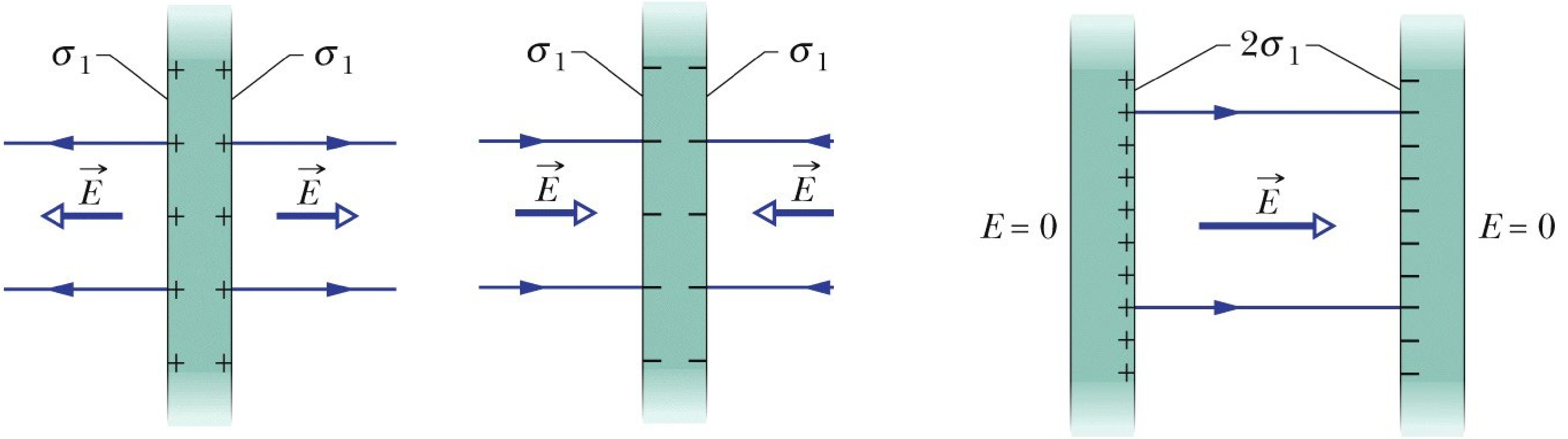
becomes $\varepsilon_0(EA) = \sigma A,$

$$E = \frac{\sigma}{\varepsilon_0} \qquad (\text{conducting surface}).$$

One conducting Sheet

23-5 Applying Gauss' Law: Planar Symmetry (7 of 7)

Two conducting Plates



$\sigma_1 = \sigma_1$ no external electric field, uniform surface charge density at two faces

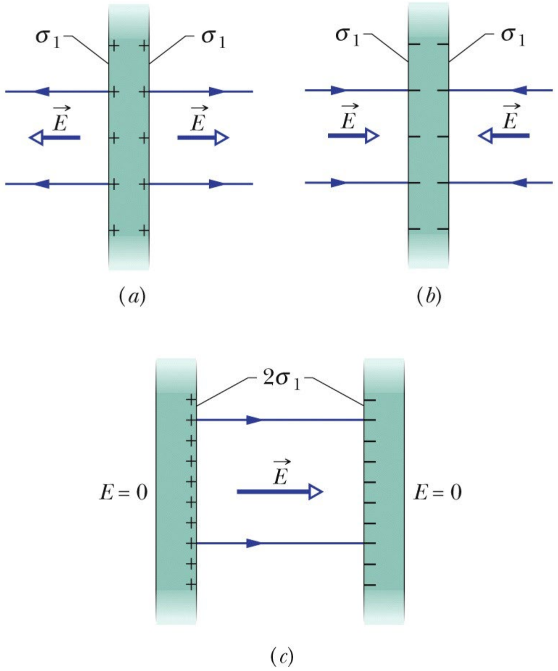
Figure (a) shows a cross section of a thin, infinite conducting plate with excess positive charge

Figure (b) shows an identical plate with excess negative charge having the same magnitude of surface charge density

the plates of **Figures. a** and **b** to be close to each other and parallel **(c)**.

23-5 Applying Gauss' Law: Planar Symmetry (7 of 7)

two conducting Sheet



$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \qquad (\text{Gauss' law}).$$

becomes $\varepsilon_0(EA) = 2\sigma_1 A,$

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

two conducting Sheet

23-6 Applying Gauss' Law: Spherical Symmetry

Lets use Gauss' law to prove the two shell theorems presented without proof in Coulomb's law

21-1 Coulomb's Law (15 of 16)

Multiple Forces: If multiple electrostatic forces act on a particle, the net force is the vector sum (not scalar sum) of the individual forces.

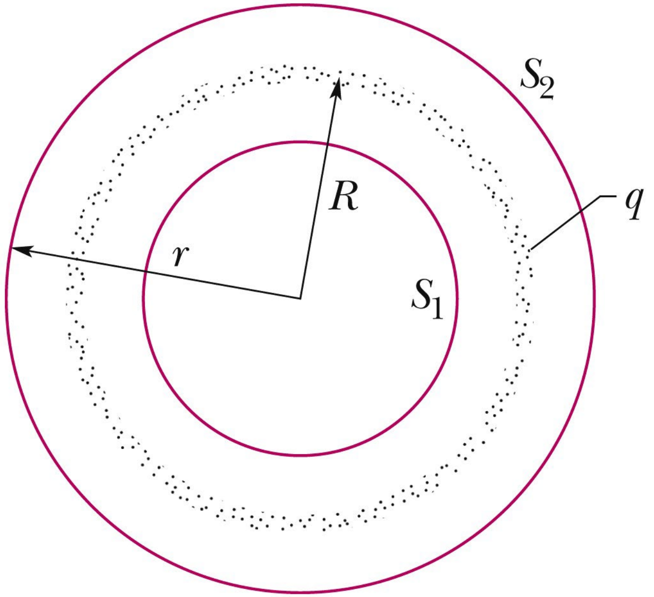
$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n},$$

Shell Theories: There are two shell theories for electrostatic force

Shell theory 1. A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

Shell theory 2. A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

23-6 Applying Gauss' Law: Spherical Symmetry (3 of 6)



A thin, uniformly charged, spherical shell with total charge q, in cross section. Two Gaussian surfaces S_1 and S_2 are also shown in cross section. Surface S_2 encloses the shell, and S_1 encloses only the empty interior of the shell.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \qquad (\text{Gauss' law}).$$

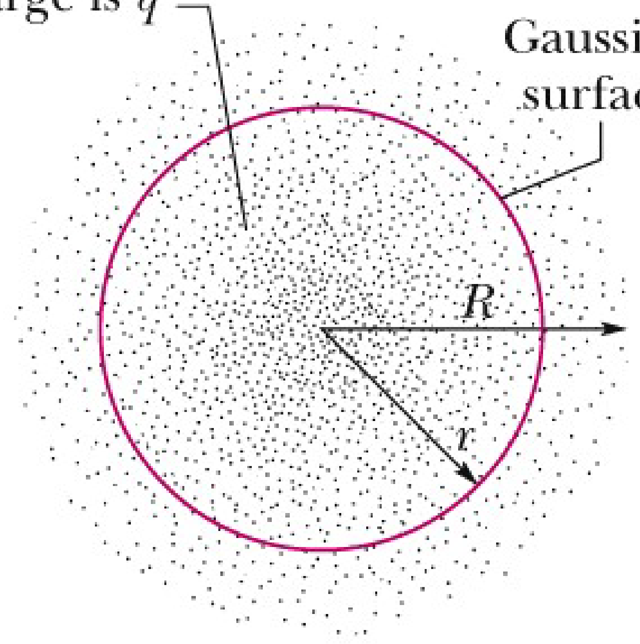
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad (\text{spherical shell, field at } r \geq R).$$

$$E = 0 \qquad (\text{spherical shell, field at } r < R), \qquad becasue\ \Phi = 0$$

Shell theory 1. A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

Shell theory 2. A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

23-6 Applying Gauss' Law: Spherical Symmetry (6 of 6)



The dots represent a spherically symmetric distribution of charge of radius R, whose volume charge density ρ is a function only of distance from the center. The charge is assumed to be fixed in position.

When r > R, treat it like a particle with charge q

$$E = \left(\frac{q}{4\pi\varepsilon_0 r^2}\right)$$

How to calculate the E when r < R?

we separately consider the charge inside it and the charge outside it.

But outside it is 0 because shell theory II.

Inside: q' represent that enclosed charge, volume charge density ρ *is same*

$$\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}. \quad \Rightarrow \quad E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r \qquad \text{(uniform charge, field at } r \le R\text{)}.$$

Summary (1 of 4)

Gauss' Law

- Gauss' law is

$$\varepsilon_0 \Phi = q_{\text{enc}}$$

Equation (23-6)

- the net flux of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

Equation (23-6)

Summary (2 of 4)

Applications of Gauss' Law

- surface of a charged conductor

$$E = \frac{\sigma}{\varepsilon_0}$$

Equation (23-11)

- Within the surface $E = 0$.
- line of charge

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Equation (23-12)

Summary (3 of 4)

- Infinite non-conducting sheet

$$E = \frac{\sigma}{2\varepsilon_0}$$

Equation (23-13)

- Outside a spherical shell of charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Equation (23-15)

Summary (4 of 4)

- Inside a uniform spherical shell

$$E = 0$$

Equation (23-16)

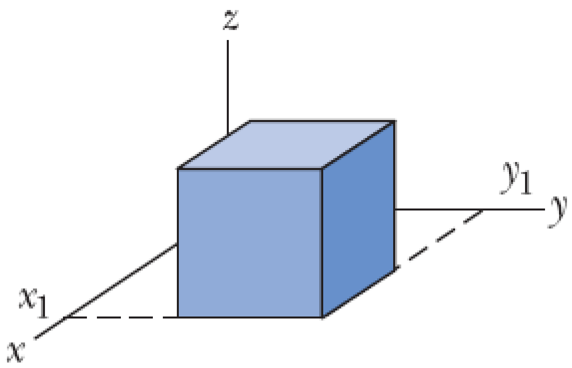
- Inside a uniform sphere of charge

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r.$$

Equation (23-20)

In-class quiz

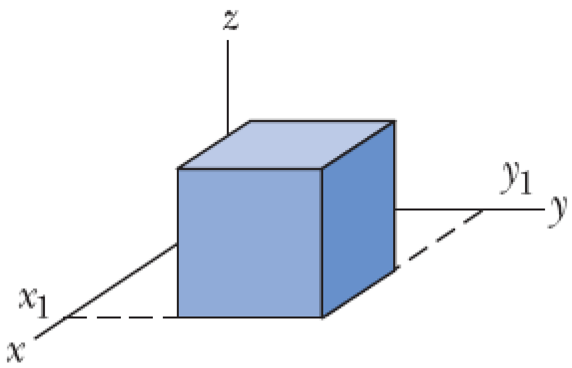
Figure shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m, with one corner at $x_1 = 5.00$ m, $y_1 = 4.00$ m. The cube lies in a region where the electric field vector is given by $\vec{E} = -3.00\hat{i} - 4.00y^2\hat{j} + 3.00\hat{k}$ N/C, with y in meters. What is the net charge contained by the cube?



In-class quiz

In-class quiz

Figure shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m, with one corner at $x_1 = 5.00$ m, $y_1 = 4.00$ m. The cube lies in a region where the electric field vector is given by $\vec{E} = -3.00\hat{i} - 4.00y^2\hat{j} + 3.00\hat{k}$ N/C, with y in meters. What is the net charge contained by the cube?



None of the constant terms will result in a nonzero contribution to the flux, so we focus on the x dependent term only:

$$E_{\text{nonconstant}} = (-4.00y^2)\,\mathrm{j}$$

$$E_{\text{nonconstant}} A = (-4)(4^2)(4) = -256\ \mathrm{N{\cdot}m/C^2}.$$

$$-E_{\text{nonconstant}} A = -\,(-4)(2^2)(4) = 64\ \mathrm{N{\cdot}m/C^2}.$$

Thus, the net flux is $\Phi = (-256 + 64)\ \mathrm{N{\cdot}m/C^2} = -192\ \mathrm{N{\cdot}m/C^2}$.

According to Gauss's law, we therefore have

$$q_{\text{enc}} = \varepsilon_0\Phi = (8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(-192\ \mathrm{N\cdot m^2/C}) = -1.70\times10^{-9}\,\mathrm{C}.$$

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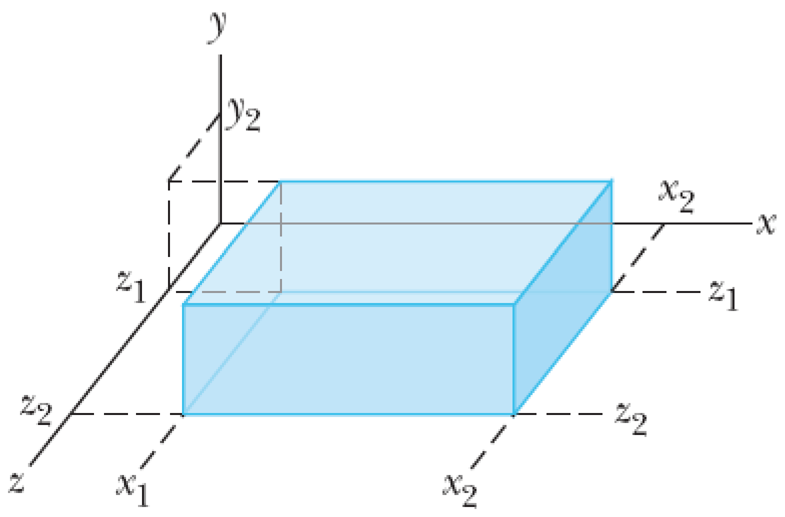
The box-like Gaussian surface shown in Fig. encloses a net charge of $+24.0\varepsilon_0$ C and lies in an electric field given by $\vec{E} = [(10.0 + 2.00x)\hat{i} - 3.00\hat{j} + bz\hat{k}]$ N/C, with x and z in meters and b a constant. The bottom face is in the xz plane; the top face is in the horizontal plane passing through $y_2 = 1.00$ m. For $x_1 = 1.00$ m, $x_2 = 4.00$ m, $z_1 = 1.00$ m, and $z_2 = 3.00$ m, what is b?



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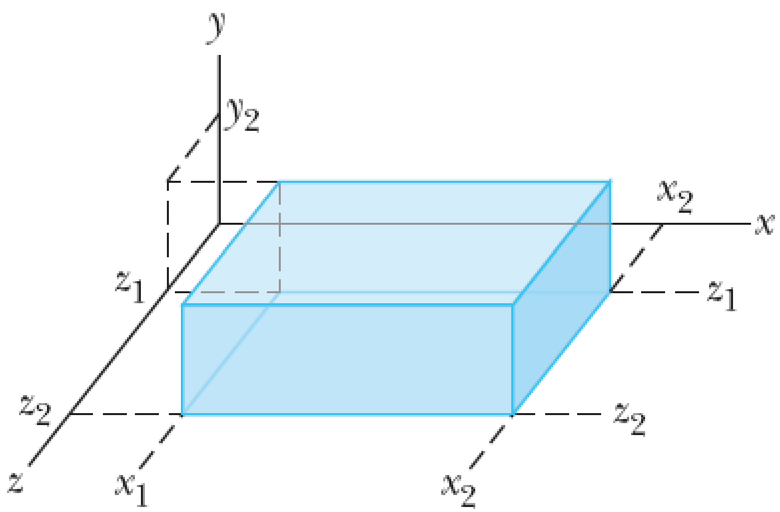

The total electric flux through the cube is

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

the yz plane

$$\Phi_{yz} = \iint [E_x(x = x_2) - E_x(x = x_1)]\,dydz = \int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz[10 + 2(4) - 10 - 2(1)]$$
$$= 6\int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz = 6(1)(2) = 12.$$

the xz plane

$$\Phi_{xz} = \iint \left[E_y(y = y_2) - E_y(y = y_1)\right] dxdz = \int_{x_1=1}^{x_2=4} dy \int_{z_1=1}^{z_2=3} dz[-3 - (-3)] = 0$$

the xy plane is

$$\Phi_{xy} = \iint [E_z(z = z_2) - E_z(z = z_1)]\,dxdy = \int_{x_1=1}^{x_2=4} dx \int_{y_1=0}^{y_2=1} dy\,(3b - b) = 2b(3)(1) = 6b.$$

$$q_{\text{enc}} = \varepsilon_0\Phi = \varepsilon_0(\Phi_{xy} + \Phi_{xz} + \Phi_{yz}) = \varepsilon_0(6.00b + 0 + 12.0) = 24.0\varepsilon_0$$

Hence $b = 2.00$