UFUG 1504: Honors General Physics II

Chapter 25

Capacitance

25 Summary (1 of 4)

Capacitor and Capacitance

- The capacitance of a capacitor is defined as:

$$q = CV$$ **Equation (25-1)**

Determining Capacitance

- Parallel-plate capacitor:

$$C = \frac{\varepsilon_0 A}{d}.$$ **Equation (25-9)**

- Cylindrical Capacitor:

$$C = 2\pi\varepsilon_0 \frac{L}{\ln\left(\frac{b}{a}\right)}.$$ **Equation (25-14)**

25 Summary (2 of 4)

- Spherical Capacitor:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}.$$

Equation (25-17)

- Isolated sphere:

$$C = 4\pi\varepsilon_0 R.$$

Equation (25-18)

Capacitor in parallel and series

- In parallel:

$$C_{\mathrm{eq}} = \sum_{j=1}^{n} C_j$$

Equation (25-19)

25 Summary (3 of 4)

- In series

$$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

Equation (25-20)

Potential Energy and Energy Density

- Electric Potential Energy (U):

$$U = \frac{q^2}{2C} = \tfrac{1}{2} CV^2$$

Equation (25-21&22)

- Energy density (u)

$$u = \tfrac{1}{2} \varepsilon_0 E^2.$$

Equation (25-25)

25 Summary (4 of 4)

Capacitance with a Dielectric

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ, called the dielectric constant, which is characteristic of the material.

Gauss' Law with a Dielectric

- When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad \text{(Gauss' law with dielectric)}$$

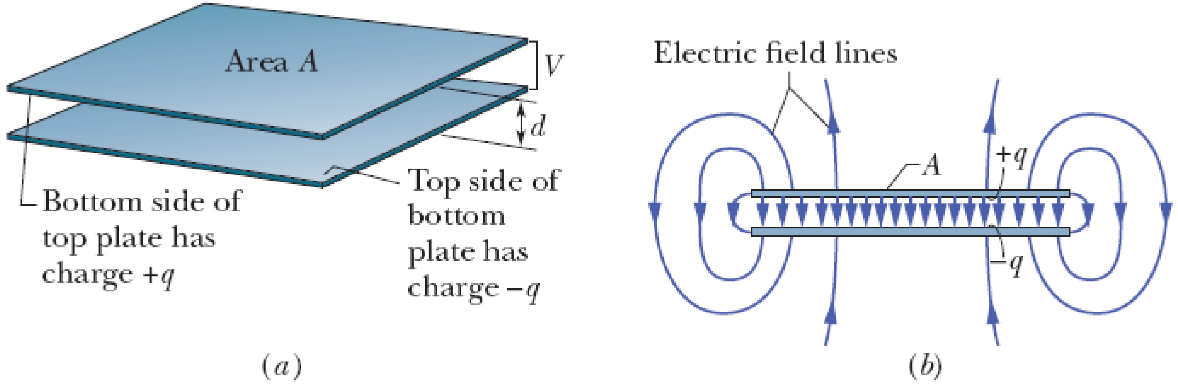
Equation (25-36)

25-1 Capacitance (2 of 6)

Paul Silvermann/Fundamental Photographs

Figure 25-1 An assortment of capacitors.

25-1 Capacitance (2 of 6)



(a) (b)

A parallel-plate capacitor, made up of two plates of area A separated by a distance d. The charges on the facing plate surfaces have the same magnitude q but opposite signs

A capacitor consists of two isolated conductors (the plates) with charges $+q$ and $-q$. Its **capacitance** C is defined from

$$q = CV.$$

where V is the potential difference between the plates.

Unit: the *farad (法拉)* (F):
1 farad = 1 F = 1 coulomb per volt = 1 C/V.

25-2 Calculating the Capacitance (2 of 12)

Calculating electric field and potential difference

To relate the electric field $\vec{E}$ between the plates of a capacitor to the charge q on either plate, we shall use Gauss' law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

25-2 Calculating the Capacitance (3 of 12)

Calculating electric field and potential difference

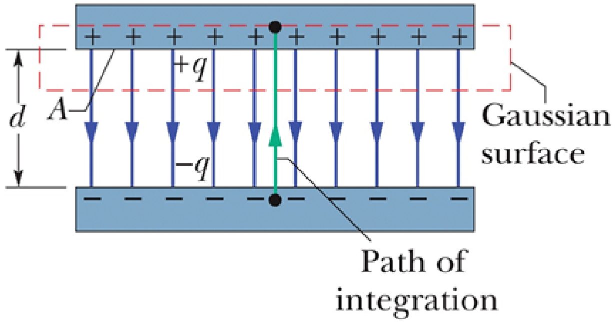
the potential difference between the plates of a capacitor is related to the field $\vec{E}$ by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

Letting V represent the difference $V_f = V_i$, we can then recast the above equation as:

$$V = \int_-^+ E\, ds$$

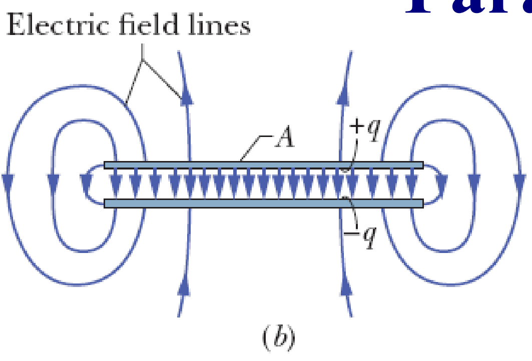
We use Gauss' law to relate q and E. Then we integrate the E to get the potential difference.



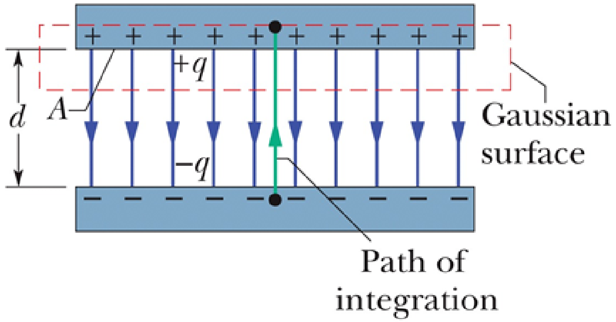
now apply above equations to some particular cases

25-2 Calculating the Capacitance (4 of 12)

Parallel-Plate Capacitor



We use Gauss' law to relate q and E. Then we integrate the E to get the potential difference.



We assume, as Figure suggests, that the plates of our parallel-plate capacitor are so large and so close together that we can neglect the fringing of the electric field at the edges of the plates, taking $\vec{E}$ to be constant throughout the region between the plates.

We draw a Gaussian surface that encloses just the charge q on the positive plate

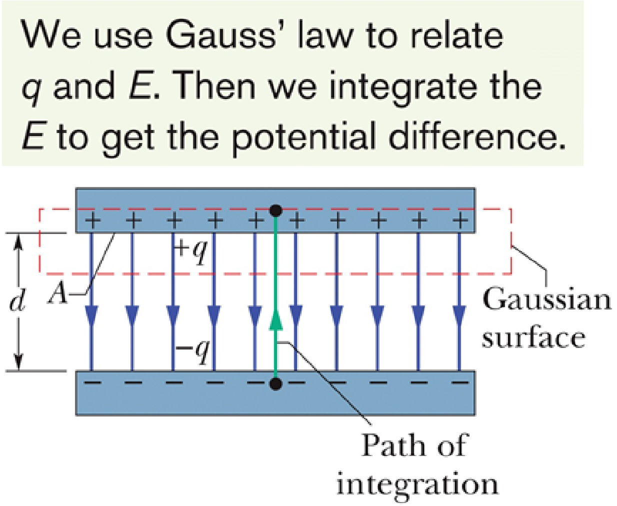
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \quad \Rightarrow \quad q = \varepsilon_0 EA$$

where A is the area of the plate.

25-2 Calculating the Capacitance (5 of 12)

potential difference

$$V = \int_{-}^{+} E\, ds = E \int_{0}^{d} ds = Ed.$$



Now if we substitute q in the above relations to $q = CV$, we get,

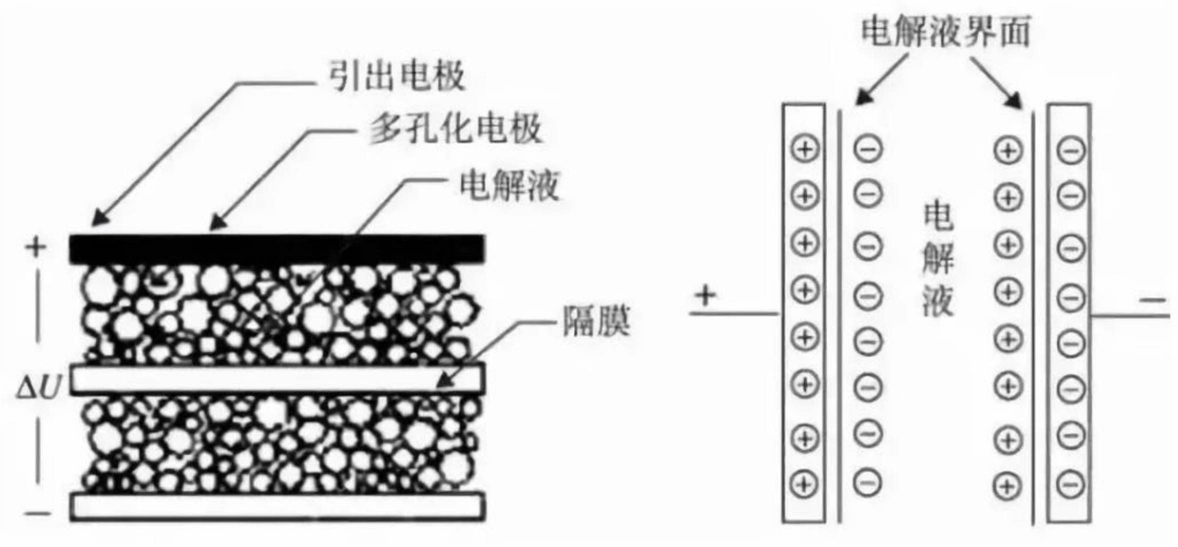
$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor)}.$$

https://phet.colorado.edu/sims/html/capacitor-lab-basics/latest/capacitor-lab-basics_all.html

25-2 Calculating the Capacitance (5 of 12)

Q1: What is supercapacitor?

$$C = \frac{\varepsilon_0 A}{d} \quad (\text{parallel-plate capacitor}).$$



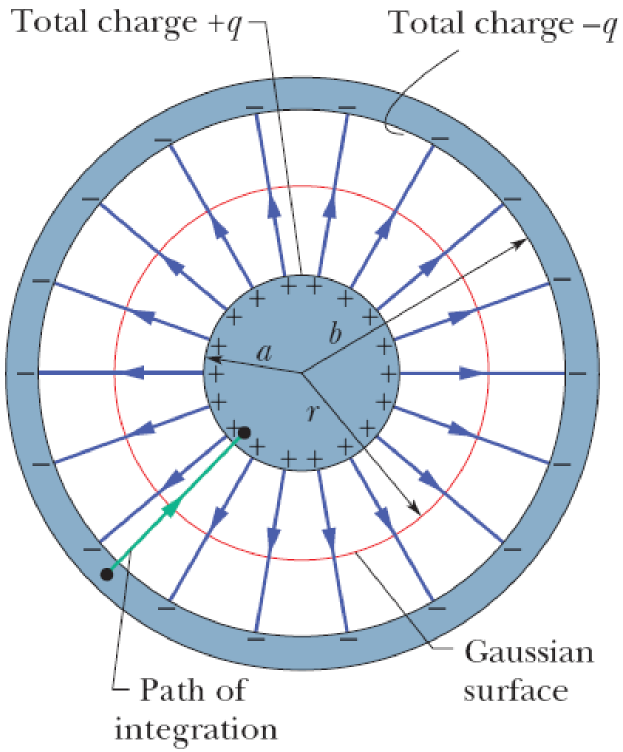
25-2 Calculating the Capacitance (6 of 12)

Why have cylindrical capacitor more than normal parallel-plate capacitor?

Paul Silvermann/Fundamental Photographs

Figure 25-1 An assortment of capacitors.

25-2 Calculating the Capacitance (7 of 12)



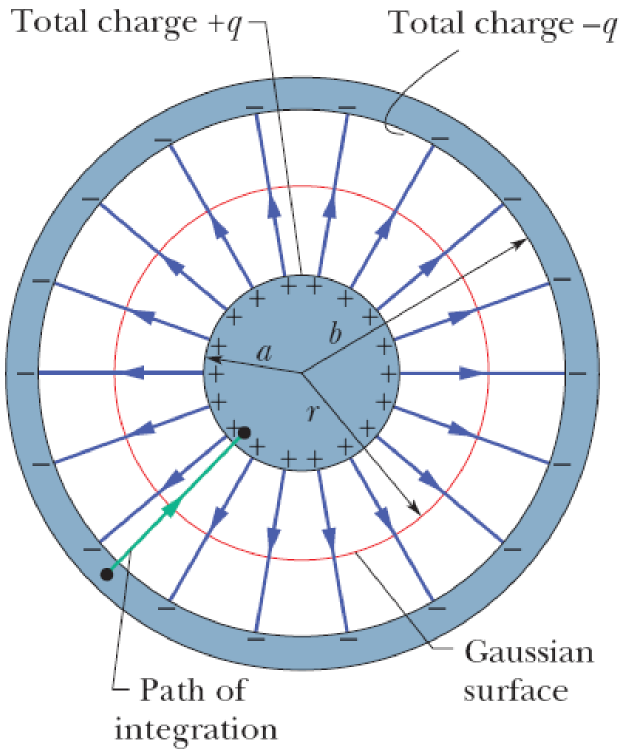
Cylindrical Capacitor

Figure shows, in cross section, a cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b. We assume that $L \gg b$ so that we can neglect the fringing of the electric field that occurs at the ends of the cylinders. Each plate contains a charge of magnitude q. Here, charge and the field magnitude E is related as follows,

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \Longrightarrow q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$$

25-2 Calculating the Capacitance (8 of 12)

$$q = \varepsilon_0 E(2\pi r L)$$



Solving for E field:

$$V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

From the relation $C = \frac{q}{V}$, we then have

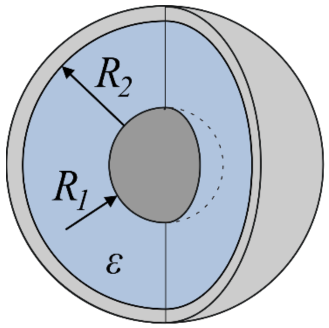
$$C = 2\pi\varepsilon_0 \frac{L}{\ln\left(\frac{b}{a}\right)} \quad \text{(cylindrical capacitor).}$$

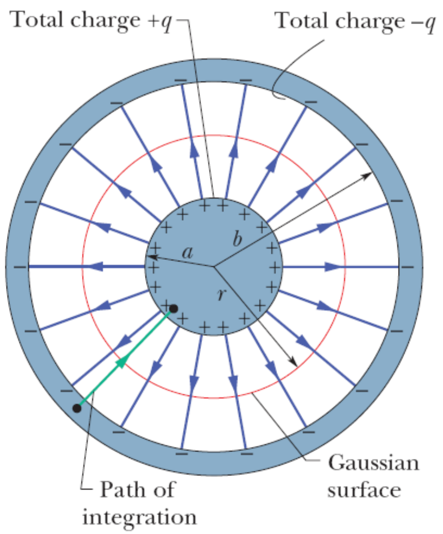
25-2 Calculating the Capacitance (10 of 12)

Others…

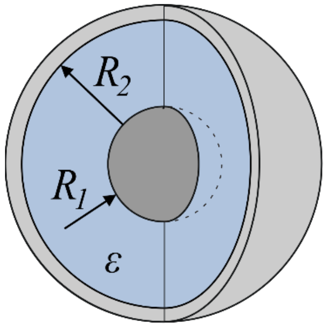
For **spherical capacitor** the capacitance is:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{(spherical capacitor).}$$





25-2 Calculating the Capacitance (10 of 12)

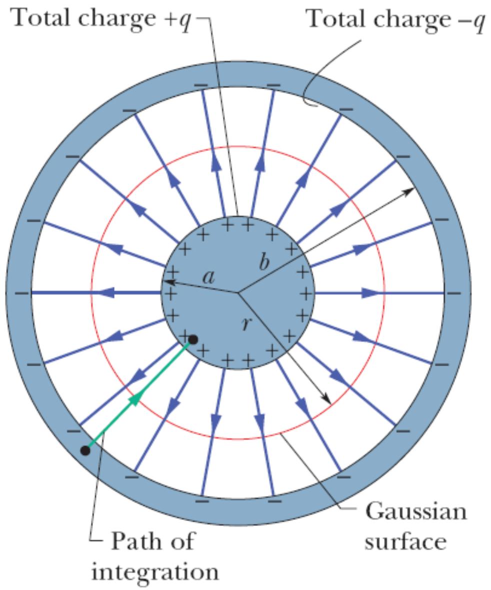


$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \Rightarrow q = \varepsilon_0 EA = \varepsilon_0 E(4\pi r^2),$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

$$V = \int_-^+ E\, ds = -\frac{q}{4\pi\varepsilon_0} \int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0} \frac{b-a}{ab},$$

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{(spherical capacitor)}.$$

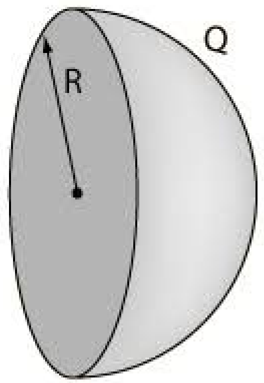


25-2 Calculating the Capacitance (10 of 12)

Others…

Capacitance of an **isolated sphere**:

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere).}$$



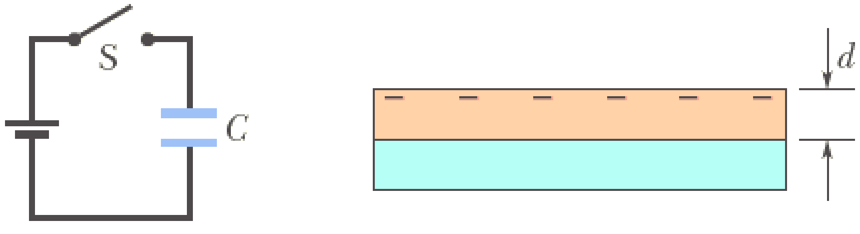
$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{(spherical capacitor).} \Longrightarrow C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}$$

$b \to \infty$, a=R

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere).}$$

25-2 Calculating the Capacitance

In Fig. 25-7*a*, switch S is closed to connect the uncharged capacitor of capacitance $C = 0.25\ \mu F$ to the battery of potential difference $V = 12$ V. The lower capacitor plate has thickness $L = 0.50$ cm and face area $A = 2.0 \times 10^{-4}\ m^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 \times 10^{28}$ electrons/m^3. From what depth d within the plate (Fig. 25-7*b*) must electrons move to the plate face as the capacitor becomes charged?



(*a*) (*b*)

$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor)}.$$

25-2 Calculating the Capacitance

$$q = CV = (0.25 \times 10^{-6}\ \text{F})(12\ \text{V})$$
$$= 3.0 \times 10^{-6}\ \text{C}.$$

Dividing this result by e gives us the number N of conduction electrons that come up to the face:

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ \text{C}}{1.602 \times 10^{-19}\ \text{C}}$$
$$= 1.873 \times 10^{13}\ \text{electrons}.$$

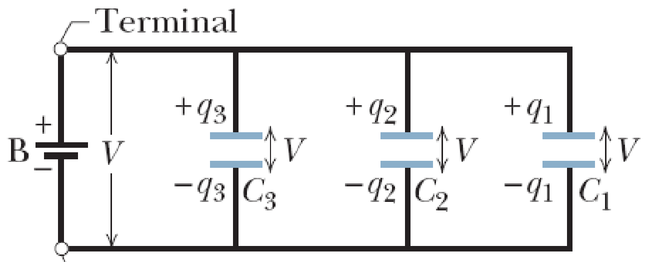
These electrons come from a volume that is the product of the face area A and the depth d we seek. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad},$$

or

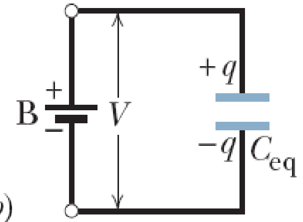
$$d = \frac{N}{An} = \frac{1.873 \times 10^{13}\ \text{electrons}}{(2.0 \times 10^{-4}\ \text{m}^2)(8.49 \times 10^{28}\ \text{electrons/m}^3)}$$
$$= 1.1 \times 10^{-12}\ \text{m} = 1.1\ \text{pm}. \qquad \text{(Answer)}$$

25-3 Capacitors in Parallel and in Series (4 of 9)



(a)

Parallel capacitors and their equivalent have the same *V* ("par-V").



(b)

Capacitors in Parallel (并联)

When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.

$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

The total charge on the parallel combination of Figure. 25-8*a* is then

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

25-3 Capacitors in Parallel and in Series (5 of 9)

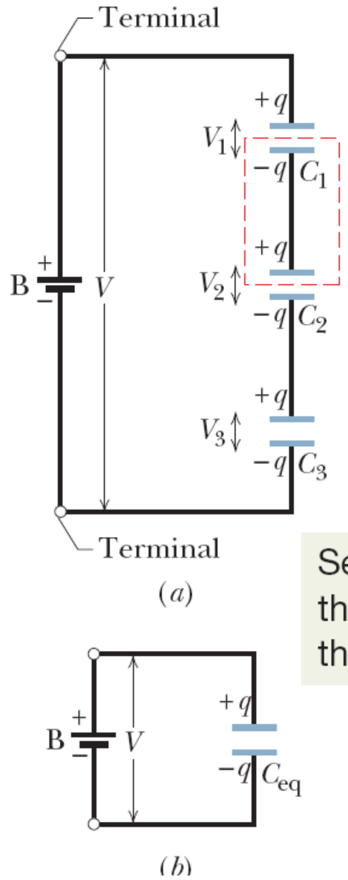
The equivalent capacitance, with the same total charge q and applied potential difference V as the combination, is then

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

a result that we can easily extend to any number n of capacitors, as

$$C_{eq} = \sum_{j=1}^{n} C_j \quad (n \text{ capacitors in parallel}).$$

25-3 Capacitors in Parallel and in Series (7 of 9)



Series capacitors and their equivalent have the same q (“seri-q”).

Capacitors in Series (串联)

When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

The total potential difference V due to the battery is the sum

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

25-3 Capacitors in Parallel and in Series (8 of 9)

The equivalent capacitance is then

$$C_{\text{eq}} = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}},$$

or

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$

25-3 Capacitors in Parallel and in Series (8 of 9)

Figure 25-30 Problem 14.

••15 GO In Fig. 25-31, a 20.0 V battery is connected across capacitors of capacitances $C_1 = C_6 = 3.00\ \mu F$ and $C_3 = C_5 = 2.00C_2 = 2.00C_4 = 4.00\ \mu F$. What are (a) the equivalent capacitance C_{eq} of the capacitors and (b) the charge stored by C_{eq}? What are (c) V_1 and (d) q_1 of capacitor 1, (e) V_2 and (f) q_2 of capacitor 2, and (g) V_3 and (h) q_3 of capacitor 3?

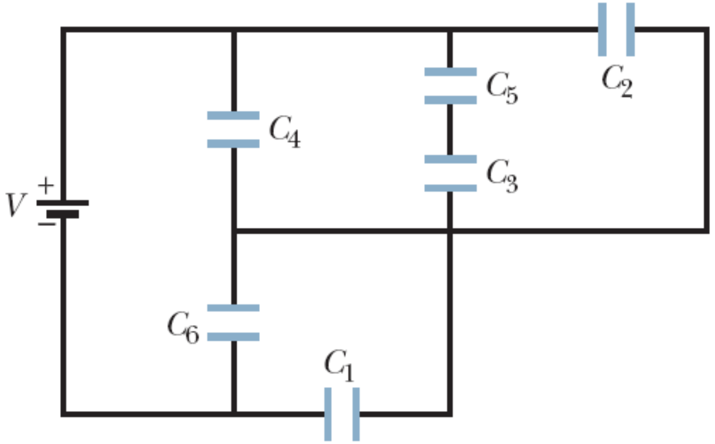


Figure 25-31 Problem 15.

25-3 Capacitors in Parallel and in Series (8 of 9)

25-3 Capacitors in Parallel and in Series (8 of 9)

Figure 25-30 Problem

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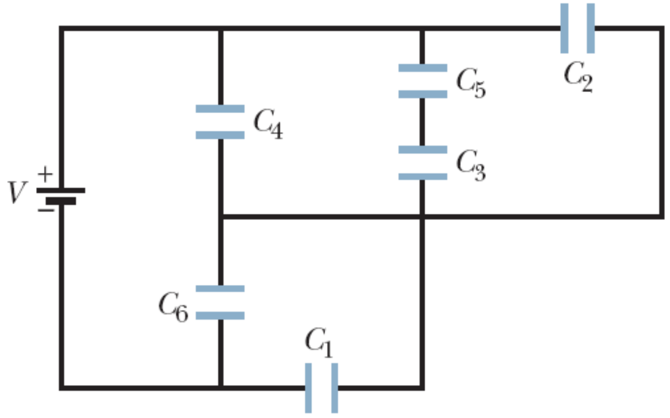


Figure 25-31 Problem 15.

15. (a) First, the equivalent capacitance of the two 4.00 μF capacitors connected in series is given by 4.00 μF/2 = 2.00 μF. This combination is then connected in parallel with two other 2.00-μF capacitors (one on each side), resulting in an equivalent capacitance C = 3(2.00 μF) = 6.00 μF. This is now seen to be in series with another combination, which consists of the two 3.0-μF capacitors connected in parallel (which are themselves equivalent to $C' = 2(3.00\ \mu F) = 6.00\ \mu F$). Thus, the equivalent capacitance of the circuit is

$$C_{eq} = \frac{CC'}{C+C'} = \frac{(6.00\,\mu F)(6.00\,\mu F)}{6.00\,\mu F + 6.00\,\mu F} = 3.00\,\mu F.$$

(b) Let $V = 20.0$ V be the potential difference supplied by the battery. Then

$$q = C_{eq}V = (3.00\ \mu F)(20.0\ V) = 6.00 \times 10^{-5}\ C.$$

(c) The potential difference across C_1 is given by

$$V_1 = \frac{CV}{C+C'} = \frac{(6.00\,\mu F)(20.0 V)}{6.00\,\mu F + 6.00\,\mu F} = 10.0 V.$$

(d) The charge carried by C_1 is $q_1 = C_1V_1 = (3.00\ \mu F)(10.0\ V) = 3.00 \times 10^{-5}$ C.

(e) The potential difference across C_2 is given by $V_2 = V - V_1 = 20.0\ V - 10.0\ V = 10.0$ V.

25-4 Energy Stored in an Electric Field (2 of 3)

The **electric potential energy** U of a charged capacitor,

$$U = \frac{q^2}{2C} \quad \text{(potential energy).}$$

$$dW = V'\,dq' = \frac{q'}{C}\,dq'$$

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

and $$U = \tfrac{1}{2}CV^2 \quad \text{(potential energy).}$$

is equal to the work required to charge the capacitor. This energy can be associated with the capacitor's electric field $\vec{E}$.

25-4 Energy Stored in an Electric Field (3 of 3)

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

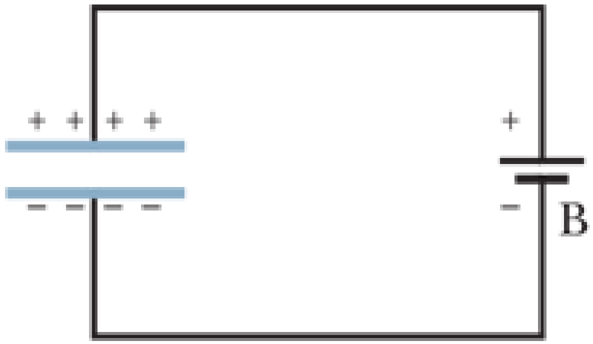
Every electric field, in a capacitor or from any other source, has an associated stored energy. In vacuum, the **energy density** u (potential energy per unit volume) in a field of magnitude E is

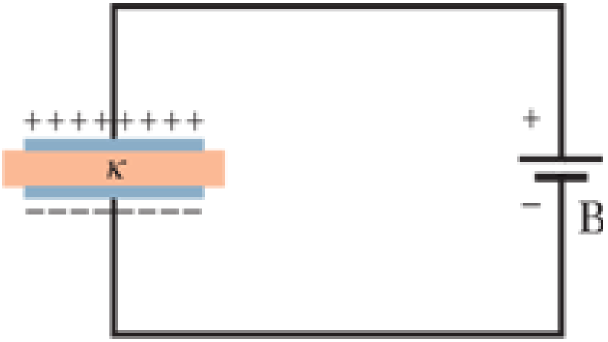
$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \quad \text{(energy density)}.$$

$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor)}.$$

25-5 Capacitor with a Dielectric (3 of 7)





If the space between the plates of a capacitor is completely filled with a **dielectric material (介电材料)**, the capacitance C in vacuum (or, effectively, in air) is

multiplied by the material's **dielectric constant** （介电常数）κ,

(Greek kappa) which is a number greater than 1.

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing

ε_0 with $\kappa\varepsilon_0$. $\mathrm{C}_{\mathrm{rel}} = \kappa C_{air}$.

25-5 Capacitor with a Dielectric (3 of 7)

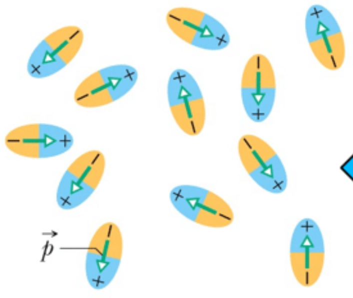
Table 25-1 Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8
For a vacuum, κ = unity.		

[a]Measured at room temperature, except for the water.

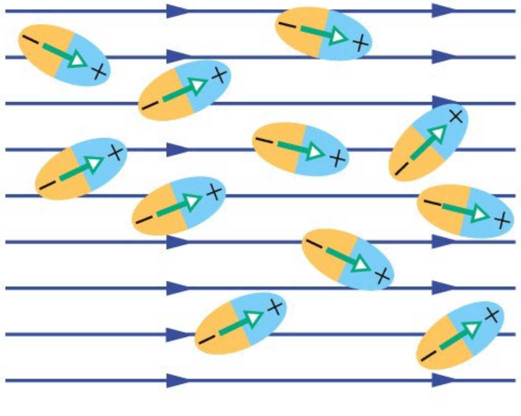
25-5 Capacitor with a Dielectric (6 of 7)

Dielectric : **An Atomic View**



Polar Dielectrics

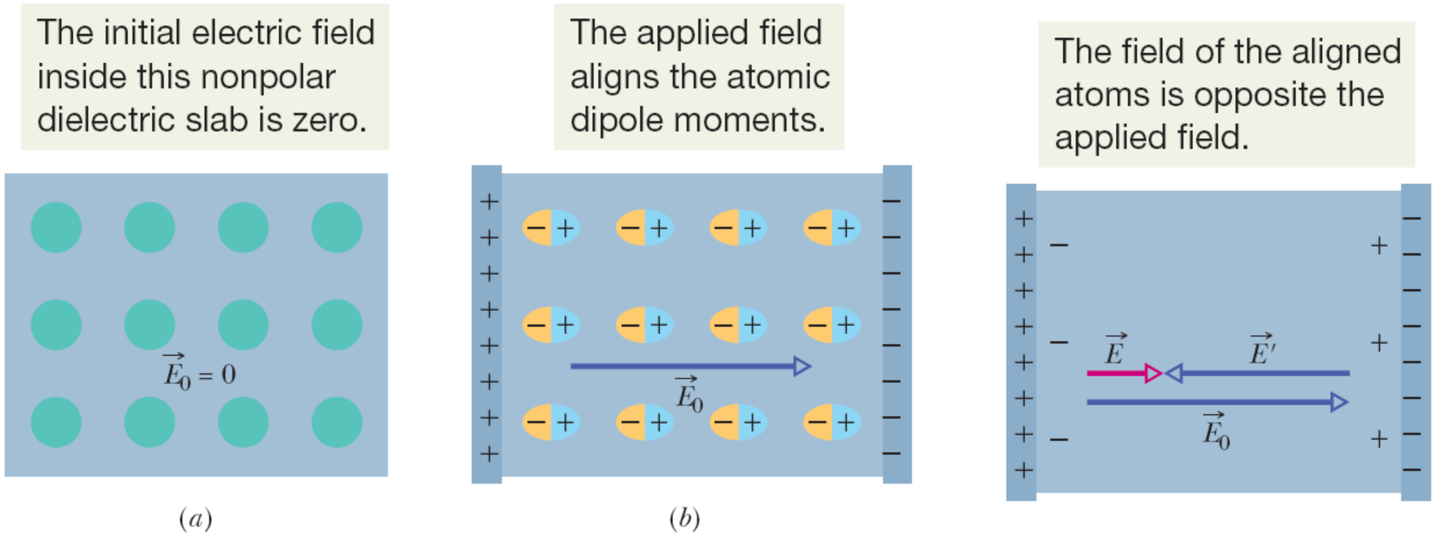
The molecules of some dielectrics, like water, have permanent electric dipole moments. eg. Water



(a) Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field.

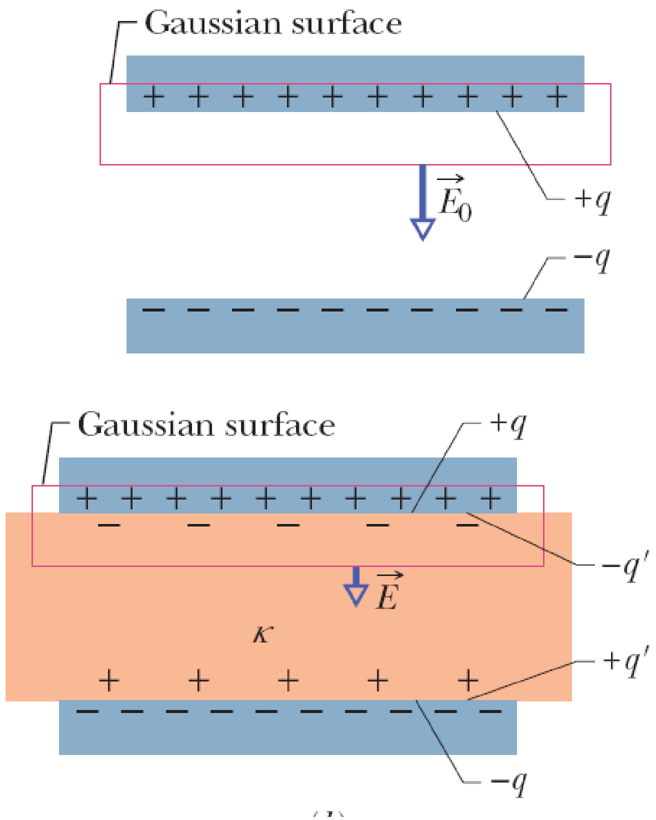
(b) An electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.

25-5 Capacitor with a Dielectric (7 of 7)



the effect of both polar and nonpolar dielectrics is to weaken any applied field within them, as between the plates of a capacitor.

25-6 Dielectrics and Gauss' Law (2 of 4)



- Inserting a dielectric into a capacitor causes induced charge to appear on the faces of the dielectric and weakens the electric field between the plates.
- The induced charge is less than the free charge on the plates.

When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \Longrightarrow \varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad \text{(Gauss' law with dielectric).}$$

where q is the free charge. Any induced surface charge is accounted for by including the dielectric constant κ inside the integral.

25-6 Dielectrics and Gauss' Law (3 of 4)

Note:

The flux integral now involves $\kappa\vec{E}$, not just $\vec{E}$. The vector $\varepsilon_0\kappa\vec{E}$ is sometimes called the electric displacement $\vec{D}$, so that the above equation can be written in the form

$$\varepsilon_0\kappa\vec{E} = \mathrm{D}$$

$$\varepsilon_0 \oint \kappa\vec{E} \cdot d\vec{A} = q \quad \text{(Gauss' law with dielectric).} \quad \Longrightarrow \quad \oint \vec{D} \cdot d\vec{A} = q.$$

25 Summary (1 of 4)

Capacitor and Capacitance

- The capacitance of a capacitor is defined as:

$$q = CV$$

Equation (25-1)

Determining Capacitance

- Parallel-plate capacitor:

$$C = \frac{\varepsilon_0 A}{d}.$$

Equation (25-9)

- Cylindrical Capacitor:

$$C = 2\pi\varepsilon_0 \frac{L}{\ln\left(\frac{b}{a}\right)}.$$

Equation (25-14)

25 Summary (2 of 4)

- Spherical Capacitor:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}.$$

Equation (25-17)

- Isolated sphere:

$$C = 4\pi\varepsilon_0 R.$$

Equation (25-18)

Capacitor in parallel and series

- In parallel:

$$C_{\text{eq}} = \sum_{j=1}^{n} C_j$$

Equation (25-19)

25 Summary (3 of 4)

- In series

$$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

Equation (25-20)

Potential Energy and Energy Density

- Electric Potential Energy (U):

$$U = \frac{q^2}{2C} = \tfrac{1}{2} C V^2$$

Equation (25-21&22)

- Energy density (u)

$$u = \tfrac{1}{2} \varepsilon_0 E^2.$$

Equation (25-25)

25 Summary (4 of 4)

Capacitance with a Dielectric

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ, called the dielectric constant, which is characteristic of the material.

Gauss' Law with a Dielectric

- When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad \text{(Gauss' law with dielectric)}$$

Equation (25-36)

Figure 25-34 Problem 19.

••20 Figure 25-35 shows a variable "air gap" capacitor for manual tuning. Alternate plates are connected together; one group of plates is fixed in position, and the other group is capable of rotation. Consider a capacitor of $n = 8$ plates of alternating polarity, each plate having area $A = 1.25\ \text{cm}^2$ and separated from adjacent plates by distance $d = 3.40$ mm. What is th maximum capacitance of the device?

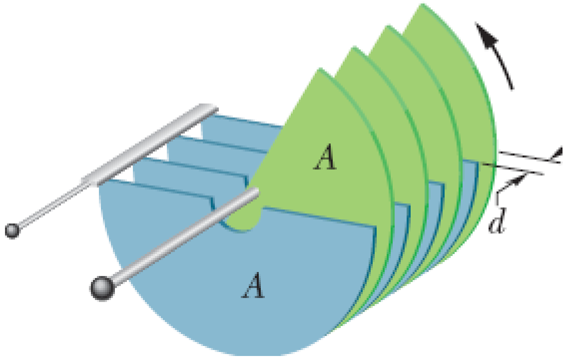


Figure 25-35 Problem 20.

$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor).}$$

20. For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of (n – 1) identical single capacitors connected in parallel. Each capacitor has surface area A and plate separation d so its capacitance is given by $C_0 = \varepsilon_0 A/d$. Thus, the total capacitance of the combination is

$$C = (n-1)C_0 = \frac{(n-1)\varepsilon_0 A}{d} = \frac{(8-1)(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(1.25\times10^{-4}\ \mathrm{m^2})}{3.40\times10^{-3}\ \mathrm{m}} = 2.28\times10^{-12}\,\mathrm{F}.$$