ENUNCIADO EXAME

CURSO:	MIAA
UNIDADE CURRICULAR:	Math Foundations for Artificial Intelligence
ANO CURRICULAR:	1º — **SEMESTRE:** 1º
DOCENTE:	Teresa Abreu

☒ **Época Normal** ☐ **Época Exames** ☐ **Época Especial** ☐ **Exame Especial (a)** ☐ **Justificação de Faltas**

☐ **Com consulta** ☐ **Sem consulta** **Duração:** 1 **hora** 30 **minutos** **Tolerância:** 0 **minutos**

ANO LECTIVO: 2023/2024 **DATA EXAME:** 13-11-2023

1. Consider in the Euclidean vector space $\mathbb{R}^5$ the subspace $U = \left\{ \begin{bmatrix} x \\ y \\ x-y \\ 3y+x \\ w \end{bmatrix} : x, y, w \in \mathbb{R} \right\}$ and the vector $u = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 4 \\ 5 \end{bmatrix}$.

   1.1. Find an base for U.

   1.2. Verify if $u \in U^{\perp}$.

2. Consider in the Euclidean vector space $\mathbb{R}^4$ with the dot product the subspace . $U = span[\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}]$ and the vector $x = \begin{bmatrix} 1 \\ 2 \\ 2 \\ -3 \end{bmatrix}$.

   2.1. Find an base for U.

   2.2. Find an orthonormal basis of U.

   2.3. Determine the orthogonal projection $\pi_U(x)$ of x onto U.

   2.4. Consider the plane in $\mathbb{R}^4$ which is defined by $P := U + \begin{bmatrix} 1 \\ 12 \\ -1 \\ 8 \end{bmatrix}$. Determine the distance between P and x.

3. Consider the matrix: $A = \begin{bmatrix} 4 & 3 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 2 & 2 \end{bmatrix}$.

   Without resorting to maple commands "eigenvalue, etc ".

   3.1. Verify if the matrix A is diagonalizable?

   3.2. Determine the eigendecomposition.

4. Let be the matrix $B = \begin{bmatrix} -1 & 10 & -5 \\ -9 & 6 & 3 \\ 11 & -2 & 1 \\ -3 & 6 & -9 \end{bmatrix}$

   4.1. Without resorting to maple commands "Singular Values" or "Singular Value Decomposition". Find the singular value decomposition of B.

   4.2. Determine the spectral norm of B.

   4.3. Find the rank-2 approximation of B.