Lab 02：

2023年1月19日 11:08

Question 1：

let measurement $C = (S + N_A + N_p)A$, where S is signal. — (1)

$N_A \to$ Zero mean amplifier noise with variance σ_A^2

$N_p \to$ Zero mean photon noise with , A is amplifier gain.

a) Variance of C is $\sigma_C^2 = \overline{C^2} - (\bar{C})^2$.

from equation (1) $\bar{C} = (\bar{S} + \overline{N_A} + \overline{N_p})A = \bar{S}A$.

as $\overline{N_A} = 0$, $\overline{N_p} = 0$.

$C^2 = (S^2 + N_A^2 + N_p^2 + 2SN_A + 2SN_p + 2N_pN_A)A^2$

$\overline{C^2} = (\overline{S^2} + \overline{N_A^2} + \overline{N_p^2} + 2\overline{SN_A} + 2\overline{SN_p} + 2\overline{N_pN_A})A^2$

Since S, N_A & N_p are independent, $\left.\begin{aligned} \overline{SN_A} &= \bar{S}\,\overline{N_A} = 0 \\ \overline{SN_p} &= \bar{S}\,\overline{N_p} = 0 \\ \overline{N_pN_A} &= \overline{N_p}\,\overline{N_A} = 0 \end{aligned}\right\}$.

$\overline{C^2} = (\overline{S^2} + \sigma_A^2 + \overline{N_p^2})A^2$, let $\overline{N_p^2} = \sigma_p^2$.

$\overline{C^2} = (\overline{S^2} + \sigma_A^2 + \sigma_p^2)A^2$.

$\sigma_C^2 = \overline{C^2} - (\bar{C})^2 \Rightarrow$

$\sigma_C^2 = A^2\overline{S^2} - (\bar{S})^2A^2 + A^2(\sigma_p^2 + \sigma_A^2)$.

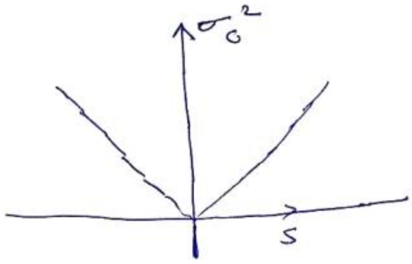
$$\boxed{\sigma_C^2 = A^2(\overline{S^2} - (\bar{S})^2) + A^2(\sigma_p^2 + \sigma_A^2)} \text{ — (1)}$$

b) the variance $\sigma_S^2 = \overline{S^2} - (\bar{S})^2$.

$\sigma_C^2 = A^2(\sigma_S^2 + \sigma_p^2 + \sigma_A^2)$ — (2)

is signal strength increase (S increase) so does it's variance σ_S^2.

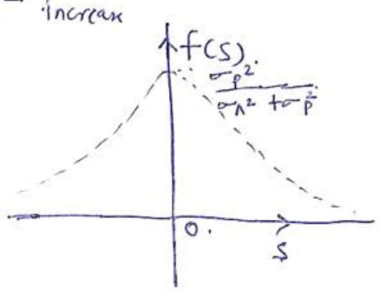
assuming σ_p^2 and σ_A^2 are constant



c) the fraction due to photon noise is.

$$f(s) = \frac{A^2\sigma_p^2}{\sigma_C^2} = \frac{\sigma_p^2}{\sigma_A^2 + \sigma_p^2 + \sigma_s^2}.$$

d) as. $|S|$ increase, σ_s^2 increase
so $f(s)$ decrease



e) $$f(s) = \frac{\sigma_p^2}{\sigma_A^2 + \sigma_p^2 + \sigma_s^2}.$$

at high Light level, $\sigma_p^2 \to \infty$ (let assume.

$$f(s) = \lim_{\sigma_p^2 \to \infty} \frac{\sigma_p^2}{\sigma_A^2 + \sigma_p^2 + \sigma_s^2} = \frac{1}{1 + \frac{\sigma_A^2}{\sigma_p^2} + \frac{\sigma_s^2}{\sigma_p^2}} = 1.$$

at Low Light level, $\sigma_p^2 \to 0$.

$$f(s) = \lim_{\sigma_p^2 \to 0} \frac{\sigma_p^2}{\sigma_A^2 + \sigma_p^2 + \sigma_s^2}. = 0.$$

hence. photon noise represent Larger fraction of the total noise. at high light level.

Question 2:

Question 3 (18 points) Suppose that in an imaging system using a lens the focal length of the lens is 6cm and the image plane is a distance 10cm behind the lens.

a) How far in front of the lens on the optical axis of the system must we place a point to get an image of the point without blur?

b) If the lens diameter is 1cm and a point is placed at 18cm in front of the lens, what will be the blur diameter of the image of the point?

3 $f = +6\,cm$

Object distance = $d_o = -10\,cm$

using lens formula

$$\frac{1}{d_i} = \frac{1}{f} + \frac{1}{d_o}$$

$d_i \rightarrow$ image distance

$$d_i = \frac{d_o \times f}{d_o + f}$$

$$d_i = \frac{-10 \times 6}{-10+6} = 15\,cm$$

$\therefore$ Required distance $= d_i = 15\,cm$

b $f = D/2 = \frac{d}{4} = \frac{1}{4} = 0.25\,cm$

$d_o = -18\,cm$

$$d_i = \frac{d_o \times f}{d_o + f} = \frac{0.25 \times -18}{-18 + 0.25}$$

$$\boxed{d_i = 0.25\,cm}$$

Question 3:

(a)

Digitised pixel value signal given by :—

$$D = (S + N_A + N_P)A + N_Q$$

Expectation value.

$$E(D) = A \cdot E(S) + E(N_A) \cdot A + A \cdot E(N_P) + E(N_Q)$$

Now, N_P, N_Q and N_A are gaussian noises

Therefore $E(N_P) = E(N_Q) = E(N_A) = 0$

{Property}

Notes

for $E(S) = \dfrac{\int_{-\infty}^{\infty} x S \, dx}{\int_{-\infty}^{\infty} x \, dx}$

$= S \dfrac{\int_{-\infty}^{\infty} x \, dx}{\int_{-\infty}^{\infty} x \, dx} = S$

It is constant therefore comes out of integration.

$\therefore \; E(D) = A \cdot E(S) = AS$

$\mu = SA$

expected value of D.

for Variance

$V(D) = \sigma_D^2 = E\left[(D - E(D))^2\right]$

$= E\left[D^2 - 2DE(D) + E(D)^2\right]$

$= E[D^2] - E[D]^2$

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$$V(D) = A^2 V(S) + A^2 V(N_A) + A^2 V(N_P) + V(N_Q).$$

Now $V(S) = 0$

$\{$ $S \rightarrow$ constant

$\therefore$ do not vary

Hence $V(S) = 0$ $\}$

Given

Variance $(N_A) = \sigma_A^2$

Variance $(N_P) = S$

Variance $(N_Q) = \sigma_Q^2$

Substituting, we get.

$$V(D) = \sigma_D^2 = 0 + A^2 \sigma_A^2 + A^2 \cdot S + \sigma_Q^2$$

$$= A \cdot (\overbrace{SA}^{\mu}) + \underbrace{A^2 \sigma_A^2 + \sigma_Q^2}$$

$$\sigma_D^2 = A \cdot \mu + \sigma_C^2$$