1.

a)

> $f := x \rightarrow \begin{cases} a \cdot e^{b \cdot x} & x > 0 \\ 0 & x \leq 0 \end{cases}$

$$f := x \mapsto \begin{cases} a \cdot e^{b \cdot x} & 0 < x \\ 0 & x \leq 0 \end{cases}$$

Since f is a probability density function then:

- $\int_{-\infty}^{+\infty} f(x)dx = 1$

$$\lim_{x \to \infty} \frac{a\left(e^{b x} - 1\right)}{b}$$

>
> $eq1 := int(f(x), x = 0 ..infinity)$

$$eq1 := \lim_{x \to \infty} \frac{a\left(e^{b x} - 1\right)}{b}$$

> $eq1 := eq1 = 1$

$$eq1 := \lim_{x \to \infty} \frac{a\left(e^{b x} - 1\right)}{b} = 1$$

$E(X) = \int_{-inf}^{+inf} x \cdot f(x)\, dx$

> $eq2 := int(x \cdot f(x), x = 0 ..infinity)$

$$eq2 := \lim_{x \to \infty} \frac{a\left(e^{b x} b x - e^{b x} + 1\right)}{b^2}$$

> $eq2 := eq2 = 5$

$$eq2 := \lim_{x \to \infty} \frac{a\left(e^{b x} b x - e^{b x} + 1\right)}{b^2} = 5$$

> $sol1 := fsolve(\{eq1, eq2\}, \{a, b\})$

$$sol1 := \{a = 0.2000000000, b = -0.2000000000\}$$

a=0.2 and b=-0.2

b)

B) Find the median of X

The median of a probability density function is:

$Median = \int_{a}^{Median} f(x)\, dx$ **or** $\int_{Median}^{b} f(x)\, dx = \frac{1}{2}$

> $eq3 := int(f(x), x = 0 ..mediana) = \frac{1}{2}$

$$eq3 := -1.\, e^{-0.2000000000\, mediana} + 1. = \frac{1}{2}$$

> $evalf(solve(eq3, mediana))$

$$3.465735903$$

Median=3.47

c)

3.465735903

C) Find the mode of X

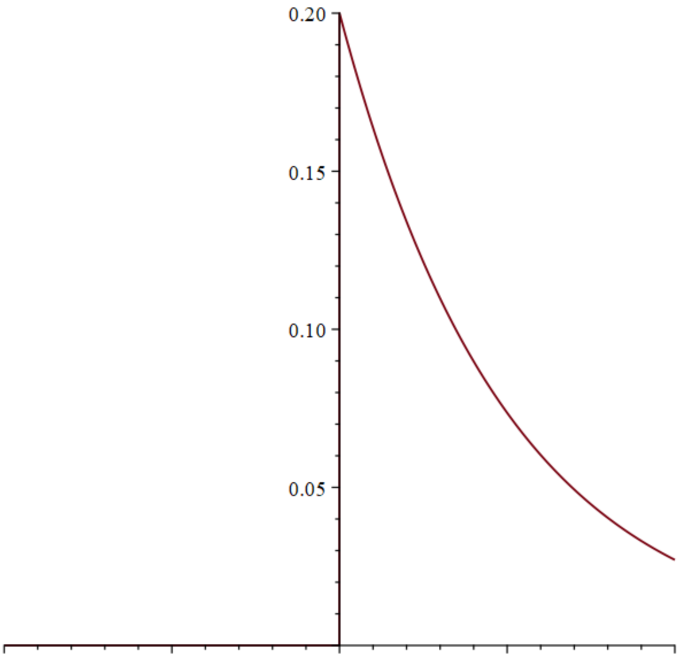
> $limit(f(x), x = 0, right)$

0.2000000000

> $f(0.0001)$

0.1999960000

▸ *plot*(**(8)**)



From the graph we can see that the function f(x) is always decreasing this means that the most common value is the one where X=0. The mode is 0.2.

d)

D) Probability a customer will queue for longer than 4 minutes

$$P(X > 4) = \int_{4}^{infinity} f(x)\,dx$$

$P(4 < X)$

> $evalf(int(f(x), x = 4..infinity))$

0.4493289641

P(X>4)=0.45

e)

E) A customer has been waiting for 4 minutes what is the probability he will wait up to 9 minutes

For a customer to wait 9 minutes this means he already has waited 4 minutes this makes A a subset of B.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}.$$

P(X>9|X>4)=P(X>9)/ P(X>4)

> $xmaior9 := evalf(int(f(x), x = 9 ..\text{infinity}))$

$$xmaior9 := 0.1652988882$$

> $xmaior4 := evalf(int(f(x), x = 4 ..\text{infinity}))$

$$xmaior4 := 0.4493289641$$

> $\frac{xmaior9}{xmaior4}$

$$0.3678794411$$

P(X>9|X>4)=0.37

f)

F) Write distribution function F

F(t) is the integral of f(t)

> $F := x \rightarrow int(f(x), x = -\text{infinity} ..x)$

$$F := x \mapsto \int_{-\infty}^{x} f(x)\, dx$$

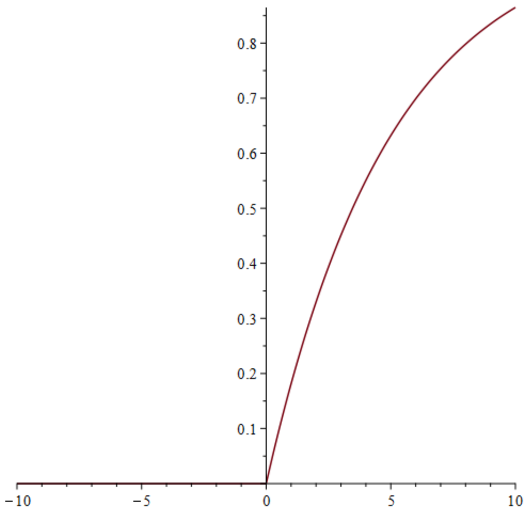
> $F(x)$

$$\begin{cases} 0. & x \le 0. \\ -1.\, e^{-0.2000000000\, x} + 1. & 0. < x \end{cases}$$

> $F := x \rightarrow \begin{cases} 0. & x \le 0. \\ -1.\, e^{-0.2000000000\, x} + 1. & 0. < x \end{cases}$

$$F := x \mapsto \begin{cases} 0. & x \le 0. \\ -1.\, e^{-0.2000000000 \cdot x} + 1. & 0. < x \end{cases}$$

> $plot(\mathbf{(18)})$



2.a)

2 A) FIND PARTIAL DERIVATIVES OF F(a,b) for (a=-2,b=1)

> $f := (x, y) \rightarrow \sin(x \cdot y) \cdot \cos\left(\frac{x}{y}\right)$

$$f := (x, y) \mapsto \sin(y \cdot x) \cdot \cos\left(\frac{x}{y}\right)$$

> $dfx := diff(f(x, y), x)$

$$dfx := y \cos(y x) \cos\left(\frac{x}{y}\right) - \frac{\sin(y x) \sin\left(\frac{x}{y}\right)}{y}$$

> $dfy := diff(f(x, y), y)$

$$dfy := x \cos(y x) \cos\left(\frac{x}{y}\right) + \frac{\sin(y x)\, x \sin\left(\frac{x}{y}\right)}{y^2}$$

> $subs(x = -2, y = 1, dfx)$

$$\cos(-2)^2 - \sin(-2)^2$$

> $evalf(\%)$

$$-0.6536436209$$

>

> $subs(x = -2, y = 1, dfy)$

$$-2\cos(-2)^2 - 2\sin(-2)^2$$

> $evalf(\%)$

$$-2.000000000$$

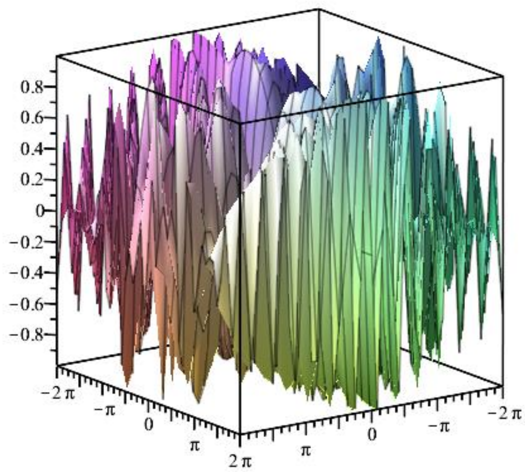
b)

B) 3D GRAPH SURFACE of f

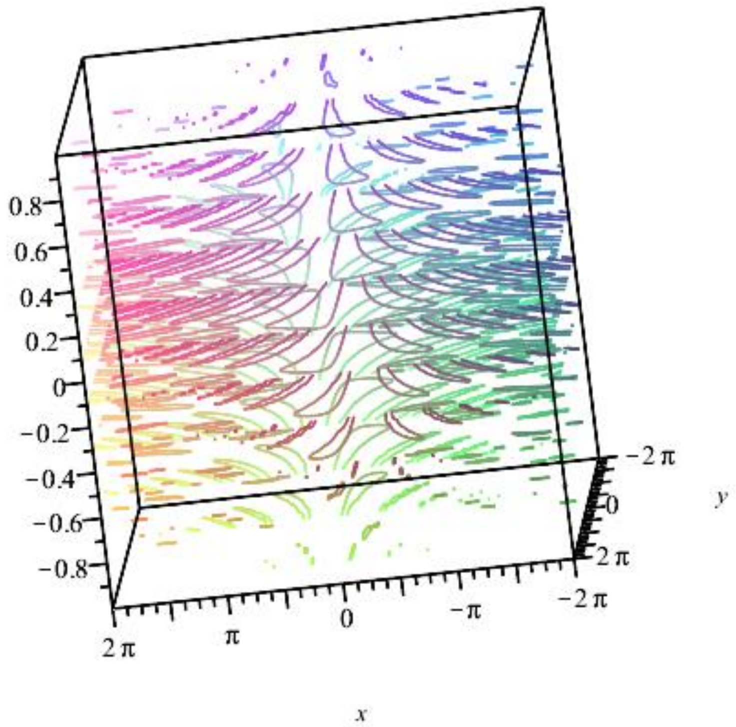
> $z := \sin(x \cdot y) \cdot \cos\left(\frac{x}{y}\right)$

$$z := \sin(y x) \cos\left(\frac{x}{y}\right)$$

> $PlotBuilder(\mathbf{(26)})$



C)

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>
> Student[MultivariateCalculus][CrossSectionTutor]( );
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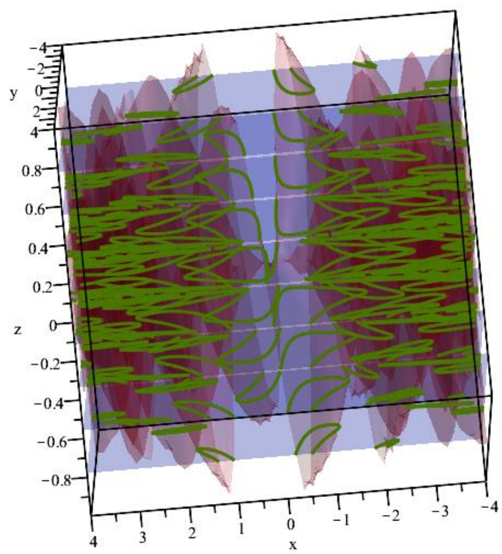
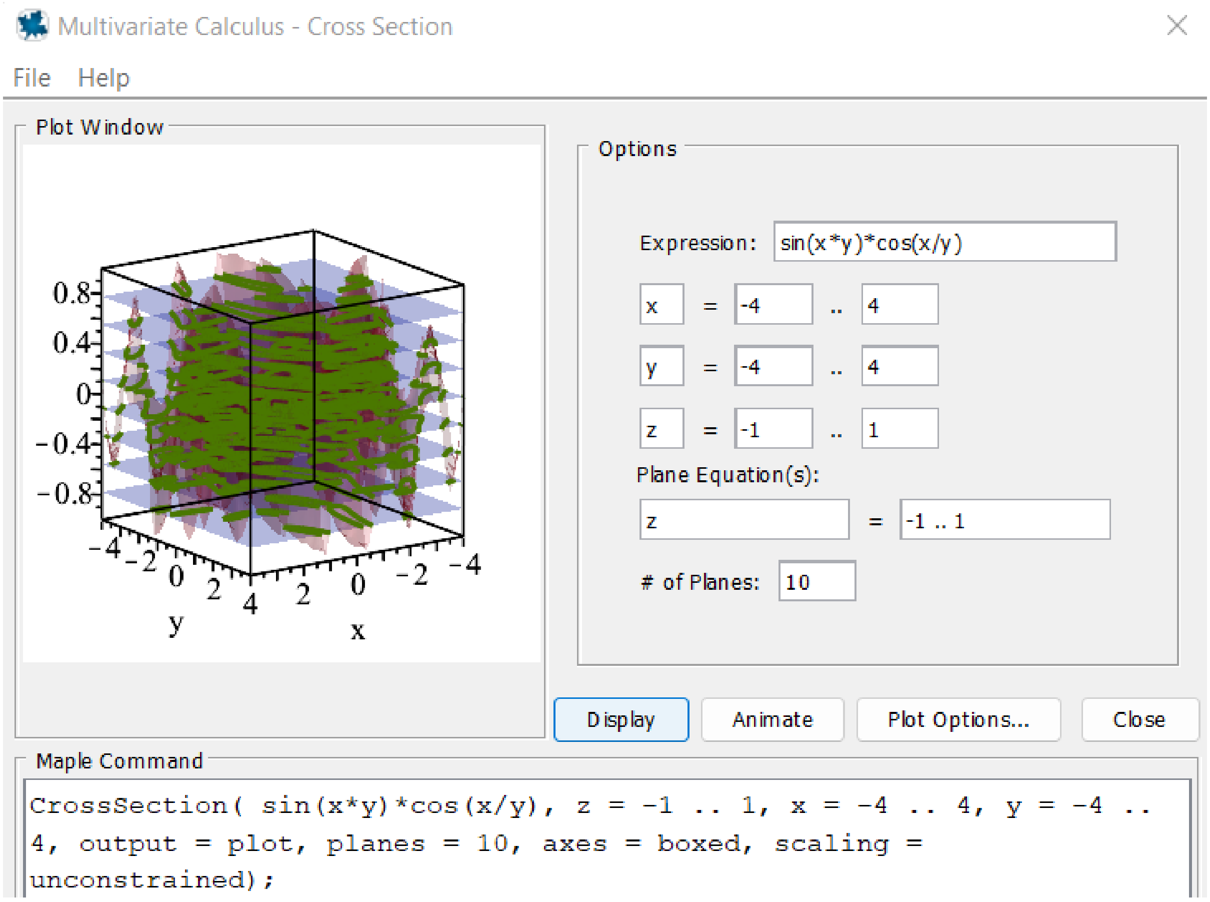
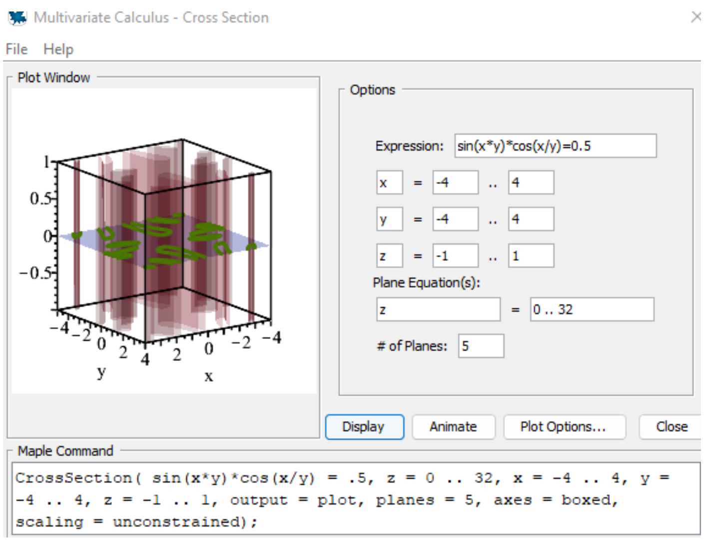



Multivariate Calculus - Cross Section
File Help
Plot Window
Options
Expression: sin(x*y)*cos(x/y)
x = -4 .. 4
y = -4 .. 4
z = -1 .. 1
Plane Equation(s):
z = -1 .. 1
# of Planes: 10
Display
Animate
Plot Options...
Close
Maple Command
CrossSection(sin(x*y)*cos(x/y), z = -1 .. 1, x = -4 .. 4, y = -4 .. 4, output = plot, planes = 10, axes = boxed, scaling = unconstrained);

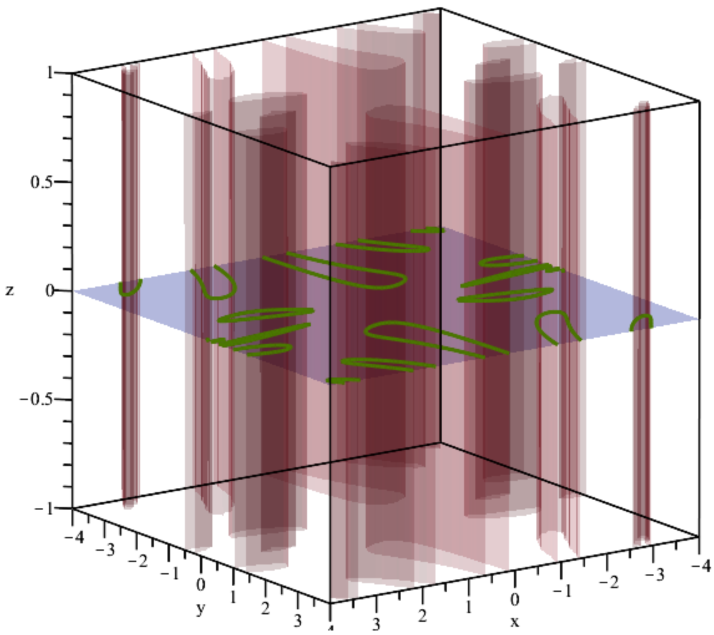
D)


Multivariate Calculus - Cross Section
File Help
Plot Window
Options
Expression: sin(x*y)*cos(x/y)=0.5
x = -4 .. 4
y = -4 .. 4
z = -1 .. 1
Plane Equation(s):
z = 0 .. 32
# of Planes: 5
Display
Animate
Plot Options...
Close
Maple Command
CrossSection(sin(x*y)*cos(x/y) = .5, z = 0 .. 32, x = -4 .. 4, y = -4 .. 4, z = -1 .. 1, output = plot, planes = 5, axes = boxed, scaling = unconstrained);

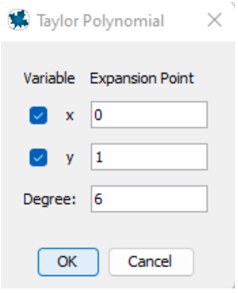
2d) OBTAIN PLANE SECTION f(x,y)=0.5

>

> *Student*[*MultivariateCalculus*][*CrossSectionTutor*]( );



E)



> $mtaylor(0, [x, y], 7)$

2E) Write f as polynomial of degree 6. Expand f for x near zero and y near one.

> $\sin(y x) \cos\left(\frac{x}{y}\right)$

$$\sin(x y) \cos\left(\frac{x}{y}\right)$$

> $mtaylor(\,\mathbf{(33)}, [x, y = 1], 6 + 1)$

$$x + x (y - 1) - \frac{2 x^3}{3} + \frac{2 x^5}{15} - x^3 (y - 1)^2 + \frac{x^3 (y - 1)^3}{3}$$

3.

3 A) FIND F explicity and find F'(t)

> $f := (x, y) \to \cos(3 \cdot x + y)$

$$f := (x, y) \mapsto \cos(3 \cdot x + y)$$

> $x := t \to t^3$

$$x := t \mapsto t^3$$

> $y := t \to t + e^{2 \cdot t}$

$$y := t \mapsto t + e^{2 \cdot t}$$

> $f(x(t), y(t))$

$$\cos\left(3\,t^3 + t + e^{2\,t}\right)$$

> $F := t \to \cos\left(3\,t^3 + t + e^{2\,t}\right)$

$$F := t \mapsto \cos\left(3 \cdot t^3 + t + e^{2 \cdot t}\right)$$

> $F(t)$

$$\cos\left(3\,t^3 + t + e^{2\,t}\right)$$

> $fnormal := diff(F(t), t)$

$$fnormal := -\left(9\,t^2 + 1 + 2\,e^{2\,t}\right) \sin\left(3\,t^3 + t + e^{2\,t}\right)$$

----

3 B) OBTAIN F'(t) using chain rule

CHAIN RULE

> $f1 := subs(x = x(t), y = y(t), diff(f(x, y), x))$

$$f1 := -3 \sin\left(3\,t^3 + t + e^{2\,t}\right)$$

> $f2 := subs(x = x(t), y = y(t), diff(f(x, y), y))$

$$f2 := -\sin\left(3\,t^3 + t + e^{2\,t}\right)$$

> $f1 \cdot x'(t) + f2 \cdot y'(t)$

$$-9 \sin\left(3\,t^3 + t + e^{2\,t}\right) t^2 - \left(2\,e^{2\,t} + 1\right) \sin\left(3\,t^3 + t + e^{2\,t}\right)$$

> $fchain := simplify(\%)$

$$fchain := \left(-9\,t^2 - 1 - 2\,e^{2\,t}\right) \sin\left(3\,t^3 + t + e^{2\,t}\right)$$