Introduction to Algorithms

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GUROBI OPTIMIZATION

The World's Fastest Solver

About the Speaker

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Gurobi Optimization

Agenda

1. Linear programming (LP)
    a. Define LP
    b. LP examples
    c. Algorithms (high-level)

2. Mixed-integer programming (MIP)
    a. Define MIP
    b. MIP example

3. Basic techniques for solving MIP
    a. Bounding the optimal solution
    b. Bound-and-bound
    c. Improvements (heuristics, cuts, etc.)
    d. Termination criteria

4. Log file interpretation
5. Outlook: Tuning

Introducing LP

What is LP?

$$
\begin{array}{lrcl}
\min\limits_{x \in \mathbb{R}^n} & f(x) & & \\
\textbf{subject to} & h_1(x) & \leq & 0 \\
 & h_2(x) & \leq & 0 \\
 & & \vdots & \\
 & h_m(x) & \leq & 0
\end{array}
$$

- **This is a Linear Programming (LP) instance if:**
  - The objective function $f(x)$ is linear
    - e.g., $f(x) = 3x1 + 2x2$
  - The functions $h_1(x), \ldots, h_m(x)$ are linear
    - e.g., $h_1(x) = x1 - 17x2 + 10$, etc.

Quiz time!

$$
\begin{array}{rrcrcrcr}
\min\limits_{x\in\mathbb{R}^2} & x_1 & - & x_2 & & & & \\
\text{subject to} & x_1 & + & 17x_2 & - & 10 & \leq & 0 \\
 & 2x_1 & + & 3x_2 & - & 5 & \leq & 0 \\
 & -x_1 & & & & & \leq & 0 \\
 & & & & & -x_2 & \leq & 0
\end{array}
$$

Quiz time!

$$
\begin{array}{rrcrcrcr}
\min\limits_{x \in \mathbb{R}^2} & \boxed{x_1^2} & - & x_2 & & & & \\
\text{subject to} & x_1 & + & 17x_2 & - & 10 & \leq & 0 \\
 & 2x_1 & + & 3x_2 & - & 5 & \leq & 0 \\
 & -x_1 & & & & & \leq & 0 \\
 & & & & & -x_2 & \leq & 0
\end{array}
$$

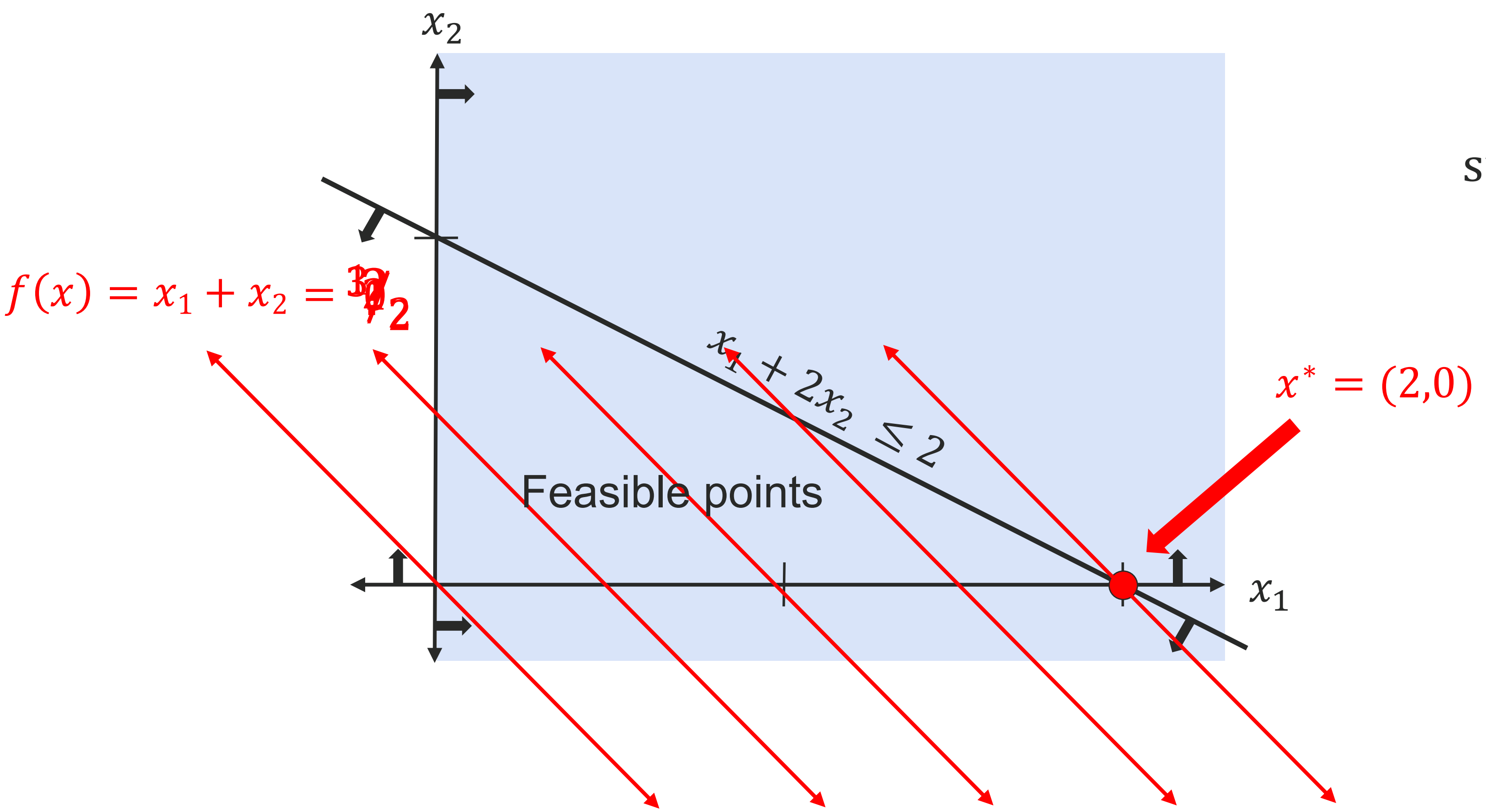
Quiz time!

$$
\begin{array}{rrcrcrcc}
\min\limits_{x\in\mathbb{R}^2} & |x_1 & - & x_2| & & & & \\
\text{subject to} & x_1 & + & 17x_2 & - & 10 & \leq & 0 \\
 & 2x_1 & + & 3x_2 & - & 5 & \leq & 0 \\
 & -x_1 & & & & & \leq & 0 \\
 & & & & & -x_2 & \leq & 0
\end{array}
$$

Quiz time!

$$
\begin{aligned}
\max_{x\in\mathbb{R}^2} \quad & x_1 - x_2 & \iff \quad & -\min_{x\in\mathbb{R}^2} -(x_1 - x_2) \\
\text{subject to} \quad & x_1 + 17x_2 \leq 10 & \iff \quad & x_1 + 17x_2 - 10 \leq 0 \\
& 2x_1 + 3x_2 - 5 \geq 0 & \iff \quad & -2x_1 - 3x_2 + 5 \leq 0 \\
& 4x_1 + x_2 - 9 = 0 & \iff \quad & \begin{cases} 4x_1 + x_2 - 9 \leq 0, \\ -4x_1 - x_2 + 9 \leq 0, \end{cases} \\
& -x_1 \leq 0 & & \\
& -x_2 \leq 0 & &
\end{aligned}
$$

LP by picture

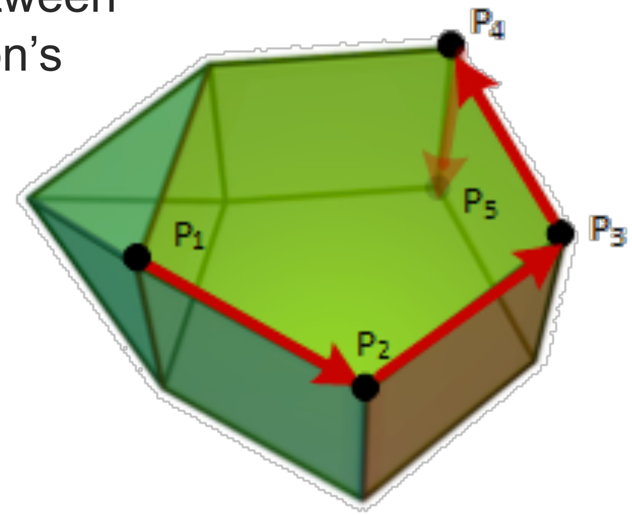


$$
\begin{array}{llllll}
\max\limits_{x\in\mathbb{R}^2} & x_1 & + & x_2 & & \\
\text{subject to} & x_1 & + & 2x_2 & \leq & 2 \\
 & x_1 & & & \geq & 0 \\
 & & & x_2 & \geq & 0
\end{array}
$$

Algorithmic LP ideas

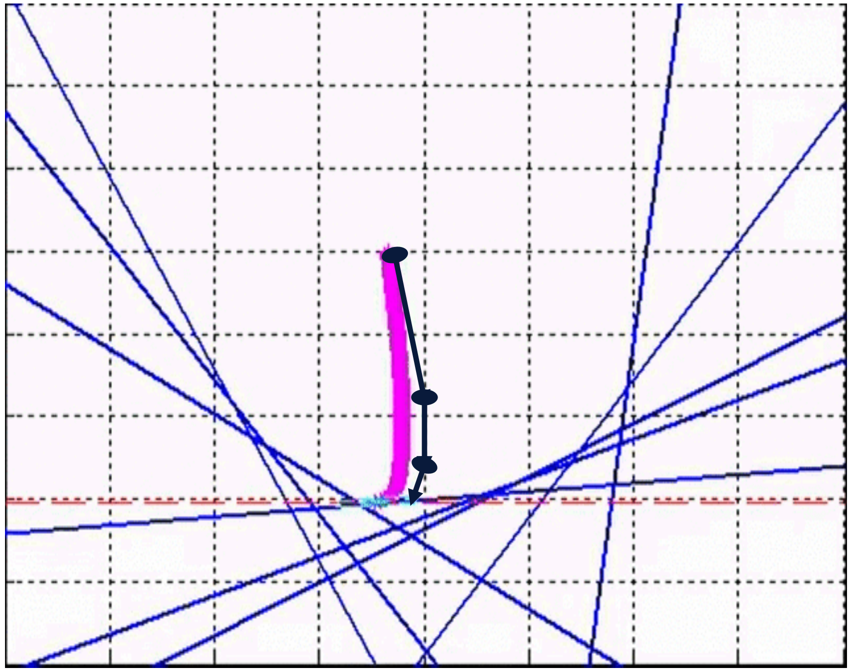
$$\min_{x \in \mathbb{R}^2} \quad c^T x$$
$$\text{subject to} \quad Ax \leq b$$

Traverses edges between polyhedron's vertices



Simplex

Generates sequence of points in interior of polyhedron



Barrier

img src: mathstools.com

Introducing MIP

What is MILP?

$$
\begin{array}{llll}
\min\limits_{x\in\mathbb{R}^n} & f(x) & & \\
\textbf{subject to} & h_1(x) & \leq & 0 \\
 & h_2(x) & \leq & 0 \\
 & & \vdots & \\
 & h_m(x) & \leq & 0 \\
\end{array}
$$

Some x_i are integer

- **This is a Mixed-Integer Linear Programming (MILP) instance if:**
  - The objective function $f(x)$ is linear
    - e.g., $f(x) = 3x1 + 2x2$
  - The functions $h_1(x), \dots, h_m(x)$ are linear
    - e.g., $h_1(x) = x1 - 17x2 + 10$, etc.
- **MILP = LP + integrality constraints!**

MIP example: furniture manufacturing

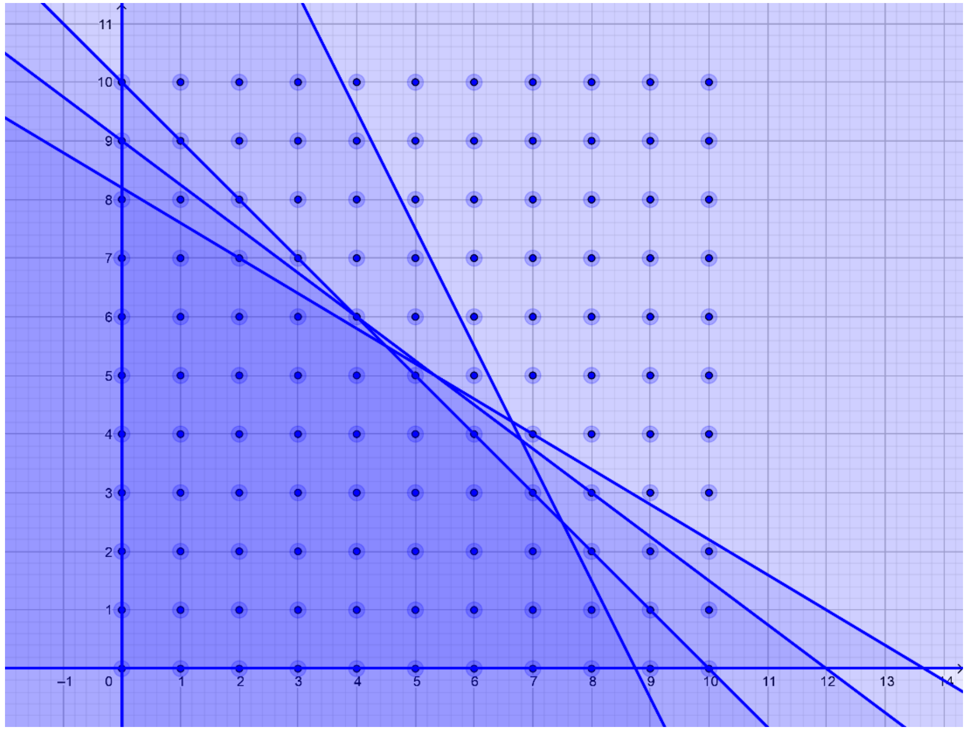
Variables:
- x: number of stools we make
- y: number of chairs we make

~~**Solution:**~~ $\cancel{x^* = 4.5, y^* = 5.5}$

	Stool (x)	Chair (y)	Available
Profit	\$10	\$11	↑↑↑

$$
\begin{array}{lrcrcl}
\Leftrightarrow & \max_{x,y} & 10x & + & 11y & \\
\Leftrightarrow & \text{subject to} & x & + & y & \leq 10 \\
\Leftrightarrow & & 3x & + & 4y & \leq 36 \\
\Leftrightarrow & & 3x & + & 5y & \leq 41 \\
\Leftrightarrow & & 4x & + & 2y & \leq 35 \\
& & \text{integer} & & x, y & \geq 0
\end{array}
$$

MIP by picture



$$
\begin{array}{rrcrcr}
\max_{x,y} & {\color{red}10x} & {\color{red}+} & {\color{red}11y} & & \\
\text{subject to} & x & + & y & \leq & 10 \\
 & 3x & + & 4y & \leq & 36 \\
 & 3x & + & 5y & \leq & 41 \\
 & 4x & + & 2y & \leq & 35 \\
 & & & \text{integer } x, y & \geq & 0
\end{array}
$$

Why not enumerate (try) all the integer feasible points?

Why not enumerate all combinations of 0-1 values?

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- **Try all 2^n points on Summit supercomputer at ORNL**
  - 200 petaflops (2×10^{17})

n	$0-1$ points to check	Time
10	1024	0 seconds
60	1.15×10^{18}	6 seconds
70	1.18×10^{21}	1.6 hours
120	1.33×10^{36}	210 billion years

Solving MIP

Bounds and MIP Gap

Let $f(x,y) \coloneqq 10x + 11y$

$$
\begin{aligned}
z^* \coloneqq \max_{x,y} \quad & 10x + 11y \\
\text{s.t.} \quad & x + y \leq 10 \\
& 3x + 4y \leq 36 \\
& 3x + 5y \leq 41 \\
& 4x + 2y \leq 35 \\
& \text{integer } x, y \geq 0
\end{aligned}
$$

$0 \leq z^* = \ ?$

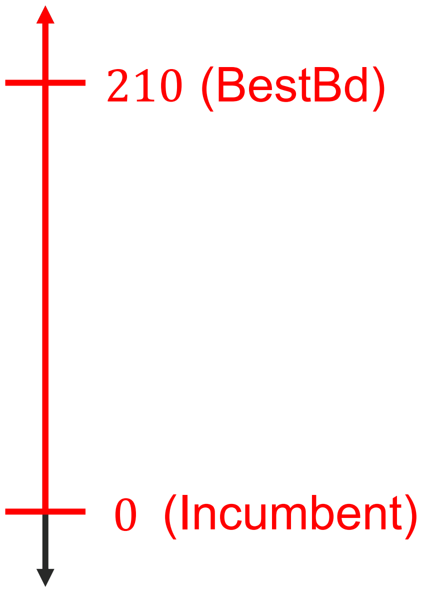
0 (Incumbent)

Can we try something simple?

- $x = 0, y = 0$ is a feasible solution!
- Then z^* must *at least as large* as f(0,0) = 0
- Currently, (0,0) is our *incumbent* solution

Bounds and MIP Gap

$$0 \le z^* \le 210$$



$$
\begin{array}{rl}
z^* := \max\limits_{x,y} & \boxed{10x + 11y} \\
\text{s.t.} & \boxed{x + y \le 10} \\
& 3x + 4y \le 36 \\
& 3x + 5y \le 41 \\
& 4x + 2y \le 35 \\
& \text{integer } x, y \ge 0
\end{array}
$$

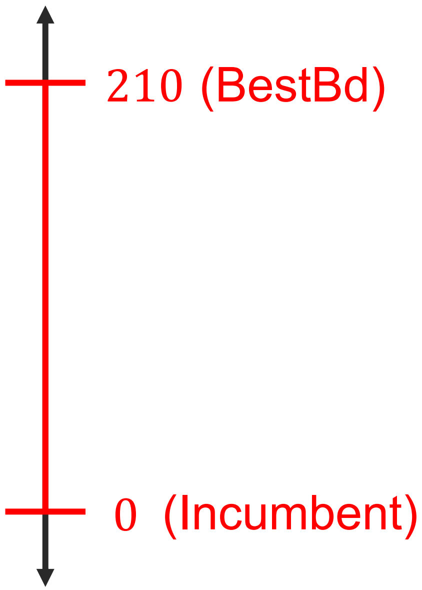
$$x + y \le 10$$
$$\Downarrow$$
$$x \le 10 \text{ and } y \le 10$$
$$\Downarrow$$
$$10x \le 100 \text{ and } 11y \le 110$$
$$\Downarrow$$
$$f(x,y) = 10x + 11y \le 210$$

What about upper bounds?

- Use linear constraints to get an upper bound on the optimal objective z^*
- *All* solutions have objective $z^* \le 210$
  - This is currently our *best (upper) bound*

Bounds and MIP Gap

$$0 \le z^* \le 210$$



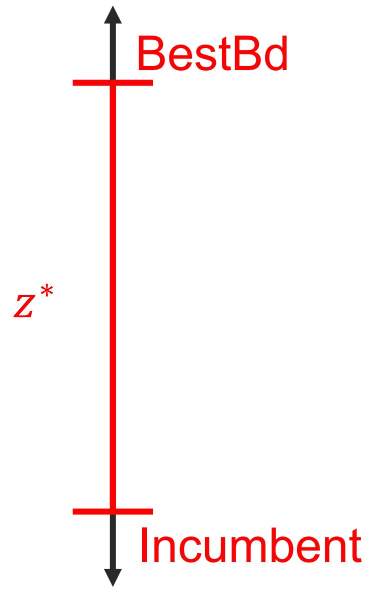
$$
\begin{array}{rrcrcrcr}
z^* := \max\limits_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \le & 10 \\
 & 3x & + & 4y & \le & 36 \\
 & 3x & + & 5y & \le & 41 \\
 & 4x & + & 2y & \le & 35 \\
\end{array}
$$

integer $x, y \ge 0$

How good is the incumbent?

- Measure the quality by the *(relative) MIP gap:* $\frac{|\text{BestBd} - \text{Incumbent}|}{|\text{Incumbent}|} \times 100\%$
- Current gap is $\frac{210-0}{0} \times 100\%$ (undefined or $+\infty$)
  - This is a very poor gap!

Bounds and MIP Gap



$$
\begin{array}{rrcrcrcr}
z^* := \max\limits_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
 & 3x & + & 4y & \leq & 36 \\
 & 3x & + & 5y & \leq & 41 \\
 & 4x & + & 2y & \leq & 35 \\
\end{array}
$$

$$\text{integer } x, y \geq 0$$

Can we do better?

- A MIPGap of 0 means the incumbent solution is optimal
  - This happens when the bound and the incumbent have equal value
- Our goal: try to improve the bound and/or incumbent

Upper bound from relaxation

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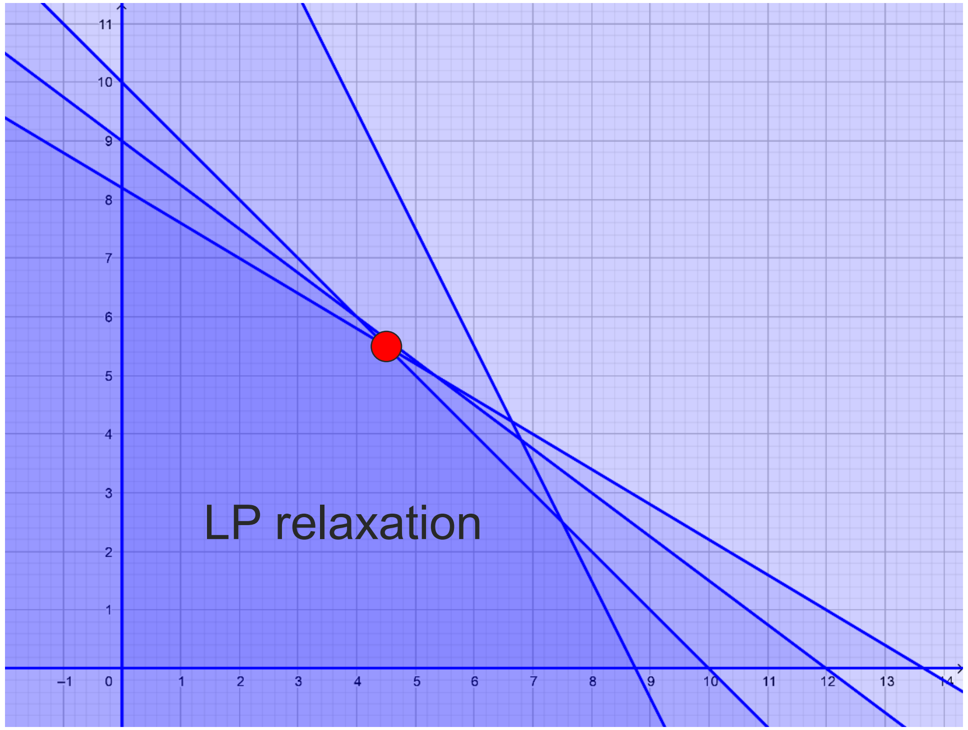
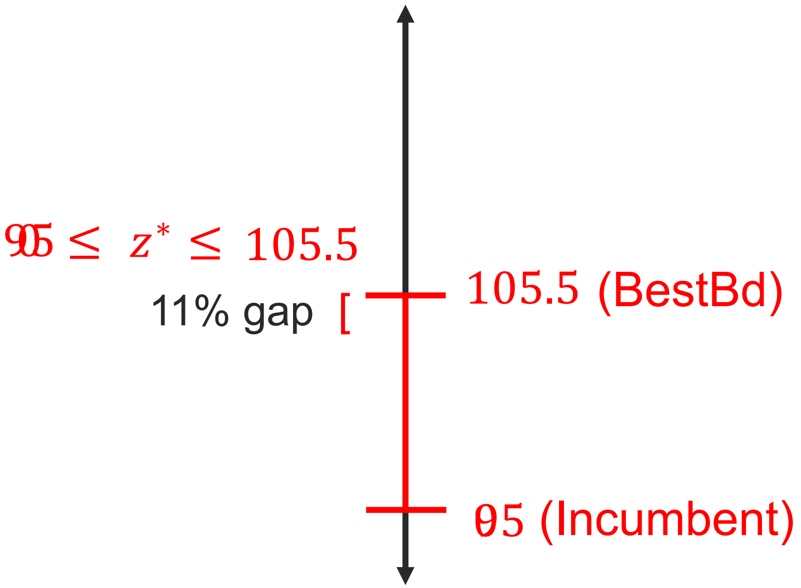


$$
\begin{array}{rrcrcr}
z^{LP} := \max\limits_{x,y} & \color{red}{10x} & \color{red}{+} & \color{red}{11y} & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
& 3x & + & 4y & \leq & 36 \\
& 3x & + & 5y & \leq & 41 \\
& 4x & + & 2y & \leq & 35 \\
& & & \cancel{\text{integer}}\ x, y & \geq & 0
\end{array}
$$

Can we improve our upper bound?

- The integrality constraints are hard…let's temporarily ignore them
- *LP (root) relaxation*: relax (remove) integrality restrictions
- Optimal solution is $(4.5, 5.5)$
  $\Longrightarrow z^{\mathrm{LP}} = f(4.5,5.5) = 105.5$ is an upper bound on z^*!

Lower bound from heuristics



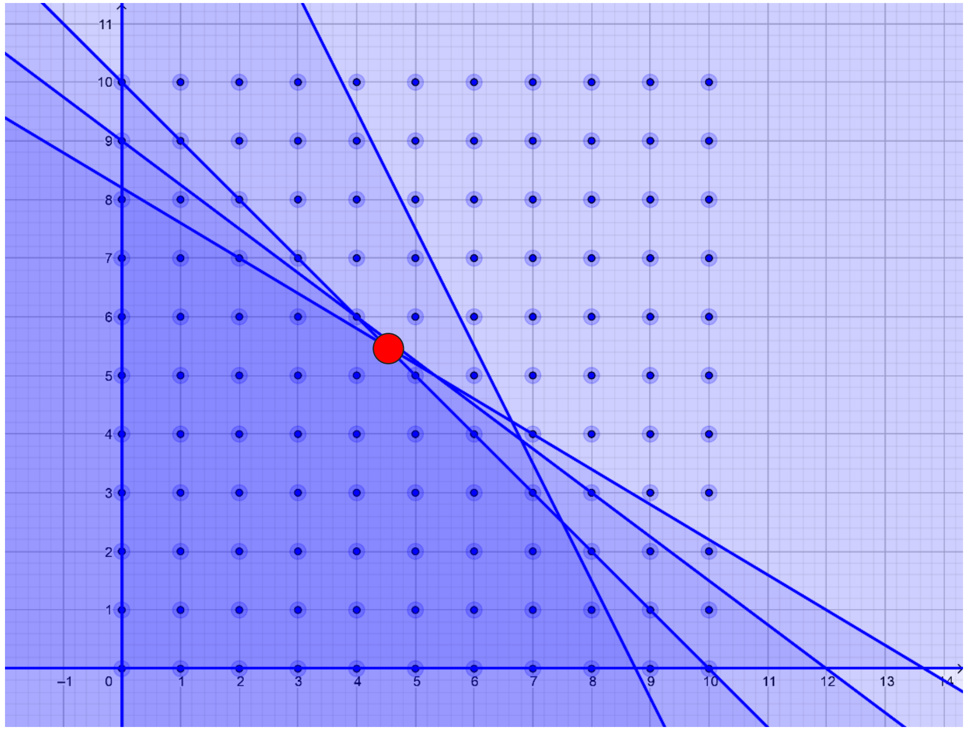
$$
\begin{array}{rrcrcl}
z^* := \max\limits_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
 & 3x & + & 4y & \leq & 36 \\
 & 3x & + & 5y & \leq & 41 \\
 & 4x & + & 2y & \leq & 35 \\
 & & & \text{integer } x, y & \geq & 0
\end{array}
$$

Can we improve our incumbent?

- Let's "guess" at a good solution
  - This is commonly called a "heuristic"
- Try: round down the relaxed solution $(4.5, 5.5)$:

  $\Longrightarrow (\lfloor 4.5 \rfloor,\ \lfloor 5.5 \rfloor) = (4, 5)$ is feasible!
- $f(4, 5)\ =\ 95$…we have improved our incumbent!

Cuts



$$
\begin{array}{rrcrcrcr}
z^* := \max\limits_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
& 3x & + & 4y & \leq & 36 \\
& 3x & + & 5y & \leq & 41 \\
& 4x & + & 2y & \leq & 35
\end{array}
$$

integer $x, y \geq 0$

LP optimum:
$x = 4.5, y = 5.5$

Fractional solutions are bad!

- Is there an inequality for our MIP that "cuts off" the LP optimum $(4.5, 5.5)$?
- *Cuts*: inequalities that tighten our relaxation of the MIP feasible region
- Let's try to derive a cut that is violated by $(4.5, 5.5)$!

Cuts

$$\max_{x, y, s_1, s_2, s_3, s_4} \quad 10x + 11y$$

$$\max_{x,y} \quad 10x + 11y$$

s. t.

$$\begin{aligned} x &+ y \le 10 \\ 3x &+ 4y \le 36 \\ 3x &+ 5y \le 41 \\ 4x &+ 2y \le 35 \end{aligned}$$

$$\begin{aligned} x &+ y + s_1 = 10 \\ 3x &+ 4y + s_2 = 36 \\ 3x &+ 5y + s_3 = 41 \\ 4x &+ 2y + s_4 = 35 \end{aligned}$$

$s_1, s_2, s_3, s_4, x, y \ge 0$

integer $x, y \ge 0$

Satisfied without slack at $(4.5, 5.5)$

LP optimum:
$x = 4.5, y = 5.5$

1. Aggregate

$$-3x - 3y - 3s_1 = -30$$

and

$$3x + 5y + s_3 = 41$$

$$\Downarrow$$

$$2y - 3s_1 + s_3 = 11$$

$$\Downarrow$$

$$y - \frac{3}{2}s_1 + \frac{1}{2}s_3 = \frac{11}{2}$$

2. Weaken

$$y - \frac{3}{2}s_1 + \frac{1}{2}s_3 = \frac{11}{2}$$

$$\Downarrow$$

$$y + \left\lfloor -\frac{3}{2} \right\rfloor s_1 + \left\lfloor \frac{1}{2} \right\rfloor s_3 \le \frac{11}{2}$$

$$y - 2(10 - x - y) \le 5$$

$$\Downarrow$$

$$y - 2s_1 \le \frac{11}{2}$$

$$2x + 3y \le 25$$

3. Strengthen

$$y - 2s_1 \le \frac{11}{2}$$

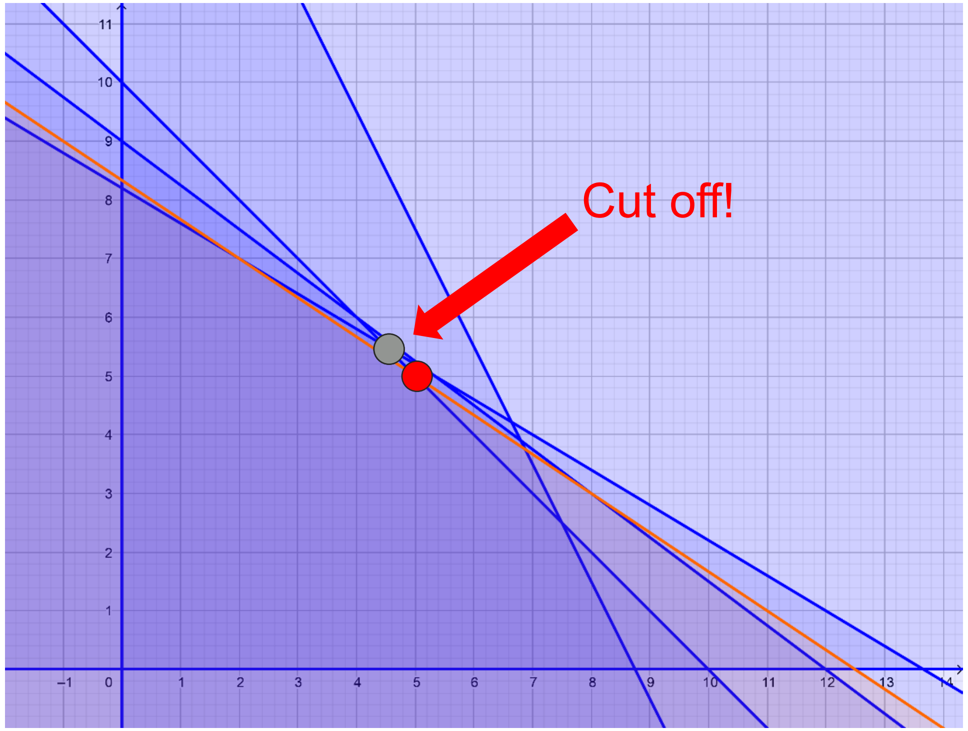
$$\Downarrow$$

$$y - 2s_1 \le \left\lfloor \frac{11}{2} \right\rfloor = 5$$

New inequality!

Cuts

GUROBI OPTIMIZATION



LP optimum:

$x = 4.5, y = 5.5$

$$
\begin{array}{rrcrcrcr}
z^* := \max\limits_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
& 3x & + & 4y & \leq & 36 \\
& 3x & + & 5y & \leq & 41 \\
& 4x & + & 2y & \leq & 35 \\
& 3x & + & 2y & \leq & 25 \\
& & & \text{integer } x, y & \geq & 0
\end{array}
$$

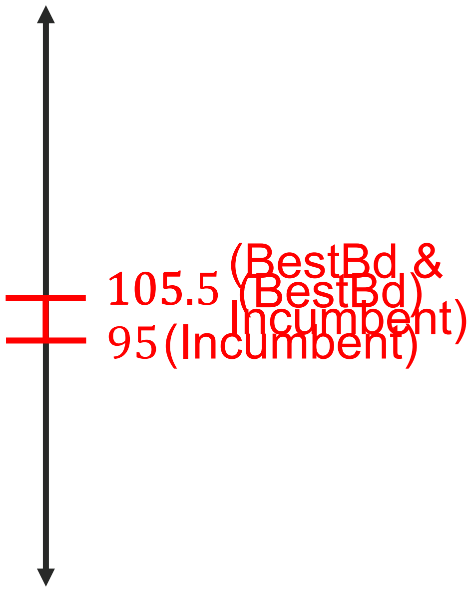
Is this new inequality any good?

- It cuts off the previous optimal solution!
- Solving the new, tightened relaxation yields a new upper bound
- New upper bound on z^* is $10(5) + 11(5) = 105$

Improving bounds

$105 \leq z^* \leq 105.5$

Gap=0%

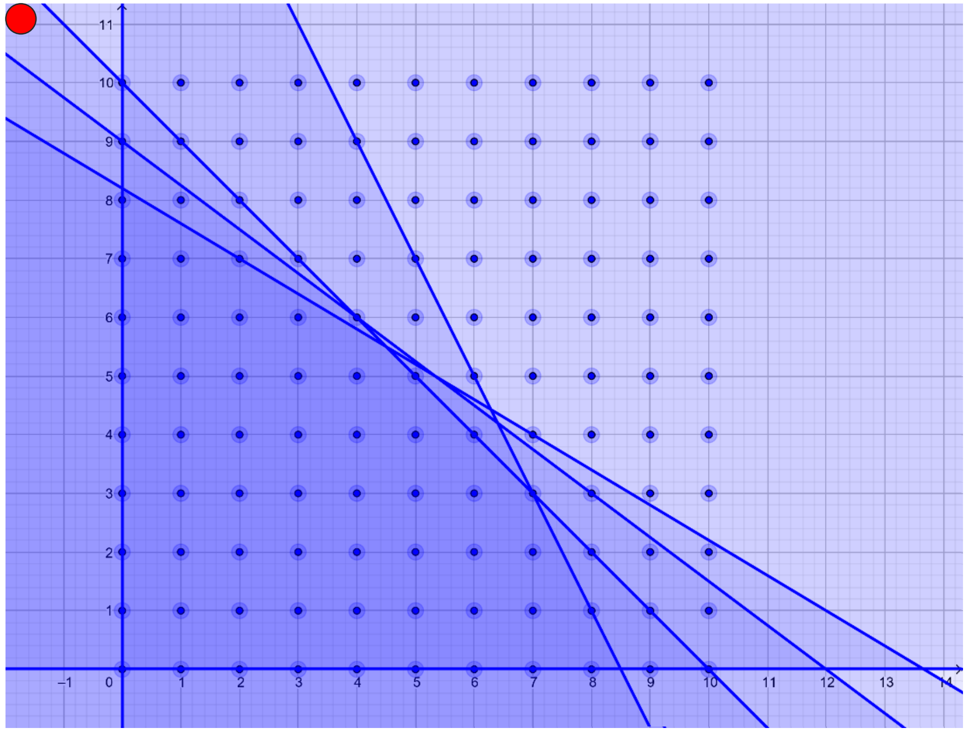


$$
\begin{array}{rrcrcr}
z^* := \max_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
 & 3x & + & 4y & \leq & 36 \\
 & 3x & + & 5y & \leq & 41 \\
 & 4x & + & 2y & \leq & 35 \\
 & & \text{integer} & x, y & \geq & 0
\end{array}
$$

What can we say about z^*?

- New upper bound: $z^* \leq 105$
- But the optimal solution to the relaxation is $(5, 5)$
    - This is feasible to the original problem!
    - 105 is a new incumbent objective value
- **We have solved the problem: $z^* = 105$**

Presolve

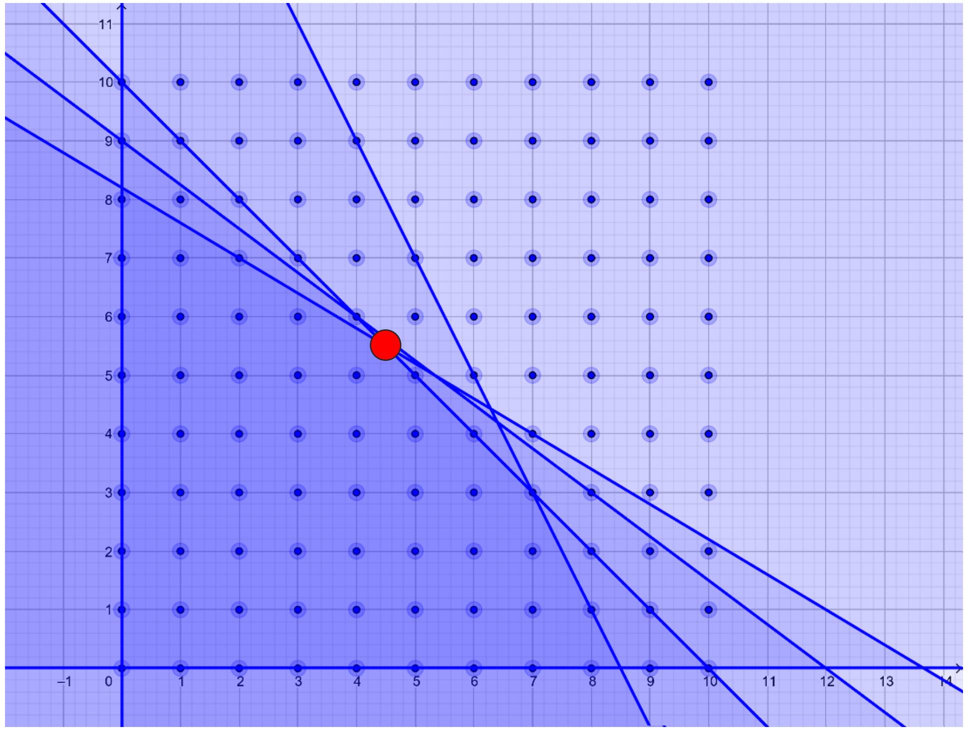


$$
\begin{array}{rlrcrcr}
z^* := \max\limits_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
 & 3x & + & 4y & \leq & 36 \\
 & 3x & + & 5y & \leq & 41 \\
 & 4x & + & 2y & \leq & 35 \\
 & & & \text{integer } x, y & \geq & 0
\end{array}
$$

Can we tighten the formulation just by looking at it?

- *Tighten*: reduce the feasible region of relaxation without removing any feasible points
- $4x$ is even, and so is $2y$…
- This doesn't change optimal solution relaxation

Branch-and-bound

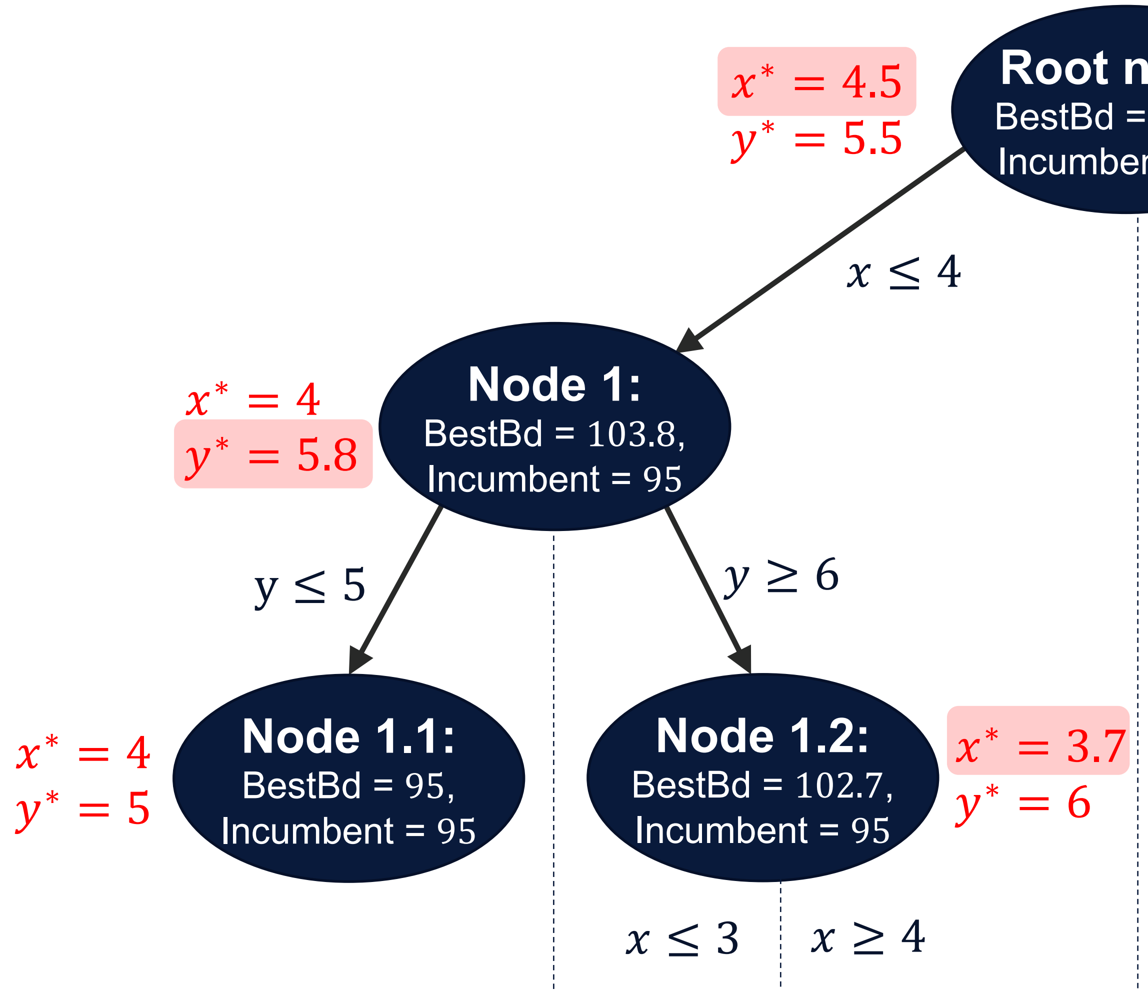


$$
\begin{array}{rrcrcr}
z^* := \max_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
& 3x & + & 4y & \leq & 36 \\
& 3x & + & 5y & \leq & 41 \\
& 4x & + & 2y & \leq & 34 \\
& & & \text{integer } x, y & \geq & 0
\end{array}
$$

Can we combine these ideas with brute-force enumeration?

- Yes! For example, consider the relaxation solution $(x, y) = (4.5, 5.5)$
  - x has to be integer. So is $x \leq 4$ or $x \geq 5$?
  - Let's investigate!

Branch-and-bound tree



$$
\begin{array}{rrcrcr}
z^* := \max\limits_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
 & 3x & + & 4y & \leq & 36 \\
 & 3x & + & 5y & \leq & 41 \\
 & 4x & + & 2y & \leq & 34 \\
 & & & \text{integer } x, y & \geq & 0 \\
 & & & & & x \leq 4 \\
 & & & & & y \geq 6
\end{array}
$$

Branch-and-bound tree

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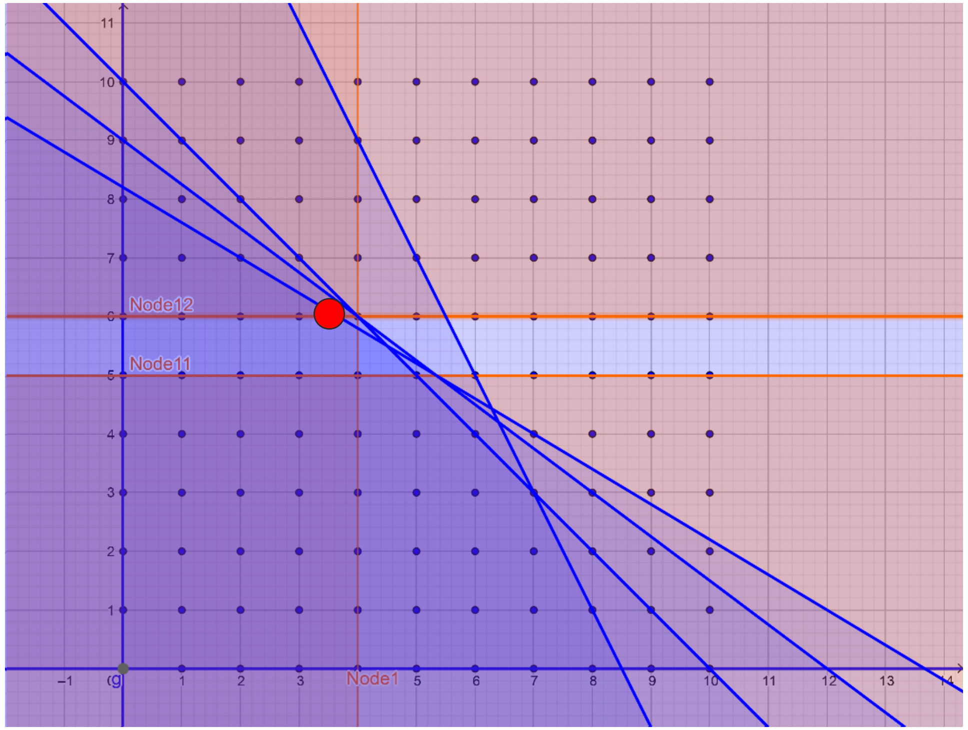


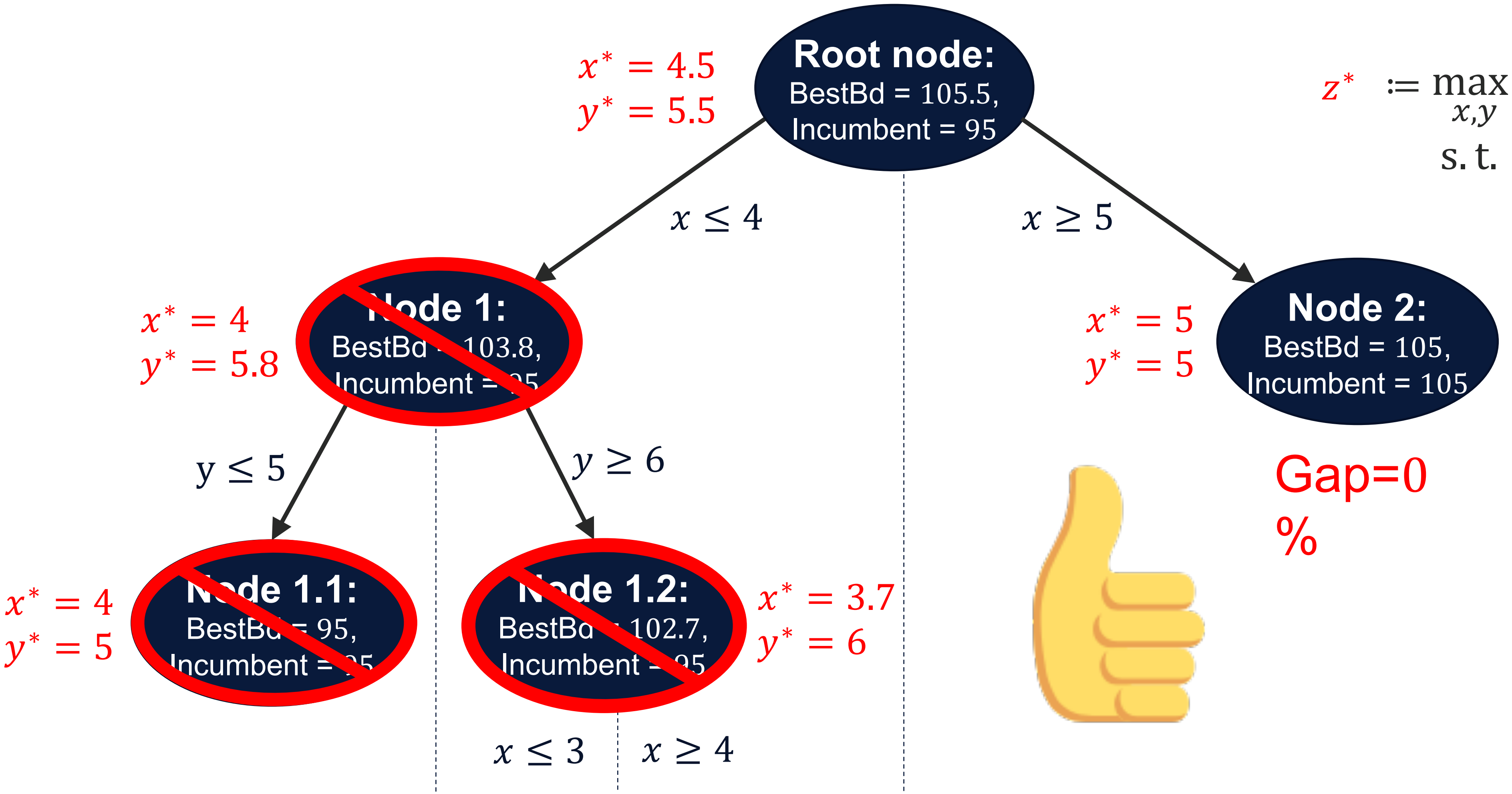
Geometrically, what does this look like?

$$
\begin{array}{llrcrcr}
\color{red}{z^*} & := \max\limits_{x,y} & \color{red}{10x} & \color{red}{+} & \color{red}{11y} & & \\
& \text{s.t.} & x & + & y & \leq & 10 \\
& & 3x & + & 4y & \leq & 36 \\
& & 3x & + & 5y & \leq & 41 \\
& & 4x & + & 2y & \leq & 34 \\
& & & & \text{integer } x, y & \geq & 0
\end{array}
$$

$x \leq 4$

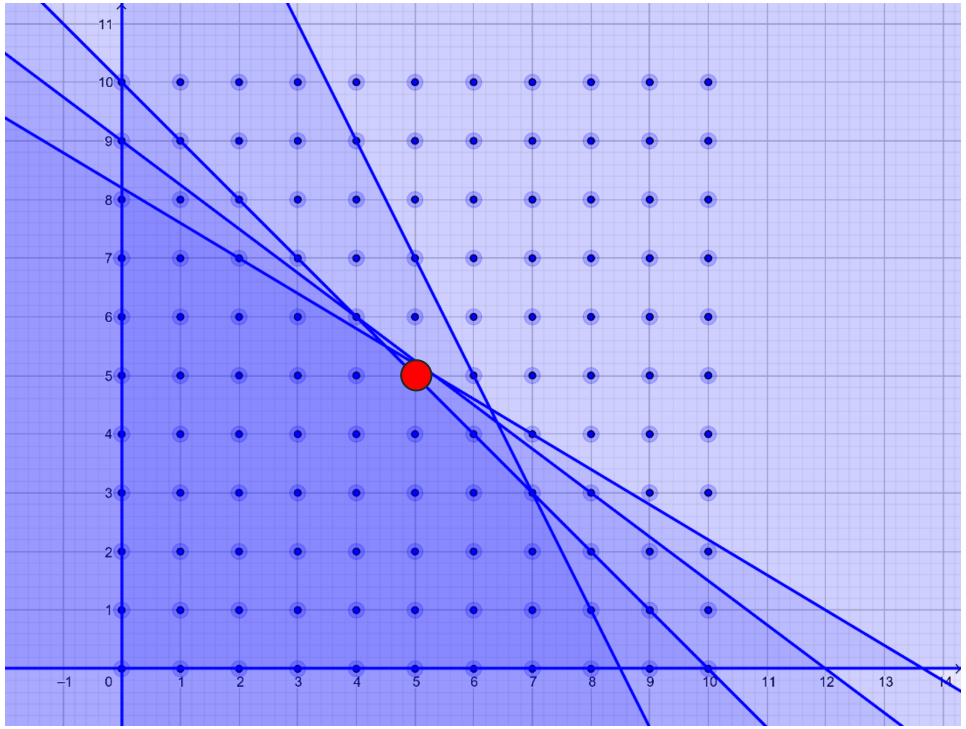
$y \geq 6$

Branch-and-bound tree



$$
\begin{array}{rrcrcll}
z^* := \max\limits_{x,y} & 10x & + & 11y & & \\
\text{s.t.} & x & + & y & \leq & 10 \\
& 3x & + & 4y & \leq & 36 \\
& 3x & + & 5y & \leq & 41 \\
& 4x & + & 2y & \leq & 34 \\
& & & \text{integer } x, y & \geq & 0 \\
& & & & & x \geq 5
\end{array}
$$

Branch-and-bound



$$
\begin{array}{rlrcrcr}
z^* := & \max_{x,y} & 10x & + & 11y & & \\
& \text{s.t.} & x & + & y & \leq & 10 \\
& & 3x & + & 4y & \leq & 36 \\
& & 3x & + & 5y & \leq & 41 \\
& & 4x & + & 2y & \leq & 34 \\
& & & & \text{integer } x, y & \geq & 0
\end{array}
$$

Looks like a lot of work!

- For the computer – not for you!
- Gurobi does presolve, cuts, heuristics, branch-and-bound and more “auto-magically”!

Termination criteria

- MIP gap
  - E.g., terminate if *relative* MIP gap $\leq$ 1% or *absolute* MIP gap $\leq$ 10,
- Time spent solving
  - E.g., terminate after 3600 seconds have elapsed
- Number of nodes explored
  - E.g., terminate after solver visits 100,000 nodes
- Number of feasible solutions found
- …and more!

See the Termination section of the **Parameters** page in the Reference Manual

Gurobi logs

Recognize these ideas in the Gurobi logs

Log file structure

LP

1. User input + model statistics
2. Presolve
3. Simplex or Barrier iterations
4. Termination statistics

MILP

1. User input + model statistics
2. Presolve
3. Root node relaxation
4. Branch-and-bound tree exploration
5. Termination statistics

Simplex log

```
Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    1.7748600e+04   6.627132e+03   0.000000e+00      0s
    9643    1.1574611e+07   1.418653e+03   0.000000e+00      5s
   14440    1.1607748e+07   4.793500e+00   0.000000e+00     10s
   15213    1.1266396e+07   0.000000e+00   0.000000e+00     11s

Solved in 15213 iterations and 10.86 seconds
Optimal objective  1.126639605e+07
```

Barrier log

```
Barrier statistics:
 AA' NZ      : 2.836e+03
 Factor NZ   : 3.551e+03 (roughly 40 MBytes of memory)
 Factor Ops  : 1.739e+05 (less than 1 second per iteration)
 Threads     : 4

                Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.30273209e+06  0.00000000e+00  5.90e+02 0.00e+00  7.32e+00   12s
   1   1.04326180e+05 -5.84079103e+02  4.84e+01 1.69e+00  5.95e-01   12s
   2   9.46325157e+03 -4.40392705e+02  2.92e+00 1.35e+00  5.46e-02   12s
   3   3.66683689e+03  9.27381244e+02  1.94e-01 5.35e-01  1.41e-02   12s
   4   3.37449982e+03  1.79938013e+03  1.29e-01 2.41e-01  7.64e-03   12s
   5   3.13244138e+03  1.90266941e+03  8.89e-02 2.07e-01  6.00e-03   12s
   6   2.71282610e+03  2.11401255e+03  3.20e-02 1.15e-01  2.96e-03   12s
   7   2.48856811e+03  2.18107490e+03  1.06e-02 7.26e-02  1.56e-03   12s
   8   2.35427593e+03  2.21183615e+03  3.20e-03 4.52e-02  7.36e-04   12s
   9   2.30239737e+03  2.22464753e+03  1.53e-03 2.38e-02  4.03e-04   12s
  10   2.25547118e+03  2.23096162e+03  3.00e-04 1.40e-02  1.30e-04   12s
  11   2.24052450e+03  2.23917612e+03  4.10e-06 6.33e-04  7.20e-06   12s
  12   2.23967243e+03  2.23966346e+03  2.01e-08 5.01e-06  4.82e-08   12s
  13   2.23966667e+03  2.23966666e+03  1.11e-10 1.14e-13  4.81e-11   13s

Barrier solved model in 13 iterations and 12.51 seconds
Optimal objective 2.23966667e+03
```

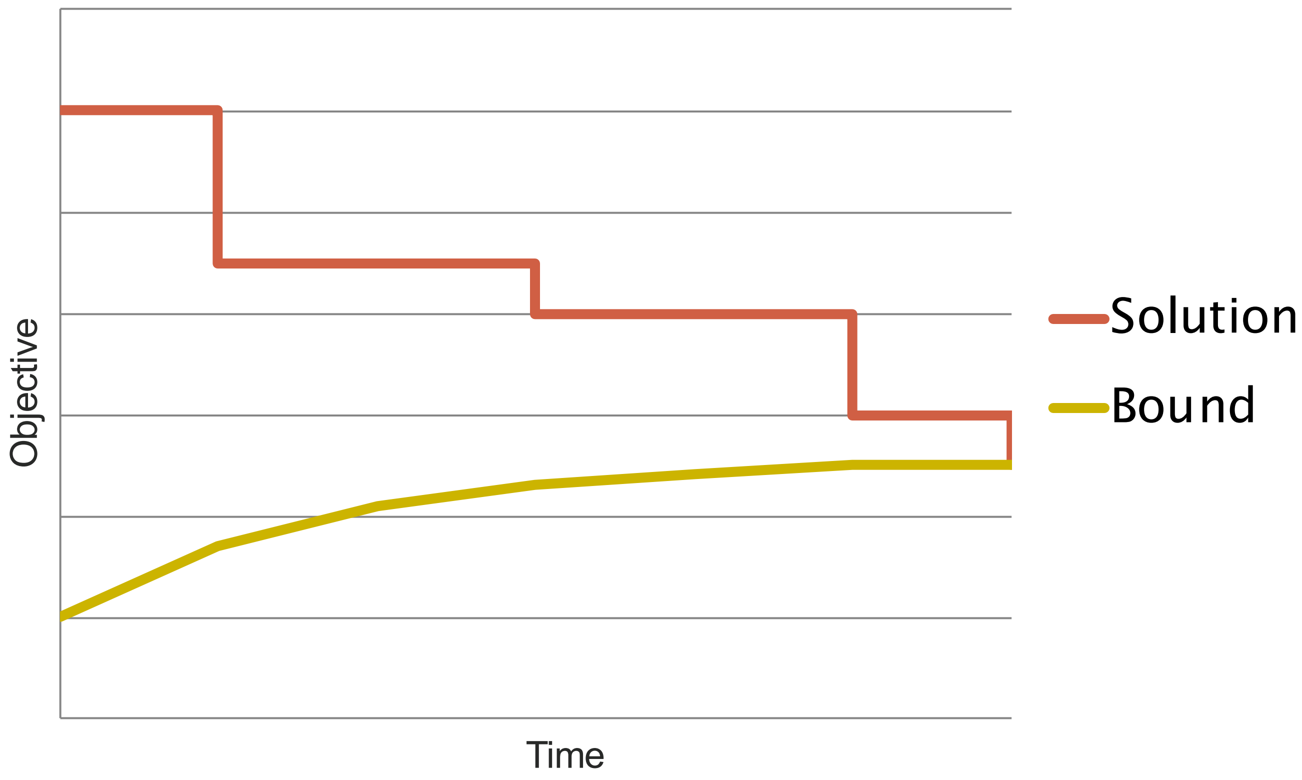
Barrier crossover log

```
Barrier solved model in 13 iterations and 12.51 seconds
Optimal objective 2.23966667e+03


Root crossover log...

      40 DPushes remaining with DInf 0.0000000e+00                13s
       0 DPushes remaining with DInf 7.8159701e-14                13s

    1176 PPushes remaining with PInf 0.0000000e+00                13s
       0 PPushes remaining with PInf 0.0000000e+00                13s

  Push phase complete: Pinf 0.0000000e+00, Dinf 1.2079227e-13     13s


Root simplex log...

Iteration    Objective       Primal Inf.    Dual Inf.      Time
    1219    2.2396667e+03   0.000000e+00   0.000000e+00     13s
    1219    2.2396667e+03   0.000000e+00   0.000000e+00     13s

Root relaxation: objective 2.239667e+03, 1219 iterations, 0.43 seconds
```

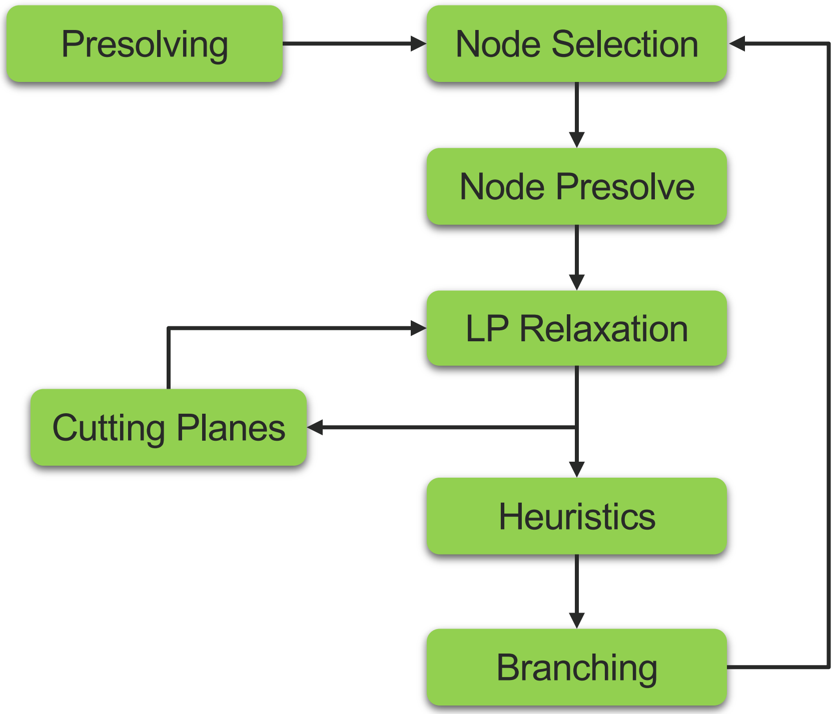
MIP tree exploration

```
    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time


     0     0 8.0000e+08    0   72 3.1334e+09 8.0000e+08  74.5%     -    0s
H    0     0                    2.400019e+09 8.0000e+08  66.7%     -    0s
     0     0 8.0000e+08    0   72 2.4000e+09 8.0000e+08  66.7%     -    0s
H    0     0                    2.066683e+09 8.0000e+08  61.3%     -    0s
     0     0 8.0000e+08    0   72 2.0667e+09 8.0000e+08  61.3%     -    0s
     0     0 8.0000e+08    0   72 2.0667e+09 8.0000e+08  61.3%     -    0s
     0     2 8.0000e+08    0   72 2.0667e+09 8.0000e+08  61.3%     -    0s
H  796   711                    2.050016e+09 8.0000e+08  61.0%   4.4    0s
H  796   675                    2.050016e+09 8.0000e+08  61.0%   4.4    0s
* 3353  2003             109    2.000021e+09 8.0000e+08  60.0%   4.5    0s
H 5097  3281                    2.000019e+09 8.0000e+08  60.0%   4.1    0s
H 5571  3544                    1.950018e+09 8.0000e+08  59.0%   4.0    0s
* 6228  4017              82    1.940019e+09 8.0000e+08  58.8%   3.9    1s
* 6347  4038              97    1.925019e+09 8.0000e+08  58.4%   3.9    1s
H 6422  3912                    1.833347e+09 8.0000e+08  56.4%   3.9    1s
* 7999  4981             104    1.816683e+09 8.0000e+08  56.0%   3.8    1s
H 9958  6572                    1.800016e+09 8.0000e+08  55.6%   3.8    1s
```

Solving a MIP model



Branch-and-cut



Branch-and-cut

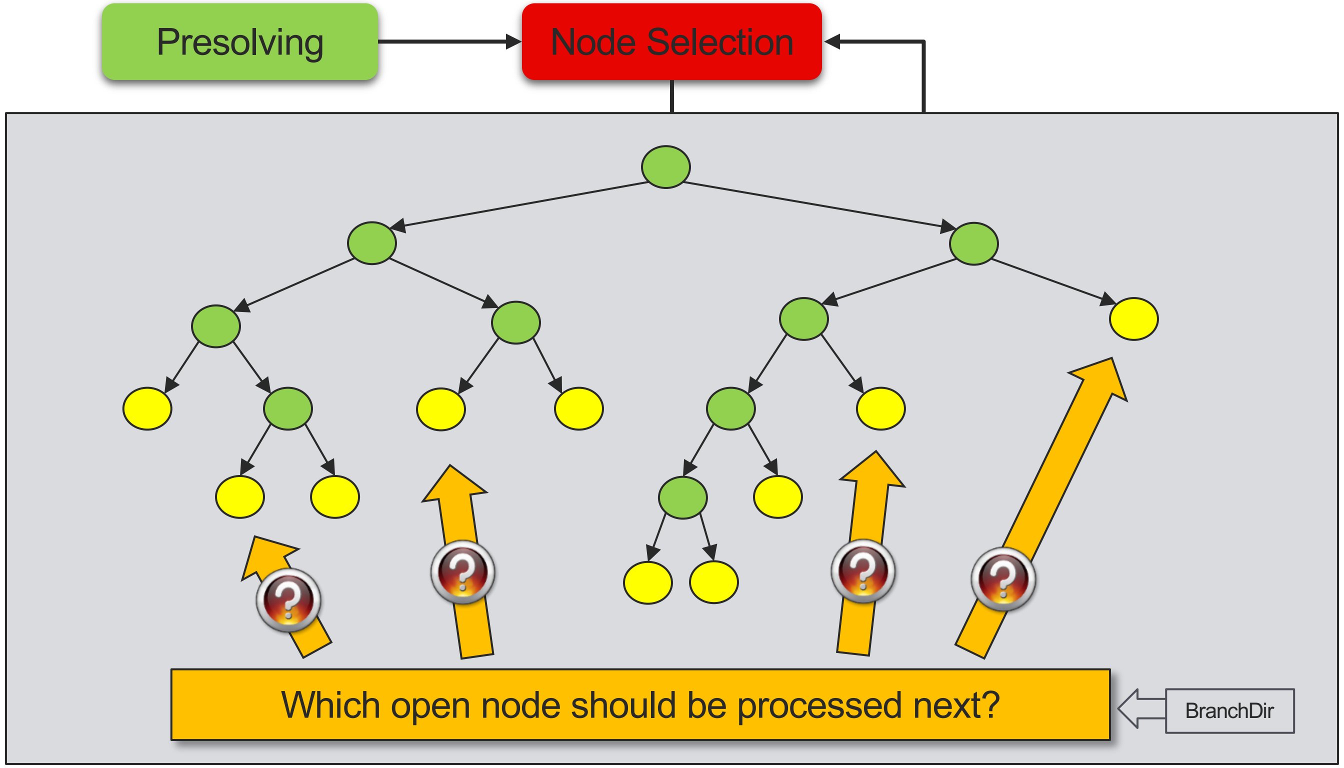
Presolving

Cutting Planes

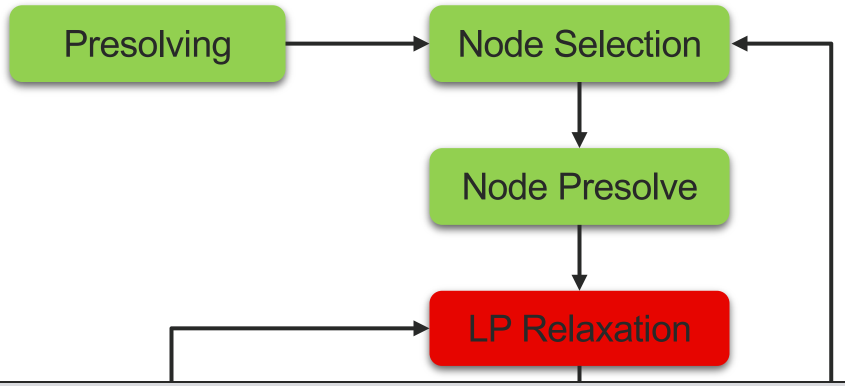
Branching

```
Gurobi Optimizer version 6.0.0 (linux64)
Copyright (c) 2014, Gurobi Optimization, Inc.

Read MPS format model from file /models/mip/roll3000.mps.bz2
Reading time = 0.03 seconds
roll3000: 2295 rows, 1166 columns, 29386 nonzeros
Optimize a model with 2295 rows, 1166 columns and 29386 nonzeros
Coefficient statistics:
  Matrix range    [2e-01, 3e+02]
  Objective range [1e+00, 1e+00]
  Bounds range    [1e+00, 1e+09]
  RHS range       [6e-01, 1e+03]
Presolve removed 1308 rows and 311 columns
Presolve time: 0.08s
Presolved: 987 rows, 855 columns, 19346 nonzeros
Variable types: 211 continuous, 644 integer (545 binary)

Root relaxation: objective 1.112003e+04, 1063 iterations, 0.03 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 11120.0279    0  154          - 11120.0279      -     -    0s
     0     0 11526.8918    0  207          - 11526.8918      -     -    0s
     0     0 11896.9710    0  190          - 11896.9710      -     -    0s
```

Branch-and-cut



Branch-and-cut




```
Presolved: 987 rows, 855 columns, 19346 nonzeros
Variable types: 211 continuous, 644 integer (545 binary)

Root relaxation: objective 1.112003e+04, 1063 iterations, 0.03 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 11120.0279    0  154          - 11120.0279      -     -    0s
     0     0 11526.8918    0  207          - 11526.8918      -     -    0s
     0     0 11896.9710    0  190          - 11896.9710      -     -    0s
...
H  327   218                    13135.000000 12455.2162  5.18%  42.6    1s
H  380   264                    13093.000000 12455.2162  4.87%  41.6    1s
H  413   286                    13087.000000 12455.2162  4.83%  41.4    1s
  1066   702 12956.2676   31  192 13087.0000 12629.5426  3.50%  37.2    5s
  1097   724 12671.8285    8  147 13087.0000 12671.8285  3.17%  41.6   10s
H 1121   702                    12890.000000 12689.9964  1.55%  40.7   13s
  1135   710 12732.5601   32  126 12890.0000 12727.1362  1.26%  44.6   15s
```

Branch-and-cut

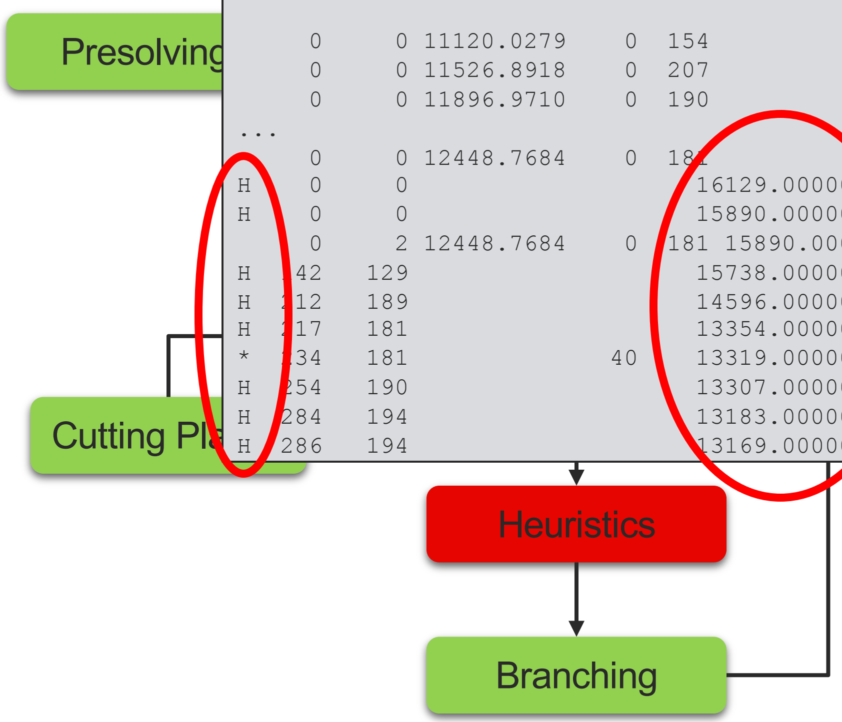
Presolving

Cutting Planes

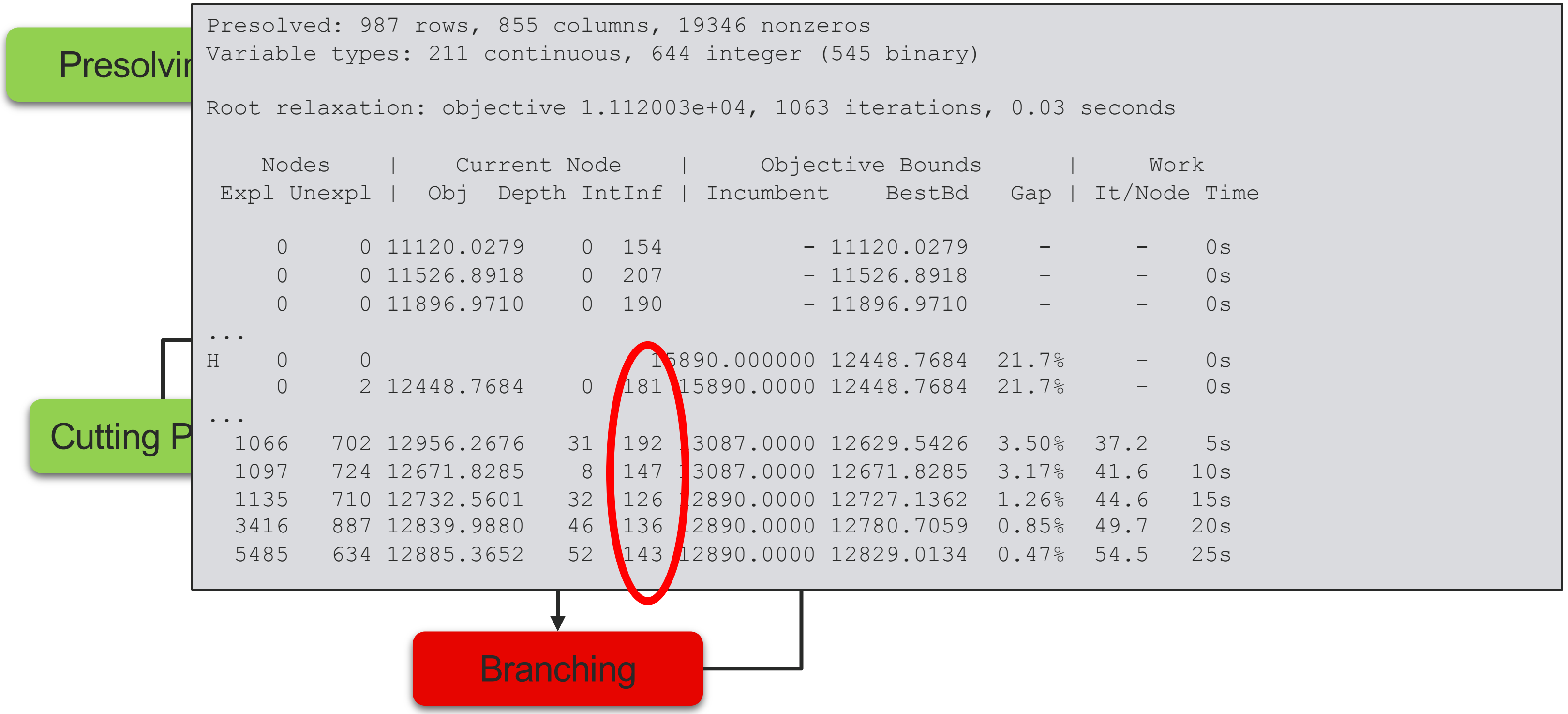
```
Presolved: 987 rows, 855 columns, 19346 nonzeros
Variable types: 211 continuous, 644 integer (545 binary)

Root relaxation: objective 1.112003e+04, 1063 iterations, 0.03 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 11120.0279    0  154          - 11120.0279      -     -    0s
     0     0 11526.8918    0  207          - 11526.8918      -     -    0s
     0     0 11896.9710    0  190          - 11896.9710      -     -    0s
     0     0 12151.4022    0  190          - 12151.4022      -     -    0s
     0     0 12278.3391    0  208          - 12278.3391      -     -    0s
...
  5485   634 12885.3652   52  143 12890.0000 12829.0134  0.47%  54.5   25s

Cutting planes:
  Learned: 4
  Gomory: 46
  Cover: 39
  Implied bound: 8
  Clique: 2
  MIR: 112
  Flow cover: 27
  GUB cover: 11
  Zero half: 91

Explored 6008 nodes (357915 simplex iterations) in 27.17 seconds
Thread count was 4 (of 8 available processors)
```

Branch-and-cut

```
Presolved: 987 rows, 855 columns, 19346 nonzeros
Variable types: 211 continuous, 644 integer (545 binary)

Root relaxation: objective 1.112003e+04, 1063 iterations, 0.03 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 11120.0279    0  154          - 11120.0279      -     -    0s
     0     0 11526.8918    0  207          - 11526.8918      -     -    0s
     0     0 11896.9710    0  190          - 11896.9710      -     -    0s
...
     0     0 12448.7684    0  181          - 12448.7684      -     -    0s
H    0     0                    16129.000000 12448.7684  22.8%     -    0s
H    0     0                    15890.000000 12448.7684  21.7%     -    0s
     0     2 12448.7684    0  181 15890.0000 12448.7684  21.7%     -    0s
H  142   129                    15738.000000 12450.7195  20.9%  43.8    1s
H  212   189                    14596.000000 12453.8870  14.7%  42.3    1s
H  217   181                    13354.000000 12453.8870  6.74%  42.6    1s
*  234   181              40    13319.000000 12453.8870  6.50%  42.1    1s
H  254   190                    13307.000000 12453.8870  6.41%  41.3    1s
H  284   194                    13183.000000 12453.8870  5.53%  42.6    1s
H  286   194                    13169.000000 12453.8870  5.43%  42.7    1s
```



Branch-and-cut




```
Presolved: 987 rows, 855 columns, 19346 nonzeros
Variable types: 211 continuous, 644 integer (545 binary)

Root relaxation: objective 1.112003e+04, 1063 iterations, 0.03 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 11120.0279    0  154          - 11120.0279      -     -    0s
     0     0 11526.8918    0  207          - 11526.8918      -     -    0s
     0     0 11896.9710    0  190          - 11896.9710      -     -    0s
...
H    0     0                    15890.000000 12448.7684  21.7%     -    0s
     0     2 12448.7684    0  181 15890.0000 12448.7684  21.7%     -    0s
...
  1066   702 12956.2676   31  192 13087.0000 12629.5426  3.50%  37.2    5s
  1097   724 12671.8285    8  147 13087.0000 12671.8285  3.17%  41.6   10s
  1135   710 12732.5601   32  126 12890.0000 12727.1362  1.26%  44.6   15s
  3416   887 12839.9880   46  136 12890.0000 12780.7059  0.85%  49.7   20s
  5485   634 12885.3652   52  143 12890.0000 12829.0134  0.47%  54.5   25s
```

Outlook: Algorithmic Performance

- **Solving optimization problems is never fast enough**

- **Great solver performance is the sum of many parts**
  - Model construction speed
  - Selection and configuration of the underlying algorithms
  - Hardware
  - …and a lot more

- **The default settings of Gurobi are carefully selected to perform best on a large variety of model instance**

- **However, there is usually some tuning potential left, e.g.**
  - The model formulation itself
  - More efficient use of the API
  - Parameter changes
  - …and a lot more

Thank you!

GUROBI OPTIMIZATION

The World's Fastest Solver

Your Next Steps

- Try Gurobi 9.0 Now!
  - www.gurobi.com/free-trial
  - Get a 30-day commercial trial license of Gurobi at academic and research licenses are free!
- For questions about Gurobi pricing, please contact sales@gurobi.com or sales@gurobi.de
- **Need help leveraging Optimization for your business?**
  - Contact us at info@gurobi.com
- A recording of this webinar, including the slides, will be available in roughly one week.

This Week!

- **Wednesday, June 24th**
  - Compute Server and Cloud Demo
  - Introduction to Algorithms
  - Advanced Algorithms
  - Ask the Experts: R&D Team
- **Thursday, June 25th**
  - Customer Case Study: KPMG
  - Customer Case Study: NFL
  - Ask the Experts: Fireside chat with Michael North, VP of Planning & Scheduling at NFL