**Problem 4.2**

xInitial = 0;

xFinal = 2;

h = 0.2;

yInitial = 0;

yFinal = -4;

xbar = [0.2, 0.1, 0.05];

e1 = zeros(3, 1);

e2 = zeros(3, 1);

e3 = zeros(3, 1);

index = 1;

for h = [0.2, 0.1, 0.05]

N = (xFinal - xInitial)/h;

% exact

fexact = @(x) 1/6\*x^3\*exp(x) - 5/3\*x\*exp(x) + 2\*exp(x) - x - 2;

y = zeros(N+1,1);

x = zeros(N+1,1);

x(1) = xInitial;

y(1) = yInitial;

x(N+1) = xFinal;

y(N+1) = yFinal;

for i = 1:N-1

x(i+1) = x(i) + h;

y(i+1) = fexact(x(i+1));

end

% FDM

p = -2;

q = 1;

f = @(x) x\*exp(x) - x;

A = zeros(N-1, N-1);

A(1,1) = -2/h^2 + q;

A(1,2) = 1/h^2 + p/(2\*h);

A(N-1,N-2) = 1/h^2 - p/(2\*h);

A(N-1,N-1) = -2/h^2 + q;

for i = 2:N-2

A(i,i-1) = 1/h^2 - p/(2\*h);

A(i,i) = -2/h^2 + q;

A(i,i+1) = 1/h^2 + p/(2\*h);

end

B = zeros(N-1,1);

B(1) = f(xInitial + h) - yInitial/h^2 + p\*yInitial/(2\*h);

B(N-1) = f(xFinal - h) - yFinal/h^2 - p\*yFinal/(2\*h);

for i = 2:N-2

B(i) = f(xInitial + i\*h);

end

FDMY = inv(A)\*B;

fy = zeros(N+1,1);

fy(1) = yInitial;

fy(N+1) = yFinal;

for i = 2:N

fy(i) = FDMY(i-1);

end

e1(index) = sum(abs(y - fy));

e2(index) = sqrt(sum((y-fy).^2));

e3(index) = max(abs(y-fy));

end

plot(xbar, e1, 'DisplayName','1-norm')

hold on

plot(xbar, e2, 'DisplayName','2-norm')

hold on

plot(xbar, e3, 'DisplayName','infinity-norm')

legend

title('Problem 4.2')

xlabel('x')

ylabel('y(x)')

Values:

|  |  |  |  |
| --- | --- | --- | --- |
| h | 1-norm | 2-norm | infinity-norm |
| 0.05 | 0.0114 | 0.0021 | 0.0006 |
| 0.1 | 0.0228 | 0.0059 | 0.0024 |
| 0.2 | 0.0451 | 0.017 | 0.0096 |

