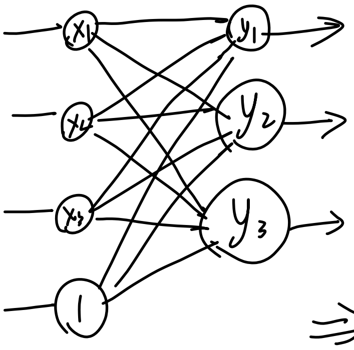
Affine space and its generation



W_{11}: weight from x_1 to y_1, b: weight from 1 to y_1

$$y_1 = W_{11}x_1 + W_{21}x_2 + W_{31}x_3 + b_1$$
$$y_2 = W_{12}x_1 + W_{22}x_2 + W_{32}x_3 + b_2$$
$$y_3 = W_{13}x_1 + W_{23}x_2 + W_{33}x_3 + b_3$$

$\Rightarrow$ compact form : $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} W_{11} & W_{21} & W_{31} \\ W_{12} & W_{22} & W_{32} \\ W_{13} & W_{23} & W_{33} \end{bmatrix}}_{W} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{\vec{b}}$

$\Rightarrow \vec{y} = W\vec{x} + \vec{b} \rightarrow$ affine mapping.

linear mapping

1. Group (群)

Consider a set G and an operation : $\otimes : G \times G \mapsto G$ defined on G. (Cartansian product)

Then, $G : (G, \otimes)$ is called a group if the following holds.

for all

① $\forall x, y \in G$, $x \otimes y \in G$ The set is closed w.r.t. the operations

② $x, y, z \in G$, $(x \otimes y) \otimes z = x \otimes (y \otimes z)$ Associativity

within

③ $\exists e \in G$, s.t. $\forall x \in G$, $x \otimes e = e \otimes x = x$ Identity element

④ $\forall x \in G$, $\exists y \in G$, s.t. $x \otimes y = y \otimes x = e$, Every element has its inverse in its group.

Specifically, G is a Abelian Group, if

① G is a group ; ② $\forall x, y \in G$, $x \otimes y = y \otimes x$ commutativity

Example $(\mathbb{R} \setminus \{0\}, \cdot)$ is Abelian.

① $\forall a, b \in \mathbb{R} \setminus \{0\}$, $a \cdot b \in \mathbb{R} \setminus \{0\}$

② $\forall a, b, c \in \mathbb{R} \setminus \{0\}$, $a \cdot b \cdot c = a \cdot (b \cdot c)$

③ consider $1 \in \mathbb{R} \setminus \{0\}$, $\forall a \in \mathbb{R} \setminus \{0\}$, $a \cdot 1 = 1 \cdot a = a$

④ $\forall a \in \mathbb{R} \setminus \{0\}$, we have $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$

⑤ $\forall a, b \in \mathbb{R} \setminus \{0\}$, $a \cdot b = b \cdot a$

Example : $(\mathbb{R}, \cdot)$ is not a group

④ : for $0 \in \mathbb{R}$, $\exists\, a \in \mathbb{R}$ $a \cdot 0 = 1$

Exercise: $(\mathbb{R}, +)$ is an Abelian Group

① $\forall a, b \in \mathbb{R}$, $a+b \in \mathbb{R}$

② $\forall a, b, c \in \mathbb{R}$, $a+b+c = a+(b+c)$

③ consider $0 \in \mathbb{R}$, $\forall a \in \mathbb{R}$, $a+0=0+a=a$

④ $\forall a \in \mathbb{R}$, we have $a+(-a) = -a+a = 0$

⑤ $\forall a, b \in \mathbb{R}$, $a+b = b+a$

2. Vector space

A real-valued vector space $V = (\mathcal{V}, +, \cdot)$ is a set with two operations.

1) $+ : V \times V \longrightarrow V$ 2) $\cdot :$ $\mathbb{R} \times V \longrightarrow V$

if: 1) $(V, +)$ is a Abelian group

2) distributivity (over $\mathbb{R}$) holds

① $\forall \lambda \in \mathbb{R}$, $\vec{x}, \vec{y} \in \mathcal{V}$, $\lambda(\vec{x}+\vec{y}) = \lambda\vec{x} + \lambda\cdot\vec{y}$

② $\forall \lambda, \psi \in \mathbb{R}$, $\vec{x} \in \mathcal{V}$, $(\lambda+\psi)\vec{x} = \lambda\vec{x} + \psi\cdot\vec{x}$

3) $\forall \lambda, \psi \in \mathbb{R}$, $\vec{x} \in V$: $\lambda(\psi \cdot \vec{x}) = (\lambda\psi)\vec{x}$

4) $\forall \vec{x} \in V$: $1 \cdot \vec{x} = \vec{x}$