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Data Structure

1.1 Segment Tree

1.1.1 Segment Tree & Lazy Propagation

```
class segtree
      \textbf{const static int } N{=}100000;
      int tr[4*N], lazy[4*N];
public:
      segtree(){};
void clear()
      {
            \begin{array}{ll} memset(\,tr\;,\;\;0\,,\;\;\mathbf{sizeof}(\,tr\;))\,;\\ memset(\,lazy\;,\;\;0\,,\;\;\mathbf{sizeof}(\,lazy\;))\,; \end{array}
      void build(int no, int l, int r, vector<int>&data)
      {
             \mathbf{i}\,\mathbf{f}\,(\;l =\!\!\!-\!\!\!r\,)
             {
                   tr [no]=data[l];
                  return;
            int nxt=no*2;
            int mid=(l+r)/2;
            build (nxt, 1, mid, data);
build (nxt+1, mid+1, r, data);
tr[no]=tr[nxt]+tr[nxt+1];
      void propagate(int no, int l, int r)
             if (! lazy [no])
                  return;
             tr[no]+=(r-l+1)*lazy[no];
             if ( l!=r )
             {
                   int nxt=no*2;
                   lazy[nxt]+=lazy[no];
                   lazy[nxt+1]+=lazy[no];
            lazy[no]=0;
```

```
void update(int no, int l, int r, int i, int j, int x)
        propagate(no, l, r);
        if(l>j || r<i)
            return;
        if(l>=i && r<=j)
        {
            lazy [no]=x;
            propagate(no, l, r);
            return;
        int nxt=no*2;
        int mid=(l+r)/2;
        update(nxt, 1, mid, i, j, x);
        update(nxt+1, mid+1, r, i, j, x);
        tr[no] = tr[nxt] + tr[nxt+1];
   int query(int no, int l, int r, int i, int j)
        propagate(no, l, r);
        if(l>j || r<i)
            return 0;
        if(l>=i && r<=j)
            return tr[no];
        int nxt=no*2;
        int mid=(l+r)/2;
        int ql = query(nxt, l, mid, i, j);
        int qr = query(nxt+1, mid+1, r, i, j);
        return (ql+qr);
   }
};
```

1.1.2 Quadtree

```
class quadtree
     //needs to be NxN
     const static int N=100000;
     int tr[16*N];
public:
     quadtree(){};
     void build (int node, int l1, int r1, int l2, int r2, vector < vector < int > data)
     {
           if(11=12 \&\& r1=r2)
           {
                tr [node]=data[l1][r1];
                return;
          int nxt=node * 4;
          int midl=(11+12)/2;
          int midr = (r1+r2)/2;
           \texttt{build} \, (\, \texttt{nxt} \, - 2 \,, \ \texttt{l1} \,\,, \ \ \texttt{r1} \,\,, \ \ \texttt{midl} \,\,, \ \ \texttt{midr} \,\,, \ \ \texttt{data} \,) \,;
           build (nxt-1, midl+1, r1, l2, midr, data);
build (nxt, l1, midr+1, midl, r2, data);
           build(nxt+1, midl+1, midr+1, 12, r2, data);
           tr[node] = tr[nxt-2] + tr[nxt-1] + tr[nxt] + tr[nxt+1];
     void update(int node, int l1, int r1, int l2, int r2, int i, int j, int x)
```

```
if(l1>l2 || r1>r2)
            return;
            if(i==l1 && i==l2 && j==r1 && j==r2)
                  tr[node]=x;
                  return;
            int nxt=node*4;
            int midl=(11+12)/2;
            int midr=(r1+r2)/2;
            update(nxt-2, l1, r1, midl, midr, i, j, x);
            update(nxt-1, midl+1, r1, l2, midr, i, j, x);
update(nxt, l1, midr+1, midl, r2, i, j, x);
            update(nxt+1, midl+1, midr+1, l2, r2, i, j, x);
            tr[node] = tr[nxt-2] + tr[nxt-1] + tr[nxt] + tr[nxt+1];
     int query(int node, int l1, int r1, int l2, int r2, int i1, int j1, int i2, int j2)
      {
            if(i1>12 || j1>r2 || i2<11 || j2<r1 || i1>i2 || j1>j2)
                  return 0;
            if(i1<=l1 && j1<=r1 && l2<=i2 && r2<=j2)
                 return tr[node];
            int nxt=node * 4;
            int midl=(11+12)/2;
            int midr=(r1+r2)/2;
            int q1=query(nxt-2, l1, r1, midl, midr, i1, j1, i2, j2);
int q2=query(nxt-1, midl+1, r1, l2, midr, i1, j1, i2, j2);
int q3=query(nxt, l1, midr+1, midl, r2, i1, j1, i2, j2);
int q4=query(nxt+1, midl+1, midr+1, l2, r2, i1, i2, i2, i2);
            \mathbf{int} \ \ \mathbf{q4} \!\!=\!\! \mathbf{query} \, (\, \mathbf{nxt} \! + \! 1, \ \mathbf{midl} \! + \! 1, \ \mathbf{midr} \! + \! 1, \ \mathbf{l2} \, , \ \mathbf{r2} \, , \ \mathbf{i1} \, , \ \mathbf{j2} \, , \ \mathbf{i2} \, , \ \mathbf{j2} \, ) \, ;
};
```

1.1.3 Mergesort Segtree

```
class mergesort_segtree
{
                      const static int N=100000;
                      vector < int > tr[4*N];
public:
                      mergesort_segtree(){};
                       void build(int no, int l, int r, vector<int>&data)
                        {
                                               if ( l==r )
                                               {
                                                                      tr[no].push_back(data[l]);
                                                                    return;
                                              int nxt=no * 2;
                                              int mid=(l+r)/2;
                                               build(nxt, l, mid, data);
                                               build(nxt+1, mid+1, r, data);
                                               tr[no].resize(tr[nxt].size()+tr[nxt+1].size());
                                              merge(tr[nxt].begin(), tr[nxt].end(), tr[nxt+1].begin(), tr[nxt+1].end(), tr[no].begin(), tr
```

```
//how many numbers in (i, j) are greater or equal than k
int query(int no, int l, int r, int i, int j, int k)
{
    if(r<i || l>j)
        return 0;
    if(l>=i && r<=j)
        return (int)(tr[no].end()-upper_bound(tr[no].begin(), tr[no].end(), k));
    int nxt=no*2;
    int mid=(l+r)/2;
    int ql=query(nxt, l, mid, i, j, k);
    int qr=query(nxt+1, mid+1, r, i, j, k);
    return ql+qr;
};
};</pre>
```

1.1.4 Persistent Segtree

```
class persistent_segtree
     \textbf{const static int } N{=}100000;
    int n;
    int tr[N];
    int root [N], L[N], R[N];
    int cnt, id;
public:
    persistent_segtree(){};
     void set(int _n)
          memset(tr, 0, sizeof(tr));
         memset(root, 0, sizeof(root));
         memset(L, 0, sizeof(L));
memset(R, 0, sizeof(R));
         id = 0:
          cnt=1;
         n=_n;
    void build(int no, int l, int r, vector<int>&data)
          if ( l==r )
          {
               tr [no]=data[1];
               return;
          int mid=(l+r)/2;
          L[no] = cnt ++;
         R[no] = cnt + +;
         build (L[no], l, mid, data);
build (R[no], mid+1, r, data);
tr[no]=tr[L[no]]+tr[R[no]];
     int update(int no, int l, int r, int i, int x)
          int newno=cnt++;
          tr[newno]=tr[no];
         L[newno]=L[no];
         R[newno]=R[no];
          if ( l==r )
          {
               tr[newno]=x;
               \textbf{return} \ \text{newno};\\
```

```
int mid=(l+r)/2;
              if(i \le mid)
                     L[newno]=update(L[newno], l, mid, i, x);
               \begin{array}{c} R[\,\mathrm{newno}] \!=\! \mathrm{update} \left( R[\,\mathrm{newno}\,] \;,\; \mathrm{mid} \!+\! 1,\; r\;,\; i\;,\; x \,\right); \\ \mathrm{tr}\left[\,\mathrm{newno}\right] \!=\! \mathrm{tr}\left[\; L[\,\mathrm{newno}\,] \;\;] \!+\! \mathrm{tr}\left[\; R[\,\mathrm{newno}\,] \;\;\right]; \end{array} 
              return newno;
       int query(int no, int l, int r, int i, int j)
              if(r < i \mid \mid l > j)
                     return 0;
              if(l>=i && r<=j)
                    return tr[no];
              \begin{array}{ll} \textbf{int} & \min = (l+r)/2; \\ \textbf{int} & ql = query\left(L \begin{bmatrix} no \end{bmatrix}, \ l \ , \ \min d, \ i \ , \ j \ \right); \end{array}
              int qr=query(R[no], mid+1, r, i, j);
              return ql+qr;
       //update the i-th value to x.
      void update(int i, int x)
              root[id+1]=update(root[id], 0, n-1, i, x);
       //returns sum(l, r) after the k-th update.
      int query(int 1, int r, int k)
              return query (root [k], 0, n-1, l, r);
      }
};
```

1.2 Fenwick Tree

1.2.1 Fenwick Tree 1D

```
class fenwicktree
   #define D(x) x&(-x)
   const static int N=100000;
   int tr[N], n;
public:
    fenwicktree(){};
    void build(int _n)
        memset(tr, 0, sizeof(tr));
    void update(int i, int x)
        for(i++; i<=n; i+=D(i))
            tr[i]+=x;
   int query(int i)
   {
        int ret = 0;
        for (i++; i>0; i-=D(i))
           ret+=tr[i];
        return ret;
```

```
    int rquery(int 1, int r)
    {
        return query(r)-query(l-1);
    }
    void set(int i, int x)
    {
          update(i, -rquery(i, i)+x);
    }
    void rset(int l, int r, int x)
    {
          update(l, x);
          update(r+1, -x);
    }
};
```

1.2.2 Fenwick Tree 2D

```
class fenwicktree
    #define D(x) x&(-x)
    const static int N=1000;
    int tr[N][N], n, m;
public:
    fenwicktree(){};
    void build(int _n, int _m)
        n=_n, m=_m;
        memset(tr, 0, sizeof(tr));
    void update(int r, int c, int x)
        for(int i=r+1; i<=n; i+=D(i))</pre>
            for(int j=c+1; j<=m; j+=D(j))
                tr[i]==x;
    int query(int r, int c)
        int ret = 0;
        for (int i=r+1; i>0; i-D(i))
            for(int j=c+1; j>0; j=D(j))
ret+=tr[i][j];
        return ret;
    int rquery(int r1, int c1, int r2, int c2)
        if ((r1>r2 && c1>c2) || (r1=r2 && c1>c2) || (r1>r2 && c1=c2))
            swap(r1, r2);
            swap(c1, c2);
        else if(r1<r2 && c1>c2)
            swap(c1,c2);
        else if (r1>r2 && c1<c2)
        {
            swap(r1, r2);
        return query (r2, c2)-query (r1-1, c2)-query (r2, c1-1)+query (r1-1, c1-1);
```

```
}
    void set(int r, int c, int x)
    {
        update(r, c, -rquery(r, c, r, c)+x);
    }
};
```

1.3 Cartesian Tree

1.3.1 Cartesian Tree

```
//srand(time(NULL))
int vrand()
{
      return abs(rand() << (rand() \%31));
{f struct} node
      //x=key, y=priority key, c=tree count
     int x, y, c;
node *L, *R;
     node(){};
      node(int _x)
             \begin{array}{ll} x \!\!=\!\! \_x \;, \;\; y \!\!=\!\! v \, r \, and \, (\,) \;, \;\; c \! = \! 0; \\ L \!\!=\!\! R \!\!=\!\! NULL; \end{array} 
      }
};
int cnt(node *root)
      return root?root\rightarrowc:0;
void upd_cnt(node *root)
{
      if (root)
           root \rightarrow c=1+cnt(root \rightarrow L)+cnt(root \rightarrow R);
void split (node *root, int x, node *&L, node *&R)
      if (!root)
           L=R=NULL;
      else if (x < root \rightarrow x)
            split (root->L, x, L, root->L), R=root;
            split(root->R, x, root->R, R), L=root;
      upd_cnt(root);
\mathbf{void} \ \mathtt{insert} \, (\, \mathtt{node} \ *\&\mathtt{root} \; , \ \mathtt{node} \ *\mathtt{it} \, )
      if (!root)
           root=it;
      else if(it -> y > root -> y)
           split(root, it \rightarrow x, it \rightarrow L, it \rightarrow R), root=it;
```

```
insert(it->x < root->x? root->L:root->R, it);
    upd_cnt(root);
}
void merge(node *&root , node *L, node *R)
    if (!L || !R)
         root=L?L:R;
    \mathbf{else} \ \mathbf{if} (L\!\!-\!\!>\!\! y > R\!\!-\!\!>\!\! y)
         merge(L\rightarrow R, L\rightarrow R, R), root=L;
         merge\,(R\!\!-\!\!>\!\!L\,,\ L\,,\ R\!\!-\!\!>\!\!L\,)\,,\ root\!=\!\!R;
    upd_cnt(root);
void erase(node *&root, int x)
    if(root->x=x)
        merge(root, root->L, root->R);
    _{
m else}
         erase(x < root \rightarrow x? root \rightarrow L: root \rightarrow R, x);
    upd_cnt(root);
node *unite(node *L, node *R)
    if (!L || !R)
         return L?L:R;
    i\,f\,(L\!\!-\!\!>\!\!y\,<\,R\!\!-\!\!>\!\!y\,)
         swap(L, R);
    L\rightarrow R=unite(L\rightarrow R, Rt);
    return L;
int find(node *root, int x)
{
    if (!root)
         return 0;
     if(root->x=x)
         return 1;
    if(x > root -> x)
         return find (root->R, x);
    _{
m else}
         return find(root->L, x);
int findkth(node *root, int x)
{
    if (!root)
         return -1;
    int Lc=cnt(root->L);
    if(x-Lc-1==0)
         return root ->x;
    if (x>Lc)
         else
         return findkth(root->L, x);
```

1.3.2 Implicit Cartesian Tree

```
//srand(time(NULL))
int vrand()
    return abs(rand()<<(rand()%31));
struct node
    //basic treap: x=key, y=priority key, c=tree count;
    int x, y, c;
    //treap operations: v=max(x), lazy=lazy value of propagation, rev=reversed
    int v, lazy, rev;
    node *L, *R;
    node(){};
    node(int _x)
         x=_x, y=vrand();
         L=R=NULL;
         v=x;
         lazy=0;
         \scriptstyle \operatorname{rev} = 0;
    }
};
//updating functions
inline int get_cnt(node *root)
    return root?root ->c:0;
inline void upd_cnt(node *root)
{
    if (root)
         root->c=1+get_cnt(root->L)+get_cnt(root->R);
inline void push(node *&root)
    if(root && root->rev)
         root \rightarrow rev = 0;
         swap(root->L, root->R);
         if (root->L)
             root \rightarrow L \rightarrow rev = 1;
         if(root->R)
             root \rightarrow R \rightarrow rev = 1;
    }
inline void propagate(node *&root)
    if (root)
         if (!root->lazy)
             return;
         int lazy=root->lazy;
         root \rightarrow x + = lazy;
         if(root->L)
```

```
root \rightarrow L \rightarrow lazy = lazy;
            if(root->R)
                 root->R->lazy=lazy;
           root \rightarrow lazy = 0;
     }
}
inline int get_max(node *root)
     return root?root \rightarrow v:-INF;
inline void upd_max(node *root)
     if (root)
           root \rightarrow v=max(root \rightarrow x, max(get_max(root \rightarrow L), get_max(root \rightarrow R)));
inline void update(node *root)
     propagate(root);
     upd_cnt(root);
     upd_max(root);
void merge(node *&root , node *L, node *R)
     push(L);
     push(R);
     if (!L || !R)
           root=L?L:R;
     \begin{array}{c} \textbf{else} \ \ \textbf{if} \ (L\!\!\rightarrow\!\! y \ > \ R\!\!\rightarrow\!\! y) \\ \ \ \ \text{merge} \ (L\!\!\rightarrow\!\! R, \ L\!\!\rightarrow\!\! R, \ R) \ , \ \ \text{root=}L; \end{array}
           merge(R->L, L, R->L), root=R;
     update(root);
}
\mathbf{void} \ \mathrm{split} \, (\, \mathrm{node} \ *\mathrm{root} \, , \ \mathrm{node} \ *\&\mathrm{L}, \ \mathrm{node} \ *\&\mathrm{R}, \ \mathbf{int} \ \mathrm{x} \, , \ \mathbf{int} \ \mathrm{add} \! = \! 0)
     if (!root)
           return void(L=R=NULL);
     push(root);
     int ix=add+get_cnt(root->L); //implicit key
     if(x \le ix)
           split (root->L, L, root->L, x, add), R=root;
           split (root->R, root->R, R, x, add+1+get_cnt(root->L)), L=root;
     update(root);
//insert function
void insert (node *&root, int pos, int x)//(insert x at position pos)
     node *R1, *R2;
     split (root, R1, R2, pos);
     merge(R1, R1, new node(x));
     merge(root, R1, R2);
//erase value x
void erase_x(node *&root, int x)
```

```
if (!root)
            return;
      if(root \rightarrow x=x)
           merge(root, root->L, root->R);
      else
            erase_x(x < root \rightarrow x? root \rightarrow L: root \rightarrow R, x);
      update(root);
//erase kth value
void erase_kth(node *&root, int x)
{
      if (!root)
           return;
      int Lc=get_cnt(root->L);
      if(x-Lc-1==0)
           merge(root, root \rightarrow L, root \rightarrow R);
      else if(x>Lc)
           erase_kth(root->R, x-Lc-1);
           erase_kth(root->L, x);
      update(root);
//add x to [1, r]
inline void paint (node *&root, int 1, int r, int x)
      node *R1, *R2, *R3;
split(root, R1, R2, 1);
      split (R2, R2, R3, r-l+1);
      R2->lazy=x;
      propagate(R2);
      \begin{array}{ll} merge (\hspace{.05cm} \texttt{root} \hspace{.1cm}, \hspace{.1cm} R1 \hspace{.05cm}, \hspace{.1cm} R2 \hspace{.05cm}) \hspace{.05cm}; \\ merge (\hspace{.05cm} \texttt{root} \hspace{.1cm}, \hspace{.1cm} \texttt{root} \hspace{.1cm}, \hspace{.1cm} R3) \hspace{.05cm}; \end{array}
}
//max range query [l,r]
inline int rquery(node *&root, int 1, int r)
      node *R1, *R2, *R3;
      split (root, R1, R2, l);
split (R2, R2, R3, r-l+1);
      int ret=R2->v;
      merge(root, R1, R2);
      merge(root, root, R3);
      return ret;
inline void reverse(node *&root, int 1, int r)//reverse elements [1, r]
      node *R1, *R2, *R3;
split(root, R1, R2, 1);
      split (R2, R2, R3, r-l+1);
      R2->rev^=1;
      merge(root, R1, R2);
merge(root, root, R3);
//output functions
int poscnt=0;
```

```
void output_all(node *root)
     if (!root)
          return;
     update(root);
     push(root);
     output_all(root->L);
     printf("[%d]_%d\n", poscnt++, root->x);
output_all(root->R);
int output_kth(node *root, int x)
{
     if (!root)
          return -1;
     update(root);
     push(root);
     int Lc=get_cnt(root->L);
     \mathbf{i} \mathbf{f} \left( \mathbf{x} - \mathbf{L} \mathbf{c} - 1 = = 0 \right)
          return root ->x;
     if(x>Lc)
          return output_kth(root->R, x-Lc-1);
          return output_kth(root->L, x);
```

1.4 Merge Sort & Swap Count

1.4.1 Merge Sort & Vector

```
#define INF 0x3F3F3F3F
int mergesort(vector<int>&data)
      if(data.size()==1)
            return 0;
      vector<int>L, R;
      \mathbf{int} \ \ \mathbf{t} {=} \mathbf{data.\, size} \ (\,) \, ;
      for (int i=0; i < t/2; i++)
           L. push_back (data [i]);
      for (int i=t/2; i < t; i++)
            R. push_back(data[i]);
      int ret=mergesort(L)+mergesort(R);
      \label{eq:formula} \mbox{for}(\,\mbox{int}\ i\!=\!\!0,\ j\!=\!\!0,\ k\!=\!\!0;\ j\!<\!\!L.\,size\,(\,)\ |\,|\ k\!<\!\!R.\,size\,(\,)\,;\ i\!+\!\!+\!\!)
            int x=j<L.size()?L[j]:INF;
            int y=k<R. size()?R[k]:INF;
            \mathbf{i}\,\mathbf{f}\,(\,x{<}y\,)
                  data[i]=x;
                  j++;
            }
            else
                  data[i]=y;
                  k++;
                  ret += (L.size()-j);
      {\bf return} \ {\rm ret} \ ;
```

}

1.4.2 Merge Sort

```
#define INF 0x3F3F3F3F
  int temp[100000];
  int mergesort(int data[], int l, int r)
                                {\bf i}\,{\bf f}\,(\,a\,b\,s\,(\,l\,{-}r\,)\!<\!=\!1)
                                                            return 0;
                               int mid=(l+r)/2;
                               \mathbf{int} \ \mathtt{ret} \! = \! \mathtt{mergesort} \, (\, \mathtt{data} \, , \ l \, , \ \mathtt{mid}) \! + \! \mathtt{mergesort} \, (\, \mathtt{data} \, , \ \mathtt{mid} \, , \ r \, ) \, ;
                              for (int i=1; i<r; i++)
temp[i]=data[i];
                               \begin{tabular}{ll} \beg
                                                            \quad \textbf{int} \ x{=}j{<}mid?temp\left[\ j\ \right]{:}INF\,;
                                                            int y=k< r?temp[k]:INF;
                                                              if(x < y) //x <= y
                                                                                           data[i]=x;
                                                                                          j++;
                                                            else
                                                              {
                                                                                           data[i]=y;
                                                                                           ret += (mid-j);
                               {\bf return} \ {\rm ret} \ ;
```

1.5 Sparse Table

```
{
    int i=abs(l-r)+1;
    int j=lbit(i);
    return max(data[1][j], data[1-(1<<j)+1][j]);
};
};</pre>
```

1.6 SQRT Decomposition

1.6.1 Array

```
const int N=100000;
int SN=sqrt(N);
{f class} mo
public:
    int l , r , i ;
mo(){};
    mo(int _l , int _r , int _i)
          l=_{-}l , r=_{-}r , i=_{-}i ;
     bool operator <(const mo &foo) const
           if ((r/SN)!=(foo.r/SN))
                return (r/SN) < (foo.r/SN);
           if (1!=foo.1)
               return ĺ<foo.l;
           \textbf{return} \hspace{0.2cm} i \!<\! foo.i;
     }
};
\mathbf{int}\ \mathrm{data}\left[N\right],\ \mathrm{freq}\left[N\right],\ \mathrm{ans}\left[N\right];
int cnt=0;
void update(int p, int s)
     int x=data[p];
     if(s==1)
           if(freq[x]==0)
                cnt++;
     else
           if(freq[x]==1)
                cnt --;
     freq[x]+=s;
int main()
     int n;
scanf("%d", &n);
     for(int i=1; i<=n; i++) scanf("%d", &data[i]);
     int q;
```

```
scanf("%d", \&q);
vector <mo>querys;
\  \, \mathbf{for} \, (\, \mathbf{int} \quad i \,{=}\, 0\,; \quad i \,{<} q\,; \quad i \,{+}{+})
{
     int l, r;
scanf("%d_%d", &l, &r);
      querys.push_back(mo(l, r, i));
sort(querys.begin(), querys.end());
int l=1, r=1;
cnt=0;
memset(freq , 0, sizeof(freq));
update(1, 1);
for (int i=0; i < q; i++)
     \mathbf{int} \ \ \mathtt{li=querys}\left[ \ i \ \right]. \ \mathtt{l};
      int ri=querys[i].r;
      int ii=querys[i].i;
      \mathbf{while}(l > li)
           update(--1, 1);
      \mathbf{while}(\mathbf{r} < \mathbf{r} \mathbf{i})
           update(++r, 1);
      while (l<li)
           update(l++, -1);
      \mathbf{while}(r > ri)
          update(r--, -1);
      ans[ii]=cnt;
for(int i=0; i<querys.size(); i++)
     printf("%d\n", ans[i]);
return 0;
```

1.6.2 Tree

```
#define pb push_back
#define ALL(x) x.begin(),x.end()
const int N=1e+5+35;
const int M=20;
const int SN=sqrt(2*N)+1;
{f class} \ {f mo}
public:
    int 1, r, i, lc;
mo(){};
    mo(int _l , int _r , int _lc , int _i )
    {
        l=_l , r=_r , lc=_lc , i=_i ;
    bool operator <(const mo &foo) const
         if ((r/SN)!=(foo.r/SN))
             return (r/SN)<(foo.r/SN);
         if(1!=foo.1)
             return l<foo.l;
         return i<foo.i;
```

```
};
\begin{array}{ll} \textbf{int} & n \,,\, q \,; \\ \textbf{int} & h \,[N] \,,\, \left. l\, c\, a \,[N] \,[M] \,; \\ \end{array}
vector < int > g[N];
int dl[N], dr[N], di[2*N], cur;
void dfs(int u, int p)
       dl[u]=++cur;
       di[cur]=u;
       lca[u][0]=p;
       for (int i=1; i\llM; i++)

lca[u][i]=lca[lca[u][i-1]][i-1];
       for(int i=0; i<g[u].size(); i++)
               \quad \textbf{int} \ \ v \hspace{-0.1cm}=\hspace{-0.1cm} g \hspace{.1cm} [\hspace{.1cm} u\hspace{.1cm}] \hspace{.1cm} [\hspace{.1cm} i\hspace{.1cm}] \hspace{.1cm};
               \mathbf{i}\,\mathbf{f}\,(\,v\!\!=\!\!\!-\!\!p\,)
                     continue;
              h[v]=h[u]+1;
              dfs(v, u);
       \mathrm{d} \, r \, [\, u] \!\! = \!\! + \!\! + \! c \, u \, r \; ;
       di [cur]=u;
inline int getLca(int u, int v)
       \mathbf{i}\,\mathbf{f}\,(\,h\,[\,u]\!>\!h\,[\,v\,]\,)
              swap(u, v);
       for(int i=M-1; i>=0; i--)
               if (h[v]-(1<<i)>=h[u])
v=lca[v][i];
       \mathbf{i}\,\mathbf{f}\,(\,\mathbf{u}\!\!=\!\!\!-\!\!\mathbf{v}\,)
              return u;
       for (int i=M-1; i>=0; i--)
       {
               \mathbf{i}\,\mathbf{f}\,(\,l\,c\,a\,[\,u\,]\,[\,\,i\,\,]!\!=\!\,l\,c\,a\,[\,v\,]\,[\,\,i\,\,]\,)
               {
                      u{=}\,l\,c\,a\;[\;u\;]\;[\;i\;]\;;
                      v=lca[v][i];
       return lca[u][0];
int x=data[u];
if(vis[u] && (--freq[ data[u] ]==0))
       else if (! vis [u] && (freq [ data[u] ]++==0))
              cnt++;
       vis[u]^=1;
}
int main()
       scanf("%d_{d}", &n, &q);
       for (int i=1; i <= n; i++)
```

```
char temp [25];
      scanf("%s", temp);
       string temp2=string(temp);
       if(!remap.count(temp2))
             remap[temp2]=remap.size();
      data[i]=remap[temp2];
for (int i=1; i<n; i++)
      \begin{array}{lll} \textbf{int} & u \,, & v \,; \\ s \, c \, a \, n \, f \, (\,\text{``%d..\%d''} \,\,, & \&u \,, & \&v \,) \,; \end{array}
      g[u].pb(v);
g[v].pb(u);
dfs(1, 0);
vector<mo>query;
\  \, \mathbf{for}\,(\,\mathbf{int}\  \  \, i\,{=}0;\  \, i\,{<}q\,;\  \  \, i\,{+}{+})
      int u, v;
scanf("%d_%d", &u, &v);
int lc=getLca(u, v);
       if (dl[u]>dl[v])
             swap(u, v);
       query.pb(mo(u=lc?dl[u]:dr[u], dl[v], lc, i));
sort(ALL(query));
\quad \mathbf{int} \ \ l {=} query \left[ \, 0 \, \right] . \, l \; , \ \ r {=} query \left[ \, 0 \, \right] . \, l \, -1;
cnt = 0;
for (int i=0; i < q; i++)
       int li=query[i].l;
      int ri=query[i].r;
int lc=query[i].lc;
      int ii=query[i].i;
      while(l>li)
             update(di[--1]);
       while (r<ri)
             update(di[++r]);
       while (l<li)
             update(di[l++]);
      \mathbf{while}\,(\,\mathbf{r}\!>\!\mathbf{r}\,\mathbf{i}\,)
             update (di [r--]);
      int u=di[1], v=di[r];
if(lc!=u && lc!=v)
             update(lc);
       ans[ii]=cnt;
       if (lc!=u && lc!=v)
             update(lc);
for(int i=0; i<q; i++)
    printf("%d\n", ans[i]);</pre>
return 0;
```

Graph

- 2.1 Components
- 2.1.1 Articulations, Bridges & Cycles
- 2.1.2 Strongly Connected Components
- 2.1.3 Semi-Strongly Connected Components
- 2.2 Single Source Shortest Path
- 2.2.1 Dijkstra
- 2.2.2 Bellmanford
- 2.3 All Pairs Shortest Path
- 2.3.1 Floyd Warshall
- 2.4 Minimum Spannig Tree
- 2.4.1 Kruskal
- 2.4.2 Prim
- 2.5 Flow
- 2.5.1 Maximum Bipartite Matching

```
const int MN=1e+3;
vector <int>g [MN];
int match [MN], rmatch [MN], vis [MN];
int findmatch (int u)
{
    if (vis [u])
        return 0;
    vis [u]=true;
    for (int v:g [u])
```

```
{
    if(match[v]==-1 || findmatch(match[v]))
    {
        match[v]=u;
        rmatch[u]=v;
        return 1;
    }
}
    return 0;
}

int maxMatch(int n)
{
    int ret=0;
    memset(match, -1, sizeof(match));
    for(int i=0; i<n; i++)
    {
        memset(vis, false, sizeof(vis));
        ret+=findmatch(i);
    }
    return ret;
}</pre>
```

2.5.2 Maximum Flow

Dinic

```
class graph
    const static int N=100000;
public:
    vector< pair<int, int> >edge;
    vector < int > adj[N];
    int ptr[N];
    int dist[N];
    graph(){};
    void clear()
        for(int i=0; i< N; i++)
            adj[i].clear();
        edge.clear();
    void add_edge(int u, int v, int c)
        adj[u].push_back(edge.size());
        \verb|edge.push_back(mp(v, c));|\\
        adj[v].push_back(edge.size());
        edge.push_back(mp(u, 0)); //(u, c) if is non-directed
    bool dinic_bfs(int s, int t)
        memset(dist, -1, sizeof(dist));
        dist[s]=0;
        queue<int>bfs;
        bfs.push(s);
        while (! bfs.empty() && dist[t]==-1)
            int u=bfs.front();
```

```
bfs.pop();
                   for (int i=0; i<adj[u].size(); i++)
                         \mathbf{int} \ \mathrm{idx} {=} \mathrm{adj} \left[ \, u \, \right] \left[ \, i \, \right];
                         int v=edge[idx].F;
                         if(dist[v]==-1 \&\& edge[idx].S>0)
                                \operatorname{dist}[v] = \operatorname{dist}[u] + 1;
                                bfs.push(v);
                  }
            return dist[t]!=-1;
     int dinic_dfs (int u, int t, int flow)
            if(u=t)
                  return flow;
            for(int &i=ptr[u]; i<adj[u].size(); i++)</pre>
                  int idx=adj[u][i];
int v=edge[idx].F;
                   if(dist[v] == dist[u] + 1 \&\& edge[idx].S > 0)
                         int cf=dinic_dfs(v, t, min(flow, edge[idx].S));
                         if(cf>0)
                               \begin{array}{l} \operatorname{edge}\left[\operatorname{idx}\right].\operatorname{S-=cf};\\ \operatorname{edge}\left[\operatorname{idx}^{\,\,}1\right].\operatorname{S+=cf}; \end{array}
                                return cf;
                   }
            return 0;
     int maxflow(int s, int t)
            int ret = 0;
            while (dinic_bfs(s, t))
                   memset(ptr, 0, sizeof(ptr));
                   int cf=dinic_dfs(s, t, INF);
                   \mathbf{i} \mathbf{f} (cf == 0)
                         break;
                   ret+=cf;
            return ret;
     }
};
```

2.5.3 Minimum Cost Maximum Flow

Dijkstra

```
/*
undirected graph:
u->uu(flow, 0)
uu->vv(flow, cost)
vv->v(flow, 0)
```

```
v \rightarrow xx (flow, 0)
vv \rightarrow u(flow, 0)
typedef int FTYPE; //type of flow
typedef int CTYPE; //type of cost
typedef pair<FTYPE,CTYPE>pfc;
const CTYPE CINF=INF;
const FTYPE FINF=INF;
void operator+=(pfc &p1, pfc &p2)
{
     p1.F+=p2.F;
     p1.S+=p2.S;
class graph
     \textbf{const static int} \ M\!N\!\!=\!\!1e\!+\!4;
public:
    int n;
    FTYPE flow [MN];
    CTYPE dist [MN], pot [MN];
     int prev[MN], eidx[MN];
     {f struct} Edge
          int to;
         FTYPE \ cap \, ;
          CTYPE cost;
          Edge(){};
          Edge(int _to , FTYPE _cap , CTYPE _cost )
          {
               to = _{-}to;
               cap=_cap;
               cost=_cost;
          }//
     };
     struct node
     {
          int u;
          CTYPE d;
          node(){};
          node(int _u, CTYPE _d)
               u=_{-}u;
               d=_d;
          bool operator <(const node &foo) const
               \mathbf{return} \ d{>} \mathbf{foo.d};
     };
     graph(){};
     vector < int > adj [MN];
     vector<Edge>edge;
     inline void set(int _n)
          n=_n;
     inline void reset()
     {
          for (int i=0; i < MN; i++)
```

```
adj[i].clear();
      edge.clear();
inline void add_edge(int u, int v, FTYPE c, FTYPE cst)
      adj[u].push_back(edge.size());
      edge.push_back(Edge(v, c, cst));
      adj[v].push_back(edge.size());
      edge.push_back(Edge(u, 0, -cst));
pfc dijkstra(int s, int t)
      for(register int i=0; i< n; i++)
             dist[i]=CINF;
      dist[s]=0;
      flow [s]=FINF;
      priority_queue < node > heap;
      heap.push(node(s, 0));
      \mathbf{while}\,(\,!\,\mathrm{heap}\,.\,\mathrm{empty}\,(\,)\,)
            int u=heap.top().u;
            CTYPE d=heap.top().d;
            heap.pop();
             if(d>dist[u])
                   {\bf continue}\,;
             for(int i=0; i<adj[u].size(); i++)
                   int idx=adj[u][i];
int v=edge[idx].to;
                   CTYPE w=edge[idx].cost;
                   \mathbf{if} \; (\,!\, \mathrm{edge}\,[\,\mathrm{idx}\,]\,.\, \mathrm{cap} \;\;|\,| \;\; \mathrm{dist}\,[\,v] \!\!<\!\! = \!\! \mathrm{d} \!\!+\!\! \mathrm{w} \!\!+\!\! \mathrm{pot}\,[\,\mathrm{u}] \!\!-\!\! \mathrm{pot}\,[\,\mathrm{v}\,]\,)
                         continue;
                   if(d+w< dist[v])
                   {
                         \mathrm{d}\hspace{.5mm} i\hspace{.5mm} s\hspace{.5mm} t\hspace{.5mm} [\hspace{.1mm} v\hspace{.1mm}] \hspace{-.5mm} = \hspace{-.5mm} d\hspace{-.5mm} +\hspace{-.5mm} w\hspace{.5mm} ;
                         prev[v]=u;
                         eidx[v]=idx;
                         flow[v]=min(flow[u], edge[idx].cap);
                         heap.push(node(v, d+w));
                   }
            }
      if (dist[t]==CINF)
            return mp(FINF, CINF);
      pfc ret = mp(flow[t], 0);
      for(int u=t; u!=s; u=prev[u])
      {
            \mathbf{int} \ \mathrm{idx}{=}\mathrm{eidx}\left[\,u\,\right];
            edge[idx].cap—flow[t];
edge[idx^1].cap+=flow[t];
ret.second+=flow[t]*edge[idx].cost;
      return ret;
inline pfc mfmc(int s, int t)
      pfc ret=mp(0, 0);
      pfc got;
      while ((got=dijkstra(s, t)).first!=FINF)
           ret+=got;
```

```
return ret;
};
```

Bellmanford

- 2.5.4 Minimum Cut
- 2.6 Tree
- 2.6.1 Lowest Common Ancestor
- 2.6.2 Centroid Decomposition

```
\mathbf{const} int N=1e+5;
const int M=\log 2(N)+1;
set<int>g[N]; //graph
int h[N]; //heigh of nodes
int trSz[N], sz; //tree subsize, size of current tree
int lca[N][M]; //lca sparse table
int cg[N]; //centroid graph
void dfs(int u, int l)
{
       lca[u][0] = 1;
       for (int i=1; i \triangleleft M; i++)
              lca[u][i]=lca[lca[u][i-1]][i-1];
       \quad \quad \mathbf{for} \, (\, \mathbf{auto} \  \, \mathbf{v} \, \colon \! \mathbf{g} \, [\, \mathbf{u} \, ] \, )
              if(v=l)
                    continue;
              h\,[\,v\,]\!=\!h\,[\,u\,]\!+\!1\,;
              dfs(v, u);
       }
}
inline int getLca(int u, int v)
       \mathbf{i}\,\mathbf{f}\,(\,h\,[\,u]\!>\!h\,[\,v\,]\,)
       swap(u, v);
for(int i=M-1; i>=0; i--)
              if(h[v]-(1<< i)>=h[u])
                    v=lca[v][i];
       i f ( u==v )
             return u;
       for (int i=M-1; i>=0; i--)
              if(lca[u][i]!=lca[v][i])
              {
                    \begin{array}{l} u \!\!=\!\! l\, c\, a \; [\, u\, ]\, [\, i\, ]\, ; \\ v \!\!=\!\! l\, c\, a \; [\, v\, ]\, [\, i\, ]\, ; \end{array}
       return lca[u][0];
inline int getDist(int u, int v)
```

```
\textbf{return} \ h\left[\,u\right] + h\left[\,v\right] - 2 * h\left[\,getLca\left(\,u\,,\ v\,\right)\,\right];
void centDfs(int u, int l)
      trSz[u]=1;
      sz++;
      \quad \textbf{for} \, (\, \textbf{auto} \  \, \textbf{v} : \textbf{g} \, [\, \textbf{u} \,] \,)
            if(v=l)
                  continue;
            centDfs(v, u);
            trSz[u]+=trSz[v];
}
int findCentroid(int u, int 1)
      \quad \quad \mathbf{for} \, (\, \mathbf{auto} \ v : g \, [\, \mathbf{u} \,] \,)
            \mathbf{i}\,\mathbf{f}\,(\,v\!\!=\!\!-l\,)
                  continue;
            if(trSz[v]*2>=sz)
                  return findCentroid(v, u);
      return u;
inline void buildCentroid(int u, int l)
      sz=0;
      centDfs(u, u);
      int c=findCentroid(u, u); //actual centroid
      cg[c]=(u=1?c:1);
      for (auto v:g[c])
            g[v].erase(g[v].find(c));
            buildCentroid(v, c);
      g[c].clear();
```

2.6.3 Heavy Light Decomposition on Edges

```
tr[no]=val;
                   return;
             int nxt = (no << 1);
             int mid=(l+r)>>1;
             update(nxt, l, mid, i, val);
update(nxt+1, mid+1, r, i, val);
             tr[no] = tr[nxt] + tr[nxt+1];
      int query(int no, int l, int r, int i, int j)
             \mathbf{i}\,\mathbf{f}\,(\,\mathrm{r}{<}\mathrm{i}\quad |\,|\quad l{>}\mathrm{j}\,)
                   return 0;
             if(l>=i && r<=j)
                  return tr[no];
             int nxt = (no << 1);
             int mid=(l+r)>>1;
             return query(nxt, l, mid, i, j)+query(nxt+1, mid+1, r, i, j);
      }
};
const int N=1e+5;
\mathbf{const} \ \mathbf{int} \ M\!\!=\!\! \log 2 \, (N) \! + \! 1;
int n;
segtree tr;
vector< pair<int, int> >g[N];
int lca[N][M];
int h[N], trSz[N];
//in - use X[], Y[] in case
//of edge weights
\quad \textbf{int} \ X[N] \ , \ Y[N] \ , \ W[N] \ ;
//hld
int chainInd [N], chainSize [N], chainHead [N], chainPos [N], chainNo, posInBase [N];
int ptr;
void dfs(int u, int l)
{
      trSz[u]=1;
      lca[u][0] = 1;
      for (int i=1; i \triangleleft M; i++)
            lca[u][i]=lca[lca[u][i-1]][i-1];
      for(int i=0; i<g[u].size(); i++)
             \mathbf{int} \ v \!\!=\!\! g \left[\, u\,\right] \left[\,\, i\,\,\right]. \ f \, i \, r \, s \, t \,\, ;
             if (v==1)
                   continue;
            h\,[\,v\,]\!=\!h\,[\,u\,]\!+\!1\,;
             dfs(v, u);
             \operatorname{tr} Sz\;[\,u] + = \operatorname{tr} Sz\;[\,v\,]\;;
      }
}
\textbf{inline int} \hspace{0.1cm} \mathtt{getLca} \hspace{0.1cm} (\hspace{0.1cm} \textbf{int} \hspace{0.1cm} u \hspace{0.1cm}, \hspace{0.1cm} \textbf{int} \hspace{0.1cm} v)
      \mathbf{i}\,\mathbf{f}\,(\,h\,[\,u\,]\!>\!h\,[\,v\,]\,)
             swap\left(\,u\,,\ v\,\right);
      for (int i=M-1; i>=0; i--)
             if(h[v]-(1<< i)>=h[u])
                   v=lca[v][i];
      i f ( u==v )
```

```
return u;
     for (int i=M-1; i>=0; i--)
            if(lca[u][i]!=lca[v][i])
                 u{=}\,l\,c\,a\;[\,u\,]\,[\,\,i\,\,]\,;
                 v=lca[v][i];
     return lca[u][0];
//dont use 'c' if the weight is on the vertex
//instead of the edge
inline void hld(int u, int l, int c)
     if (chainHead [chainNo]==-1)
           chainHead [chainNo]=u;
     chainInd [u]=chainNo;
     chainPos [u]=chainSize [chainNo]++;
      tr.update(1, 0, n, ptr, c);
     posInBase[u]=ptr++;
     int msf, idx;
     msf = idx = -1;
     for (int i=0; i < g[u]. size (); i++)
           \mathbf{int} \ v{=}g\,[\,u\,]\,[\,\,i\,\,]\,.\,\,f\,i\,r\,s\,t\,\,;
            if (v==1)
                 continue;
            if(trSz[v]>msf)
            {
                 msf=trSz[v];
                 idx=i;
           }
      if(idx >= 0)
           hld\,(\,g\,[\,u\,]\,[\,idx\,]\,.\,\,first\ ,\ u\,,\ g\,[\,u\,]\,[\,idx\,]\,.\,\,second\,)\,;
     \label{eq:for_int} \  \, \text{for} \, (\, \text{int} \  \  \, \text{i} = \! 0; \  \, \text{i} \! < \! \! \text{g} \, [\, u \, ] \, . \, \, \text{size} \, (\, ) \, ; \  \, \text{i} \! + \! + \! )
            if(i=idx)
                 continue;
           \mathbf{int} \ v \!\!=\!\! g\left[\, u\,\right]\left[\,\, i\,\,\right]. \ first \ ;
           int w=g[u][i].second;
            if (v==1)
                continue;
           {\rm chainNo}++;
           hld(v, u, w);
     }
inline int query_up(int u, int v)
     int uchain=chainInd[u];
     int vchain=chainInd[v];
     int ret = 0;
     \mathbf{while}(\mathbf{true})
            uchain=chainInd[u];
            if (uchain=vchain)
            {
                  ret+=tr.query(1, 0, n, posInBase[v]+1, posInBase[u]);
```

```
break;
             int head=chainHead[uchain];
ret+=tr.query(1, 0, n, posInBase[head],posInBase[u]);
             u=lca[u][0];
      return ret;
//returns sum of all edges weights //from 'u' to 'v'
inline int query(int u, int v)
      \mathbf{i} \mathbf{f} (\mathbf{u} = \mathbf{v})
             return 0;
      \quad \textbf{int} \quad l \!=\! \text{getLca}\left(u\,,\ v\,\right);
      return query_up(u, l)+query_up(v, l);
//set and edge to value 'val'
inline void update(int u, int val)
{
      \begin{array}{ll} \mbox{int} & x\!\!=\!\!X[\,u\,] \;, \;\; y\!\!=\!\!Y[\,u\,] \;; \\ \mbox{if} \; (\;\!1c\,a\,[\,x\,][\,0\,] \!=\! =\! y \;) \end{array}
            tr.update(1, 0, n, posInBase[x], val);
             tr.update(1,\ 0,\ n,\ posInBase[y],\ val);
void clearHld()
       //tr.reset();
      for (int i=0; i <= n; i++)
             g[i].clear();
             chainHead[i]=-1;
             chainSize[i]=0;
      ptr=1;
      chainNo=0;
int main()
{
      scanf("%d", \&n);
      clearHld();
      for (int i=1; i < n; i++)
      {
              \begin{array}{l} scanf(\mbox{``%d\_\%d\_\%d''}\,,\ \&X[\,i\,]\,,\ \&Y[\,i\,]\,,\ \&W[\,i\,]\,)\,;\\ g[\ X[\,i\,]\ ]\,.push\_back(\{Y[\,i\,]\,,\ W[\,i\,]\})\,;\\ g[\ Y[\,i\,]\ ]\,.push\_back(\{X[\,i\,]\,,\ W[\,i\,]\})\,; \end{array} 
      dfs(1, 0);

hld(1, 0, 0);
      int q;
scanf("%d", &q);
      \mathbf{while} (q--)
             if (o==1)
```

- ${\bf 2.6.4}\quad {\bf Heavy\ Light\ Decomposition\ on\ Vertex}$
- 2.6.5 All-Pairs Distance Sum
- 2.7 MISC
- 2.7.1 2-SAT

Dynamic Programming

3.1 Optimizations

3.1.1 Divide and Counquer

```
/// David Mateus Batista <david.batista3010@gmail.com>
/// Computer Science - Federal University of Itajuba - Brazil
/// Uri Online Judge - 2475
#include <bits/stdc++.h>
 using namespace std;
 typedef long long ll;
typedef unsigned long long ull; typedef long double ld;
 typedef pair<int,int> pii;
typedef pair<ll, ll> pll;
#define INF 0x3F3F3F3F
#define LINF 0x3F3F3F3F3F3F3F3F1LL
#define DINF (double)1e+30
#define EPS (double)1e-9
#define PI (double) acos(-1.0)
#define RAD(x) (double)(x*PI)/180.0
#define PCT(x,y) (double)x*100.0/y
#define pb push_back
#define mp make_pair
#define pq priority_queue
#define F first
#define S second
#define D(x) x&(-x)
#define ALL(x) x.begin(),x.end()
#define SET(a,b) memset(a, b, sizeof(a))
\#define DEBUG(x,y) cout << x << y << endl
#define gcd(x,y) = gcd(x, y)
#define lcm(x,y) = (x/gcd(x,y))*y
#define bitcnt(x) __builtin_popcountll(x)
#define lbit(x) 63-_builtin_clzll(x)
\#define zerosbitll(x) __builtin_ctzll(x)
#define zerosbit(x) __builtin_ctz(x)
\mathbf{enum} \ \left\{ \mathrm{North} \ , \ \mathrm{East} \ , \ \mathrm{South} \ , \ \mathrm{West} \right\};
//\{0, 1, 2, 3\}
```

```
//{Up, Right, Down, Left}
const int MN=1e+4+35;
const int MN2=535;
int p, a;
ll data [MN];
inline ll getValue(int l, int r)
{
      return (r-l+1)*(data[r]-data[l-1]);
 11 dp[MN2][MN];
inline void solve(int k, int l, int r, int L, int R)
{
      if(l>r)
            return;
      int m=(l+r)/2;
      int s=L;
      dp[k][m]=LINF;
      for(int i=max(m, L); i<=R; i++)
            if(dp[k][m]>dp[k-1][i+1]+getValue(m+1, i+1))
                  dp\,[\,k\,]\,[m] \!=\! dp\,[\,k-1]\,[\,i+1] \!+\! get\,V\,alu\,e\,(m\!+\!1,\ i+\!1)\,;
      solve(k, l, m-1, L, s);
solve(k, m+1, r, s, R);
int main()
{
      \begin{array}{l} {\rm s\,c\,a\,n\,f\,(\,''\%d\,.\%d''}\;,\;\;\&p\,,\;\;\&a\,)\,;\\ {\rm \bf for\,(\,int}\ \ i\,=\!1;\;\;i\,<\!\!=\!\!p\,;\;\;i\,+\!+\!)} \end{array}
            11 x;
            scanf("%lld", &x);
            data[i] = data[i-1] + x;
      \  \, \mathbf{for} \, (\, \mathbf{int} \quad i = \! 0; \quad i \! < \! \! = \! \! p \, ; \quad i + \! \! +)
            dp[0][i]=LINF;
      for (int i=0; i <=a; i++)
            dp[i][p]=0;
      for(int i=1; i<=a; i++)
      solve(i, 0, p-1, 0, p-1); printf("%lld\n", dp[a][0]);
      return 0;
```

3.1.2 Convex Hull I

Linear

```
//Original recurrence:
```

```
dp[i]=min(dp[j]+b[j]*a[i]) for j<i
// Condition:
       b\left[\;j\right]>=b\left[\;j+1\right]
       a[i] \le a[i+1]
   Solution:
// Hull cht=Hull();
// cht.insertLine(b[0], dp[0])
// for (int i=1; i<n; i++)
       dp[i]=cht.query(a[i]);
       cht.insertLine(b[i], dp[i])
// answer is dp[n-1]
class Hull
{
     const static int CN=1e+5+35;
public:
     \begin{array}{ll} \textbf{long long} \ \ a\,[CN] \ , \ \ b\,[CN] \ ; \\ \textbf{double} \ \ x\,[CN] \ ; \end{array}
     int head, tail;
     Hull(): head(1), tail(0){};
     long long query(long long xx)
          if (head>tail)
               return 0;
          \mathbf{while} (\text{head} < \text{tail && } x [\text{head} + 1] < = xx)
               head++;
          x[head]=xx;
          return a [head] * xx+b [head];
     }
     void insertLine(long long aa, long long bb)
          double xx=-1e18;
          while (head <= tail)
                if(aa==a[tail])
                    return;
               xx=1.0*(b[tail]-bb)/(aa-a[tail]);
                if(head==tail || xx>=x[tail])
                    break;
                tail --;
          a[++tail]=aa;
          b[tail]=bb;
          x[tail]=xx;
     }
};
```

Dynamic

```
//Original recurrence:
// dp[i]=min(dp[j]+b[j]*a[i]) for j<i
//Condition:
// b[j]>=b[j+1]
// a[i]<=a[i+1]
// Solution:
// HullDynamic cht;
// cht.insertLine(b[0], dp[0])
```

```
// for(int i=1; i< n; i++)
     dp[i]=cht.query(a[i]);
cht.insertLine(b[i], dp[i])
// answer is dp[n-1]
const long long is_query=-(1LL<<62);</pre>
class Line
public:
   long long m, b;
    mutable function < const Line *() > succ;
    bool operator < (const Line &rhs) const
        if(rhs.b!=is_query)
            return m<rhs.m;
        const Line *s=succ();
        if(!s)
           return 0;
        long long x=rhs.m;
        {f return} \ (b-s->b)<((s->m-m)*x);
   }
};
class HullDynamic: public multiset <Line>
public:
   void clear()
    {
        clear();
   bool bad(iterator y)
        auto z=next(y);
        if(y==begin())
        {
            if (z==end())
                return 0;
            return (y->m=z->m && y->b<=z->b);
        auto x=prev(y);
        if (z=end())
            void insertLine(ll m, ll b)
        auto y=insert(\{m, b\});
        y \rightarrow succ = [=]
            return next(y)==end()?0:&*next(y);
        if (bad(y))
            erase(y);
            return;
        \mathbf{while}(\text{next}(y)!=\text{end}() \&\& \text{bad}(\text{next}(y)))
        erase(next(y));
while(y!=begin() && bad(prev(y)))
            erase(prev(y));
```

```
}
long long query(long long x)
{
    auto ret=*lower_bound((Line){x, is_query});
    return ret.m*x+ret.b;
}
};
```

- 3.1.3 Convex Hull II
- 3.1.4 Knuth Optimization
- 3.2 Digits
- 3.3 Grundy Numbers

String

- 4.1 Hash
- 4.2 KMP
- 4.3 Aho Corasick
- 4.4 Manacher
- 4.5 Z-Algorithm
- 4.6 Suffix Array & LCP
- 4.7 Suffix Tree

Mathematic

- 5.1 Prime Numbers
- 5.1.1 Erastotenes Sieve
- 5.1.2 Linear Sieve
- 5.1.3 Miller Rabin
- 5.1.4 BPSW
- 5.1.5 Primality Test
- 5.2 Chinese Remainder Theorem
- 5.3 Fast Fourier Transformation

```
#define PI (double) acos(-1.0)
typedef complex<double> base;
void fft(vector<base>&data, bool invert)
      int n=data.size();
      \label{eq:formalized} \mbox{for}\,(\,\mbox{i}\,\mbox{nt}\,\ i\,{=}1\,,\ j\,{=}0\,;\ i\,{<}n\,;\ i\,{+}{+})
            int bit=n>>1;
            for(; j>=bit; bit>>=1)
                j-=bit;
            j+=bit;
            \mathbf{i}\mathbf{f}(i < j)
                 swap(data[i], data[j]);
      for(int len=2; len <=n; len <<=1)
            \label{eq:double_ang} \textbf{double} \ \ \text{ang=2*PI/len*(invert?-1:1);}
            base wlen(cos(ang), sin(ang));
for(int i=0; i<n; i+=len)
                  base w(1);
                  for (int j=0; j < len / 2; j++)
```

```
base \ u \!\!=\!\! data\,[\,i \!+\! j\,]\,, \ v \!\!=\!\! data\,[\,i \!\!+\! j \!+\! len\,/\,2\,] \!*\! w;
                          data[i+j]=u+v;
                          data[i+j+len/2]=u-v;
                         w = wlen;
                   }
             }
      if (invert)
            for (int i=0; i < n; i++)
                   data[i]/=n;
vector<int>fft_multiply(vector<int>&a, vector<int>&b)
       \begin{array}{l} {\rm vector}{<}{\rm base}{>}{\rm fa}\left({\rm a.begin}\left(\right),\ {\rm a.end}\left(\right)\right); \\ {\rm vector}{<}{\rm base}{>}{\rm fb}\left({\rm b.begin}\left(\right),\ {\rm b.end}\left(\right)\right); \end{array} 
      int n=1;
      while(n<max(a.size(), b.size()))
           n < < =1;
      n << =1;
      fa.resize(n);
      fb.resize(n);
      fft(fa, false);
      fft (fb, false);

for (int i=0; i<n; i++)

fa[i]*=fb[i];
      fft(fa, true);
      vector < int > ret (n);
for (int i = 0; i < n; i++)</pre>
             ret[i]=(int)(fa[i].real()+0.5);
      int carry=0;
      for (int i=0; i< n; i++)
             \mathtt{ret}\;[\;i] += \mathtt{carry}\;;
             carry=ret[i]/10;
             ret[i]%=10;
      return ret;
}
int main()
      int n, m;
scanf("%d_%d", &n, &m);
      vector < int > a, b;
      for (int i=0; i < n; i++)
             int x;
            scanf("%d", &x);
             a.pb(x);
      }
      for (int i=0; i \le m; i++)
            scanf("%d", &x);
            b.pb(x);
      reverse(a.begin(), a.end());
```

```
reverse(b.begin(), b.end());

vector<int>ans=fft_multiply(a, b);
reverse(ans.begin(), ans.end());
bool flag=false;
for(int i=0; i<ans.size(); i++)
{
    if(ans[i])
        flag=true;
    if(flag)
        printf("%d", ans[i]);
}
printf("\n");
return 0;
}</pre>
```

5.4 Modular Math

- 5.4.1 Multiplicative Inverse
- 5.4.2 Linear All Multiplicative Inverse
- 5.5 Gaussian Elimination
- 5.6 Combinatorics

Geometry

- 6.1 2d Template
- 6.2 3d Template
- 6.3 Polygon Template
- 6.4 Convex Hull
- 6.4.1 Graham Scan
- 6.4.2 Monotone Chain
- 6.5 Rotating Calipers
- 6.6 KD Tree
- 6.7 Range Tree
- 6.8 Circle Sweep