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A comparative analysis of optimization solvers

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Abstract

Optimization software provides better design and development of optimization solutions for real-life problems. The software generates different solutions under different constraints. An attempt has been made to compare three well-known optimization solvers. The comparisons have been done in terms of capabilities and problem domain. The results reveal that CPLEX and GuRoBi provide competitive optimization solutions. However, CPLEX performs better than GuRoBi under high dimensionality problems. Besides this, CPLEX is able to solve Non-convex mixed integer quadratic problem.

Keywords*: CPLEX, Optimization solver, Linear programming, GuRoBi, XPRESS, Mixed integer programming.*

1. Introduction

In the last few years, optimization solvers have gained attention from research communities due to their capability of handling large number of constraints. They provide a better way to handle the conflicting constraints as well as objective functions. A large number of optimization solvers are developed for solving complex problems. The design and development of these solvers depends upon the nature of particular problem to be handled. Only one optimization solver is unable to solve all types of real-life problems. Hence, there is a need to study the performance of

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optimization solvers. The motivation of this paper is to analyze the performance of different optimization solvers.

In this paper, we study various different solvers from a new perspective: how they are able to handle multiple constraints and objectives. The three well-known optimization solvers are discussed with their capabilities. These are CPLEX, GuRoBi and XPRESS. These are evaluated on basis of their problem domain and capabilities. The rest of the paper is structured as follows. Section 2 briefly describes the basic concepts and work done in the field of optimization. Section 3 introduces the working of solvers. In Section 4, the optimization solvers are discussed. Section 5 describes the performance analysis of solvers. Finally, the concluding remarks are drawn in Section 6.

2. Background

2.1 *Basic Concepts of Optimization*

Optimization consists of maximizing and minimizing a function by selectively choosing input values from within defined set of values and then computing the value of the function. The mathematical formulation of optimization is given as.

$$\begin{aligned} &\text{Minimize} \quad F_o(x) \\ &\text{Subject to} \quad F_i(x) \le b_i \quad \forall\, i = 1, \ldots n \end{aligned} \tag{1}$$

where $x = (x_1, \ldots, x_n)$ be the optimization variables, and $F_o : R^n \to R$ be the objective function to be minimised and $F_i : R^n \to R, i = 1, \ldots, n$ are the various constraint functions defined as equalities or inequalities and these functions are defined from Euclidean space R^n to real numbers. Optimal solution x^* has the smallest value of F_o among all vectors that specify the constraints.

The optimization problems are classified into eight categories such as Integer Programming (IP), Linear Programming (LP), Mixed Integer Programming (MIP), Mixed Integer Linear Programming (MILP), Mixed Integer Quadratic Programming (MIQP), Non- Linear Programming (NLP), Constraint Programming (CP), Mixed Integer Second order cone Programming (MISOCP).

2.2 *Literature Review*

In 1979, General Algebraic Modeling System (GAMS) was used for solving linear and non-linear programming [1]. Bixby [2] provided a comprehensive review on linear and mixed integer programming that utilized the CPLEX as optimization solver. The first version of CPLEX (i.e., 1.0) was developed in 1988. It was used to handle 401,640 constraints and 1,584,000 variables. MOSEK was developed in 1997. It was first to utilize to solve quadratic and conic problems. Another solver named as KNITRO (Nonlinear Integer point Trust Region Optimization) was developed in 2001. It was used to solve nonlinear programming [4]. Linear Interactive and Discrete Optimizer (LINDO) was first commercial optimization solver that utilize the branch-and-bound algorithm [5]. It was used to solve LP, IP, NLP, Stochastic programming and global optimisation. For the global solution of non-convex MINLP, the general purpose solvers such as α-Branch-and-Bound (alphaBB) [6], Branch-And-Reduce Optimization Navigator (BARON) [7], and Google's linear programming system (GLOP) [8] was designed.

Mixed Integer Distributed Ant Colony Optimization (MIDACO) was initially developed in 2010 for MINLP problems that were arising from aerospace design problem [9]. It was further modified to solve general optimization problems. Another solver named as GuRoBi was released in 2009. Its performance was equivalent to CPLEX 11.1 version. It was capable to solve both linear and mixed integer programming. Jeroslow [10] has discussed the quadratic convex problems that were unsolvable from last 20 years. These problems are now solvable using CPLEX [11], GuRoBi, MOSEK to some extent due to advancement in solvers. Based on the above-mentioned facts, there is a need to study the optimization solvers with their strengths and weaknesses.

3. Workflow of Optimisation Solver

The general framework of optimization solver consists of five main procedures. These are problem formulation, model creation, model configuration, optimization, solution generation. The objective functions and constraints are defined in problem formulation. In model generation, all the functions and constraints identified are modeled using modeling language. The decision variables are integrated in the defined model. The model is configured according to the constraints and objective functions. The objective functions are optimized using the designed model. Thereafter, the optimized solutions are produced.

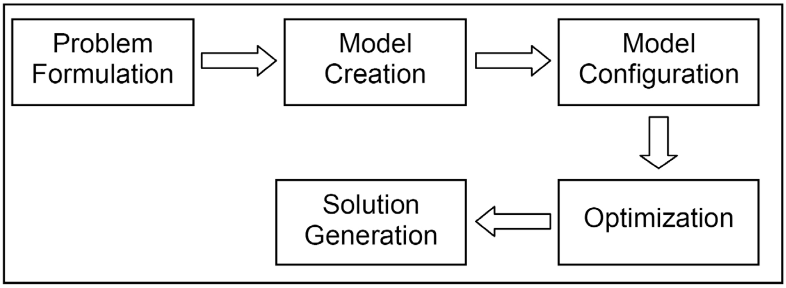

Figure 1

Workflow of Optimization Solvers

4. Optimization Solvers

The optimization solvers are broadly classified into two categories based on licensing. These are commercial and open-source software.

Table 1

Comparison among different solvers in terms of Problem Domain

Solver	Developer	LP	CP	MIP	MILP	MIQP	MISOCP	NLP
CPLEX	IBM	✓	✓	✓	✓	✓	✓	✓
XPRESS	FICO	✓	✓	✓	✓	✓	✓	✓
GuRoBi	GuRoBi	✓		✓	✓	✓	✓	
XA	Sunset Software Technologies	✓		✓				
KNITRO	Ziena Opt. Inc	✓						✓
lp_solve	Michel Berkelaar	✓		✓	✓	✓		
GLPK	GNU project	✓		✓				
CBC	-	✓			✓	✓		
Lindo	Lindo Systems Inc.	✓		✓	✓	✓		✓
OML	Ketron Management	✓			✓	✓		
FortMP	OptiRisk Sytems	✓			✓	✓		
FrontLine	FrontLine Systems			✓				
CONOPT	ARKI consulting	✓			✓	✓		✓
MOSEK	Mosek Aps	✓		✓	✓		✓	
MIDACO	ASTER Labs			✓				✓

4.1 *Open Source Solvers*

Lindo is used to solve LP, IP, NLP, and Stochastic programming problems. Coin-or Branch and Cut (CBC) is a used for mixed integer programming. It is written in C++ and can be used as either stand-alone executable or callable library. GNU Linear Programming Kit (GLPK) is a math programming project that is part of the GNU project. It was developed to solve large scale LP problems. Generic Constraint Development Environment (Gecode) is a software library for solving constraint satisfaction problems. It is implemented in C++ and strictly follows C++ standards. lp_solve is written in ANSI C and solves linear, semi-continuous, mixed- integer as well as binary, and special ordered sets models. MIDACO is a solver for general optimization problems. It can be applied on continuous (NLP), discrete/integer (IP) and mixed integer (MINLP) problems. Table 1 describes the problem domain in which solvers can perform well.

4.2 *Commercial Solvers*

FrontLine is from FrontLine Systems, which provides number of Excel Add-ons along with solving large optimization problems. CONOPT from ARKI Consulting in Denmark is a large-scale solver which is specialized in solving complicated nonlinear models; KNITRO, a solver by Ziena Inc, is capable of solving linear, nonlinear, and quadratic optimization problems, either convex or non-convex. FortMP from OptiRisk Systems supports quadratic mixed integer programming and stochastic programming, XA from Sunset Software Technologies is a relatively fast solver. Optimization Modeling Language (OML) is optimization solver that was developed by Ketron Management. MOSEK solver was developed by MOSEK ApS which is capable for solving convex MIQCPs and mixed-integer conic programs. The well-known commercial solvers are CPLEX, GuRoBi, and XPRESS. These are described in preceding subsections.

4.2.1 CPLEX

CPLEX is one of the most advanced and accepted optimization solvers. After version 12.2, CPLEX and CPL Optimizer plus all former OPL CPLEX development bundles now all equally entitle you to CPLEX Optimization Studio. It is a complete package for large scale problems as it offers every feature from heuristics to lazy constraints, from branch and cuts to call backs to Distributed or parallel optimization with ease of

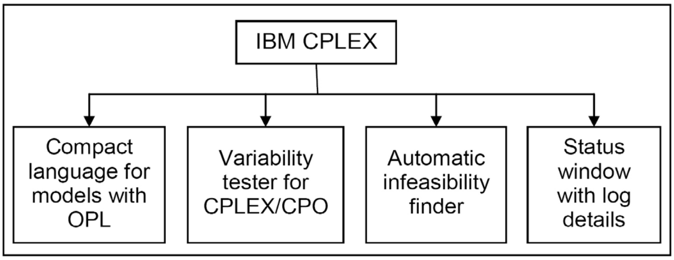

Figure 2

CPLEX Solver's Features

defining a complex problem into model and rest are the results shown. The features of CPLEX are described in Figure 2.

CPLEX with its latest release 12.7 in 2016, it has the most significant gains for MIP, MIQP, MIQCP and non-convex QP and MIQP models. For CP Optimizer, the main improvement area is scheduling models, but no such gain in combinatorial integer models. For certain MIPs with a decomposable structure, CPLEX can apply a Benders' decomposition technique which has a number of levels of control indicated through the new "benders strategy" parameter [12]. Table 2 describes the evolution of CPLEX.

Table 2

Evolution of CPLEX Solver

Version	Year	Improvements	Problem Domain
CPLEX 1.0	1988	LP solver	Simplex LP
CPLEX 2.0	1992	Simple Branch and bound, Limited cuts	MIP
CPLEX 6.0	1998	Simple Branch and bound, Limited cuts, Simple heuristics, Faster dual simplex	
CPLEX 6.5	1999	Five different node heuristics, Six types of cutting planes	
CPLEX 7.0	2000	Default LP method-dual simplex, Preprocessing, Semi-Continuous and semi-Integer values	
CPLEX 8.0	2002	New method for LP model sitting, Concurrent optimization	QP Simplex Convex MIQP

Contd...

CPLEX 9.0	2003	Relaxation induced Neighborhood search(RNS)	Convex QCP, MIQCP,SOCP
CPLEX 10.0	2006	Improvements for MIQPs, Changes in MIP start behavior, Feasible Relaxation, Indicators, Solution Polishing	
CPLEX 11.0	2007	Solution pool,Tuning tool, Parallel mode	
CPLEX 12.1	2009	Connector for mathWorks, MATLAB, Excel, CPLEX python API, Deterministic parallel barrier algorithm, Parallel algorithms	
CPLEX 12.2	2010	New functionality for MATLAB and PYTHON, MIP kappa, MSF	
CPLEX 12.4	2011	Quadratic expression interface, SOCP duals and reduced costs, Deterministic time limit, New file format-Annotated Linear Program(ALP)	Non Convex QP, MIQP
CPLEX 12.6	2013	Global Solution of Non Convex MQP, Distributed parallel MIP optimization	
CPLEX 12.7	2016	Benders Algorithm, Modeling assistance, Annotating the model, New error codes	

4.2.2 GuRoBi

GuRoBi, from GuRoBi Optimisation, Inc., is a powerful optimizer which is designed from scratch to run in multi core with capability of running in parallel mode. Figure 3 shows the features of GuRoBi.

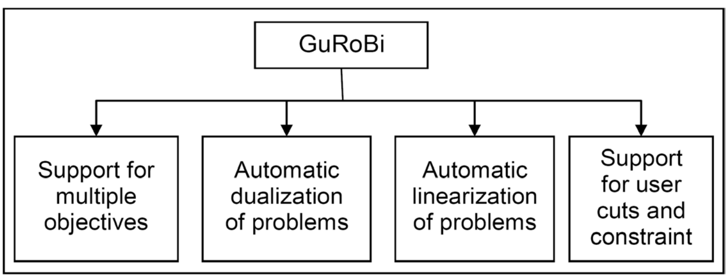

Figure 3

GuRoBi Solver's Features

It has achieved incredible performance increase with each version. With its recent release as GuRoBi 7.0, it has covered few more gaps between CPLEX as it has provided pool support for solutions along with multiple objectives, which were present earlier in CPLEX [13]. The evolution of GuRoBi is described in Table 3.

Table 3

Evolution of GuRoBi Solver

Version	Year	Improvements	Problem Domain
GuRoBi 1.0	2009	LP solver	Simplex LP
GuRoBi 4.0	2010	Continued performance improvements in MIP, dual simplex and barrier, Visual studio 2010 support	QP and MIP solvers, concurrent LP
GuRoBi 4.6	2011	Expanded python mdeling interface, Improved MIP performance,Support for user branching priorities in MIP, support for .zip and .7zip files, New xero objective heuristics	
GuRoBi 5.0	2010	Support for MATLAB and R, Overall performance enhancements, New FeasRelax for finding solutions.	quadratic constraints
GuRoBi 5.1	2013	Improved MIP Performance, User control of random number seed, MIP start files from the command line interface, Visual Studio 2012 support	
GuRoBi 5.5	2013	Client server functionality, Automatic Parameter tuning tool, Barrier crossover factorization improvement	
GuRoBi 5.6	2013	Improved LP and MIP performance, Distributed tuning, Distributed concurrent optimization, Additional user control of asynchronous optimization	QCP
GuRoBi 6.0	2014	Distributed tuning, Distributed concurrent, Piecewise linear Objective Support, Explicit support for lazy constraints	Distributed MIP

Contd...

GuRoBi 6.5	2015	Records sequence of GuRoBi commands, Variable hints, API simplification	
GuRoBi 7.0	2016	Support for Multi objective, MIP solution pool support, Annotating a model, Enhanced .Net Property support, Support for tuning criterion	

4.2.3 XPRESS

XPRESS is worldwide known solver originally developed by Dash Optimization and was acquired by FICO in 2008. It is capable of solving very large optimisation problems especially mixed integer. Xpress-Solvers are often able to solve problems that other solvers can't solve and is first to support true 64bit for modeling and optimization [14]. Figure 4 shows some useful features of XPRESS.

The evolution of XPRESS is depicted in Table 4. New Version Xpress 8.0 has provided new improved MIP parallel code along with addition of java and .NET API's[15].

5. Performance Analysis

Due to advancements in solvers, most of the real-life problems can be solved in reasonable time bound. However, these are unable to deliver the optimal solutions for some special situations. There is a need to develop a hybrid model using the existing solvers for these situations. Therefore, the capabilities of three well-known solvers are need to be explored for designing a hybrid model. The capabilities of optimization solvers are described in Table 5.

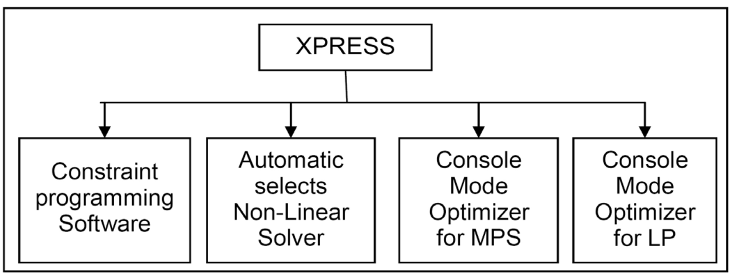

Figure 4

XPRESS Solver's Features

Table 4

Evolution of XPRESS solver

Version	Year	Improvement	Problem Domain
XPRESS-MP	1983	Xpress-MP LP solver released, general purpose algebraic modeling language (mp-model) Dash optimisation founded by Bob Daniel and Robert Ashford	LP
XPRESS	1985	MP-Model-Modeling language	
XPRESS	1992	Parallel MIP (1997 on distributed PC/ Linux networks)	MIP
XPRESS 2001 A	2001	Mosel Modeling language, Xpress-IVE	QP
XPRESS 2003B	2003	XSLP non linear solver added	Non-Linear Optimization With MINLP
XPRESS 2008A	2008	Dash optimisation bought by Fair Isaac(Now FICO)	MIQP
XPRESS 7.1	2009	Multi-threaded performance, Parallel heuristics	
XPRESS	2012	Xpress Optimisation Modeller	
XPRESS	2013	automatic solver selection for NLP	
XPRESS 8.0	2016	Parallelization For MIP	

Present Day solver support various means for creating custom-made solution approaches that can incorporate specific techniques, for example, dynamic cut and column generation, specialized branching, or heuristics to help discover optimal solutions more rapidly. At the most elevated amount is the modeling language which enable the implementation of various complex algorithms by utilising presolve, primal heuristics features and various successive call backs to a solver. At the minimal level, the application programming interfaces (APIs) or subroutine libraries enable interaction with the solver through different low level functions within a programming language. Lately, we have seen the development of conditions that fall between modeling language and an API. In these solvers, distributed parallel environments focus on making the model development process done by modelling languages an easy task while maximising the efficiency and maintaining low level controls of an API.

Table 5

Capability Comparison among CPLEX, GuRoBi, and Xpress

	CPLEX	**GuRoBi**	**XPRESS**
Broad range of API supported	Yes	Yes	Yes
Unique Language accepted	OPL	-	Mosel
Works on cloud	Yes	Yes	Yes
Convex LP/MIQP	Yes	Yes	Yes
Migration to Others	Yes	Yes	-
Supports Multi Objectives	Yes	Yes	-
Non-Convex LP/MIQP	Yes	-	-
GUI	Yes	-	Yes
Perform Better	Single Core	Multi Core	Single Core

6. Conclusion

In this paper, the performance of three well known optimization solver has been analyzed. The results are evaluated in terms of their capabilities and problem domain. It has been observed that there is no single best solver for all types of problems or for all quality measures. The appropriateness of an optimization solver depends upon the nature of problem and computation time. On the basis of our analysis, we have reported that CPLEX and GuRoBi provide competitive results on real life problems. Besides this, CPLEX is able to solve MIQP. XPRESS performs better than CPLEX and GUROBI on complex and high scalability due to their multithreading capabilities.

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