CHAPTER 28

# Magnetic Fields

## 28-1 MAGNETIC FIELDS AND THE DEFINITION OF $\vec{B}$

### Learning Objectives

*After reading this module, you should be able to . . .*

28.01 Distinguish an electromagnet from a permanent magnet.

28.02 Identify that a magnetic field is a vector quantity and thus has both magnitude and direction.

28.03 Explain how a magnetic field can be defined in terms of what happens to a charged particle moving through the field.

28.04 For a charged particle moving through a uniform magnetic field, apply the relationship between force magnitude $F_B$, charge $q$, speed $v$, field magnitude $B$, and the angle $\phi$ between the directions of the velocity vector $\vec{v}$ and the magnetic field vector $\vec{B}$.

28.05 For a charged particle sent through a uniform magnetic field, find the direction of the magnetic force $\vec{F}_B$ by (1) applying the right-hand rule to find the direction of the cross product $\vec{v} \times \vec{B}$ and (2) determining what effect the charge $q$ has on the direction.

28.06 Find the magnetic force $\vec{F}_B$ acting on a moving charged particle by evaluating the cross product $q(\vec{v} \times \vec{B})$ in unit-vector notation and magnitude-angle notation.

28.07 Identify that the magnetic force vector $\vec{F}_B$ must always be perpendicular to both the velocity vector $\vec{v}$ and the magnetic field vector $\vec{B}$.

28.08 Identify the effect of the magnetic force on the particle's speed and kinetic energy.

28.09 Identify a magnet as being a magnetic dipole.

28.10 Identify that opposite magnetic poles attract each other and like magnetic poles repel each other.

28.11 Explain magnetic field lines, including where they originate and terminate and what their spacing represents.

### Key Ideas

- When a charged particle moves through a magnetic field $\vec{B}$, a magnetic force acts on the particle as given by

$$\vec{F}_B = q(\vec{v} \times \vec{B}),$$

where $q$ is the particle's charge (sign included) and $\vec{v}$ is the particle's velocity.

- The right-hand rule for cross products gives the direction of $\vec{v} \times \vec{B}$. The sign of $q$ then determines whether $\vec{F}_B$ is in the same direction as $\vec{v} \times \vec{B}$ or in the opposite direction.

- The magnitude of $\vec{F}_B$ is given by

$$F_B = |q|vB \sin \phi,$$

where $\phi$ is the angle between $\vec{v}$ and $\vec{B}$.

## What Is Physics?

As we have discussed, one major goal of physics is the study of how an *electric field* can produce an *electric force* on a charged object. A closely related goal is the study of how a *magnetic field* can produce a *magnetic force* on a (moving) charged particle or on a magnetic object such as a magnet. You may already have a hint of what a magnetic field is if you have ever attached a note to a refrigerator door with a small magnet or accidentally erased a credit card by moving it near a magnet. The magnet acts on the door or credit card via its magnetic field.

The applications of magnetic fields and magnetic forces are countless and changing rapidly every year. Here are just a few examples. For decades, the entertainment industry depended on the magnetic recording of music and images on audiotape and videotape. Although digital technology has largely replaced

Digital Vision/Getty Images, Inc.

**Figure 28-1** Using an electromagnet to collect and transport scrap metal at a steel mill.

magnetic recording, the industry still depends on the magnets that control CD and DVD players and computer hard drives; magnets also drive the speaker cones in headphones, TVs, computers, and telephones. A modern car comes equipped with dozens of magnets because they are required in the motors for engine ignition, automatic window control, sunroof control, and windshield wiper control. Most security alarm systems, doorbells, and automatic door latches employ magnets. In short, you are surrounded by magnets.

The science of magnetic fields is physics; the application of magnetic fields is engineering. Both the science and the application begin with the question "What produces a magnetic field?"

## What Produces a Magnetic Field?

Because an electric field $\vec{E}$ is produced by an electric charge, we might reasonably expect that a magnetic field $\vec{B}$ is produced by a magnetic charge. Although individual magnetic charges (called *magnetic monopoles*) are predicted by certain theories, their existence has not been confirmed. How then are magnetic fields produced? There are two ways.

One way is to use moving electrically charged particles, such as a current in a wire, to make an **electromagnet.** The current produces a magnetic field that can be used, for example, to control a computer hard drive or to sort scrap metal (Fig. 28-1). In Chapter 29, we discuss the magnetic field due to a current.

The other way to produce a magnetic field is by means of elementary particles such as electrons because these particles have an *intrinsic* magnetic field around them. That is, the magnetic field is a basic characteristic of each particle just as mass and electric charge (or lack of charge) are basic characteristics. As we discuss in Chapter 32, the magnetic fields of the electrons in certain materials add together to give a net magnetic field around the material. Such addition is the reason why a **permanent magnet**, the type used to hang refrigerator notes, has a permanent magnetic field. In other materials, the magnetic fields of the electrons cancel out, giving no net magnetic field surrounding the material. Such cancellation is the reason you do not have a permanent field around your body, which is good because otherwise you might be slammed up against a refrigerator door every time you passed one.

Our first job in this chapter is to define the magnetic field $\vec{B}$. We do so by using the experimental fact that when a charged particle moves through a magnetic field, a magnetic force $\vec{F}_B$ acts on the particle.

## The Definition of $\vec{B}$

We determined the electric field $\vec{E}$ at a point by putting a test particle of charge $q$ at rest at that point and measuring the electric force $\vec{F}_E$ acting on the particle. We then defined $\vec{E}$ as

$$\vec{E} = \frac{\vec{F}_E}{q}. \tag{28-1}$$

If a magnetic monopole were available, we could define $\vec{B}$ in a similar way. Because such particles have not been found, we must define $\vec{B}$ in another way, in terms of the magnetic force $\vec{F}_B$ exerted on a moving electrically charged test particle.

***Moving Charged Particle.*** In principle, we do this by firing a charged particle through the point at which $\vec{B}$ is to be defined, using various directions and speeds for the particle and determining the force $\vec{F}_B$ that acts on the particle at that point. After many such trials we would find that when the particle's velocity

$\vec{v}$ is along a particular axis through the point, force $\vec{F}_B$ is zero. For all other directions of $\vec{v}$, the magnitude of $\vec{F}_B$ is always proportional to $v \sin \phi$, where $\phi$ is the angle between the zero-force axis and the direction of $\vec{v}$. Furthermore, the direction of $\vec{F}_B$ is always perpendicular to the direction of $\vec{v}$. (These results suggest that a cross product is involved.)

***The Field.*** We can then define a **magnetic field** $\vec{B}$ to be a vector quantity that is directed along the zero-force axis. We can next measure the magnitude of $\vec{F}_B$ when $\vec{v}$ is directed perpendicular to that axis and then define the magnitude of $\vec{B}$ in terms of that force magnitude:

$$B = \frac{F_B}{|q|v},$$

where $q$ is the charge of the particle.

We can summarize all these results with the following vector equation:

$$\vec{F}_B = q\vec{v} \times \vec{B}; \tag{28-2}$$

that is, the force $\vec{F}_B$ on the particle is equal to the charge $q$ times the cross product of its velocity $\vec{v}$ and the field $\vec{B}$ (all measured in the same reference frame). Using Eq. 3-24 for the cross product, we can write the magnitude of $\vec{F}_B$ as

$$F_B = |q|vB \sin \phi, \tag{28-3}$$

where $\phi$ is the angle between the directions of velocity $\vec{v}$ and magnetic field $\vec{B}$.

## Finding the Magnetic Force on a Particle

Equation 28-3 tells us that the magnitude of the force $\vec{F}_B$ acting on a particle in a magnetic field is proportional to the charge $q$ and speed $v$ of the particle. Thus, the force is equal to zero if the charge is zero or if the particle is stationary. Equation 28-3 also tells us that the magnitude of the force is zero if $\vec{v}$ and $\vec{B}$ are either parallel ($\phi = 0°$) or antiparallel ($\phi = 180°$), and the force is at its maximum when $\vec{v}$ and $\vec{B}$ are perpendicular to each other.

***Directions.*** Equation 28-2 tells us all this plus the direction of $\vec{F}_B$. From Module 3-3, we know that the cross product $\vec{v} \times \vec{B}$ in Eq. 28-2 is a vector that is perpendicular to the two vectors $\vec{v}$ and $\vec{B}$. The right-hand rule (Figs. 28-2*a* through *c*) tells us that the thumb of the right hand points in the direction of $\vec{v} \times \vec{B}$ when the fingers sweep $\vec{v}$ into $\vec{B}$. If $q$ is positive, then (by Eq. 28-2) the force $\vec{F}_B$ has the same sign as $\vec{v} \times \vec{B}$ and thus must be in the same direction; that is, for positive $q$, $\vec{F}_B$ is directed along the thumb (Fig. 28-2*d*). If $q$ is negative, then

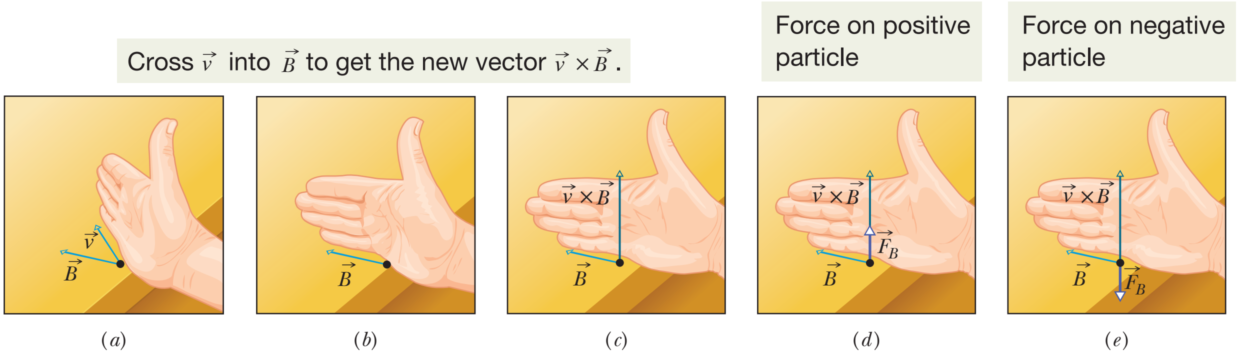



**Figure 28-2** (*a*)–(*c*) The right-hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle $\phi$ between them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb. (*d*) If $q$ is positive, then the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$. (*e*) If $q$ is negative, then the direction of $\vec{F}_B$ is opposite that of $\vec{v} \times \vec{B}$.

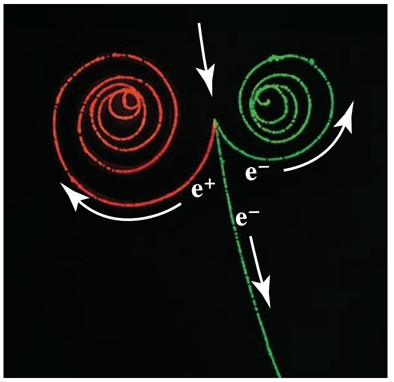
the force $\vec{F}_B$ and cross product $\vec{v} \times \vec{B}$ have opposite signs and thus must be in opposite directions. For negative $q$, $\vec{F}_B$ is directed opposite the thumb (Fig. 28-2*e*). *Heads up:* Neglect of this effect of negative $q$ is a very common error on exams.

Regardless of the sign of the charge, however,

> The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

Thus, $\vec{F}_B$ *never* has a component parallel to $\vec{v}$. This means that $\vec{F}_B$ cannot change the particle's speed $v$ (and thus it cannot change the particle's kinetic energy). The force can change only the direction of $\vec{v}$ (and thus the direction of travel); only in this sense can $\vec{F}_B$ accelerate the particle.

To develop a feeling for Eq. 28-2, consider Fig. 28-3, which shows some tracks left by charged particles moving rapidly through a *bubble chamber.* The chamber, which is filled with liquid hydrogen, is immersed in a strong uniform magnetic field that is directed out of the plane of the figure. An incoming gamma ray particle — which leaves no track because it is uncharged — transforms into an electron (spiral track marked $e^-$) and a positron (track marked $e^+$) while it knocks an electron out of a hydrogen atom (long track marked $e^-$). Check with Eq. 28-2 and Fig. 28-2 that the three tracks made by these two negative particles and one positive particle curve in the proper directions.




Lawrence Berkeley Laboratory/Science Source

**Figure 28-3** The tracks of two electrons ($e^-$) and a positron ($e^+$) in a bubble chamber that is immersed in a uniform magnetic field that is directed out of the plane of the page.

***Unit.*** The SI unit for $\vec{B}$ that follows from Eqs. 28-2 and 28-3 is the newton per coulomb-meter per second. For convenience, this is called the **tesla** (T):

$$1 \text{ tesla} = 1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})}.$$

Recalling that a coulomb per second is an ampere, we have

$$1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb/second})(\text{meter})} = 1 \frac{\text{N}}{\text{A} \cdot \text{m}}. \qquad (28\text{-}4)$$

An earlier (non-SI) unit for $\vec{B}$, still in common use, is the *gauss* (G), and

$$1 \text{ tesla} = 10^4 \text{ gauss}. \qquad (28\text{-}5)$$

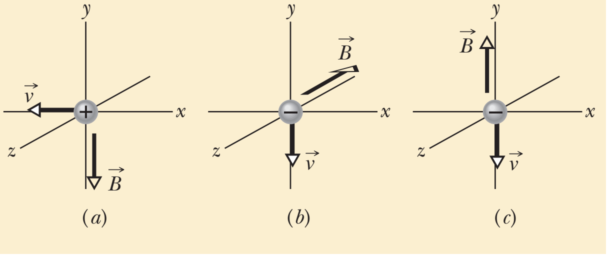
Table 28-1 lists the magnetic fields that occur in a few situations. Note that Earth's magnetic field near the planet's surface is about $10^{-4}$ T (= 100 $\mu$T or 1 G).

**Table 28-1** Some Approximate Magnetic Fields

| | |
|---|---|
| At surface of neutron star | $10^8$ T |
| Near big electromagnet | 1.5 T |
| Near small bar magnet | $10^{-2}$ T |
| At Earth's surface | $10^{-4}$ T |
| In interstellar space | $10^{-10}$ T |
| Smallest value in magnetically shielded room | $10^{-14}$ T |

### Checkpoint 1

The figure shows three situations in which a charged particle with velocity $\vec{v}$ travels through a uniform magnetic field $\vec{B}$. In each situation, what is the direction of the magnetic force $\vec{F}_B$ on the particle?




## Magnetic Field Lines

We can represent magnetic fields with field lines, as we did for electric fields. Similar rules apply: (1) the direction of the tangent to a magnetic field line at any point gives the direction of $\vec{B}$ at that point, and (2) the spacing of the lines represents the magnitude of $\vec{B}$ — the magnetic field is stronger where the lines are closer together, and conversely.

Figure 28-4*a* shows how the magnetic field near a *bar magnet* (a permanent magnet in the shape of a bar) can be represented by magnetic field lines. The lines all pass through the magnet, and they all form closed loops (even those that are not shown closed in the figure). The external magnetic effects of a bar magnet are strongest near its ends, where the field lines are most closely spaced. Thus, the magnetic field of the bar magnet in Fig. 28-4*b* collects the iron filings mainly near the two ends of the magnet.

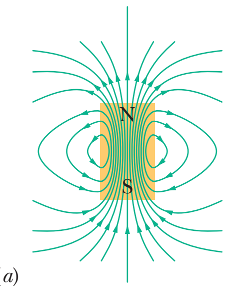
***Two Poles.*** The (closed) field lines enter one end of a magnet and exit the other end. The end of a magnet from which the field lines emerge is called the *north pole* of the magnet; the other end, where field lines enter the magnet, is called the *south pole.* Because a magnet has two poles, it is said to be a **magnetic dipole.** The magnets we use to fix notes on refrigerators are short bar magnets. Figure 28-5 shows two other common shapes for magnets: a *horseshoe magnet* and a magnet that has been bent around into the shape of a **C** so that the *pole faces* are facing each other. (The magnetic field between the pole faces can then be approximately uniform.) Regardless of the shape of the magnets, if we place two of them near each other we find:

> Opposite magnetic poles attract each other, and like magnetic poles repel each other.

When you hold two magnets near each other with your hands, this attraction or repulsion seems almost magical because there is no contact between the two to visibly justify the pulling or pushing. As we did with the electrostatic force between two charged particles, we explain this noncontact force in terms of a field that you cannot see, here the magnetic field.

Earth has a magnetic field that is produced in its core by still unknown mechanisms. On Earth's surface, we can detect this magnetic field with a compass, which is essentially a slender bar magnet on a low-friction pivot. This bar magnet, or this needle, turns because its north-pole end is attracted toward the Arctic region of Earth. Thus, the *south* pole of Earth's magnetic field must be located toward the Arctic. Logically, we then should call the pole there a south pole. However, because we call that direction north, we are trapped into the statement that Earth has a *geomagnetic north pole* in that direction.

With more careful measurement we would find that in the Northern Hemisphere, the magnetic field lines of Earth generally point down into Earth and toward the Arctic. In the Southern Hemisphere, they generally point up out of Earth and away from the Antarctic — that is, away from Earth's *geomagnetic south pole.*

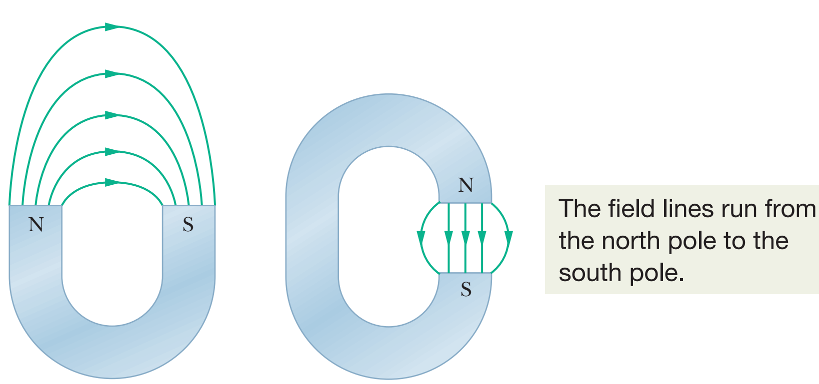



(*a*)

(*b*)

Courtesy Dr. Richard Cannon, Southeast Missouri State University, Cape Girardeau

**Figure 28-4** (*a*) The magnetic field lines for a bar magnet. (*b*) A "cow magnet" — a bar magnet that is intended to be slipped down into the rumen of a cow to prevent accidentally ingested bits of scrap iron from reaching the cow's intestines. The iron filings at its ends reveal the magnetic field lines.




(*a*) (*b*)

**Figure 28-5** (*a*) A horseshoe magnet and (*b*) a **C**-shaped magnet. (Only some of the external field lines are shown.)

### Sample Problem 28.01 Magnetic force on a moving charged particle

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is $1.67 \times 10^{-27}$ kg. (Neglect Earth's magnetic field.)

#### KEY IDEAS

Because the proton is charged and moving through a magnetic field, a magnetic force $\vec{F}_B$ can act on it. Because the initial direction of the proton's velocity is not along a magnetic field line, $\vec{F}_B$ is not simply zero.

***Magnitude:*** To find the magnitude of $\vec{F}_B$, we can use Eq. 28-3 ($F_B = |q|vB \sin \phi$) provided we first find the proton's speed $v$. We can find $v$ from the given kinetic energy because $K = \frac{1}{2}mv^2$. Solving for $v$, we obtain

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{1.67 \times 10^{-27} \text{ kg}}}$$
$$= 3.2 \times 10^7 \text{ m/s}.$$

Equation 28-3 then yields

$$\begin{aligned} F_B &= |q|vB \sin \phi \\ &= (1.60 \times 10^{-19} \text{ C})(3.2 \times 10^7 \text{ m/s}) \\ &\quad \times (1.2 \times 10^{-3} \text{ T})(\sin 90°) \\ &= 6.1 \times 10^{-15} \text{ N}. \end{aligned} \qquad \text{(Answer)}$$

This may seem like a small force, but it acts on a particle of small mass, producing a large acceleration; namely,

$$a = \frac{F_B}{m} = \frac{6.1 \times 10^{-15} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 3.7 \times 10^{12} \text{ m/s}^2.$$

***Direction:*** To find the direction of $\vec{F}_B$, we use the fact that $\vec{F}_B$ has the direction of the cross product $q\vec{v} \times \vec{B}$. Because the charge $q$ is positive, $\vec{F}_B$ must have the same direction as $\vec{v} \times \vec{B}$, which can be determined with the right-hand rule for cross products (as in Fig. 28-2*d*). We know that $\vec{v}$ is directed horizontally from south to north and $\vec{B}$ is directed vertically up. The right-hand rule shows us that the deflecting force $\vec{F}_B$ must be directed horizontally from west to east, as Fig. 28-6 shows. (The array of dots in the figure represents a magnetic field directed out of the plane of the figure. An array of **X**s would have represented a magnetic field directed into that plane.)

If the charge of the particle were negative, the magnetic deflecting force would be directed in the opposite direction — that is, horizontally from east to west. This is predicted automatically by Eq. 28-2 if we substitute a negative value for $q$.

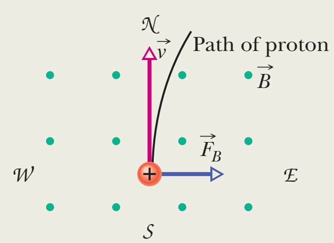



**Figure 28-6** An overhead view of a proton moving from south to north with velocity $\vec{v}$ in a chamber. A magnetic field is directed vertically upward in the chamber, as represented by the array of dots (which resemble the tips of arrows). The proton is deflected toward the east.

Additional examples, video, and practice available at *WileyPLUS*

# 28-2 CROSSED FIELDS: DISCOVERY OF THE ELECTRON

## Learning Objectives

*After reading this module, you should be able to . . .*

**28.12** Describe the experiment of J. J. Thomson.

**28.13** For a charged particle moving through a magnetic field and an electric field, determine the net force on the particle in both magnitude-angle notation and unit-vector notation.

**28.14** In situations where the magnetic force and electric force on a particle are in opposite directions, determine the speeds at which the forces cancel, the magnetic force dominates, and the electric force dominates.

## Key Ideas

- If a charged particle moves through a region containing both an electric field and a magnetic field, it can be affected by both an electric force and a magnetic force.
- If the fields are perpendicular to each other, they are said to be *crossed fields*.
- If the forces are in opposite directions, a particular speed will result in no deflection of the particle.

## Crossed Fields: Discovery of the Electron

Both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ can produce a force on a charged particle. When the two fields are perpendicular to each other, they are said to be *crossed fields.* Here we shall examine what happens to charged particles—namely, electrons—as they move through crossed fields. We use as our example the experiment that led to the discovery of the electron in 1897 by J. J. Thomson at Cambridge University.

***Two Forces.*** Figure 28-7 shows a modern, simplified version of Thomson's experimental apparatus—a *cathode ray tube* (which is like the picture tube in an old-type television set). Charged particles (which we now know as electrons) are emitted by a hot filament at the rear of the evacuated tube and are accelerated by an applied potential difference $V$. After they pass through a slit in screen C, they form a narrow beam. They then pass through a region of crossed $\vec{E}$ and $\vec{B}$ fields, headed toward a fluorescent screen S, where they produce a spot of light (on a television screen the spot is part of the picture). The forces on the charged particles in the crossed-fields region can deflect them from the center of the screen. By controlling the magnitudes and directions of the fields, Thomson could thus control where the spot of light appeared on the screen. Recall that the force on a negatively charged particle due to an electric field is directed opposite the field. Thus, for the arrangement of Fig. 28-7, electrons are forced up the page by electric field $\vec{E}$ and down the page by magnetic field $\vec{B}$; that is, the forces are *in opposition.* Thomson's procedure was equivalent to the following series of steps.

1. Set $E = 0$ and $B = 0$ and note the position of the spot on screen S due to the undeflected beam.
2. Turn on $\vec{E}$ and measure the resulting beam deflection.
3. Maintaining $\vec{E}$, now turn on $\vec{B}$ and adjust its value until the beam returns to the undeflected position. (With the forces in opposition, they can be made to cancel.)

We discussed the deflection of a charged particle moving through an electric field $\vec{E}$ between two plates (step 2 here) in Sample Problem 22.04. We found that the deflection of the particle at the far end of the plates is

$$y = \frac{|q|EL^2}{2mv^2}, \tag{28-6}$$

where $v$ is the particle's speed, $m$ its mass, and $q$ its charge, and $L$ is the length of the plates. We can apply this same equation to the beam of electrons in Fig. 28-7; if need be, we can calculate the deflection by measuring the deflection of the beam on screen S and then working back to calculate the deflection $y$ at the end of the plates. (Because the direction of the deflection is set by the sign of the particle's charge, Thomson was able to show that the particles that were lighting up his screen were negatively charged.)

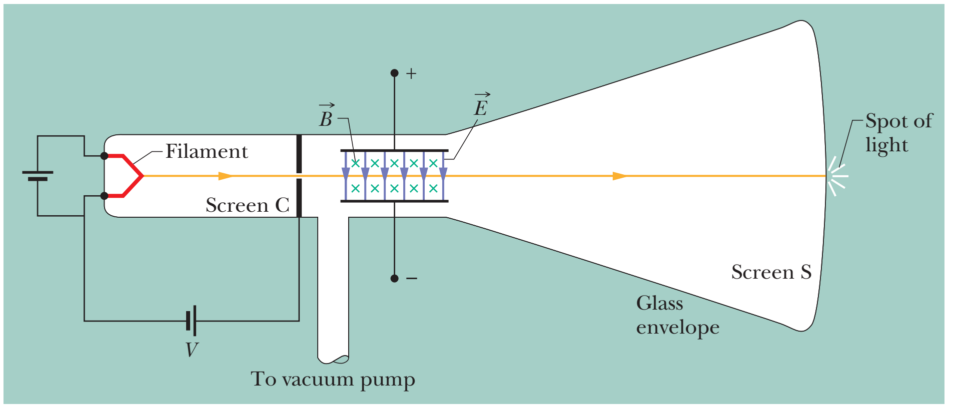



**Figure 28-7** A modern version of J. J. Thomson's apparatus for measuring the ratio of mass to charge for the electron. An electric field $\vec{E}$ is established by connecting a battery across the deflecting-plate terminals. The magnetic field $\vec{B}$ is set up by means of a current in a system of coils (not shown). The magnetic field shown is into the plane of the figure, as represented by the array of Xs (which resemble the feathered ends of arrows).

***Canceling Forces.*** When the two fields in Fig. 28-7 are adjusted so that the two deflecting forces cancel (step 3), we have from Eqs. 28-1 and 28-3

$$|q|E = |q|vB\sin(90°) = |q|vB$$

or
$$v = \frac{E}{B} \quad \text{(opposite forces canceling).} \tag{28-7}$$

Thus, the crossed fields allow us to measure the speed of the charged particles passing through them. Substituting Eq. 28-7 for $v$ in Eq. 28-6 and rearranging yield

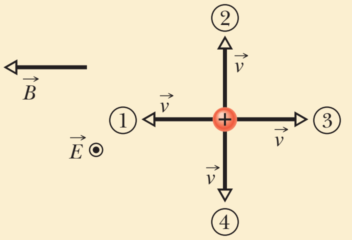
$$\frac{m}{|q|} = \frac{B^2L^2}{2yE}, \tag{28-8}$$

in which all quantities on the right can be measured. Thus, the crossed fields allow us to measure the ratio $m/|q|$ of the particles moving through Thomson's apparatus. (*Caution:* Equation 28-7 applies only when the electric and magnetic forces are in opposite directions. You might see other situations in the homework problems.)

Thomson claimed that these particles are found in all matter. He also claimed that they are lighter than the lightest known atom (hydrogen) by a factor of more than 1000. (The exact ratio proved later to be 1836.15.) His $m/|q|$ measurement, coupled with the boldness of his two claims, is considered to be the "discovery of the electron."

**Checkpoint 2**

The figure shows four directions for the velocity vector $\vec{v}$ of a positively charged particle moving through a uniform electric field $\vec{E}$ (directed out of the page and represented with an encircled dot) and a uniform magnetic field $\vec{B}$. (a) Rank directions 1, 2, and 3 according to the magnitude of the net force on the particle, greatest first. (b) Of all four directions, which might result in a net force of zero?




# 28-3 CROSSED FIELDS: THE HALL EFFECT

## Learning Objectives

*After reading this module, you should be able to . . .*

**28.15** Describe the Hall effect for a metal strip carrying current, explaining how the electric field is set up and what limits its magnitude.

**28.16** For a conducting strip in a Hall-effect situation, draw the vectors for the magnetic field and electric field. For the conduction electrons, draw the vectors for the velocity, magnetic force, and electric force.

**28.17** Apply the relationship between the Hall potential difference $V$, the electric field magnitude $E$, and the width of the strip $d$.

**28.18** Apply the relationship between charge-carrier number density $n$, magnetic field magnitude $B$, current $i$, and Hall-effect potential difference $V$.

**28.19** Apply the Hall-effect results to a conducting object moving through a uniform magnetic field, identifying the width across which a Hall-effect potential difference $V$ is set up and calculating $V$.

## Key Ideas

- When a uniform magnetic field $\vec{B}$ is applied to a conducting strip carrying current $i$, with the field perpendicular to the direction of the current, a Hall-effect potential difference $V$ is set up across the strip.
- The electric force $\vec{F}_E$ on the charge carriers is then balanced by the magnetic force $\vec{F}_B$ on them.
- The number density $n$ of the charge carriers can then be determined from

$$n = \frac{Bi}{Vle},$$

where $l$ is the thickness of the strip (parallel to $\vec{B}$).
- When a conductor moves through a uniform magnetic field $\vec{B}$ at speed $v$, the Hall-effect potential difference $V$ across it is

$$V = vBd,$$

where $d$ is the width perpendicular to both velocity $\vec{v}$ and field $\vec{B}$.

## Crossed Fields: The Hall Effect

As we just discussed, a beam of electrons in a vacuum can be deflected by a magnetic field. Can the drifting conduction electrons in a copper wire also be deflected by a magnetic field? In 1879, Edwin H. Hall, then a 24-year-old graduate student at the Johns Hopkins University, showed that they can. This **Hall effect** allows us to find out whether the charge carriers in a conductor are positively or negatively charged. Beyond that, we can measure the number of such carriers per unit volume of the conductor.

Figure 28-8*a* shows a copper strip of width $d$, carrying a current $i$ whose conventional direction is from the top of the figure to the bottom. The charge carriers are electrons and, as we know, they drift (with drift speed $v_d$) in the opposite direction, from bottom to top. At the instant shown in Fig. 28-8*a*, an external magnetic field $\vec{B}$, pointing into the plane of the figure, has just been turned on. From Eq. 28-2 we see that a magnetic deflecting force $\vec{F}_B$ will act on each drifting electron, pushing it toward the right edge of the strip.

As time goes on, electrons move to the right, mostly piling up on the right edge of the strip, leaving uncompensated positive charges in fixed positions at the left edge. The separation of positive charges on the left edge and negative charges on the right edge produces an electric field $\vec{E}$ within the strip, pointing from left to right in Fig. 28-8*b*. This field exerts an electric force $\vec{F}_E$ on each electron, tending to push it to the left. Thus, this electric force on the electrons, which opposes the magnetic force on them, begins to build up.

***Equilibrium.*** An equilibrium quickly develops in which the electric force on each electron has increased enough to match the magnetic force. When this happens, as Fig. 28-8*b* shows, the force due to $\vec{B}$ and the force due to $\vec{E}$ are in balance. The drifting electrons then move along the strip toward the top of the page at velocity $\vec{v}_d$ with no further collection of electrons on the right edge of the strip and thus no further increase in the electric field $\vec{E}$.

A *Hall potential difference V* is associated with the electric field across strip width $d$. From Eq. 24-21, the magnitude of that potential difference is

$$V = Ed. \tag{28-9}$$

By connecting a voltmeter across the width, we can measure the potential difference between the two edges of the strip. Moreover, the voltmeter can tell us which edge is at higher potential. For the situation of Fig. 28-8*b*, we would find that the left edge is at higher potential, which is consistent with our assumption that the charge carriers are negatively charged.

For a moment, let us make the opposite assumption, that the charge carriers in current $i$ are positively charged (Fig. 28-8*c*). Convince yourself that as these charge carriers move from top to bottom in the strip, they are pushed to the right edge by $\vec{F}_B$ and thus that the *right* edge is at higher potential. Because that last statement is contradicted by our voltmeter reading, the charge carriers must be negatively charged.

***Number Density.*** Now for the quantitative part. When the electric and magnetic forces are in balance (Fig. 28-8*b*), Eqs. 28-1 and 28-3 give us

$$eE = ev_dB. \tag{28-10}$$

From Eq. 26-7, the drift speed $v_d$ is

$$v_d = \frac{J}{ne} = \frac{i}{neA}, \tag{28-11}$$

in which $J$ (= $i/A$) is the current density in the strip, $A$ is the cross-sectional area of the strip, and $n$ is the *number density* of charge carriers (number per unit volume).

In Eq. 28-10, substituting for $E$ with Eq. 28-9 and substituting for $v_d$ with Eq. 28-11, we obtain

$$n = \frac{Bi}{Vle}, \tag{28-12}$$

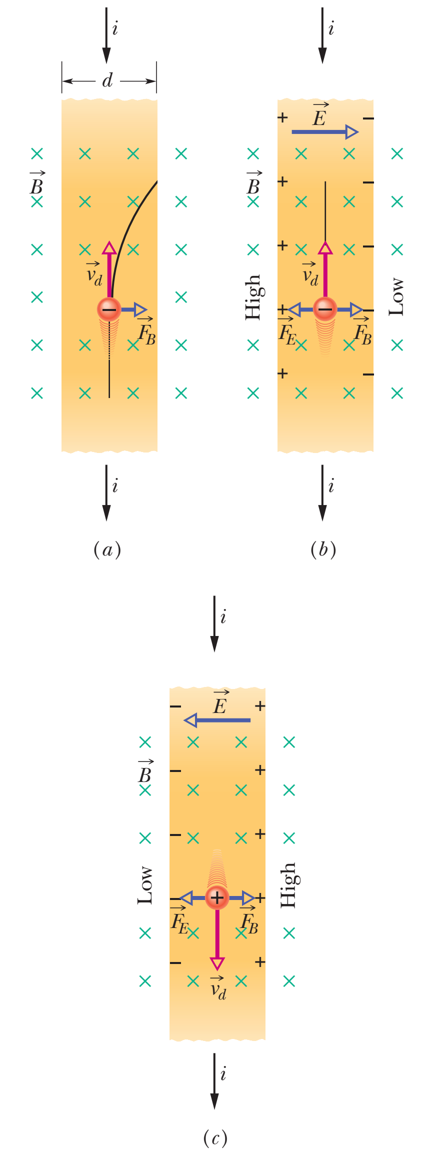



**Figure 28-8** A strip of copper carrying a current $i$ is immersed in a magnetic field $\vec{B}$. (*a*) The situation immediately after the magnetic field is turned on. The curved path that will then be taken by an electron is shown. (*b*) The situation at equilibrium, which quickly follows. Note that negative charges pile up on the right side of the strip, leaving uncompensated positive charges on the left. Thus, the left side is at a higher potential than the right side. (*c*) For the same current direction, if the charge carriers were positively charged, *they* would pile up on the right side, and the right side would be at the higher potential.

in which $l$ $(= A/d)$ is the thickness of the strip. With this equation we can find $n$ from measurable quantities.

***Drift Speed.*** It is also possible to use the Hall effect to measure directly the drift speed $v_d$ of the charge carriers, which you may recall is of the order of centimeters per hour. In this clever experiment, the metal strip is moved mechanically through the magnetic field in a direction opposite that of the drift velocity of the charge carriers. The speed of the moving strip is then adjusted until the Hall potential difference vanishes. At this condition, with no Hall effect, the velocity of the charge carriers *with respect to the laboratory frame* must be zero, so the velocity of the strip must be equal in magnitude but opposite the direction of the velocity of the negative charge carriers.

***Moving Conductor.*** When a conductor begins to move at speed $v$ through a magnetic field, its conduction electrons do also. They are then like the moving conduction electrons in the current in Figs. 28-8*a* and *b*, and an electric field $\vec{E}$ and potential difference $V$ are quickly set up. As with the current, equilibrium of the electric and magnetic forces is established, but we now write that condition in terms of the conductor's speed $v$ instead of the drift speed $v_d$ in a current as we did in Eq. 28-10:

$$eE = evB.$$

Substituting for $E$ with Eq. 28-9, we find that the potential difference is

$$V = vBd. \tag{28-13}$$

Such a motion-caused circuit potential difference can be of serious concern in some situations, such as when a conductor in an orbiting satellite moves through Earth's magnetic field. However, if a conducting line (said to be an *electrodynamic tether*) dangles from the satellite, the potential produced along the line might be used to maneuver the satellite.

## Sample Problem 28.02 Potential difference set up across a moving conductor

Figure 28-9*a* shows a solid metal cube, of edge length $d = 1.5$ cm, moving in the positive $y$ direction at a constant velocity $\vec{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\vec{B}$ of magnitude 0.050 T in the positive $z$ direction.

(a) Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

### KEY IDEA

Because the cube is moving through a magnetic field $\vec{B}$, a magnetic force $\vec{F}_B$ acts on its charged particles, including its conduction electrons.

*Reasoning:* When the cube first begins to move through the magnetic field, its electrons do also. Because each electron has charge $q$ and is moving through a magnetic field with velocity $\vec{v}$, the magnetic force $\vec{F}_B$ acting on the electron is given by Eq. 28-2. Because $q$ is negative, the direction of $\vec{F}_B$ is opposite the cross product $\vec{v} \times \vec{B}$, which is in the positive direction of the $x$ axis (Fig. 28-9*b*). Thus, $\vec{F}_B$ acts in the negative direction of the $x$ axis, toward the left face of the cube (Fig. 28-9*c*).

Most of the electrons are fixed in place in the atoms of the cube. However, because the cube is a metal, it contains conduction electrons that are free to move. Some of those conduction electrons are deflected by $\vec{F}_B$ to the left cube face, making that face negatively charged and leaving the right face positively charged (Fig. 28-9*d*). This charge separation produces an electric field $\vec{E}$ directed from the positively charged right face to the negatively charged left face (Fig. 28-9*e*). Thus, the left face is at a lower electric potential, and the right face is at a higher electric potential.

(b) What is the potential difference between the faces of higher and lower electric potential?

### KEY IDEAS

1. The electric field $\vec{E}$ created by the charge separation produces an electric force $\vec{F}_E = q\vec{E}$ on each electron

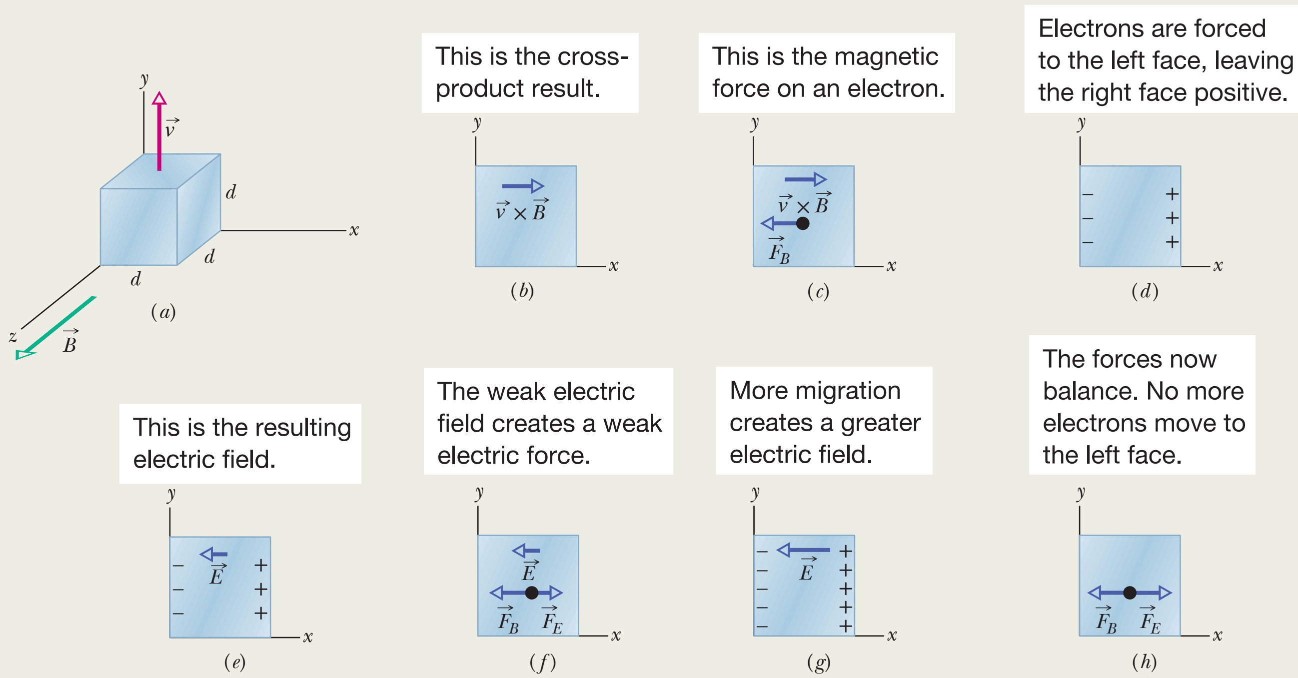



**Figure 28-9** (*a*) A solid metal cube moves at constant velocity through a uniform magnetic field. (*b*)–(*d*) In these front views, the magnetic force acting on an electron forces the electron to the left face, making that face negative and leaving the opposite face positive. (*e*)–(*f*) The resulting weak electric field creates a weak electric force on the next electron, but it too is forced to the left face. Now (*g*) the electric field is stronger and (*h*) the electric force matches the magnetic force.

(Fig. 28-9*f*). Because $q$ is negative, this force is directed opposite the field $\vec{E}$ — that is, rightward. Thus on each electron, $\vec{F}_E$ acts toward the right and $\vec{F}_B$ acts toward the left.

2. When the cube had just begun to move through the magnetic field and the charge separation had just begun, the magnitude of $\vec{E}$ began to increase from zero. Thus, the magnitude of $\vec{F}_E$ also began to increase from zero and was initially smaller than the magnitude of $\vec{F}_B$. During this early stage, the net force on any electron was dominated by $\vec{F}_B$, which continuously moved additional electrons to the left cube face, increasing the charge separation between the left and right cube faces (Fig. 28-9*g*).
3. However, as the charge separation increased, eventually magnitude $F_E$ became equal to magnitude $F_B$ (Fig. 28-9*h*). Because the forces were in opposite directions, the net force on any electron was then zero, and no additional electrons were moved to the left cube face. Thus, the magnitude of $\vec{F}_E$ could not increase further, and the electrons were then in equilibrium.

***Calculations:*** We seek the potential difference $V$ between the left and right cube faces after equilibrium was reached (which occurred quickly). We can obtain $V$ with Eq. 28-9 ($V = Ed$) provided we first find the magnitude $E$ of the electric field at equilibrium. We can do so with the equation for the balance of forces ($F_E = F_B$).

For $F_E$, we substitute $|q|E$, and then for $F_B$, we substitute $|q|vB \sin \phi$ from Eq. 28-3. From Fig. 28-9*a*, we see that the angle $\phi$ between velocity vector $\vec{v}$ and magnetic field vector $\vec{B}$ is 90°; thus $\sin \phi = 1$ and $F_E = F_B$ yields

$$|q|E = |q|vB \sin 90° = |q|vB.$$

This gives us $E = vB$; so $V = Ed$ becomes

$$V = vBd.$$

Substituting known values tells us that the potential difference between the left and right cube faces is

$$V = (4.0 \text{ m/s})(0.050 \text{ T})(0.015 \text{ m})$$
$$= 0.0030 \text{ V} = 3.0 \text{ mV}. \qquad \text{(Answer)}$$

# 28-4 A CIRCULATING CHARGED PARTICLE

## Learning Objectives

*After reading this module, you should be able to . . .*

**28.20** For a charged particle moving through a uniform magnetic field, identify under what conditions it will travel in a straight line, in a circular path, and in a helical path.

**28.21** For a charged particle in uniform circular motion due to a magnetic force, start with Newton's second law and derive an expression for the orbital radius $r$ in terms of the field magnitude $B$ and the particle's mass $m$, charge magnitude $q$, and speed $v$.

**28.22** For a charged particle moving along a circular path in a uniform magnetic field, calculate and relate speed, centripetal force, centripetal acceleration, radius, period, frequency, and angular frequency, and identify which of the quantities do not depend on speed.

**28.23** For a positive particle and a negative particle moving along a circular path in a uniform magnetic field, sketch the path and indicate the magnetic field vector, the velocity vector, the result of the cross product of the velocity and field vectors, and the magnetic force vector.

**28.24** For a charged particle moving in a helical path in a magnetic field, sketch the path and indicate the magnetic field, the pitch, the radius of curvature, the velocity component parallel to the field, and the velocity component perpendicular to the field.

**28.25** For helical motion in a magnetic field, apply the relationship between the radius of curvature and one of the velocity components.

**28.26** For helical motion in a magnetic field, identify pitch $p$ and relate it to one of the velocity components.

## Key Ideas

- A charged particle with mass $m$ and charge magnitude $|q|$ moving with velocity $\vec{v}$ perpendicular to a uniform magnetic field $\vec{B}$ will travel in a circle.
- Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r},$$

from which we find the radius $r$ of the circle to be

$$r = \frac{mv}{|q|B}.$$

- The frequency of revolution $f$, the angular frequency $\omega$, and the period of the motion $T$ are given by

$$f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{|q|B}{2\pi m}.$$

- If the velocity of the particle has a component parallel to the magnetic field, the particle moves in a helical path about field vector $\vec{B}$.

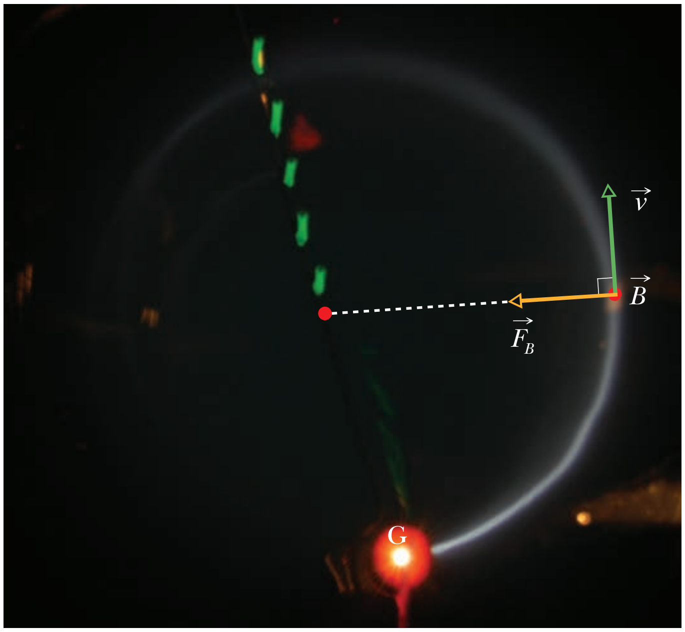
## A Circulating Charged Particle

If a particle moves in a circle at constant speed, we can be sure that the net force acting on the particle is constant in magnitude and points toward the center of the circle, always perpendicular to the particle's velocity. Think of a stone tied to a string and whirled in a circle on a smooth horizontal surface, or of a satellite moving in a circular orbit around Earth. In the first case, the tension in the string provides the necessary force and centripetal acceleration. In the second case, Earth's gravitational attraction provides the force and acceleration.

Figure 28-10 shows another example: A beam of electrons is projected into a chamber by an *electron gun* G. The electrons enter in the plane of the page with speed $v$ and then move in a region of uniform magnetic field $\vec{B}$ directed out of that plane. As a result, a magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ continuously deflects the electrons, and because $\vec{v}$ and $\vec{B}$ are always perpendicular to each other, this deflection causes the electrons to follow a circular path. The path is visible in the photo because atoms of gas in the chamber emit light when some of the circulating electrons collide with them.

We would like to determine the parameters that characterize the circular motion of these electrons, or of any particle of charge magnitude $|q|$ and mass $m$ moving perpendicular to a uniform magnetic field $\vec{B}$ at speed $v$. From Eq. 28-3, the force acting on the particle has a magnitude of $|q|vB$. From Newton's second law $(\vec{F} = m\vec{a})$ applied to uniform circular motion (Eq. 6-18),

$$F = m\frac{v^2}{r}, \tag{28-14}$$




Courtesy Jearl Walker

**Figure 28-10** Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field $\vec{B}$, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force $\vec{F}_B$; for circular motion to occur, $\vec{F}_B$ *must* point toward the center of the circle. Use the right-hand rule for cross products to confirm that $\vec{F}_B = q\vec{v} \times \vec{B}$ gives $\vec{F}_B$ the proper direction. (Don't forget the sign of $q$.)

we have

$$|q|vB = \frac{mv^2}{r}. \tag{28-15}$$

Solving for $r$, we find the radius of the circular path as

$$r = \frac{mv}{|q|B} \quad \text{(radius)}. \tag{28-16}$$

The period $T$ (the time for one full revolution) is equal to the circumference divided by the speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad \text{(period)}. \tag{28-17}$$

The frequency $f$ (the number of revolutions per unit time) is

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \quad \text{(frequency)}. \tag{28-18}$$

The angular frequency $\omega$ of the motion is then

$$\omega = 2\pi f = \frac{|q|B}{m} \quad \text{(angular frequency)}. \tag{28-19}$$

The quantities $T$, $f$, and $\omega$ do not depend on the speed of the particle (provided the speed is much less than the speed of light). Fast particles move in large circles and slow ones in small circles, but all particles with the same charge-to-mass ratio $|q|/m$ take the same time $T$ (the period) to complete one round trip. Using Eq. 28-2, you can show that if you are looking in the direction of $\vec{B}$, the direction of rotation for a positive particle is always counterclockwise, and the direction for a negative particle is always clockwise.

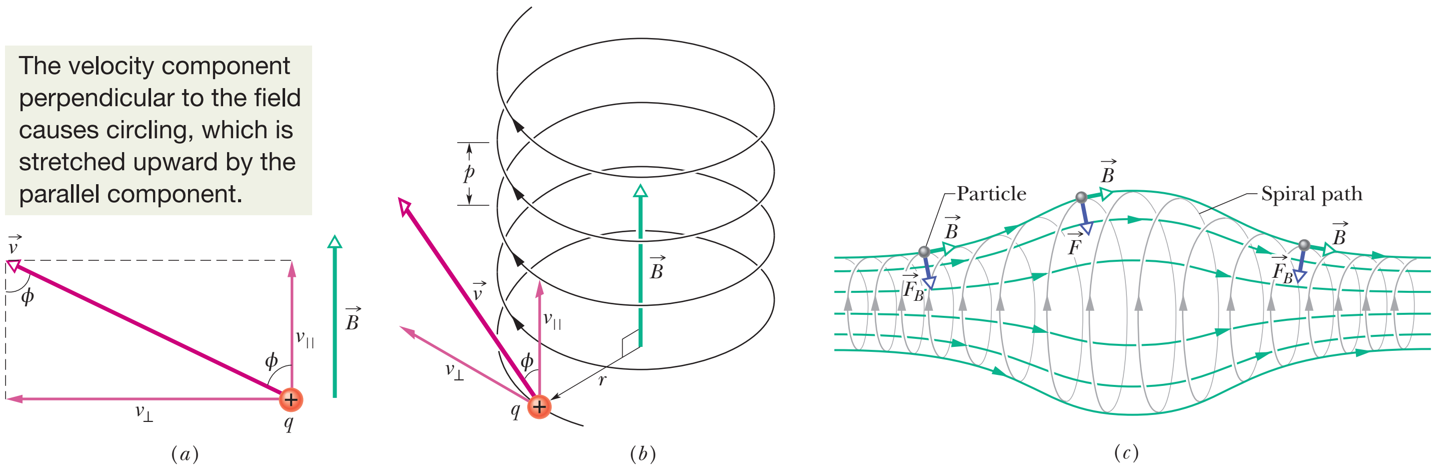



**Figure 28-11** (*a*) A charged particle moves in a uniform magnetic field $\vec{B}$, the particle's velocity $\vec{v}$ making an angle $\phi$ with the field direction. (*b*) The particle follows a helical path of radius $r$ and pitch $p$. (*c*) A charged particle spiraling in a nonuniform magnetic field. (The particle can become trapped in this *magnetic bottle*, spiraling back and forth between the strong field regions at either end.) Note that the magnetic force vectors at the left and right sides have a component pointing toward the center of the figure.

## Helical Paths

If the velocity of a charged particle has a component parallel to the (uniform) magnetic field, the particle will move in a helical path about the direction of the field vector. Figure 28-11*a*, for example, shows the velocity vector $\vec{v}$ of such a particle resolved into two components, one parallel to $\vec{B}$ and one perpendicular to it:

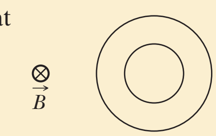
$$v_\parallel = v\cos\phi \quad \text{and} \quad v_\perp = v\sin\phi. \tag{28-20}$$

The parallel component determines the *pitch* $p$ of the helix — that is, the distance between adjacent turns (Fig. 28-11*b*). The perpendicular component determines the radius of the helix and is the quantity to be substituted for $v$ in Eq. 28-16.

Figure 28-11*c* shows a charged particle spiraling in a nonuniform magnetic field. The more closely spaced field lines at the left and right sides indicate that the magnetic field is stronger there. When the field at an end is strong enough, the particle "reflects" from that end.

### Checkpoint 3

The figure here shows the circular paths of two particles that travel at the same speed in a uniform magnetic field $\vec{B}$, which is directed into the page. One particle is a proton; the other is an electron (which is less massive). (a) Which particle follows the smaller circle, and (b) does that particle travel clockwise or counterclockwise?




## Sample Problem 28.03 Helical motion of a charged particle in a magnetic field

An electron with a kinetic energy of 22.5 eV moves into a region of uniform magnetic field $\vec{B}$ of magnitude $4.55 \times 10^{-4}$ T. The angle between the directions of $\vec{B}$ and the electron's velocity $\vec{v}$ is 65.5°. What is the pitch of the helical path taken by the electron?

### KEY IDEAS

(1) The pitch $p$ is the distance the electron travels parallel to the magnetic field $\vec{B}$ during one period $T$ of circulation. (2) The period $T$ is given by Eq. 28-17 for any nonzero angle between $\vec{v}$ and $\vec{B}$.

***Calculations:*** Using Eqs. 28-20 and 28-17, we find

$$p = v_\parallel T = (v\cos\phi)\frac{2\pi m}{|q|B}. \tag{28-21}$$

Calculating the electron's speed $v$ from its kinetic energy, we find that $v = 2.81 \times 10^6$ m/s, and so Eq. 28-21 gives us

$$\begin{aligned} p &= (2.81 \times 10^6 \text{ m/s})(\cos 65.5°) \\ &\quad \times \frac{2\pi(9.11 \times 10^{-31}\text{ kg})}{(1.60 \times 10^{-19}\text{ C})(4.55 \times 10^{-4}\text{ T})} \\ &= 9.16 \text{ cm.} \end{aligned} \qquad \text{(Answer)}$$

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### Sample Problem 28.04 Uniform circular motion of a charged particle in a magnetic field

Figure 28-12 shows the essentials of a *mass spectrometer,* which can be used to measure the mass of an ion; an ion of mass $m$ (to be measured) and charge $q$ is produced in source $S$. The initially stationary ion is accelerated by the electric field due to a potential difference $V$. The ion leaves $S$ and enters a separator chamber in which a uniform magnetic field $\vec{B}$ is perpendicular to the path of the ion. A wide detector lines the bottom wall of the chamber, and the $\vec{B}$ causes the ion to move in a semicircle and thus strike the detector. Suppose that $B = 80.000$ mT, $V = 1000.0$ V, and ions of charge $q = +1.6022 \times 10^{-19}$ C strike the detector at a point that lies at $x = 1.6254$ m. What is the mass $m$ of the individual ions, in atomic mass units (Eq. 1-7: 1 u $= 1.6605 \times 10^{-27}$ kg)?

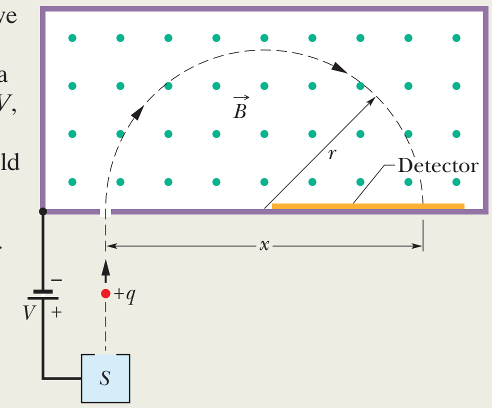


**Figure 28-12** A positive ion is accelerated from its source $S$ by a potential difference $V$, enters a chamber of uniform magnetic field $\vec{B}$, travels through a semicircle of radius $r$, and strikes a detector at a distance $x$.

#### KEY IDEAS

(1) Because the (uniform) magnetic field causes the (charged) ion to follow a circular path, we can relate the ion's mass $m$ to the path's radius $r$ with Eq. 28-16 ($r = mv/|q|B$). From Fig. 28-12 we see that $r = x/2$ (the radius is half the diameter). From the problem statement, we know the magnitude $B$ of the magnetic field. However, we lack the ion's speed $v$ in the magnetic field after the ion has been accelerated due to the potential difference $V$. (2) To relate $v$ and $V$, we use the fact that mechanical energy ($E_{mec} = K + U$) is conserved during the acceleration.

***Finding speed:*** When the ion emerges from the source, its kinetic energy is approximately zero. At the end of the acceleration, its kinetic energy is $\frac{1}{2}mv^2$. Also, during the acceleration, the positive ion moves through a change in potential of $-V$. Thus, because the ion has positive charge $q$, its potential energy changes by $-qV$. If we now write the conservation of mechanical energy as

$$\Delta K + \Delta U = 0,$$

we get

$$\tfrac{1}{2}mv^2 - qV = 0$$

or

$$v = \sqrt{\frac{2qV}{m}}. \tag{28-22}$$

***Finding mass:*** Substituting this value for $v$ into Eq. 28-16 gives us

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

Thus,

$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}.$$

Solving this for $m$ and substituting the given data yield

$$m = \frac{B^2qx^2}{8V}$$

$$= \frac{(0.080000\ \text{T})^2(1.6022 \times 10^{-19}\ \text{C})(1.6254\ \text{m})^2}{8(1000.0\ \text{V})}$$

$$= 3.3863 \times 10^{-25}\ \text{kg} = 203.93\ \text{u}. \qquad \text{(Answer)}$$

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# 28-5 CYCLOTRONS AND SYNCHROTRONS

## Learning Objectives

*After reading this module, you should be able to . . .*

28.27 Describe how a cyclotron works, and in a sketch indicate a particle's path and the regions where the kinetic energy is increased.

28.28 Identify the resonance condition.

28.29 For a cyclotron, apply the relationship between the particle's mass and charge, the magnetic field, and the frequency of circling.

28.30 Distinguish between a cyclotron and a synchrotron.

## Key Ideas

- In a cyclotron, charged particles are accelerated by electric forces as they circle in a magnetic field.
- A synchrotron is needed for particles accelerated to nearly the speed of light.

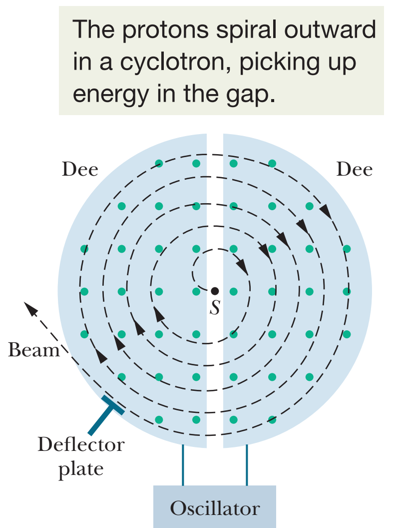



**Figure 28-13** The elements of a cyclotron, showing the particle source $S$ and the dees. A uniform magnetic field is directed up from the plane of the page. Circulating protons spiral outward within the hollow dees, gaining energy every time they cross the gap between the dees.

# Cyclotrons and Synchrotrons

Beams of high-energy particles, such as high-energy electrons and protons, have been enormously useful in probing atoms and nuclei to reveal the fundamental structure of matter. Such beams were instrumental in the discovery that atomic nuclei consist of protons and neutrons and in the discovery that protons and neutrons consist of quarks and gluons. Because electrons and protons are charged, they can be accelerated to the required high energy if they move through large potential differences. The required acceleration distance is reasonable for electrons (low mass) but unreasonable for protons (greater mass).

A clever solution to this problem is first to let protons and other massive particles move through a modest potential difference (so that they gain a modest amount of energy) and then use a magnetic field to cause them to circle back and move through a modest potential difference again. If this procedure is repeated thousands of times, the particles end up with a very large energy.

Here we discuss two *accelerators* that employ a magnetic field to repeatedly bring particles back to an accelerating region, where they gain more and more energy until they finally emerge as a high-energy beam.

## The Cyclotron

Figure 28-13 is a top view of the region of a *cyclotron* in which the particles (protons, say) circulate. The two hollow D-shaped objects (each open on its straight edge) are made of sheet copper. These *dees,* as they are called, are part of an electrical oscillator that alternates the electric potential difference across the gap between the dees. The electrical signs of the dees are alternated so that the electric field in the gap alternates in direction, first toward one dee and then toward the other dee, back and forth. The dees are immersed in a large magnetic field directed out of the plane of the page. The magnitude $B$ of this field is set via a control on the electromagnet producing the field.

Suppose that a proton, injected by source $S$ at the center of the cyclotron in Fig. 28-13, initially moves toward a negatively charged dee. It will accelerate toward this dee and enter it. Once inside, it is shielded from electric fields by the copper walls of the dee; that is, the electric field does not enter the dee. The magnetic field, however, is not screened by the (nonmagnetic) copper dee, so the proton moves in a circular path whose radius, which depends on its speed, is given by Eq. 28-16 ($r = mv/|q|B$).

Let us assume that at the instant the proton emerges into the center gap from the first dee, the potential difference between the dees is reversed. Thus, the proton *again* faces a negatively charged dee and is *again* accelerated. This process continues, the circulating proton always being in step with the oscillations of the dee potential, until the proton has spiraled out to the edge of the dee system. There a deflector plate sends it out through a portal.

***Frequency.*** The key to the operation of the cyclotron is that the frequency $f$ at which the proton circulates in the magnetic field (and that does *not* depend on its speed) must be equal to the fixed frequency $f_{osc}$ of the electrical oscillator, or

$$f = f_{osc} \quad \text{(resonance condition)}. \tag{28-23}$$

This *resonance condition* says that, if the energy of the circulating proton is to increase, energy must be fed to it at a frequency $f_{osc}$ that is equal to the natural frequency $f$ at which the proton circulates in the magnetic field.

Combining Eqs. 28-18 ($f = |q|B/2\pi m$) and 28-23 allows us to write the resonance condition as

$$|q|B = 2\pi m f_{osc}. \tag{28-24}$$

The oscillator (we assume) is designed to work at a single fixed frequency $f_{osc}$. We then "tune" the cyclotron by varying $B$ until Eq. 28-24 is satisfied, and then many protons circulate through the magnetic field, to emerge as a beam.

### The Proton Synchrotron

At proton energies above 50 MeV, the conventional cyclotron begins to fail because one of the assumptions of its design — that the frequency of revolution of a charged particle circulating in a magnetic field is independent of the particle's speed — is true only for speeds that are much less than the speed of light. At greater proton speeds (above about 10% of the speed of light), we must treat the problem relativistically. According to relativity theory, as the speed of a circulating proton approaches that of light, the proton's frequency of revolution decreases steadily. Thus, the proton gets out of step with the cyclotron's oscillator — whose frequency remains fixed at $f_{osc}$ — and eventually the energy of the still circulating proton stops increasing.

There is another problem. For a 500 GeV proton in a magnetic field of 1.5 T, the path radius is 1.1 km. The corresponding magnet for a conventional cyclotron of the proper size would be impossibly expensive, the area of its pole faces being about $4 \times 10^6\ \text{m}^2$.

The *proton synchrotron* is designed to meet these two difficulties. The magnetic field $B$ and the oscillator frequency $f_{osc}$, instead of having fixed values as in the conventional cyclotron, are made to vary with time during the accelerating cycle. When this is done properly, (1) the frequency of the circulating protons remains in step with the oscillator at all times, and (2) the protons follow a circular — not a spiral — path. Thus, the magnet need extend only along that circular path, not over some $4 \times 10^6\ \text{m}^2$. The circular path, however, still must be large if high energies are to be achieved.

### Sample Problem 28.05 Accelerating a charged particle in a cyclotron

Suppose a cyclotron is operated at an oscillator frequency of 12 MHz and has a dee radius $R = 53$ cm.

(a) What is the magnitude of the magnetic field needed for deuterons to be accelerated in the cyclotron? The deuteron mass is $m = 3.34 \times 10^{-27}$ kg (twice the proton mass).

#### KEY IDEA

For a given oscillator frequency $f_{osc}$, the magnetic field magnitude $B$ required to accelerate any particle in a cyclotron depends on the ratio $m/|q|$ of mass to charge for the particle, according to Eq. 28-24 ($|q|B = 2\pi m f_{osc}$).

***Calculation:*** For deuterons and the oscillator frequency $f_{osc} = 12$ MHz, we find

$$B = \frac{2\pi m f_{osc}}{|q|} = \frac{(2\pi)(3.34 \times 10^{-27}\ \text{kg})(12 \times 10^6\ \text{s}^{-1})}{1.60 \times 10^{-19}\ \text{C}}$$

$$= 1.57\ \text{T} \approx 1.6\ \text{T}. \qquad \text{(Answer)}$$

Note that, to accelerate protons, $B$ would have to be reduced by a factor of 2, provided the oscillator frequency remained fixed at 12 MHz.

(b) What is the resulting kinetic energy of the deuterons?

#### KEY IDEAS

(1) The kinetic energy ($\frac{1}{2}mv^2$) of a deuteron exiting the cyclotron is equal to the kinetic energy it had just before exiting, when it was traveling in a circular path with a radius approximately equal to the radius $R$ of the cyclotron dees. (2) We can find the speed $v$ of the deuteron in that circular path with Eq. 28-16 ($r = mv/|q|B$).

***Calculations:*** Solving that equation for $v$, substituting $R$ for $r$, and then substituting known data, we find

$$v = \frac{R|q|B}{m} = \frac{(0.53\ \text{m})(1.60 \times 10^{-19}\ \text{C})(1.57\ \text{T})}{3.34 \times 10^{-27}\ \text{kg}}$$

$$= 3.99 \times 10^7\ \text{m/s}.$$

This speed corresponds to a kinetic energy of

$$K = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}(3.34 \times 10^{-27}\ \text{kg})(3.99 \times 10^7\ \text{m/s})^2$$

$$= 2.7 \times 10^{-12}\ \text{J}, \qquad \text{(Answer)}$$

or about 17 MeV.

# 28-6 MAGNETIC FORCE ON A CURRENT-CARRYING WIRE

## Learning Objectives

*After reading this module, you should be able to . . .*

**28.31** For the situation where a current is perpendicular to a magnetic field, sketch the current, the direction of the magnetic field, and the direction of the magnetic force on the current (or wire carrying the current).

**28.32** For a current in a magnetic field, apply the relationship between the magnetic force magnitude $F_B$, the current $i$, the length of the wire $L$, and the angle $\phi$ between the length vector $\vec{L}$ and the field vector $\vec{B}$.

**28.33** Apply the right-hand rule for cross products to find the direction of the magnetic force on a current in a magnetic field.

**28.34** For a current in a magnetic field, calculate the magnetic force $\vec{F}_B$ with a cross product of the length vector $\vec{L}$ and the field vector $\vec{B}$, in magnitude-angle and unit-vector notations.

**28.35** Describe the procedure for calculating the force on a current-carrying wire in a magnetic field if the wire is not straight or if the field is not uniform.

## Key Ideas

- A straight wire carrying a current $i$ in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}.$$

- The force acting on a current element $i\,d\vec{L}$ in a magnetic field is

$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}.$$

- The direction of the length vector $\vec{L}$ or $d\vec{L}$ is that of the current $i$.

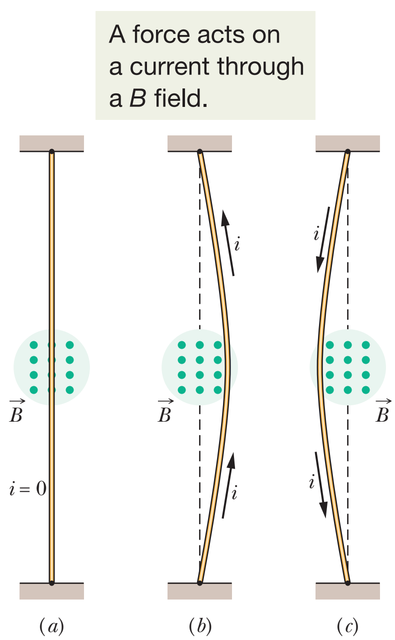



**Figure 28-14** A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (*a*) Without current in the wire, the wire is straight. (*b*) With upward current, the wire is deflected rightward. (*c*) With downward current, the deflection is leftward. The connections for getting the current into the wire at one end and out of it at the other end are not shown.

## Magnetic Force on a Current-Carrying Wire

We have already seen (in connection with the Hall effect) that a magnetic field exerts a sideways force on electrons moving in a wire. This force must then be transmitted to the wire itself, because the conduction electrons cannot escape sideways out of the wire.

In Fig. 28-14*a*, a vertical wire, carrying no current and fixed in place at both ends, extends through the gap between the vertical pole faces of a magnet. The magnetic field between the faces is directed outward from the page. In Fig. 28-14*b*, a current is sent upward through the wire; the wire deflects to the right. In Fig. 28-14*c*, we reverse the direction of the current and the wire deflects to the left.

Figure 28-15 shows what happens inside the wire of Fig. 28-14*b*. We see one of the conduction electrons, drifting downward with an assumed drift speed $v_d$. Equation 28-3, in which we must put $\phi = 90°$, tells us that a force $\vec{F}_B$ of magnitude $ev_dB$ must act on each such electron. From Eq. 28-2 we see that this force must be directed to the right. We expect then that the wire as a whole will experience a force to the right, in agreement with Fig. 28-14*b*.

If, in Fig. 28-15, we were to reverse *either* the direction of the magnetic field *or* the direction of the current, the force on the wire would reverse, being directed now to the left. Note too that it does not matter whether we consider negative

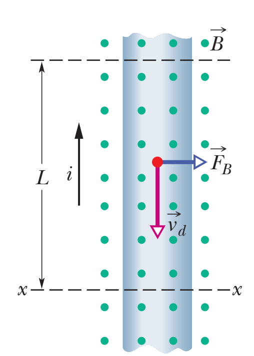



**Figure 28-15** A close-up view of a section of the wire of Fig. 28-14*b*. The current direction is upward, which means that electrons drift downward. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

charges drifting downward in the wire (the actual case) or positive charges drifting upward. The direction of the deflecting force on the wire is the same. We are safe then in dealing with a current of positive charge, as we usually do in dealing with circuits.

***Find the Force.*** Consider a length $L$ of the wire in Fig. 28-15. All the conduction electrons in this section of wire will drift past plane $xx$ in Fig. 28-15 in a time $t = L/v_d$. Thus, in that time a charge given by

$$q = it = i\frac{L}{v_d}$$

will pass through that plane. Substituting this into Eq. 28-3 yields

$$F_B = qv_dB \sin\phi = \frac{iL}{v_d} v_dB \sin 90°$$

or

$$F_B = iLB. \tag{28-25}$$

Note that this equation gives the magnetic force that acts on a length $L$ of straight wire carrying a current $i$ and immersed in a uniform magnetic field $\vec{B}$ that is *perpendicular* to the wire.

If the magnetic field is *not* perpendicular to the wire, as in Fig. 28-16, the magnetic force is given by a generalization of Eq. 28-25:

$$\vec{F}_B = i\vec{L} \times \vec{B} \quad \text{(force on a current).} \tag{28-26}$$

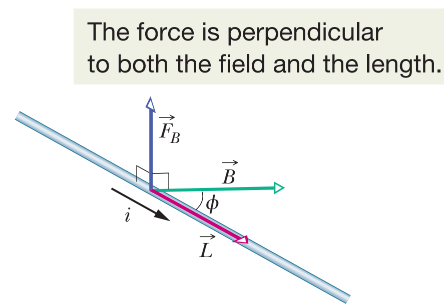



**Figure 28-16** A wire carrying current $i$ makes an angle $\phi$ with magnetic field $\vec{B}$. The wire has length $L$ in the field and length vector $\vec{L}$ (in the direction of the current). A magnetic force $\vec{F}_B = i\vec{L} \times \vec{B}$ acts on the wire.

Here $\vec{L}$ is a *length vector* that has magnitude $L$ and is directed along the wire segment in the direction of the (conventional) current. The force magnitude $F_B$ is

$$F_B = iLB \sin\phi, \tag{28-27}$$

where $\phi$ is the angle between the directions of $\vec{L}$ and $\vec{B}$. The direction of $\vec{F}_B$ is that of the cross product $\vec{L} \times \vec{B}$ because we take current $i$ to be a positive quantity. Equation 28-26 tells us that $\vec{F}_B$ is always perpendicular to the plane defined by vectors $\vec{L}$ and $\vec{B}$, as indicated in Fig. 28-16.

Equation 28-26 is equivalent to Eq. 28-2 in that either can be taken as the defining equation for $\vec{B}$. In practice, we define $\vec{B}$ from Eq. 28-26 because it is much easier to measure the magnetic force acting on a wire than that on a single moving charge.

***Crooked Wire.*** If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments and apply Eq. 28-26 to each segment. The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write

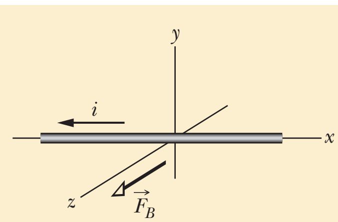
$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}, \tag{28-28}$$

and we can find the resultant force on any given arrangement of currents by integrating Eq. 28-28 over that arrangement.

In using Eq. 28-28, bear in mind that there is no such thing as an isolated current-carrying wire segment of length $dL$. There must always be a way to introduce the current into the segment at one end and take it out at the other end.

## Checkpoint 4

The figure shows a current $i$ through a wire in a uniform magnetic field $\vec{B}$, as well as the magnetic force $\vec{F}_B$ acting on the wire. The field is oriented so that the force is maximum. In what direction is the field?



### Sample Problem 28.06 Magnetic force on a wire carrying current

A straight, horizontal length of copper wire has a current $i = 28$ A through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire — that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

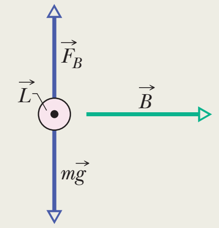



**Figure 28-17** A wire (shown in cross section) carrying current out of the page.

#### KEY IDEAS

(1) Because the wire carries a current, a magnetic force $\vec{F}_B$ can act on the wire if we place it in a magnetic field $\vec{B}$. To balance the downward gravitational force $\vec{F}_g$ on the wire, we want $\vec{F}_B$ to be directed upward (Fig. 28-17). (2) The direction of $\vec{F}_B$ is related to the directions of $\vec{B}$ and the wire's length vector $\vec{L}$ by Eq. 28-26 ($\vec{F}_B = i\vec{L} \times \vec{B}$).

***Calculations:*** Because $\vec{L}$ is directed horizontally (and the current is taken to be positive), Eq. 28-26 and the right-hand rule for cross products tell us that $\vec{B}$ must be horizontal and rightward (in Fig. 28-17) to give the required upward $\vec{F}_B$.

The magnitude of $\vec{F}_B$ is $F_B = iLB \sin \phi$ (Eq. 28-27). Because we want $\vec{F}_B$ to balance $\vec{F}_g$, we want

$$iLB \sin \phi = mg, \tag{28-29}$$

where $mg$ is the magnitude of $\vec{F}_g$ and $m$ is the mass of the wire. We also want the minimal field magnitude $B$ for $\vec{F}_B$ to balance $\vec{F}_g$. Thus, we need to maximize $\sin \phi$ in Eq. 28-29. To do so, we set $\phi = 90°$, thereby arranging for $\vec{B}$ to be perpendicular to the wire. We then have $\sin \phi = 1$, so Eq. 28-29 yields

$$B = \frac{mg}{iL \sin \phi} = \frac{(m/L)g}{i}. \tag{28-30}$$

We write the result this way because we know $m/L$, the linear density of the wire. Substituting known data then gives us

$$B = \frac{(46.6 \times 10^{-3} \text{ kg/m})(9.8 \text{ m/s}^2)}{28 \text{ A}}$$
$$= 1.6 \times 10^{-2} \text{ T.} \qquad \text{(Answer)}$$

This is about 160 times the strength of Earth's magnetic field.

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# 28-7 TORQUE ON A CURRENT LOOP

## Learning Objectives

*After reading this module, you should be able to . . .*

**28.36** Sketch a rectangular loop of current in a magnetic field, indicating the magnetic forces on the four sides, the direction of the current, the normal vector $\vec{n}$, and the direction in which a torque from the forces tends to rotate the loop.

**28.37** For a current-carrying coil in a magnetic field, apply the relationship between the torque magnitude $\tau$, the number of turns $N$, the area of each turn $A$, the current $i$, the magnetic field magnitude $B$, and the angle $\theta$ between the normal vector $\vec{n}$ and the magnetic field vector $\vec{B}$.

## Key Ideas

- Various magnetic forces act on the sections of a current-carrying coil lying in a uniform external magnetic field, but the net force is zero.
- The net torque acting on the coil has a magnitude given by

$$\tau = NiAB \sin \theta,$$

where $N$ is the number of turns in the coil, $A$ is the area of each turn, $i$ is the current, $B$ is the field magnitude, and $\theta$ is the angle between the magnetic field $\vec{B}$ and the normal vector to the coil $\vec{n}$.

## Torque on a Current Loop

Much of the world's work is done by electric motors. The forces behind this work are the magnetic forces that we studied in the preceding section — that is, the forces that a magnetic field exerts on a wire that carries a current.

Figure 28-18 shows a simple motor, consisting of a single current-carrying loop immersed in a magnetic field $\vec{B}$. The two magnetic forces $\vec{F}$ and $-\vec{F}$ produce a torque on the loop, tending to rotate it about its central axis. Although many essential details have been omitted, the figure does suggest how the action of a magnetic field on a current loop produces rotary motion. Let us analyze that action.

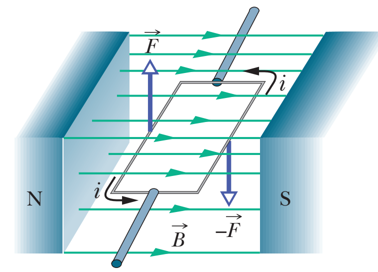



**Figure 28-18** The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.

Figure 28-19*a* shows a rectangular loop of sides $a$ and $b$, carrying current $i$ through uniform magnetic field $\vec{B}$. We place the loop in the field so that its long sides, labeled 1 and 3, are perpendicular to the field direction (which is into the page), but its short sides, labeled 2 and 4, are not. Wires to lead the current into and out of the loop are needed but, for simplicity, are not shown.

To define the orientation of the loop in the magnetic field, we use a normal vector $\vec{n}$ that is perpendicular to the plane of the loop. Figure 28-19*b* shows a right-hand rule for finding the direction of $\vec{n}$. Point or curl the fingers of your right hand in the direction of the current at any point on the loop. Your extended thumb then points in the direction of the normal vector $\vec{n}$.

In Fig. 28-19*c*, the normal vector of the loop is shown at an arbitrary angle $\theta$ to the direction of the magnetic field $\vec{B}$. We wish to find the net force and net torque acting on the loop in this orientation.

***Net Torque.*** The net force on the loop is the vector sum of the forces acting on its four sides. For side 2 the vector $\vec{L}$ in Eq. 28-26 points in the direction of the current and has magnitude $b$. The angle between $\vec{L}$ and $\vec{B}$ for side 2 (see Fig. 28-19*c*) is $90° - \theta$. Thus, the magnitude of the force acting on this side is

$$F_2 = ibB\sin(90° - \theta) = ibB\cos\theta. \tag{28-31}$$

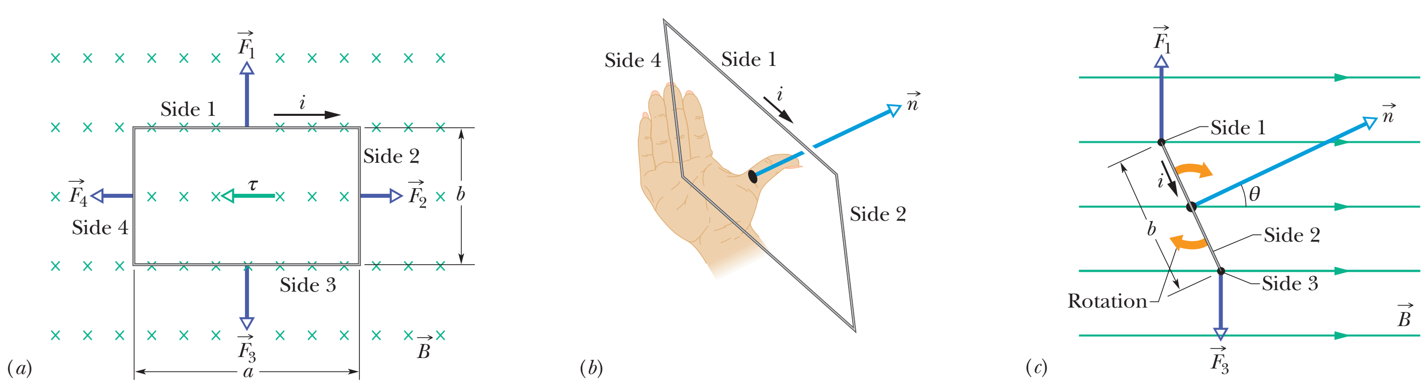
You can show that the force $\vec{F}_4$ acting on side 4 has the same magnitude as $\vec{F}_2$ but the opposite direction. Thus, $\vec{F}_2$ and $\vec{F}_4$ cancel out exactly. Their net force is zero and, because their common line of action is through the center of the loop, their net torque is also zero.

The situation is different for sides 1 and 3. For them, $\vec{L}$ is perpendicular to $\vec{B}$, so the forces $\vec{F}_1$ and $\vec{F}_3$ have the common magnitude $iaB$. Because these two forces have opposite directions, they do not tend to move the loop up or down. However, as Fig. 28-19*c* shows, these two forces do *not* share the same line of action; so they *do* produce a net torque. The torque tends to rotate the loop so as to align its normal vector $\vec{n}$ with the direction of the magnetic field $\vec{B}$. That torque has moment arm $(b/2)\sin\theta$ about the central axis of the loop. The magnitude $\tau'$ of the torque due to forces $\vec{F}_1$ and $\vec{F}_3$ is then (see Fig. 28-19*c*)

$$\tau' = \left(iaB\,\frac{b}{2}\sin\theta\right) + \left(iaB\,\frac{b}{2}\sin\theta\right) = iabB\sin\theta. \tag{28-32}$$

***Coil.*** Suppose we replace the single loop of current with a *coil* of $N$ loops, or *turns.* Further, suppose that the turns are wound tightly enough that they can be

**Figure 28-19** A rectangular loop, of length $a$ and width $b$ and carrying a current $i$, is located in a uniform magnetic field. A torque $\tau$ acts to align the normal vector $\vec{n}$ with the direction of the field. (*a*) The loop as seen by looking in the direction of the magnetic field. (*b*) A perspective of the loop showing how the right-hand rule gives the direction of $\vec{n}$, which is perpendicular to the plane of the loop. (*c*) A side view of the loop, from side 2. The loop rotates as indicated.



approximated as all having the same dimensions and lying in a plane. Then the turns form a *flat coil,* and a torque $\tau'$ with the magnitude given in Eq. 28-32 acts on each of them. The total torque on the coil then has magnitude

$$\tau = N\tau' = NiabB \sin\theta = (NiA)B \sin\theta, \tag{28-33}$$

in which $A$ (= $ab$) is the area enclosed by the coil. The quantities in parentheses ($NiA$) are grouped together because they are all properties of the coil: its number of turns, its area, and the current it carries. Equation 28-33 holds for all flat coils, no matter what their shape, provided the magnetic field is uniform. For example, for the common circular coil, with radius $r$, we have

$$\tau = (Ni\pi r^2)B \sin\theta. \tag{28-34}$$

***Normal Vector.*** Instead of focusing on the motion of the coil, it is simpler to keep track of the vector $\vec{n}$, which is normal to the plane of the coil. Equation 28-33 tells us that a current-carrying flat coil placed in a magnetic field will tend to rotate so that $\vec{n}$ has the same direction as the field. In a motor, the current in the coil is reversed as $\vec{n}$ begins to line up with the field direction, so that a torque continues to rotate the coil. This automatic reversal of the current is done via a commutator that electrically connects the rotating coil with the stationary contacts on the wires that supply the current from some source.

# 28-8 THE MAGNETIC DIPOLE MOMENT

## Learning Objectives

*After reading this module, you should be able to . . .*

**28.38** Identify that a current-carrying coil is a magnetic dipole with a magnetic dipole moment $\vec{\mu}$ that has the direction of the normal vector $\vec{n}$, as given by a right-hand rule.

**28.39** For a current-carrying coil, apply the relationship between the magnitude $\mu$ of the magnetic dipole moment, the number of turns $N$, the area $A$ of each turn, and the current $i$.

**28.40** On a sketch of a current-carrying coil, draw the direction of the current, and then use a right-hand rule to determine the direction of the magnetic dipole moment vector $\vec{\mu}$.

**28.41** For a magnetic dipole in an external magnetic field, apply the relationship between the torque magnitude $\tau$, the dipole moment magnitude $\mu$, the magnetic field magnitude $B$, and the angle $\theta$ between the dipole moment vector $\vec{\mu}$ and the magnetic field vector $\vec{B}$.

**28.42** Identify the convention of assigning a plus or minus sign to a torque according to the direction of rotation.

**28.43** Calculate the torque on a magnetic dipole by evaluating a cross product of the dipole moment vector $\vec{\mu}$ and the external magnetic field vector $\vec{B}$, in magnitude-angle notation and unit-vector notation.

**28.44** For a magnetic dipole in an external magnetic field, identify the dipole orientations at which the torque magnitude is minimum and maximum.

**28.45** For a magnetic dipole in an external magnetic field, apply the relationship between the orientation energy $U$, the dipole moment magnitude $\mu$, the external magnetic field magnitude $B$, and the angle $\theta$ between the dipole moment vector $\vec{\mu}$ and the magnetic field vector $\vec{B}$.

**28.46** Calculate the orientation energy $U$ by taking a dot product of the dipole moment vector $\vec{\mu}$ and the external magnetic field vector $\vec{B}$, in magnitude-angle and unit-vector notations.

**28.47** Identify the orientations of a magnetic dipole in an external magnetic field that give the minimum and maximum orientation energies.

**28.48** For a magnetic dipole in a magnetic field, relate the orientation energy $U$ to the work $W_a$ done by an external torque as the dipole rotates in the magnetic field.

## Key Ideas

- A coil (of area $A$ and $N$ turns, carrying current $i$) in a uniform magnetic field $\vec{B}$ will experience a torque $\vec{\tau}$ given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

Here $\vec{\mu}$ is the magnetic dipole moment of the coil, with magnitude $\mu = NiA$ and direction given by the right-hand rule.

- The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

- If an external agent rotates a magnetic dipole from an initial orientation $\theta_i$ to some other orientation $\theta_f$ and the dipole is stationary both initially and finally, the work $W_a$ done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i.$$

# The Magnetic Dipole Moment

As we have just discussed, a torque acts to rotate a current-carrying coil placed in a magnetic field. In that sense, the coil behaves like a bar magnet placed in the magnetic field. Thus, like a bar magnet, a current-carrying coil is said to be a *magnetic dipole.* Moreover, to account for the torque on the coil due to the magnetic field, we assign a **magnetic dipole moment** $\vec{\mu}$ to the coil. The direction of $\vec{\mu}$ is that of the normal vector $\vec{n}$ to the plane of the coil and thus is given by the same right-hand rule shown in Fig. 28-19. That is, grasp the coil with the fingers of your right hand in the direction of current $i$; the outstretched thumb of that hand gives the direction of $\vec{\mu}$. The magnitude of $\vec{\mu}$ is given by

$$\mu = NiA \quad \text{(magnetic moment)}, \tag{28-35}$$

in which $N$ is the number of turns in the coil, $i$ is the current through the coil, and $A$ is the area enclosed by each turn of the coil. From this equation, with $i$ in amperes and $A$ in square meters, we see that the unit of $\vec{\mu}$ is the ampere–square meter ($A \cdot m^2$).

***Torque.*** Using $\vec{\mu}$, we can rewrite Eq. 28-33 for the torque on the coil due to a magnetic field as

$$\tau = \mu B \sin\theta, \tag{28-36}$$

in which $\theta$ is the angle between the vectors $\vec{\mu}$ and $\vec{B}$.

We can generalize this to the vector relation

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \tag{28-37}$$

which reminds us very much of the corresponding equation for the torque exerted by an *electric* field on an *electric* dipole — namely, Eq. 22-34:

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

In each case the torque due to the field — either magnetic or electric — is equal to the vector product of the corresponding dipole moment and the field vector.

***Energy.*** A magnetic dipole in an external magnetic field has an energy that depends on the dipole's orientation in the field. For electric dipoles we have shown (Eq. 22-38) that

$$U(\theta) = -\vec{p} \cdot \vec{E}.$$

In strict analogy, we can write for the magnetic case

$$U(\theta) = -\vec{\mu} \cdot \vec{B}. \tag{28-38}$$

In each case the energy due to the field is equal to the negative of the scalar product of the corresponding dipole moment and the field vector.

A magnetic dipole has its lowest energy ($= -\mu B \cos 0 = -\mu B$) when its dipole moment $\vec{\mu}$ is lined up with the magnetic field (Fig. 28-20). It has its highest energy ($= -\mu B \cos 180° = +\mu B$) when $\vec{\mu}$ is directed opposite the field. From Eq. 28-38, with $U$ in joules and $\vec{B}$ in teslas, we see that the unit of $\vec{\mu}$ can be the joule per tesla (J/T) instead of the ampere–square meter as suggested by Eq. 28-35.

***Work.*** If an applied torque (due to "an external agent") rotates a magnetic dipole from an initial orientation $\theta_i$ to another orientation $\theta_f$, then work $W_a$ is done on the dipole by the applied torque. *If the dipole is stationary* before and after the change in its orientation, then work $W_a$ is

$$W_a = U_f - U_i, \tag{28-39}$$

where $U_f$ and $U_i$ are calculated with Eq. 28-38.

The magnetic moment vector attempts to align with the magnetic field.

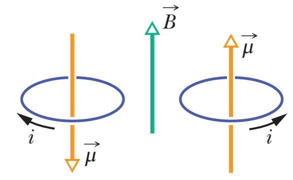



**Figure 28-20** The orientations of highest and lowest energy of a magnetic dipole (here a coil carrying current) in an external magnetic field $\vec{B}$. The direction of the current $i$ gives the direction of the magnetic dipole moment $\vec{\mu}$ via the right-hand rule shown for $\vec{n}$ in Fig. 28-19*b*.

**Table 28-2** Some Magnetic Dipole Moments

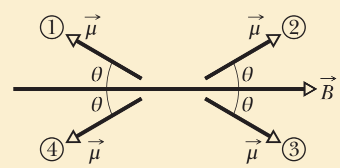
| | |
|---|---|
| Small bar magnet | 5 J/T |
| Earth | $8.0 \times 10^{22}$ J/T |
| Proton | $1.4 \times 10^{-26}$ J/T |
| Electron | $9.3 \times 10^{-24}$ J/T |

So far, we have identified only a current-carrying coil and a permanent magnet as a magnetic dipole. However, a rotating sphere of charge is also a magnetic dipole, as is Earth itself (approximately). Finally, most subatomic particles, including the electron, the proton, and the neutron, have magnetic dipole moments. As you will see in Chapter 32, all these quantities can be viewed as current loops. For comparison, some approximate magnetic dipole moments are shown in Table 28-2.

***Language.*** Some instructors refer to $U$ in Eq. 28-38 as a potential energy and relate it to work done by the magnetic field when the orientation of the dipole changes. Here we shall avoid the debate and say that $U$ is an energy associated with the dipole orientation.

### Checkpoint 5

The figure shows four orientations, at angle $\theta$, of a magnetic dipole moment $\vec{\mu}$ in a magnetic field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the orientation energy of the dipole, greatest first.




## Sample Problem 28.07 Rotating a magnetic dipole in a magnetic field

Figure 28-21 shows a circular coil with 250 turns, an area $A$ of $2.52 \times 10^{-4}$ m$^2$, and a current of 100 $\mu$A. The coil is at rest in a uniform magnetic field of magnitude $B = 0.85$ T, with its magnetic dipole moment $\vec{\mu}$ initially aligned with $\vec{B}$.

(a) In Fig. 28-21, what is the direction of the current in the coil?

***Right-hand rule:*** Imagine cupping the coil with your right hand so that your right thumb is outstretched in the direction of $\vec{\mu}$. The direction in which your fingers curl around the coil is the direction of the current in the coil. Thus, in the wires on the near side of the coil — those we see in Fig. 28-21 — the current is from top to bottom.

(b) How much work would the torque applied by an external agent have to do on the coil to rotate it 90° from its initial orientation, so that $\vec{\mu}$ is perpendicular to $\vec{B}$ and the coil is again at rest?

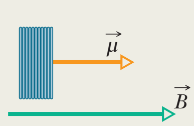



**Figure 28-21** A side view of a circular coil carrying a current and oriented so that its magnetic dipole moment  is aligned with magnetic field $\vec{B}$.

### KEY IDEA

The work $W_a$ done by the applied torque would be equal to the change in the coil's orientation energy due to its change in orientation.

***Calculations:*** From Eq. 28-39 ($W_a = U_f - U_i$), we find

$$\begin{aligned} W_a &= U(90°) - U(0°) \\ &= -\mu B \cos 90° - (-\mu B \cos 0°) = 0 + \mu B \\ &= \mu B. \end{aligned}$$

Substituting for $\mu$ from Eq. 28-35 ($\mu = NiA$), we find that

$$\begin{aligned} W_a &= (NiA)B \\ &= (250)(100 \times 10^{-6}\text{ A})(2.52 \times 10^{-4}\text{ m}^2)(0.85\text{ T}) \\ &= 5.355 \times 10^{-6}\text{ J} \approx 5.4\ \mu\text{J}. \qquad \text{(Answer)} \end{aligned}$$

Similarly, we can show that to change the orientation by another 90°, so that the dipole moment is opposite the field, another 5.4 $\mu$J is required.

# Review & Summary

**Magnetic Field $\vec{B}$** A **magnetic field** $\vec{B}$ is defined in terms of the force $\vec{F}_B$ acting on a test particle with charge $q$ moving through the field with velocity $\vec{v}$:

$$\vec{F}_B = q\vec{v} \times \vec{B}. \tag{28-2}$$

The SI unit for $\vec{B}$ is the **tesla** (T): $1\ \text{T} = 1\ \text{N/(A}\cdot\text{m)} = 10^4$ gauss.

**The Hall Effect** When a conducting strip carrying a current $i$ is placed in a uniform magnetic field $\vec{B}$, some charge carriers (with charge $e$) build up on one side of the conductor, creating a potential difference $V$ across the strip. The polarities of the sides indicate the sign of the charge carriers.

**A Charged Particle Circulating in a Magnetic Field** A charged particle with mass $m$ and charge magnitude $|q|$ moving with velocity $\vec{v}$ perpendicular to a uniform magnetic field $\vec{B}$ will travel in a circle. Applying Newton's second law to the circular motion yields

$$|q|vB = \frac{mv^2}{r}, \tag{28-15}$$

from which we find the radius $r$ of the circle to be

$$r = \frac{mv}{|q|B}. \tag{28-16}$$

The frequency of revolution $f$, the angular frequency $\omega$, and the period of the motion $T$ are given by

$$f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{|q|B}{2\pi m}. \qquad \text{(28-19, 28-18, 28-17)}$$

**Magnetic Force on a Current-Carrying Wire** A straight wire carrying a current $i$ in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}. \tag{28-26}$$

The force acting on a current element $i\,d\vec{L}$ in a magnetic field is

$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}. \tag{28-28}$$

The direction of the length vector $\vec{L}$ or $d\vec{L}$ is that of the current $i$.

**Torque on a Current-Carrying Coil** A coil (of area $A$ and $N$ turns, carrying current $i$) in a uniform magnetic field $\vec{B}$ will experience a torque $\vec{\tau}$ given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}. \tag{28-37}$$

Here $\vec{\mu}$ is the **magnetic dipole moment** of the coil, with magnitude $\mu = NiA$ and direction given by the right-hand rule.

**Orientation Energy of a Magnetic Dipole** The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}. \tag{28-38}$$

If an external agent rotates a magnetic dipole from an initial orientation $\theta_i$ to some other orientation $\theta_f$ and the dipole is stationary both initially and finally, the work $W_a$ done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i. \tag{28-39}$$

# Questions

**1** Figure 28-22 shows three situations in which a positively charged particle moves at velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$ and experiences a magnetic force $\vec{F}_B$. In each situation, determine whether the orientations of the vectors are physically reasonable.

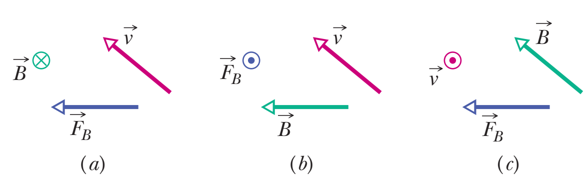



**Figure 28-22** Question 1.

**2** Figure 28-23 shows a wire that carries current to the right through a uniform magnetic field. It also shows four choices for the direction of that field. (a) Rank the choices according to the magnitude of the electric potential difference that would be set up across the width of the wire, greatest first. (b) For which choice is the top side of the wire at higher potential than the bottom side of the wire?

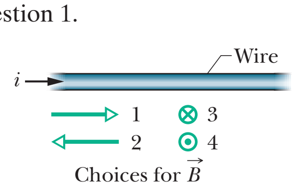



**Figure 28-23** Question 2.

**3** Figure 28-24 shows a metallic, rectangular solid that is to move at a certain speed $v$ through the uniform magnetic field $\vec{B}$. The dimensions of the solid are multiples of $d$, as shown. You have six choices for the direction of the velocity: parallel to $x$, $y$, or $z$ in either the positive or negative direction. (a) Rank the six choices according to the potential difference set up across the solid, greatest first. (b) For which choice is the front face at lower potential?

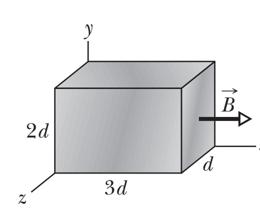



**Figure 28-24** Question 3.

**4** Figure 28-25 shows the path of a particle through six regions of uniform magnetic field, where the path is either a half-circle or a quarter-circle. Upon leaving the last region, the particle travels between two charged, parallel plates and is deflected toward the plate of higher potential. What is the direction of the magnetic field in each of the six regions?

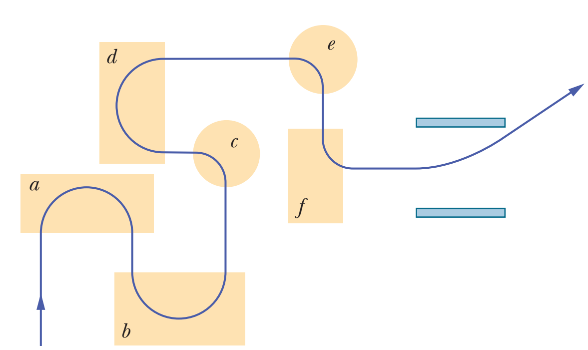



**Figure 28-25** Question 4.

**5** In Module 28-2, we discussed a charged particle moving through crossed fields with the forces $\vec{F}_E$ and $\vec{F}_B$ in opposition. We found that the particle moves in a straight line (that is, neither force dominates the motion) if its speed is given by Eq. 28-7 ($v = E/B$). Which of the two forces dominates if the speed of the particle is (a) $v < E/B$ and (b) $v > E/B$?

**6** Figure 28-26 shows crossed uniform electric and magnetic fields $\vec{E}$ and $\vec{B}$ and, at a certain instant, the velocity vectors of the 10 charged particles listed in Table 28-3. (The vectors are not drawn to scale.) The speeds given in the table are either less than or greater than $E/B$ (see Question 5). Which particles will move out of the page toward you after the instant shown in Fig. 28-26?

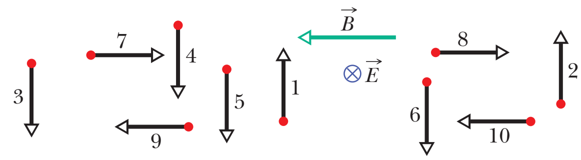



**Figure 28-26** Question 6.

**Table 28-3** Question 6

| Particle | Charge | Speed | Particle | Charge | Speed |
|---|---|---|---|---|---|
| 1 | + | Less | 6 | – | Greater |
| 2 | + | Greater | 7 | + | Less |
| 3 | + | Less | 8 | + | Greater |
| 4 | + | Greater | 9 | – | Less |
| 5 | – | Less | 10 | – | Greater |

**7** Figure 28-27 shows the path of an electron that passes through two regions containing uniform magnetic fields of magnitudes $B_1$ and $B_2$. Its path in each region is a half-circle. (a) Which field is stronger? (b) What is the direction of each field? (c) Is the time spent by the electron in the $\vec{B}_1$ region greater than, less than, or the same as the time spent in the $\vec{B}_2$ region?

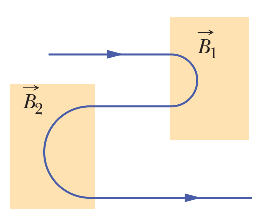



**Figure 28-27** Question 7.

**8** Figure 28-28 shows the path of an electron in a region of uniform magnetic field. The path consists of two straight sections, each between a pair of uniformly charged plates, and two half-circles. Which plate is at the higher electric potential in (a) the top pair of plates and (b) the bottom pair? (c) What is the direction of the magnetic field?

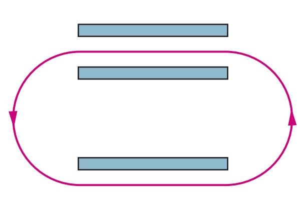


**Figure 28-28** Question 8.

**9** (a) In Checkpoint 5, if the dipole moment $\vec{\mu}$ is rotated from orientation 2 to orientation 1 by an external agent, is the work done on the dipole by the agent positive, negative, or zero? (b) Rank the work done on the dipole by the agent for these three rotations, greatest first: $2 \to 1$, $2 \to 4$, $2 \to 3$.

**10** *Particle roundabout.* Figure 28-29 shows 11 paths through a region of uniform magnetic field. One path is a straight line; the rest are half-circles. Table 28-4 gives the masses, charges, and speeds of 11 particles that take these paths through the field in the directions shown. Which path in the figure corresponds to which particle in the table? (The direction of the magnetic field can be determined by means of one of the paths, which is unique.)

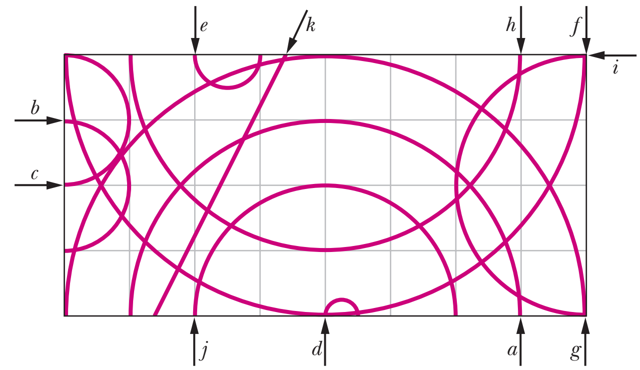



**Figure 28-29** Question 10.

**Table 28-4** Question 10

| Particle | Mass | Charge | Speed |
|---|---|---|---|
| 1 | $2m$ | $q$ | $v$ |
| 2 | $m$ | $2q$ | $v$ |
| 3 | $m/2$ | $q$ | $2v$ |
| 4 | $3m$ | $3q$ | $3v$ |
| 5 | $2m$ | $q$ | $2v$ |
| 6 | $m$ | $-q$ | $2v$ |
| 7 | $m$ | $-4q$ | $v$ |
| 8 | $m$ | $-q$ | $v$ |
| 9 | $2m$ | $-2q$ | $3v$ |
| 10 | $m$ | $-2q$ | $8v$ |
| 11 | $3m$ | 0 | $3v$ |

**11** In Fig. 28-30, a charged particle enters a uniform magnetic field $\vec{B}$ with speed $v_0$, moves through a half-circle in time $T_0$, and then leaves the field. (a) Is the charge positive or negative? (b) Is the final speed of the particle greater than, less than, or equal to $v_0$? (c) If the initial speed had been $0.5v_0$, would the time spent in field $\vec{B}$ have been greater than, less than, or equal to $T_0$? (d) Would the path have been a half-circle, more than a half-circle, or less than a half-circle?

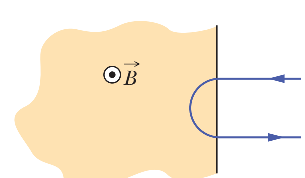



**Figure 28-30** Question 11.

**12** Figure 28-31 gives snapshots for three situations in which a positively charged particle passes through a uniform magnetic field $\vec{B}$. The velocities $\vec{v}$ of the particle differ in orientation in the three snapshots but not in magnitude. Rank the situations according to (a) the period, (b) the frequency, and (c) the pitch of the particle's motion, greatest first.

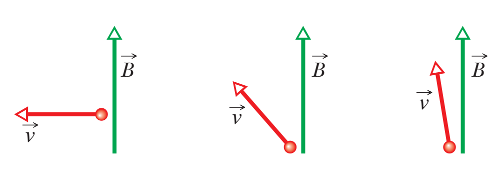



**Figure 28-31** Question 12.

# Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS* and WebAssign
SSM Worked-out solution available in Student Solutions Manual
• – ••• Number of dots indicates level of problem difficulty
Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com
WWW Worked-out solution is at
ILW Interactive solution is at
http://www.wiley.com/college/halliday

## Module 28-1 Magnetic Fields and the Definition of $\vec{B}$

**•1** SSM ILW A proton traveling at 23.0° with respect to the direction of a magnetic field of strength 2.60 mT experiences a magnetic force of $6.50 \times 10^{-17}$ N. Calculate (a) the proton's speed and (b) its kinetic energy in electron-volts.

**•2** A particle of mass 10 g and charge 80 $\mu$C moves through a uniform magnetic field, in a region where the free-fall acceleration is $-9.8\hat{j}$ m/s$^2$. The velocity of the particle is a constant $20\hat{i}$ km/s, which is perpendicular to the magnetic field. What, then, is the magnetic field?

**•3** An electron that has an instantaneous velocity of

$$\vec{v} = (2.0 \times 10^6 \text{ m/s})\hat{i} + (3.0 \times 10^6 \text{ m/s})\hat{j}$$

is moving through the uniform magnetic field $\vec{B} = (0.030 \text{ T})\hat{i} - (0.15 \text{ T})\hat{j}$. (a) Find the force on the electron due to the magnetic field. (b) Repeat your calculation for a proton having the same velocity.

**•4** An alpha particle travels at a velocity $\vec{v}$ of magnitude 550 m/s through a uniform magnetic field $\vec{B}$ of magnitude 0.045 T. (An alpha particle has a charge of $+3.2 \times 10^{-19}$ C and a mass of $6.6 \times 10^{-27}$ kg.) The angle between $\vec{v}$ and $\vec{B}$ is 52°. What is the magnitude of (a) the force $\vec{F}_B$ acting on the particle due to the field and (b) the acceleration of the particle due to $\vec{F}_B$? (c) Does the speed of the particle increase, decrease, or remain the same?

**••5** GO An electron moves through a uniform magnetic field given by $\vec{B} = B_x\hat{i} + (3.0B_x)\hat{j}$. At a particular instant, the electron has velocity $\vec{v} = (2.0\hat{i} + 4.0\hat{j})$ m/s and the magnetic force acting on it is $(6.4 \times 10^{-19} \text{ N})\hat{k}$. Find $B_x$.

**••6** GO A proton moves through a uniform magnetic field given by $\vec{B} = (10\hat{i} - 20\hat{j} + 30\hat{k})$ mT. At time $t_1$, the proton has a velocity given by $\vec{v} = v_x\hat{i} + v_y\hat{j} + (2.0 \text{ km/s})\hat{k}$ and the magnetic force on the proton is $\vec{F}_B = (4.0 \times 10^{-17} \text{ N})\hat{i} + (2.0 \times 10^{-17} \text{ N})\hat{j}$. At that instant, what are (a) $v_x$ and (b) $v_y$?

## Module 28-2 Crossed Fields: Discovery of the Electron

**•7** An electron has an initial velocity of $(12.0\hat{j} + 15.0\hat{k})$ km/s and a constant acceleration of $(2.00 \times 10^{12} \text{ m/s}^2)\hat{i}$ in a region in which uniform electric and magnetic fields are present. If $\vec{B} = (400 \,\mu\text{T})\hat{i}$, find the electric field $\vec{E}$.

**•8** An electric field of 1.50 kV/m and a perpendicular magnetic field of 0.400 T act on a moving electron to produce no net force. What is the electron's speed?

**•9** ILW In Fig. 28-32, an electron accelerated from rest through potential difference $V_1 = 1.00$ kV enters the gap between two parallel plates having separation $d = 20.0$ mm and potential difference $V_2 = 100$ V. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?

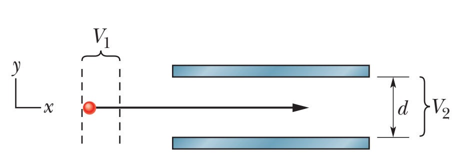



**Figure 28-32** Problem 9.

**••10** A proton travels through uniform magnetic and electric fields. The magnetic field is $\vec{B} = -2.50\hat{i}$ mT. At one instant the velocity of the proton is $\vec{v} = 2000\hat{j}$ m/s. At that instant and in unit-vector notation, what is the net force acting on the proton if the electric field is (a) $4.00\hat{k}$ V/m, (b) $-4.00\hat{k}$ V/m, and (c) $4.00\hat{i}$ V/m?

**••11** GO An ion source is producing $^6$Li ions, which have charge $+e$ and mass $9.99 \times 10^{-27}$ kg. The ions are accelerated by a potential difference of 10 kV and pass horizontally into a region in which there is a uniform vertical magnetic field of magnitude $B = 1.2$ T. Calculate the strength of the electric field, to be set up over the same region, that will allow the $^6$Li ions to pass through without any deflection.

**•••12** GO At time $t_1$, an electron is sent along the positive direction of an $x$ axis, through both an electric field $\vec{E}$ and a magnetic field $\vec{B}$, with $\vec{E}$ directed parallel to the $y$ axis. Figure 28-33 gives the $y$ component $F_{\text{net},y}$ of the net force on the electron due to the two fields, as a function of the electron's speed $v$ at time $t_1$. The scale of the velocity axis is set by $v_s = 100.0$ m/s. The $x$ and $z$ components of the net force are zero at $t_1$. Assuming $B_x = 0$, find (a) the magnitude $E$ and (b) $\vec{B}$ in unit-vector notation.

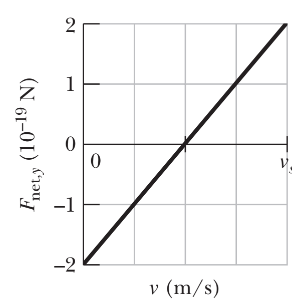



**Figure 28-33** Problem 12.

## Module 28-3 Crossed Fields: The Hall Effect

**•13** A strip of copper 150 $\mu$m thick and 4.5 mm wide is placed in a uniform magnetic field $\vec{B}$ of magnitude 0.65 T, with $\vec{B}$ perpendicular to the strip. A current $i = 23$ A is then sent through the strip such that a Hall potential difference $V$ appears across the width of the strip. Calculate $V$. (The number of charge carriers per unit volume for copper is $8.47 \times 10^{28}$ electrons/m$^3$.)

**•14** A metal strip 6.50 cm long, 0.850 cm wide, and 0.760 mm thick moves with constant velocity $\vec{v}$ through a uniform magnetic field $B = 1.20$ mT directed perpendicular to the strip, as shown in Fig. 28-34. A potential difference of 3.90 $\mu$V is measured between points $x$ and $y$ across the strip. Calculate the speed $v$.

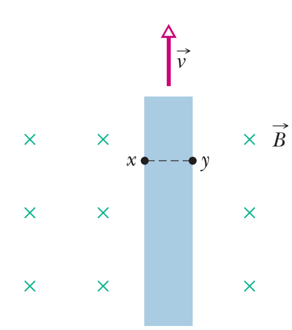



**Figure 28-34** Problem 14.

**••15** GO A conducting rectangular solid of dimensions $d_x = 5.00$ m, $d_y = 3.00$ m, and $d_z = 2.00$ m moves with a constant velocity $\vec{v} = (20.0 \text{ m/s})\hat{i}$ through a uniform magnetic field $\vec{B} = (30.0 \text{ mT})\hat{j}$ (Fig. 28-35). What are the resulting (a) electric field within the solid, in unit-vector notation, and (b) potential difference across the solid?

**•••16** GO Figure 28-35 shows a metallic block, with its faces parallel to coordinate axes. The block is in a uniform magnetic field of magnitude 0.020 T. One edge length of the block is 25 cm; the block is *not* drawn to scale. The block is moved at 3.0 m/s parallel to each axis, in turn, and the resulting potential difference $V$ that appears across the block is measured. With the motion parallel to the $y$ axis, $V = 12$ mV; with the motion parallel to the $z$ axis, $V = 18$ mV; with the motion parallel to the $x$ axis, $V = 0$. What are the block lengths (a) $d_x$, (b) $d_y$, and (c) $d_z$?

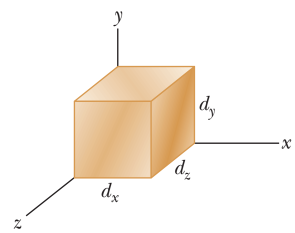


**Figure 28-35** Problems 15 and 16.

## Module 28-4 A Circulating Charged Particle

**•17** An alpha particle can be produced in certain radioactive decays of nuclei and consists of two protons and two neutrons. The particle has a charge of $q = +2e$ and a mass of 4.00 u, where u is the atomic mass unit, with $1 \text{ u} = 1.661 \times 10^{-27}$ kg. Suppose an alpha particle travels in a circular path of radius 4.50 cm in a uniform magnetic field with $B = 1.20$ T. Calculate (a) its speed, (b) its period of revolution, (c) its kinetic energy, and (d) the potential difference through which it would have to be accelerated to achieve this energy.

**•18** GO In Fig. 28-36, a particle moves along a circle in a region of uniform magnetic field of magnitude $B = 4.00$ mT. The particle is either a proton or an electron (you must decide which). It experiences a magnetic force of magnitude $3.20 \times 10^{-15}$ N. What are (a) the particle's speed, (b) the radius of the circle, and (c) the period of the motion?

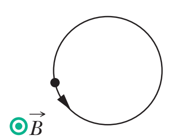


**Figure 28-36** Problem 18.

**•19** A certain particle is sent into a uniform magnetic field, with the particle's velocity vector perpendicular to the direction of the field. Figure 28-37 gives the period $T$ of the particle's motion versus the *inverse* of the field magnitude $B$. The vertical axis scale is set by $T_s = 40.0$ ns, and the horizontal axis scale is set by $B_s^{-1} = 5.0 \text{ T}^{-1}$. What is the ratio $m/q$ of the particle's mass to the magnitude of its charge?

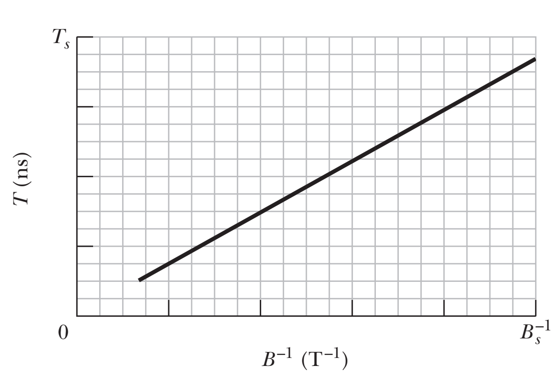


**Figure 28-37** Problem 19.

**•20** An electron is accelerated from rest through potential difference $V$ and then enters a region of uniform magnetic field, where it undergoes uniform circular motion. Figure 28-38 gives the radius $r$ of that motion versus $V^{1/2}$. The vertical axis scale is set by $r_s = 3.0$ mm, and the horizontal axis scale is set by $V_s^{1/2} = 40.0 \text{ V}^{1/2}$. What is the magnitude of the magnetic field?

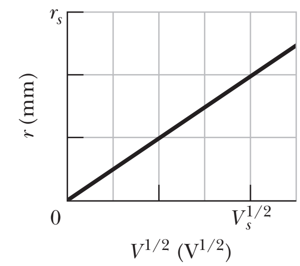


**Figure 28-38** Problem 20.

**•21** SSM An electron of kinetic energy 1.20 keV circles in a plane perpendicular to a uniform magnetic field. The orbit radius is 25.0 cm. Find (a) the electron's speed, (b) the magnetic field magnitude, (c) the circling frequency, and (d) the period of the motion.

**•22** In a nuclear experiment a proton with kinetic energy 1.0 MeV moves in a circular path in a uniform magnetic field. What energy must (a) an alpha particle ($q = +2e$, $m = 4.0$ u) and (b) a deuteron ($q = +e$, $m = 2.0$ u) have if they are to circulate in the same circular path?

**•23** What uniform magnetic field, applied perpendicular to a beam of electrons moving at $1.30 \times 10^6$ m/s, is required to make the electrons travel in a circular arc of radius 0.350 m?

**•24** An electron is accelerated from rest by a potential difference of 350 V. It then enters a uniform magnetic field of magnitude 200 mT with its velocity perpendicular to the field. Calculate (a) the speed of the electron and (b) the radius of its path in the magnetic field.

**•25** (a) Find the frequency of revolution of an electron with an energy of 100 eV in a uniform magnetic field of magnitude 35.0 $\mu$T. (b) Calculate the radius of the path of this electron if its velocity is perpendicular to the magnetic field.

**••26** In Fig. 28-39, a charged particle moves into a region of uniform magnetic field $\vec{B}$, goes through half a circle, and then exits that region. The particle is either a proton or an electron (you must decide which). It spends 130 ns in the region. (a) What is the magnitude of $\vec{B}$? (b) If the particle is sent back through the magnetic field (along the same initial path) but with 2.00 times its previous kinetic energy, how much time does it spend in the field during this trip?

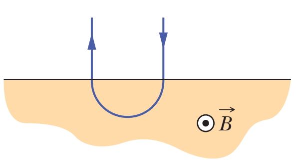


**Figure 28-39** Problem 26.

**••27** A mass spectrometer (Fig. 28-12) is used to separate uranium ions of mass $3.92 \times 10^{-25}$ kg and charge $3.20 \times 10^{-19}$ C from related species. The ions are accelerated through a potential difference of 100 kV and then pass into a uniform magnetic field, where they are bent in a path of radius 1.00 m. After traveling through 180° and passing through a slit of width 1.00 mm and height 1.00 cm, they are collected in a cup. (a) What is the magnitude of the (perpendicular) magnetic field in the separator? If the machine is used to separate out 100 mg of material per hour, calculate (b) the current of the desired ions in the machine and (c) the thermal energy produced in the cup in 1.00 h.

**••28** A particle undergoes uniform circular motion of radius 26.1 $\mu$m in a uniform magnetic field. The magnetic force on the particle has a magnitude of $1.60 \times 10^{-17}$ N. What is the kinetic energy of the particle?

**••29** An electron follows a helical path in a uniform magnetic field of magnitude 0.300 T. The pitch of the path is 6.00 $\mu$m, and the magnitude of the magnetic force on the electron is $2.00 \times 10^{-15}$ N. What is the electron's speed?

**••30** GO In Fig. 28-40, an electron with an initial kinetic energy of 4.0 keV enters region 1 at time $t = 0$. That region contains a uniform magnetic field directed into the page, with magnitude 0.010 T. The electron goes through a half-circle and then exits region 1, headed toward region 2 across a gap of 25.0 cm. There is an electric potential difference $\Delta V = 2000$ V across the gap, with a polarity such that the electron's speed increases uniformly as it traverses the gap. Region 2 contains a uniform magnetic field directed out of the page, with magnitude 0.020 T. The electron goes through a half-circle and then leaves region 2. At what time $t$ does it leave?

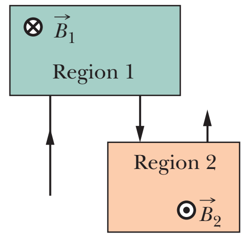


**Figure 28-40** Problem 30.

**••31** A particular type of fundamental particle decays by transforming into an electron $e^-$ and a positron $e^+$. Suppose the decaying particle is at rest in a uniform magnetic field $\vec{B}$ of magnitude 3.53 mT and the $e^-$ and $e^+$ move away from the decay point in paths lying in a plane perpendicular to $\vec{B}$. How long after the decay do the $e^-$ and $e^+$ collide?

**••32** A source injects an electron of speed $v = 1.5 \times 10^7$ m/s into a uniform magnetic field of magnitude $B = 1.0 \times 10^{-3}$ T. The velocity of the electron makes an angle $\theta = 10°$ with the direction of the magnetic field. Find the distance $d$ from the point of injection at which the electron next crosses the field line that passes through the injection point.

**••33** SSM WWW A positron with kinetic energy 2.00 keV is projected into a uniform magnetic field $\vec{B}$ of magnitude 0.100 T, with its velocity vector making an angle of 89.0° with $\vec{B}$. Find (a) the period, (b) the pitch $p$, and (c) the radius $r$ of its helical path.

**••34** An electron follows a helical path in a uniform magnetic field given by $\vec{B} = (20\hat{i} - 50\hat{j} - 30\hat{k})$ mT. At time $t = 0$, the electron's velocity is given by $\vec{v} = (20\hat{i} - 30\hat{j} + 50\hat{k})$ m/s. (a) What is the angle $\phi$ between $\vec{v}$ and $\vec{B}$? The electron's velocity changes with time. Do (b) its speed and (c) the angle $\phi$ change with time? (d) What is the radius of the helical path?

## Module 28-5 Cyclotrons and Synchrotrons

**••35** A proton circulates in a cyclotron, beginning approximately at rest at the center. Whenever it passes through the gap between dees, the electric potential difference between the dees is 200 V. (a) By how much does its kinetic energy increase with each passage through the gap? (b) What is its kinetic energy as it completes 100 passes through the gap? Let $r_{100}$ be the radius of the proton's circular path as it completes those 100 passes and enters a dee, and let $r_{101}$ be its next radius, as it enters a dee the next time. (c) By what percentage does the radius increase when it changes from $r_{100}$ to $r_{101}$? That is, what is

$$\text{percentage increase} = \frac{r_{101} - r_{100}}{r_{100}} 100\%?$$

**••36** A cyclotron with dee radius 53.0 cm is operated at an oscillator frequency of 12.0 MHz to accelerate protons. (a) What magnitude $B$ of magnetic field is required to achieve resonance? (b) At that field magnitude, what is the kinetic energy of a proton emerging from the cyclotron? Suppose, instead, that $B = 1.57$ T. (c) What oscillator frequency is required to achieve resonance now? (d) At that frequency, what is the kinetic energy of an emerging proton?

**••37** Estimate the total path length traveled by a deuteron in a cyclotron of radius 53 cm and operating frequency 12 MHz during the (entire) acceleration process. Assume that the accelerating potential between the dees is 80 kV.

**••38** In a certain cyclotron a proton moves in a circle of radius 0.500 m. The magnitude of the magnetic field is 1.20 T. (a) What is the oscillator frequency? (b) What is the kinetic energy of the proton, in electron-volts?

## Module 28-6 Magnetic Force on a Current-Carrying Wire

**•39** SSM A horizontal power line carries a current of 5000 A from south to north. Earth's magnetic field (60.0 μT) is directed toward the north and inclined downward at 70.0° to the horizontal. Find the (a) magnitude and (b) direction of the magnetic force on 100 m of the line due to Earth's field.

**•40** A wire 1.80 m long carries a current of 13.0 A and makes an angle of 35.0° with a uniform magnetic field of magnitude $B = 1.50$ T. Calculate the magnetic force on the wire.

**•41** ILW A 13.0 g wire of length $L = 62.0$ cm is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T (Fig. 28-41). What are the (a) magnitude and (b) direction (left or right) of the current required to remove the tension in the supporting leads?

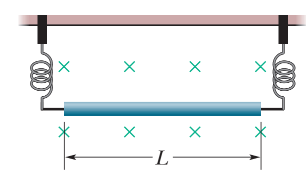


**Figure 28-41** Problem 41.

**•42** The bent wire shown in Fig. 28-42 lies in a uniform magnetic field. Each straight section is 2.0 m long and makes an angle of $\theta = 60°$ with the $x$ axis, and the wire carries a current of 2.0 A. What is the net magnetic force on the wire in unit-vector notation if the magnetic field is given by (a) $4.0\hat{k}$ T and (b) $4.0\hat{i}$ T?

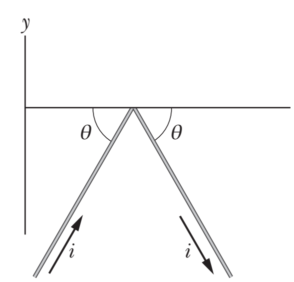


**Figure 28-42** Problem 42.

**•43** A single-turn current loop, carrying a current of 4.00 A, is in the shape of a right triangle with sides 50.0, 120, and 130 cm. The loop is in a uniform magnetic field of magnitude 75.0 mT whose direction is parallel to the current in the 130 cm side of the loop. What is the magnitude of the magnetic force on (a) the 130 cm side, (b) the 50.0 cm side, and (c) the 120 cm side? (d) What is the magnitude of the net force on the loop?

**••44** Figure 28-43 shows a wire ring of radius $a = 1.8$ cm that is perpendicular to the general direction of a radially symmetric, diverging magnetic field. The magnetic field at the ring is everywhere of the same magnitude $B = 3.4$ mT, and its direction at the ring everywhere makes an angle $\theta = 20°$ with a normal to the plane of the ring. The twisted lead wires have no effect on the problem. Find the magnitude of the force the field exerts on the ring if the ring carries a current $i = 4.6$ mA.

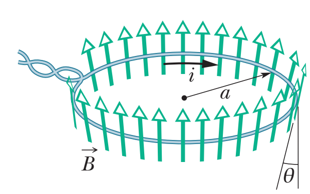


**Figure 28-43** Problem 44.

**••45** A wire 50.0 cm long carries a 0.500 A current in the positive direction of an $x$ axis through a magnetic field $\vec{B} = (3.00\text{ mT})\hat{j} + (10.0\text{ mT})\hat{k}$. In unit-vector notation, what is the magnetic force on the wire?

**••46** In Fig. 28-44, a metal wire of mass $m = 24.1$ mg can slide with negligible friction on two horizontal parallel rails separated by distance $d = 2.56$ cm. The track lies in a vertical uniform magnetic field of magnitude 56.3 mT. At time $t = 0$, device $G$ is connected to the rails, producing a constant current $i = 9.13$ mA in the wire and rails (even as the wire moves). At $t = 61.1$ ms, what are the wire's (a) speed and (b) direction of motion (left or right)?

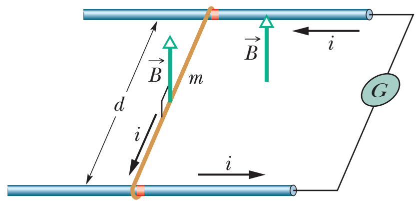



**Figure 28-44** Problem 46.

**•••47** GO A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What are the (a) magnitude and (b) angle (relative to the vertical) of the smallest magnetic field that puts the rod on the verge of sliding?

**•••48** GO A long, rigid conductor, lying along an $x$ axis, carries a current of 5.0 A in the negative $x$ direction. A magnetic field $\vec{B}$ is present, given by $\vec{B} = 3.0\hat{i} + 8.0x^2\hat{j}$, with $x$ in meters and $\vec{B}$ in milliteslas. Find, in unit-vector notation, the force on the 2.0 m segment of the conductor that lies between $x = 1.0$ m and $x = 3.0$ m.

## Module 28-7 Torque on a Current Loop

**•49** SSM Figure 28-45 shows a rectangular 20-turn coil of wire, of dimensions 10 cm by 5.0 cm. It carries a current of 0.10 A and is hinged along one long side. It is mounted in the $xy$ plane, at angle $\theta = 30°$ to the direction of a uniform magnetic field of magnitude 0.50 T. In unit-vector notation, what is the torque acting on the coil about the hinge line?

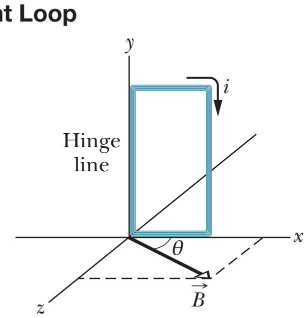



**Figure 28-45** Problem 49.

**••50** An electron moves in a circle of radius $r = 5.29 \times 10^{-11}$ m with speed $2.19 \times 10^6$ m/s. Treat the circular path as a current loop with a constant current equal to the ratio of the electron's charge magnitude to the period of the motion. If the circle lies in a uniform magnetic field of magnitude $B = 7.10$ mT, what is the maximum possible magnitude of the torque produced on the loop by the field?

**••51** Figure 28-46 shows a wood cylinder of mass $m = 0.250$ kg and length $L = 0.100$ m, with $N = 10.0$ turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle $\theta$ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.500 T, what is the least current $i$ through the coil that keeps the cylinder from rolling down the plane?

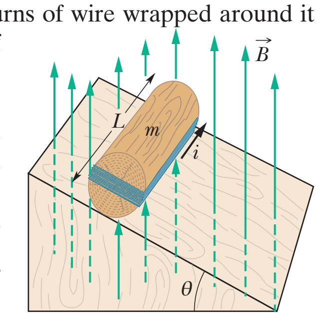



**Figure 28-46** Problem 51.

**••52** In Fig. 28-47, a rectangular loop carrying current lies in the plane of a uniform magnetic field of magnitude 0.040 T. The loop consists of a single turn of flexible conducting wire that is wrapped around a flexible mount such that the dimensions of the rectangle can be changed. (The total length of the wire is not changed.) As edge length $x$ is varied from approximately zero to its maximum value of approximately 4.0 cm, the magnitude $\tau$ of the torque on the loop changes. The maximum value of $\tau$ is $4.80 \times 10^{-8}$ N·m. What is the current in the loop?

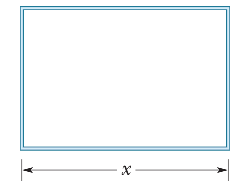



**Figure 28-47** Problem 52.

**••53** Prove that the relation $\tau = NiAB \sin\theta$ holds not only for the rectangular loop of Fig. 28-19 but also for a closed loop of any shape. (*Hint:* Replace the loop of arbitrary shape with an assembly of adjacent long, thin, approximately rectangular loops that are nearly equivalent to the loop of arbitrary shape as far as the distribution of current is concerned.)

## Module 28-8 The Magnetic Dipole Moment

**•54** A magnetic dipole with a dipole moment of magnitude 0.020 J/T is released from rest in a uniform magnetic field of magnitude 52 mT. The rotation of the dipole due to the magnetic force on it is unimpeded. When the dipole rotates through the orientation where its dipole moment is aligned with the magnetic field, its kinetic energy is 0.80 mJ. (a) What is the initial angle between the dipole moment and the magnetic field? (b) What is the angle when the dipole is next (momentarily) at rest?

**•55** SSM Two concentric, circular wire loops, of radii $r_1 = 20.0$ cm and $r_2 = 30.0$ cm, are located in an $xy$ plane; each carries a clockwise current of 7.00 A (Fig. 28-48). (a) Find the magnitude of the net magnetic dipole moment of the system. (b) Repeat for reversed current in the inner loop.

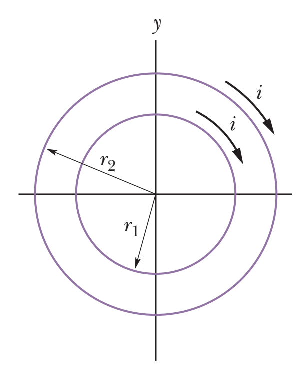



**Figure 28-48** Problem 55.

**•56** A circular wire loop of radius 15.0 cm carries a current of 2.60 A. It is placed so that the normal to its plane makes an angle of 41.0° with a uniform magnetic field of magnitude 12.0 T. (a) Calculate the magnitude of the magnetic dipole moment of the loop. (b) What is the magnitude of the torque acting on the loop?

**•57** SSM A circular coil of 160 turns has a radius of 1.90 cm. (a) Calculate the current that results in a magnetic dipole moment of magnitude 2.30 A·m². (b) Find the maximum magnitude of the torque that the coil, carrying this current, can experience in a uniform 35.0 mT magnetic field.

**•58** The magnetic dipole moment of Earth has magnitude $8.00 \times 10^{22}$ J/T. Assume that this is produced by charges flowing in Earth's molten outer core. If the radius of their circular path is 3500 km, calculate the current they produce.

**•59** A current loop, carrying a current of 5.0 A, is in the shape of a right triangle with sides 30, 40, and 50 cm. The loop is in a uniform magnetic field of magnitude 80 mT whose direction is parallel to the current in the 50 cm side of the loop. Find the magnitude of (a) the magnetic dipole moment of the loop and (b) the torque on the loop.

**••60** Figure 28-49 shows a current loop $ABCDEFA$ carrying a current $i = 5.00$ A. The sides of the loop are parallel to the coordinate axes shown, with $AB = 20.0$ cm, $BC = 30.0$ cm, and $FA = 10.0$ cm. In unit-vector notation, what is the magnetic dipole moment of this loop? (*Hint:* Imagine equal and opposite currents $i$ in the line segment $AD$; then treat the two rectangular loops $ABCDA$ and $ADEFA$.)

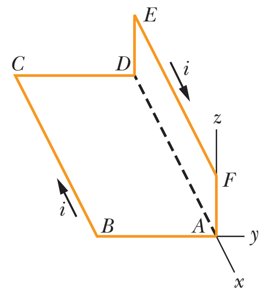


**Figure 28-49** Problem 60.

**••61** SSM The coil in Fig. 28-50 carries current $i = 2.00$ A in the direction indicated, is parallel to an $xz$ plane, has 3.00 turns and an area of $4.00 \times 10^{-3}$ m$^2$, and lies in a uniform magnetic field $\vec{B} = (2.00\hat{i} - 3.00\hat{j} - 4.00\hat{k})$ mT. What are (a) the orientation energy of the coil in the magnetic field and (b) the torque (in unit-vector notation) on the coil due to the magnetic field?

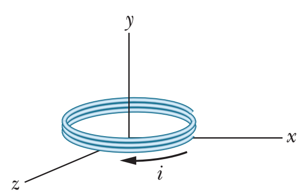


**Figure 28-50** Problem 61.

**••62** GO In Fig. 28-51*a*, two concentric coils, lying in the same plane, carry currents in opposite directions. The current in the larger coil 1 is fixed. Current $i_2$ in coil 2 can be varied. Figure 28-51*b* gives the net magnetic moment of the two-coil system as a function of $i_2$. The vertical axis scale is set by $\mu_{net,s} = 2.0 \times 10^{-5}$ A · m$^2$, and the horizontal axis scale is set by $i_{2s} = 10.0$ mA. If the current in coil 2 is then reversed, what is the magnitude of the net magnetic moment of the two-coil system when $i_2 = 7.0$ mA?

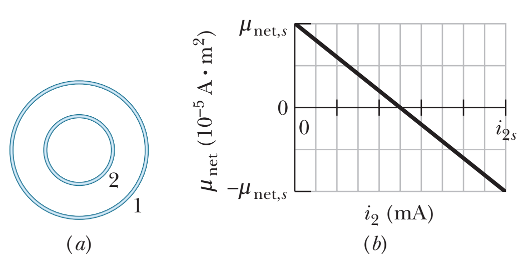


**Figure 28-51** Problem 62.

**••63** A circular loop of wire having a radius of 8.0 cm carries a current of 0.20 A. A vector of unit length and parallel to the dipole moment $\vec{\mu}$ of the loop is given by $0.60\hat{i} - 0.80\hat{j}$. (This unit vector gives the orientation of the magnetic dipole moment vector.) If the loop is located in a uniform magnetic field given by $\vec{B} = (0.25\text{ T})\hat{i} + (0.30\text{ T})\hat{k}$, find (a) the torque on the loop (in unit-vector notation) and (b) the orientation energy of the loop.

**••64** GO Figure 28-52 gives the orientation energy $U$ of a magnetic dipole in an external magnetic field $\vec{B}$, as a function of angle $\phi$ between the directions of $\vec{B}$ and the dipole moment. The vertical axis scale is set by $U_s = 2.0 \times 10^{-4}$ J. The dipole can be rotated about an axle with negligible friction in order to change $\phi$. Counterclockwise rotation from $\phi = 0$ yields positive values of $\phi$, and clockwise rotations yield negative values. The dipole is to be released at angle $\phi = 0$ with a rotational kinetic energy of $6.7 \times 10^{-4}$ J, so that it rotates counterclockwise. To what maximum value of $\phi$ will it rotate? (In the language of Module 8-3, what value $\phi$ is the turning point in the potential well of Fig. 28-52?)

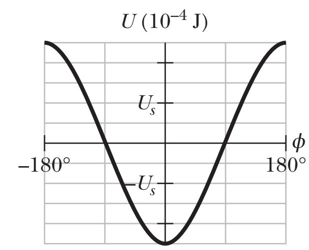


**Figure 28-52** Problem 64.

**••65** SSM ILW A wire of length 25.0 cm carrying a current of 4.51 mA is to be formed into a circular coil and placed in a uniform magnetic field $\vec{B}$ of magnitude 5.71 mT. If the torque on the coil from the field is maximized, what are (a) the angle between $\vec{B}$ and the coil's magnetic dipole moment and (b) the number of turns in the coil? (c) What is the magnitude of that maximum torque?

### Additional Problems

**66** A proton of charge $+e$ and mass $m$ enters a uniform magnetic field $\vec{B} = B\hat{i}$ with an initial velocity $\vec{v} = v_{0x}\hat{i} + v_{0y}\hat{j}$. Find an expression in unit-vector notation for its velocity $\vec{v}$ at any later time $t$.

**67** A stationary circular wall clock has a face with a radius of 15 cm. Six turns of wire are wound around its perimeter; the wire carries a current of 2.0 A in the clockwise direction. The clock is located where there is a constant, uniform external magnetic field of magnitude 70 mT (but the clock still keeps perfect time). At exactly 1:00 P.M., the hour hand of the clock points in the direction of the external magnetic field. (a) After how many minutes will the minute hand point in the direction of the torque on the winding due to the magnetic field? (b) Find the torque magnitude.

**68** A wire lying along a $y$ axis from $y = 0$ to $y = 0.250$ m carries a current of 2.00 mA in the negative direction of the axis. The wire fully lies in a nonuniform magnetic field that is given by $\vec{B} = (0.300\text{ T/m})y\hat{i} + (0.400\text{ T/m})y\hat{j}$. In unit-vector notation, what is the magnetic force on the wire?

**69** Atom 1 of mass 35 u and atom 2 of mass 37 u are both singly ionized with a charge of $+e$. After being introduced into a mass spectrometer (Fig. 28-12) and accelerated from rest through a potential difference $V = 7.3$ kV, each ion follows a circular path in a uniform magnetic field of magnitude $B = 0.50$ T. What is the distance $\Delta x$ between the points where the ions strike the detector?

**70** An electron with kinetic energy 2.5 keV moving along the positive direction of an $x$ axis enters a region in which a uniform electric field of magnitude 10 kV/m is in the negative direction of the $y$ axis. A uniform magnetic field $\vec{B}$ is to be set up to keep the electron moving along the $x$ axis, and the direction of $\vec{B}$ is to be chosen to minimize the required magnitude of $\vec{B}$. In unit-vector notation, what $\vec{B}$ should be set up?

**71** Physicist S. A. Goudsmit devised a method for measuring the mass of heavy ions by timing their period of revolution in a known magnetic field. A singly charged ion of iodine makes 7.00 rev in a 45.0 mT field in 1.29 ms. Calculate its mass in atomic mass units.

**72** A beam of electrons whose kinetic energy is $K$ emerges from a thin-foil "window" at the end of an accelerator tube. A metal plate at distance $d$ from this window is perpendicular to the direction of the emerging beam (Fig. 28-53). (a) Show that we can prevent the beam from hitting the plate if we apply a uniform magnetic field such that

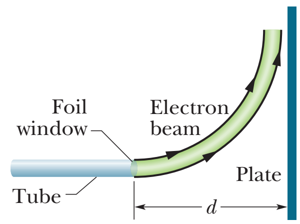



**Figure 28-53** Problem 72.

$$B \geq \sqrt{\frac{2mK}{e^2 d^2}},$$

in which $m$ and $e$ are the electron mass and charge. (b) How should $\vec{B}$ be oriented?

**73** SSM At time $t = 0$, an electron with kinetic energy 12 keV moves through $x = 0$ in the positive direction of an $x$ axis that is parallel to the horizontal component of Earth's magnetic field $\vec{B}$. The field's vertical component is downward and has magnitude $55.0\ \mu$T. (a) What is the magnitude of the electron's acceleration due to $\vec{B}$? (b) What is the electron's distance from the $x$ axis when the electron reaches coordinate $x = 20$ cm?

**74** GO A particle with charge 2.0 C moves through a uniform magnetic field. At one instant the velocity of the particle is $(2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k})$ m/s and the magnetic force on the particle is $(4.0\hat{i} - 20\hat{j} + 12\hat{k})$ N. The $x$ and $y$ components of the magnetic field are equal. What is $\vec{B}$?

**75** A proton, a deuteron ($q = +e$, $m = 2.0$ u), and an alpha particle ($q = +2e$, $m = 4.0$ u) all having the same kinetic energy enter a region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. What is the ratio of (a) the radius $r_d$ of the deuteron path to the radius $r_p$ of the proton path and (b) the radius $r_\alpha$ of the alpha particle path to $r_p$?

**76** Bainbridge's mass spectrometer, shown in Fig. 28-54, separates ions having the same velocity. The ions, after entering through slits, $S_1$ and $S_2$, pass through a velocity selector composed of an electric field produced by the charged plates P and P′, and a magnetic field $\vec{B}$ perpendicular to the electric field and the ion path. The ions that then pass undeviated through the crossed $\vec{E}$ and $\vec{B}$ fields enter into a region where a second magnetic field $\vec{B}'$ exists, where they are made to follow circular paths. A photographic plate (or a modern detector) registers their arrival. Show that, for the ions, $q/m = E/rBB'$, where $r$ is the radius of the circular orbit.

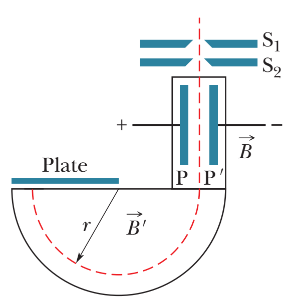



**Figure 28-54** Problem 76.

**77** SSM In Fig. 28-55, an electron moves at speed $v = 100$ m/s along an $x$ axis through uniform electric and magnetic fields. The magnetic field $\vec{B}$ is directed into the page and has magnitude 5.00 T. In unit-vector notation, what is the electric field?

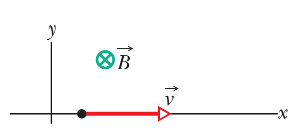



**Figure 28-55** Problem 77.

**78** (a) In Fig. 28-8, show that the ratio of the Hall electric field magnitude $E$ to the magnitude $E_C$ of the electric field responsible for moving charge (the current) along the length of the strip is

$$\frac{E}{E_C} = \frac{B}{ne\rho},$$

where $\rho$ is the resistivity of the material and $n$ is the number density of the charge carriers. (b) Compute this ratio numerically for Problem 13. (See Table 26-1.)

**79** SSM A proton, a deuteron ($q = +e$, $m = 2.0$ u), and an alpha particle ($q = +2e$, $m = 4.0$ u) are accelerated through the same potential difference and then enter the same region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. What is the ratio of (a) the proton's kinetic energy $K_p$ to the alpha particle's kinetic energy $K_\alpha$ and (b) the deuteron's kinetic energy $K_d$ to $K_\alpha$? If the radius of the proton's circular path is 10 cm, what is the radius of (c) the deuteron's path and (d) the alpha particle's path?

**80** An electron is moving at $7.20 \times 10^6$ m/s in a magnetic field of strength 83.0 mT. What is the (a) maximum and (b) minimum magnitude of the force acting on the electron due to the field? (c) At one point the electron has an acceleration of magnitude $4.90 \times 10^{14}$ m/s$^2$. What is the angle between the electron's velocity and the magnetic field?

**81** A $5.0\ \mu$C particle moves through a region containing the uniform magnetic field $-20\hat{i}$ mT and the uniform electric field $300\hat{j}$ V/m. At a certain instant the velocity of the particle is $(17\hat{i} - 11\hat{j} + 7.0\hat{k})$ km/s. At that instant and in unit-vector notation, what is the net electromagnetic force (the sum of the electric and magnetic forces) on the particle?

**82** In a Hall-effect experiment, a current of 3.0 A sent lengthwise through a conductor 1.0 cm wide, 4.0 cm long, and $10\ \mu$m thick produces a transverse (across the width) Hall potential difference of $10\ \mu$V when a magnetic field of 1.5 T is passed perpendicularly through the thickness of the conductor. From these data, find (a) the drift velocity of the charge carriers and (b) the number density of charge carriers. (c) Show on a diagram the polarity of the Hall potential difference with assumed current and magnetic field directions, assuming also that the charge carriers are electrons.

**83** SSM A particle of mass 6.0 g moves at 4.0 km/s in an $xy$ plane, in a region with a uniform magnetic field given by $5.0\hat{i}$ mT. At one instant, when the particle's velocity is directed 37° counterclockwise from the positive direction of the $x$ axis, the magnetic force on the particle is $0.48\hat{k}$ N. What is the particle's charge?

**84** A wire lying along an $x$ axis from $x = 0$ to $x = 1.00$ m carries a current of 3.00 A in the positive $x$ direction. The wire is immersed in a nonuniform magnetic field that is given by $\vec{B} = (4.00\ \text{T/m}^2)x^2\hat{i} - (0.600\ \text{T/m}^2)x^2\hat{j}$. In unit-vector notation, what is the magnetic force on the wire?

**85** At one instant, $\vec{v} = (-2.00\hat{i} + 4.00\hat{j} - 6.00\hat{k})$ m/s is the velocity of a proton in a uniform magnetic field $\vec{B} = (2.00\hat{i} - 4.00\hat{j} + 8.00\hat{k})T$. At that instant, what are (a) the magnetic force $\vec{F}$ acting on the proton, in unit-vector notation, (b) the angle between $\vec{v}$ and $\vec{F}$, and (c) the angle between $\vec{v}$ and $\vec{B}$?

**86** An electron has velocity $\vec{v} = (32\hat{i} + 40\hat{j})$ km/s as it enters a uniform magnetic field $\vec{B} = 60\hat{i}\ \mu$T. What are (a) the radius of the helical path taken by the electron and (b) the pitch of that path? (c) To an observer looking into the magnetic field region from the entrance point of the electron, does the electron spiral clockwise or counterclockwise as it moves?

**87** Figure 28-56 shows a *homopolar generator*, which has a solid conducting disk as rotor and which is rotated by a motor (not shown). Conducting brushes connect this emf device to a circuit through which the device drives current. The device can produce a greater emf than wire loop rotors because they can spin at a much higher angular speed without rupturing. The disk has radius $R = 0.250$ m and rotation frequency $f = 4000$ Hz, and the device is in a uniform magnetic field of magnitude $B = 60.0$ mT that is perpendicular to the disk. As the disk is rotated, conduction electrons along the conducting path (dashed line) are forced to move through the magnetic field. (a) For the indicated rotation, is the magnetic force on those electrons up or down in the figure? (b) Is the magnitude of that force greater at the rim or near the center of the disk? (c) What is the work per unit charge done in moving charge along the radial line, between the rim and the center? (d) What, then, is the emf of the device? (e) If the current is 50.0 A, what is the power at which electrical energy is being produced?

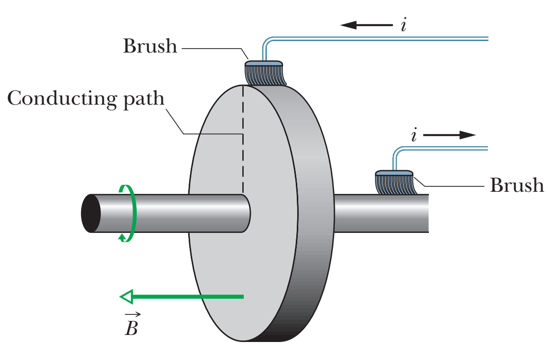



**Figure 28-56** Problem 87.

**88** In Fig. 28-57, the two ends of a **U**-shaped wire of mass $m = 10.0$ g and length $L = 20.0$ cm are immersed in mercury (which is a conductor). The wire is in a uniform field of magnitude $B = 0.100$ T. A switch (unshown) is rapidly closed and then reopened, sending a pulse of current through the wire, which causes the wire to jump upward. If jump height $h = 3.00$ m, how much charge was in the pulse? Assume that the duration of the pulse is much less than the time of flight. Consider the definition of impulse (Eq. 9-30) and its relationship with momentum (Eq. 9-31). Also consider the relationship between charge and current (Eq. 26-2).

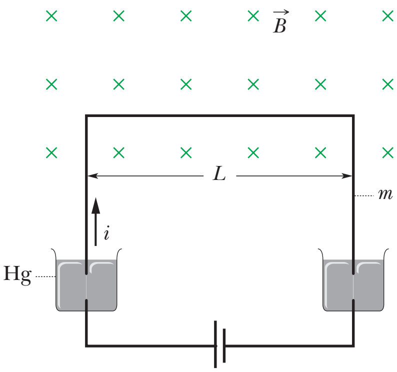



**Figure 28-57** Problem 88.

**89** In Fig. 28-58, an electron of mass $m$, charge $-e$, and low (negligible) speed enters the region between two plates of potential difference $V$ and plate separation $d$, initially headed directly toward the top plate. A uniform magnetic field of magnitude $B$ is normal to the plane of the figure. Find the minimum value of $B$ such that the electron will not strike the top plate.

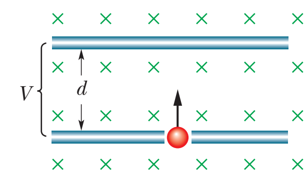



**Figure 28-58** Problem 89.

**90** A particle of charge $q$ moves in a circle of radius $r$ with speed $v$. Treating the circular path as a current loop with an average current, find the maximum torque exerted on the loop by a uniform field of magnitude $B$.

**91** In a Hall-effect experiment, express the number density of charge carriers in terms of the Hall-effect electric field magnitude $E$, the current density magnitude $J$, and the magnetic field magnitude $B$.

**92** An electron that is moving through a uniform magnetic field has velocity $\vec{v} = (40 \text{ km/s})\hat{i} + (35 \text{ km/s})\hat{j}$ when it experiences a force $\vec{F} = -(4.2 \text{ fN})\hat{i} + (4.8 \text{ fN})\hat{j}$ due to the magnetic field. If $B_x = 0$, calculate the magnetic field $\vec{B}$.

CHAPTER 29

# Magnetic Fields Due to Currents

## 29-1 MAGNETIC FIELD DUE TO A CURRENT

### Learning Objectives

*After reading this module, you should be able to . . .*

**29.01** Sketch a current-length element in a wire and indicate the direction of the magnetic field that it sets up at a given point near the wire.

**29.02** For a given point near a wire and a given current-length element in the wire, determine the magnitude and direction of the magnetic field due to that element.

**29.03** Identify the magnitude of the magnetic field set up by a current-length element at a point in line with the direction of that element.

**29.04** For a point to one side of a long straight wire carrying current, apply the relationship between the magnetic field magnitude, the current, and the distance to the point.

**29.05** For a point to one side of a long straight wire carrying current, use a right-hand rule to determine the direction of the field vector.

**29.06** Identify that around a long straight wire carrying current, the magnetic field lines form circles.

**29.07** For a point to one side of the end of a semi-infinite wire carrying current, apply the relationship between the magnetic field magnitude, the current, and the distance to the point.

**29.08** For the center of curvature of a circular arc of wire carrying current, apply the relationship between the magnetic field magnitude, the current, the radius of curvature, and the angle subtended by the arc (in radians).

**29.09** For a point to one side of a short straight wire carrying current, integrate the Biot–Savart law to find the magnetic field set up at the point by the current.

### Key Ideas

- The magnetic field set up by a current-carrying conductor can be found from the Biot–Savart law. This law asserts that the contribution $d\vec{B}$ to the field produced by a current-length element $i\,d\vec{s}$ at a point $P$ located a distance $r$ from the current element is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2} \quad \text{(Biot–Savart law).}$$

Here $\hat{r}$ is a unit vector that points from the element toward $P$. The quantity $\mu_0$, called the permeability constant, has the value

$$4\pi\times10^{-7}\ \mathrm{T\cdot m/A} \approx 1.26\times10^{-6}\ \mathrm{T\cdot m/A}.$$

- For a long straight wire carrying a current $i$, the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance $R$ from the wire,

$$B = \frac{\mu_0 i}{2\pi R} \quad \text{(long straight wire).}$$

- The magnitude of the magnetic field at the center of a circular arc, of radius $R$ and central angle $\phi$ (in radians), carrying current $i$, is

$$B = \frac{\mu_0 i\phi}{4\pi R} \quad \text{(at center of circular arc).}$$

## What Is Physics?

One basic observation of physics is that a moving charged particle produces a magnetic field around itself. Thus a current of moving charged particles produces a magnetic field around the current. This feature of *electromagnetism,* which is the combined study of electric and magnetic effects, came as a surprise to the people who discovered it. Surprise or not, this feature has become enormously important in everyday life because it is the basis of countless electromagnetic devices. For example, a magnetic field is produced in maglev trains and other devices used to lift heavy loads.

Our first step in this chapter is to find the magnetic field due to the current in a very small section of current-carrying wire. Then we shall find the magnetic field due to the entire wire for several different arrangements of the wire.

## Calculating the Magnetic Field Due to a Current

Figure 29-1 shows a wire of arbitrary shape carrying a current $i$. We want to find the magnetic field $\vec{B}$ at a nearby point $P$. We first mentally divide the wire into differential elements $ds$ and then define for each element a length vector $d\vec{s}$ that has length $ds$ and whose direction is the direction of the current in $ds$. We can then define a differential *current-length element* to be $i\,d\vec{s}$; we wish to calculate the field $d\vec{B}$ produced at $P$ by a typical current-length element. From experiment we find that magnetic fields, like electric fields, can be superimposed to find a net field. Thus, we can calculate the net field $\vec{B}$ at $P$ by summing, via integration, the contributions $d\vec{B}$ from all the current-length elements. However, this summation is more challenging than the process associated with electric fields because of a complexity; whereas a charge element $dq$ producing an electric field is a scalar, a current-length element $i\,d\vec{s}$ producing a magnetic field is a vector, being the product of a scalar and a vector.

***Magnitude.*** The magnitude of the field $d\vec{B}$ produced at point $P$ at distance $r$ by a current-length element $i\,d\vec{s}$ turns out to be

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}, \tag{29-1}$$

where $\theta$ is the angle between the directions of $d\vec{s}$ and $\hat{r}$, a unit vector that points from $ds$ toward $P$. Symbol $\mu_0$ is a constant, called the *permeability constant,* whose value is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T\cdot m/A}. \tag{29-2}$$

***Direction.*** The direction of $d\vec{B}$, shown as being into the page in Fig. 29-1, is that of the cross product $d\vec{s} \times \hat{r}$. We can therefore write Eq. 29-1 in vector form as

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2} \quad \text{(Biot–Savart law)}. \tag{29-3}$$

This vector equation and its scalar form, Eq. 29-1, are known as the **law of Biot and Savart** (rhymes with "Leo and bazaar"). The law, which is experimentally deduced, is an inverse-square law. We shall use this law to calculate the net magnetic field $\vec{B}$ produced at a point by various distributions of current.

Here is one easy distribution: If current in a wire is either directly toward or directly away from a point $P$ of measurement, can you see from Eq. 29-1 that the magnetic field at $P$ from the current is simply zero (the angle $\theta$ is either 0° for *toward* or 180° for *away*, and both result in $\sin\theta = 0$)?

This element of current creates a magnetic field at $P$, into the page.

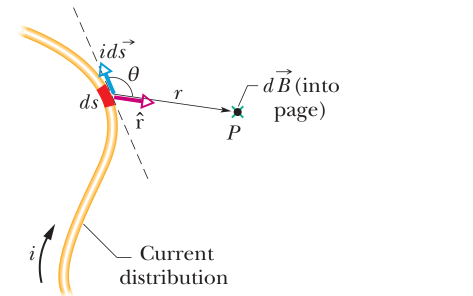



**Figure 29-1** A current-length element $i\,d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point $P$. The green × (the tail of an arrow) at the dot for point $P$ indicates that $d\vec{B}$ is directed *into* the page there.

### Magnetic Field Due to a Current in a Long Straight Wire

Shortly we shall use the law of Biot and Savart to prove that the magnitude of the magnetic field at a perpendicular distance $R$ from a long (infinite) straight wire carrying a current $i$ is given by

$$B = \frac{\mu_0 i}{2\pi R} \quad \text{(long straight wire)}. \tag{29-4}$$

The field magnitude $B$ in Eq. 29-4 depends only on the current and the perpendicular distance $R$ of the point from the wire. We shall show in our derivation that the field lines of $\vec{B}$ form concentric circles around the wire, as Fig. 29-2 shows

The magnetic field vector at any point is tangent to a circle.

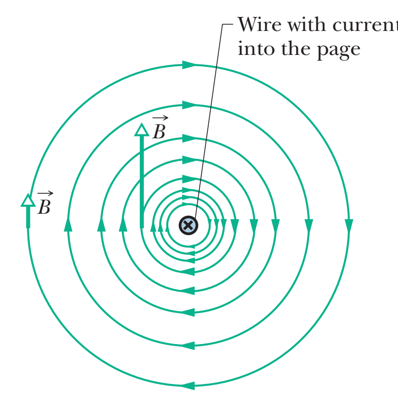



**Figure 29-2** The magnetic field lines produced by a current in a long straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the ×.

Courtesy Education Development Center

**Figure 29-3** Iron filings that have been sprinkled onto cardboard collect in concentric circles when current is sent through the central wire. The alignment, which is along magnetic field lines, is caused by the magnetic field produced by the current.

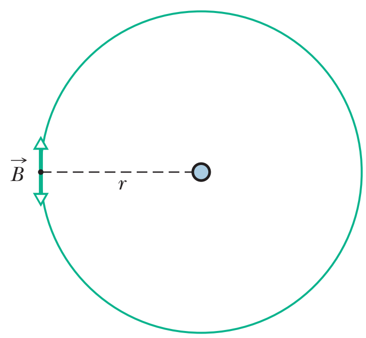



**Figure 29-4** The magnetic field vector $\vec{B}$ is perpendicular to the radial line extending from a long straight wire with current, but which of the two perpendicular vectors is it?

and as the iron filings in Fig. 29-3 suggest. The increase in the spacing of the lines in Fig. 29-2 with increasing distance from the wire represents the $1/R$ decrease in the magnitude of $\vec{B}$ predicted by Eq. 29-4. The lengths of the two vectors $\vec{B}$ in the figure also show the $1/R$ decrease.

***Directions.*** Plugging values into Eq. 29-4 to find the field magnitude $B$ at a given radius is easy. What is difficult for many students is finding the direction of a field vector $\vec{B}$ at a given point. The field lines form circles around a long straight wire, and the field vector at any point on a circle must be tangent to the circle. That means it must be perpendicular to a radial line extending to the point from the wire. But there are two possible directions for that perpendicular vector, as shown in Fig. 29-4. One is correct for current into the figure, and the other is correct for current out of the figure. How can you tell which is which? Here is a simple right-hand rule for telling which vector is correct:

> *Curled–straight right-hand rule:* Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

The result of applying this right-hand rule to the current in the straight wire of Fig. 29-2 is shown in a side view in Fig. 29-5*a*. To determine the direction of the magnetic field $\vec{B}$ set up at any particular point by this current, mentally wrap your right hand around the wire with your thumb in the direction of the current. Let your fingertips pass through the point; their direction is then the direction of the magnetic field at that point. In the view of Fig. 29-2, $\vec{B}$ at any point is *tangent to a magnetic field line;* in the view of Fig. 29-5, it is *perpendicular to a dashed radial line connecting the point and the current.*

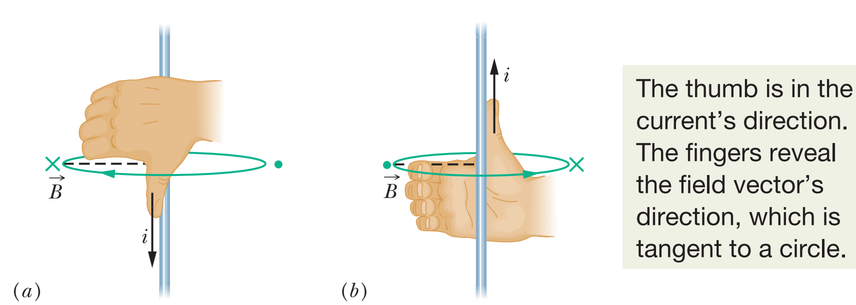



**Figure 29-5** A right-hand rule gives the direction of the magnetic field due to a current in a wire. (*a*) The situation of Fig. 29-2, seen from the side. The magnetic field $\vec{B}$ at any point to the left of the wire is perpendicular to the dashed radial line and directed into the page, in the direction of the fingertips, as indicated by the ×. (*b*) If the current is reversed, $\vec{B}$ at any point to the left is still perpendicular to the dashed radial line but now is directed out of the page, as indicated by the dot.

## Proof of Equation 29-4

Figure 29-6, which is just like Fig. 29-1 except that now the wire is straight and of infinite length, illustrates the task at hand. We seek the field $\vec{B}$ at point $P$, a perpendicular distance $R$ from the wire. The magnitude of the differential magnetic field produced at $P$ by the current-length element $i\,d\vec{s}$ located a distance $r$ from $P$ is given by Eq. 29-1:

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}.$$

The direction of $d\vec{B}$ in Fig. 29-6 is that of the vector $d\vec{s} \times \hat{r}$—namely, directly into the page.

Note that $d\vec{B}$ at point $P$ has this same direction for all the current-length elements into which the wire can be divided. Thus, we can find the magnitude of the magnetic field produced at $P$ by the current-length elements in the upper half of the infinitely long wire by integrating $dB$ in Eq. 29-1 from 0 to $\infty$.

Now consider a current-length element in the lower half of the wire, one that is as far below $P$ as $d\vec{s}$ is above $P$. By Eq. 29-3, the magnetic field produced at $P$ by this current-length element has the same magnitude and direction as that from element $i\,d\vec{s}$ in Fig. 29-6. Further, the magnetic field produced by the lower half of the wire is exactly the same as that produced by the upper half. To find the magnitude of the *total* magnetic field $\vec{B}$ at $P$, we need only multiply the result of our integration by 2. We get

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta\,ds}{r^2}. \tag{29-5}$$

The variables $\theta$, $s$, and $r$ in this equation are not independent; Fig. 29-6 shows that they are related by

$$r = \sqrt{s^2 + R^2}$$

and

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}.$$

With these substitutions and integral 19 in Appendix E, Eq. 29-5 becomes

$$\begin{aligned} B &= \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R\,ds}{(s^2 + R^2)^{3/2}} \\ &= \frac{\mu_0 i}{2\pi R}\left[\frac{s}{(s^2 + R^2)^{1/2}}\right]_0^\infty = \frac{\mu_0 i}{2\pi R}, \end{aligned} \tag{29-6}$$

as we wanted. Note that the magnetic field at $P$ due to either the lower half or the upper half of the infinite wire in Fig. 29-6 is half this value; that is,

$$B = \frac{\mu_0 i}{4\pi R} \quad \text{(semi-infinite straight wire).} \tag{29-7}$$

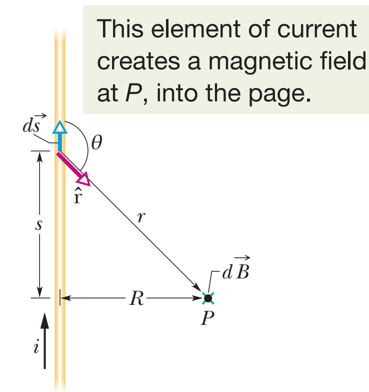



**Figure 29-6** Calculating the magnetic field produced by a current $i$ in a long straight wire. The field $d\vec{B}$ at $P$ associated with the current-length element $i\,d\vec{s}$ is directed into the page, as shown.

## Magnetic Field Due to a Current in a Circular Arc of Wire

To find the magnetic field produced at a point by a current in a curved wire, we would again use Eq. 29-1 to write the magnitude of the field produced by a single current-length element, and we would again integrate to find the net field produced by all the current-length elements. That integration can be difficult, depending on the shape of the wire; it is fairly straightforward, however, when the wire is a circular arc and the point is the center of curvature.

Figure 29-7*a* shows such an arc-shaped wire with central angle $\phi$, radius $R$, and center $C$, carrying current $i$. At $C$, each current-length element $i\,d\vec{s}$ of the wire produces a magnetic field of magnitude $dB$ given by Eq. 29-1. Moreover, as Fig. 29-7*b* shows, no matter where the element is located on the wire, the angle $\theta$

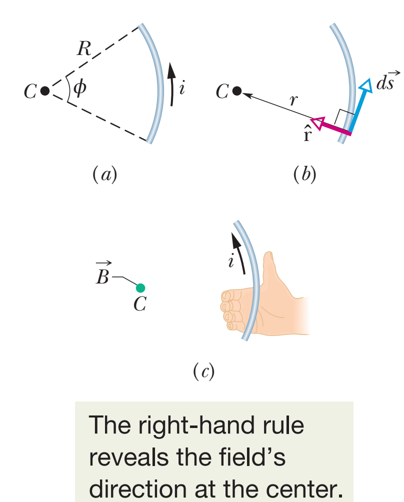



**Figure 29-7** (*a*) A wire in the shape of a circular arc with center $C$ carries current $i$. (*b*) For any element of wire along the arc, the angle between the directions of $d\vec{s}$ and $\hat{r}$ is 90°. (*c*) Determining the direction of the magnetic field at the center $C$ due to the current in the wire; the field is out of the page, in the direction of the fingertips, as indicated by the colored dot at $C$.

between the vectors $d\vec{s}$ and $\hat{r}$ is 90°; also, $r = R$. Thus, by substituting $R$ for $r$ and 90° for $\theta$ in Eq. 29-1, we obtain

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds \sin 90°}{R^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}. \tag{29-8}$$

The field at $C$ due to each current-length element in the arc has this magnitude.

***Directions.*** How about the direction of the differential field $d\vec{B}$ set up by an element? From above we know that the vector must be perpendicular to a radial line extending through point $C$ from the element, either into the plane of Fig. 29-7*a* or out of it. To tell which direction is correct, we use the right-hand rule for any of the elements, as shown in Fig. 29-7*c*. Grasping the wire with the thumb in the direction of the current and bringing the fingers into the region near $C$, we see that the vector $d\vec{B}$ due to any of the differential elements is out of the plane of the figure, not into it.

***Total Field.*** To find the total field at $C$ due to all the elements on the arc, we need to add all the differential field vectors $d\vec{B}$. However, because the vectors are all in the same direction, we do not need to find components. We just sum the magnitudes $dB$ as given by Eq. 29-8. Since we have a vast number of those magnitudes, we sum via integration. We want the result to indicate how the total field depends on the angle $\phi$ of the arc (rather than the arc length). So, in Eq. 29-8 we switch from $ds$ to $d\phi$ by using the identity $ds = R\,d\phi$. The summation by integration then becomes

$$B = \int dB = \int_0^{\phi} \frac{\mu_0}{4\pi}\frac{iR\,d\phi}{R^2} = \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi.$$

Integrating, we find that

$$B = \frac{\mu_0 i\phi}{4\pi R} \quad \text{(at center of circular arc)}. \tag{29-9}$$

***Heads Up.*** Note that this equation gives us the magnetic field *only* at the center of curvature of a circular arc of current. When you insert data into the equation, you must be careful to express $\phi$ in radians rather than degrees. For example, to find the magnitude of the magnetic field at the center of a full circle of current, you would substitute $2\pi$ rad for $\phi$ in Eq. 29-9, finding

$$B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R} \quad \text{(at center of full circle)}. \tag{29-10}$$

## Sample Problem 29.01 Magnetic field at the center of a circular arc of current

The wire in Fig. 29-8*a* carries a current $i$ and consists of a circular arc of radius $R$ and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center $C$ of the arc. What magnetic field $\vec{B}$ (magnitude and direction) does the current produce at $C$?

### KEY IDEAS

We can find the magnetic field $\vec{B}$ at point $C$ by applying the Biot–Savart law of Eq. 29-3 to the wire, point by point along the full length of the wire. However, the application of Eq. 29-3 can be simplified by evaluating $\vec{B}$ separately for the three distinguishable sections of the wire — namely, (1) the straight section at the left, (2) the straight section at the right, and (3) the circular arc.

*Straight sections:* For any current-length element in section 1, the angle $\theta$ between $d\vec{s}$ and $\hat{r}$ is zero (Fig. 29-8*b*); so Eq. 29-1 gives us

$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds \sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds \sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at $C$:

$$B_1 = 0.$$

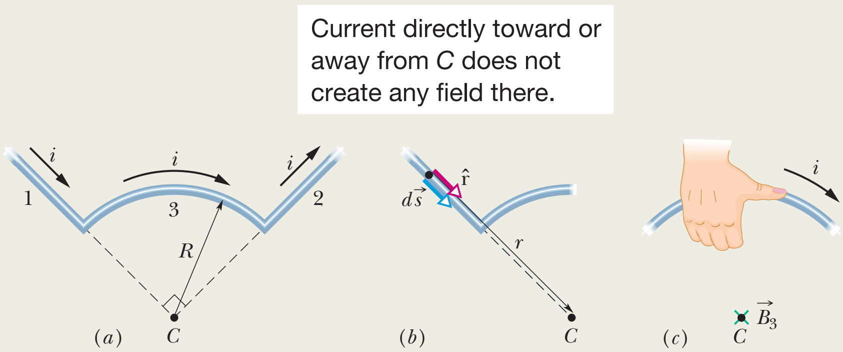



**Figure 29-8** (*a*) A wire consists of two straight sections (1 and 2) and a circular arc (3), and carries current *i*. (*b*) For a current-length element in section 1, the angle between $d\vec{s}$ and $\hat{r}$ is zero. (*c*) Determining the direction of magnetic field $\vec{B}_3$ at $C$ due to the current in the circular arc; the field is into the page there.

The same situation prevails in straight section 2, where the angle $\theta$ between $d\vec{s}$ and $\hat{r}$ for any current-length element is 180°. Thus,

$$B_2 = 0.$$

***Circular arc:*** Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc leads to Eq. 29-9 ($B = \mu_0 i\phi/4\pi R$). Here the central angle $\phi$ of the arc is $\pi/2$ rad. Thus from Eq. 29-9, the magnitude of the magnetic field $\vec{B}_3$ at the arc's center $C$ is

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

To find the direction of $\vec{B}_3$, we apply the right-hand rule displayed in Fig. 29-5. Mentally grasp the circular arc with your right hand as in Fig. 29-8*c*, with your thumb in the direction of the current. The direction in which your fingers curl around the wire indicates the direction of the magnetic field lines around the wire. They form circles around the wire, coming out of the page above the arc and going into the page inside the arc. In the region of point $C$ (inside the arc), your fingertips point *into the plane* of the page. Thus, $\vec{B}_3$ is directed into that plane.

***Net field:*** Generally, we combine multiple magnetic fields as vectors. Here, however, only the circular arc produces a magnetic field at point $C$. Thus, we can write the magnitude of the net field $\vec{B}$ as

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}. \qquad \text{(Answer)}$$

The direction of $\vec{B}$ is the direction of $\vec{B}_3$— namely, into the plane of Fig. 29-8.

## Sample Problem 29.02 Magnetic field off to the side of two long straight currents

Figure 29-9*a* shows two long parallel wires carrying currents $i_1$ and $i_2$ in opposite directions. What are the magnitude and direction of the net magnetic field at point $P$? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and $d = 5.3$ cm.

### KEY IDEAS

(1) The net magnetic field $\vec{B}$ at point $P$ is the vector sum of the magnetic fields due to the currents in the two wires. (2) We can find the magnetic field due to any current by applying the Biot–Savart law to the current. For points near the current in a long straight wire, that law leads to Eq. 29-4.

***Finding the vectors:*** In Fig. 29-9*a*, point $P$ is distance $R$ from both currents $i_1$ and $i_2$. Thus, Eq. 29-4 tells us that at point $P$ those currents produce magnetic fields $\vec{B}_1$ and $\vec{B}_2$ with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29-9*a*, note that the base angles (between sides $R$ and $d$) are both 45°. This allows us to write

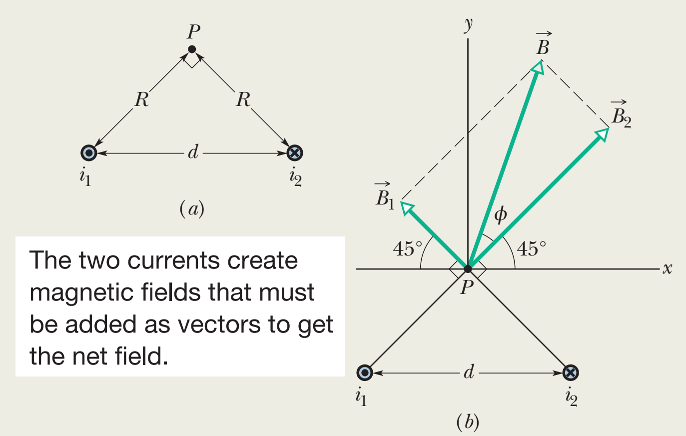



**Figure 29-9** (*a*) Two wires carry currents $i_1$ and $i_2$ in opposite directions (out of and into the page). Note the right angle at $P$. (*b*) The separate fields $\vec{B}_1$ and $\vec{B}_2$ are combined vectorially to yield the net field $\vec{B}$.

$\cos 45° = R/d$ and replace $R$ with $d \cos 45°$. Then the field magnitudes $B_1$ and $B_2$ become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45°} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45°}.$$

We want to combine $\vec{B}_1$ and $\vec{B}_2$ to find their vector sum, which is the net field $\vec{B}$ at $P$. To find the directions of $\vec{B}_1$ and $\vec{B}_2$, we apply the right-hand rule of Fig. 29-5 to each current in Fig. 29-9*a*. For wire 1, with current out of the page, we mentally grasp the wire with the right hand, with the thumb pointing out of the page. Then the curled fingers indicate that the field lines run counterclockwise. In particular, in the region of point $P$, they are directed upward to the left. Recall that the magnetic field at a point near a long, straight current-carrying wire must be directed perpendicular to a radial line between the point and the current. Thus, $\vec{B}_1$ must be directed upward to the left as drawn in Fig. 29-9*b*. (Note carefully the perpendicular symbol between vector $\vec{B}_1$ and the line connecting point $P$ and wire 1.)

Repeating this analysis for the current in wire 2, we find that $\vec{B}_2$ is directed upward to the right as drawn in Fig. 29-9*b*.

***Adding the vectors:*** We can now vectorially add $\vec{B}_1$ and $\vec{B}_2$ to find the net magnetic field $\vec{B}$ at point $P$, either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of $\vec{B}$. However, in Fig. 29-9*b*, there is a third method: Because $\vec{B}_1$ and $\vec{B}_2$ are perpendicular to each other, they form the legs of a right triangle, with $\vec{B}$ as the hypotenuse. So,

$$\begin{aligned} B &= \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45^\circ)}\sqrt{i_1^2 + i_2^2} \\ &= \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})\sqrt{(15\ \mathrm{A})^2 + (32\ \mathrm{A})^2}}{(2\pi)(5.3 \times 10^{-2}\ \mathrm{m})(\cos 45^\circ)} \\ &= 1.89 \times 10^{-4}\ \mathrm{T} \approx 190\ \mu\mathrm{T}. \qquad \text{(Answer)} \end{aligned}$$

The angle $\phi$ between the directions of $\vec{B}$ and $\vec{B}_2$ in Fig. 29-9*b* follows from

$$\phi = \tan^{-1}\frac{B_1}{B_2},$$

which, with $B_1$ and $B_2$ as given above, yields

$$\phi = \tan^{-1}\frac{i_1}{i_2} = \tan^{-1}\frac{15\ \mathrm{A}}{32\ \mathrm{A}} = 25^\circ.$$

The angle between $\vec{B}$ and the $x$ axis shown in Fig. 29-9*b* is then

$$\phi + 45^\circ = 25^\circ + 45^\circ = 70^\circ. \qquad \text{(Answer)}$$

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# 29-2 FORCE BETWEEN TWO PARALLEL CURRENTS

## Learning Objectives

*After reading this module, you should be able to . . .*

**29.10** Given two parallel or antiparallel currents, find the magnetic field of the first current at the location of the second current and then find the force acting on that second current.

**29.11** Identify that parallel currents attract each other, and antiparallel currents repel each other.

**29.12** Describe how a rail gun works.

## Key Ideas

- Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length $L$ of either wire is

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d},$$

where $d$ is the wire separation, and $i_a$ and $i_b$ are the currents in the wires.

## Force Between Two Parallel Currents

Two long parallel wires carrying currents exert forces on each other. Figure 29-10 shows two such wires, separated by a distance $d$ and carrying currents $i_a$ and $i_b$. Let us analyze the forces on these wires due to each other.

We seek first the force on wire $b$ in Fig. 29-10 due to the current in wire $a$. That current produces a magnetic field $\vec{B}_a$, and it is this magnetic field that actually causes the force we seek. To find the force, then, we need the magnitude and direction of the field $\vec{B}_a$ *at the site of wire b*. The magnitude of $\vec{B}_a$ at every point of wire $b$ is, from Eq. 29-4,

$$B_a = \frac{\mu_0 i_a}{2\pi d}. \qquad \text{(29-11)}$$

The curled–straight right-hand rule tells us that the direction of $\vec{B}_a$ at wire $b$ is down, as Fig. 29-10 shows. Now that we have the field, we can find the force it produces on wire $b$. Equation 28-26 tells us that the force $\vec{F}_{ba}$ on a length $L$ of wire $b$ due to the external magnetic field $\vec{B}_a$ is

$$\vec{F}_{ba} = i_b\vec{L} \times \vec{B}_a, \qquad (29\text{-}12)$$

where $\vec{L}$ is the length vector of the wire. In Fig. 29-10, vectors $\vec{L}$ and $\vec{B}_a$ are perpendicular to each other, and so with Eq. 29-11, we can write

$$F_{ba} = i_bLB_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}. \qquad (29\text{-}13)$$

The direction of $\vec{F}_{ba}$ is the direction of the cross product $\vec{L} \times \vec{B}_a$. Applying the right-hand rule for cross products to $\vec{L}$ and $\vec{B}_a$ in Fig. 29-10, we see that $\vec{F}_{ba}$ is directly toward wire $a$, as shown.

The general procedure for finding the force on a current-carrying wire is this:

> To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

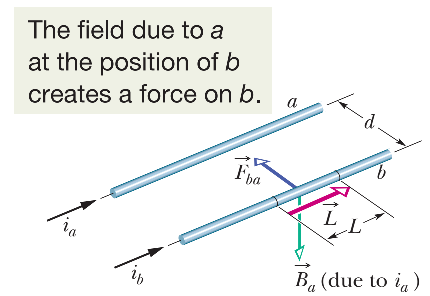



**Figure 29-10** Two parallel wires carrying currents in the same direction attract each other. $\vec{B}_a$ is the magnetic field at wire $b$ produced by the current in wire $a$. $\vec{F}_{ba}$ is the resulting force acting on wire $b$ because it carries current in $\vec{B}_a$.

We could now use this procedure to compute the force on wire $a$ due to the current in wire $b$. We would find that the force is directly toward wire $b$; hence, the two wires with parallel currents attract each other. Similarly, if the two currents were antiparallel, we could show that the two wires repel each other. Thus,

> Parallel currents attract each other, and antiparallel currents repel each other.

The force acting between currents in parallel wires is the basis for the definition of the ampere, which is one of the seven SI base units. The definition, adopted in 1946, is this: The ampere is that constant current which, if maintained in two straight, parallel conductors of infinite length, of negligible circular cross section, and placed 1 m apart in vacuum, would produce on each of these conductors a force of magnitude $2 \times 10^{-7}$ newton per meter of wire length.

## Rail Gun

The basics of a rail gun are shown in Fig. 29-11*a*. A large current is sent out along one of two parallel conducting rails, across a conducting "fuse" (such as a narrow piece of copper) between the rails, and then back to the current source along the second rail. The projectile to be fired lies on the far side of the fuse and fits loosely between the rails. Immediately after the current begins, the fuse element melts and vaporizes, creating a conducting gas between the rails where the fuse had been.

The curled–straight right-hand rule of Fig. 29-5 reveals that the currents in the rails of Fig. 29-11*a* produce magnetic fields that are directed downward between the rails. The net magnetic field $\vec{B}$ exerts a force $\vec{F}$ on the gas due to the current $i$ through the gas (Fig. 29-11*b*). With Eq. 29-12 and the right-hand rule for cross products, we find that $\vec{F}$ points outward along the rails. As the gas is forced outward along the rails, it pushes the projectile, accelerating it by as much as $5 \times 10^6 g$, and then launches it with a speed of 10 km/s, all within 1 ms. Someday rail guns may be used to launch materials into space from mining operations on the Moon or an asteroid.

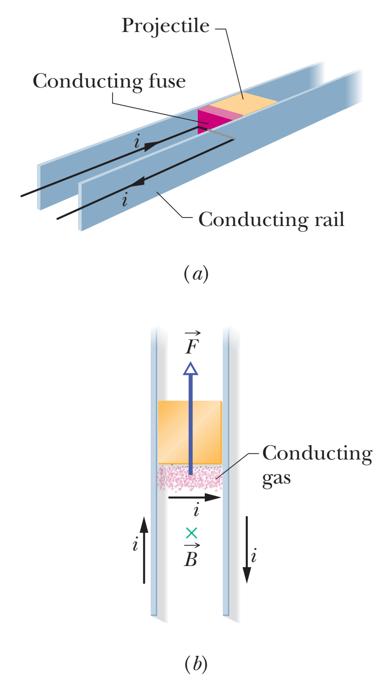



**Figure 29-11** (*a*) A rail gun, as a current $i$ is set up in it. The current rapidly causes the conducting fuse to vaporize. (*b*) The current produces a magnetic field $\vec{B}$ between the rails, and the field causes a force $\vec{F}$ to act on the conducting gas, which is part of the current path. The gas propels the projectile along the rails, launching it.

### Checkpoint 1

The figure here shows three long, straight, parallel, equally spaced wires with identical currents either into or out of the page. Rank the wires according to the magnitude of the force on each due to the currents in the other two wires, greatest first.

# 29-3 AMPERE'S LAW

## Learning Objectives

*After reading this module, you should be able to . . .*

**29.13** Apply Ampere's law to a loop that encircles current.

**29.14** With Ampere's law, use a right-hand rule for determining the algebraic sign of an encircled current.

**29.15** For more than one current within an Amperian loop, determine the net current to be used in Ampere's law.

**29.16** Apply Ampere's law to a long straight wire with current, to find the magnetic field magnitude inside and outside the wire, identifying that only the current encircled by the Amperian loop matters.

## Key Idea

- Ampere's law states that

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere's law).}$$

The line integral in this equation is evaluated around a closed loop called an Amperian loop. The current $i$ on the right side is the *net* current encircled by the loop.

## Ampere's Law

We can find the net electric field due to *any* distribution of charges by first writing the differential electric field $d\vec{E}$ due to a charge element and then summing the contributions of $d\vec{E}$ from all the elements. However, if the distribution is complicated, we may have to use a computer. Recall, however, that if the distribution has planar, cylindrical, or spherical symmetry, we can apply Gauss' law to find the net electric field with considerably less effort.

Similarly, we can find the net magnetic field due to *any* distribution of currents by first writing the differential magnetic field $d\vec{B}$ (Eq. 29-3) due to a current-length element and then summing the contributions of $d\vec{B}$ from all the elements. Again we may have to use a computer for a complicated distribution. However, if the distribution has some symmetry, we may be able to apply **Ampere's law** to find the magnetic field with considerably less effort. This law, which can be derived from the Biot–Savart law, has traditionally been credited to André-Marie Ampère (1775–1836), for whom the SI unit of current is named. However, the law actually was advanced by English physicist James Clerk Maxwell. Ampere's law is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere's law).} \tag{29-14}$$

The loop on the integral sign means that the scalar (dot) product $\vec{B} \cdot d\vec{s}$ is to be integrated around a *closed* loop, called an *Amperian loop*. The current $i_{\text{enc}}$ is the *net* current encircled by that closed loop.

To see the meaning of the scalar product $\vec{B} \cdot d\vec{s}$ and its integral, let us first apply Ampere's law to the general situation of Fig. 29-12. The figure shows cross sections of three long straight wires that carry currents $i_1$, $i_2$, and $i_3$ either directly into or directly out of the page. An arbitrary Amperian loop lying in the plane of the page encircles two of the currents but not the third. The counterclockwise direction marked on the loop indicates the arbitrarily chosen direction of integration for Eq. 29-14.

To apply Ampere's law, we mentally divide the loop into differential vector elements $d\vec{s}$ that are everywhere directed along the tangent to the loop in the direction of integration. Assume that at the location of the element $d\vec{s}$ shown in Fig. 29-12, the net magnetic field due to the three currents is $\vec{B}$. Because the wires are perpendicular to the page, we know that the magnetic field at $d\vec{s}$ due to each current is in the plane of Fig. 29-12; thus, their net magnetic field $\vec{B}$ at $d\vec{s}$ must also be in that plane. However, we do not know the orientation of $\vec{B}$ within the plane. In Fig. 29-12, $\vec{B}$ is arbitrarily drawn at an angle $\theta$ to the direction of $d\vec{s}$. The scalar product $\vec{B} \cdot d\vec{s}$ on the left side of Eq. 29-14 is equal to $B \cos\theta\, ds$. Thus, Ampere's law can be written as

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta\, ds = \mu_0 i_{\text{enc}}. \tag{29-15}$$

We can now interpret the scalar product $\vec{B} \cdot d\vec{s}$ as being the product of a length $ds$ of the Amperian loop and the field component $B \cos\theta$ tangent to the loop. Then we can interpret the integration as being the summation of all such products around the entire loop.

***Signs.*** When we can actually perform this integration, we do not need to know the direction of $\vec{B}$ before integrating. Instead, we arbitrarily assume $\vec{B}$ to be generally in the direction of integration (as in Fig. 29-12). Then we use the following curled–straight right-hand rule to assign a plus sign or a minus sign to each of the currents that make up the net encircled current $i_{\text{enc}}$:

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

Finally, we solve Eq. 29-15 for the magnitude of $\vec{B}$. If $B$ turns out positive, then the direction we assumed for $\vec{B}$ is correct. If it turns out negative, we neglect the minus sign and redraw $\vec{B}$ in the opposite direction.

***Net Current.*** In Fig. 29-13 we apply the curled–straight right-hand rule for Ampere's law to the situation of Fig. 29-12. With the indicated counterclockwise direction of integration, the net current encircled by the loop is

$$i_{\text{enc}} = i_1 - i_2.$$

(Current $i_3$ is not encircled by the loop.) We can then rewrite Eq. 29-15 as

$$\oint B \cos\theta\, ds = \mu_0(i_1 - i_2). \tag{29-16}$$

You might wonder why, since current $i_3$ contributes to the magnetic-field magnitude $B$ on the left side of Eq. 29-16, it is not needed on the right side. The answer is that the contributions of current $i_3$ to the magnetic field cancel out because the integration in Eq. 29-16 is made around the full loop. In contrast, the contributions of an encircled current to the magnetic field do not cancel out.

We cannot solve Eq. 29-16 for the magnitude $B$ of the magnetic field because for the situation of Fig. 29-12 we do not have enough information to simplify and solve the integral. However, we do know the outcome of the integration; it must be equal to $\mu_0(i_1 - i_2)$, the value of which is set by the net current passing through the loop.

We shall now apply Ampere's law to two situations in which symmetry does allow us to simplify and solve the integral, hence to find the magnetic field.

## Magnetic Field Outside a Long Straight Wire with Current

Figure 29-14 shows a long straight wire that carries current $i$ directly out of the page. Equation 29-4 tells us that the magnetic field $\vec{B}$ produced by the current has the same magnitude at all points that are the same distance $r$ from the wire; that is, the field $\vec{B}$ has cylindrical symmetry about the wire. We can take advantage of that symmetry to simplify the integral in Ampere's law (Eqs. 29-14 and 29-15) if we encircle the wire with a concentric circular Amperian loop of radius $r$, as in Fig. 29-14. The magnetic field then has the same magnitude $B$ at every point on the loop. We shall integrate counterclockwise, so that $d\vec{s}$ has the direction shown in Fig. 29-14.

Only the currents encircled by the loop are used in Ampere's law.

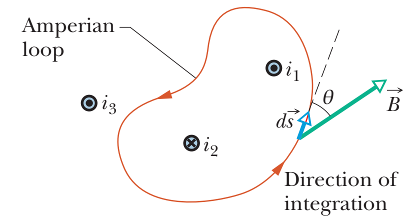



**Figure 29-12** Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

This is how to assign a sign to a current used in Ampere's law.

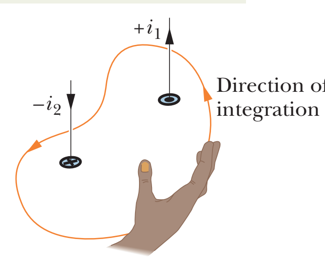



**Figure 29-13** A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop. The situation is that of Fig. 29-12.

All of the current is encircled and thus all is used in Ampere's law.

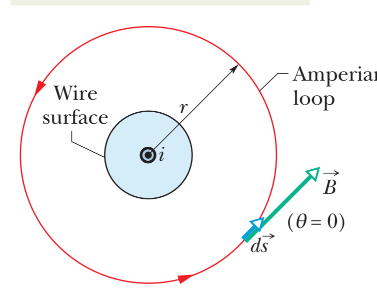



**Figure 29-14** Using Ampere's law to find the magnetic field that a current $i$ produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

We can further simplify the quantity $B\cos\theta$ in Eq. 29-15 by noting that $\vec{B}$ is tangent to the loop at every point along the loop, as is $d\vec{s}$. Thus, $\vec{B}$ and $d\vec{s}$ are either parallel or antiparallel at each point of the loop, and we shall arbitrarily assume the former. Then at every point the angle $\theta$ between $d\vec{s}$ and $\vec{B}$ is $0°$, so $\cos\theta = \cos 0° = 1$. The integral in Eq. 29-15 then becomes

$$\oint \vec{B}\cdot d\vec{s} = \oint B\cos\theta\, ds = B\oint ds = B(2\pi r).$$

Note that $\oint ds$ is the summation of all the line segment lengths $ds$ around the circular loop; that is, it simply gives the circumference $2\pi r$ of the loop.

Our right-hand rule gives us a plus sign for the current of Fig. 29-14. The right side of Ampere's law becomes $+\mu_0 i$, and we then have

$$B(2\pi r) = \mu_0 i$$

or

$$B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straight wire).} \tag{29-17}$$

With a slight change in notation, this is Eq. 29-4, which we derived earlier — with considerably more effort — using the law of Biot and Savart. In addition, because the magnitude $B$ turned out positive, we know that the correct direction of $\vec{B}$ must be the one shown in Fig. 29-14.

Only the current encircled by the loop is used in Ampere's law.

## Magnetic Field Inside a Long Straight Wire with Current

Figure 29-15 shows the cross section of a long straight wire of radius $R$ that carries a uniformly distributed current $i$ directly out of the page. Because the current is uniformly distributed over a cross section of the wire, the magnetic field $\vec{B}$ produced by the current must be cylindrically symmetrical. Thus, to find the magnetic field at points inside the wire, we can again use an Amperian loop of radius $r$, as shown in Fig. 29-15, where now $r < R$. Symmetry again suggests that $\vec{B}$ is tangent to the loop, as shown; so the left side of Ampere's law again yields

$$\oint \vec{B}\cdot d\vec{s} = B\oint ds = B(2\pi r). \tag{29-18}$$

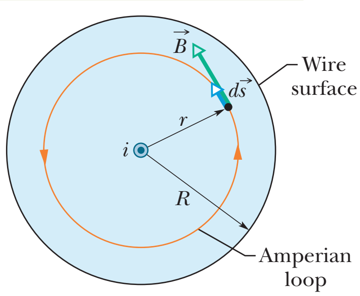



**Figure 29-15** Using Ampere's law to find the magnetic field that a current $i$ produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

Because the current is uniformly distributed, the current $i_{enc}$ encircled by the loop is proportional to the area encircled by the loop; that is,

$$i_{enc} = i\,\frac{\pi r^2}{\pi R^2}. \tag{29-19}$$

Our right-hand rule tells us that $i_{enc}$ gets a plus sign. Then Ampere's law gives us

$$B(2\pi r) = \mu_0 i\,\frac{\pi r^2}{\pi R^2}$$

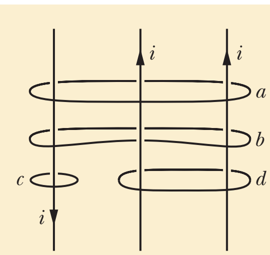
or

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \quad \text{(inside straight wire).} \tag{29-20}$$

Thus, inside the wire, the magnitude $B$ of the magnetic field is proportional to $r$, is zero at the center, and is maximum at $r = R$ (the surface). Note that Eqs. 29-17 and 29-20 give the same value for $B$ at the surface.

### Checkpoint 2

The figure here shows three equal currents $i$ (two parallel and one antiparallel) and four Amperian loops. Rank the loops according to the magnitude of $\oint \vec{B}\cdot d\vec{s}$ along each, greatest first.



### Sample Problem 29.03 Ampere's law to find the field inside a long cylinder of current

Figure 29-16*a* shows the cross section of a long conducting cylinder with inner radius $a = 2.0$ cm and outer radius $b = 4.0$ cm. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $J = cr^2$, with $c = 3.0 \times 10^6$ A/m$^4$ and $r$ in meters. What is the magnetic field $\vec{B}$ at the dot in Fig. 29-16*a*, which is at radius $r = 3.0$ cm from the central axis of the cylinder?

#### KEY IDEAS

The point at which we want to evaluate $\vec{B}$ is inside the material of the conducting cylinder, between its inner and outer radii. We note that the current distribution has cylindrical symmetry (it is the same all around the cross section for any given radius). Thus, the symmetry allows us to use Ampere's law to find $\vec{B}$ at the point. We first draw the Amperian loop shown in Fig. 29-16*b*. The loop is concentric with the cylinder and has radius $r = 3.0$ cm because we want to evaluate $\vec{B}$ at that distance from the cylinder's central axis.

Next, we must compute the current $i_{enc}$ that is encircled by the Amperian loop. However, we *cannot* set up a proportionality as in Eq. 29-19, because here the current is not uniformly distributed. Instead, we must integrate the current density magnitude from the cylinder's inner radius $a$ to the loop radius $r$, using the steps shown in Figs. 29-16*c* through *h*.

**Calculations:** We write the integral as

$$\begin{aligned} i_{enc} &= \int J\,dA = \int_a^r cr^2(2\pi r\,dr) \\ &= 2\pi c \int_a^r r^3\,dr = 2\pi c \left[\frac{r^4}{4}\right]_a^r \\ &= \frac{\pi c(r^4 - a^4)}{2}. \end{aligned}$$

Note that in these steps we took the differential area $dA$ to be the area of the thin ring in Figs. 29-16*d*–*f* and then replaced it with its equivalent, the product of the ring's circumference $2\pi r$ and its thickness $dr$.

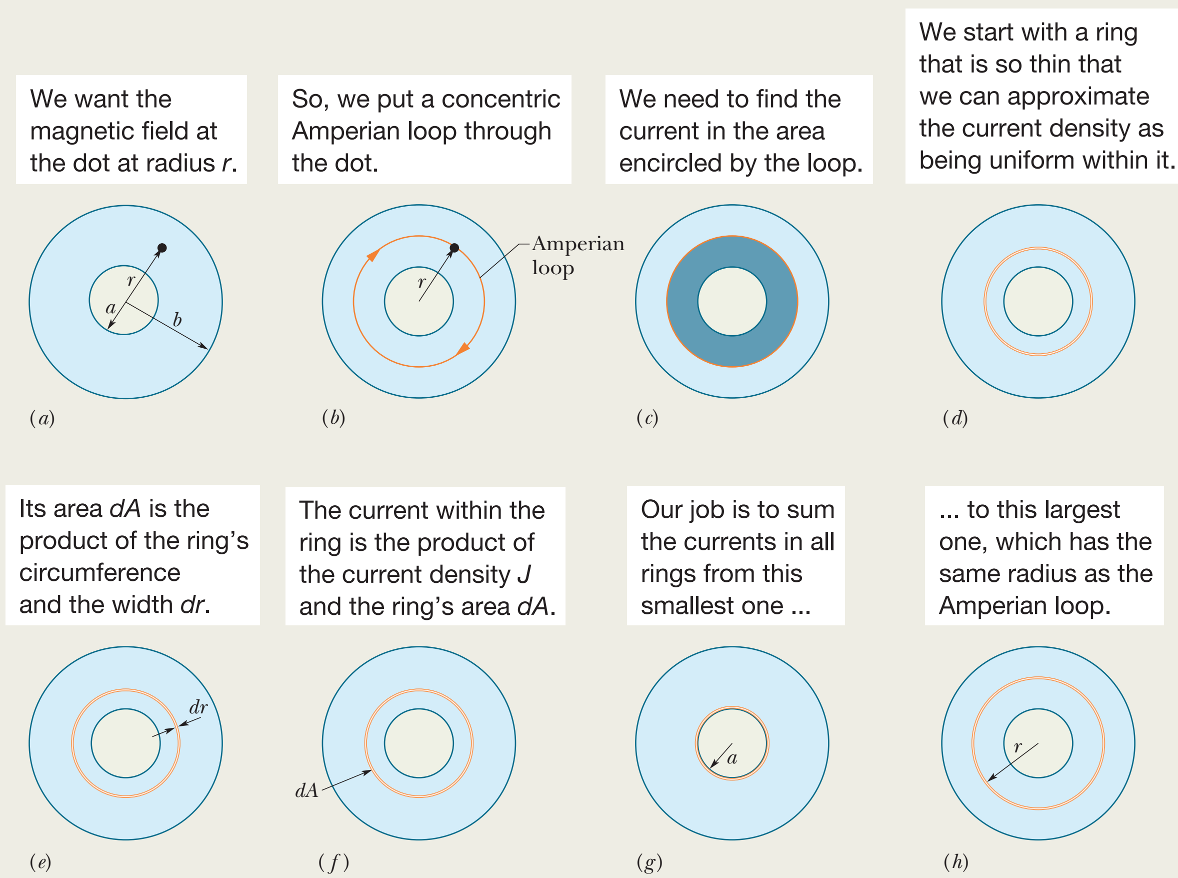



**Figure 29-16** (*a*)–(*b*) To find the magnetic field at a point within this conducting cylinder, we use a concentric Amperian loop through the point. We then need the current encircled by the loop. (*c*)–(*h*) Because the current density is nonuniform, we start with a thin ring and then sum (via integration) the currents in all such rings in the encircled area.

For the Amperian loop, the direction of integration indicated in Fig. 29-16*b* is (arbitrarily) clockwise. Applying the right-hand rule for Ampere's law to that loop, we find that we should take $i_{enc}$ as negative because the current is directed out of the page but our thumb is directed into the page.

We next evaluate the left side of Ampere's law as we did in Fig. 29-15, and we again obtain Eq. 29-18. Then Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc},$$

gives us

$$B(2\pi r) = -\frac{\mu_0 \pi c}{2}(r^4 - a^4).$$

Solving for $B$ and substituting known data yield

$$\begin{aligned} B &= -\frac{\mu_0 c}{4r}(r^4 - a^4) \\ &= -\frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(3.0 \times 10^6\ \mathrm{A/m^4})}{4(0.030\ \mathrm{m})} \\ &\quad \times [(0.030\ \mathrm{m})^4 - (0.020\ \mathrm{m})^4] \\ &= -2.0 \times 10^{-5}\ \mathrm{T}. \end{aligned}$$

Thus, the magnetic field $\vec{B}$ at a point 3.0 cm from the central axis has magnitude

$$B = 2.0 \times 10^{-5}\ \mathrm{T} \qquad \text{(Answer)}$$

and forms magnetic field lines that are directed opposite our direction of integration, hence counterclockwise in Fig. 29-16*b*.

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# 29-4 SOLENOIDS AND TOROIDS

## Learning Objectives

*After reading this module, you should be able to . . .*

29.17 Describe a solenoid and a toroid and sketch their magnetic field lines.

29.18 Explain how Ampere's law is used to find the magnetic field inside a solenoid.

29.19 Apply the relationship between a solenoid's internal magnetic field $B$, the current $i$, and the number of turns per unit length $n$ of the solenoid.

29.20 Explain how Ampere's law is used to find the magnetic field inside a toroid.

29.21 Apply the relationship between a toroid's internal magnetic field $B$, the current $i$, the radius $r$, and the total number of turns $N$.

## Key Ideas

- Inside a long solenoid carrying current $i$, at points not near its ends, the magnitude $B$ of the magnetic field is

$$B = \mu_0 i n \quad \text{(ideal solenoid)},$$

where $n$ is the number of turns per unit length.

- At a point inside a toroid, the magnitude $B$ of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r} \quad \text{(toroid)},$$

where $r$ is the distance from the center of the toroid to the point.

## Solenoids and Toroids

### Magnetic Field of a Solenoid

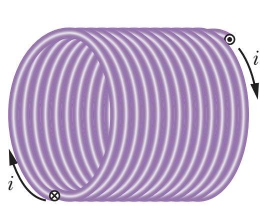



**Figure 29-17** A solenoid carrying current $i$.

We now turn our attention to another situation in which Ampere's law proves useful. It concerns the magnetic field produced by the current in a long, tightly wound helical coil of wire. Such a coil is called a **solenoid** (Fig. 29-17). We assume that the length of the solenoid is much greater than the diameter.

Figure 29-18 shows a section through a portion of a "stretched-out" solenoid. The solenoid's magnetic field is the vector sum of the fields produced by the individual turns (*windings*) that make up the solenoid. For points very close to a turn,

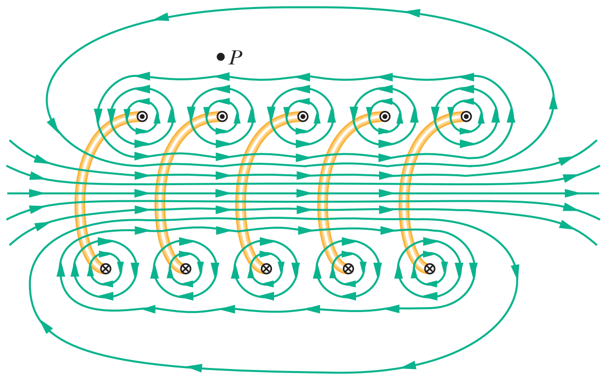



**Figure 29-18** A vertical cross section through the central axis of a "stretched-out" solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid. Each turn produces circular magnetic field lines near itself. Near the solenoid's axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.

the wire behaves magnetically almost like a long straight wire, and the lines of $\vec{B}$ there are almost concentric circles. Figure 29-18 suggests that the field tends to cancel between adjacent turns. It also suggests that, at points inside the solenoid and reasonably far from the wire, $\vec{B}$ is approximately parallel to the (central) solenoid axis. In the limiting case of an *ideal solenoid,* which is infinitely long and consists of tightly packed (*close-packed*) turns of square wire, the field inside the coil is uniform and parallel to the solenoid axis.

At points above the solenoid, such as $P$ in Fig. 29-18, the magnetic field set up by the upper parts of the solenoid turns (these upper turns are marked $\odot$) is directed to the left (as drawn near $P$) and tends to cancel the field set up at $P$ by the lower parts of the turns (these lower turns are marked $\otimes$), which is directed to the right (not drawn). In the limiting case of an ideal solenoid, the magnetic field outside the solenoid is zero. Taking the external field to be zero is an excellent assumption for a real solenoid if its length is much greater than its diameter and if we consider external points such as point $P$ that are not at either end of the solenoid. The direction of the magnetic field along the solenoid axis is given by a curled–straight right-hand rule: Grasp the solenoid with your right hand so that your fingers follow the direction of the current in the windings; your extended right thumb then points in the direction of the axial magnetic field.

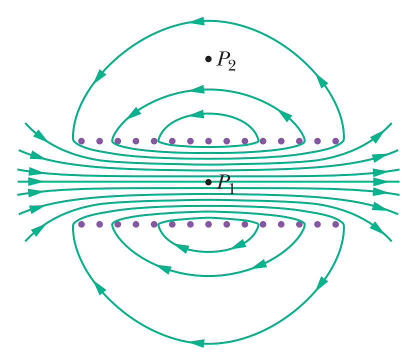



**Figure 29-19** Magnetic field lines for a real solenoid of finite length. The field is strong and uniform at interior points such as $P_1$ but relatively weak at external points such as $P_2$.

Figure 29-19 shows the lines of $\vec{B}$ for a real solenoid. The spacing of these lines in the central region shows that the field inside the coil is fairly strong and uniform over the cross section of the coil. The external field, however, is relatively weak.

***Ampere's Law.*** Let us now apply Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, i_{\text{enc}}, \tag{29-21}$$

to the ideal solenoid of Fig. 29-20, where $\vec{B}$ is uniform within the solenoid and zero outside it, using the rectangular Amperian loop *abcda.* We write $\oint \vec{B} \cdot d\vec{s}$ as the sum of four integrals, one for each loop segment:

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}. \tag{29-22}$$

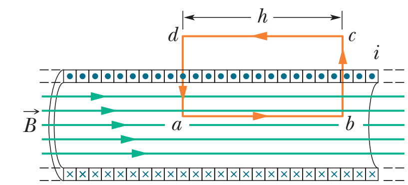



**Figure 29-20** Application of Ampere's law to a section of a long ideal solenoid carrying a current $i$. The Amperian loop is the rectangle *abcda*.

The first integral on the right of Eq. 29-22 is $Bh$, where $B$ is the magnitude of the uniform field $\vec{B}$ inside the solenoid and $h$ is the (arbitrary) length of the segment from $a$ to $b$. The second and fourth integrals are zero because for every element $ds$ of these segments, $\vec{B}$ either is perpendicular to $ds$ or is zero, and thus the product $\vec{B}\cdot d\vec{s}$ is zero. The third integral, which is taken along a segment that lies outside the solenoid, is zero because $B = 0$ at all external points. Thus, $\oint \vec{B}\cdot d\vec{s}$ for the entire rectangular loop has the value $Bh$.

***Net Current.*** The net current $i_{enc}$ encircled by the rectangular Amperian loop in Fig. 29-20 is not the same as the current $i$ in the solenoid windings because the windings pass more than once through this loop. Let $n$ be the number of turns per unit length of the solenoid; then the loop encloses $nh$ turns and

$$i_{enc} = i(nh).$$

Ampere's law then gives us

$$Bh = \mu_0 inh$$

or
$$B = \mu_0 in \quad \text{(ideal solenoid).} \tag{29-23}$$

Although we derived Eq. 29-23 for an infinitely long ideal solenoid, it holds quite well for actual solenoids if we apply it only at interior points and well away from the solenoid ends. Equation 29-23 is consistent with the experimental fact that the magnetic field magnitude $B$ within a solenoid does not depend on the diameter or the length of the solenoid and that $B$ is uniform over the solenoidal cross section. A solenoid thus provides a practical way to set up a known uniform magnetic field for experimentation, just as a parallel-plate capacitor provides a practical way to set up a known uniform electric field.

## Magnetic Field of a Toroid

Figure 29-21$a$ shows a **toroid,** which we may describe as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet. What magnetic field $\vec{B}$ is set up inside the toroid (inside the hollow of the bracelet)? We can find out from Ampere's law and the symmetry of the bracelet.

From the symmetry, we see that the lines of $\vec{B}$ form concentric circles inside the toroid, directed as shown in Fig. 29-21$b$. Let us choose a concentric circle of radius $r$ as an Amperian loop and traverse it in the clockwise direction. Ampere's law (Eq. 29-14) yields

$$(B)(2\pi r) = \mu_0 iN,$$

where $i$ is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and $N$ is the total number of turns. This gives

$$B = \frac{\mu_0 iN}{2\pi}\frac{1}{r} \quad \text{(toroid).} \tag{29-24}$$

In contrast to the situation for a solenoid, $B$ is not constant over the cross section of a toroid.

It is easy to show, with Ampere's law, that $B = 0$ for points outside an ideal toroid (as if the toroid were made from an ideal solenoid). The direction of the magnetic field within a toroid follows from our curled–straight right-hand rule: Grasp the toroid with the fingers of your right hand curled in the direction of the current in the windings; your extended right thumb points in the direction of the magnetic field.

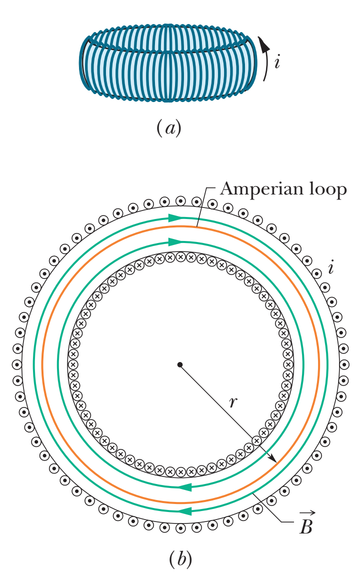



**Figure 29-21** ($a$) A toroid carrying a current $i$. ($b$) A horizontal cross section of the toroid. The interior magnetic field (inside the bracelet-shaped tube) can be found by applying Ampere's law with the Amperian loop shown.

### Sample Problem 29.04 The field inside a solenoid (a long coil of current)

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length $L$. What is $B$ at its center?

#### KEY IDEA

The magnitude $B$ of the magnetic field along the solenoid's central axis is related to the solenoid's current $i$ and number of turns per unit length $n$ by Eq. 29-23 ($B = \mu_0 in$).

***Calculation:*** Because $B$ does not depend on the diameter of the windings, the value of $n$ for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

$$B = \mu_0 in = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.57\ \text{A})\frac{5 \times 850\ \text{turns}}{1.23\ \text{m}}$$

$$= 2.42 \times 10^{-2}\ \text{T} = 24.2\ \text{mT}. \qquad \text{(Answer)}$$

To a good approximation, this is the field magnitude throughout most of the solenoid.



# 29-5 A CURRENT-CARRYING COIL AS A MAGNETIC DIPOLE

## Learning Objectives

*After reading this module, you should be able to . . .*

**29.22** Sketch the magnetic field lines of a flat coil that is carrying current.

**29.23** For a current-carrying coil, apply the relationship between the dipole moment magnitude $\mu$ and the coil's current $i$, number of turns $N$, and area per turn $A$.

**29.24** For a point along the central axis, apply the relationship between the magnetic field magnitude $B$, the magnetic moment $\mu$, and the distance $z$ from the center of the coil.

## Key Idea

- The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point $P$ located a distance $z$ along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3},$$

where $\vec{\mu}$ is the dipole moment of the coil. This equation applies only when $z$ is much greater than the dimensions of the coil.

## A Current-Carrying Coil as a Magnetic Dipole

So far we have examined the magnetic fields produced by current in a long straight wire, a solenoid, and a toroid. We turn our attention here to the field produced by a coil carrying a current. You saw in Module 28-8 that such a coil behaves as a magnetic dipole in that, if we place it in an external magnetic field $\vec{B}$, a torque $\vec{\tau}$ given by

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad (29\text{-}25)$$

acts on it. Here $\vec{\mu}$ is the magnetic dipole moment of the coil and has the magnitude $NiA$, where $N$ is the number of turns, $i$ is the current in each turn, and $A$ is the area enclosed by each turn. (*Caution:* Don't confuse the magnetic dipole moment $\vec{\mu}$ with the permeability constant $\mu_0$.)

Recall that the direction of $\vec{\mu}$ is given by a curled–straight right-hand rule: Grasp the coil so that the fingers of your right hand curl around it in the direction of the current; your extended thumb then points in the direction of the dipole moment $\vec{\mu}$.

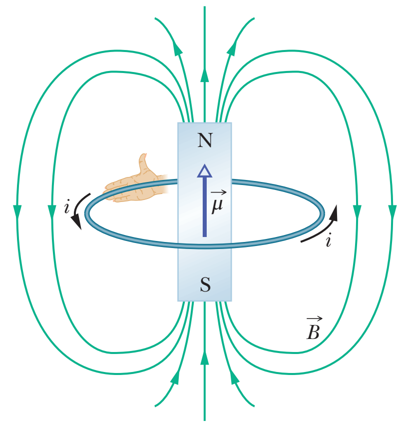



**Figure 29-22** A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment $\vec{\mu}$ of the loop, its direction given by a curled–straight right-hand rule, points from the south pole to the north pole, in the direction of the field $\vec{B}$ within the loop.

## Magnetic Field of a Coil

We turn now to the other aspect of a current-carrying coil as a magnetic dipole. What magnetic field does *it* produce at a point in the surrounding space? The problem does not have enough symmetry to make Ampere's law useful; so we must turn to the law of Biot and Savart. For simplicity, we first consider only a coil with a single circular loop and only points on its perpendicular central axis, which we take to be a $z$ axis. We shall show that the magnitude of the magnetic field at such points is

$$B(z) = \frac{\mu_0 iR^2}{2(R^2 + z^2)^{3/2}}, \tag{29-26}$$

in which $R$ is the radius of the circular loop and $z$ is the distance of the point in question from the center of the loop. Furthermore, the direction of the magnetic field $\vec{B}$ is the same as the direction of the magnetic dipole moment $\vec{\mu}$ of the loop.

***Large z.*** For axial points far from the loop, we have $z \gg R$ in Eq. 29-26. With that approximation, the equation reduces to

$$B(z) \approx \frac{\mu_0 iR^2}{2z^3}.$$

Recalling that $\pi R^2$ is the area $A$ of the loop and extending our result to include a coil of $N$ turns, we can write this equation as

$$B(z) = \frac{\mu_0}{2\pi}\frac{NiA}{z^3}.$$

Further, because $\vec{B}$ and $\vec{\mu}$ have the same direction, we can write the equation in vector form, substituting from the identity $\mu = NiA$:

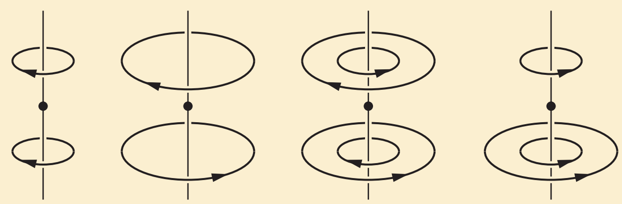
$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \quad \text{(current-carrying coil).} \tag{29-27}$$

Thus, we have two ways in which we can regard a current-carrying coil as a magnetic dipole: (1) it experiences a torque when we place it in an external magnetic field; (2) it generates its own intrinsic magnetic field, given, for distant points along its axis, by Eq. 29-27. Figure 29-22 shows the magnetic field of a current loop; one side of the loop acts as a north pole (in the direction of $\vec{\mu}$)

and the other side as a south pole, as suggested by the lightly drawn magnet in the figure. If we were to place a current-carrying coil in an external magnetic field, it would tend to rotate just like a bar magnet would.

## Checkpoint 3

The figure here shows four arrangements of circular loops of radius $r$ or $2r$, centered on vertical axes (perpendicular to the loops) and carrying identical currents in the directions indicated. Rank the arrangements according to the magnitude of the net magnetic field at the dot, midway between the loops on the central axis, greatest first.




## Proof of Equation 29-26

Figure 29-23 shows the back half of a circular loop of radius $R$ carrying a current $i$. Consider a point $P$ on the central axis of the loop, a distance $z$ from its plane. Let us apply the law of Biot and Savart to a differential element $ds$ of the loop, located at the left side of the loop. The length vector $d\vec{s}$ for this element points perpendicularly out of the page. The angle $\theta$ between $d\vec{s}$ and $\hat{r}$ in Fig. 29-23 is 90°; the plane formed by these two vectors is perpendicular to the plane of the page and contains both $\hat{r}$ and $d\vec{s}$. From the law of Biot and Savart (and the right-hand rule), the differential field $d\vec{B}$ produced at point $P$ by the current in this element is perpendicular to this plane and thus is directed in the plane of the figure, perpendicular to $\hat{r}$, as indicated in Fig. 29-23.

Let us resolve $d\vec{B}$ into two components: $dB_\parallel$ along the axis of the loop and $dB_\perp$ perpendicular to this axis. From the symmetry, the vector sum of all the perpendicular components $dB_\perp$ due to all the loop elements $ds$ is zero. This leaves only the axial (parallel) components $dB_\parallel$ and we have

$$B = \int dB_\parallel.$$

For the element $d\vec{s}$ in Fig. 29-23, the law of Biot and Savart (Eq. 29-1) tells us that the magnetic field at distance $r$ is

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{r^2}.$$

We also have

$$dB_\parallel = dB\cos\alpha.$$

Combining these two relations, we obtain

$$dB_\parallel = \frac{\mu_0\, i\cos\alpha\, ds}{4\pi r^2}. \tag{29-28}$$

Figure 29-23 shows that $r$ and $\alpha$ are related to each other. Let us express each in terms of the variable $z$, the distance between point $P$ and the center of the loop. The relations are

$$r = \sqrt{R^2 + z^2} \tag{29-29}$$

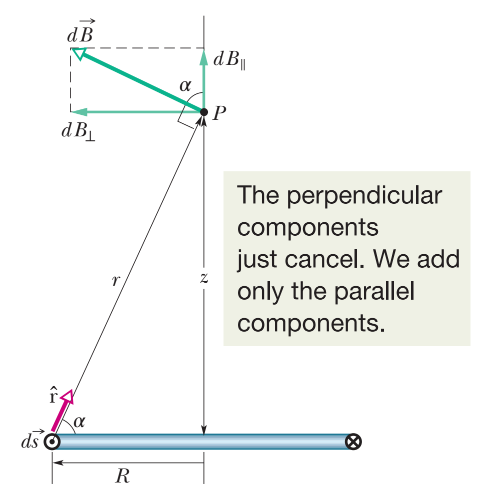



**Figure 29-23** Cross section through a current loop of radius $R$. The plane of the loop is perpendicular to the page, and only the back half of the loop is shown. We use the law of Biot and Savart to find the magnetic field at point $P$ on the central perpendicular axis of the loop.

and $$\cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}. \tag{29-30}$$

Substituting Eqs. 29-29 and 29-30 into Eq. 29-28, we find

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi(R^2 + z^2)^{3/2}}\, ds.$$

Note that $i$, $R$, and $z$ have the same values for all elements $ds$ around the loop; so when we integrate this equation, we find that

$$B = \int dB_{\parallel} = \frac{\mu_0\, iR}{4\pi(R^2 + z^2)^{3/2}} \int ds$$

or, because $\int ds$ is simply the circumference $2\pi R$ of the loop,

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.$$

This is Eq. 29-26, the relation we sought to prove.

## Review & Summary

**The Biot–Savart Law** The magnetic field set up by a current-carrying conductor can be found from the *Biot–Savart law*. This law asserts that the contribution $d\vec{B}$ to the field produced by a current-length element $i\, d\vec{s}$ at a point $P$ located a distance $r$ from the current element is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{r}}{r^2} \quad \text{(Biot–Savart law).} \tag{29-3}$$

Here $\hat{r}$ is a unit vector that points from the element toward $P$. The quantity $\mu_0$, called the permeability constant, has the value

$$4\pi \times 10^{-7}\ \mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T\cdot m/A}.$$

**Magnetic Field of a Long Straight Wire** For a long straight wire carrying a current $i$, the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance $R$ from the wire,

$$B = \frac{\mu_0 i}{2\pi R} \quad \text{(long straight wire).} \tag{29-4}$$

**Magnetic Field of a Circular Arc** The magnitude of the magnetic field at the center of a circular arc, of radius $R$ and central angle $\phi$ (in radians), carrying current $i$, is

$$B = \frac{\mu_0 i \phi}{4\pi R} \quad \text{(at center of circular arc).} \tag{29-9}$$

**Force Between Parallel Currents** Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length $L$ of either wire is

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}, \tag{29-13}$$

where $d$ is the wire separation, and $i_a$ and $i_b$ are the currents in the wires.

**Ampere's Law** **Ampere's law** states that

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{(Ampere's law).} \tag{29-14}$$

The line integral in this equation is evaluated around a closed loop called an *Amperian loop*. The current $i$ on the right side is the *net* current encircled by the loop. For some current distributions, Eq. 29-14 is easier to use than Eq. 29-3 to calculate the magnetic field due to the currents.

**Fields of a Solenoid and a Toroid** Inside a *long solenoid* carrying current $i$, at points not near its ends, the magnitude $B$ of the magnetic field is

$$B = \mu_0 i n \quad \text{(ideal solenoid),} \tag{29-23}$$

where $n$ is the number of turns per unit length. Thus the internal magnetic field is uniform. Outside the solenoid, the magnetic field is approximately zero.

At a point inside a *toroid*, the magnitude $B$ of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r} \quad \text{(toroid),} \tag{29-24}$$

where $r$ is the distance from the center of the toroid to the point.

**Field of a Magnetic Dipole** The magnetic field produced by a current-carrying coil, which is a *magnetic dipole,* at a point $P$ located a distance $z$ along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}, \tag{29-27}$$

where $\vec{\mu}$ is the dipole moment of the coil. This equation applies only when $z$ is much greater than the dimensions of the coil.

# Questions

**1** Figure 29-24 shows three circuits, each consisting of two radial lengths and two concentric circular arcs, one of radius $r$ and the other of radius $R > r$. The circuits have the same current through them and the same angle between the two radial lengths. Rank the circuits according to the magnitude of the net magnetic field at the center, greatest first.

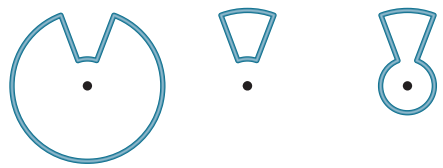



**Figure 29-24** Question 1.

**2** Figure 29-25 represents a snapshot of the velocity vectors of four electrons near a wire carrying current $i$. The four velocities have the same magnitude; velocity $\vec{v}_2$ is directed into the page. Electrons 1 and 2 are at the same distance from the wire, as are electrons 3 and 4. Rank the electrons according to the magnitudes of the magnetic forces on them due to current $i$, greatest first.

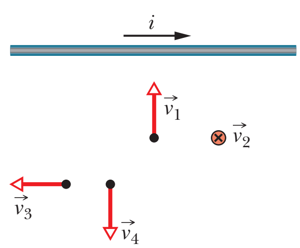



**Figure 29-25** Question 2.

**3** Figure 29-26 shows four arrangements in which long parallel wires carry equal currents directly into or out of the page at the corners of identical squares. Rank the arrangements according to the magnitude of the net magnetic field at the center of the square, greatest first.

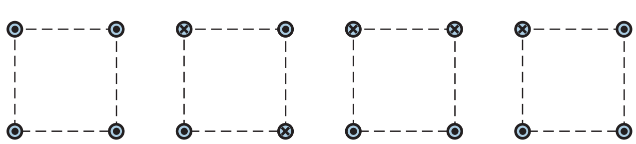



**Figure 29-26** Question 3.

**4** Figure 29-27 shows cross sections of two long straight wires; the left-hand wire carries current $i_1$ directly out of the page. If the net magnetic field due to the two currents is to be zero at point $P$, (a) should the direction of current $i_2$ in the right-hand wire be directly into or out of the page and (b) should $i_2$ be greater than, less than, or equal to $i_1$?

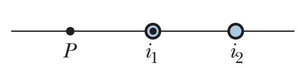



**Figure 29-27** Question 4.

**5** Figure 29-28 shows three circuits consisting of straight radial lengths and concentric circular arcs (either half- or quarter-circles of radii $r$, $2r$, and $3r$). The circuits carry the same current. Rank them according to the magnitude of the magnetic field produced at the center of curvature (the dot), greatest first.

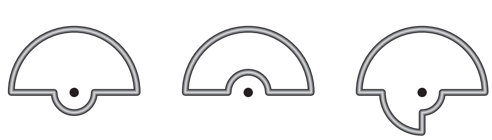



**Figure 29-28** Question 5.

**6** Figure 29-29 gives, as a function of radial distance $r$, the magnitude $B$ of the magnetic field inside and outside four wires ($a$, $b$, $c$, and $d$), each of which carries a current that is uniformly distributed across the wire's cross section. Overlapping portions of the plots (drawn slightly separated) are indicated by double labels. Rank the wires according to (a) radius, (b) the magnitude of the magnetic field on the surface, and (c) the value of the current, greatest first. (d) Is the magnitude of the current density in wire $a$ greater than, less than, or equal to that in wire $c$?

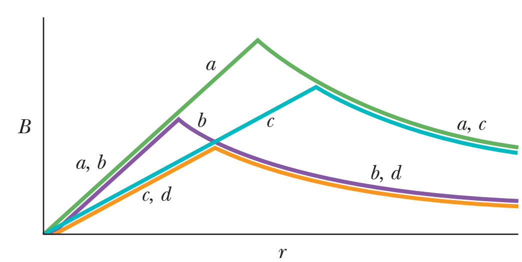



**Figure 29-29** Question 6.

**7** Figure 29-30 shows four circular Amperian loops ($a$, $b$, $c$, $d$) concentric with a wire whose current is directed out of the page. The current is uniform across the wire's circular cross section (the shaded region). Rank the loops according to the magnitude of $\oint \vec{B} \cdot d\vec{s}$ around each, greatest first.

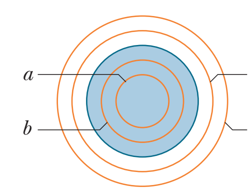



**Figure 29-30** Question 7.

**8** Figure 29-31 shows four arrangements in which long, parallel, equally spaced wires carry equal currents directly into or out of the page. Rank the arrangements according to the magnitude of the net force on the central wire due to the currents in the other wires, greatest first.

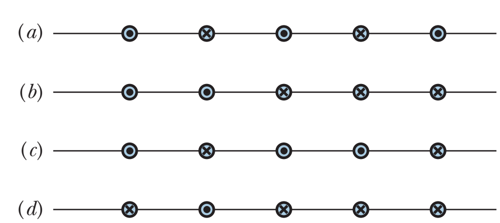



**Figure 29-31** Question 8.

**9** Figure 29-32 shows four circular Amperian loops ($a$, $b$, $c$, $d$) and, in cross section, four long circular conductors (the shaded regions), all of which are concentric. Three of the conductors are hollow cylinders; the central conductor is a solid cylinder. The currents in the conductors are, from smallest radius to largest radius, 4 A out of the page,

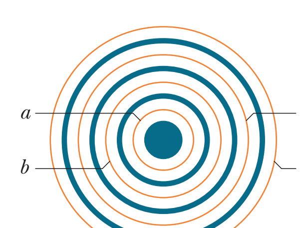



**Figure 29-32** Question 9.

9 A into the page, 5 A out of the page, and 3 A into the page. Rank the Amperian loops according to the magnitude of $\oint \vec{B} \cdot d\vec{s}$ around each, greatest first.

**10** Figure 29-33 shows four identical currents $i$ and five Amperian paths ($a$ through $e$) encircling them. Rank the paths according to the value of $\oint \vec{B} \cdot d\vec{s}$ taken in the directions shown, most positive first.

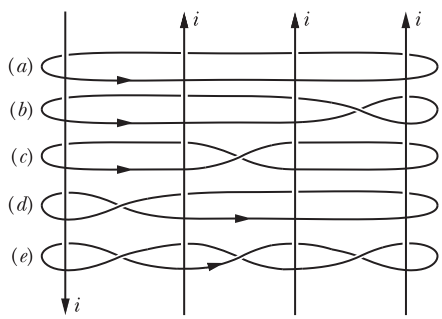



**Figure 29-33** Question 10.

**11** Figure 29-34 shows three arrangements of three long straight wires carrying equal currents directly into or out of the page. (a) Rank the arrangements according to the magnitude of the net force on wire $A$ due to the currents in the other wires, greatest first. (b) In arrangement 3, is the angle between the net force on wire $A$ and the dashed line equal to, less than, or more than 45°?

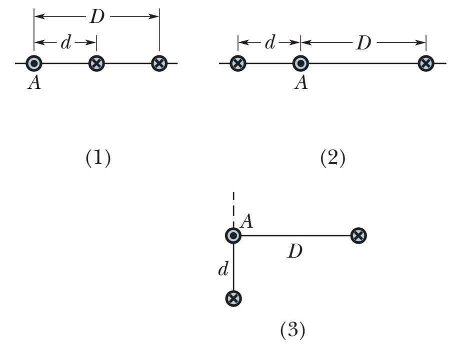



**Figure 29-34** Question 11.

# Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS* and WebAssign

SSM Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at

• – ••• Number of dots indicates level of problem difficulty

ILW Interactive solution is at

http://www.wiley.com/college/halliday

Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

## Module 29-1 Magnetic Field Due to a Current

**•1** A surveyor is using a magnetic compass 6.1 m below a power line in which there is a steady current of 100 A. (a) What is the magnetic field at the site of the compass due to the power line? (b) Will this field interfere seriously with the compass reading? The horizontal component of Earth's magnetic field at the site is 20 $\mu$T.

**•2** Figure 29-35$a$ shows an element of length $ds = 1.00$ $\mu$m in a very long straight wire carrying current. The current in that element sets up a differential magnetic field $d\vec{B}$ at points in the surrounding space. Figure 29-35$b$ gives the magnitude $dB$ of the field for points 2.5 cm from the element, as a function of angle $\theta$ between the wire and a straight line to the point. The vertical scale is set by $dB_s = 60.0$ pT. What is the magnitude of the magnetic field set up by the entire wire at perpendicular distance 2.5 cm from the wire?

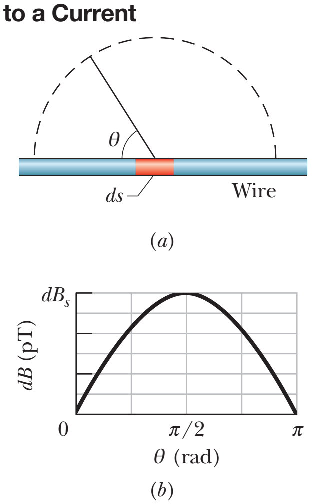



**Figure 29-35** Problem 2.

**•3** SSM At a certain location in the Philippines, Earth's magnetic field of 39 $\mu$T is horizontal and directed due north. Suppose the net field is zero exactly 8.0 cm above a long, straight, horizontal wire that carries a constant current. What are the (a) magnitude and (b) direction of the current?

**•4** A straight conductor carrying current $i = 5.0$ A splits into identical semicircular arcs as shown in Fig. 29-36. What is the magnetic field at the center $C$ of the resulting circular loop?

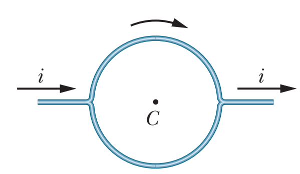



**Figure 29-36** Problem 4.

**•5** In Fig. 29-37, a current $i = 10$ A is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius $R = 5.0$ mm. Point $b$ is midway between the straight sections and so distant from the semicircle that each straight section can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of $\vec{B}$ at $a$ and the (c) magnitude and (d) direction of $\vec{B}$ at $b$?

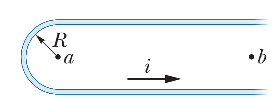



**Figure 29-37** Problem 5.

**•6** In Fig. 29-38, point $P$ is at perpendicular distance $R = 2.00$ cm from a very long straight wire carrying a current. The magnetic field $\vec{B}$ set up at point $P$ is due to contributions from all the identical current-length elements $i\,d\vec{s}$ along the wire. What is the distance $s$

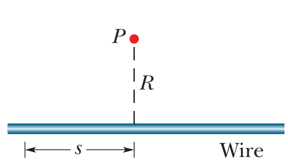



**Figure 29-38** Problem 6.

to the element making (a) the greatest contribution to field $\vec{B}$ and (b) 10.0% of the greatest contribution?

**•7** GO In Fig. 29-39, two circular arcs have radii $a = 13.5$ cm and $b = 10.7$ cm, subtend angle $\theta = 74.0°$, carry current $i = 0.411$ A, and share the same center of curvature $P$. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at $P$?

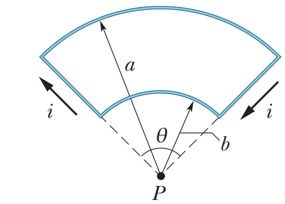


**Figure 29-39** Problem 7.

**•8** In Fig. 29-40, two semicircular arcs have radii $R_2 = 7.80$ cm and $R_1 = 3.15$ cm, carry current $i = 0.281$ A, and have the same center of curvature $C$. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at $C$?

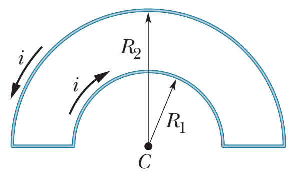


**Figure 29-40** Problem 8.

**•9** SSM Two long straight wires are parallel and 8.0 cm apart. They are to carry equal currents such that the magnetic field at a point halfway between them has magnitude 300 $\mu$T. (a) Should the currents be in the same or opposite directions? (b) How much current is needed?

**•10** In Fig. 29-41, a wire forms a semicircle of radius $R = 9.26$ cm and two (radial) straight segments each of length $L = 13.1$ cm. The wire carries current $i = 34.8$ mA. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the semicircle's center of curvature $C$?

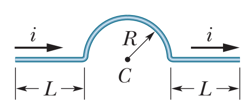


**Figure 29-41** Problem 10.

**•11** In Fig. 29-42, two long straight wires are perpendicular to the page and separated by distance $d_1 = 0.75$ cm. Wire 1 carries 6.5 A into the page. What are the (a) magnitude and (b) direction (into or out of the page) of the current in wire 2 if the net magnetic field due to the two currents is zero at point $P$ located at distance $d_2 = 1.50$ cm from wire 2?

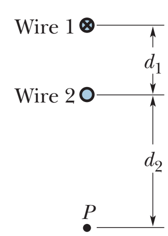


**Figure 29-42** Problem 11.

**•12** In Fig. 29-43, two long straight wires at separation $d = 16.0$ cm carry currents $i_1 = 3.61$ mA and $i_2 = 3.00i_1$ out of the page. (a) Where on the $x$ axis is the net magnetic field equal to zero? (b) If the two currents are doubled, is the zero-field point shifted toward wire 1, shifted toward wire 2, or unchanged?

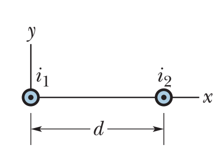


**Figure 29-43** Problem 12.

**••13** In Fig. 29-44, point $P_1$ is at distance $R = 13.1$ cm on the perpendicular bisector of a straight wire of length $L = 18.0$ cm carrying current $i = 58.2$ mA. (Note that the wire is *not* long.) What is the magnitude of the magnetic field at $P_1$ due to $i$?

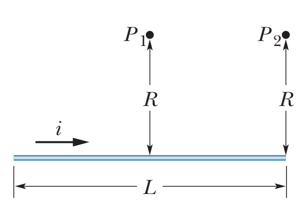


**Figure 29-44** Problems 13 and 17.

**••14** Equation 29-4 gives the magnitude $B$ of the magnetic field set up by a current in an *infinitely long* straight wire, at a point $P$ at perpendicular distance $R$ from the wire. Suppose that point $P$ is actually at perpendicular distance $R$ from the midpoint of a wire with a *finite* length $L$. Using Eq. 29-4 to calculate $B$ then results in a certain percentage error. What value must the ratio $L/R$ exceed if the percentage error is to be less than 1.00%? That is, what $L/R$ gives

$$\frac{(B \text{ from Eq. 29-4}) - (B \text{ actual})}{(B \text{ actual})}(100\%) = 1.00\%?$$

**••15** Figure 29-45 shows two current segments. The lower segment carries a current of $i_1 = 0.40$ A and includes a semicircular arc with radius 5.0 cm, angle 180°, and center point $P$. The upper segment carries current $i_2 = 2i_1$ and includes a circular arc with radius 4.0 cm, angle 120°, and the same center point $P$. What are the (a) magnitude and (b) direction of the net magnetic field $\vec{B}$ at $P$ for the indicated current directions? What are the (c) magnitude and (d) direction of $\vec{B}$ if $i_1$ is reversed?

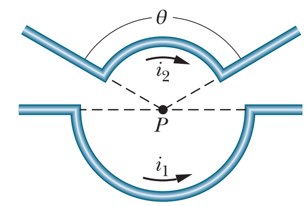


**Figure 29-45** Problem 15.

**••16** GO In Fig. 29-46, two concentric circular loops of wire carrying current in the same direction lie in the same plane. Loop 1 has radius 1.50 cm and carries 4.00 mA. Loop 2 has radius 2.50 cm and carries 6.00 mA. Loop 2 is to be rotated about a diameter while the net magnetic field $\vec{B}$ set up by the two loops at their common center is measured. Through what angle must loop 2 be rotated so that the magnitude of that net field is 100 nT?

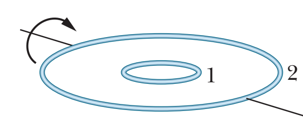


**Figure 29-46** Problem 16.

**••17** SSM In Fig. 29-44, point $P_2$ is at perpendicular distance $R = 25.1$ cm from one end of a straight wire of length $L = 13.6$ cm carrying current $i = 0.693$ A. (Note that the wire is *not* long.) What is the magnitude of the magnetic field at $P_2$?

**••18** A current is set up in a wire loop consisting of a semicircle of radius 4.00 cm, a smaller concentric semicircle, and two radial straight lengths, all in the same plane. Figure 29-47*a* shows the arrangement but is not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is 47.25 $\mu$T. The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29-47*b*). The magnetic field produced at the (same) center of curvature now has magnitude 15.75 $\mu$T, and its direction is reversed from the initial magnetic field. What is the radius of the smaller semicircle?

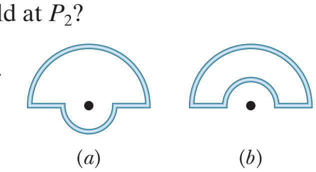


**Figure 29-47** Problem 18.

**••19** One long wire lies along an $x$ axis and carries a current of 30 A in the positive $x$ direction. A second long wire is perpendicular to the $xy$ plane, passes through the point (0, 4.0 m, 0), and carries a current of 40 A in the positive $z$ direction. What is the magnitude of the resulting magnetic field at the point (0, 2.0 m, 0)?

**••20** In Fig. 29-48, part of a long insulated wire carrying current $i = 5.78$ mA is bent into a circular section of radius $R = 1.89$ cm. In unit-vector notation, what is the magnetic field at the center of curvature $C$ if the circular section (a) lies in the plane of the page as shown and (b) is perpendicular to the plane of the page after being rotated 90° counterclockwise as indicated?

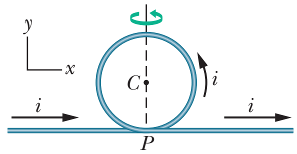


**Figure 29-48** Problem 20.

**••21** GO Figure 29-49 shows two very long straight wires (in cross section) that each carry a current of 4.00 A directly out of the page. Distance $d_1 = 6.00$ m and distance $d_2 = 4.00$ m. What is the magnitude of the net magnetic field at point $P$, which lies on a perpendicular bisector to the wires?

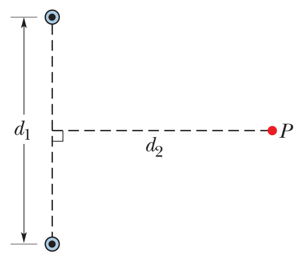


**Figure 29-49** Problem 21.

**••22** GO Figure 29-50*a* shows, in cross section, two long, parallel wires carrying current and separated by distance $L$. The ratio $i_1/i_2$ of their currents is 4.00; the directions of the currents are not indicated. Figure 29-50*b* shows the $y$ component $B_y$ of their net magnetic field along the $x$ axis to the right of wire 2. The vertical scale is set by $B_{ys} = 4.0$ nT, and the horizontal scale is set by $x_s = 20.0$ cm. (a) At what value of $x > 0$ is $B_y$ maximum? (b) If $i_2 = 3$ mA, what is the value of that maximum? What is the direction (into or out of the page) of (c) $i_1$ and (d) $i_2$?

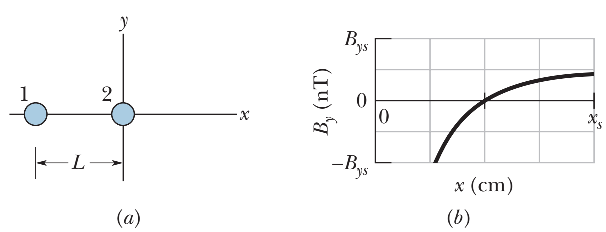


**Figure 29-50** Problem 22.

**••23** ILW Figure 29-51 shows a snapshot of a proton moving at velocity $\vec{v} = (-200 \text{ m/s})\hat{j}$ toward a long straight wire with current $i = 350$ mA. At the instant shown, the proton's distance from the wire is $d = 2.89$ cm. In unit-vector notation, what is the magnetic force on the proton due to the current?

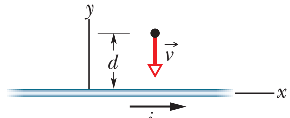


**Figure 29-51** Problem 23.

**••24** GO Figure 29-52 shows, in cross section, four thin wires that are parallel, straight, and very long. They carry identical currents in the directions indicated. Initially all four wires are at distance $d = 15.0$ cm from the origin of the coordinate system, where they create a net magnetic field $\vec{B}$. (a) To what value of $x$ must you move wire 1 along the $x$ axis in order to rotate $\vec{B}$ counterclockwise by 30°? (b) With wire 1 in that new position, to what value of $x$ must you move wire 3 along the $x$ axis to rotate $\vec{B}$ by 30° back to its initial orientation?

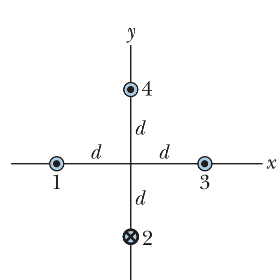


**Figure 29-52** Problem 24.

**••25** SSM A wire with current $i = 3.00$ A is shown in Fig. 29-53. Two semi-infinite straight sections, both tangent to the same circle, are connected by a circular arc that has a central angle $\theta$ and runs along the circumference of the circle. The arc and the two straight sections all lie in the same plane. If $B = 0$ at the circle's center, what is $\theta$?

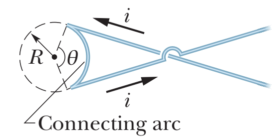


**Figure 29-53** Problem 25.

**••26** GO In Fig. 29-54*a*, wire 1 consists of a circular arc and two radial lengths; it carries current $i_1 = 0.50$ A in the direction indicated. Wire 2, shown in cross section, is long, straight, and perpendicular to the plane of the figure. Its distance from the center of the arc is equal to the radius $R$ of the arc, and it carries a current $i_2$ that can be varied. The two currents set up a net magnetic field $\vec{B}$ at the center of the arc. Figure 29-54*b* gives the square of the field's magnitude $B^2$ plotted versus the square of the current $i_2^2$. The vertical scale is set by $B_s^2 = 10.0 \times 10^{-10}\,\text{T}^2$. What angle is subtended by the arc?

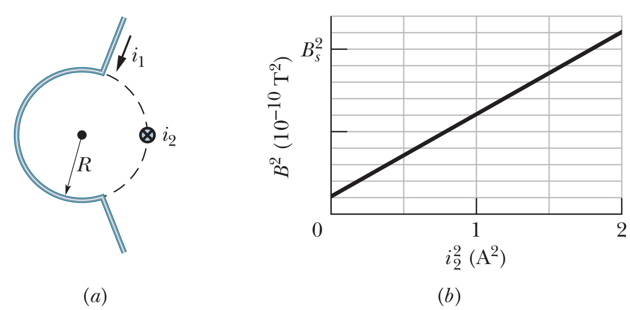


**Figure 29-54** Problem 26.

**••27** In Fig. 29-55, two long straight wires (shown in cross section) carry the currents $i_1 = 30.0$ mA and $i_2 = 40.0$ mA directly out of the page. They are equal distances from the origin, where they set up a magnetic field $\vec{B}$. To what value must current $i_1$ be changed in order to rotate $\vec{B}$ 20.0° clockwise?

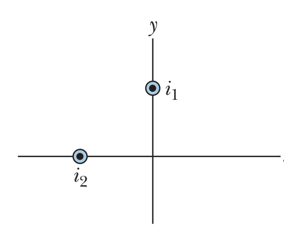


**Figure 29-55** Problem 27.

**••28** GO Figure 29-56*a* shows two wires, each carrying a current. Wire 1 consists of a circular arc of radius $R$ and two radial lengths;

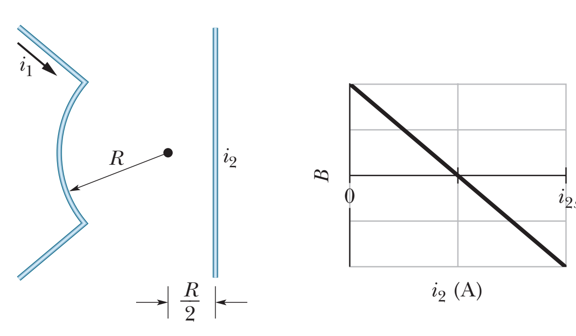


**Figure 29-56** Problem 28.

it carries current $i_1 = 2.0$ A in the direction indicated. Wire 2 is long and straight; it carries a current $i_2$ that can be varied; and it is at distance $R/2$ from the center of the arc. The net magnetic field $\vec{B}$ due to the two currents is measured at the center of curvature of the arc. Figure 29-56*b* is a plot of the component of $\vec{B}$ in the direction perpendicular to the figure as a function of current $i_2$. The horizontal scale is set by $i_{2s} = 1.00$ A. What is the angle subtended by the arc?

**••29** SSM In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a = 20$ cm. The currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3, and each wire carries 20 A. In unit-vector notation, what is the net magnetic field at the square's center?

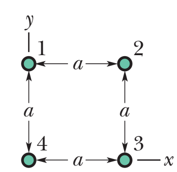



**Figure 29-57** Problems 29, 37, and 40.

**•••30** GO Two long straight thin wires with current lie against an equally long plastic cylinder, at radius $R = 20.0$ cm from the cylinder's central axis. Figure 29-58*a* shows, in cross section, the cylinder and wire 1 but not wire 2. With wire 2 fixed in place, wire 1 is moved around the cylinder, from angle $\theta_1 = 0°$ to angle $\theta_1 = 180°$, through the first and second quadrants of the $xy$ coordinate system. The net magnetic field $\vec{B}$ at the center of the cylinder is measured as a function of $\theta_1$. Figure 29-58*b* gives the $x$ component $B_x$ of that field as a function of $\theta_1$ (the vertical scale is set by $B_{xs} = 6.0\ \mu$T), and Fig. 29-58*c* gives the $y$ component $B_y$ (the vertical scale is set by $B_{ys} = 4.0\ \mu$T). (a) At what angle $\theta_2$ is wire 2 located? What are the (b) size and (c) direction (into or out of the page) of the current in wire 1 and the (d) size and (e) direction of the current in wire 2?

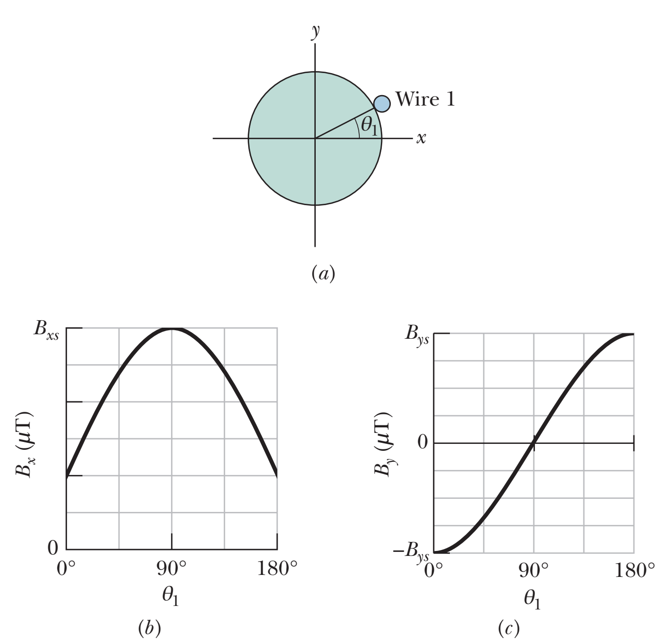



**Figure 29-58** Problem 30.

**•••31** In Fig. 29-59, length $a$ is 4.7 cm (short) and current $i$ is 13 A. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at point $P$?

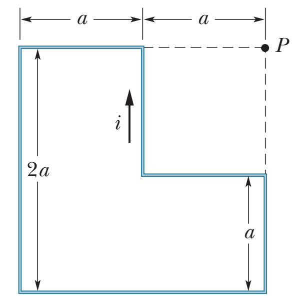



**Figure 29-59** Problem 31.

**•••32** GO The current-carrying wire loop in Fig. 29-60*a* lies all in one plane and consists of a semicircle of radius 10.0 cm, a smaller semicircle with the same center, and two radial lengths. The smaller semicircle is rotated out of that plane by angle $\theta$, until it is perpendicular to the plane (Fig. 29-60*b*). Figure 29-60*c* gives the magnitude of the net magnetic field at the center of curvature versus angle $\theta$. The vertical scale is set by $B_a = 10.0\ \mu$T and $B_b = 12.0\ \mu$T. What is the radius of the smaller semicircle?

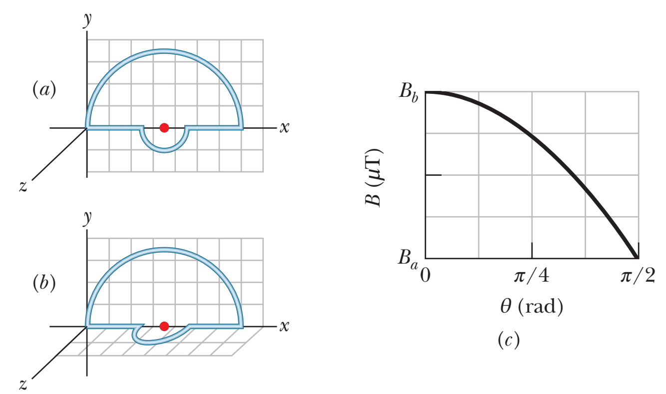



**Figure 29-60** Problem 32.

**•••33** SSM ILW Figure 29-61 shows a cross section of a long thin ribbon of width $w = 4.91$ cm that is carrying a uniformly distributed total current $i = 4.61\ \mu$A into the page. In unit-vector notation, what is the magnetic field $\vec{B}$ at a point $P$ in the plane of the ribbon at a distance $d = 2.16$ cm from its edge? (*Hint:* Imagine the ribbon as being constructed from many long, thin, parallel wires.)

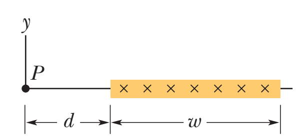



**Figure 29-61** Problem 33.

**•••34** GO Figure 29-62 shows, in cross section, two long straight wires held against a plastic cylinder of radius 20.0 cm. Wire 1 carries current $i_1 = 60.0$ mA out of the page and is fixed in place at the left side of the cylinder. Wire 2 carries current $i_2 = 40.0$ mA out of the page and can be moved around the cylinder. At what (positive) angle $\theta_2$ should wire 2 be positioned such that, at the origin, the net magnetic field due to the two currents has magnitude 80.0 nT?

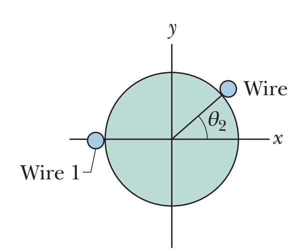



**Figure 29-62** Problem 34.

## Module 29-2 Force Between Two Parallel Currents

**•35** SSM Figure 29-63 shows wire 1 in cross section; the wire is long

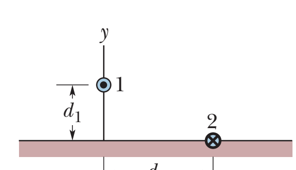



**Figure 29-63** Problem 35.

and straight, carries a current of 4.00 mA out of the page, and is at distance $d_1 = 2.40$ cm from a surface. Wire 2, which is parallel to wire 1 and also long, is at horizontal distance $d_2 = 5.00$ cm from wire 1 and carries a current of 6.80 mA into the page. What is the $x$ component of the magnetic force *per unit length* on wire 2 due to wire 1?

**••36** In Fig. 29-64, five long parallel wires in an $xy$ plane are separated by distance $d = 8.00$ cm, have lengths of 10.0 m, and carry identical currents of 3.00 A out of the page. Each wire experiences a magnetic force due to the currents in the other wires. In unit-vector notation, what is the net magnetic force on (a) wire 1, (b) wire 2, (c) wire 3, (d) wire 4, and (e) wire 5?

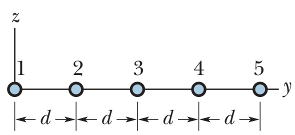



**Figure 29-64** Problems 36 and 39.

**••37** GO In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a = 13.5$ cm. Each wire carries 7.50 A, and the currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3. In unit-vector notation, what is the net magnetic force *per meter of wire length* on wire 4?

**••38** GO Figure 29-65*a* shows, in cross section, three current-carrying wires that are long, straight, and parallel to one another. Wires 1 and 2 are fixed in place on an $x$ axis, with separation $d$. Wire 1 has a current of 0.750 A, but the direction of the current is not given. Wire 3, with a current of 0.250 A out of the page, can be moved along the $x$ axis to the right of wire 2. As wire 3 is moved, the magnitude of the net magnetic force $\vec{F}_2$ on wire 2 due to the currents in wires 1 and 3 changes. The $x$ component of that force is $F_{2x}$ and the value per unit length of wire 2 is $F_{2x}/L_2$. Figure 29-65*b* gives $F_{2x}/L_2$ versus the position $x$ of wire 3. The plot has an asymptote $F_{2x}/L_2 = -0.627\ \mu$N/m as $x \to \infty$. The horizontal scale is set by $x_s = 12.0$ cm. What are the (a) size and (b) direction (into or out of the page) of the current in wire 2?

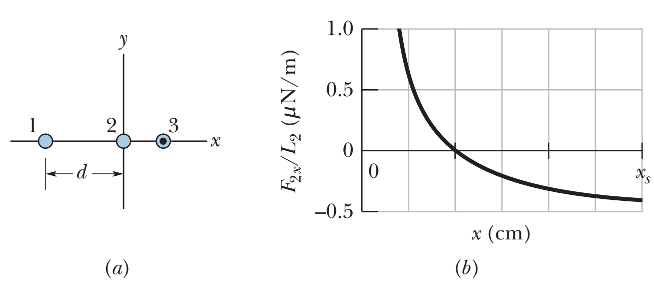



**Figure 29-65** Problem 38.

**••39** GO In Fig. 29-64, five long parallel wires in an $xy$ plane are separated by distance $d = 50.0$ cm. The currents into the page are $i_1 = 2.00$ A, $i_3 = 0.250$ A, $i_4 = 4.00$ A, and $i_5 = 2.00$ A; the current out of the page is $i_2 = 4.00$ A. What is the magnitude of the net force *per unit length* acting on wire 3 due to the currents in the other wires?

**••40** In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a = 8.50$ cm. Each wire carries 15.0 A, and all the currents are out of the page. In unit-vector notation, what is the net magnetic force *per meter of wire length* on wire 1?

**•••41** ILW In Fig. 29-66, a long straight wire carries a current $i_1 = 30.0$ A and a rectangular loop carries current $i_2 = 20.0$ A. Take the dimensions to be $a = 1.00$ cm, $b = 8.00$ cm, and $L = 30.0$ cm. In unit-vector notation, what is the net force on the loop due to $i_1$?

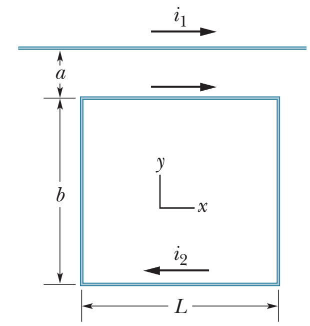



**Figure 29-66** Problem 41.

## Module 29-3 Ampere's Law

**•42** In a particular region there is a uniform current density of 15 A/m² in the positive $z$ direction. What is the value of $\oint \vec{B} \cdot d\vec{s}$ when that line integral is calculated along a closed path consisting of the three straight-line segments from $(x, y, z)$ coordinates $(4d, 0, 0)$ to $(4d, 3d, 0)$ to $(0, 0, 0)$ to $(4d, 0, 0)$, where $d = 20$ cm?

**•43** Figure 29-67 shows a cross section across a diameter of a long cylindrical conductor of radius $a = 2.00$ cm carrying uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) 0, (b) 1.00 cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm?

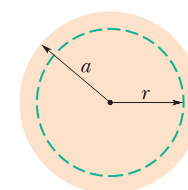



**Figure 29-67** Problem 43.

**•44** Figure 29-68 shows two closed paths wrapped around two conducting loops carrying currents $i_1 = 5.0$ A and $i_2 = 3.0$ A. What is the value of the integral $\oint \vec{B} \cdot d\vec{s}$ for (a) path 1 and (b) path 2?

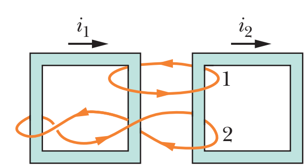



**Figure 29-68** Problem 44.

**•45** SSM Each of the eight conductors in Fig. 29-69 carries 2.0 A of current into or out of the page. Two paths are indicated for the line integral $\oint \vec{B} \cdot d\vec{s}$. What is the value of the integral for (a) path 1 and (b) path 2?

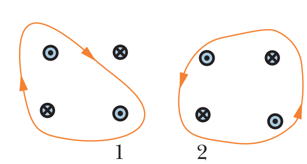



**Figure 29-69** Problem 45.

**•46** Eight wires cut the page perpendicularly at the points shown in Fig. 29-70. A wire labeled with the integer $k$ $(k = 1, 2, \ldots, 8)$ carries the current $ki$, where $i = 4.50$ mA. For those wires with odd $k$, the current is out of the page; for those with even $k$, it is into the page. Evaluate $\oint \vec{B} \cdot d\vec{s}$ along the closed path indicated and in the direction shown.

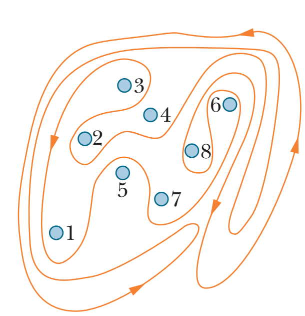



**Figure 29-70** Problem 46.

**••47** ILW The current density $\vec{J}$ inside a long, solid, cylindrical wire of radius $a = 3.1$ mm is in the direction of the central axis, and its magnitude varies linearly with radial distance $r$ from the axis according to $J = J_0 r/a$, where $J_0 = 310$ A/m$^2$. Find the magnitude of the magnetic field at (a) $r = 0$, (b) $r = a/2$, and (c) $r = a$.

**••48** In Fig. 29-71, a long circular pipe with outside radius $R = 2.6$ cm carries a (uniformly distributed) current $i = 8.00$ mA into the page. A wire runs parallel to the pipe at a distance of $3.00R$ from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point $P$ has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.

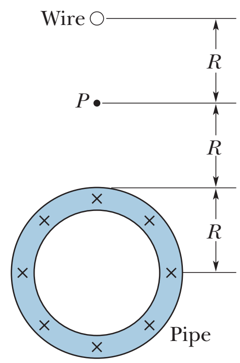



**Figure 29-71** Problem 48.

## Module 29-4 Solenoids and Toroids

**•49** A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of 0.800 A. (It is made up of a square solenoid — instead of a round one as in Fig. 29-17 — bent into a doughnut shape.) What is the magnetic field inside the toroid at (a) the inner radius and (b) the outer radius?

**•50** A solenoid that is 95.0 cm long has a radius of 2.00 cm and a winding of 1200 turns; it carries a current of 3.60 A. Calculate the magnitude of the magnetic field inside the solenoid.

**•51** A 200-turn solenoid having a length of 25 cm and a diameter of 10 cm carries a current of 0.29 A. Calculate the magnitude of the magnetic field $\vec{B}$ inside the solenoid.

**•52** A solenoid 1.30 m long and 2.60 cm in diameter carries a current of 18.0 A. The magnetic field inside the solenoid is 23.0 mT. Find the length of the wire forming the solenoid.

**••53** A long solenoid has 100 turns/cm and carries current $i$. An electron moves within the solenoid in a circle of radius 2.30 cm perpendicular to the solenoid axis. The speed of the electron is $0.0460c$ ($c$ = speed of light). Find the current $i$ in the solenoid.

**••54** An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is 800 m/s and its velocity vector makes an angle of 30° with the central axis of the solenoid. The solenoid carries 4.0 A and has 8000 turns along its length. How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid's opposite end? (In a real solenoid, where the field is not uniform at the two ends, the number of revolutions would be slightly less than the answer here.)

**••55** SSM ILW WWW A long solenoid with 10.0 turns/cm and a radius of 7.00 cm carries a current of 20.0 mA. A current of 6.00 A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at 45.0° to the axial direction? (b) What is the magnitude of the magnetic field there?

## Module 29-5 A Current-Carrying Coil as a Magnetic Dipole

**•56** Figure 29-72 shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of 200 turns and radius $R = 25.0$ cm, separated by a distance $s = R$. The two coils carry equal currents $i = 12.2$ mA in the same direction. Find the magnitude of the net magnetic field at $P$, midway between the coils.

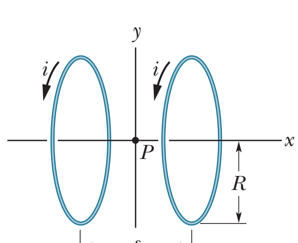



**Figure 29-72** Problem 56.

**•57** SSM A student makes a short electromagnet by winding 300 turns of wire around a wooden cylinder of diameter $d = 5.0$ cm. The coil is connected to a battery producing a current of 4.0 A in the wire. (a) What is the magnitude of the magnetic dipole moment of this device? (b) At what axial distance $z \gg d$ will the magnetic field have the magnitude 5.0 $\mu$T (approximately one-tenth that of Earth's magnetic field)?

**•58** Figure 29-73$a$ shows a length of wire carrying a current $i$ and bent into a circular coil of one turn. In Fig. 29-73$b$ the same length of wire has been bent to give a coil of two turns, each of half the original radius. (a) If $B_a$ and $B_b$ are the magnitudes of the magnetic fields at the centers of the two coils, what is the ratio $B_b/B_a$? (b) What is the ratio $\mu_b/\mu_a$ of the dipole moment magnitudes of the coils?

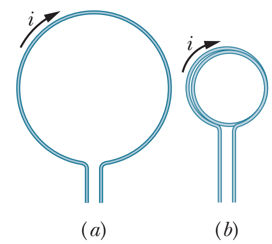



**Figure 29-73** Problem 58.

**•59** SSM What is the magnitude of the magnetic dipole moment $\vec{\mu}$ of the solenoid described in Problem 51?

**••60** GO In Fig. 29-74$a$, two circular loops, with different currents but the same radius of 4.0 cm, are centered on a $y$ axis. They are initially separated by distance $L = 3.0$ cm, with loop 2 positioned at the origin of the axis. The currents in the two loops produce a net magnetic field at the origin, with $y$ component $B_y$. That component is to be measured as loop 2 is gradually moved in the positive direction of the $y$ axis. Figure 29-74$b$ gives $B_y$ as a function of the position $y$ of loop 2. The curve approaches an asymptote of $B_y = 7.20\ \mu$T as $y \to \infty$. The horizontal scale is set by $y_s = 10.0$ cm. What are (a) current $i_1$ in loop 1 and (b) current $i_2$ in loop 2?

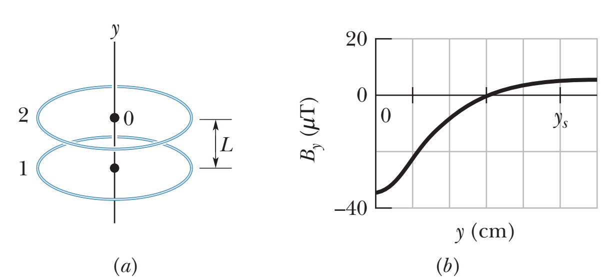



**Figure 29-74** Problem 60.

**••61** A circular loop of radius 12 cm carries a current of 15 A. A flat coil of radius 0.82 cm, having 50 turns and a current of 1.3 A, is concentric with the loop. The plane of the loop is perpendicular to the plane of the coil. Assume the loop's magnetic field is uniform across the coil. What is the magnitude of (a) the magnetic field produced by the loop at its center and (b) the torque on the coil due to the loop?

**••62** In Fig. 29-75, current $i = 56.2$ mA is set up in a loop having two radial lengths and two semicircles of radii $a = 5.72$ cm

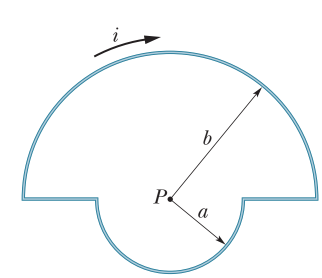



**Figure 29-75** Problem 62.

and $b = 9.36$ cm with a common center $P$. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at $P$ and the (c) magnitude and (d) direction of the loop's magnetic dipole moment?

**••63** In Fig. 29-76, a conductor carries 6.0 A along the closed path *abcdefgha* running along 8 of the 12 edges of a cube of edge length 10 cm. (a) Taking the path to be a combination of three square current loops (*bcfgb*, *abgha*, and *cdefc*), find the net magnetic moment of the path in unit-vector notation. (b) What is the magnitude of the net magnetic field at the *xyz* coordinates of (0, 5.0 m, 0)?

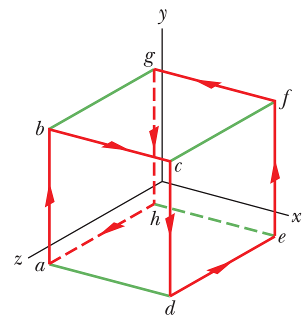



**Figure 29-76** Problem 63.

## Additional Problems

**64** In Fig. 29-77, a closed loop carries current $i = 200$ mA. The loop consists of two radial straight wires and two concentric circular arcs of radii 2.00 m and 4.00 m. The angle $\theta$ is $\pi/4$ rad. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the center of curvature $P$?

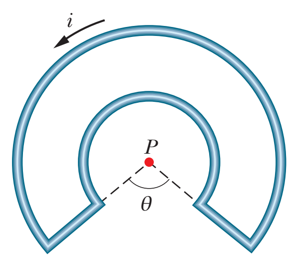



**Figure 29-77** Problem 64.

**65** A cylindrical cable of radius 8.00 mm carries a current of 25.0 A, uniformly spread over its cross-sectional area. At what distance from the center of the wire is there a point within the wire where the magnetic field magnitude is 0.100 mT?

**66** Two long wires lie in an $xy$ plane, and each carries a current in the positive direction of the $x$ axis. Wire 1 is at $y = 10.0$ cm and carries 6.00 A; wire 2 is at $y = 5.00$ cm and carries 10.0 A. (a) In unit-vector notation, what is the net magnetic field $\vec{B}$ at the origin? (b) At what value of $y$ does $\vec{B} = 0$? (c) If the current in wire 1 is reversed, at what value of $y$ does $\vec{B} = 0$?

**67** Two wires, both of length $L$, are formed into a circle and a square, and each carries current $i$. Show that the square produces a greater magnetic field at its center than the circle produces at its center.

**68** A long straight wire carries a current of 50 A. An electron, traveling at $1.0 \times 10^7$ m/s, is 5.0 cm from the wire. What is the magnitude of the magnetic force on the electron if the electron velocity is directed (a) toward the wire, (b) parallel to the wire in the direction of the current, and (c) perpendicular to the two directions defined by (a) and (b)?

**69** Three long wires are parallel to a $z$ axis, and each carries a current of 10 A in the positive $z$ direction. Their points of intersection with the $xy$ plane form an equilateral triangle with sides of 50 cm, as shown in Fig. 29-78. A fourth wire (wire $b$) passes through the midpoint of the base of the triangle and is parallel to the other three wires. If the net magnetic force on wire $a$ is zero, what are the (a) size and (b) direction ($+z$ or $-z$) of the current in wire $b$?

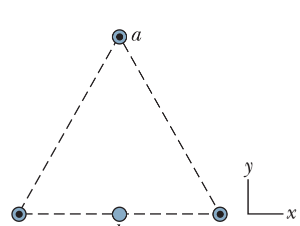



**Figure 29-78** Problem 69.

**70** Figure 29-79 shows a closed loop with current $i = 2.00$ A. The loop consists of a half-circle of radius 4.00 m, two quarter-circles each of radius 2.00 m, and three radial straight wires. What is the magnitude of the net magnetic field at the common center of the circular sections?

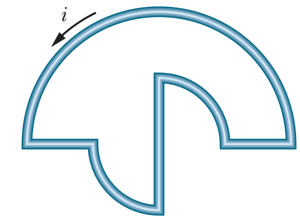



**Figure 29-79** Problem 70.

**71** A 10-gauge bare copper wire (2.6 mm in diameter) can carry a current of 50 A without overheating. For this current, what is the magnitude of the magnetic field at the surface of the wire?

**72** A long vertical wire carries an unknown current. Coaxial with the wire is a long, thin, cylindrical conducting surface that carries a current of 30 mA upward. The cylindrical surface has a radius of 3.0 mm. If the magnitude of the magnetic field at a point 5.0 mm from the wire is 1.0 $\mu$T, what are the (a) size and (b) direction of the current in the wire?

**73** Figure 29-80 shows a cross section of a long cylindrical conductor of radius $a = 4.00$ cm containing a long cylindrical hole of radius $b = 1.50$ cm. The central axes of the cylinder and hole are parallel and are distance $d = 2.00$ cm apart; current $i = 5.25$ A is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? (b) Discuss the two special cases $b = 0$ and $d = 0$.

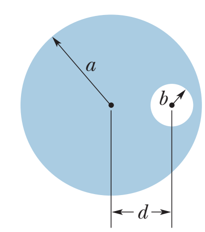



**Figure 29-80** Problem 73.

**74** The magnitude of the magnetic field at a point 88.0 cm from the central axis of a long straight wire is 7.30 $\mu$T. What is the current in the wire?

**75** SSM Figure 29-81 shows a wire segment of length $\Delta s = 3.0$ cm, centered at the origin, carrying current $i = 2.0$ A in the positive $y$ direction (as part of some complete circuit). To calculate the magnitude of the magnetic field $\vec{B}$ produced by the segment at a point several meters from the origin, we can use $B = (\mu_0/4\pi)i\,\Delta s\,(\sin\theta)/r^2$ as the Biot–Savart law. This is because $r$ and $\theta$ are essentially constant over the segment. Calculate $\vec{B}$ (in unit-vector notation) at the $(x, y, z)$ coordinates (a) (0, 0, 5.0 m), (b) (0, 6.0 m, 0), (c) (7.0 m, 7.0 m, 0), and (d) (–3.0 m, –4.0 m, 0).

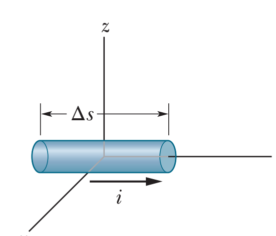



**Figure 29-81** Problem 75.

**76** GO Figure 29-82 shows, in cross section, two long parallel wires spaced by distance $d = 10.0$ cm; each carries 100 A, out of the page in wire 1. Point $P$ is on a perpendicular bisector of the line connecting the wires. In unit-vector notation, what is the net magnetic field at $P$ if the current in wire 2 is (a) out of the page and (b) into the page?

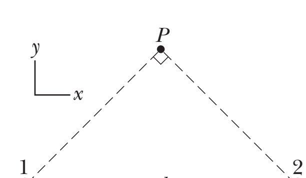



**Figure 29-82** Problem 76.

**77** In Fig. 29-83, two infinitely long wires carry equal currents $i$. Each follows a 90° arc on the circumference of the same circle of radius $R$. Show that the magnetic field $\vec{B}$ at the center of the circle is the same as the field $\vec{B}$ a distance $R$ below an infinite straight wire carrying a current $i$ to the left.

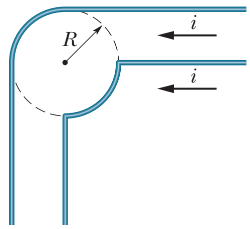


**Figure 29-83** Problem 77.

**78** A long wire carrying 100 A is perpendicular to the magnetic field lines of a uniform magnetic field of magnitude 5.0 mT. At what distance from the wire is the net magnetic field equal to zero?

**79** A long, hollow, cylindrical conductor (with inner radius 2.0 mm and outer radius 4.0 mm) carries a current of 24 A distributed uniformly across its cross section. A long thin wire that is coaxial with the cylinder carries a current of 24 A in the opposite direction. What is the magnitude of the magnetic field (a) 1.0 mm, (b) 3.0 mm, and (c) 5.0 mm from the central axis of the wire and cylinder?

**80** A long wire is known to have a radius greater than 4.0 mm and to carry a current that is uniformly distributed over its cross section. The magnitude of the magnetic field due to that current is 0.28 mT at a point 4.0 mm from the axis of the wire, and 0.20 mT at a point 10 mm from the axis of the wire. What is the radius of the wire?

**81** SSM Figure 29-84 shows a cross section of an infinite conducting sheet carrying a current per unit $x$-length of $\lambda$; the current emerges perpendicularly out of the page. (a) Use the Biot–Savart law and symmetry to show that for all points $P$ above the sheet and all points $P'$ below it, the magnetic field $\vec{B}$ is parallel to the sheet and directed as shown. (b) Use Ampere's law to prove that $B = \frac{1}{2}\mu_0\lambda$ at all points $P$ and $P'$.

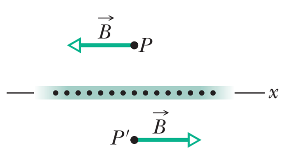


**Figure 29-84** Problem 81.

**82** Figure 29-85 shows, in cross section, two long parallel wires that are separated by distance $d = 18.6$ cm. Each carries 4.23 A, out of the page in wire 1 and into the page in wire 2. In unit-vector notation, what is the net magnetic field at point $P$ at distance $R = 34.2$ cm, due to the two currents?

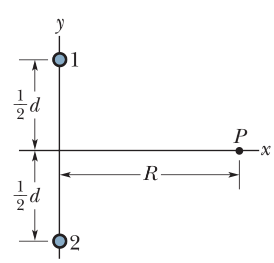


**Figure 29-85** Problem 82.

**83** SSM In unit-vector notation, what is the magnetic field at point $P$ in Fig. 29-86 if $i = 10$ A and $a = 8.0$ cm? (Note that the wires are *not* long.)

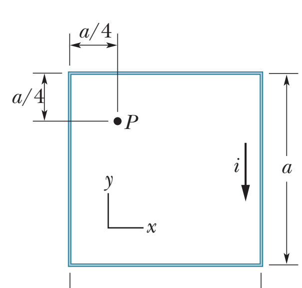


**Figure 29-86** Problem 83.

**84** Three long wires all lie in an $xy$ plane parallel to the $x$ axis. They are spaced equally, 10 cm apart. The two outer wires each carry a current of 5.0 A in the positive $x$ direction. What is the magnitude of the force on a 3.0 m section of either of the outer wires if the current in the center wire is 3.2 A (a) in the positive $x$ direction and (b) in the negative $x$ direction?

**85** SSM Figure 29-87 shows a cross section of a hollow cylindrical conductor of radii $a$ and $b$, carrying a uniformly distributed current $i$. (a) Show that the magnetic field magnitude $B(r)$ for the radial distance $r$ in the range $b < r < a$ is given by

$$B = \frac{\mu_0 i}{2\pi(a^2 - b^2)} \frac{r^2 - b^2}{r}.$$

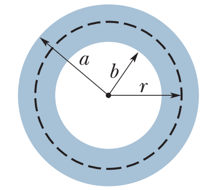


**Figure 29-87** Problem 85.

(b) Show that when $r = a$, this equation gives the magnetic field magnitude $B$ at the surface of a long straight wire carrying current $i$; when $r = b$, it gives zero magnetic field; and when $b = 0$, it gives the magnetic field inside a solid conductor of radius $a$ carrying current $i$. (c) Assume that $a = 2.0$ cm, $b = 1.8$ cm, and $i = 100$ A, and then plot $B(r)$ for the range $0 < r < 6$ cm.

**86** Show that the magnitude of the magnetic field produced at the center of a rectangular loop of wire of length $L$ and width $W$, carrying a current $i$, is

$$B = \frac{2\mu_0 i}{\pi} \frac{(L^2 + W^2)^{1/2}}{LW}.$$

**87** Figure 29-88 shows a cross section of a long conducting coaxial cable and gives its radii ($a$, $b$, $c$). Equal but opposite currents $i$ are uniformly distributed in the two conductors. Derive expressions for $B(r)$ with radial distance $r$ in the ranges (a) $r < c$, (b) $c < r < b$, (c) $b < r < a$, and (d) $r > a$. (e) Test these expressions for all the special cases that occur to you. (f) Assume that $a = 2.0$ cm, $b = 1.8$ cm, $c = 0.40$ cm, and $i = 120$ A and plot the function $B(r)$ over the range $0 < r < 3$ cm.

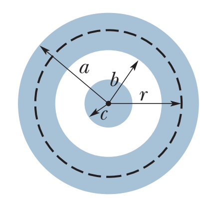


**Figure 29-88** Problem 87.

**88** Figure 29-89 is an idealized schematic drawing of a rail gun. Projectile $P$ sits between two wide rails of circular cross section; a source of current sends current through the rails and through the (conducting) projectile (a fuse is not used). (a) Let $w$ be the distance between the rails, $R$ the radius of each rail, and $i$ the current. Show that the force on the projectile is directed to the right along the rails and is given approximately by

$$F = \frac{i^2\mu_0}{2\pi} \ln \frac{w + R}{R}.$$

(b) If the projectile starts from the left end of the rails at rest, find the speed $v$ at which it is expelled at the right. Assume that $i = 450$ kA, $w = 12$ mm, $R = 6.7$ cm, $L = 4.0$ m, and the projectile mass is 10 g.

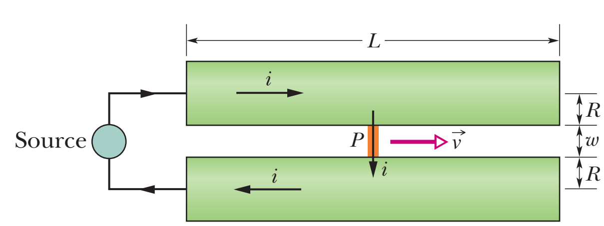


**Figure 29-89** Problem 88.