1.a. Using R. $\hat{B_0} = 0.289523$ $\hat{B_1} = 17.367688$

$Var(\hat{B_0} - 3\hat{B_1} - 15) = Var(\hat{B_0}) + Var(3\hat{B_1}) - 6cov(\hat{B_0}, \hat{B_1})$

$$= \frac{\sigma^2}{n} + \frac{\bar{x}^2\sigma^2}{\Sigma(x_i-\bar{x})^2} + \frac{9\sigma^2}{\Sigma(x_i-\bar{x})^2} + \frac{6\bar{x}\sigma^2}{\Sigma(x_i-\bar{x})^2}$$

$$\therefore \frac{\dfrac{\hat{B_0}-3\hat{B_1}-15-0}{\sqrt{Var(\hat{B_0}-3\hat{B_1}-15)}}}{\sqrt{\dfrac{(n-2)S_e^2}{\sigma^2(n-2)}}} = \frac{\hat{B_0} - 3\hat{B_1} - 15}{\sqrt{S_e^2 \cdot \dfrac{\Sigma x_i^2 + 6\bar{x} + 9}{\Sigma(x_i-\bar{x})^2}}}$$

$t = 0.002240767$ using R

1.b.

$$\frac{(SSE_R - SSE_F)/(df_R - df_F)}{SSE_F/df_F} \sim F_{df_R - df_F,\, df_F}$$

$SSE_F = \Sigma e_i^2$ $SSE_R = \Sigma e_i^{*2}$

$B_0 - 3B_1 - 15 = 0.$ $B_0 = 3B_1 + 15$

$y_i = 3B_1 + 15 + B_1 x_i + \varepsilon_i$

$y_i - 15 = B_1(x_i + 3) + \varepsilon_i$

$\hat{B_1}^* = \frac{\Sigma x_i^+ y_i^+}{\Sigma x_i^{+2}}$ $\Sigma x_i^+ = (x_i + 3)$ $y_i^+ = y_i - 15$

$dF_R = n-1$ $dF_F = n-2$ by previous hw.

$$\therefore \frac{(SSE_R - SSE_F)}{SSE_F/(n-2)} = \frac{\Sigma e_i^{*2} - \Sigma e_i^2}{\Sigma e_i^2/(n-2)} \sim F_{1,n-2}$$

c. The part $\frac{(\hat{B_0} - 3\hat{B_1} - 15)^2}{Var(\hat{B_0} - 3\hat{B_1} - 15)}$ is equal to $(SSE_R - SSE_F)$

2. $E(\hat{y_{g_i}}) = \hat{B_0} + \hat{B_1} x_g$. unbiased

$Var(\hat{y_{g_i}}) = Var(\hat{B_0} + \hat{B_1} x_g)$

$= Var(\bar{y} - \hat{B_1}\bar{x} + \hat{B_1} x_g)$

$= Var(\bar{y}) + Var(\hat{B_1})(x_g - \bar{x})^2 + cov(\bar{y}, \hat{B_1})(x_g - \bar{x})$

$= \frac{6^2}{n} + \frac{6^2(x_g-\bar{x})^2}{\Sigma(X_i-\bar{x})^2}$

$$\frac{E(\hat{y_{g_i}}) - E(y_{g_i})}{\sqrt{\frac{6^2}{n} + \frac{6^2(x_g-\bar{x})^2}{\Sigma(X_i-\bar{x})^2}}} \sim t_{n-2}$$

$\therefore$ let significance level be α.

$$\left(E(\hat{y_{g_i}}) - t_{\frac{\alpha}{2}, n-2}\sqrt{6^2\left(\frac{1}{n} + \frac{(x_g-\bar{x})^2}{\Sigma(x_i-\bar{x})^2}\right)},\ E(\hat{y_{g_i}}) + t_{\frac{\alpha}{2}, n-2}\sqrt{6^2\left(\frac{1}{n} + \frac{(x_g-\bar{x})^2}{\Sigma(X_i-\bar{x})^2}\right)} \right)$$

Exe 3.

$$\left.\begin{aligned} y_i &= \beta_0 + \varepsilon_i \\ x_i &= \gamma_0 + \eta_i \end{aligned}\right\} \Longrightarrow \begin{aligned} \hat{y}_i &= \hat{\beta}_0 = \bar{y} \\ \hat{x}_i &= \hat{\gamma}_0 = \bar{x} \end{aligned}$$

$e_i = y_i - \bar{y}$

$e_i^* = x_i - \bar{x}$

$e_i = \alpha_0 + \alpha_1 e_i^* + r_i$

$$\hat{\alpha}_1 = \frac{\Sigma(e_i^* - \bar{e_i^*})(e_i - \bar{e_i})}{\Sigma(e_i^* - \bar{e})^2} = \frac{\Sigma e_i^* e_i}{\Sigma e_i^{*2}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{\Sigma(x_i - \bar{x}) y_i - 0}{\Sigma(x_i - \bar{x})^2}$$

$\hat{\alpha}_0 = \bar{e}_i - \hat{\alpha}_1 e_i^*$

$\therefore \hat{\alpha}_1 = \hat{\beta}_1$

4 a. $\begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix} = \frac{6^2}{n\Sigma(x_i-\bar{x})^2}\begin{pmatrix}\Sigma x_i^2 & -\Sigma x_i\\ -\Sigma x_i & n\end{pmatrix}\begin{pmatrix}1\\ x_0\end{pmatrix}$

$$= \begin{pmatrix} \frac{6^2}{n} + \frac{6^2\bar{x}^2}{\Sigma(x_i-\bar{x})^2} - \frac{6^2\bar{x}x_0}{\Sigma(x_i-\bar{x})^2} \\ \frac{-\bar{x}6^2}{\Sigma(x_i-\bar{x})^2} + \frac{x_0 6^2}{\Sigma(x_i-\bar{x})^2} \end{pmatrix}$$

$\therefore \lambda_1 = 6^2\frac{\Sigma x_i^2 - n\bar{x}x_0}{n\Sigma(x_i-\bar{x})^2}$ $\quad \lambda_2 = 6^2\frac{x_0-\bar{x}}{\Sigma(x_i-\bar{x})^2}$

b. $\Sigma a_i y_i = \Sigma \frac{\lambda_1+\lambda_2 x_i}{6^2} y_i$

$$= \Sigma \frac{\Sigma x_i^2 - n\bar{x}x_0 + nx_0 x_i - n\bar{x}x_i}{n\Sigma(x_i-\bar{x})^2} y_i$$

$$= \frac{n\bar{y}\Sigma x_i^2 - n\bar{x}x_0\Sigma y_i + nx_0\Sigma x_i y_i - n\bar{x}\Sigma x_i y_i \pm n^2\bar{x}^2\bar{y}}{n\Sigma(x_i-\bar{x})^2}$$

$$= \frac{\bar{y}(\Sigma x_i^2 - n\bar{x}^2) + \Sigma x_i y_i(x_0-\bar{x}) - \bar{x}x_0\Sigma y_i + n\bar{x}^2\bar{y}}{\Sigma(x_i-\bar{x})^2}$$

$$= \bar{y} + \frac{(x_0-\bar{x})\Sigma x_i y_i + \bar{x}(x_0-\bar{x})\Sigma y_i}{\Sigma(x_i-\bar{x})^2}$$

$$= \bar{y} + (x_0-\bar{x})\frac{\Sigma(x_i-\bar{x})y_i}{\Sigma(x_i-\bar{x})^2}$$

$$= \bar{y} + \hat{B}_1(x_0-\bar{x})$$