a. $r^2_{yx_1} = \dfrac{SSE(x_2 \cdots x_k) - SSE(x_1 \cdots x_k)}{SSE(x_2 \cdots x_k)}$

$= \dfrac{SSE_R - SSE_F}{SSE_R} = \dfrac{t^2}{t^2 + n-k-1}$

$t^2 = F_{n-k-1} = \dfrac{(SSE_R - SSE_F)/1}{SSE_F/n-k-1}$

under H_0.
$Y \sim N(X_R B_R, I)$

$\dfrac{SSE_R - SSE_F}{\sigma^2} = Y'(I-H_R)Y - Y'(I-H)Y = \dfrac{Y'(H-H_R)Y}{\sigma^2}$

$(H-H_R)(H-H_R) = HH - HH_R - H_RH + H_RH_R$ $\quad H_RH = X_R(X_R'X_R)^{-1}X_R'H = H_R$

$= H - H_R - H_R + H_R = H - H_R$

$\dfrac{SSE_R - SSE_F}{\sigma^2} \sim \chi^2_{tr(H-H_R)} = tr(H) - tr(H_R) = k+1-k = 1$

$(H-H_R)X_RB_R = HX_RB_R - H_RX_RB_R = 0$

$\dfrac{(n-k-1)S_e^2}{\sigma^2} \sim \chi^2_{n-k-1}$

$t^2 = \dfrac{\frac{SSE_R - SSE_F}{\sigma^2}/1}{\frac{SSE_F}{\sigma^2}/(n-k-1)} = \dfrac{SSE_R - SSE_F}{SSE_F/(n-k-1)} \sim F_{1,n-k-1}$

$SSE_R - SSE_F = \dfrac{t^2 SSE_F}{n-k-1}$ $\quad SSE_R = SSE_F + \dfrac{t^2 SSE_F}{n-k-1}$

$r^2_{yx_1} = \dfrac{\frac{t^2(SSE_F)}{n-k-1}}{SSE(1+\frac{t^2}{n-k-1})} = \dfrac{t^2}{t^2+n-k-1}$

b. $C = (0, \cdots 0\ 1)$

$r = 0$

$H_0: CB = r.$

$H_0: B_k = 0$

$H_a: B_k \neq 0$

$$\frac{\hat{B}'X'Y - \hat{B}_1'X_1'Y}{SSE/(n-k-1)} \sim F_{1,n-k-1}$$

$$SSR = \Sigma(\hat{y}_i - \bar{y})^2 = \Sigma \hat{y}_i^2 - n\bar{y}^2$$

$$= \hat{Y}'\hat{Y} - n\bar{y}^2$$

$$= \frac{\hat{B}'X'X\hat{B} - \hat{B}_1'X_1'X_1\hat{B}_1}{SSE/(n-k-1)}$$

$$= \frac{\hat{Y}'\hat{Y} - \hat{Y}_1'\hat{Y}_1}{SSE/(n-k-1)}$$

$$= \frac{SSR_F + n\bar{y}^2 - SSR_R - n\bar{y}^2}{SSE/(n-k-1)}$$

$$= \frac{SST - SSE_F - SST + SSE_R}{SSE/(n-k-1)}$$

$$= \frac{SSE_R - SSE_F}{SSE/(n-k-1)} \sim F_{1,n-k-1}$$

c. $$\frac{\hat{B}'X'Y}{(k+1)S_e^2} = \frac{\hat{B}'X'Y/(k+1)}{S_e^2} = \frac{\hat{B}'X'Y/(k+1)}{SSE/(n-k-1)} \sim F_{k+1,n-k-1}$$

$H_0: B = 0$

$H_a: B \neq 0$

$Y \sim N(0, \sigma^2 I)$ $\quad \hat{B} \sim N(B, \sigma^2(X'X)^{-1})$ $\quad (X'X) = P\Lambda P'$

$V = (X'X)^{\frac{1}{2}}\hat{B} \sim N((X'X)^{\frac{1}{2}}B, \sigma^2 I)$

$\frac{V}{\sigma} \sim N\left(\frac{(X'X)^{\frac{1}{2}}B}{\sigma}, I\right)$

$$\frac{V'V}{\sigma^2} = \frac{\hat{B}'(X'X)\hat{B}}{\sigma^2} \sim \chi^2_{k+1}$$

$$NCP = \frac{B'X'XB}{\sigma^2}$$

d. $H_0: \beta_{(0)} = 0$

$H_a: \beta_{(0)} \neq 0$.

$$F = \frac{(SSE_R - SSE_F)/m}{SSE/(n-k-1)}$$

$$SSE_R = Y'(I-H_R)Y.$$

$$SSE_R^* = (Y-c1)'(I-H_R)(Y-c1)$$

$$= Y'(I-H_R)Y - c1'(I-H_R)Y - cY'(I-H_R)1 + c^2 1'(I-H_R)1$$

$$= SSE_R - 0 - 0 + 0 \quad \text{since } (I-H_R)1 = 1 - H_R 1 = 1-1 = 0$$

$\therefore$ F is unchanged since SSE_R is unchanged.

But is intercept is not included, SSE_R' will not be the same $\therefore$ F will change.