UFUG 1504: Honors General Physics II

Chapter 22

Electric Fields

Summary (1 of 5)

Definition of Electric Field

- The electric field at any point

$$\vec{E} = \frac{\vec{F}}{q_0}. \qquad \textbf{Equation (22-1)}$$

Electric Field Lines

- Provide a means for visualizing the directions and the magnitudes of electric fields

Summary (2 of 5)

Field due to a Point Charge

- The magnitude of the electric field $\vec{E}$ set up by a point charge q at a distance r from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$

Equation (22-3)

Summary (3 of 5)

Field due to an Electric Dipole

- The magnitude of the electric field set up by the dipole at a distant point on the dipole axis is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

Equation (22-9)

Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Equation (22-26)

Summary (4 of 5)

Force on a Point Charge in an Electric Field

- When a point charge q is placed in an external electric field $\vec{E}$

$$\vec{F} = q\vec{E}.$$

Equation (22-28)

Dipole in an Electric Field

- The electric field exerts a torque on a dipole

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

Equation (22-34)

Summary (5 of 5)

- The dipole has a potential energy *U* associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}.$$

Equation (22-38)

Summary (2 of 5)

Field due to a Point Charge

- The magnitude of the electric field $\vec{E}$ set up by a point charge q at a distance r from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}. \qquad \textbf{Equation (22-3)}$$

22-1 The Electric Field (3 of 8)

How does particle 1 “know” of the presence of particle 2?

That is, since the particles do not touch, how can particle 2 push on particle 1—how can there be such an action at a distance?

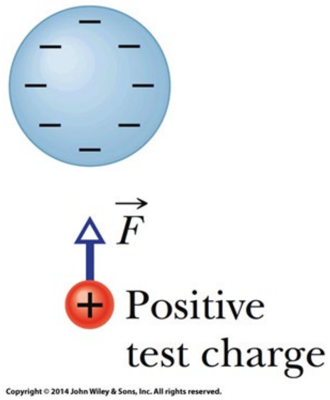
22-1 The Electric Field (4 of 8)

Electric Field

The explanation that we shall examine here is this: Particle 2 sets up an electric field at all points in the surrounding space, even if the space is a vacuum. If we place particle 1 at any point in that space, particle 1 knows of the presence of particle 2 because it is affected by the electric field particle 2 has already set up at that point. Thus, particle 2 pushes on particle 1 not by touching it as you would push on a coffee mug by making contact. Instead, particle 2 pushes by means of the electric field it has set up.

22-1 The Electric Field (5 of 8)

Electric Field



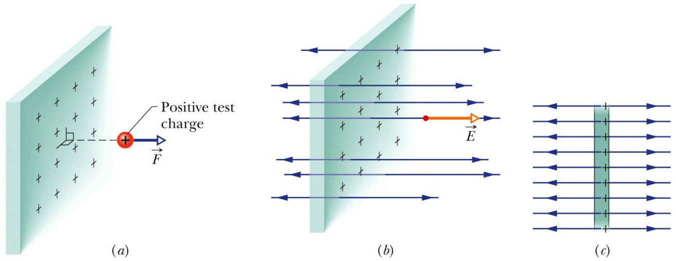
The electric field $\vec{E}$ at any point is defined in terms of the electrostatic force $\vec{F}$ that would be exerted on a positive test charge q_0 placed there:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

22-1 The Electric Field (6 of 8)

Electric Field Lines

Electric field lines help us visualize the direction and magnitude of electric fields. The electric field vector at any point is tangent to the field line through that point. The density of field lines in that region is proportional to the magnitude of the electric field there.



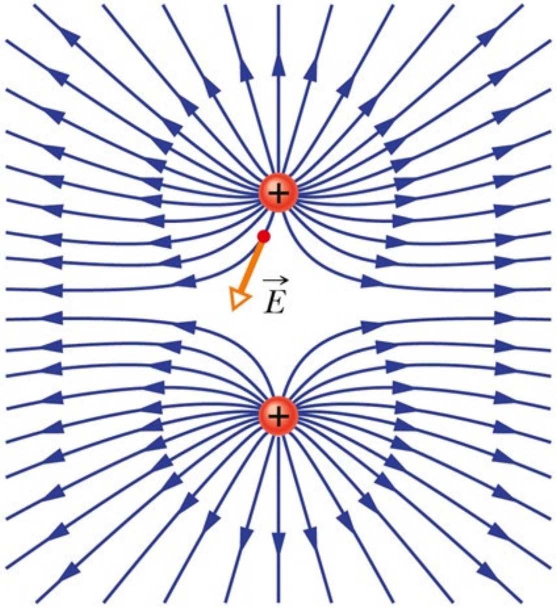
(a) The force on a positive test charge near a very large, non-conducting sheet with uniform positive charge on one side. (b) The electric field vector

$\vec{E}$ at the test charge's location, and the nearby electric field lines, extending away from the sheet. (c) Side view.

22-1 The Electric Field (7 of 8)

Electric Field Lines

Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).



1) The electric field vector at any given point must be tangent to the field line at that point and in the same direction, as shown for one vector.
2) A closer spacing means a larger field magnitude.

22-2 The Electric Field Due to a Charged Particle (3 of 4)

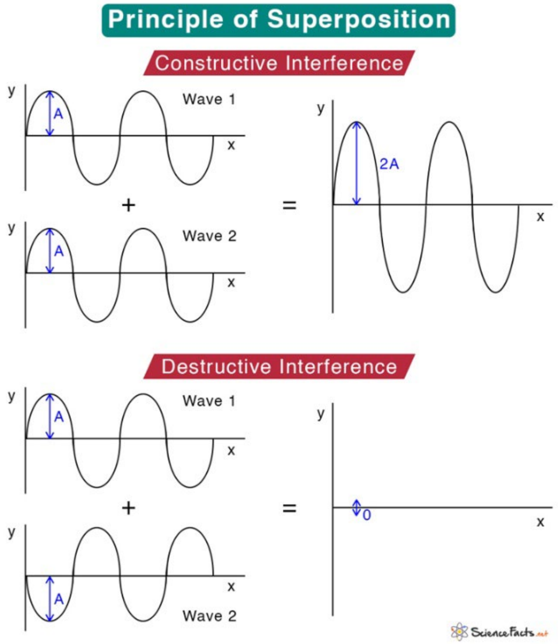
The magnitude of the electric field $\vec{E}$ set up by a particle with charge q at distance r from the particle is

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad f = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law)},$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

The **electric field vectors** set up by a positively charged particle all point directly away from the particle. Those set up by a negatively charged particle all point directly toward the particle.

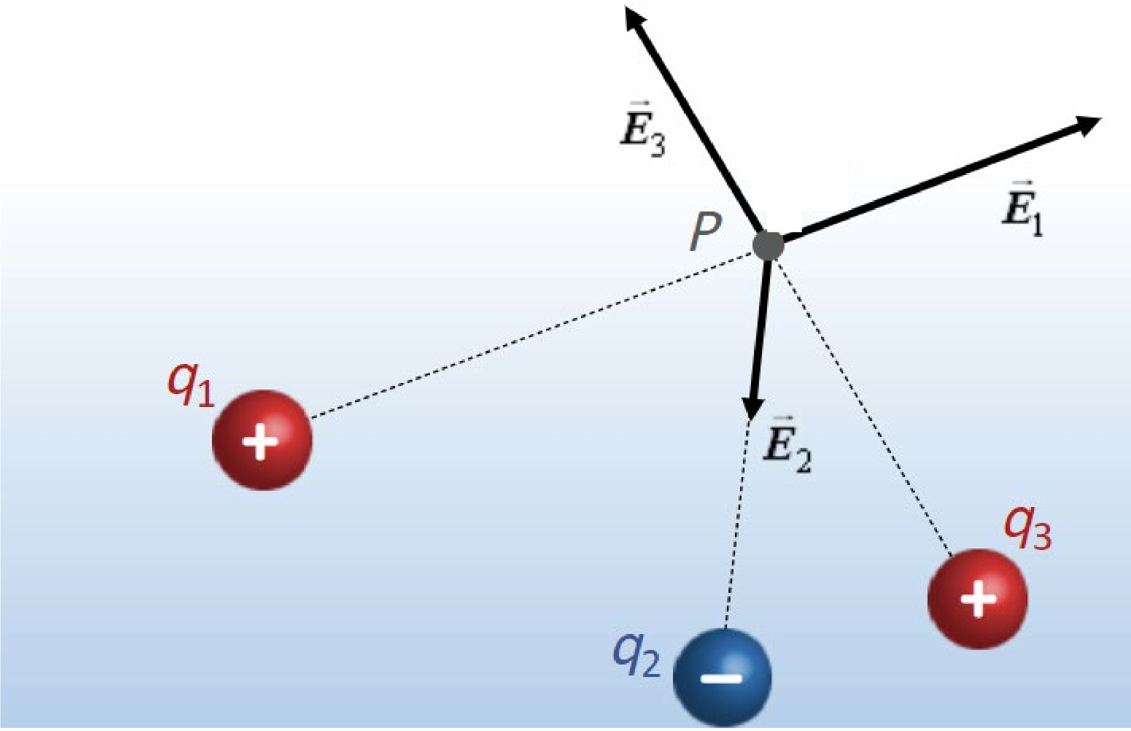
If more than one charged particle sets up an electric field at a point, the net electric field is the vector **sum** of the individual electric fields—**electric fields obey the superposition principle （叠加原理）**.

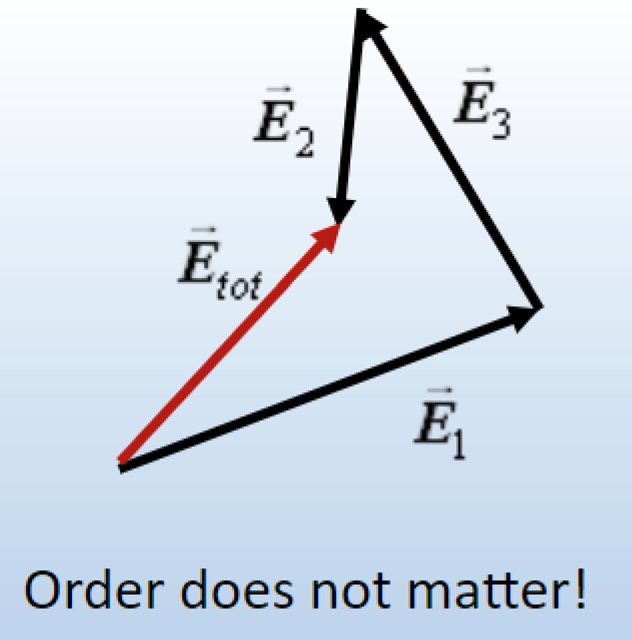


22-2 The Electric Field Due to a Charged Particle (3 of 4)

what is the E-field at point P due to q_1, q_2, and q_3?

$\vec{E}$





Week	Topic	Briefly outline what this topic will cover	Indicate which courese ILOs this topic is related to
1	Introduction, Coulomb's law	Coulomb's law describes the electrostatic force between two charged particles, which is fundamental in understanding the behavior of electric charges and the principles of electromagnetism.	ILO1, ILO2, ILO3, ILO4
2	Electric field, Gauss's Law,	Electric fields are the regions around charged particles where other charges experience a force. Gauss's Law, a fundamental law of electromagnetism, relates the electric flux through a closed surface to the charge enclosed by that surface.	ILO1, ILO2, ILO3, ILO4
3	Electric potential, Capacitance,	Electric potential is a scalar quantity that describes the work done per unit charge in moving a test charge from one point to another in an electric field. Capacitanceis a measure of a system's ability to store electric charge and energy in an electric field.	ILO1, ILO2, ILO3, ILO4
4		Current is the flow of electric	

Summary (3 of 5)

Field due to an Electric Dipole

- The magnitude of the electric field set up by the dipole at a distant point on the dipole axis is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

Equation (22-9)

Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

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Summary (4 of 5)

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Summary (5 of 5)

- The dipole has a potential energy U associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}. \qquad \textbf{Equation (22-38)}$$

Summary (2 of 5)

Field due to a Point Charge

- The magnitude of the electric field $\vec{E}$ set up by a point charge q at a distance r from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$

Equation (22-3)

22-3 The Electric Field Due to a Dipole (偶极子)

What is the E at P?

P

Electric Dipole （电偶极子）

An electric dipole consists of two particles with charges of equal magnitude q but opposite signs, separated by a small distance d.

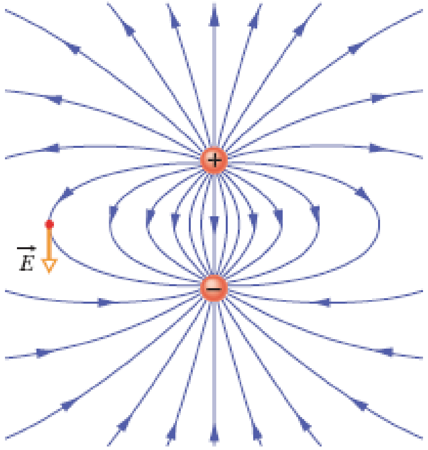
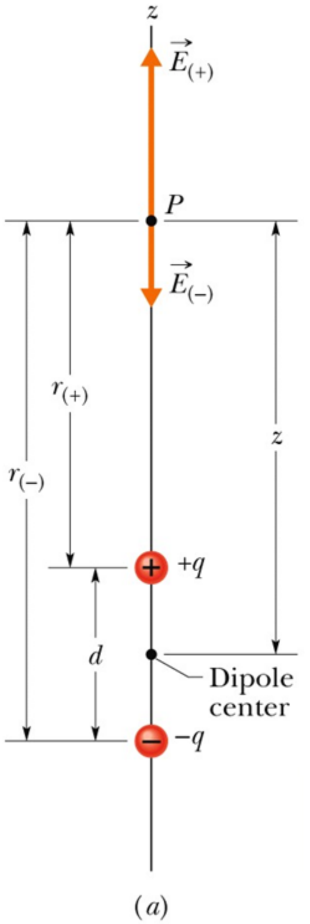


Figure 22-8 The pattern of electric field lines around an electric dipole, with an electric field vector $\vec{E}$ shown at one point (tangent to the field line through that point).

22-3 The Electric Field Due to a Dipole (偶极子) (4 of 5)



$$E = E_{(+)} - E_{(-)}$$

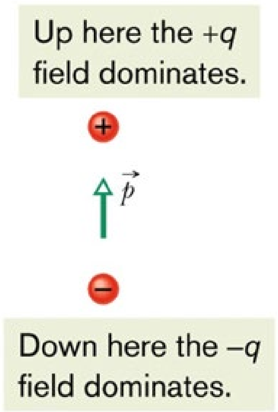
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2}.$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2}\right).$$

$$z \gg d. \qquad d/2z \ll 1$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3},$$



electric dipole moment $\vec{p}$

电偶极矩

$$E = 2k\frac{qd}{z^3} = 2k\frac{p}{z^3},$$

$$k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}. \quad k = \frac{1}{4\pi\varepsilon_0}.$$

22-3 The Electric Field Due to a Dipole (偶极子)

Electric Dipole （电偶极子）

An electric dipole consists of two particles with charges of equal magnitude q but opposite signs, separated by a small distance d.

The magnitude of the electric field set up by an electric dipole at a distant point on the dipole axis (which runs through both particles) can be written in terms of either the product qd or the magnitude p of the dipole moment(电偶极矩):

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}, \qquad z \gg d.$$

where z is the distance between the point and the center of the dipole.

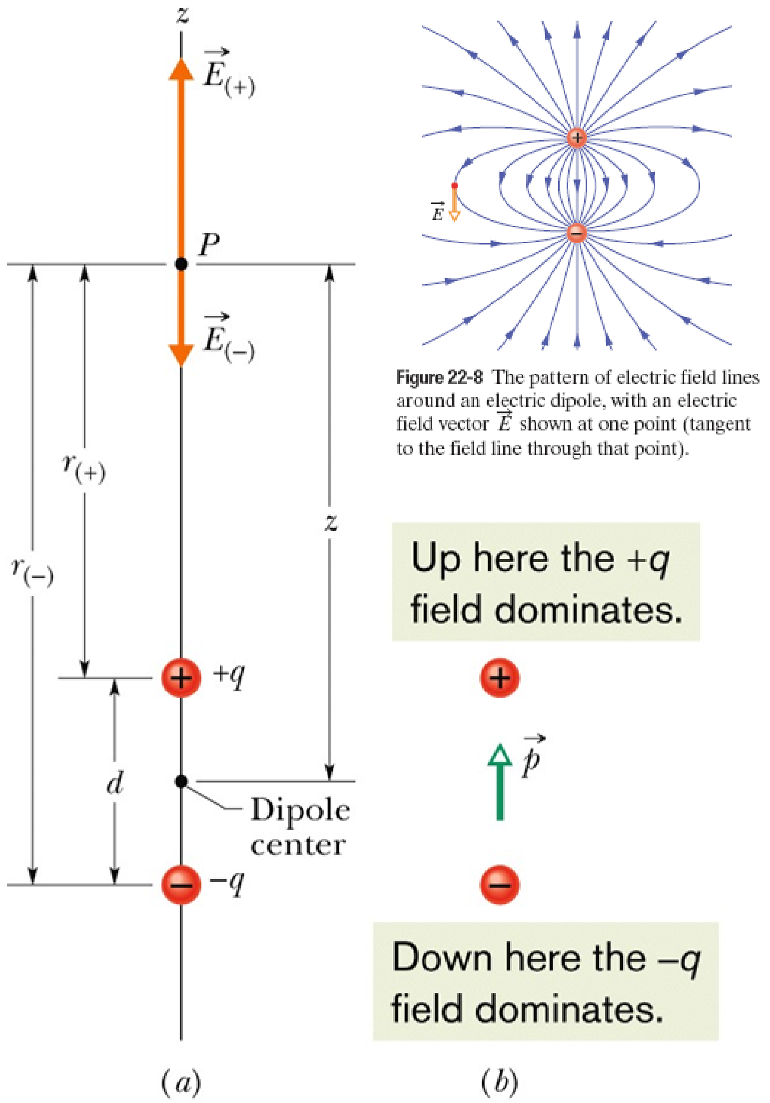
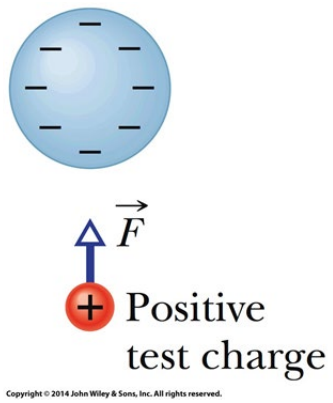


Figure 22-8 The pattern of electric field lines around an electric dipole, with an electric field vector $\vec{E}$ shown at one point (tangent to the field line through that point).

22-3 The Electric Field Due to a Dipole

Electric Field Due to a Particle, how about due to other?



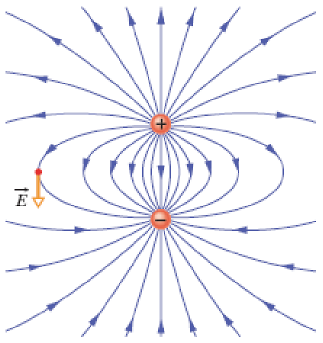
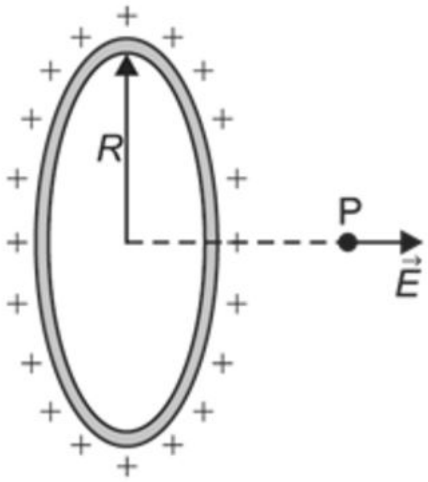


Figure 22-8 The pattern of electric field lines around an electric dipole, with an electric field vector $\vec{E}$ shown at one point (tangent to the field line through that point).

Charged Ring



$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}, \qquad z \gg d.$$

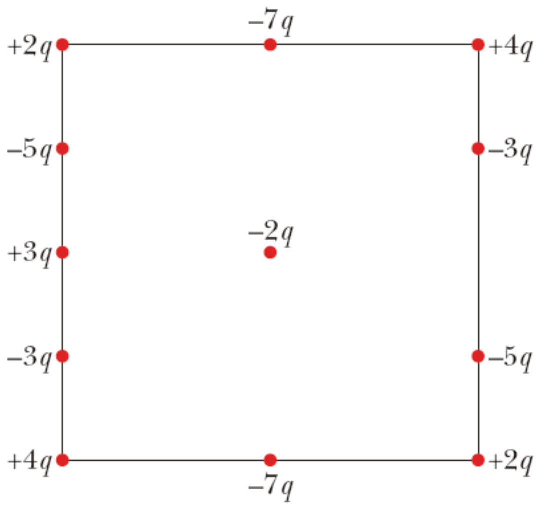
22-4 The Electric Field Due to a Line of Charge

Key Concepts

continuous charge distribution

- To find the electric field of an extended object at a point, we first consider the electric field set up by a charge element dq in the object, where the element is small enough for us to apply the equation for a particle. Then we sum, via integration, components of the electric fields $d\vec{E}$ from all the charge elements.

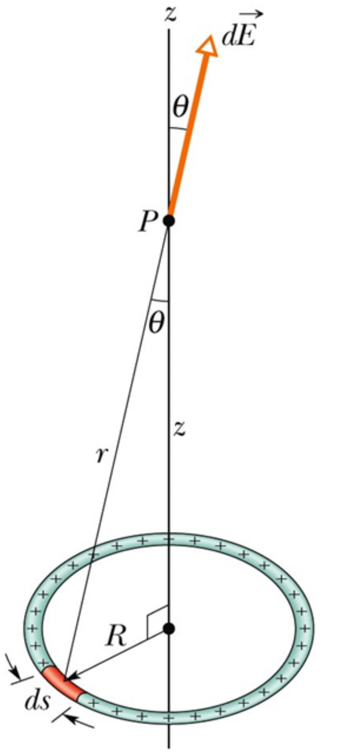
- Because the individual electric fields $d\vec{E}$ have different magnitudes and point in different directions, we first see if symmetry allows us to cancel out any of the components of the fields, to simplify the integration.



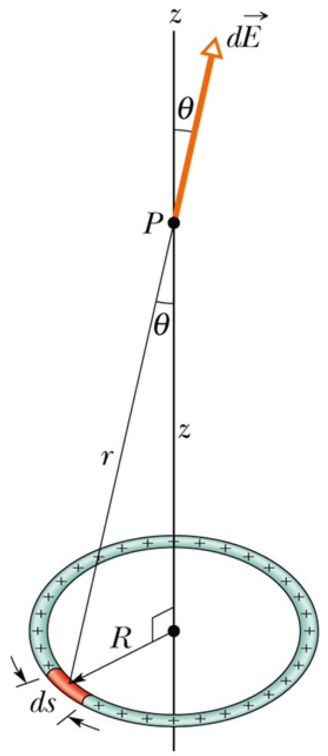
22-4 The Electric Field Due to a Line of Charge

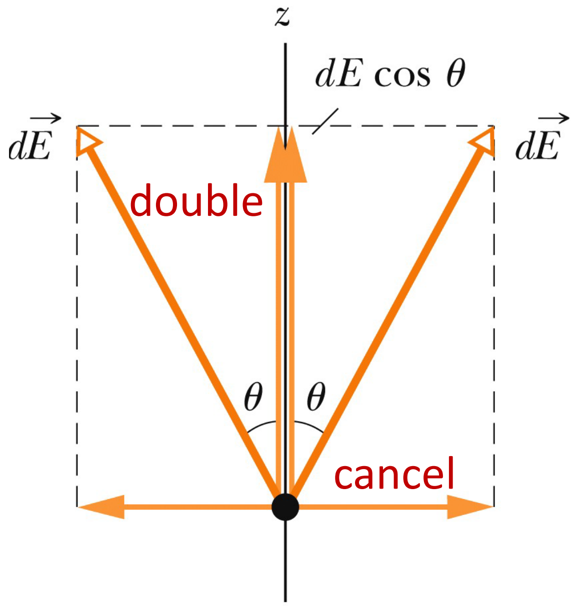
Charged Ring

Canceling Components - Point *P* is on the axis: In the Figure, consider the charge element on the opposite side of the ring. It too contributes the field magnitude $d\vec{E}$ but the field vector leans at angle θ in the opposite direction from the vector from our first charge element, as indicated in the side view of Figure (bottom). Thus the two perpendicular components cancel. All around the ring, this cancelation occurs for every charge element and its symmetric partner on the opposite side of the ring. So we can neglect all the perpendicular components.



22-4 The Electric Field Due to a Line of Charge





1. A ring of uniform positive charge. A differential element of charge occupies a length *ds*. This element sets up an electric field dE at point

2. The components perpendicular to the *z* axis cancel; the parallel components add.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2} \qquad dq = \lambda\, ds$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}.$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\left(z^2+R^2\right)^{\frac{1}{2}}}$$

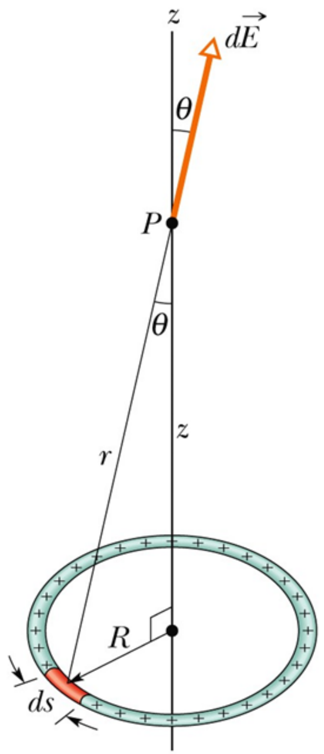
$$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{\left(z^2+R^2\right)^{\frac{3}{2}}}ds,$$

Table 22-1 Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

Actually, here we didn't use cancel rule textbook is not right.

22-4 The Electric Field Due to a Line of Charge (11 of 13)



Charged Ring

Integrating.

$$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{\left(z^2+R^2\right)^{\frac{3}{2}}}ds,$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0\left(z^2+R^2\right)^{\frac{3}{2}}}\int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}$$

$$;\ \lambda = q/(2\pi R);$$

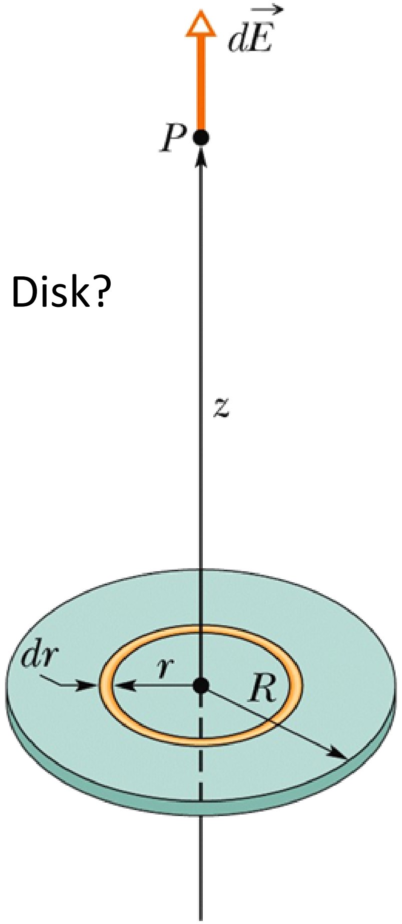
Finally, $\boxed{E = \frac{qz}{4\pi\varepsilon_0(z^2+R^2)^{\frac{3}{2}}}}$ (charged ring).

If $z \gg R$. particle again

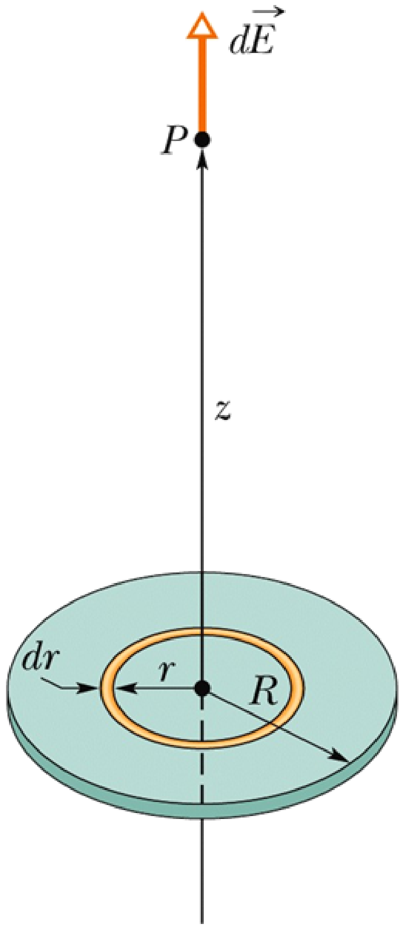
$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

22-5 The Electric Field Due to a Charged Disk (3 of 5)

How about the Electric Field Due to a Charged Disk?



22-5 The Electric Field Due to a Charged Disk



$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{\frac{3}{2}}} \quad \text{(charged ring).}$$

$$dE = \frac{dqz}{4\pi\varepsilon_0\left(z^2 + r^2\right)^{\frac{3}{2}}}. \qquad r \le R.$$

$$dq = \sigma\, dA = \sigma\,(2\pi r\, dr)$$

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0}\int_0^R \left(z^2 + r^2\right)^{\frac{-3}{2}}(2r)\, dr.$$

$$X = (z^2 + r^2),\ m = -\tfrac{3}{2}; \qquad dX = (2r)\, dr. \qquad \int X^m\, dX = \frac{X^{m+1}}{m+1}$$

$$E = \frac{\sigma z}{4\varepsilon_0}\left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)} \quad \text{(charged disk)}$$

Table 22-1 Some Measures of Electric Charge

Name	Symbol	SI Unit
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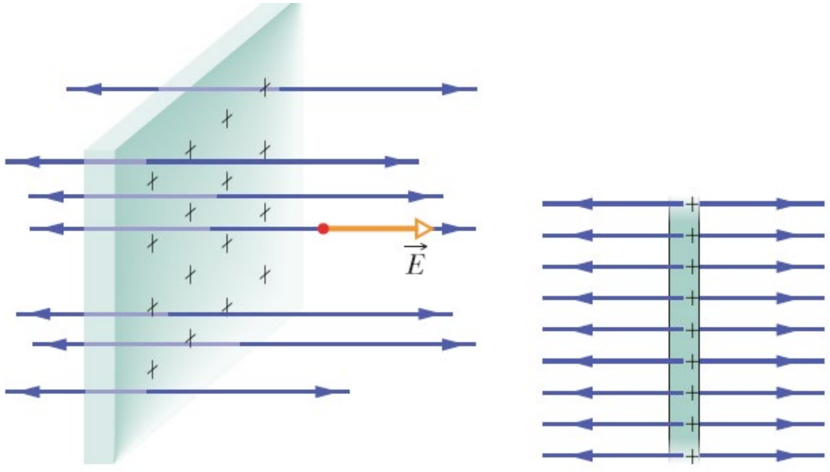
22-5 The Electric Field Due to a Charged Disk

$$\boxed{E=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)}\quad \text{(charged disk)}$$



(b) (c)

R=∞

$$E=\frac{\sigma}{2\varepsilon_0}\qquad \text{(infinite sheet).}$$

22-6 A Point Charge in an Electric Field (3 of 4)

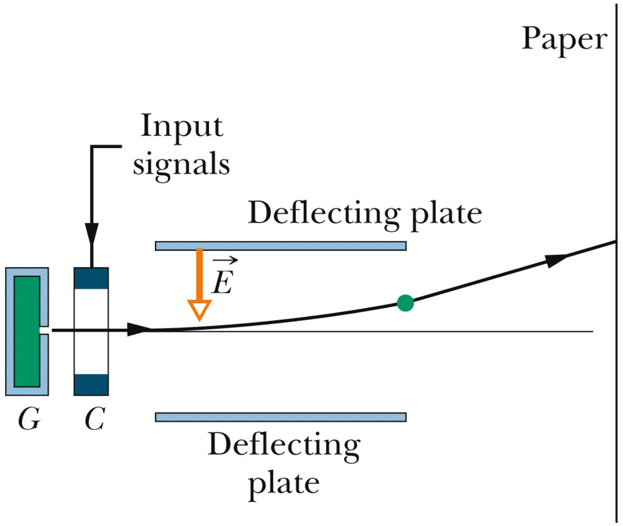
If a particle with charge q is placed in an external electric field $\vec{E}$, an electrostatic force $\vec{F}$ acts on the particle:

$$\boxed{\vec{F} = q\vec{E}.}$$

The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

22-6 A Point Charge in an Electric Field
(4 of 4)

How ink-jet printer work?

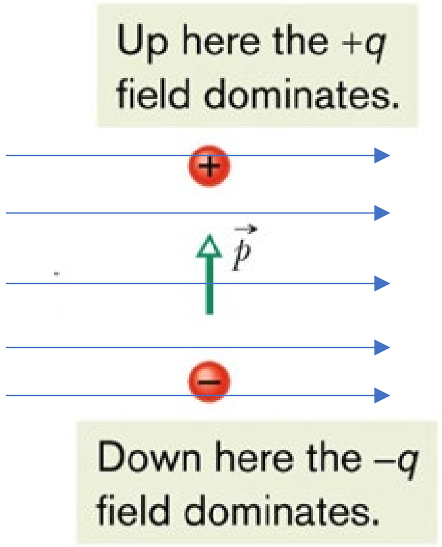


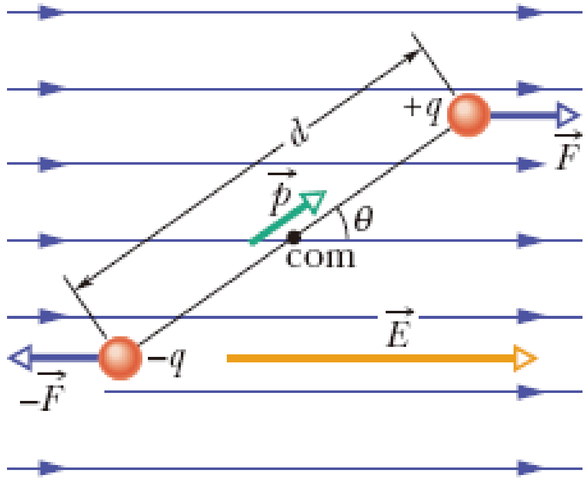
Ink-jet printer. Drops shot from generator *G* receive a charge in charging unit *C*. An input signal from a computer controls the charge and thus the effect of field E on where the drop lands on the paper.

Other application?

22-7 A Dipole in an Electric Field (4 of 6)

What if add electric field on a dipole?



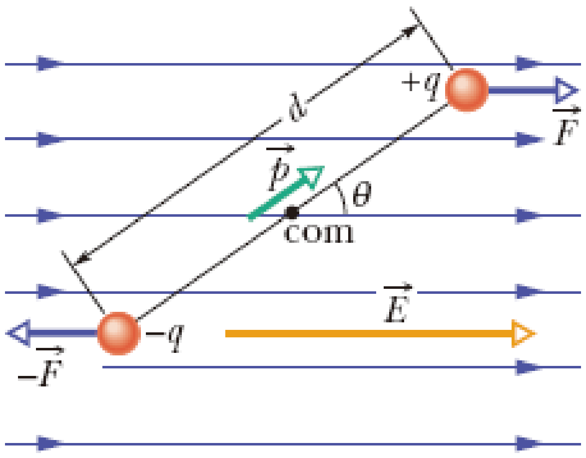


(a)

22-7 A Dipole in an Electric Field (4 of 6)

电偶极子的扭矩 偶极矩

The torque on an electric dipole of dipole moment $\vec{p}$ when placed in an external electric field $\vec{E}$ is given by a cross product:

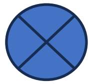


(a)

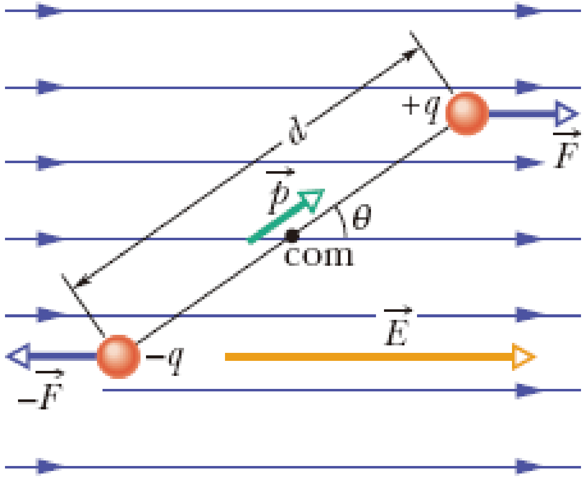
$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on dipole)}.$$

$$\tau = pE \sin\theta.$$

Direction: right-hand rule



22-7 A Dipole in an Electric Field (4 of 6)



(a)

A potential energy U is associated with the orientation of the dipole moment in the field, as given by a dot product:

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}$$

- **Work and Rotational Kinetic Energy**

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

- **Equation (10-53)**

$$U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta. = -pE\cos\theta.$$

Summary (1 of 5)

Definition of Electric Field

- The electric field at any point

$$\vec{E} = \frac{\vec{F}}{q_0}. \qquad \textbf{Equation (22-1)}$$

Electric Field Lines

- Provide a means for visualizing the directions and the magnitudes of electric fields

Summary (2 of 5)

Field due to a Point Charge

- The magnitude of the electric field $\vec{E}$ set up by a point charge q at a distance r from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$

Equation (22-3)

Summary (3 of 5)

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Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

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Equation (22-26)

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Dipole in an Electric Field

- The electric field exerts a torque on a dipole

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Summary (5 of 5)

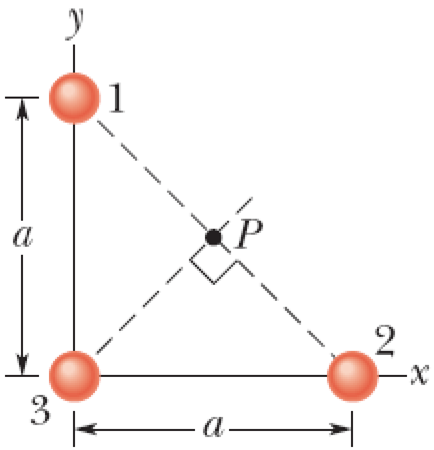
- The dipole has a potential energy U associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}.$$

Equation (22-38)

In-class quiz

The three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance a = 6.00 μm. What are the (a) magnitude and (b) direction of the net electric field at point P due to the particles?



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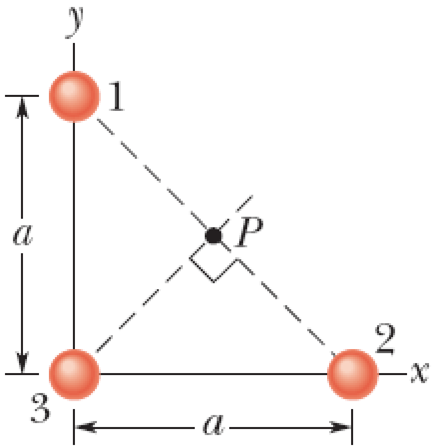


(a) two charges $q_1 = q_2 = +e$ cancel each other, the field only due to $q_3 = +2e$.

$$|\vec{E}_{\text{net}}| = \frac{1}{4\pi\varepsilon_0}\frac{2e}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{2e}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{a^2}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.60\times10^{-19}\ \text{C})}{(6.00\times10^{-6}\ \text{m})^2} = 160\ \text{N/C}.$$

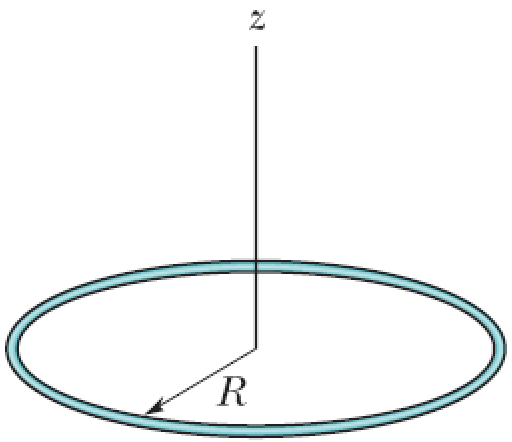
(b) This field points at 45.0°, counterclockwise from the x axis.

In-class quiz

A thin nonconducting rod with a uniform distribution of positive charge Q is bent into a complete circle of radius R, (Fig. 22-48). The central perpendicular axis through the ring is a z axis, with the origin at the center of the ring.

(a) In terms of R, at what positive value of z is that magnitude maximum?

(b) If R = 2.00 cm and Q = 4.00 μC, what is the maximum magnitude?



Hint: Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

$$\boxed{E = \frac{qz}{4\pi\varepsilon_0 (z^2 + R^2)^{\frac{3}{2}}}} \quad \text{(charged ring).}$$

In-class quiz

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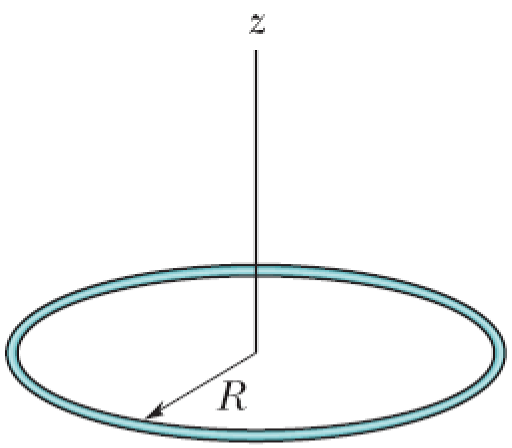


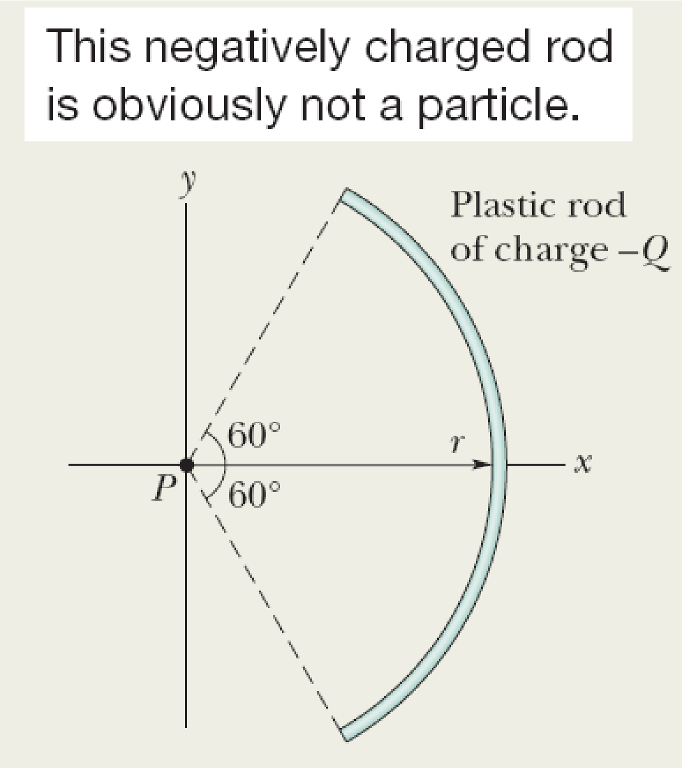
(a) Differentiating Eq. 22-16 and setting equal to zero (to obtain the location where it is maximum) leads to

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right) = \frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}} = 0 \quad \Rightarrow \quad z = +\frac{R}{\sqrt{2}} = 0.707R$$

(b) Plugging this value back into Eq. 22-16 with the values stated in the problem, we find $E_{max} = 3.46 \times 10^7$ N/C.

In-class quiz (do together)

Figure shows a plastic rod with a uniform charge $-Q$. It is bent in a 120° circular arc of radius r and symmetrically placed across an x axis with the origin at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?



In-class quiz (do together)

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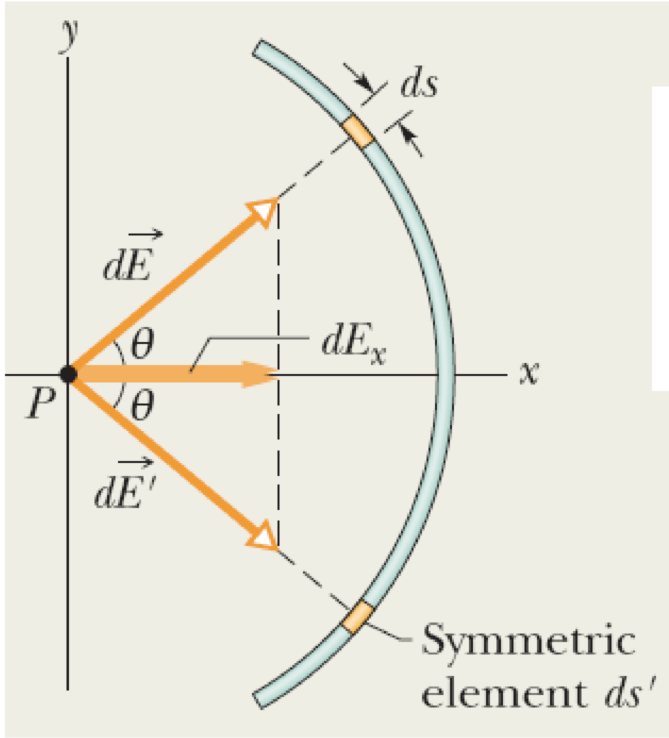

$$dE_x = dE \cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, ds.$$

has two variables, θ and s.

$$ds = r\, d\theta,$$

$$\begin{aligned} E &= \int dE_x = \int_{-60^\circ}^{60^\circ} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\, d\theta \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-60^\circ}^{60^\circ}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-60^\circ}^{60^\circ} \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin 60^\circ - \sin(-60^\circ)] \\ &= \frac{1.73\lambda}{4\pi\varepsilon_0 r}. \end{aligned}$$

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}$$

$$\vec{E} = \frac{0.83Q}{4\pi\varepsilon_0 r^2}\,\hat{\mathrm{i}}.$$