①

1. 0.196226
2. 0.039
3. 3.646×10^{-4}
4. 4
5. 3
6. 40.74374817
7. −6.595127425
8. 1.324550981

② a. $\hat{\beta} = \beta + A\varepsilon$.

$\hat{\beta} = (X'X)^{-1}X'y = (X'X)^{-1}X'(X\beta + \varepsilon) = (X'X)^{-1}X'X\beta + (X'X)^{-1}X'\varepsilon = \beta + (X'X)^{-1}X'\varepsilon$

$\therefore A = (X'X)^{-1}X'$

$\hat{y} = Hy = H(X\beta + \varepsilon) = HX\beta + H\varepsilon = X\beta + H\varepsilon.$

$\therefore B = H$

b. $SSE = \Sigma e_i^2 = e'e = (y - X\hat{\beta})'(y - X\hat{\beta})$

$= [(I-H)y]'[(I-H)y]$

$= [(I-H)(X\beta+\varepsilon)]'[(I-H)(X\beta+\varepsilon)]$

$= [(I-H)\varepsilon]'[(I-H)\varepsilon]$

$= \varepsilon'(I-H)(I-H)\varepsilon$

$= \varepsilon'(I-H)\varepsilon$

c. $e'X = [(I-H)y]'X = y'(I-H)'X = y'(X-X) = 0$

d. $e'\hat{y} = [(I-H)y]'\hat{y} = y'(I-H)'Hy = y'(H-H)y = 0.$

3. $x = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix}$ $x' = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$ $y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$

$x'x = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ $(x'x)^{-1} \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{6} \end{pmatrix}$

$x'y = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} y_1 + y_2 + y_3 \\ -y_1 + y_3 \\ y_1 - 2y_2 + y_3 \end{pmatrix}$

$Var(\hat{B}) = \sigma^2 (x'x)^{-1} = \begin{pmatrix} \frac{1}{3}\sigma^2 & 0 & 0 \\ 0 & \frac{1}{2}\sigma^2 & 0 \\ 0 & 0 & \frac{1}{6}\sigma^2 \end{pmatrix}$

$\hat{B} = (x'x)^{-1} x'y$

$= \begin{pmatrix} \frac{1}{3}(y_1 + y_2 + y_3) \\ \frac{1}{2}(-y_1 + y_3) \\ \frac{1}{6}(y_1 - 2y_2 + y_3) \end{pmatrix}$

4. $Hx = x$

$Hx = \begin{pmatrix} | & | & & | \\ H1 & Hx_1 & \cdots & Hx_k \\ | & | & & | \end{pmatrix} = \begin{pmatrix} | & | & & | \\ 1 & x_1 & \cdots & x_k \\ | & | & & | \end{pmatrix}$

$H = \begin{pmatrix} | & | & & | \\ h_1 & h_2 & \cdots & h_n \\ | & | & & | \end{pmatrix} = \begin{pmatrix} — h_1 — \\ \vdots \\ — h_n — \end{pmatrix}$

$H1 = \begin{pmatrix} h_1^T 1 \\ \vdots \\ h_n^T 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$

$\therefore h_i^T 1 = 1 \Rightarrow \sum_{j=1}^{n} h_{ij} = 1$

If intercept is dropped, then we can not assume $\sum_{j=1}^{n} h_{ij} = 1$