习题7-8

1. $f_x' = 4x-y-6$，$f_x'|_{\substack{x=1\\y=1}} = -3$

$f_y' = -x-2y-3$，$f_y'|_{\substack{x=1\\y=1}} = -6$

$f''_{xy} = -1$，$f''_{x^2} = 4$，$f''_{y^2} = -2$.

$\therefore \cancel{f(0,0)=-4}\ f(1,1) = -4$

$\therefore f(x,y) = -4 - 3(x-1) - 6(y-1) + \frac{1}{2!}[4(x-1)^2 - 2(x-1)(y-1) - 2(y-1)^2]$

$= -4 - 3(x-1) - 6(y-1) + 2(x-1)^2 - (x-1)(y-1) - (y-1)^2$

2. $f(0,0)=0$

$f'_x = 2x\cos(x^2+y^2)$，$f''_{x^2} = 2\cos(x^2+y^2) - 4x^2\sin(x^2+y^2)$

$f'_y = 2y\cos(x^2+y^2)$，$f''_{y^2} = 2\cos(x^2+y^2) - 4y^2\sin(x^2+y^2)$

$f''_{xy} = -4xy\sin(x^2+y^2)$，$f'''_{x^2y} = -4y\sin(x^2+y^2) - x^2y\cos(x^2+y^2)$

$f'''_{xy^2} = -4x\sin(x^2+y^2) - xy^2\cos(x^2+y^2)$

$f'''_{x^3} = -12x\sin(x^2+y^2) - 8x - 4x^2\cos(x^2+y^2)$

$f'''_{y^3} = -12y\sin(x^2+y^2) - 4y^2\cos(x^2+y^2)$

$\therefore f'_x(0,0)=0$，$f'_y(0,0)=0$，$f''_{x^2}(0,0)=2$，$f''_{y^2}(0,0)=2$.

$f''_{xy}=0$，$f'''_{x^2y}(0,0)=0$，$f'''_{xy^2}(0,0)=0$，$f'''_{x^3}(0,0)=0$

$f'''_{y^3}(0,0)=0$

$\therefore f(x,y) = 0+0+0+\frac{1}{2!}(x^2+0+y^2)+o(\rho^2)$

3. 由题有

$f(0,0)=1$.

$f'_x = f'_y = f''_{xy} = e^{x+y}$.

$f''_{x^2} = f''_{y^2} = f'''_{x^2y} = f'''_{xy^2} = e^{x+y}$.

$f'''_{x^3} = f'''_{y^3} = e^{x+y}$.

$\therefore$ 代入$(0,0)$，$e^{x+y}=1$.

$\therefore f(x,y) = f(0,0) + f'_x(0,0)\cdot x + f'_y(0,0)y + \frac{1}{2!}(f''_{x^2}(0,0)\cdot x^2 + f''_{xy}(0,0)xy + f''_{y^2}(0,0)\cdot y^2) + \frac{1}{3!}(f'''_{x^3}(\theta x,\theta y)\cdot x^3 + 3x^2y f'''_{x^2y}(\theta x,\theta y) + 3xy^2 f'''_{xy^2}(0,0) + f'''_{y^3}(0,0)\cdot y^3)$

$= 1+x+y+\frac{1}{2}(x^2+2xy+y^2) + \frac{1}{3!}(x^3+3x^2y+3xy^2+y^3)\cdot e^{\theta(x+y)}$

$(0<\theta<1)$

4. $f'_x = f''_{x^2} = f'''_{x^3} = e^x\cdot\ln(1+y)$；$f'_x|_{(0,0)} = f''_{x^2}|_{(0,0)} = f'''_{x^3}|_{(0,0)} = 0$

$f'_y = f''_{xy} = \frac{e^x}{1+y}$；$f'_y|_{(0,0)} = f''_{xy}|_{(0,0)} = 1$

$f''_{y^2} = -\frac{e^x}{(1+y)^2}$　　$f''_{y^2}|_{(0,0)} = -1$

$f'''_{y^3} = 2\cdot\frac{e^x}{(1+y)^3}$　　$f'''_{y^3}|_{(0,0)} = 2$

$f'''_{x^2y} = \frac{e^x}{1+y}$　　$f'''_{x^2y}|_{(0,0)} = 1$

$f'''_{xy^2} = -\frac{e^x}{(1+y)^2}$　　$f'''_{xy^2}|_{(0,0)} = -1$

$\therefore$ 二阶麦克劳林展开式

$f(x,y) = f(0,0) + y + \frac{1}{2!}[2xy-y^2] + \frac{1}{3!}[3x^2y-3xy^2+2y^3] + o(\rho^3)$

$f(x,y) = y + \frac{1}{2!}[2xy-y^2] + \frac{1}{3!}[3x^2y-3xy^2+2y^3] + o(\rho^3)$

5. $f'_x = \cos x\sin y$

$f'_y = \cos y\sin x$

$f''_{x^2} = -\sin x\cdot\sin y$

$f''_{y^2} = -\sin y\sin x$

$f''_{xy} = -\cos x\cos y$

代入$(\frac{\pi}{4},\frac{\pi}{4})$

有$f(\frac{\pi}{4},\frac{\pi}{4}) = \frac{1}{2}$

$f'_x(\frac{\pi}{4},\frac{\pi}{4}) = f'_y(\frac{\pi}{4},\frac{\pi}{4}) = \frac{1}{2}$

$f''_{x^2}(\frac{\pi}{4},\frac{\pi}{4}) = f''_{y^2}(\frac{\pi}{4},\frac{\pi}{4}) = f''_{xy}(\frac{\pi}{4},\frac{\pi}{4}) = -\frac{1}{2}$

$\therefore f(x,y) = \frac{1}{2}[1+(x-\frac{\pi}{4})+(y-\frac{\pi}{4}) + \frac{1}{2!}((x-\frac{\pi}{4})^2 - 2(x-\frac{\pi}{4})(y-\frac{\pi}{4}) + (y-\frac{\pi}{4})^2)] + o(\rho^2)$

$= \frac{1}{2}\{1+x+y-\frac{\pi}{2}+\frac{1}{2!}[(x-y)^2]\} + o(\rho^2)$

$= \frac{1}{2} + \frac{1}{2}(x+y) - \frac{\pi}{4} + \frac{1}{4}(x-y)^2 + o(\rho^2)$

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