(2)

No-signalling : $A \not\Rightarrow B$. $A \not\Leftarrow B$

$$\begin{cases} p_B(b|y) \overset{NS}{=} p_B(b|x,y) = \sum_a p(ab|xy) \\ \quad \hookrightarrow \text{marginal} \\ p_A(a|x) \overset{NS}{=} p_A(a|x,y) = \sum_b p(ab|xy) \end{cases}$$

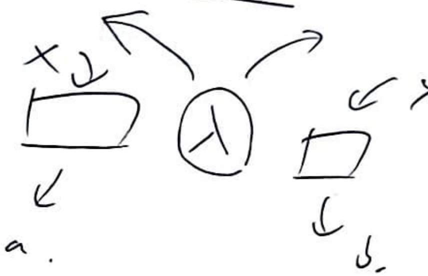
Exc.

$$p(ab|xy) = \begin{cases} 1/2 & a \oplus b = xy \\ 0 & \text{oth.} \end{cases}$$

Q
① NS ?
② L/NL ?

Non local correlations. 벨부등식 벨상태 (1) [1].

A.



$p_{AB}(ab|xy) = p_A(a|x)\, p_B(b|y)$; indep.

$p_A(a|x)$ $\quad p_B(b|y)$

λ : local hidden variable.
(LHV) $\longrightarrow$ shared randomness (SR)

$p_A(a|x,\lambda)$ $\quad p_B(b|y,\lambda)$ $\rightarrow$ Common Cause

$$p(ab|xy) = \sum_\lambda p(\lambda)\, p_A(a|x,\lambda)\, p_B(b|y,\lambda)$$

$\hookrightarrow$ local probability.

$\mathcal{E}$ CVX Combination.

$p(ab|xy) \neq$ (Non local).

$\rho^{AB} \neq$ nentangled state.

$|\phi^+\rangle\langle\phi^+|$

- sep. by. LOCC.

- Quantum interaction (Direct interaction)

$U = e^{-iHt}$

$H = A \otimes B$

- Quantum Commication (Indirect Interaction)

Quantum Correlation

C.C { Classical Communication

Q.C { Quantum

LO $\sim p_i$ LO

local Quantum operation

ρ^A ρ^B

$\otimes$

곱해서 얻어진다.

$\rho^{AB} = \rho^A \otimes \rho^B$ ~ product state

more Generally,

ρ_i^A ρ_i^B

$\rho^{AB} = \sum_i p_i \, \rho_i^A \otimes \rho_i^B$ prod. Mixture

~ separable state

(LOCC)

pre preparation.

EW : $W = W^{+}$ { 에르미트 op. / sel adjoint opt. }

: observable.

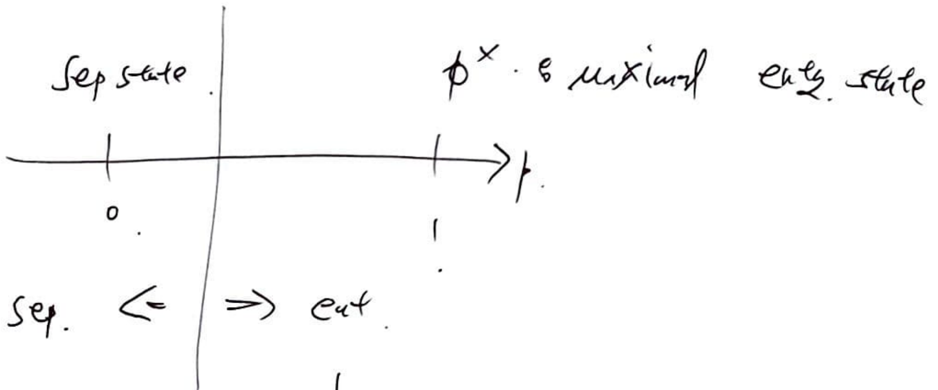
$\forall \sigma_{sep}$ $tr[W \sigma_{sep}] \geq 0$.

ex. $\underbrace{W = \frac{1}{2} I \times I - |\phi^{+}\rangle\langle\phi^{+}|}_{}$: $\min_{\sigma_{sep}} tr|W\sigma_{sep}| = 0$.

pf. Exc. witness

NTC : $|e\rangle|+\rangle$

$\rho = p|\phi^{+}\rangle\langle\phi^{+}| + (1-p)\frac{1}{4} I \otimes I$.



$p^{*} \in \frac{1}{3}$.

$tr[W_p] = \frac{1}{4}(1-3p) < 0 \Leftrightarrow p > \frac{1}{3}$.

② ③ ①

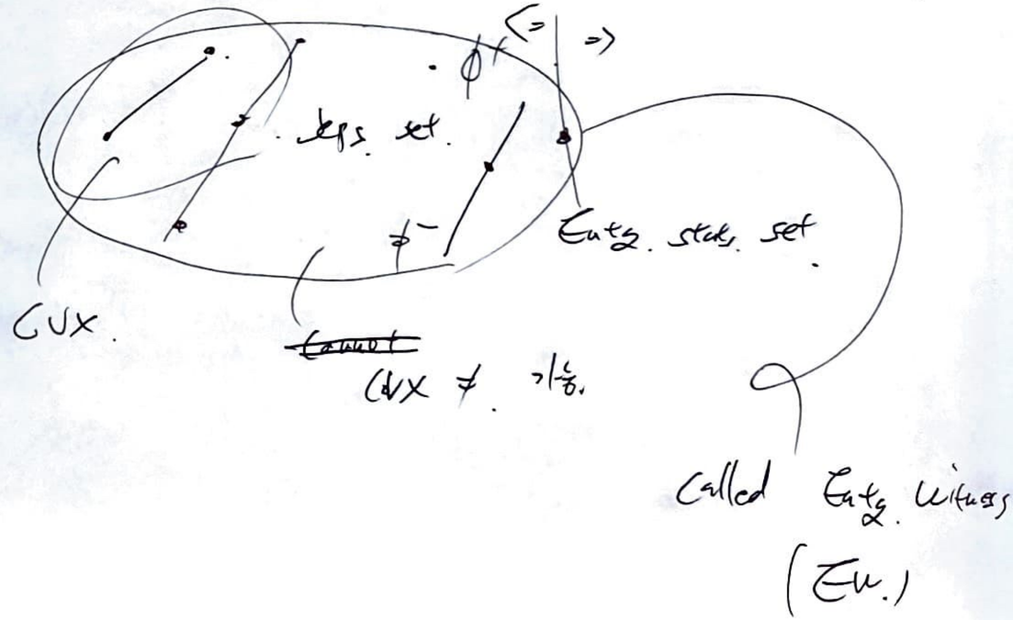
Non local probs.

$p_{AB}^{(H}(ab|xy) = \sum_{\lambda} p(\lambda)\, p(a|x,\lambda)\, p(b|y,\lambda)$

Entang. states.

$\sigma^{sep} = \sum_i p_i \rho_i^{A} \otimes \rho_i^{B}$.

$p\,\sigma_1^{sep} + (1-p)\,\sigma_2^{sep} = \sigma \in sep.$



④

$S = \{|0\rangle, |1\rangle, |\pm\rangle,$
$|\pm i\rangle\}$.

i.e. $\frac{1}{6}\sum_{|i\rangle \in S} |ii\rangle\langle ii|$.

2-design. (In coding theory)

---

4. $|\phi^+\rangle$ $|\phi^-\rangle$ $|\psi^+\rangle$ $|\psi^-\rangle$.
$1/4$ $1/4$ $1/4$ $1/4$

$\rho = I \otimes I/4$.

Sexs

$\rho \to$ ent? / sep?

NP-Hard class. (In $O(n)$)

(Can not treat on present Computer Science skill.)

③

Ex. Ion trap. $\to$ ....

Exs.

A B.

1. $|\phi^+\rangle$, $|\phi^-\rangle$. $\rho_1 = \frac{1}{2}|\phi^+\rangle\langle\phi^+| + \frac{1}{2}|\phi^-\rangle\langle\phi^-|$.

2. $|00\rangle$ $|11\rangle$. $\rho_2^{loc} = \frac{1}{2}|0\rangle\langle 0|_A \otimes |0\rangle\langle 0|_B + \frac{1}{2}|1\rangle\langle 1|_A \otimes |1\rangle\langle 1|_B$.

$\in$ Sep.

$\forall M_{AB}$, $\mathrm{tr}[M_{AB}\rho_1]$ $\mathrm{tr}[M_{AB}\rho_2]$.

---

3. $|\phi^+\rangle$, $|\phi^-\rangle$, $|\psi^+\rangle$. $\rho_3 = \frac{1}{3}|\phi^+\rangle\langle\phi^+| + \frac{1}{3}|\phi^-\rangle\langle\phi^-| + \frac{1}{3}|\psi^+\rangle\langle\psi^+|$.

$\in$ Sep.

$= \frac{1}{6}\sum_{i\in S} |ii\rangle\langle ii|$. (:

②

$w = 3/4$ $\quad$ $w = \cos^2 \pi/8 \sim 0.854$ $\quad$ $w = 1$

L. $\quad$ Q. $\quad$ NS.

: Models range.

$$p_{NC} = \begin{cases} 1/2 & a+b = xy \\ 0 & \text{otw} \end{cases}$$

Called PR box.

$|\phi^+\rangle$.

	$a=0$	$a=1$
$x=0$	$\|0\rangle$	$\|1\rangle$
$x=1$	$\|+\rangle$	$\|-\rangle$

	$b=0$	$b=1$
$y=0$	$\cos\frac{\pi}{8}\|0\rangle + \sin\frac{\pi}{8}\|1\rangle$	$(\ )^\perp$
$y=1$	$\cos\frac{\pi}{8}\|0\rangle - \sin\frac{\pi}{8}\|1\rangle$	$(\ )^\perp$

④, ①

Nonlocal game.

$x \to A \to a$ $\quad$ $y \to B \to b$

$$w(p) = \sum_{abxy} \underbrace{\pi(x,y)}_{1/4} V(ab|xy) \underbrace{p(ab|xy)}$$

Quantum prob.

$$p(a|x) = \mathrm{tr}[\rho\, M^x_a]$$

$$p(ab|xy) = \mathrm{tr}[\rho\, M^x_a \otimes M^y_b]$$

state $\quad$ mea.

$S_{CHSH}(p) = 4$

V: payoff score.

$$= \frac{1}{4}\sum_{abxy} \underbrace{V(ab|xy)}_{\text{score}} p(ab|xy).$$

$$V = \begin{cases} 1 & a+b=xy \\ 0 & \text{otw.} \end{cases}$$

$$= \frac{1}{4}\sum_{a+b=xy} p(ab|xy).$$

L, Q, NC.

4

? :

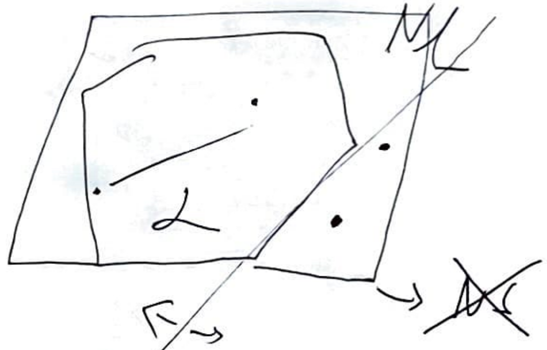
cf. Information Causality (why $2\sqrt{2}$) Explained.

" 정보 처리 "

3, 3

Recall non-local pols.

given $\mu\, p^{(L)} + (1-\mu)\, \Sigma^{(L)} \in \mathcal{L}$



Γ : witness.

The line : Bell - Ineq.

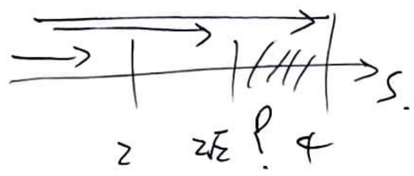
CHSH : $\sum_{abxy} (-1)^{a\,b\,xy}\, p(ab|xy) = S_{CHSH}$ polynomial.

o local ( )
o nonlocal ( )
o quantum ( )

$\overset{L}{\leq} 2$. (local bd).

$\overset{Q}{\leq} 2\sqrt{2}$. (quantum bd).

$\overset{NS}{\leq} 4$



$A(x) = 1$. $B(y) = 1$. $C(z) = 1$.

$w_c \le \frac{3}{4}$. $w_q = 1$.

Quantum strategy :

$|\psi\rangle = \frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right)$.

$x = 0 \to Z$. $y = 0 \to Z$.

$x = 1 \to X$. $y = 1 \to X$. ...

Ex.

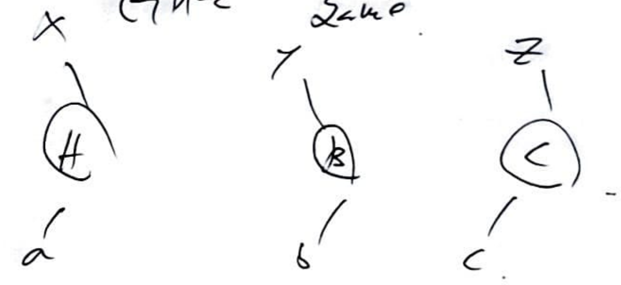
$p(\text{win} \mid 000) = p(a+b+c=0 \mid 000)$

$= \sum_{a+b+c=0} \mathrm{tr}\left[|\psi\rangle\langle\psi| M_a^0 \otimes M_b^0 \otimes M_c^0\right]$

$= \langle\psi| \underbrace{\sum_{a+b+c=0} M_a^0 \otimes M_b^0 \otimes M_c^0}_{|000\rangle\langle 000| + |011\rangle\langle 011| + |101\rangle\langle 101| + |110\rangle\langle 110|} |\psi\rangle$

$= 1$.

GHZ Game.



$a = A(x)$ $b = B(y)$ $c = C(z)$.

They win if.

x y z	a+b+c
0 0 0	0
0 1 1	1
1 0 1	1
1 1 0	1

R "Referee"

Ex. $(x, y, z) = (0, 0, 1)$

$= (0, 0, 0)$

if $a+b+c = 1$, R win.

$= 0$, abc win.

$1 = 0 + 1 + 1 + 1$

$= (A(0) + B(0) + C(0)) + (A(0) + B(1) + C(1))$

$+ \cdots$

$= 2(A(0) + A(1) + \cdots) \ne 0 \Rightarrow$ ※.