ESCOLA SUPERIOR DE TECNOLOGIA IPCA

Math Foundations for Artificial Intelligence

2023/2024

Matrix Decomposition

1. Consider the transformation matrix:

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}.$$

Without resorting to maple commands "eigenvalue, etc ".

1.1. Compute the Characteristic Polynomial and indicate the algebraic multiplicity of each eigenvalue.

1.2. For each eigenvalue, determinate the respective eigenspace.

1.3. Verify if the matrix A is diagonalizable?

2. Consider the transformation matrices:

$$A = \begin{bmatrix} 0 & -1 & 1 & 1 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}.$$

Without resorting to maple commands "eigenvalue, etc ".

2.1. Compute the Characteristic Polynomial and indicate the algebraic multiplicity of each eigenvalue.

2.2. For each eigenvalue, determinate the respective eigenspace.

2.3. Verify if the matrix A is diagonalizable?

3. Consider the transformation matrix:

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 1 & 3 & 2 \\ 5 & 0 & 0 & 2 \end{bmatrix}.$$

Without resorting to maple commands "eigenvalue, etc ".

3.1. Verify if the matrix A is diagonalizable?

3.2. Determine the eigendecomposition.

4. Consider the transformation matrices:

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 0 \end{bmatrix}.$$

Without resorting to maple commands "eigenvalue, etc ".

4.1. Verify if the matrix A is diagonalizable?

4.2. Determine the eigendecomposition.

5. Without resorting to maple commands "Singular Values" or “Singular Value Decomposition”. Find the singular value decomposition of the following matrices:

5.1. $A = \begin{bmatrix} 5 & 5 \\ -3 & 3 \end{bmatrix}$

5.2. $B = \begin{bmatrix} -6 & 6 \\ 6 & -6 \\ -2 & 2 \\ 2 & 2 \end{bmatrix}$

5.3. $C = \begin{bmatrix} -1 & -9 & 11 & -3 \\ 10 & 6 & -2 & 6 \\ -5 & 3 & 1 & -9 \end{bmatrix}$

6. Consider the matrix $C = \begin{bmatrix} -1 & -9 & 11 & -3 \\ 10 & 6 & -2 & 6 \\ -5 & 3 & 1 & -9 \end{bmatrix}$

6.1. Determine the spectral norm of C.

6.2. Find the rank-2 approximation of C.

7. Without resorting to maple commands "Singular Values" or “Singular Value Decomposition”. Find the singular value decomposition of the following matrices:

7.1. $A = \begin{bmatrix} 3 & -3 \\ 2 & 2 \end{bmatrix}$

7.2. $B = \begin{bmatrix} 3 & 6 & 9 \\ -1 & -10 & -5 \\ 11 & 2 & 1 \\ 9 & 6 & -3 \end{bmatrix}$

7.3. $C = \begin{bmatrix} 4 & 4 & -4 & 4 \\ -6 & 6 & 6 & 6 \end{bmatrix}$

8. Consider the matrix $B = \begin{bmatrix} 3 & 6 & 9 \\ -1 & -10 & -5 \\ 11 & 2 & 1 \\ 9 & 6 & -3 \end{bmatrix}$.

Determine the spectral norm of B and the rank-2 approximation of B.