1. $E(b) = E(AY) = E(A(XB+\varepsilon)) = AXB + EA\varepsilon = AXB$

b Unbiase $\Rightarrow$ $E(b) = B \Rightarrow AXB = B \Rightarrow AX = I$

$q'[Var(b) - Var(\hat{B})]q$

$= q'[A6^2A' - 6^2(X'X)^{-1}]q$

$= 6^2 q'[AA' - (X'X)^{-1}]q$

$= 6^2 q'[AA' - AX(X'X)^{-1}X'A']q$ as $AX = I$. $AX(X'X)^{-1}X'A' = I(X'X)^{-1}I = (X'X)^{-1}$

$= 6^2 q'A[I-H]A'q$ Let $w' = q'A$ then $w = A'q$

$= 6^2 w'[I-H]w \geq 0$ as $I-H$ is positive semi-definite

Since $q'[Var(b) - Var(\hat{B})]q \geq 0$. $q \in R^n$.

$\therefore$ $Var(b) - Var(\hat{B})$ is positive semi-definit.

2b. $SSE = e'e = y'(I-H)y = (y-X\hat{\beta})'(y-X\hat{\beta})$

$$= y'y - y'X\hat{\beta} - \hat{\beta}'X'y + \hat{\beta}'X'X\hat{\beta}$$

$$= y'y - 2\hat{\beta}'X'y + \hat{\beta}'X'X(X'X)^{-1}X'y$$

$$= y'y - \hat{\beta}'X'y$$

$$= y'y - \hat{\beta}'X'X\hat{\beta}$$

$$= y'y - \hat{\beta}'X'\hat{y}$$

2c. $(y-X\beta)'(y-X\beta) = (y - X\hat{\beta} + X\hat{\beta} - X\beta)'(y - X\hat{\beta} + X\hat{\beta} - X\beta)$ PS: $y - X\hat{\beta} = e$

$$= e'e + e'(X\hat{\beta} - X\beta) + (\hat{\beta}-\beta)'X'e + (\hat{\beta}-\beta)'X'X(\hat{\beta}-\beta)$$

$$= e'e + (\hat{\beta}-\beta)'X'X(\hat{\beta}-\beta)$$

Since $e'X = 0$ $X'e = (e'X)' = 0$.

2a. $H = \begin{pmatrix} | & | & & | \\ h_1 & h_2 & \cdots & h_n \\ | & | & & | \end{pmatrix}$

$$HH = \begin{pmatrix} - h_1^T - \\ \vdots \\ - h_n^T - \end{pmatrix} \cdot \begin{pmatrix} | & & | \\ h_1 & \cdots & h_n \\ | & & | \end{pmatrix} = \begin{pmatrix} h_1^T h_1 & h_2^T h_2 & \cdots & h_n^T h_n \\ \vdots & \vdots & & \vdots \\ h_n^T h_1 & \vdots & & h_n^T h_n \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{12} \\ \\ h_{n1} & h_{n2} & \cdots & h_{nn} \end{pmatrix}$$

$\therefore h_{ij} = h_i^T h_j$

$$h_{ii} = h_i^T h_i$$

$$= \sum_{j=1}^{n} h_{ij}^2$$