UFUG 1504: Honors General Physics II

Chapter 27

Circuits

27 Summary (1 of 4)

Note: all emf symbol use $\mathcal{E}$ in your homework and exam

Emf

- The **emf** (work per unit charge) of the device is

$$\mathscr{E} = \frac{dw}{dq} \quad \left(\text{definition of } \mathscr{E}\right). \qquad \textbf{Equation (27-1)}$$

Single-Loop Circuits

- Current in a single-loop circuit:

$$i = \frac{\mathscr{E}}{R + r}, \qquad \textbf{Equation (27-4)}$$

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27 Summary (2 of 4)

Series Resistance

- When resistances are in series

$$R_{\text{eq}} = \sum_{j=1}^{n} R_j$$

Equation (27-7)

Power

- The rate P of energy transfer to the charge carriers is

$$P = iV$$

Equation (27-14)

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27 Summary (3 of 4)

- The rate P_r at which energy is dissipated as thermal energy in the battery is

$$P_r = i^2 r. \qquad \textbf{Equation (27-16)}$$

- The rate P_{emf} at which the chemical energy in the battery changes is

$$P_{\text{emf}} = i\mathscr{E}. \qquad \textbf{Equation (27-17)}$$

Parallel Resistance

- When resistances are in parallel

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j} \qquad \textbf{Equation (27-24)}$$

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27 Summary (4 of 4)

RC Circuits

- The charge on the capacitor increases according to

$$q = C\mathscr{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

Equation (27-33)

- During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-\frac{t}{RC}}$$

Equation (27-34)

- During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-\frac{t}{RC}}$$

Equation (27-40)

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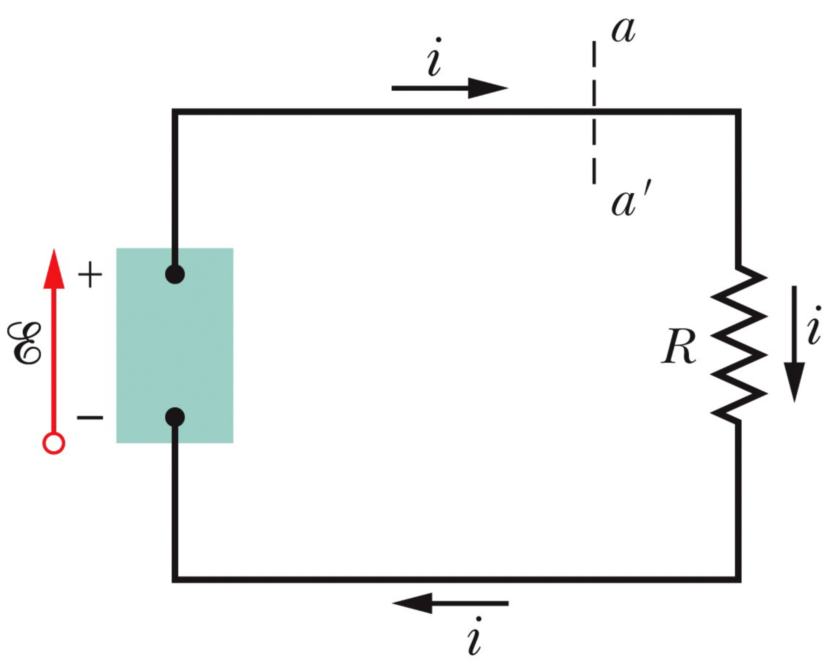
27-1 Single-Loop Circuits (5 of 21)

emf: Electromotive force.

$\mathscr{E}$, in textbook and slide
$\mathcal{E}$ in textbook and slide



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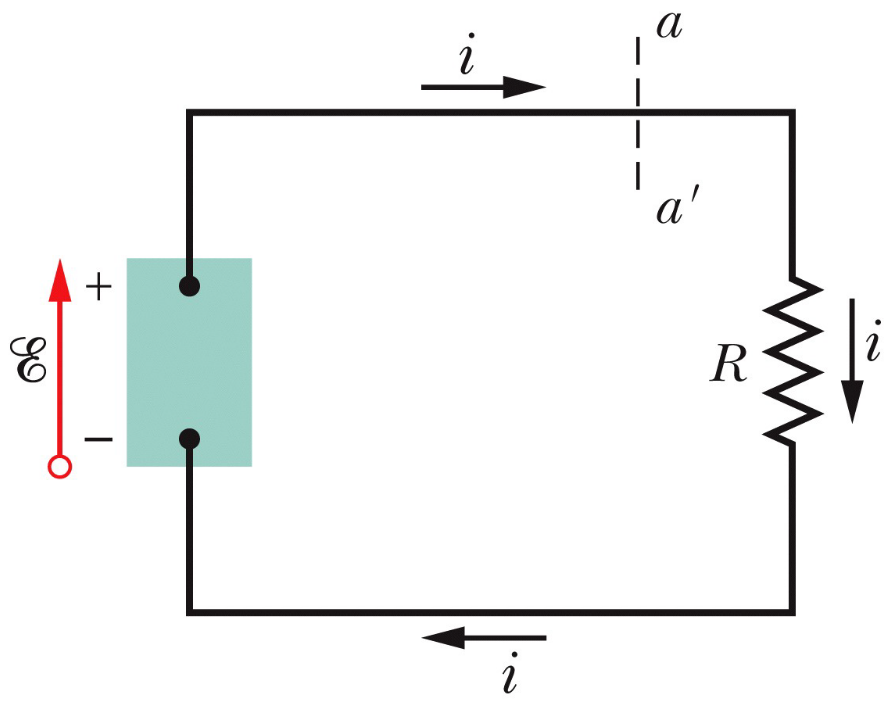
In chapter 26, we discussed the motion of charge carriers through a circuit in terms of the electric field set up in the circuit—the field produces forces that move the charge carriers.
In this chapter we take a different approach: We discuss the motion of the charge carriers in terms of the required energy—an emf device supplies the energy for the motion via the work it does.

To produce a steady flow of charge, you need a "charge pump," a device that—by doing work on the charge carriers—maintains a potential difference between a pair of terminals. We call such a device an **emf device**, and the device is said to provide an emf $\mathscr{E}$, which means that it does work on charge carriers.

27-1 Single-Loop Circuits (5 of 21)



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A small circle on the tail of the emf arrow distinguishes it from the arrows that indicate current direction.

Figure shows an emf device (consider it to be a battery) that is part of a simple circuit containing a single resistance R. The emf device keeps one of its terminals (called the positive terminal and often labeled +) at a higher electric potential than the other terminal (called the negative terminal and labeled −). We can represent the emf of the device with an arrow that points from the negative terminal toward the positive terminal as in Figure.

27-1 Single-Loop Circuits (7 of 21)

An emf device does work on charges to maintain a potential difference between its output terminals. If dW is the work the device does to force positive charge dq from the negative to the positive terminal, then the emf (work per unit charge) of the device is

$$\mathscr{E} = \frac{dW}{dq} \quad (\text{definition of } \mathscr{E}).$$

An **ideal emf device** is one that lacks any internal resistance. The potential difference (V) between its terminals is equal to the emf.

A **real emf device** has internal resistance. The potential difference between its terminals is equal to the emf only if there is no current through the device.

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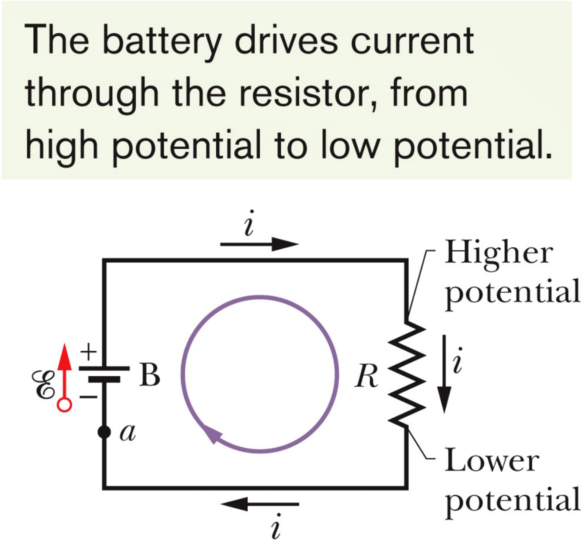
27-1 Single-Loop Circuits (9 of 21)

Calculating Current in a Single-Loop Circuits

Energy Method



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Equation, $P = i^2 R$, tells us that in a time interval dt an amount of energy given by $i^2 R\, dt$ will appear in the resistor (shown in the figure) as thermal energy. This energy is said to be **dissipated**. (Because we assume the wires to have negligible resistance, no thermal energy will appear in them.)

During the same interval, a charge $dq = i\, dt$ will have moved through battery B, and the work that the battery will have done on this charge is

$$dW = \mathscr{E}\, dq = \mathscr{E}\, i\, dt.$$

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27-1 Single-Loop Circuits (10 of 21)

From the principle of conservation of energy, the work done by the (ideal) battery must equal the thermal energy that appears in the resistor:

$$\mathscr{E}\, i\, dt = i^2 R\, dt.$$

Which gives us

$$i = \frac{\mathscr{E}}{R}.$$

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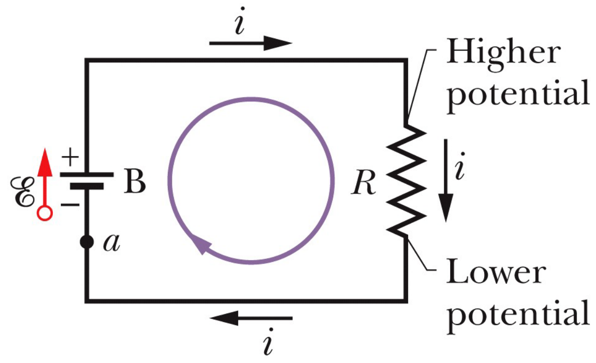
27-1 Single-Loop Circuits (11 of 21)

Calculating Current in a Single-Loop Circuits

Potential Method

The battery drives current through the resistor, from high potential to low potential.



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In the figure, let us start at point a, whose potential is V_a, and mentally walk clockwise around the circuit until we are back at point a, keeping track of potential changes as we move. Our starting point is at the low-potential terminal of the battery. Because the battery is ideal, the potential difference between its terminals is equal to $\mathscr{E}$. When we pass through the battery to the high-potential terminal, the change in potential is $+\mathscr{E}$.

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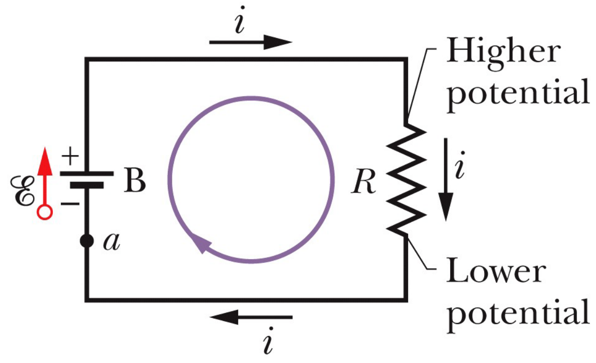
27-1 Single-Loop Circuits (12 of 21)

The battery drives current through the resistor, from high potential to low potential.



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After making a complete loop, our initial potential, as modified for potential changes along the way, must be equal to our final potential; that is,

$$V_a + \mathscr{E} - iR = V_a.$$

The value of V_a cancels from this equation, which becomes

$$\mathscr{E} - iR = 0.$$

Which gives us

$$i = \frac{\mathscr{E}}{R}.$$

27-1 Single-Loop Circuits (13 of 21)

Calculating Current in a Single-Loop Circuits

Loop Rule: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

Resistance Rule: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

Emf Rule: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$ in the opposite direction it is $-\mathscr{E}$.

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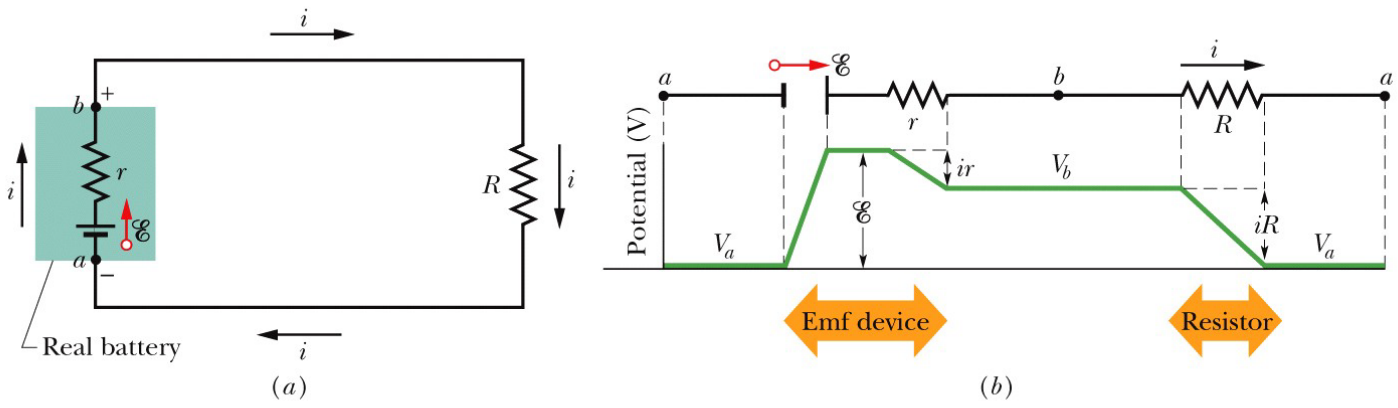
Internal Resistance



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Loop Rule: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

Now if we apply the loop rule clockwise beginning at point *a*, the changes in potential give us

$$\mathscr{E} - ir - iR = 0.$$

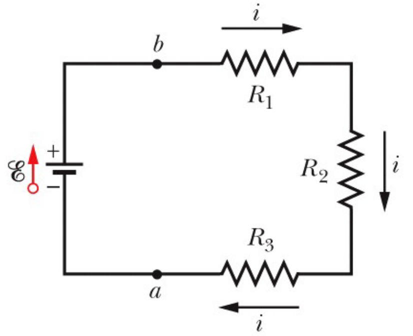
$$i = \frac{\mathscr{E}}{R + r}.$$

Figure (a) shows a real battery, with internal resistance *r*, wired to an external resistor of resistance *R*. The internal resistance of the battery is the electrical resistance of the conducting materials of the battery and thus is an un removable feature of the battery. Figure (b) shows graphically the changes in electric potential around the circuit.

27-1 Single-Loop Circuits (16 of 21)

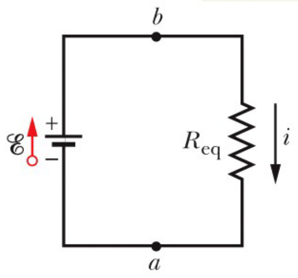
Resistance in Series

Figure (a) shows three resistances connected in series to an ideal battery with emf $\mathscr{E}$. The resistances are connected one after another between *a* and *b*, and a potential difference is maintained across *a* and *b* by the battery. The potential differences that then exist across the resistances in the series produce identical currents *i* in them.



(*a*)

Series resistors and their equivalent have the same current ("ser-i").



(*b*)

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To find total resistance R_{eq} in Figure. (b), we apply the loop rule to both circuits. For Figure. (a), starting at *a* and going clockwise around the circuit, we find

Figure. (a), $$\mathscr{E} - iR_1 - iR_2 - iR_3 = 0,$$

Figure. (b), $$\mathscr{E} - iR_{eq} = 0, \quad \text{or} \quad i = \frac{\mathscr{E}}{R_{eq}}.$$

Equating them, we get,

$$R_{eq} = R_1 + R_2 + R_3. \Rightarrow R_{eq} = \sum_{j=1}^{n} R_j$$

27-1 Single-Loop Circuits (18 of 21)

Resistance in Series

When a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V.

Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same total potential difference V as the actual resistances.

$$V_1 : V_2 : V_3 = R_1 : R_2 : R_3$$

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27-1 Single-Loop Circuits (19 of 21)

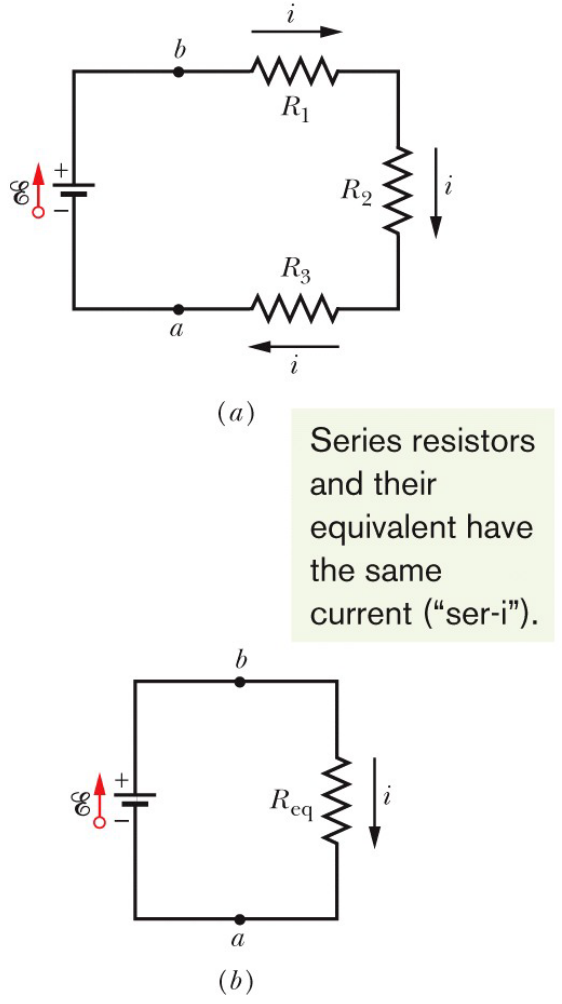
Checkpoint 2

In Figure. a, if $R_1 > R_2 > R_3$, rank the three resistances according to (a) the current through them and (b) the potential difference across them, greatest first.

Answer:

(a) current is same for all resistors in series.

(b) V_1, V_2, and V_3



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27-1 Single-Loop Circuits (20 of 21)

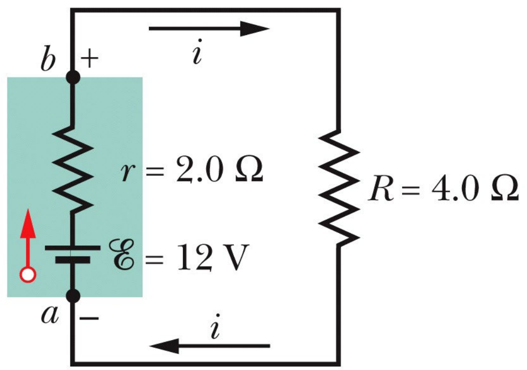
Potential Difference

To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.

Potential Difference across a real battery: In the Figure, points a and b are located at the terminals of the battery. Thus, the potential difference $V_b - V_a$ is the terminal-to-terminal potential difference V across the batter and is given by:

$$V = \mathscr{E} - ir.$$

The internal resistance reduces the potential difference between the terminals.



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27-1 Single-Loop Circuits (21 of 21)

Grounding a Circuit: Grounding a circuit usually means connecting one point in the circuit to a conducting path to Earth's surface (actually to the electrically conducting moist dirt and rock below ground)

Power of emf Device: The rate P_{emf} at which the emf device transfers energy both to the charge carriers and to internal thermal energy is

$$P_{emf} = i\mathscr{E} \quad \text{(power of emf device).}$$

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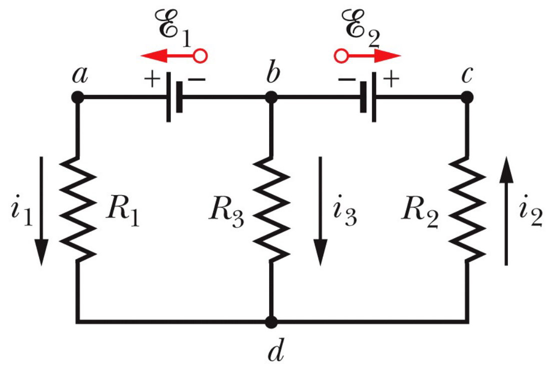
27-2 Multi-Loop Circuits (4 of 11)

Junction Rule: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

Figure shows a circuit containing more than one loop. If we traverse the left-hand loop in a counterclockwise direction from point *b*, the loop rule gives us

$$\mathscr{E}_1 - i_1 R_1 + i_3 R_3 = 0.$$

The current into the junction must equal the current out (charge is conserved).



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27-2 Multi-Loop Circuits (5 of 11)

If we traverse the right-hand loop in a counterclockwise direction from point *b*, the loop rule gives us

$$-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0.$$

If we had applied the loop rule to the big loop, we would have obtained (moving counterclockwise from *b*) the equation

$$\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0.$$

which is the sum of two small loops equations.

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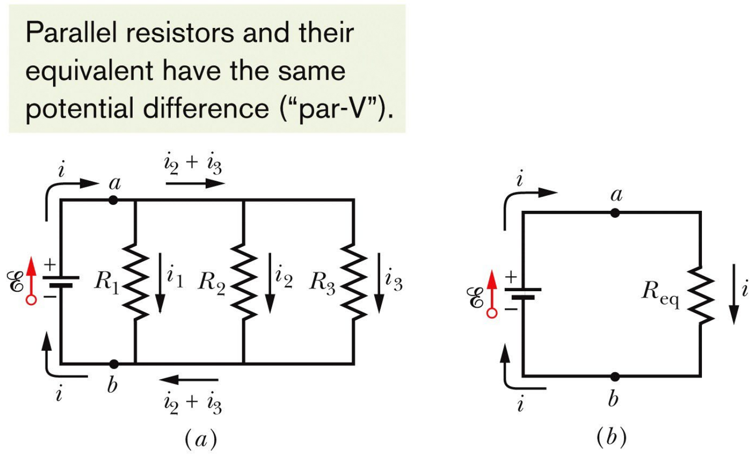
27-2 Multi-Loop Circuits (6 of 11)

Resistances in Parallel

Figure (a) shows three resistances connected in parallel to an ideal battery of emf $\mathscr{E}$. The applied potential difference V is maintained the battery. Figure. b, the three parallel resistances have been replaced with an equivalent resistance R_{eq}.



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27-2 Multi-Loop Circuits (7 of 11)

To derive an expression for R_{eq} in Figure. (b), we first write the current in each actual resistance in Figure. (a) as

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

where V is the potential difference between a and b. If we apply the junction rule at point a in Figure. (a) and then substitute these values, we find

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

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27-2 Multi-Loop Circuits (8 of 11)

If we replaced the parallel combination with the equivalent resistance R_{eq} (Figure. *b*), we would have $i = \dfrac{V}{R_{\text{eq}}}$.
and thus substituting the value of *i* from above equation we get,

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \Longrightarrow \frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad (n \text{ resistances in parallel}).$$

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27-2 Multi-Loop Circuits (9 of 11)

Resistance and capacitors

Table 27-1 Series and Parallel Resistors and Capacitors

Series		Parallel	
Resistors		Resistors	
$R_{\text{eq}} = \sum_{j=1}^{n} R_j$	**Equation (27-7)**	$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j}$	**Equation (27-24)**
Same current through all resistors		Same potential difference across all resistors	

27-2 Multi-Loop Circuits (10 of 11)

Series		Parallel	
Capacitors		Capacitors	
$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{C_j}$	**Equation (25-20)**	$C_{\text{eq}} = \sum_{j=1}^{n} C_j$	**Equation (25-19)**
Same charge on all capacitors		Same potential difference across all capacitors	

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27-2 Multi-Loop Circuits (11 of 11)

Checkpoint 4

A battery, with potential V across it, is connected to a combination of two identical resistors and then has current i through it. What are the potential difference across and the current through either resistor if the resistors are (a) in series and (b) in parallel?

Answer:

(a) Potential difference across each resistor: $\frac{V}{2}$

Current through each resistor: i

(b) Potential difference across each resistor: V

Current through each resistor: $\frac{i}{2}$

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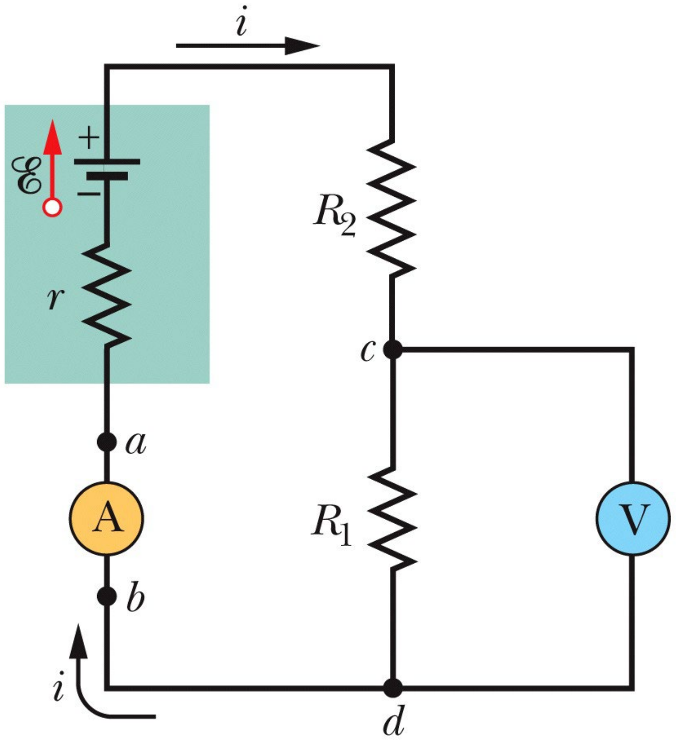
27-3 The Ammeter and The Voltmeter (2 of 3)

An instrument used to measure currents is called an **ammeter (电流表)**. To measure the current in a wire, you usually have to break or cut the wire and insert the ammeter so that the current to be measured passes through the meter. In the figure, ammeter A is set up to measure current i. It is essential that the resistance R_A of the ammeter be very much smaller than other resistances in the circuit. Otherwise, the very presence of the meter will change the current to be measured.



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27-3 The Ammeter and The Voltmeter (3 of 3)

A meter used to measure potential differences is called a **voltmeter (电压表)**. To find the potential difference between any two points in the circuit, the voltmeter terminals are connected between those points without breaking or cutting the wire. In the Figure, voltmeter *V* is set up to measure the voltage across R_1. It is essential that the resistance R_V of a voltmeter be very much larger than the resistance of any circuit element across which the voltmeter is connected. This is to insure that only a negligible current passes through the voltmeter, otherwise, the meter alters the potential difference that is to be measured.

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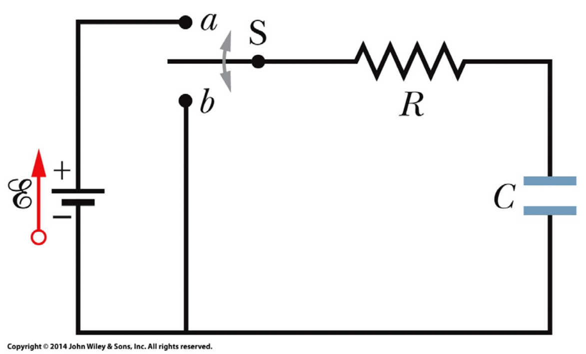
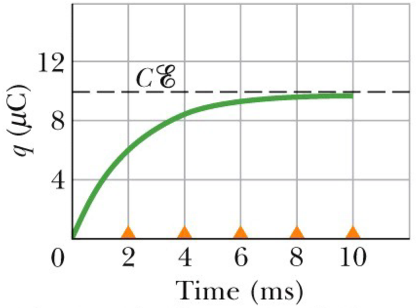
27-4 RC Circuits (3 of 8)



RC circuit



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The plot shows the buildup of charge on the capacitor of the above figure.

Charging a capacitor: The capacitor of capacitance C in the figure is initially uncharged. To charge it, we close switch S on point a. This completes an RC series circuit consisting of the capacitor, an ideal battery of emf $\mathscr{E}$, and a resistance R.

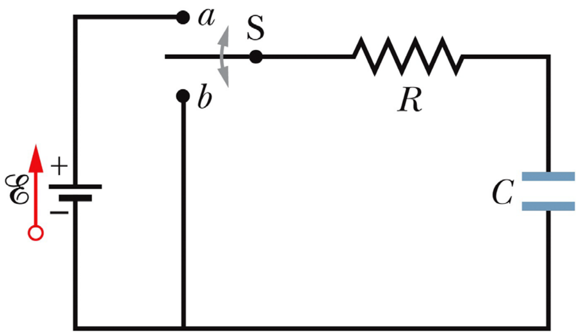
The charge on the capacitor increases according to

$$q = C\mathscr{E}\left(1 - e^{-\frac{t}{RC}}\right) \quad \text{(charging a capacitor).}$$

in which $C\mathscr{E} = q_0$ is the equilibrium (final) charge and $RC = \tau$ is the capacitive time constant of the circuit. During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-\frac{t}{RC}} \quad \text{(charging a capacitor).}$$

27-4 RC Circuits (3 of 8)



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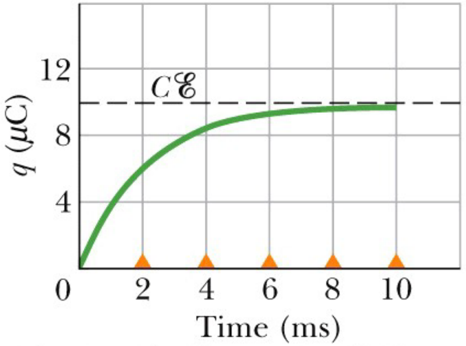

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The plot shows the buildup of charge on the capacitor of the above figure.

$$q = C\mathscr{E}\left(1 - e^{-\frac{t}{RC}}\right) \quad \text{(charging a capacitor).}$$

$$q = CV$$

$$\mathscr{E} - iR - \frac{q}{C} = 0 \quad i = \frac{dq}{dt}$$

$$R\frac{dq}{dt} + \frac{q}{C} = \mathscr{E}$$

Solving firstly $\quad dq/dt + q/RC = 0$

五．线性方程

一阶线性常微分方程的一般形式为

$$\frac{dy}{dx} + f(x)y = g(x)\text{。}$$

利用分离变量法，易知齐次线性方程

$$\frac{dy}{dx} + f(x)y = 0$$

的通解为

$$y = C\mathrm{e}^{-\int f(x)dx}\text{。}$$

$$q_1 = K\,e^{-\frac{t}{RC}}$$

Solving secondly $\quad q_2 = Q(x)$

$$q_2 = C\varepsilon$$

$$q = q_1 + q_2 = K\,e^{-\frac{t}{RC}} + \varepsilon$$

Initial condition $\quad q = 0$ at $t = 0$. $\quad K = -C\varepsilon$

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right).$$

27-4 RC Circuits (4 of 8)

And the voltage is:

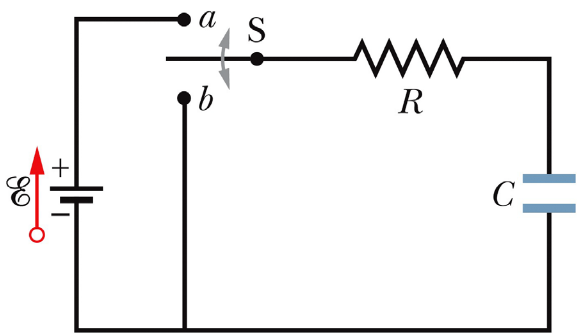
$$V_C = \frac{q}{C} = \mathscr{E}\left(1 - e^{-\frac{t}{RC}}\right) \quad \text{(charging a capacitor).}$$

The product RC is called the capacitive time constant (时间常数) of the circuit and is represented with the symbol τ.

$$\tau = RC \quad \text{(time constant).}$$

27-4 RC Circuits (4 of 8)

$$V_C = \frac{q}{C} = \mathscr{E}\left(1 - e^{-\frac{t}{RC}}\right) \quad \text{(charging a capacitor).}$$



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$U_R + U_c = \mathcal{E}$

$Ri_c + U_c = \mathcal{E}$

$q = CV \qquad i = \frac{dq}{dt}$

$RC\, dU_c/dt + U_c = \mathcal{E}$

Solving firstly $\quad RC\, dU_c/dt + U_c = 0$

$\Downarrow$

$U_{c1} = K\, e^{-\frac{t}{RC}}$

Solving secondly $\quad U_{c2} = Q$

$\Downarrow$

$U_{c2} = \mathcal{E}$

$U_c = U_{c1} + U_{c2} = e^{-\frac{t}{RC}} K + \mathcal{E}$

initial condition $\quad t=0,\ U_c = 0$

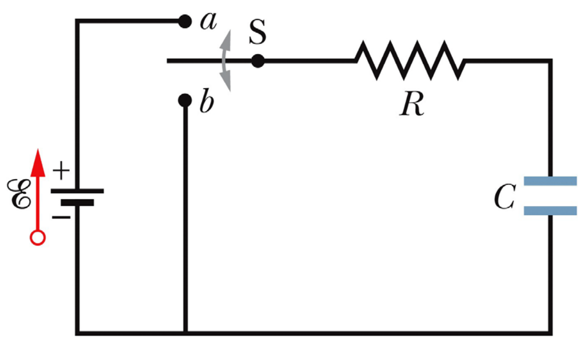
$\Downarrow$

$K = -\mathcal{E}$

$\Downarrow$

$U_c = \mathcal{E}\,(1 - e^{-\frac{t}{RC}})$

27-4 RC Circuits (6 of 8)



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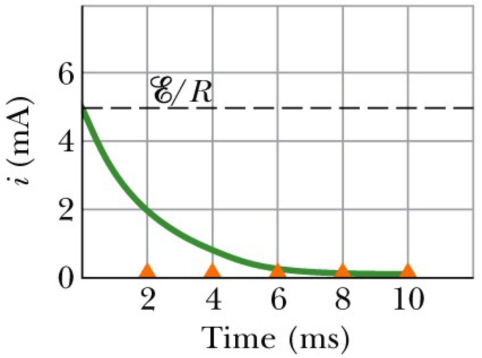

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Discharging a capacitor: Assume now that the capacitor of the figure is fully charged to a potential V_0 equal to the emf $\mathscr{E}$ of the battery. At a new time $t = 0$, switch S is thrown from a to b so that the capacitor can discharge through resistance R.

When a capacitor discharges through a resistance R, the charge on the capacitor decays according to

$$q = q_0 e^{-\frac{t}{RC}} \quad \text{(discharging a capacitor)},$$

where $q_0 (= CV_0)$ is the initial charge on the capacitor.

27-4 RC Circuits (8 of 8)

During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-\frac{t}{RC}} \quad \text{(discharging a capacitor)}.$$

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

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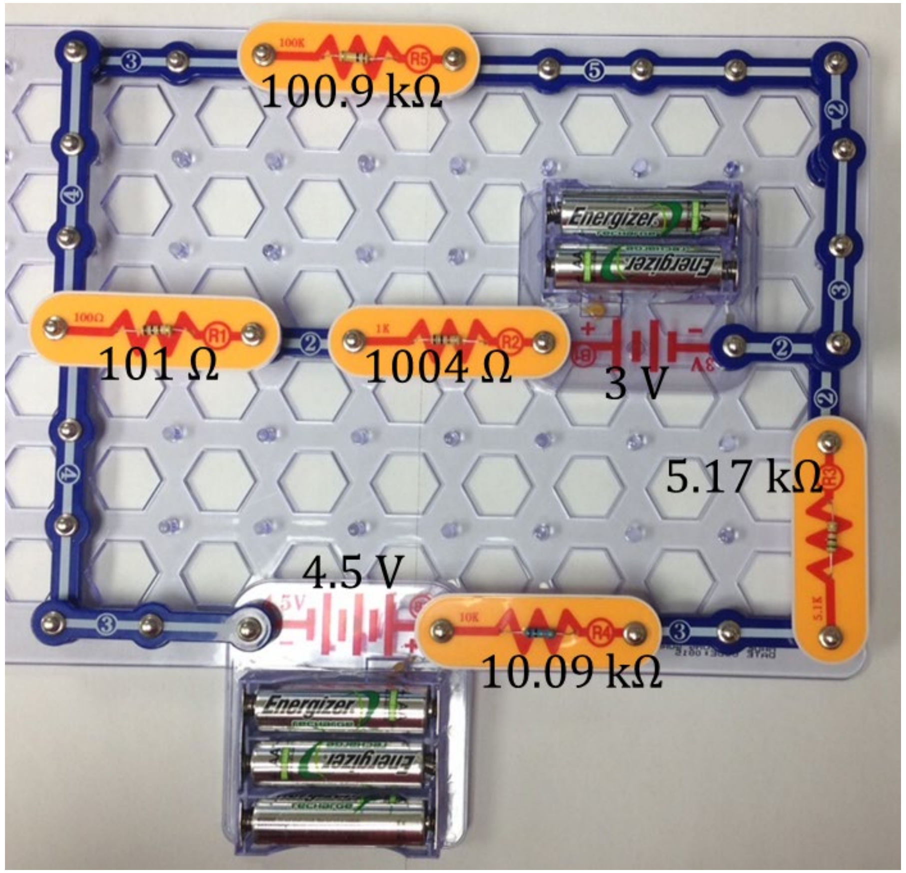
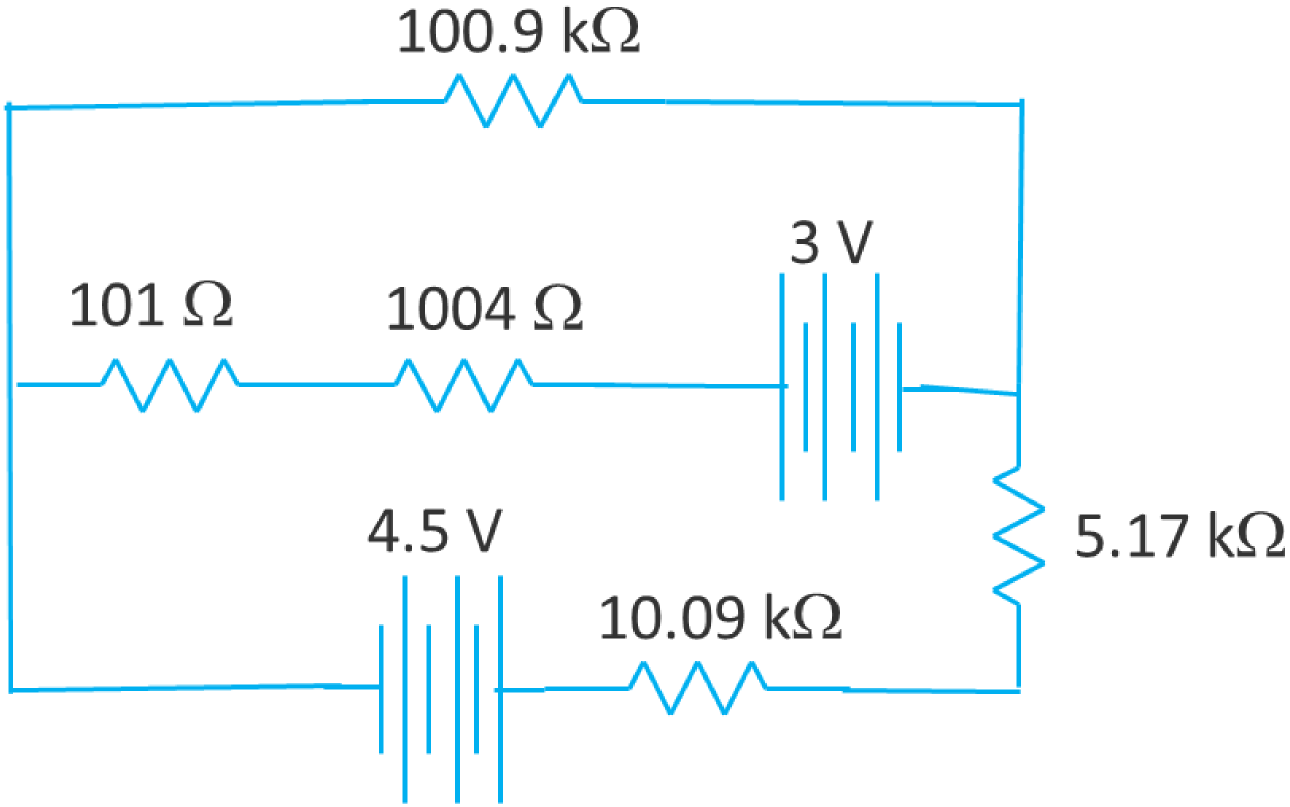
Kirchhoff’s Rules/基尔霍夫定理（Extra knowledge）





How to get the current of each branch?

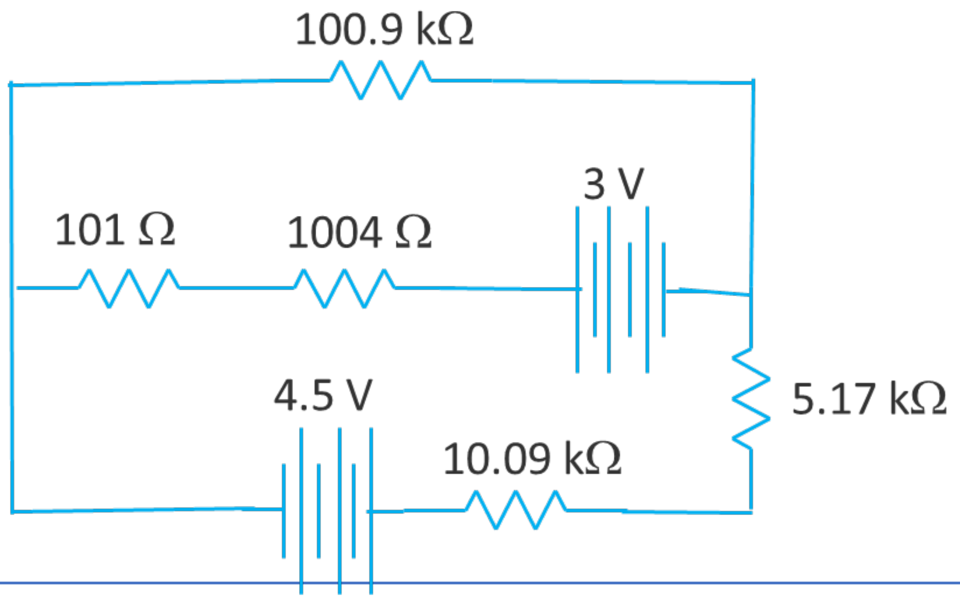
Kirchhoff’s Rules/基尔霍夫定理（Extra knowledge)

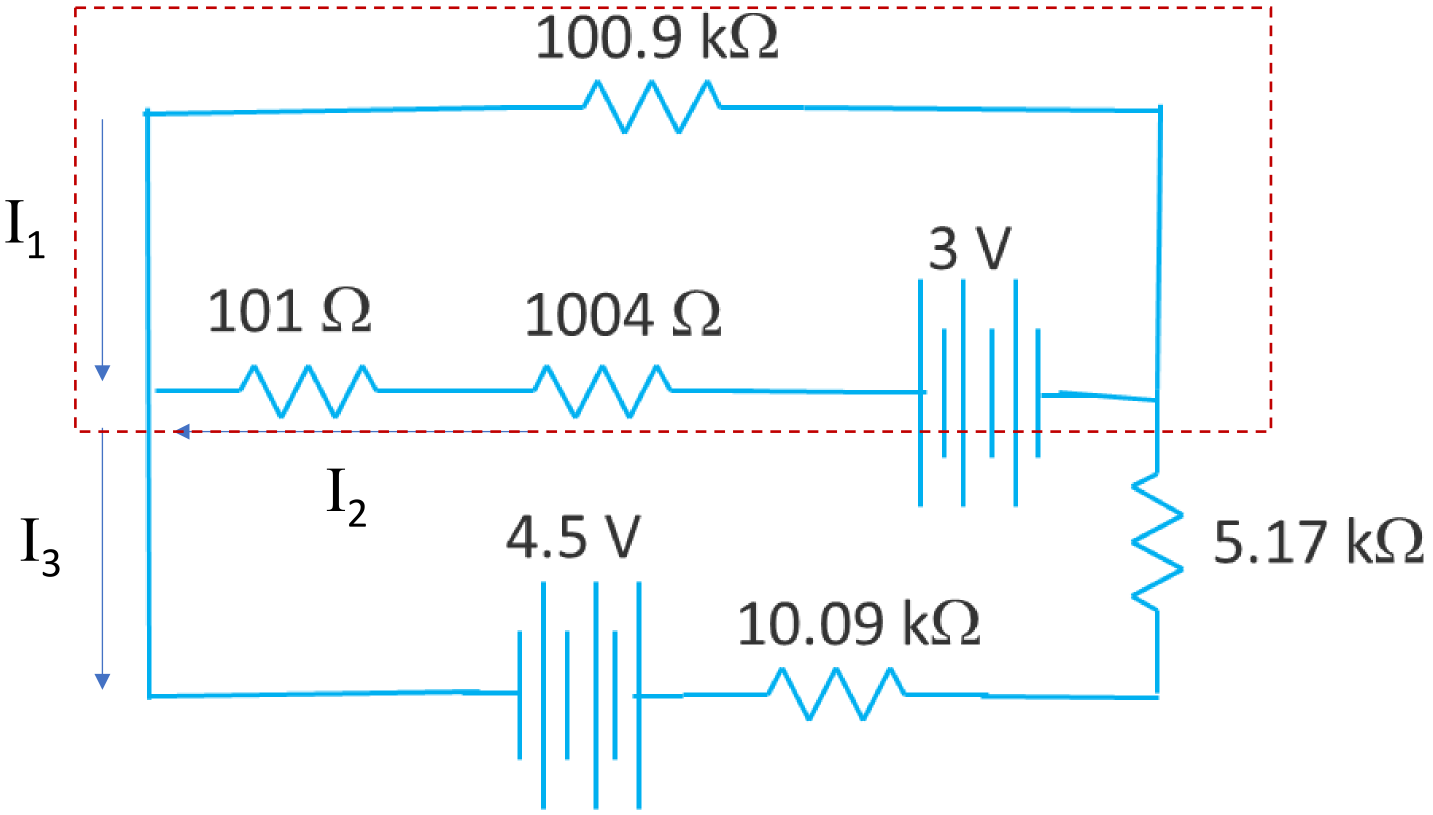
- Junction Rule

  Total current into a junction must equal the total current out of a junction
- Loop Rule

  For a closed-circuit loop,

  the total of all the potential rises – total of all potential drops = 0





$$(Junction\ Rule)\ System \qquad I_3 = I_1 + I_2$$

$$Top\ Loop \qquad 3\ V = (I_2)(1004\ \Omega) + (I_2)(101\ \Omega) - (I_1)(100900\ \Omega)$$

$$Bottom\ Loop \qquad 4.5\ V + 3\ V = (I_3)(10090\ \Omega) + (I_3)(5170\ \Omega) + (I_2)(1004\ \Omega) + (I_2)(101\ \Omega)$$

$$I_1 = -2.45 \times 10^{-5}\ A, I_2 = 4.81 \times 10^{-4}\ A, I_3 = 4.57 \times 10^{-4}\ A$$

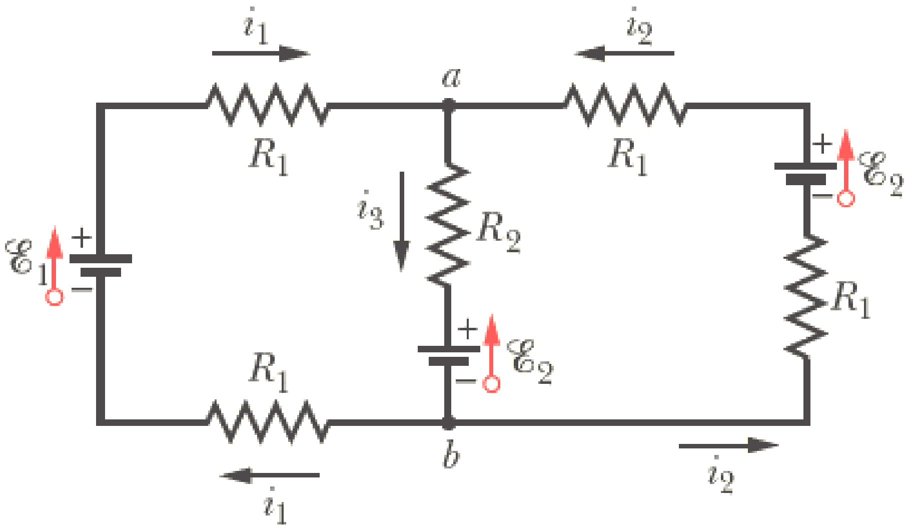
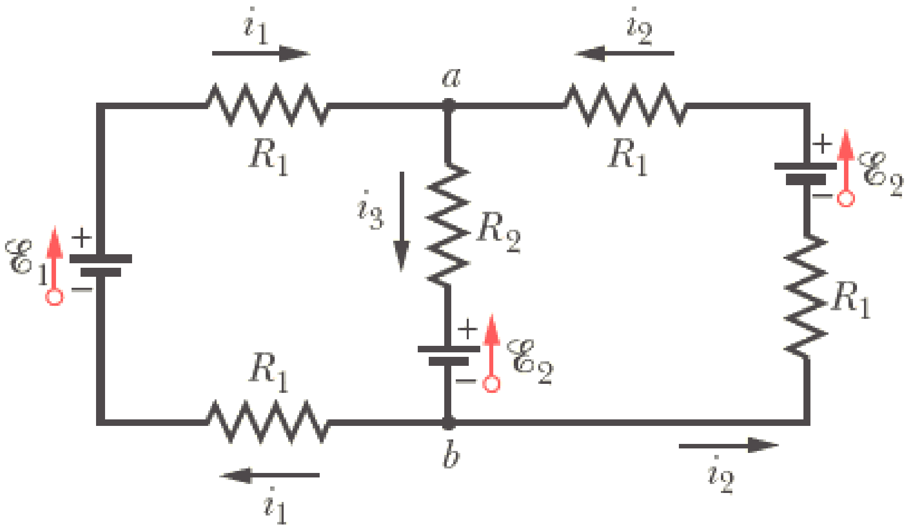


Figure shows a circuit whose elements have the following values: $\mathcal{E}_1 = 3.0$ V, $\mathcal{E}_2 = 6.0$ V, $R_1 = 2.0\ \Omega$, $R_2 = 4.0\ \Omega$. The three batteries are ideal batteries. Find the i_3.



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junction rule at *a* $\quad i_3 = i_1 + i_2$

Left-hand loop $\quad -i_1R_1 + \mathcal{E}_1 - i_1R_1 - (i_1 + i_2)R_2 - \mathcal{E}_2 = 0,$

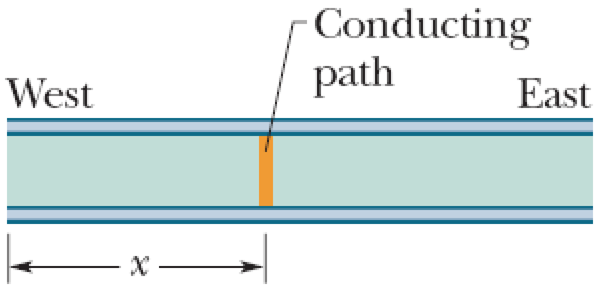
Right-hand loop $\quad -i_2R_1 + \mathcal{E}_2 - i_2R_1 - (i_1 + i_2)R_2 - \mathcal{E}_2 = 0.$

$$i_1 = -0.50 \text{ A}.$$

$$i_2 = 0.25 \text{ A}.$$

$$i_3 = i_1 + i_2 = -0.50 \text{ A} + 0.25 \text{ A} = -0.25 \text{ A}.$$

A 10-km-long underground cable extends east to west and consists of two parallel wires, each of which has resistance 13 Ω/km. An electrical short develops at distance x from the west end when a conducting path of resistance R connects the wires. The resistance of the wires and the short is then 100 Ω when measured from the east end and 200 Ω when measured from the west end. What are (a) x and (b) R?



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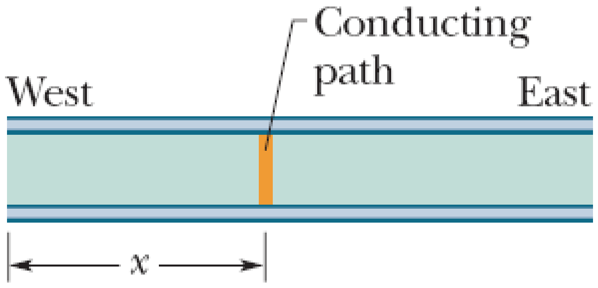


13. (a) We denote L = 10 km and α = 13 Ω/km. Measured from the east end we have

$$R_1 = 100\ \Omega = 2\alpha(L - x) + R,$$

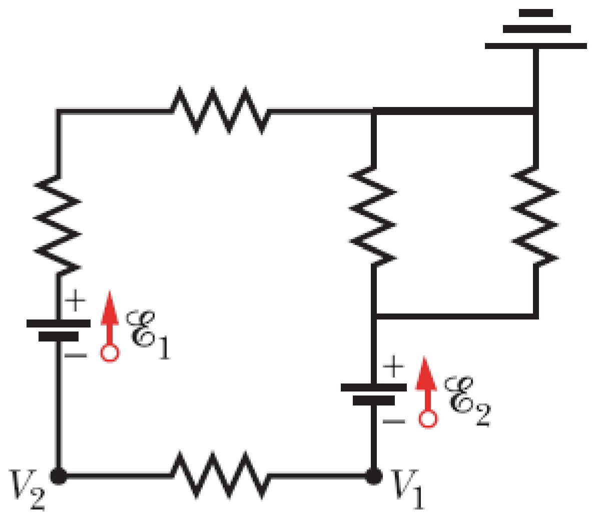
and measured from the west end $R_2 = 200\ \Omega = 2\alpha x + R$. Thus,

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\,\Omega - 100\,\Omega}{4(13\,\Omega/\text{km})} + \frac{10\,\text{km}}{2} = 6.9\,\text{km}.$$

(b) Also, we obtain

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\,\Omega + 200\,\Omega}{2} - (13\,\Omega/\text{km})(10\,\text{km}) = 20\,\Omega.$$

the ideal batteries have emfs $\mathcal{E}_1 = 5.0$ V and $\mathcal{E}_2 = 12$ V, the resistances are each 2.0 Ω, and the potential is defined to be zero at the grounded point of the circuit. What are potentials (a) V_1 and (b) V_2 at the indicated points?



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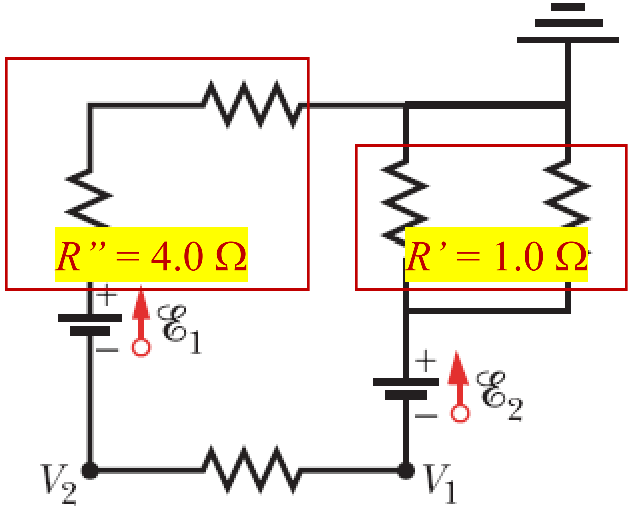

$$R_{eq} = R + R' + R'' = 7.0\ \Omega$$

$$i = \frac{\varepsilon_2 - \varepsilon_1}{R_{eq}} = \frac{7.0\text{ V}}{7.0\ \Omega} = 1.0\text{ A}$$

The voltage across R' is (1.0 A)(1.0 Ω) = 1.0 V

So, the voltage difference between *ground* and V_1 is 12 V – 1.0 V = 11 V. Noting the orientation of the battery, we conclude that

$$V_1 = -11\text{ V}$$

The voltage across R'' is (1.0 A)(4.0 Ω) = 4.0 V,

Voltage difference between *ground* and V_2 is 5.0 V + 4.0 V = 9.0 V. Noting the orientation of the battery, we conclude V_2 = –9.0 V.

verification

$$V_2 - V_1 = -9.0\text{ V} - (-11.0\text{ V}) = 2.0\text{ V},$$

$$iR = (1.0\text{ A})(2.0\ \Omega) = 2.0\text{ V}.$$

••63 SSM WWW In the circuit of Fig. 27-65, $\mathscr{E} = 1.2$ kV, $C = 6.5\ \mu$F, $R_1 = R_2 = R_3 = 0.73$ MΩ. With C completely uncharged, switch S is suddenly closed (at $t = 0$). At $t = 0$, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2, and (c) current i_3 in resistor 3? At $t = \infty$ (that is, after many time constants), what are (d) i_1, (e) i_2, and (f) i_3? What is the potential difference V_2 across resistor 2 at (g) $t = 0$ and (h) $t = \infty$? (i) Sketch V_2 versus t between these two extreme times.

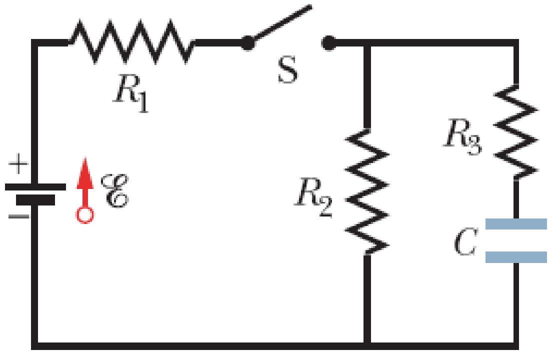

Figure 27-65 Problem 63.

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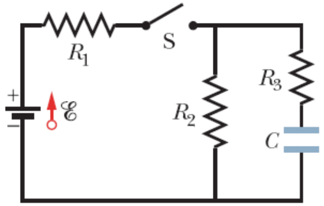


Figure 27-65 Problem 63.

The junction rule produces $i_1 = i_2 + i_3$,

the loop rule applied to the left-hand loop produces

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0.$$

replacing R_1, R_2, and R_3 with R.

ANALYZE (a) Solving the three simultaneous equations, we find

$$i_1 = \frac{2\varepsilon}{3R} = \frac{2(1.2\times10^3\ \text{V})}{3(0.73\times10^6\ \Omega)} = 1.1\times10^{-3}\ \text{A},$$

(b) $i_2 = \dfrac{\varepsilon}{3R} = \dfrac{1.2\times10^3\ \text{V}}{3(0.73\times10^6\ \Omega)} = 5.5\times10^{-4}\ \text{A},$

(c) and $i_3 = i_2 = 5.5\times10^{-4}\ \text{A}.$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields $\varepsilon - i_1 R_1 - i_1 R_2 = 0$.

(d) The solution is $i_1 = \dfrac{\varepsilon}{2R} = \dfrac{1.2\times10^3\ \text{V}}{2(0.73\times10^6\ \Omega)} = 8.2\times10^{-4}\ \text{A}$

(e) and $i_2 = i_1 = 8.2\times10^{-4}\ \text{A}.$

(f) As stated before, the current in the capacitor branch is $i_3 = 0$.

We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\varepsilon - i_1 R - i_2 R = 0$$

$$-\frac{q}{C} - i_3 R + i_2 R = 0.$$

We use the first equation to substitute for i_1 in the second and obtain

$$\varepsilon - 2i_2 R - i_3 R = 0.$$

Thus $i_2 = (\varepsilon - i_3 R)/2R$. We substitute this expression into the third equation above to obtain

$$-(q/C) - (i_3 R) + (\varepsilon/2) - (i_3 R/2) = 0.$$

Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\varepsilon}{2}.$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\varepsilon/2$. The solution is

$$q = \frac{C\varepsilon}{2}\left(1 - e^{-2t/3RC}\right).$$

The current in the capacitor branch is

$$i_3(t) = \frac{dq}{dt} = \frac{\varepsilon}{3R} e^{-2t/3RC}.$$

The current in the center branch is

$$i_2(t) = \frac{\varepsilon}{2R} - \frac{i_3}{2} = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R} e^{-2t/3RC} = \frac{\varepsilon}{6R}\left(3 - e^{-2t/3RC}\right)$$

••63 SSM WWW In the circuit of Fig. 27-65, $\mathscr{E} = 1.2$ kV, $C = 6.5$ μF, $R_1 = R_2 = R_3 = 0.73$ MΩ. With C completely uncharged, switch S is suddenly closed (at $t = 0$). At $t = 0$, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2, and (c) current i_3 in resistor 3? At $t = \infty$ (that is, after many time constants), what are (d) i_1, (e) i_2, and (f) i_3? What is the potential difference V_2 across resistor 2 at (g) $t = 0$ and (h) $t = \infty$? (i) Sketch V_2 versus t between these two extreme times.

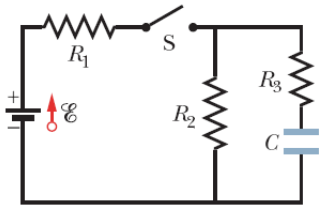


Figure 27-65 Problem 63.

The current in the center branch is

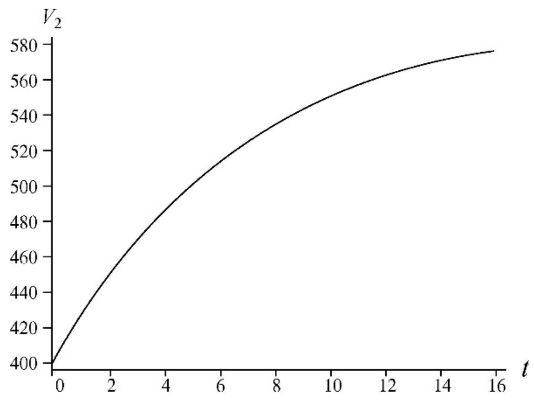
$$i_2(t) = \frac{\varepsilon}{2R} - \frac{i_3}{2} = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R}e^{-2t/3RC} = \frac{\varepsilon}{6R}\left(3 - e^{-2t/3RC}\right)$$

and the potential difference across R_2 is $V_2(t) = i_2 R = \frac{\varepsilon}{6}\left(3 - e^{-2t/3RC}\right)$.

(g) For $t = 0$, $e^{-2t/3RC} = 1$ and $V_2 = \varepsilon/3 = \left(1.2\times10^3\text{ V}\right)/3 = 4.0\times10^2\text{ V}$.

(h) For $t = \infty$, $e^{-2t/3RC} \to 0$ and $V_2 = \varepsilon/2 = \left(1.2\times20^3\text{ V}\right)/2 = 6.0\times10^2\text{ V}$.

(i) A plot of V_2 as a function of time is shown in the following graph.



LEARN A capacitor that is being charged initially behaves like an ordinary connecting wire relative to the charging current. However, a long time later after it's fully charged, it acts like a broken wire.

Honors General Physics II Module 1 exam

添加时间 受影响时段 修改预定 取消预定

教学区, 实验楼W1, 1楼, 101, Honor

预定时段：2024-10-19 星期六 09:00-12:00

已预定

编辑 取消

M1 exam 9:30-11:30，10月19号， 星期六，

记得带计算器，常数会给，公式不会

21 Summary (1 of 2)

Electric Charge

- The strength of a particle's electrical interaction with objects around it depends on its electric charge, which can be either positive or negative.

Conductors and Insulators

- Conductors are materials in which a significant number of electrons are free to move. The charged particles in nonconductors (insulators) are not free to move.

Conservation of Charge

- The net electric charge of any isolated system is always conserved.

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21 Summary (2 of 2)

Coulomb's Law

- The magnitude of the electrical force between two charged particles is proportional to the product of their charges and inversely proportional to the square of their separation distance.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

Equation 21-4

The Elementary Charge

- Electric charge is quantized (restricted to certain values).
- e is the elementary charge

$$e = 1.602 \times 10^{-19}\,\text{C}.$$

Equation 20-21

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22 Summary (1 of 5)

Definition of Electric Field

- The electric field at any point

$$\vec{E} = \frac{\vec{F}}{q_0}. \qquad \textbf{Equation (22-1)}$$

Electric Field Lines

- Provide a means for visualizing the directions and the magnitudes of electric fields

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22 Summary (2 of 5)

Field due to a Point Charge

- The magnitude of the electric field $\vec{E}$ set up by a point charge q at a distance r from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$

Equation (22-3)

22 Summary (3 of 5)

Field due to an Electric Dipole

- The magnitude of the electric field set up by the dipole at a distant point on the dipole axis is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

Equation (22-9)

Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Equation (22-26)

22 Summary (4 of 5)

Force on a Point Charge in an Electric Field

- When a point charge q is placed in an external electric field $\vec{E}$

$$\vec{F} = q\vec{E}. \qquad \textbf{Equation (22-28)}$$

Dipole in an Electric Field

- The electric field exerts a torque on a dipole

$$\vec{\tau} = \vec{p} \times \vec{E}. \qquad \textbf{Equation (22-34)}$$

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22 Summary (5 of 5)

- The dipole has a potential energy U associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}. \qquad \textbf{Equation (22-38)}$$

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23 Summary (1 of 4)

Gauss' Law

- Gauss' law is

$$\varepsilon_0 \Phi = q_{\text{enc}}$$

Equation (23-6)

- the net flux of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

Equation (23-6)

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23 Summary (2 of 4)

Applications of Gauss' Law

- surface of a charged conductor

$$E = \frac{\sigma}{\varepsilon_0}$$

Equation (23-11)

- Within the surface $E = 0$.
- line of charge

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Equation (23-12)

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23 Summary (3 of 4)

- Infinite non-conducting sheet

$$E = \frac{\sigma}{2\varepsilon_0}$$

Equation (23-13)

- Outside a spherical shell of charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Equation (23-15)

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23 Summary (4 of 4)

- Inside a uniform spherical shell

$$E = 0$$

Equation (23-16)

- Inside a uniform sphere of charge

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3} \right) r.$$

Equation (23-20)

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24 Summary (1 of 5)

Electric Potential

- The electric potential V at point P in the electric field of a charged object:

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}, \qquad \textbf{Equation (24-2)}$$

Electric Potential Energy

- Electric potential energy U of the particle-object system:

$$U = qV. \qquad \textbf{Equation (24-3)}$$

- If the particle moves through potential ΔV :

$$\Delta U = q\Delta V = q\left(V_f - V_i\right). \qquad \textbf{Equation (24-4)}$$

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24 Summary (2 of 5)

Mechanical Energy

- Applying the conservation of mechanical energy gives the change in kinetic energy:

$$\Delta K = -q\Delta V.$$

Equation (24-9)

- In case of an applied force in a particle

$$\Delta K = -q\Delta V + W_{\text{app}}.$$

Equation (24-11)

- In a special case when $\Delta K = 0$:

$$W_{\text{app}} = q\Delta V \quad \left(\text{for } K_i = K_f\right).$$

Equation (24-12)

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24 Summary (3 of 5)

Finding *V* from $\vec{E}$

- The electric potential difference between two point *I* and *f* is:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$ **Equation (24-18)**

Potential due to a Charged Particle

- due to a single charged particle at a distance r from that particle :

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$ **Equation (24-26)**

- due to a collection of charged particles

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}.$$ **Equation (24-27)**

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24 Summary (4 of 5)

Potential due to an Electric Dipole

- The electric potential of the dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

Equation (24-30)

Potential due to a Continuous Charge Distribution

- For a continuous distribution of charge:

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

Equation (24-32)

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24 Summary (5 of 5)

Calculating $\vec{E}$ from V

- The component of $\vec{E}$ in any direction is:

$$E_s = -\frac{\partial V}{\partial s}.$$

Equation (24-40)

Electric Potential Energy of a System of Charged Particle

- For two particles at separation r:

$$U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}.$$

Equation (24-46)

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25 Summary (1 of 4)

Capacitor and Capacitance

- The capacitance of a capacitor is defined as:

$$q = CV \qquad \textbf{Equation (25-1)}$$

Determining Capacitance

- Parallel-plate capacitor:

$$C = \frac{\varepsilon_0 A}{d}. \qquad \textbf{Equation (25-9)}$$

- Cylindrical Capacitor:

$$C = 2\pi\varepsilon_0 \frac{L}{\ln\left(\frac{b}{a}\right)}. \qquad \textbf{Equation (25-14)}$$

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25 Summary (2 of 4)

- Spherical Capacitor:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}. \qquad \textbf{Equation (25-17)}$$

- Isolated sphere:

$$C = 4\pi\varepsilon_0 R. \qquad \textbf{Equation (25-18)}$$

Capacitor in parallel and series

- In parallel:

$$C_{\text{eq}} = \sum_{j=1}^{n} C_j \qquad \textbf{Equation (25-19)}$$

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25 Summary (3 of 4)

- In series

$$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{C_j}$$ **Equation (25-20)**

Potential Energy and Energy Density

- Electric Potential Energy (U):

$$U = \frac{q^2}{2C} = \tfrac{1}{2} C V^2$$ **Equation (25-21&22)**

- Energy density (u)

$$u = \tfrac{1}{2} \varepsilon_0 E^2.$$ **Equation (25-25)**

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25 Summary (4 of 4)

Capacitance with a Dielectric

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ, called the dielectric constant, which is characteristic of the material.

Gauss' Law with a Dielectric

- When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad \text{(Gauss' law with dielectric)}$$

Equation (25-36)

26 Summary (1 of 6)

Current

- The electric current i in a conductor is defined by

$$i = \frac{dq}{dt}.$$

Equation 26-1

Current Density

- Current is related to current density by

$$i = \int \vec{J} \cdot d\vec{A},$$

Equation 26-4

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26 Summary (2 of 6)

Drift Speed of the Charge Carriers

- Drift speed of the charge carriers in an applied electric field is related to current density by

$$\vec{J} = (ne)\vec{v}_d, \qquad \textbf{Equation 26-7}$$

Resistance of a Conductor

- Resistance R of a conductor is defined by

$$R = \frac{V}{i} \qquad \textbf{Equation 26-8}$$

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26 Summary (3 of 6)

- Similarly the resistivity and conductivity of a material is defined by

$$\rho = \frac{1}{\sigma} = \frac{E}{J}$$

Equation 26-10&12

- Resistance of a conducting wire of length L and uniform cross section is

$$R = \rho \frac{L}{A}$$

Equation 26-16

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26 Summary (4 of 6)

Change of ρ with Temperature

- The resistivity of most material changes with temperature and is given as

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$

Equation 26-17

Ohm's Law

- A given device (conductor, resistor, or any other electrical device) obeys Ohm's law if its resistance R (defined by **Eq. 26-8** as $\frac{V}{i}$) is independent of the applied potential difference V.

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26 Summary (5 of 6)

Resistivity of a Metal

- By assuming that the conduction electrons in a metal are free to move like the molecules of a gas, it is possible to derive an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n\tau}. \qquad \textbf{Equation 26-22}$$

26 Summary (6 of 6)

Power

- The power P, or rate of energy transfer, in an electrical device across which a potential difference V is maintained is

$$P = iV$$

Equation 26-26

- If the device is a resistor, we can write

$$P = i^2 R = \frac{V^2}{R}$$

Equation 26-27&28

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27 Summary (1 of 4)

Emf

- The **emf** (work per unit charge) of the device is

$$\mathscr{E} = \frac{dw}{dq} \quad (\text{definition of } \mathscr{E}). \qquad \textbf{Equation (27-1)}$$

Single-Loop Circuits

- Current in a single-loop circuit:

$$i = \frac{\mathscr{E}}{R + r}, \qquad \textbf{Equation (27-4)}$$

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27 Summary (2 of 4)

Series Resistance

- When resistances are in series

$$R_{\text{eq}} = \sum_{j=1}^{n} R_j$$

Equation (27-7)

Power

- The rate P of energy transfer to the charge carriers is

$$P = iV$$

Equation (27-14)

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27 Summary (3 of 4)

- The rate P_r at which energy is dissipated as thermal energy in the battery is

$$P_r = i^2 r. \qquad \textbf{Equation (27-16)}$$

- The rate P_{emf} at which the chemical energy in the battery changes is

$$P_{\text{emf}} = i\mathscr{E}. \qquad \textbf{Equation (27-17)}$$

Parallel Resistance

- When resistances are in parallel

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j} \qquad \textbf{Equation (27-24)}$$

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27 Summary (4 of 4)

RC Circuits

- The charge on the capacitor increases according to

$$q = C\mathscr{E}\left(1 - e^{-\frac{t}{RC}}\right)$$ **Equation (27-33)**

- During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-\frac{t}{RC}}$$ **Equation (27-34)**

- During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-\frac{t}{RC}}$$ **Equation (27-40)**

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