EECS 101: HOMEWORK #8

Due: March 17, 2023

1. Consider an image irradiance sequence $E(x, y, t)$ in continuous space (x, y) and time (t)

$$E(x, y, t) = cos(y - t) + 1 \qquad 0 \leq x \leq 2\pi, \quad 0 \leq y \leq 2\pi, \quad t \geq 0$$

a) Determine the components of the optical flow vector $u(x, y, t)$ and $v(x, y, t)$ for this sequence. Explain your reasoning.

b) Write down the optical flow constraint equation.

c) Compute the time and space derivatives of image irradiance that appear in the optical flow constraint equation for the function given above.

d) Use your answers to the previous parts to show that the optical flow constraint equation holds for all x, y, t.

e) For the optical flow computed in part a), compute the smoothness integral

$$e_s(t) = \int\int (u_x^2 + u_y^2 + v_x^2 + v_y^2) dxdy$$

2. Assume the stereo geometry given in class with origin at (0,0,0), a baseline of b=2cm separating the two lens centers, and a distance between each lens and corresponding image plane of f=0.5cm. Suppose the left and right image planes are each a 1cm x 1cm square with the origin of each local coordinate system at the center of the square. Assume that we are viewing a black plane with three bright white spots. The three spots are observed in the left image at coordinates $(x_l', y_l') = \{(0.0, 0.2), (-0.3, 0.2), (0.0, 0.1)\}$ and in the right image at coordinates $(x_r', y_r') = \{(-0.1, 0.2), (0.1, 0.2), (0.2, 0.1)\}$

a) Determine the image correspondences for the points.

b) Determine the coordinates (x, y, z) of the three bright points in the scene.

c) Determine the gradient space representation (p, q) for the plane in the scene.

3. Consider a pattern classification problem where we would like to discriminate between two materials M_1 and M_2 using a measured color vector (R, G, B) of the unknown material. Assume that the *a priori* probability of M_1 is 2/5 and that the *a priori* probability of M_2 is 3/5. Suppose that the probability density for M_1 is uniform on the cube $50 \leq R \leq 100, 30 \leq G \leq 60, 40 \leq B \leq 80$. Suppose that the probability density for M_2 is uniform on the cube $80 \leq R \leq 140, 30 \leq G \leq 80, 60 \leq B \leq 100$.

a) What is the conditional pdf $p(R, G, B|M_1)$ as a function of (R, G, B)?

b) What is the conditional pdf $p(R, G, B|M_2)$ as a function of (R, G, B)?

c) What is the *a posteriori* probability $P(M_1|R, G, B)$ as a function of (R, G, B)?

d) What is the *a posteriori* probability $P(M_2|R, G, B)$ as a function of (R, G, B)?

e) What is the best guess for what material we are looking at as a function of (R, G, B)?

f) What is the probability of error as a function of (R, G, B) if we take the guess in part e)?