UFUG 1504: Honors General Physics II

Chapter 36

Diffraction

36 Summary (1 of 7)

Diffraction

• When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference.

Single-Slit Diffraction

A single-slit diffraction patterns satisfy

$$a \sin \theta = m\lambda$$
, for $m = 1, 2, 3, ...$ Equation 36-3

36 Summary (2 of 7)

• The intensity of the diffraction pattern at any given angle θ is

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2,$$

Equation 36-5

where

$$\alpha = \frac{\pi a}{\lambda} \sin \theta$$

36 Summary (3 of 7)

Circular Aperture Diffraction

• Diffraction by a circular aperture or a lens with diameter d produces a central maximum and concentric maxima and minima, with the first minimum at an angle θ given by

$$\sin\theta = 1.22 \frac{\lambda}{d}$$

36 Summary (4 of 7)

Rayleigh's Criterion

• Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\theta_{\rm R} = 1.22 \frac{\lambda}{d}$$

36 Summary (5 of 7)

Double-Slit Diffraction

• Waves passing through two slits, each of width a, whose centers are a distance *d* apart, display diffraction patterns whose intensity *I* at angle whose intensity *I* at angle

$$I(\theta) = I_m \left(\cos^2 \beta\right) \left(\frac{\sin \alpha}{\alpha}\right)^2$$
 Equation 36-19

36 Summary (6 of 7)

Diffraction Gratings

• Diffraction by N (multiple) slits results in maxima (lines) at angles θ such that

$$d \sin \theta = m\lambda$$
, for $m = 0, 1, 2, \dots$ Equation 36-25

with the half-widths of the lines given by

and
$$\Delta\theta_{\rm hw} = \frac{\lambda}{Nd\cos\theta}$$
 Equation 36-28
$$D = \frac{\Delta\theta}{\Delta\lambda} - \frac{m}{d\cos\theta}$$
 Equation 36-29&30
$$R = \frac{\lambda_{\rm avg}}{\Delta\lambda} = Nm.$$
 Equation 36-31&32

36 Summary (7 of 7)

X-Ray Diffraction

• Diffraction maxima (due to constructive interference) occur if the incident direction of the wave, measured from the surfaces of these planes, and the wavelength 1 of the radiation satisfy Bragg's law:

$$2d\sin\theta = m\lambda$$
, for $m = 1, 2, 3, ...$ Equation 36-12

36-1 Single-Slit Diffraction (3 of 4)

When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference. This type of interference is called diffraction.

Waves passing through a long narrow slit of width a produce, on a viewing screen, a single-slit diffraction pattern that includes a central maximum (bright fringe) and other maxima. They are separated by minima that are located relative to the central axis by angles θ :

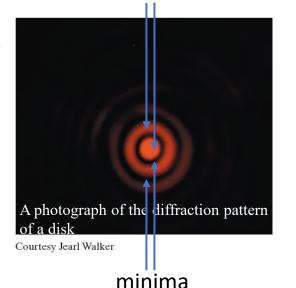
$$a \sin \theta = m\lambda$$
, for $m = 1, 2, 3...$ (minima—dark fringes).

The maxima are located approximately halfway between minima.



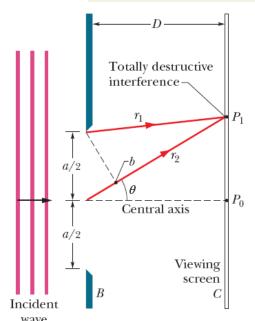
Light passed through a narrow vertical slit

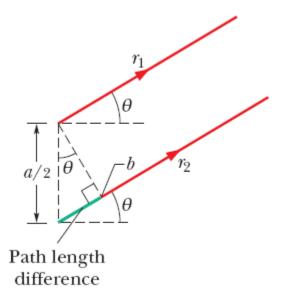
maxima



36-1 Single-Slit Diffraction (4)

This pair of rays cancel each other at P_1 . So do all such pairings.





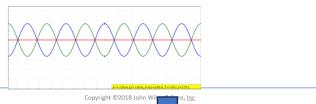
35-1 Light as a Wave (9 of 9)

Path Length Difference

the path length difference ΔL compares to the wavelength λ of the waves

maximum brightness
$$\frac{\Delta L}{\lambda} = 0, 1, 2, \dots$$
 (fully constructive interference).

darkness
$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \dots$$
 (fully destructive interference).



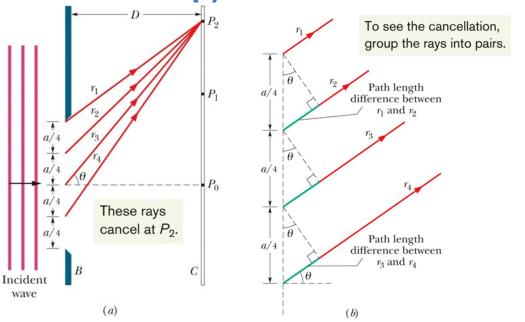
$$\left(\frac{a}{2}\right) \sin\theta = \frac{\lambda}{2}$$
 (fully destructive).

$$a \sin \theta = \lambda$$
 (first minimum).

- (a) Waves from the top points of four zones of width $\frac{a}{2}$ undergo fully destructive interference at point P_1 .
- destructive interference at point P_1 . (b) For D >> a, we can approximate rays r_1 , r_2 being parallel, at angle θ to the central axis.

na)

36-1 Single-Slit Diffraction (4 of 4)



$$a \sin \theta = m\lambda$$
, for $m = 1, 2, 3...$ (minima)

- (a) Waves from the top points of four zones of width $\frac{a}{4}$ undergo fully destructive interference at point P_2 .
- (b) For D >> a, we can approximate rays r_1 , r_2 , r_3 , and r_4 as being parallel, at angle θ to the central axis.

36-2 Intensity in Single-Slit Diffraction (3 of 15)

The intensity of the diffraction pattern at any given angle θ is

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2,$$

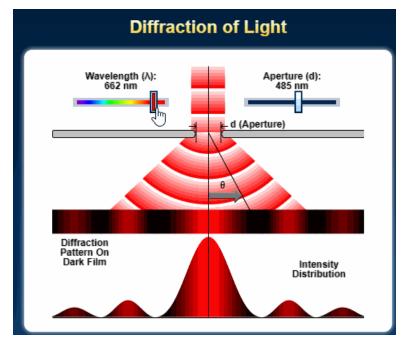
where, I_m is the intensity at the center of the pattern and $\frac{a}{\lambda}$.

$$\alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda}\sin\theta.$$

 ϕ is the phase difference (in radians)

a is the slit of width.

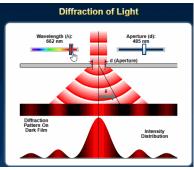
 α is just a convenient connection between the angle θ

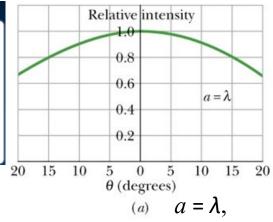


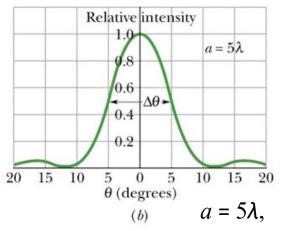
36-2 Intensity in Single-Slit Diffraction (4 of 15)

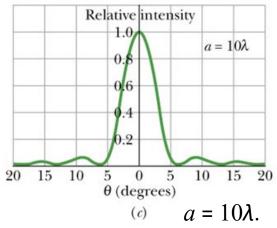
$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2, \qquad \alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda}\sin \theta.$$

• The plots show the relative intensity in single-slit diffraction for three values of the ratio $\frac{a}{\lambda}$. The wider the slit is, the narrower is the central diffraction maximum.



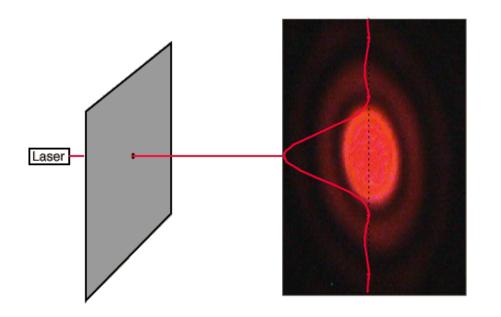


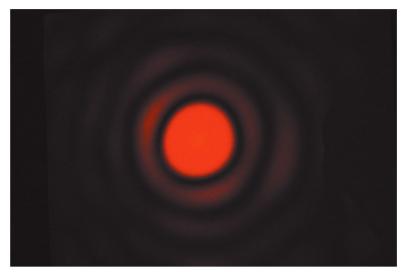




- the slit width (a) increases, the width of the *central diffraction maximum* decreases; the secondary maxima also decrease in width
- a being much greater than wavelength λ , the secondary maxima due to the slit disappear; we then no longer have single-slit diffraction

36-3 Diffraction by a Circular Aperture (9 of 15)





Courtesy Jearl Walker

The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

36-3 Diffraction by a Circular Aperture (8 of 15)

Diffraction by a circular aperture or a lens with diameter d produces a central maximum and concentric maxima and minima, given by

$$\sin \theta = 1.22 \frac{\lambda}{d}$$
 (first minimum — cricular aperture).

The angle θ here is the angle from the central axis to any point on that (circular) minimum.

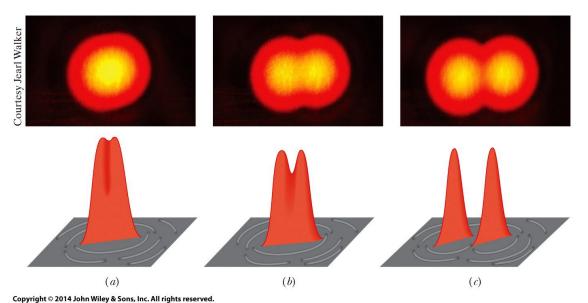
$$\sin \theta = \frac{\lambda}{a}$$
 (frist minimum – single slit),

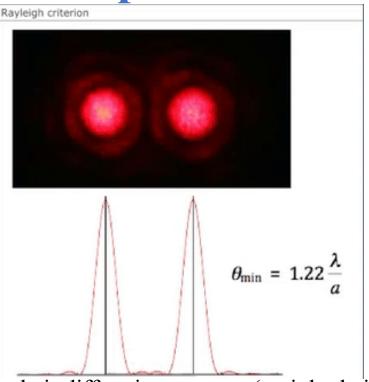
which locates the first minimum for a long narrow slit of width a.

The main difference is the factor 1.22, which enters because of the circular shape of the aperture.

36-3 Diffraction by a Circular Aperture (10 of 15)

Resolvability





- In a, the objects are not resolved because of diffraction; that is, their diffraction patterns (mainly their central maxima) overlap so much that the two objects cannot be distinguished from a single point object.
- In b the objects are barely resolved, and in c they are fully resolved.
- Rayleigh's criterion (瑞利判据) is satisfied in (b), with the central maximum of one diffraction pattern coinciding with the first minimum of the other.

36-3 Diffraction by a Circular Aperture (12 of 15)

Rayleigh's criterion (瑞利判据) suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

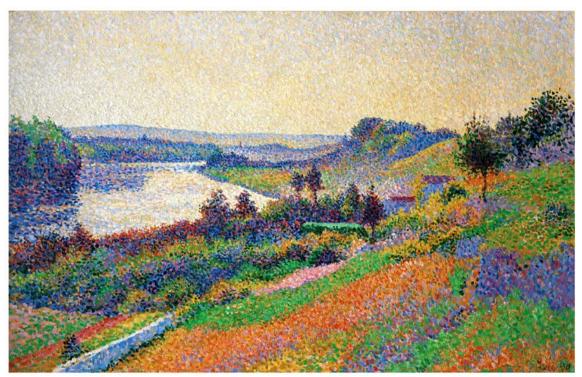
$$\theta_{\rm R} = 1.22 \frac{\lambda}{d}$$
 (Rayleigh's critreion)

in which d is the diameter of the aperture through which the light passes.

Human Vision. If the angular separation θ between the sources is greater than θ_R , we can visually resolve the sources; if it is less, we cannot.

36-3 Diffraction by a Circular Aperture (13 of 15)

Pointillism

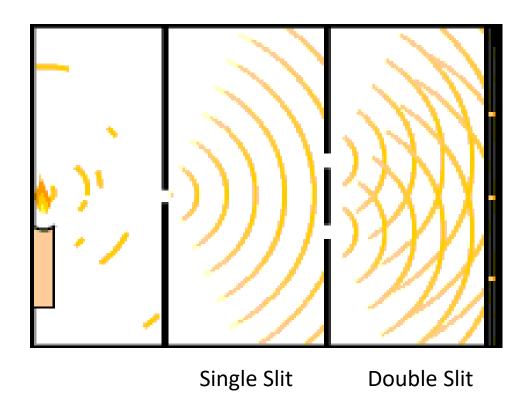


Maximilien Luce, The Seine at Herblay, 1890. Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

The pointillistic painting 点描绘法

The Seine at Herblay by Maximilien Luce consists of thousands of colored dots. With the viewer very close to the canvas, the dots and their true colors are visible. At normal viewing distances, the dots are irresolvable and thus blend

36-4 Diffraction by a Double Slit (3 of 6)

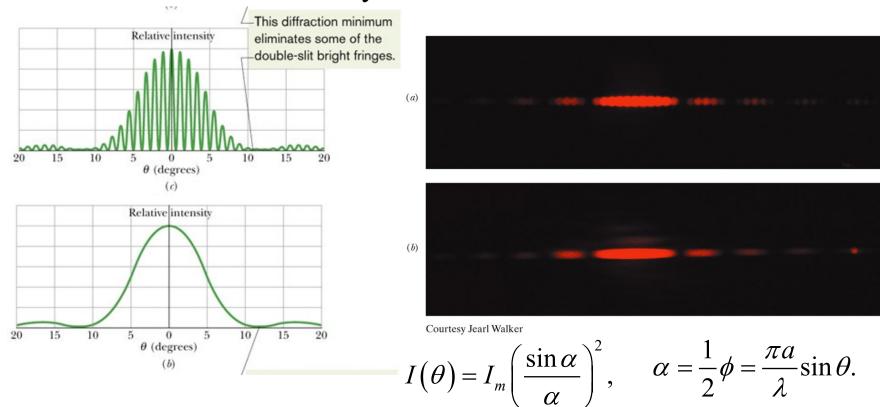


36-4 Diffraction by a Double Slit (3 of 6)

Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.

The intensity plot to be expected for two slits of width *a*

The intensity plot for diffraction by a slit of width a



36-4 Diffraction by a Double Slit (5 of 6)

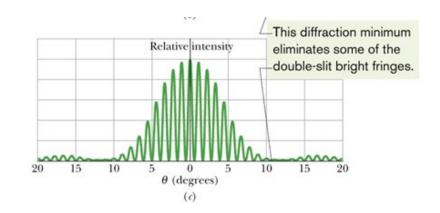
For identical slits with width a and center-to-center separation d, the intensity in the pattern varies with the angle θ from the central axis as

$$I(\theta) = I_m \left(\cos^2 \beta\right) \left(\frac{\sin \alpha}{\alpha}\right)^2$$
 (double slit),

in which
$$\beta = \frac{\pi d}{\lambda} \sin \theta$$
 and $\alpha = \frac{\pi a}{\lambda} \sin \theta$.

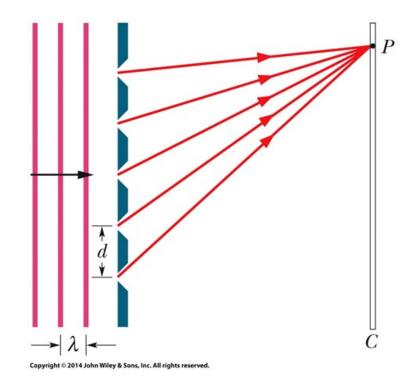
Note carefully that the right side of double slit equation is the product of I_m and two factors.

- (1) The interference factor $\cos^2 \beta$ is due to the interference between two slits with slit separation d.
- (2) The diffraction factor $\left[\frac{(\sin a)}{a}\right]^2$ is due to diffraction by a single slit of width a.

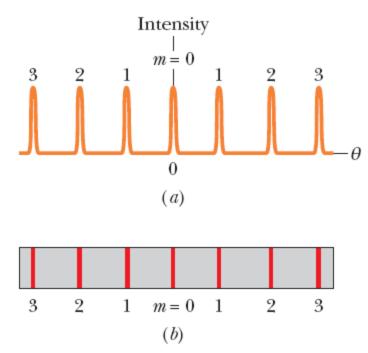




36-5 Diffraction Gratings 衍射光栅



An idealized diffraction grating, consisting of only five rulings, that produces an interference pattern on a distant viewing screen C.

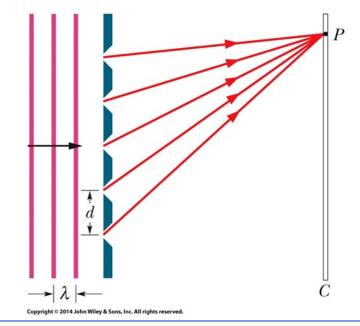


(a) The intensity plot produced by a diffraction grating with a great many rulings consists of narrow peaks, here labeled with their order numbers m. (b) The corresponding bright fringes seen on the screen are called lines and are here also labeled with order numbers m.

36-5 Diffraction Gratings 衍射光栅

A diffraction grating is a series of "slits" used to separate an incident wave into its component wavelengths by separating and displaying their diffraction maxima. Diffraction by N (multiple) slits results in maxima (lines) at angles θ such that

$$d \sin \theta = m\lambda$$
, for $m = 0,1,2,...$ (maxium — lines),



35-2 Young's Interference (8 of 8)

The phase difference between two waves can change if the waves travel paths of different lengths.

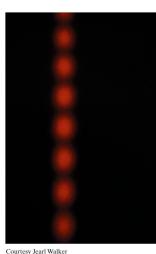
The conditions for maximum and minimum intensity are

For a bright fringe, we saw that ΔL must be either zero or an integer number of wavelengths.

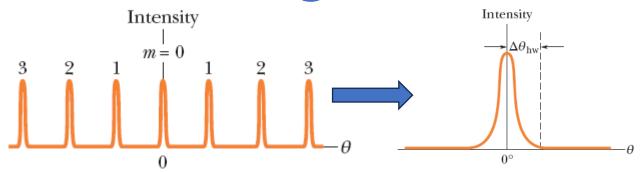
$$\Delta L = d \sin \theta = m\lambda$$
, for $m = 0, 1, 2, ...$

For a dark fringe, ΔL must be an odd multiple of half a wavelength.

$$\Delta L = d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$$
, for $m = 0, 1, 2, ...$



36-5 Diffraction Gratings 衍射光栅



A line's **half-width** (半高宽) is the angle from its center to the point where it disappears into the darkness and is given by

$$\Delta \theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta}$$
 (half-width of line at θ).

Note that for light of a given wavelength λ and a given d, the widths of the lines decrease with an increase in the number N of rulings. Thus, of two diffraction gratings, the grating with the larger value of N is better able to distinguish between wavelengths because its diffraction lines are narrower and so produce less overlap.

36-6 Gratings: Dispersion and Resolving Power (4 of 7)



Kristen Brochmann/Fundamental Photographs

The fine rulings, each $0.5\mu m$ wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms colored "lanes" that are the composite of the diffraction patterns from the rulings.

36-6 Gratings: Dispersion and Resolving Power (3 of 7)

The dispersion \overline{m} D of a diffraction grating is a measure of the angular separation $\Delta\theta$ of the lines it produces for two wavelengths differing by $\Delta\lambda$.

For order number m, at angle θ , the dispersion is given by

$$D = \frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$$
 (dispersion).

Thus, to achieve higher dispersion we must use a grating of smaller grating spacing d and work in a higher-order m. Note that the dispersion does not depend on the number of rulings N in the grating. The SI unit for D is the degree per meter or the radian per meter.

Proof of Eq. $d \sin \theta = m\lambda, \qquad \text{(maxium-lines)},$ $d(\cos \theta) d\theta = m d\lambda.$

36-6 Gratings: Dispersion and Resolving Power (5 of 7)

The resolving power R of a diffraction grating is a measure of its ability to make the emission lines of two close wavelengths distinguishable. For two wavelengths differing by $\Delta\lambda$ and with an average value of λ_{avg} , the resolving power is given by

$$R = \frac{\lambda_{\text{avg}}}{\Delta \lambda} = Nm$$

The greater *R* is, the closer two emission lines can be and still be resolved.

36-6 Gratings: Dispersion and Resolving **Power** (6 of 7)

$$D = \frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta} \quad \text{(dispersion)}. \qquad R = \frac{\lambda_{\text{avg}}}{\Delta \lambda} = Nm$$

$$R = \frac{\lambda_{\text{avg}}}{\Delta \lambda} = Nm$$

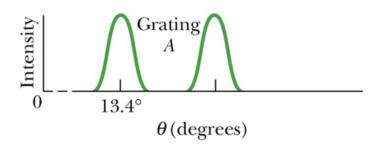
Table 36-1 Three Gratings^a

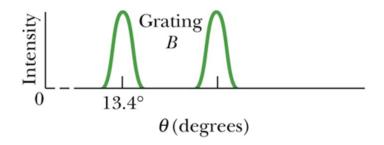
Grating	N	d(nm)	θ	$D(^{o}/\mu\mathrm{m})$	R
A	10 000	2540	13.4°	23.2	10 000
B	20 000	2540	13.4°	23.2	20 000
C	10 000	1360	25.5°	46.3	10 000

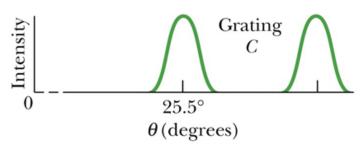
^aData are for $\lambda = 589$ nm and m = 1.

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Grating *B* has the highest resolving power, and grating C the highest dispersion.







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36-7 X-Ray Diffraction (XRD) (3 of 6)

X rays are electromagnetic radiation whose wavelengths are of the order of $1\text{Å}(=10^{-10}m)$.

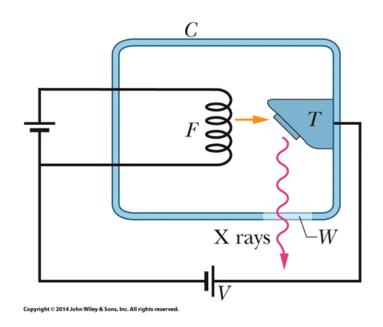
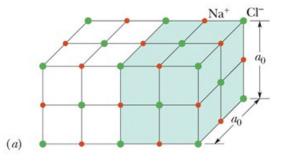
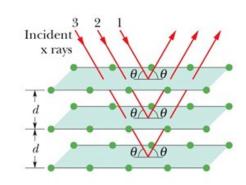
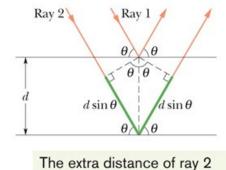


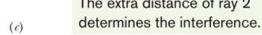
Figure (right) shows that x rays are produced when electrons escaping from a heated filament F are accelerated by a potential difference V and strike a metal target T.

- (a) The cubic structure of NaCl, showing the sodium and chlorine ions and a unit cell (shaded).
- (b) Incident x rays undergo diffraction by the structure of
- (a). The x rays are diffracted as if they were reflected by a family of parallel planes, with angles measured relative to the planes (not relative to a normal as in optics).
- (c) The path length difference between waves effectively reflected by two adjacent planes is $2d \sin \theta$.
- (d) A different orientation of the incident x rays relative to the structure. A different family of parallel planes now effectively reflects the x rays.



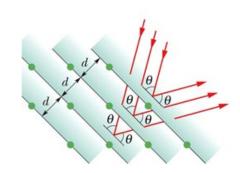






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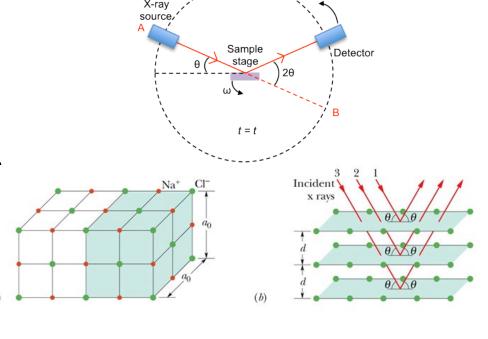
(d)

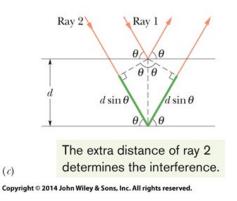


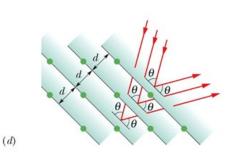
As shown in figure below if x rays are directed toward a crystal structure, they undergo Bragg scattering 布拉格散射. which is easiest to visualize if the crystal atoms are considered to be in parallel planes.

For x rays of wavelength λ scattering from crystal planes with ω separation d, the angles u at which the scattered intensity is maximum are given by

$$2d \sin \theta = m\lambda$$
, for $m = 1, 2, 3, ...$



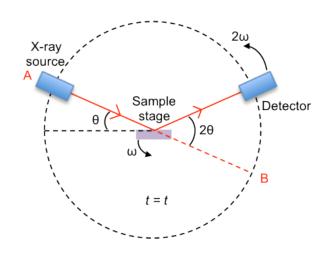




(扩展内容,不考/ Extended content, not in test)

Bragg scattering 布拉格散射,

$$2d \sin \theta = m\lambda$$
, for $m = 1, 2, 3, ...$



Crystal System Table

System	Axial length	Axial Angle	Unit Cell Geometry
Cubic	a = b = c	$\alpha = \beta = \gamma = 90^{\circ}$	
Tetragonal	a = b ≠ c	$\alpha = \beta = \gamma = 90^{\circ}$	
Orthorhombic	a≠b≠c	$\alpha = \beta = \gamma = 90^{\circ}$	
Rhombohedral	a = b = c	$\alpha = \beta = \gamma \neq 90^{\circ}$	
Hexagonal	a = b ≠ c	α = β = 90°, γ=120°	
Monoclinic	a≠b≠c	α = γ = 90°, β≠90°	
Triclinic	a≠b≠c	α≠β≠γ	

(扩展内容,不考/Extended content, not in test)

Peak Position

d-spacings and lattice parameters

$$\lambda = 2d_{hkl}sin\theta_{hkl}$$

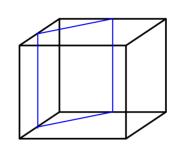
Fix
$$\lambda$$
 (Cu k α) = 1.54Å d_{hkl} = 1.54Å/2sin θ_{hkl}

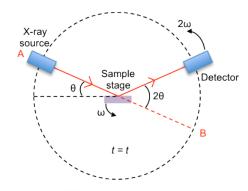
(Most accurate d-spacings are those calculated from high-angle peaks)

For a simple cubic ($a = b = c = a_0$)

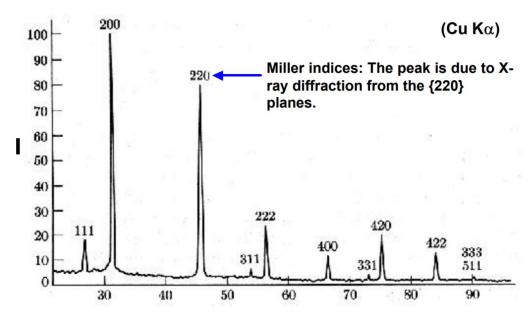
$$d_{hkl} = \frac{a_0}{\sqrt{h^2 + k^2 + l^2}}$$

 \rightarrow a₀ = d_{hkl} /(h²+k²+l²)^{1/2} e.g., for NaCl, 2 θ_{220} =46°, θ_{220} =23°, d₂₂₀=1.9707Å, a₀=5.5739Å





XRD Pattern of NaCl Powder



Diffraction angle 2θ (degrees)

Difference between Diffraction and Interference					
Interference	Diffraction				
Interference may be defined as waves emerging from two different sources, producing different wavefronts.	Diffraction, on the other hand, can be termed as secondary waves that emerge from the different parts of the same wave.				
The contrast between maxima and minima is very good.	The contrast between maxima and minima is poor.				
The width of the fringes in interference is equal.	The width of the fringes is not equal in diffraction.				
The sources are referred to as interference sources if the number of sources is as few as two sources	If the number of sources is more than two the sources are referred to as diffraction sources.				
https://phet.colorado.edu/en/simulations/wave-interference					

36 Summary (1 of 7)

Diffraction

• When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference.

Single-Slit Diffraction

A single-slit diffraction patterns satisfy

$$a \sin \theta = m\lambda$$
, for $m = 1, 2, 3, ...$ Equation 36-3

36 Summary (2 of 7)

• The intensity of the diffraction pattern at any given angle θ is

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2,$$

Equation 36-5

where

$$\alpha = \frac{\pi a}{\lambda} \sin \theta$$

36 Summary (3 of 7)

Circular Aperture Diffraction

• Diffraction by a circular aperture or a lens with diameter d produces a central maximum and concentric maxima and minima, with the first minimum at an angle θ given by

$$\sin\theta = 1.22 \frac{\lambda}{d}$$

Equation 36-12

36 Summary (4 of 7)

Rayleigh's Criterion

• Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\theta_{\rm R} = 1.22 \frac{\lambda}{d}$$

Equation 36-14

36 Summary (5 of 7)

Double-Slit Diffraction

• Waves passing through two slits, each of width a, whose centers are a distance *d* apart, display diffraction patterns whose intensity *I* at angle whose intensity *I* at angle

$$I(\theta) = I_m \left(\cos^2 \beta\right) \left(\frac{\sin \alpha}{\alpha}\right)^2$$
 Equation 36-19

36 Summary (6 of 7)

Diffraction Gratings

• Diffraction by N (multiple) slits results in maxima (lines) at angles θ such that

$$d \sin \theta = m\lambda$$
, for $m = 0, 1, 2, \dots$ Equation 36-25

with the half-widths of the lines given by

and
$$\Delta \theta_{\text{hw}} = \frac{\lambda}{Nd\cos\theta}$$
 Equation 36-28
$$D = \frac{\Delta \theta}{\Delta \lambda} - \frac{m}{d\cos\theta}$$
 Equation 36-29&30
$$R = \frac{\lambda_{\text{avg}}}{\Delta \lambda} = Nm.$$
 Equation 36-31&32

36 Summary (7 of 7)

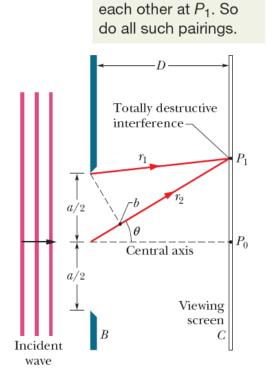
X-Ray Diffraction

• Diffraction maxima (due to constructive interference) occur if the incident direction of the wave, measured from the surfaces of these planes, and the wavelength 1 of the radiation satisfy Bragg's law:

$$2d\sin\theta = m\lambda$$
, for $m = 1, 2, 3, ...$ Equation 36-12

- 36.3.2. In a single slit experiment, what effect on the diffraction pattern would result as the slit width is decreased?
- a) The width of the central band would increase.
- b) The width of the central band would decrease.
- c) The width of the central band would not change.

 $a \sin \theta = m\lambda$, for m = 1, 2, 3...



This pair of rays cancel

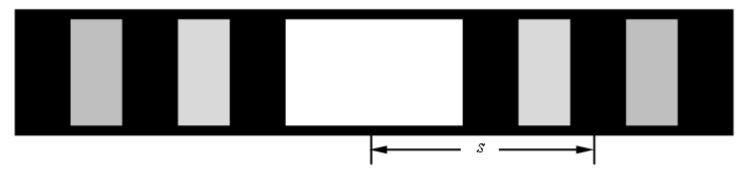
- 36.3.2. In a single slit experiment, what effect on the diffraction pattern would result as the slit width is decreased?
- a) The width of the central band would increase.
- b) The width of the central band would decrease.
- c) The width of the central band would not change.

- 36.3.3. In a single slit experiment, what effect on the first two minima in the diffraction pattern would result as the slit width is decreased?
- a) The width of the two minima would increase.
- b) The width of the two minima would decrease.
- c) The width of the two minima would not change.

$$a \sin \theta = m\lambda$$
, for $m = 1, 2, 3...$

- 36.3.3. In a single slit experiment, what effect on the first two minima in the diffraction pattern would result as the slit width is decreased?
- a) The width of the two minima would increase.
- b) The width of the two minima would decrease.
- c) The width of the two minima would not change.

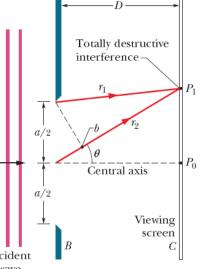
36.3.6. Light of wavelength 600 nm is incident upon a single slit with width 4×10^{-4} m. The figure shows the pattern observed on a screen positioned 2 m from the slits. Determine the distance s.



- a) 0.002 m
- b) 0.003 m
- c) 0.004 m
- d) 0.006 m

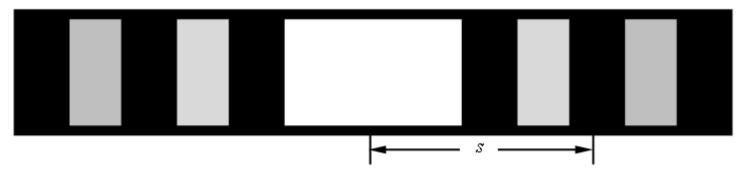
 $a \sin \theta = m\lambda$, for m = 1, 2, 3...

This pair of rays cancel each other at P_1 . So do all such pairings.



e) 0.008 m

36.3.6. Light of wavelength 600 nm is incident upon a single slit with width 4×10 –4 m. The figure shows the pattern observed on a screen positioned 2 m from the slits. Determine the distance s.



- a) 0.002 m
- b) 0.003 m
- c) 0.004 m
- d) 0.006 m

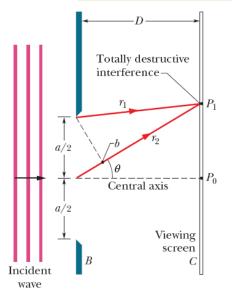
 $a \sin \theta = m\lambda$, for m = 1, 2, 3...(4x10⁻⁴)sin θ = 2 x600 nm=0.003 m

 $\sin \theta = 0.003$

 $\sin \theta \sim \tan \theta = s/D = 0.003$

s = 0.006 m

This pair of rays cancel each other at P_1 . So do all such pairings.



36.6.2. A special microscope has been set up that allows the user to view a specimen using light from among the colors listed below. Which of these would you choose to use for the best resolution?

- a) yellow
- b) red
- c) violet
- d) blue
- e) green

36.6.2. A special microscope has been set up that allows the user to view a specimen using light from among the colors listed below. Which of these would you choose to use for the best resolution?

a) yellow

To achieve the best **resolution**, choose the color with the shortest **wavelength**, which is typically blue or violet light.

b) red

c) violet

Microscope resolution depends on the wavelength of the **light** being used to view the specimens. Shorter wavelengths provide better resolution because they can distinguish smaller details. Among the colors, blue and violet have the shortest wavelengths, with violet typically having the shortest (approximately 400nm) and red having the longest (approximately 700nm).

d) blue

e) green

- 36.10.1. Why is it not beneficial to use a typical diffraction grating when the incident waves are x-rays?
- a) Most materials do not reflect x-rays.
- b) Most materials do not defract x-rays.
- c) X-rays are not electromagnetic waves.
- d) The line spacing is much larger than the x-ray wavelengths.
- e) The resolving power of diffraction gratings is too small.

- 36.10.1. Why is it not beneficial to use a typical diffraction grating when the incident waves are x-rays?
- a) Most materials do not reflect x-rays.
- b) Most materials do not defract x-rays.
- c) X-rays are not electromagnetic waves.
- The correct reason is that X-rays have a very short wavelength (1 to 10 nm), which is not comparable to the size of the obstacles in an ordinary grating, making diffraction impossible

- d) The line spacing is much larger than the x-ray wavelengths.
- e) The resolving power of diffraction gratings is too small.