ACM ICPC - Code Notebook

David Batista david.batista3010@gmail.com Federal University of Itajuba - Brazil

 $March\ 20,\ 2017$

Contents

		. ~.						_
1		ta Structure						3
	1.1	Segment Tree						3
		1.1.1 Segment Tree & Lazy Propaga						3
		1.1.2 Quadtree						4
		1.1.3 Mergesort Segtree			 			5
		1.1.4 Persistent Segtree			 			6
	1.2	Fenwick Tree						7
		1.2.1 Fenwick Tree 1D			 	 		7
		1.2.2 Fenwick Tree 2D			 	 		8
	1.3	Cartesian Tree			 			9
		1.3.1 Cartesian Tree			 			9
		1.3.2 Implicit Cartesian Tree			 	 		11
	1.4	Merge Sort & Swap Count			 	 		14
		1.4.1 Merge Sort & Vector			 	 		14
		1.4.2 Merge Sort			 			15
	1.5	Sparse Table			 			15
	1.6	SQRT Decomposition			 			16
		1.6.1 Array			 	 		16
		1.6.2 Tree			 	 		17
2	Gra	-						20
	2.1	Components						20
		2.1.1 Articulations, Bridges & Cycl						20
		2.1.2 Strongly Connected Compone						20
		2.1.3 Semi-Strongly Connected Con	nponent	s.	 			20
	2.2	Single Source Shortest Path			 	 		20
		2.2.1 Dijkstra			 	 		20
		2.2.2 Bellmanford			 	 		20
	2.3	All Pairs Shortest Path			 			20
		2.3.1 Floyd Warshall			 			20
	2.4	Minimum Spannig Tree			 	 		20
		2.4.1 Kruskal			 	 		20
		2.4.2 Prim			 			20
	2.5	Flow			 			20
		2.5.1 Maximum Bipartite Matching	g		 	 		20
		2.5.2 Maximum Flow			 	 		20
		2.5.3 Minimum Cost Maximum Flo						22
		2.5.4 Minimum Cut						24

	2.6	Tree	4
		2.6.1 Lowest Common Ancestor	4
		2.6.2 Centroid Decomposition	4
		2.6.3 Heavy Light Decomposition on Edges	6
		2.6.4 Heavy Light Decomposition on Vertex	9
		2.6.5 All-Pairs Distance Sum	9
	2.7	MISC	
		2.7.1 2-SAT	
3	Dyr	amic Programming 30	0
	3.1	Optimizations	0
		3.1.1 Divide and Counquer	0
		3.1.2 Convex Hull I	
		3.1.3 Convex Hull II	
		3.1.4 Knuth Optimization	
	3.2	Digits	
	3.3	Grundy Numbers	
	ა.ა	Grundy Numbers	+
4	Stri	35 35	5
4	4.1	Hash	
	4.1	KMP	
	4.3	Aho Corasick	
	_		
	4.4	Manacher	
	4.5	Z-Algorithm	
	4.6	Suffix Array & LCP	
	4.7	Suffix Tree	5
5	Mad	nematic 36	c
J	5.1	Prime Numbers	
	0.1		
		5.1.1 Erastotenes Sieve	
		5.1.2 Linear Sieve	
		5.1.3 Miller Rabin	
		5.1.4 BPSW	
		5.1.5 Primality Test	6
	5.2	Chinese Remainder Theorem	6
	5.3	Fast Fourier Transformation	6
	5.4	Modular Math	8
		5.4.1 Multiplicative Inverse	8
		5.4.2 Linear All Multiplicative Inverse	8
	5.5	Gaussian Elimination	8
	5.6	Combinatorics	8
6	Geo	netry 39	9
	6.1	2d Template	9
	6.2	3d Template	9
	6.3	Polygon Template	
	6.4	Convex Hull	
		6.4.1 Graham Scan	
		6.4.2 Monotone Chain	
	6.5	Rotating Calipers	
	0.0		_

6.6	KD Tree		 														39
6.7	Range Tree .		 														39
6.8	Circle Sweep		 														39

Data Structure

1.1 Segment Tree

1.1.1 Segment Tree & Lazy Propagation

```
class segtree
      \textbf{const static int } N{=}100000;
      int tr[4*N], lazy[4*N];
public:
      segtree(){};
void clear()
      {
            \begin{array}{ll} memset(\,tr\;,\;\;0\,,\;\;\mathbf{sizeof}(\,tr\;))\,;\\ memset(\,lazy\;,\;\;0\,,\;\;\mathbf{sizeof}(\,lazy\;))\,; \end{array}
      void build(int no, int l, int r, vector<int>&data)
      {
            \mathbf{i}\,\mathbf{f}\,(\;l =\!\!\!-\!\!\!r\,)
            {
                   tr [no]=data[l];
                  return;
            int nxt=no*2;
            int mid=(l+r)/2;
            build (nxt, 1, mid, data);
build (nxt+1, mid+1, r, data);
tr[no]=tr[nxt]+tr[nxt+1];
      void propagate(int no, int l, int r)
            if (! lazy [no])
                  return;
            tr[no]+=(r-l+1)*lazy[no];
            if(l!=r)
            {
                   int nxt=no*2;
                   lazy[nxt]+=lazy[no];
                   lazy[nxt+1]+=lazy[no];
            lazy[no]=0;
```

```
void update(int no, int l, int r, int i, int j, int x)
        propagate(no, l, r);
        if(l>j || r<i)
            return;
        if(l>=i && r<=j)
        {
            lazy [no]=x;
            propagate(no, l, r);
            return;
        int nxt=no*2;
        int mid=(l+r)/2;
        update(nxt, 1, mid, i, j, x);
        update(nxt+1, mid+1, r, i, j, x);
        tr[no] = tr[nxt] + tr[nxt+1];
   int query(int no, int l, int r, int i, int j)
        propagate(no, l, r);
        if(l>j || r<i)
            return 0;
        if(l>=i && r<=j)
            return tr[no];
        int nxt=no*2;
        int mid=(l+r)/2;
        int ql = query(nxt, l, mid, i, j);
        int qr = query(nxt+1, mid+1, r, i, j);
        return (ql+qr);
   }
};
```

1.1.2 Quadtree

```
class quadtree
     //needs to be NxN
     const static int N=100000;
     int tr[16*N];
public:
     quadtree(){};
     void build (int node, int l1, int r1, int l2, int r2, vector < vector < int > data)
     {
           if(11=12 \&\& r1=r2)
           {
                tr [node]=data[l1][r1];
                return;
          int nxt=node * 4;
          int midl=(11+12)/2;
          int midr = (r1+r2)/2;
           \texttt{build} \, (\, \texttt{nxt} \, - 2 \,, \ \texttt{l1} \,\,, \ \ \texttt{r1} \,\,, \ \ \texttt{midl} \,\,, \ \ \texttt{midr} \,\,, \ \ \texttt{data} \,) \,;
           build (nxt-1, midl+1, r1, l2, midr, data);
build (nxt, l1, midr+1, midl, r2, data);
           build(nxt+1, midl+1, midr+1, 12, r2, data);
           tr[node] = tr[nxt-2] + tr[nxt-1] + tr[nxt] + tr[nxt+1];
     void update(int node, int l1, int r1, int l2, int r2, int i, int j, int x)
```

```
if(l1>l2 || r1>r2)
            return;
            if(i==l1 && i==l2 && j==r1 && j==r2)
                  tr[node]=x;
                  return;
            int nxt=node*4;
            int midl=(11+12)/2;
            int midr=(r1+r2)/2;
            update(nxt-2, l1, r1, midl, midr, i, j, x);
            update(nxt-1, midl+1, r1, l2, midr, i, j, x);
update(nxt, l1, midr+1, midl, r2, i, j, x);
            update(nxt+1, midl+1, midr+1, l2, r2, i, j, x);
            tr[node] = tr[nxt-2] + tr[nxt-1] + tr[nxt] + tr[nxt+1];
     int query(int node, int l1, int r1, int l2, int r2, int i1, int j1, int i2, int j2)
      {
            if(i1>12 || j1>r2 || i2<11 || j2<r1 || i1>i2 || j1>j2)
                  return 0;
            if(i1<=l1 && j1<=r1 && l2<=i2 && r2<=j2)
                 return tr[node];
            int nxt=node * 4;
            int midl=(11+12)/2;
            int midr=(r1+r2)/2;
            int q1=query(nxt-2, l1, r1, midl, midr, i1, j1, i2, j2);
int q2=query(nxt-1, midl+1, r1, l2, midr, i1, j1, i2, j2);
int q3=query(nxt, l1, midr+1, midl, r2, i1, j1, i2, j2);
int q4=query(nxt+1, midl+1, midr+1, l2, r2, i1, i2, i2, i2);
            \mathbf{int} \ \ \mathbf{q4} \!\!=\!\! \mathbf{query} \, (\, \mathbf{nxt} \! + \! 1, \ \mathbf{midl} \! + \! 1, \ \mathbf{midr} \! + \! 1, \ \mathbf{l2} \, , \ \mathbf{r2} \, , \ \mathbf{i1} \, , \ \mathbf{j2} \, , \ \mathbf{i2} \, , \ \mathbf{j2} \, ) \, ;
};
```

1.1.3 Mergesort Segtree

```
class mergesort_segtree
{
                      const static int N=100000;
                      vector < int > tr[4*N];
public:
                      mergesort_segtree(){};
                       void build(int no, int l, int r, vector<int>&data)
                        {
                                               if ( l==r )
                                               {
                                                                      tr[no].push_back(data[l]);
                                                                    return;
                                              int nxt=no * 2;
                                              int mid=(l+r)/2;
                                               build(nxt, l, mid, data);
                                               build(nxt+1, mid+1, r, data);
                                               tr[no].resize(tr[nxt].size()+tr[nxt+1].size());
                                              merge(tr[nxt].begin(), tr[nxt].end(), tr[nxt+1].begin(), tr[nxt+1].end(), tr[no].begin(), tr
```

```
//how many numbers in (i, j) are greater or equal than k
int query(int no, int l, int r, int i, int j, int k)
{
    if(r<i || l>j)
        return 0;
    if(l>=i && r<=j)
        return (int)(tr[no].end()-upper_bound(tr[no].begin(), tr[no].end(), k));
    int nxt=no*2;
    int mid=(l+r)/2;
    int ql=query(nxt, l, mid, i, j, k);
    int qr=query(nxt+1, mid+1, r, i, j, k);
    return ql+qr;
};
};</pre>
```

1.1.4 Persistent Segtree

```
class persistent_segtree
     \textbf{const static int } N{=}100000;
    int n;
    int tr[N];
    int root [N], L[N], R[N];
    int cnt, id;
public:
    persistent_segtree(){};
     void set(int _n)
          memset(tr, 0, sizeof(tr));
         memset(root, 0, sizeof(root));
         memset(L, 0, sizeof(L));
memset(R, 0, sizeof(R));
         id = 0:
          cnt=1;
         n=_n;
    void build(int no, int l, int r, vector<int>&data)
          if ( l==r )
          {
               tr [no]=data[1];
               return;
          int mid=(l+r)/2;
          L[no] = cnt ++;
         R[no] = cnt + +;
         build (L[no], l, mid, data);
build (R[no], mid+1, r, data);
tr[no]=tr[L[no]]+tr[R[no]];
     int update(int no, int l, int r, int i, int x)
          int newno=cnt++;
          tr[newno]=tr[no];
         L[newno]=L[no];
         R[newno]=R[no];
          if ( l==r )
          {
               tr[newno]=x;
               \textbf{return} \ \text{newno};\\
```

```
int mid=(l+r)/2;
              if(i \le mid)
                     L[newno]=update(L[newno], l, mid, i, x);
               \begin{array}{c} R[\,\mathrm{newno}] \!=\! \mathrm{update} \left( R[\,\mathrm{newno}\,] \;,\; \mathrm{mid} \!+\! 1,\; r\;,\; i\;,\; x \,\right); \\ \mathrm{tr}\left[\,\mathrm{newno}\right] \!=\! \mathrm{tr}\left[\; L[\,\mathrm{newno}\,] \;\;] \!+\! \mathrm{tr}\left[\; R[\,\mathrm{newno}\,] \;\;\right]; \end{array} 
              return newno;
       int query(int no, int l, int r, int i, int j)
              if(r < i \mid \mid l > j)
                     return 0;
              if(l>=i && r<=j)
                    return tr[no];
              \begin{array}{ll} \textbf{int} & \min = (l+r)/2; \\ \textbf{int} & ql = query\left(L \begin{bmatrix} no \end{bmatrix}, \ l \ , \ \min d, \ i \ , \ j \ \right); \end{array}
              int qr=query(R[no], mid+1, r, i, j);
              return ql+qr;
       //update the i-th value to x.
      void update(int i, int x)
              root[id+1]=update(root[id], 0, n-1, i, x);
       //returns sum(l, r) after the k-th update.
      int query(int 1, int r, int k)
              return query (root [k], 0, n-1, l, r);
      }
};
```

1.2 Fenwick Tree

1.2.1 Fenwick Tree 1D

```
class fenwicktree
   #define D(x) x&(-x)
   const static int N=100000;
   int tr[N], n;
public:
    fenwicktree(){};
    void build(int _n)
        memset(tr, 0, sizeof(tr));
    void update(int i, int x)
        for(i++; i<=n; i+=D(i))
            tr[i]+=x;
   int query(int i)
   {
        int ret = 0;
        for (i++; i>0; i-=D(i))
           ret+=tr[i];
        return ret;
```

```
    int rquery(int 1, int r)
    {
        return query(r)-query(l-1);
    }
    void set(int i, int x)
    {
          update(i, -rquery(i, i)+x);
    }
    void rset(int l, int r, int x)
    {
          update(l, x);
          update(r+1, -x);
    }
};
```

1.2.2 Fenwick Tree 2D

```
class fenwicktree
    #define D(x) x&(-x)
    const static int N=1000;
    int tr[N][N], n, m;
public:
    fenwicktree(){};
    void build(int _n, int _m)
        n=_n, m=_m;
        memset(tr, 0, sizeof(tr));
    void update(int r, int c, int x)
        for(int i=r+1; i<=n; i+=D(i))</pre>
            for(int j=c+1; j<=m; j+=D(j))
                 tr[i][j]+=x;
    int query(int r, int c)
        int ret = 0;
        for (int i=r+1; i>0; i-D(i))
            for(int j=c+1; j>0; j=D(j))
ret+=tr[i][j];
        return ret;
    int rquery(int r1, int c1, int r2, int c2)
        if ((r1>r2 && c1>c2) || (r1=r2 && c1>c2) || (r1>r2 && c1=c2))
            swap(r1, r2);
            swap(c1, c2);
        else if(r1<r2 && c1>c2)
            swap(c1,c2);
        else if (r1>r2 && c1<c2)
        {
            swap(r1, r2);
        return query (r2, c2)-query (r1-1, c2)-query (r2, c1-1)+query (r1-1, c1-1);
```

```
}
    void set(int r, int c, int x)
    {
        update(r, c, -rquery(r, c, r, c)+x);
    }
};
```

1.3 Cartesian Tree

1.3.1 Cartesian Tree

```
//srand(time(NULL))
int vrand()
{
      return abs(rand() << (rand() \%31));
{f struct} node
      //x=key, y=priority key, c=tree count
     int x, y, c;
node *L, *R;
     node(){};
      node(int _x)
            \begin{array}{ll} x \!\!=\!\! \_x \;, \;\; y \!\!=\!\! v \, rand \, (\,) \;, \;\; c \!\!=\!\! 0; \\ L \!\!=\!\! R \!\!=\!\! NULL; \end{array}
      }
};
int cnt(node *root)
      return root?root\rightarrowc:0;
void upd_cnt(node *root)
{
      if (root)
           root \rightarrow c=1+cnt(root \rightarrow L)+cnt(root \rightarrow R);
void split (node *root, int x, node *&L, node *&R)
      if (!root)
           L=R=NULL;
      else if (x < root \rightarrow x)
            split (root->L, x, L, root->L), R=root;
            split(root->R, x, root->R, R), L=root;
      upd_cnt(root);
\mathbf{void} \ \mathtt{insert} \, (\, \mathtt{node} \ *\&\mathtt{root} \; , \ \mathtt{node} \ *\mathtt{it} \, )
      if (!root)
           root=it;
      else if(it -> y > root -> y)
           split(root, it \rightarrow x, it \rightarrow L, it \rightarrow R), root=it;
```

```
insert(it->x < root->x? root->L:root->R, it);
    upd_cnt(root);
}
void merge(node *&root , node *L, node *R)
    if (!L || !R)
         root=L?L:R;
    \mathbf{else} \ \mathbf{if} (L\!\!-\!\!>\!\! y > R\!\!-\!\!>\!\! y)
         merge(L\rightarrow R, L\rightarrow R, R), root=L;
         merge\,(R\!\!-\!\!>\!\!L\,,\ L\,,\ R\!\!-\!\!>\!\!L\,)\,,\ root\!=\!\!R;
    upd_cnt(root);
void erase(node *&root, int x)
    if(root->x=x)
        merge(root, root->L, root->R);
    _{
m else}
         erase(x < root \rightarrow x? root \rightarrow L: root \rightarrow R, x);
    upd_cnt(root);
node *unite(node *L, node *R)
    if (!L || !R)
         return L?L:R;
    i\,f\,(L\!\!-\!\!>\!\!y\,<\,R\!\!-\!\!>\!\!y\,)
         swap(L, R);
    L\rightarrow R=unite(L\rightarrow R, Rt);
    return L;
int find(node *root, int x)
{
    if (!root)
         return 0;
     if(root->x=x)
         return 1;
    if(x > root -> x)
         return find (root->R, x);
    _{
m else}
         return find(root->L, x);
int findkth(node *root, int x)
{
    if (!root)
         return -1;
    int Lc=cnt(root->L);
    if(x-Lc-1==0)
         return root ->x;
    if (x>Lc)
         else
         return findkth(root->L, x);
```

1.3.2 Implicit Cartesian Tree

```
//srand(time(NULL))
int vrand()
    return abs(rand()<<(rand()%31));
struct node
    //basic treap: x=key, y=priority key, c=tree count;
    int x, y, c;
    //treap operations: v=max(x), lazy=lazy value of propagation, rev=reversed
    int v, lazy, rev;
    node *L, *R;
    node(){};
    node(int _x)
         x=_x, y=vrand();
         L=R=NULL;
         v=x;
         lazy=0;
         \scriptstyle \operatorname{rev} = 0;
    }
};
//updating functions
inline int get_cnt(node *root)
    return root?root ->c:0;
inline void upd_cnt(node *root)
{
    if (root)
         root->c=1+get_cnt(root->L)+get_cnt(root->R);
inline void push(node *&root)
    if(root && root->rev)
         root \rightarrow rev = 0;
         swap(root->L, root->R);
         if (root->L)
             root \rightarrow L \rightarrow rev = 1;
         if(root->R)
             root \rightarrow R \rightarrow rev = 1;
    }
inline void propagate(node *&root)
    if (root)
         if (!root->lazy)
             return;
         int lazy=root->lazy;
         root \rightarrow x + = lazy;
         if(root->L)
```

```
root \rightarrow L \rightarrow lazy = lazy;
            if(root->R)
                 root->R->lazy=lazy;
           root \rightarrow lazy = 0;
     }
}
inline int get_max(node *root)
     return root?root \rightarrow v:-INF;
inline void upd_max(node *root)
     if (root)
           root \rightarrow v=max(root \rightarrow x, max(get_max(root \rightarrow L), get_max(root \rightarrow R)));
inline void update(node *root)
     propagate(root);
     upd_cnt(root);
     upd_max(root);
void merge(node *&root , node *L, node *R)
     push(L);
     push(R);
     if (!L || !R)
           root=L?L:R;
     \begin{array}{c} \textbf{else} \ \ \textbf{if} \ (L\!\!\rightarrow\!\! y \ > \ R\!\!\rightarrow\!\! y) \\ \ \ \ \text{merge} \ (L\!\!\rightarrow\!\! R, \ L\!\!\rightarrow\!\! R, \ R) \ , \ \ \text{root=}L; \end{array}
           merge(R->L, L, R->L), root=R;
     update(root);
}
\mathbf{void} \ \mathrm{split} \, (\, \mathrm{node} \ *\mathrm{root} \, , \ \mathrm{node} \ *\&\mathrm{L}, \ \mathrm{node} \ *\&\mathrm{R}, \ \mathbf{int} \ \mathrm{x} \, , \ \mathbf{int} \ \mathrm{add} \! = \! 0)
     if (!root)
           return void(L=R=NULL);
     push(root);
     int ix=add+get_cnt(root->L); //implicit key
     if(x \le ix)
           split (root->L, L, root->L, x, add), R=root;
           split (root->R, root->R, R, x, add+1+get_cnt(root->L)), L=root;
     update(root);
//insert function
void insert (node *&root, int pos, int x)//(insert x at position pos)
     node *R1, *R2;
     split (root, R1, R2, pos);
     merge(R1, R1, new node(x));
     merge(root, R1, R2);
//erase value x
void erase_x(node *&root, int x)
```

```
if (!root)
            return;
      if(root->x==x)
           merge(root, root->L, root->R);
      else
            erase_x(x < root \rightarrow x? root \rightarrow L: root \rightarrow R, x);
      update(root);
//erase kth value
void erase_kth(node *&root, int x)
{
      if (!root)
           return;
      int Lc=get_cnt(root->L);
      if(x-Lc-1==0)
           merge(root, root \rightarrow L, root \rightarrow R);
      else if(x>Lc)
           erase_kth(root->R, x-Lc-1);
           erase_kth(root->L, x);
      update(root);
//add x to [1, r]
inline void paint (node *&root, int 1, int r, int x)
      node *R1, *R2, *R3;
split(root, R1, R2, 1);
      split (R2, R2, R3, r-l+1);
      R2->lazy=x;
      propagate(R2);
      \begin{array}{ll} merge (\hspace{.05cm} \texttt{root} \hspace{.1cm}, \hspace{.1cm} R1 \hspace{.05cm}, \hspace{.1cm} R2 \hspace{.05cm}) \hspace{.05cm}; \\ merge (\hspace{.05cm} \texttt{root} \hspace{.1cm}, \hspace{.1cm} \texttt{root} \hspace{.1cm}, \hspace{.1cm} R3) \hspace{.05cm}; \end{array}
}
//max range query [l,r]
inline int rquery(node *&root, int 1, int r)
      node *R1, *R2, *R3;
      split (root, R1, R2, l);
split (R2, R2, R3, r-l+1);
      int ret=R2->v;
      merge(root, R1, R2);
      merge(root, root, R3);
      return ret;
inline void reverse(node *&root, int 1, int r)//reverse elements [1, r]
      node *R1, *R2, *R3;
split(root, R1, R2, 1);
      split (R2, R2, R3, r-l+1);
      R2 \rightarrow rev^=1;
      merge(root, R1, R2);
merge(root, root, R3);
//output functions
int poscnt=0;
```

```
void output_all(node *root)
     if (!root)
          return;
     update(root);
     push(root);
     output_all(root->L);
     printf("[%d]_%d\n", poscnt++, root->x);
output_all(root->R);
int output_kth(node *root, int x)
{
     if (!root)
          return -1;
     update(root);
     push(root);
     int Lc=get_cnt(root->L);
     \mathbf{i} \mathbf{f} \left( \mathbf{x} - \mathbf{L} \mathbf{c} - 1 = = 0 \right)
          return root ->x;
     if(x>Lc)
          return output_kth(root->R, x-Lc-1);
          return output_kth(root->L, x);
```

1.4 Merge Sort & Swap Count

1.4.1 Merge Sort & Vector

```
#define INF 0x3F3F3F3F
int mergesort (vector<int>&data)
      if(data.size()==1)
            return 0;
      vector<int>L, R;
      \mathbf{int} \ \ \mathbf{t} {=} \mathbf{data.\, size} \ (\,) \, ;
      for (int i=0; i < t/2; i++)
           L. push_back (data [i]);
      for (int i=t/2; i < t; i++)
           R. push_back(data[i]);
      int ret=mergesort(L)+mergesort(R);
      \label{eq:formula} \mbox{for}(\,\mbox{int}\ i\!=\!0,\ j\!=\!0,\ k\!=\!0;\ j\!<\!\!L.\,size\,(\,)\ |\,|\ k\!<\!\!R.\,size\,(\,)\,;\ i\!+\!+\!)
            int x=j<L.size()?L[j]:INF;
            int y=k<R. size()?R[k]:INF;
            \mathbf{i}\,\mathbf{f}\,(\,x{<}y\,)
                  data[i]=x;
                  j++;
            }
            else
                  data[i]=y;
                  k++;
                  ret += (L.size()-j);
      {\bf return} \ {\rm ret} \ ;
```

}

1.4.2 Merge Sort

```
#define INF 0x3F3F3F3F
  int temp[100000];
  int mergesort(int data[], int l, int r)
                                {\bf i}\,{\bf f}\,(\,a\,b\,s\,(\,l\,{-}r\,)\!<\!=\!1)
                                                            return 0;
                               int mid=(l+r)/2;
                               \mathbf{int} \ \mathtt{ret} \! = \! \mathtt{mergesort} \, (\, \mathtt{data} \, , \ l \, , \ \mathtt{mid}) \! + \! \mathtt{mergesort} \, (\, \mathtt{data} \, , \ \mathtt{mid} \, , \ r \, ) \, ;
                              for (int i=1; i<r; i++)
temp[i]=data[i];
                               \begin{tabular}{ll} \beg
                                                            \quad \textbf{int} \ x{=}j{<}mid?temp\left[\ j\ \right]{:}INF\,;
                                                            int y=k< r?temp[k]:INF;
                                                              if(x < y) //x < = y
                                                                                           data[i]=x;
                                                                                          j++;
                                                            else
                                                              {
                                                                                           data[i]=y;
                                                                                           ret += (mid-j);
                               {\bf return} \ {\rm ret} \ ;
```

1.5 Sparse Table

```
{
    int i=abs(l-r)+1;
    int j=lbit(i);
    return max(data[1][j], data[1-(1<<j)+1][j]);
};
};</pre>
```

1.6 SQRT Decomposition

1.6.1 Array

```
const int N=100000;
int SN=sqrt(N);
{f class} mo
public:
    int l , r , i ;
mo(){};
    mo(int _l , int _r , int _i)
          l=_{-}l , r=_{-}r , i=_{-}i ;
     bool operator <(const mo &foo) const
           if ((r/SN)!=(foo.r/SN))
                return (r/SN) < (foo.r/SN);
           if (1!=foo.1)
               return ĺ<foo.l;
           \textbf{return} \hspace{0.2cm} i \!<\! foo.i;
     }
};
\mathbf{int}\ \mathrm{data}\left[N\right],\ \mathrm{freq}\left[N\right],\ \mathrm{ans}\left[N\right];
int cnt=0;
void update(int p, int s)
     int x=data[p];
     if(s==1)
           if(freq[x]==0)
                cnt++;
     else
           if(freq[x]==1)
                cnt --;
     freq[x]+=s;
int main()
     int n;
scanf("%d", &n);
     for(int i=1; i<=n; i++) scanf("%d", &data[i]);
     int q;
```

```
scanf("%d", \&q);
vector <mo>querys;
\  \, \mathbf{for} \, (\, \mathbf{int} \quad i \,{=}\, 0\,; \quad i \,{<} q\,; \quad i \,{+}{+})
{
     int l, r;
scanf("%d_%d", &l, &r);
      querys.push_back(mo(l, r, i));
sort(querys.begin(), querys.end());
int l=1, r=1;
cnt=0;
memset(freq , 0, sizeof(freq));
update(1, 1);
for (int i=0; i < q; i++)
     \mathbf{int} \ \ \mathtt{li=querys}\left[ \ i \ \right]. \ \mathtt{l};
      int ri=querys[i].r;
      int ii=querys[i].i;
      \mathbf{while}(l > li)
           update(--1, 1);
      \mathbf{while}(\mathbf{r} < \mathbf{r} \mathbf{i})
           update(++r, 1);
      while (l<li)
           update(l++, -1);
      \mathbf{while}(r > ri)
          update(r--, -1);
      ans[ii]=cnt;
for(int i=0; i<querys.size(); i++)
     printf("%d\n", ans[i]);
return 0;
```

1.6.2 Tree

```
#define pb push_back
#define ALL(x) x.begin(),x.end()
const int N=1e+5+35;
const int M=20;
const int SN=sqrt(2*N)+1;
{f class} mo
public:
    int 1, r, i, lc;
mo(){};
    mo(int _l , int _r , int _lc , int _i )
    {
        l=_l , r=_r , lc=_lc , i=_i ;
    bool operator <(const mo &foo) const
         if ((r/SN)!=(foo.r/SN))
            return (r/SN)<(foo.r/SN);
         if(1!=foo.1)
            return l<foo.l;
        return i<foo.i;
```

```
};
\begin{array}{ll} \textbf{int} & n \,, \, q \,; \\ \textbf{int} & h \, [N] \,, \, \, lca \, [N] \, [M] \,; \end{array}
vector < int > g[N];
int dl[N], dr[N], di[2*N], cur;
void dfs(int u, int p)
       dl[u]=++cur;
       di[cur]=u;
       lca[u][0] = p;
       for (int i=1; i\llM; i++)

lca[u][i]=lca[lca[u][i-1]][i-1];
       for(int i=0; i<g[u].size(); i++)
              \quad \textbf{int} \ \ v \hspace{-0.1cm} = \hspace{-0.1cm} g \hspace{.05cm} [\hspace{.05cm} u\hspace{.05cm}] \hspace{.05cm} [\hspace{.05cm} i\hspace{.05cm}] \hspace{.05cm} ;
               \mathbf{i}\,\mathbf{f}\,(\,v\!\!=\!\!\!-\!\!p\,)
                    continue;
              h[v]=h[u]+1;
              dfs(v, u);
       \mathrm{d} \, r \, [\, u] \!\! = \!\! + \!\! + \! c \, u \, r \; ;
       di [cur]=u;
inline int getLca(int u, int v)
       \mathbf{i}\,\mathbf{f}\,(\,h\,[\,u]\!>\!h\,[\,v\,]\,)
              swap(u, v);
       for(int i=M-1; i>=0; i--)
              if (h[v]-(1<<i)>=h[u])
v=lca[v][i];
       \mathbf{i}\,\mathbf{f}\,(\,u\!\!=\!\!\!-\!\!v\,)
              return u;
       for (int i=M-1; i>=0; i--)
       {
               \mathbf{i}\,\mathbf{f}\,(\,l\,c\,a\,[\,u\,]\,[\,\,i\,\,]!\!=\!\,l\,c\,a\,[\,v\,]\,[\,\,i\,\,]\,)
               {
                      u{=}\,l\,c\,a\;[\;u\;]\;[\;i\;]\;;
                     v=lca[v][i];
       return lca[u][0];
int x=data[u];
if(vis[u] && (--freq[ data[u] ]==0))
       else if (! vis [u] && (freq [ data[u] ]++==0))
              cnt++;
       vis[u]^=1;
}
int main()
       scanf("%d_{d}", &n, &q);
       for (int i=1; i <= n; i++)
```

```
char temp [25];
       scanf("%s", temp);
       string temp2=string(temp);
       if(!remap.count(temp2))
             remap[temp2]=remap.size();
       data[i]=remap[temp2];
for (int i=1; i<n; i++)
      \begin{array}{lll} \textbf{int} & u \,, & v \,; \\ s \, c \, a \, n \, f \, (\,\text{``%d..\%d''} \,\,, & \&u \,, & \&v \,) \,; \end{array}
      g[u].pb(v);
g[v].pb(u);
dfs(1, 0);
vector<mo>query;
\  \, \mathbf{for}\,(\,\mathbf{int}\  \  \, i\,{=}0;\  \, i\,{<}q\,;\  \  \, i\,{+}{+})
      int u, v;
scanf("%d_%d", &u, &v);
int lc=getLca(u, v);
       if (dl[u]>dl[v])
             swap(u, v);
       query.pb(mo(u=lc?dl[u]:dr[u], dl[v], lc, i));
sort(ALL(query));
\quad \mathbf{int} \quad l \!=\! \mathbf{query} \; [\; 0\; ] \; . \; l \; , \quad \mathbf{r} \!=\! \mathbf{query} \; [\; 0\; ] \; . \; l \; -1;
cnt = 0;
for (int i=0; i < q; i++)
       int li=query[i].l;
      int ri=query[i].r;
int lc=query[i].lc;
       int ii=query[i].i;
       while(l>li)
             update(di[--1]);
       while (r<ri)
             update(di[++r]);
       while (l<li)
             update(di[l++]);
       \mathbf{while}\,(\,\mathbf{r}\!>\!\mathbf{r}\,\mathbf{i}\,)
             update (di [r--]);
      int u=di[1], v=di[r];
if(lc!=u && lc!=v)
             update(lc);
       ans[ii]=cnt;
       if (lc!=u && lc!=v)
             update(lc);
for(int i=0; i<q; i++)
    printf("%d\n", ans[i]);</pre>
return 0;
```

Graph

- 2.1 Components
- 2.1.1 Articulations, Bridges & Cycles
- 2.1.2 Strongly Connected Components
- 2.1.3 Semi-Strongly Connected Components
- 2.2 Single Source Shortest Path
- 2.2.1 Dijkstra
- 2.2.2 Bellmanford
- 2.3 All Pairs Shortest Path
- 2.3.1 Floyd Warshall
- 2.4 Minimum Spannig Tree
- 2.4.1 Kruskal
- 2.4.2 Prim
- 2.5 Flow
- 2.5.1 Maximum Bipartite Matching
- 2.5.2 Maximum Flow

Dinic

```
class graph
{
    const static int N=100000;
public:
    vector< pair<int, int> >edge;
```

```
vector < int > adj[N];
int ptr[N];
int dist[N];
graph(){};
void clear()
{
     for(int i=0; i<N; i++)
         adj[i].clear();
     edge.clear();
void add_edge(int u, int v, int c)
     adj[u].push_back(edge.size());
     edge.push_back(mp(v, c));
     adj[v].push_back(edge.size());
     edge.push\_back(mp(u,\ 0));\ /\!/(u,\ c)\ if\ is\ non-directed
bool dinic_bfs(int s, int t)
     memset(dist, -1, sizeof(dist));
     dist[s]=0;
     queue<int>bfs;
     bfs.push(s);
     while (! bfs.empty() && dist [t]==-1)
          int u=bfs.front();
          bfs.pop();
          for (int i=0; i<adj[u].size(); i++)
               int idx=adj[u][i];
              int v=edge[idx].F;
               if(dist[v]==-1 \&\& edge[idx].S>0)
                    dist[v] = dist[u] + 1;
                   bfs.push(v);
          }
     return dist [t]! = -1;
int dinic_dfs(int u, int t, int flow)
{
     if ( u==t )
         return flow;
     for(int &i=ptr[u]; i<adj[u].size(); i++)</pre>
          int idx=adj[u][i];
          int v=edge[idx].F;
          \label{eq:if} \textbf{if} \, (\, \text{dist} \, [\, v] == \, \text{dist} \, [\, u] + 1 \, \, \&\& \, \, \text{edge} \, [\, \text{idx} \, ] \, . \, S \! > \! 0)
              int cf = dinic_dfs(v, t, min(flow, edge[idx].S));
              if(cf>0)
              {
                   edge[idx].S-=cf;
edge[idx^1].S+=cf;
                   return cf;
              }
          }
```

```
return 0;
}
int maxflow(int s, int t)
{
    int ret=0;
    while(dinic_bfs(s, t))
    {
        memset(ptr, 0, sizeof(ptr));
        int cf=dinic_dfs(s, t, INF);
        if(cf==0)
            break;
        ret+=cf;
    }
    return ret;
}
```

2.5.3 Minimum Cost Maximum Flow

Dijkstra

```
undirected graph:
u\rightarrow uu(flow, 0)
uu->v(flow, cost)
vv->v(flow, 0)
v \rightarrow xx (flow, 0)
vv->u(flow, 0)
typedef int FTYPE; //type of flow
typedef int CTYPE; //type of cost
typedef pair<FTYPE,CTYPE>pfc;
const CTYPE CINF=INF;
const FTYPE FINF=INF;
\mathbf{void} \ \mathbf{operator} + = (\mathtt{pfc} \ \&\mathtt{p1} \,, \ \mathtt{pfc} \ \&\mathtt{p2})
      p1.F+=p2.F;
      p1.S+=p2.S;
{\bf class} \ {\rm graph}
      const static int MN=1e+4;
public:
      int n;
      FTYPE flow [MN];
CTYPE dist [MN], pot [MN];
int prev [MN], eidx [MN];
      {f struct} Edge
              int to;
             FTYPE cap;
             CTYPE cost;
             Edge(){};
              Edge(int _to , FTYPE _cap , CTYPE _cost )
                     to=_to;
                    cap = \_cap;
```

```
cost = \_cost;
     }//
};
struct node
{
      int u:
     CTYPE d;
     node(){};
     node(int \_u, CTYPE \_d)
      {
            u=_{-}u;
            d=_{-}d;
      bool operator <(const node &foo) const
            return d>foo.d;
};
graph(){};
vector<int>adj [MN];
vector < Edge > edge;
inline void set(int _n)
{
     n=_n;
inline void reset()
      \quad \mathbf{for} \left( \begin{array}{lll} \mathbf{int} & i = 0; & i < \!\! M\! N; & i + \!\! + \!\! ) \end{array} \right.
            adj[i].clear();
      edge.clear();
inline void add_edge(int u, int v, FTYPE c, FTYPE cst)
      adj[u].push_back(edge.size());
      edge.push_back(Edge(v, c, cst));
      adj[v].push_back(edge.size());
      edge.push_back(Edge(u, 0, -cst));
pfc dijkstra(int s, int t)
      for (register int i=0; i< n; i++)
            dist[i]=CINF;
      dist[s]=0;
flow[s]=FINF;
      priority_queue<node>heap;
      heap.push(node(s, 0));
      while (! heap.empty())
            int u=heap.top().u;
            CTYPE d=heap.top().d;
            heap.pop();
            if(d>dist[u])
                  continue;
            for(int i=0; i<adj[u].size(); i++)
            {
                  \begin{array}{ll} \textbf{int} & idx = adj\left[u\right]\left[i\right];\\ \textbf{int} & v = edge\left[idx\right].\ to; \end{array}
                 CTYPE w=edge[idx].cost;
                  \mathbf{if} \; (\,!\, \mathrm{edge}\,[\,\mathrm{id}\,x\,\,]\,.\, \mathrm{cap} \quad |\,| \quad \mathrm{dist}\,[\,v] \!\!<\!\! = \!\! \mathrm{d} \!\!+\!\! w \!\!+\!\! \mathrm{pot}\,[\,u] \!\!-\!\! \mathrm{pot}\,[\,v\,]\,)
                        continue;
                  if(d+w< dist[v])
```

```
dist[v]=d+w;
                       prev[v]=u;
                       eidx[v]=idx;
                       flow [v]=min(flow[u], edge[idx].cap);
                       heap.push(node(v, d+w));
             }
         if (dist[t]==CINF)
             return mp(FINF, CINF);
         pfc ret=mp(flow[t], 0);
for(int u=t; u!=s; u=prev[u])
              int idx=eidx[u];
             edge[idx].cap-=flow[t];
edge[idx^1].cap+=flow[t];
              ret.second+=flow[t]*edge[idx].cost;
         return ret;
    }
    inline pfc mfmc(int s, int t)
         pfc ret=mp(0, 0);
         pfc got;
         while ((got=dijkstra(s, t)). first!=FINF)
             ret+=got;
         return ret;
    }
};
```

Bellmanford

2.5.4 Minimum Cut

2.6 Tree

2.6.1 Lowest Common Ancestor

2.6.2 Centroid Decomposition

```
const int N=1e+5;
const int M=log2(N)+1;

set <int>g[N]; //graph
int h[N]; //heigh of nodes
int trSz[N], sz; //tree subsize, size of current tree
int lca[N][M]; //lca sparse table
int cg[N]; //centroid graph

void dfs(int u, int l)
{
    lca[u][0]=l;
    for(int i=1; i<M; i++)
        lca[u][i]=lca[ lca[u][i-1] ][i-1];
    for(auto v:g[u])
    {
        if (v==1)</pre>
```

```
{\bf continue}\,;
          h\,[\,v\,]\!=\!h\,[\,u\,]\!+\!1\,;
          dfs(v, u);
    }
}
inline int getLca(int u, int v)
     \mathbf{i}\,\mathbf{f}\,(\,h\,[\,u\,]\!>\!h\,[\,v\,]\,)
    swap(u, v);
for(int i=M-1; i>=0; i--)
          if(h[v]-(1<< i)>=h[u])
              v=lca[v][i];
     i f ( u==v )
          return u;
     for (int i=M-1; i>=0; i--)
          if (lca[u][i]!=lca[v][i])
          {
               u{=}\,l\,c\,a\;[\,u\,]\,[\,\,i\,\,]\,;
               v=lca[v][i];
    return lca[u][0];
inline int getDist(int u, int v)
     \textbf{return} \ h\,[\,u\,] + h\,[\,v\,] - 2*h\,[\,\text{getLca}\,(\,u\,,\ v\,)\,]\,;
void centDfs(int u, int l)
    trSz[u]=1;
    sz++;
    for(auto v:g[u])
    {
          if (v==1)
               continue;
          centDfs(v, u);
          trSz[u]+=trSz[v];
int findCentroid(int u, int l)
     for (auto v:g[u])
     {
          if(v=l)
               continue;
          if(trSz[v]*2>=sz)
               return findCentroid(v, u);
    return u;
inline void buildCentroid(int u, int 1)
{
    sz=0;
    centDfs(u, u);
     int c=findCentroid(u, u); //actual centroid
    cg[c]=(u=1?c:1);
```

2.6.3 Heavy Light Decomposition on Edges

```
class segtree
      const static int N=1e+5;
public:
      int tr[4*N];
      segtree(){};
      void reset()
      {
             memset(tr, 0, sizeof(tr));
      void update(int no, int l, int r, int i, int val)
             \boldsymbol{i}\,\boldsymbol{f}\,(\,r{<}i\quad|\,|\quad l{>}i\,)
                   return;
             if(l>=i && r<=i)
                   tr[no] = val;
                   return;
             int nxt = (no << 1);
             int mid=(l+r)>>1;
             update(nxt, l, mid, i, val);
             update(nxt+1, mid+1, r, i, val);
             tr[no] = tr[nxt] + tr[nxt+1];
      int query(int no, int l, int r, int i, int j)
             \mathbf{i}\,\mathbf{f}\,(\,\mathrm{r}{<}\mathrm{i}\ \mid\!\mid\ l{>}\mathrm{j}\,)
                   return 0;
             if(l>=i && r<=j)
                  return tr[no];
             int nxt = (no < <1);
             int mid=(l+r)>>1;
            return query(nxt, 1, mid, i, j)+query(nxt+1, mid+1, r, i, j);
      }
};
const int N=1e+5;
const int M=\log 2(N)+1;
int n;
segtree tr;
vector < pair < int, int > >g[N];
\mathbf{int} \ \operatorname{lca}\left[N\right]\left[M\right];
\mathbf{int}\ h\left[N\right],\ \mathrm{trSz}\left[N\right];
\label{eq:continuous_problem} \begin{split} // &\inf - use \ X[] \ , \ Y[] \ in \ case \\ // &of \ edge \ weights \\ &\inf \ X[N] \ , \ Y[N] \ , \ W[N] \ ; \end{split}
//hld
```

```
\mathbf{int} \ \ \mathsf{chainInd} \ [N] \ , \ \ \mathsf{chainSize} \ [N] \ , \ \ \mathsf{chainHead} \ [N] \ , \ \ \mathsf{chainPos} \ [N] \ , \ \ \mathsf{chainNo} \ , \ \ \mathsf{posInBase} \ [N] \ ;
int ptr;
void dfs(int u, int l)
{
              \operatorname{tr} Sz\;[\;u\,]\!=\!1\,;
              lca[u][0] = 1;
              for (int i=1; i \triangleleft M; i++)
                             lca[u][i]=lca[lca[u][i-1]][i-1];
              for (int i=0; i < g[u]. size(); i++)
              {
                             int v=g[u][i].first;
                              if (v==1)
                                          continue;
                             h\,[\,v\,]\!=\!h\,[\,u\,]\!+\!1\,;
                             dfs(v, u);
trSz[u]+=trSz[v];
              }
}
inline int getLca(int u, int v)
              if (h [u]>h [v])
              swap(u, v);
for(int i=M-1; i>=0; i--)
                              if(h[v]-(1<<i)>=h[u])
                                          v=lca[v][i];
              \mathbf{i} \, \mathbf{f} \, (\mathbf{u} = \mathbf{v})
                              return u;
              for (int i=M-1; i>=0; i--)
                              if (lca[u][i]!=lca[v][i])
                                            u=lca[u][i];
                                           v=lca[v][i];
              return lca[u][0];
//dont use 'c' if the weight is on the vertex
//instead of the edge
inline void hld(int u, int l, int c)
               if (chainHead [chainNo]==-1)
                            chainHead [chainNo]=u;
              chainInd [u]=chainNo;
              chainPos [u]=chainSize [chainNo]++;
              tr.update(1, 0, n, ptr, c);
              posInBase[u]=ptr++;
              int msf, idx;
              msf=idx=-1;
               \begin{tabular}{ll} \be
              {
                              int v=g[u][i].first;
                              i f ( v==1 )
                                           {\bf continue}\,;
                              \mathbf{i}\,\mathbf{f}\,(\,\mathrm{t}\,\mathrm{r}\,\mathrm{S}\,\mathrm{z}\,[\,\mathrm{v}]\!>\!\mathrm{msf}\,)
                                             msf=trSz[v];
                                            idx=i;
```

```
if(idx >= 0)
          hld(g[u][idx].first, u, g[u][idx].second);
     for (int i=0; i < g[u]. size (); i++)
          if(i=idx)
              continue;
          int v=g[u][i].first;
int w=g[u][i].second;
          if (v==1)
               continue;
          chainNo++;
          hld\left( v\,,\ u\,,\ w\right) ;
     }
}
inline int query_up(int u, int v)
     int uchain=chainInd[u];
     int vchain=chainInd[v];
     int ret=0;
     \mathbf{while}(\mathbf{true})
     {
          uchain=chainInd[u];
          if (uchain=vchain)
          {
               ret +\!\!=\!\! tr.query\left(1\,,\ 0\,,\ n\,,\ posInBase\left[v\right]\!+\!1\,,\ posInBase\left[u\right]\right);
          int head=chainHead[uchain];
          u=head:
          u=lca[u][0];
     return ret;
}
//returns sum of all edges weights
//from 'u' to 'v'
inline int query(int u, int v)
{
     \mathbf{i} \mathbf{f} (\mathbf{u} = \mathbf{v})
          return 0;
     int l=getLca(u, v);
     return query_up(u, l)+query_up(v, l);
//set and edge to value 'val'
inline void update(int u, int val)
{
     \begin{array}{ll} \textbf{int} & x\!\!=\!\!\!X[\,u\,]\;,\;\; y\!\!=\!\!\!Y[\,u\,]\;;\\ \textbf{if}\,(\,l\,c\,a\,[\,x\,][\,0\,]\!==\!y\,) \end{array}
          tr.update(1, 0, n, posInBase[x], val);
          tr.update(1, 0, n, posInBase[y], val);
}
void clearHld()
     //tr.reset();
     for (int i = 0; i <= n; i++)
```

```
g[i].clear();
                   chainHead [i]=-1;
chainSize [i]=0;
         ptr=1;
         chainNo=0;
int main()
{
         scanf("%d", &n);
         clearHld();
         for (int i=1; i < n; i++)
                   \begin{array}{l} {\rm scanf}\left( \text{``%d\_\%d\_\%d''} \,, \,\, \&\!X[\,i\,] \,, \,\, \&\!W[\,i\,] \,\right); \\ {\rm g}\left[ \,\, X[\,i\,] \,\, \right] \,. \, {\rm push\_back}\left( \left\{ Y[\,i\,] \,, \,\, W[\,i\,] \right\} \right); \\ {\rm g}\left[ \,\, Y[\,i\,] \,\, \right] \,. \, {\rm push\_back}\left( \left\{ X[\,i\,] \,, \,\, W[\,i\,] \right\} \right); \end{array} 
         \begin{array}{ll} dfs (1, 0); \\ hld (1, 0, 0); \end{array}
        int q;
scanf("%d", &q);
         \mathbf{while} (q--)
                   if (o==1)
                           printf("%d\n", query(x, y));
                            update(x, y);
         return 0;
```

- 2.6.4 Heavy Light Decomposition on Vertex
- 2.6.5 All-Pairs Distance Sum
- 2.7 MISC
- 2.7.1 2-SAT

Dynamic Programming

3.1 Optimizations

3.1.1 Divide and Counquer

```
/// David Mateus Batista <david.batista3010@gmail.com>
/// Computer Science - Federal University of Itajuba - Brazil
/// Uri Online Judge - 2475
#include <bits/stdc++.h>
 using namespace std;
 typedef long long ll;
typedef unsigned long long ull; typedef long double ld;
 typedef pair<int,int> pii;
 typedef pair<ll, ll> pll;
#define INF 0x3F3F3F3F
#define LINF 0x3F3F3F3F3F3F3F3F1LL
#define DINF (double)1e+30
#define EPS (double)1e-9
#define PI (double) acos(-1.0)
#define RAD(x) (double)(x*PI)/180.0
#define PCT(x,y) (double)x*100.0/y
#define pb push_back
#define mp make_pair
#define pq priority_queue
#define F first
#define S second
#define D(x) x&(-x)
#define ALL(x) x.begin(),x.end()
#define SET(a,b) memset(a, b, sizeof(a))
\#define DEBUG(x,y) cout << x << y << endl
#define gcd(x,y) = gcd(x, y)
#define lcm(x,y) = (x/gcd(x,y))*y
#define bitcnt(x) __builtin_popcountll(x)
#define lbit(x) 63-_builtin_clzll(x)
#define zerosbitll(x) __builtin_ctzll(x)
#define zerosbit(x) __builtin_ctz(x)
\mathbf{enum} \ \left\{ \mathrm{North} \ , \ \mathrm{East} \ , \ \mathrm{South} \ , \ \mathrm{West} \right\};
//\{0, 1, 2, 3\}
```

```
//{Up, Right, Down, Left}
const int MN=1e+4+35;
const int MN2=535;
int p, a;
ll data [MN];
inline ll getValue(int l, int r)
{
      return (r-l+1)*(data[r]-data[l-1]);
 11 dp[MN2][MN];
inline void solve(int k, int l, int r, int L, int R)
{
      if (l>r)
            return;
      int m=(l+r)/2;
      int s=L;
      dp[k][m]=LINF;
      for(int i=max(m, L); i<=R; i++)
            if(dp[k][m]>dp[k-1][i+1]+getValue(m+1, i+1))
                  dp\,[\,k\,]\,[m] \!=\! dp\,[\,k-1]\,[\,i+1] \!+\! get\,V\,alu\,e\,(m\!+\!1,\ i+\!1)\,;
      solve(k, l, m-1, L, s);
solve(k, m+1, r, s, R);
int main()
{
      \begin{array}{l} {\rm s\,c\,a\,n\,f\,(\,''\%d\,.\%d''}\;,\;\;\&p\,,\;\;\&a\,)\,;\\ {\rm \bf for\,(\,int}\ \ i\,=\!1;\;\;i\,<\!\!=\!\!p\,;\;\;i\,+\!+\!)} \end{array}
            11 x;
            scanf("%lld", &x);
            data[i] = data[i-1] + x;
      \  \, \mathbf{for} \, (\, \mathbf{int} \quad i = \! 0; \quad i \! < \! \! = \! \! p \, ; \quad i + \! \! +)
            dp[0][i]=LINF;
      for (int i=0; i <=a; i++)
            dp[i][p]=0;
      for(int i=1; i<=a; i++)
      solve(i, 0, p-1, 0, p-1); printf("%lld\n", dp[a][0]);
      return 0;
```

3.1.2 Convex Hull I

Linear

```
//Original recurrence:
```

```
dp[i]=min(dp[j]+b[j]*a[i]) for j<i
// Condition:
        \begin{array}{l} b \, [ \, j ] > = b \, [ \, j+1 ] \\ a \, [ \, i \, ] < = a \, [ \, i+1 ] \end{array}
   Solution:
// Hull cht=Hull();
// cht.insertLine(b[0], dp[0])
// for (int i=1; i<n; i++)
        dp[i]=cht.query(a[i]);
        cht.insertLine(b[i], dp[i])
// answer is dp[n-1]
class Hull
{
     const static int CN=1e+5+35;
public:
     \begin{array}{ll} \textbf{long long} \ \ a\,[CN] \ , \ \ b\,[CN] \ ; \\ \textbf{double} \ \ x\,[CN] \ ; \end{array}
     int head, tail;
     Hull(): head(1), tail(0){};
     long long query(long long xx)
           if (head>tail)
                return 0;
           \mathbf{while} (\text{head} < \text{tail && } x [\text{head} + 1] < = xx)
                 head++;
           x[head]=xx;
           return a [head] * xx+b [head];
     }
     void insertLine(long long aa, long long bb)
           double xx=-1e18;
           while (head <= tail)
                 if(aa==a[tail])
                     return;
                 xx=1.0*(b[tail]-bb)/(aa-a[tail]);
                 if(head==tail || xx>=x[tail])
                      break;
                 tail --;
           a[++tail]=aa;
           b[tail]=bb;
           x[tail]=xx;
     }
};
```

Dynamic

```
//Original recurrence:
// dp[i]=min(dp[j]+b[j]*a[i]) for j<i
//Condition:
// b[j]>=b[j+1]
// a[i]<=a[i+1]
// Solution:
// HullDynamic cht;
// cht.insertLine(b[0], dp[0])
```

```
// for(int i=1; i< n; i++)
      dp[i]=cht.query(a[i]);
cht.insertLine(b[i], dp[i])
// answer is dp[n-1]
const long long is_query=-(1LL<<62);</pre>
class Line
public:
    long long m, b;
    mutable function < const Line *() > succ;
    bool operator < (const Line &rhs) const
        if(rhs.b!=is_query)
            return m<rhs.m;
        const Line *s=succ();
        if(!s)
           return 0;
        long long x=rhs.m;
        {f return} \ (b-s->b)<((s->m-m)*x);
    }
};
class HullDynamic: public multiset <Line>
public:
    void clear()
    {
        clear();
    bool bad(iterator y)
        auto z=next(y);
        if(y==begin())
        {
            if (z==end())
                return 0;
            return (y->m=z->m && y->b<=z->b);
        auto x=prev(y);
        if (z=end())
            void insertLine(ll m, ll b)
        auto y=insert(\{m, b\});
        y \rightarrow succ = [=]
            return next(y)==end()?0:&*next(y);
        if (bad(y))
            erase(y);
            return;
        \mathbf{while}(\text{next}(y)! = \text{end}() \&\& \text{bad}(\text{next}(y)))
        erase(next(y));
while(y!=begin() && bad(prev(y)))
            erase(prev(y));
```

```
} long long query(long long x)
{
    auto ret=*lower_bound((Line){x, is_query});
    return ret.m*x+ret.b;
}
};
```

- 3.1.3 Convex Hull II
- 3.1.4 Knuth Optimization
- 3.2 Digits
- 3.3 Grundy Numbers

String

- 4.1 Hash
- 4.2 KMP
- 4.3 Aho Corasick
- 4.4 Manacher
- 4.5 Z-Algorithm
- 4.6 Suffix Array & LCP
- 4.7 Suffix Tree

Mathematic

- 5.1 Prime Numbers
- 5.1.1 Erastotenes Sieve
- 5.1.2 Linear Sieve
- 5.1.3 Miller Rabin
- 5.1.4 BPSW
- 5.1.5 Primality Test
- 5.2 Chinese Remainder Theorem
- 5.3 Fast Fourier Transformation

```
#define PI (double) acos(-1.0)
typedef complex<double> base;
void fft(vector<base>&data, bool invert)
      int n=data.size();
      \label{eq:formalized} \mbox{for}\,(\,\mbox{i}\,\mbox{nt}\,\ i\,{=}1\,,\ j\,{=}0\,;\ i\,{<}n\,;\ i\,{+}{+})
            int bit=n>>1;
            for(; j>=bit; bit>>=1)
                j-=bit;
            j+=bit;
            \mathbf{i}\mathbf{f}(i < j)
                 swap(data[i], data[j]);
      for(int len=2; len <=n; len <<=1)
            \label{eq:double_ang} \textbf{double} \ \ \text{ang=2*PI/len*(invert?-1:1);}
            base wlen(cos(ang), sin(ang));
for(int i=0; i<n; i+=len)
                  base w(1);
                  for (int j=0; j < len / 2; j++)
```

```
base \ u \!\!=\!\! data\,[\,i \!+\! j\,]\,, \ v \!\!=\!\! data\,[\,i \!\!+\! j \!+\! len\,/\,2\,] \!*\! w;
                         data[i+j]=u+v;
                         data[i+j+len/2]=u-v;
                         w = wlen;
                   }
            }
      if (invert)
            for (int i=0; i < n; i++)
                   data[i]/=n;
vector<int>fft_multiply(vector<int>&a, vector<int>&b)
       \begin{array}{l} {\rm vector}{<}{\rm base}{>}{\rm fa}\left({\rm a.begin}\left(\right),\ {\rm a.end}\left(\right)\right); \\ {\rm vector}{<}{\rm base}{>}{\rm fb}\left({\rm b.begin}\left(\right),\ {\rm b.end}\left(\right)\right); \end{array} 
      int n=1;
      while(n<max(a.size(), b.size()))
           n < < =1;
      n << =1;
      fa.resize(n);
      fb.resize(n);
      fft(fa, false);
      fft (fb, false);

for (int i=0; i<n; i++)

fa[i]*=fb[i];
      fft(fa, true);
      vector < int > ret (n);
for (int i = 0; i < n; i ++)</pre>
            ret[i]=(int)(fa[i].real()+0.5);
      int carry=0;
      for (int i=0; i< n; i++)
            \mathtt{ret}\;[\;i] += \mathtt{carry}\;;
            carry=ret[i]/10;
            ret[i]%=10;
      return ret;
}
int main()
      int n, m;
scanf("%d_%d", &n, &m);
      vector < int > a, b;
      for (int i=0; i < n; i++)
            int x;
            scanf("%d", &x);
            a.pb(x);
      }
      for (int i=0; i < m; i++)
            scanf("%d", &x);
            b.pb(x);
      reverse(a.begin(), a.end());
```

```
reverse(b.begin(), b.end());

vector<int>ans=fft_multiply(a, b);
reverse(ans.begin(), ans.end());
bool flag=false;
for(int i=0; i<ans.size(); i++)
{
    if(ans[i])
        flag=true;
    if(flag)
        printf("%d", ans[i]);
}
printf("\n");
return 0;
}</pre>
```

5.4 Modular Math

- 5.4.1 Multiplicative Inverse
- 5.4.2 Linear All Multiplicative Inverse
- 5.5 Gaussian Elimination
- 5.6 Combinatorics

Geometry

- 6.1 2d Template
- 6.2 3d Template
- 6.3 Polygon Template
- 6.4 Convex Hull
- 6.4.1 Graham Scan
- 6.4.2 Monotone Chain
- 6.5 Rotating Calipers
- 6.6 KD Tree
- 6.7 Range Tree
- 6.8 Circle Sweep