Examination

Duration: 1 h 30 m **Tolerance:** 30 minutos

Teacher: Mário Basto **Date:** 10 / 01 / 2022

1. The probability density function of a random variable X that represents the length of time, in minutes, a customer queues in a Post Office is given by:

$$f(x) = \begin{cases} a \cdot e^{bx}, & x > 0 \\ 0, & otherwise \end{cases}$$

   a) Given that $\mathrm{E}(\mathrm{X}) = 5$, find the values of a and b.

   b) Find the median of X.

   c) Find the mode of X.

   d) Find the probability that a customer will queue for longer than 4 minutes.

   e) A customer has been queueing for 4 minutes. Find the probability that this customer will be queueing for at least an additional 5 minutes (queueing for longer than 9 minutes).

   f) Write the distribution function F.

2. Consider the function $f(x,y) = \sin(xy)\cos(x/y)$

   a) Find the partial derivatives of f at (a,b) and at $(-2,1)$.

   b) Graph the 3-dimensional surface determined by f.

   c) Obtain a contour plot of the function f.

   d) Obtain the plane section $f(x,y) = 0.5$.

   e) Write f as a polynomial of degree 6. Expand f for x near zero and y near one.

3. The composition of $f(x,y) = \cos(3x + y)$,with $x(t) = t^3$ and $y(t) = t + e^{2t}$ forms the function $F(t) = (x(t), y(t))$

   a) Write F explicitly and find $F'(t)$.

   b) Obtain $F'(t)$ by applying the chain rule. Show that the results agree.