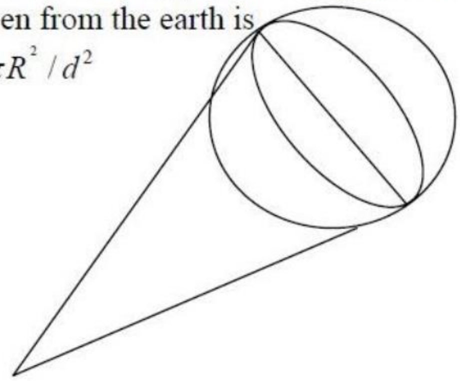
# EECS101: HOMEWORK #6 SOLUTION

1. The moon is a sphere with radius R at a distance d. The disc that is seen from the earth has an area of $\pi R^2$ and has a normal along the viewing axis. Thus we have $\theta = 0$. So the solid angle of the moon as seen from the earth is

$$\Omega = \pi R^2 / d^2$$



For a circular plate, the angle $\theta$ ranges between 0 and 90 degrees. Therefore the range of possible solid angles is 0 to $\frac{\pi R^2}{d^2}$.

2.

Lambertian plane. $\sqrt{15}x + 3y + z + 5 = 0$.

a). $z = -\sqrt{15}x - 3y - 5$

$p = \frac{\partial z}{\partial x} = -\sqrt{15} \qquad q = \frac{\partial z}{\partial y} = -3$

$\vec{n} = (-p, -q, 1) = (\sqrt{15}, 3, 1)$.

unit surface normal $\hat{n} = \frac{1}{5}(\sqrt{15}, 3, 1)$.

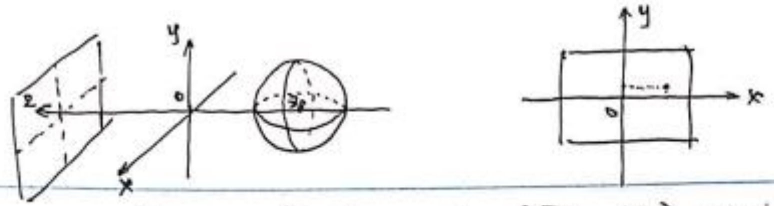
b) Since the plane is Lambertian, the reflected radiance will be largest if the incoming rays are along the same direction as the surface normal.

The location for the source is $(0,0,-5) + k(\sqrt{15}, 3, 1)$ where $k$ satisfies $\sqrt{k^2(15+9+1)} = 15 \Rightarrow k = \pm 3$

Since $(\sqrt{15}, 3, 1)$ points in the direction of $(0,0,0)$, use $k=3$ so that the source and $(0,0,0)$ are on the same side of the plane.

Location for the source is $(0,0,-5) + 3(\sqrt{15}, 3, 1)$
$= (3\sqrt{15}, 9, -2)$.

3.




a). coordinate of the point $(52, 18) = (x_0, y_0)$.

$$z = +z_0 + \sqrt{r^2 - x^2 - y^2}.$$

$$p = \frac{\partial z}{\partial x} = \frac{-x}{\sqrt{r^2 - x^2 - y^2}} \qquad q = \frac{\partial z}{\partial y} = \frac{-y}{\sqrt{r^2 - x^2 - y^2}}$$

the unit surface normal $(n_x, n_y, n_z)$.

the normal vector at this point

$$n_0 = (-p_0, -q_0, 1)$$

$$= \left( \frac{x_0}{\sqrt{r^2 - x_0^2 - y_0^2}}, \frac{y_0}{\sqrt{r^2 - x_0^2 - y_0^2}}, 1 \right) = \left( \frac{52}{\sqrt{6972}}, \frac{18}{\sqrt{6972}}, 1 \right).$$

$$n_x = \frac{52}{100}, \quad n_y = \frac{18}{100}, \quad n_z = \frac{\sqrt{6972}}{100}.$$

b). $$R(p,q) = \frac{pp_s + qq_s + 1}{\sqrt{p^2+q^2+1}\sqrt{p_s^2+q_s^2+1}} = \frac{(-p_s, -q_s, 1)}{\sqrt{p_s^2+q_s^2+1}} \cdot \frac{(-p, -q, 1)}{\sqrt{p^2+q^2+1}}$$

since the direction of incident light is parallel to $(n_x, n_y, n_z)$, $p_s = -52/\sqrt{6972}$, $q_s = -18/\sqrt{6972}$

$$R(p,q) = \frac{1}{\sqrt{p^2+q^2+1}} \left( \frac{\sqrt{6972}}{100} \left( -\frac{52}{\sqrt{6972}} p - \frac{18}{\sqrt{6972}} q + 1 \right) \right)$$

$$= \frac{1}{\sqrt{p^2+q^2+1}} \left( -\frac{52}{100} p - \frac{18}{100} q + \frac{\sqrt{6972}}{100} \right).$$

c). $\begin{cases} r = 128 - y \\ c = 128 + x \end{cases}$ $p = \frac{-x}{\sqrt{r^2-x^2-y^2}}$ $q = \frac{-y}{\sqrt{r^2-x^2-y^2}}$

$R(p,q) > 0$

$$\Rightarrow \frac{52}{100}\frac{x}{\sqrt{r^2-x^2-y^2}} + \frac{18}{100}\frac{y}{\sqrt{r^2-x^2-y^2}} + \frac{\sqrt{6972}}{100} > 0.$$

$$6972(100^2-x^2-y^2) > (52x+18y)^2$$

$$6972\left(100^2-(c-128)^2-(128-r)^2\right) > \left(52(c-128)+18(128-r)\right)^2 \quad (*)$$

d) $I(x,y) = R(p,q) = R(x,y)$.

$\begin{cases} y = 128 - r \\ x = c - 128 \end{cases}$ $= \frac{52x + 18y + \sqrt{6972}\sqrt{100^2-x^2-y^2}}{100^2}$

$$I(r,c) = \frac{52(c-128)+18(128-r)+\sqrt{6972}\sqrt{100^2-(c-128)^2-(128-r)^2}}{100^2}.$$

satisfying (*).

e) row = 110, ~~colom~~ column = 180.