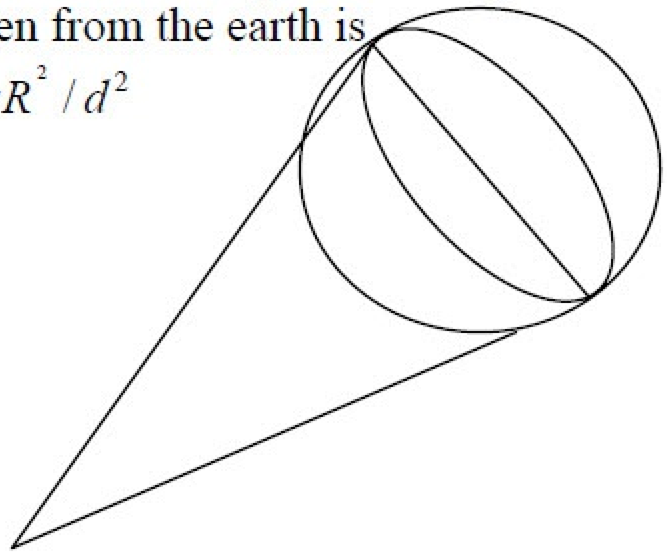
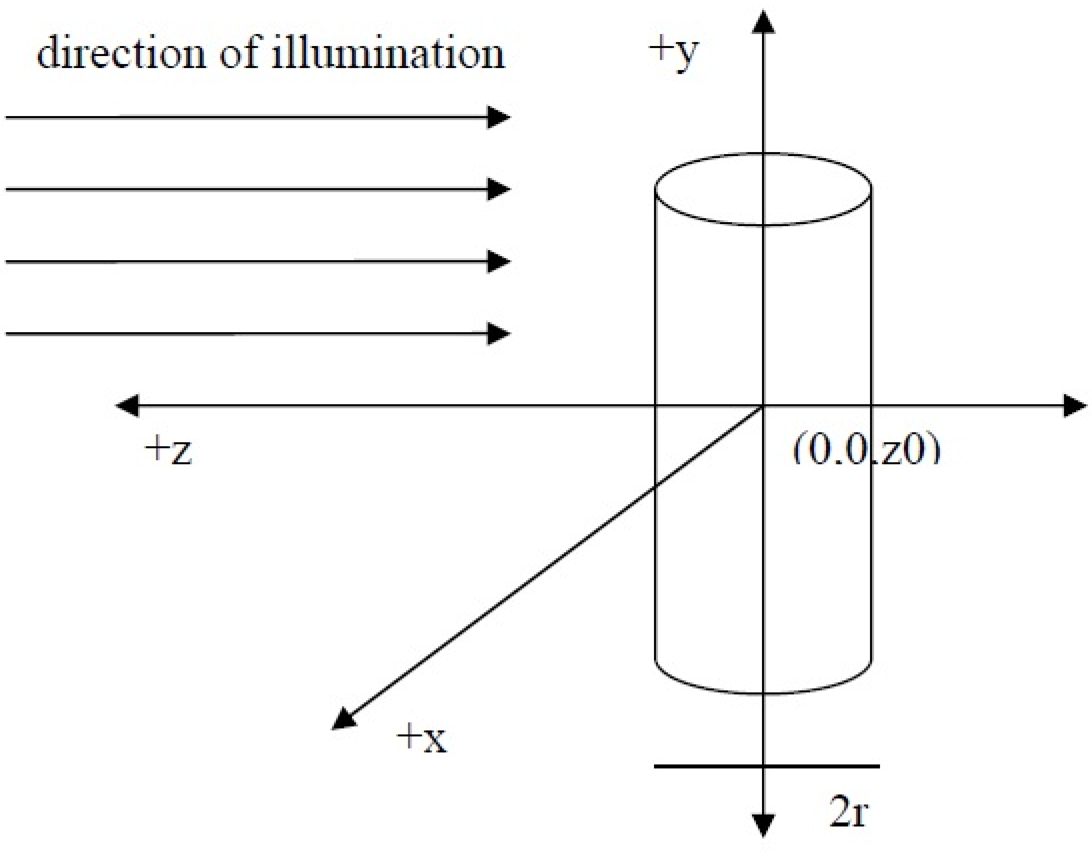
1. The moon is a sphere with radius R at a distance d. The disc that is seen from the earth has an area of πR^2 and has a normal along the viewing axis. Thus we have $\theta = 0$. So the solid angle of the moon as seen from the earth is

$$\Omega = \pi R^2 / d^2$$



For a circular plate, the angle θ ranges between 0 and 90 degrees. Therefore the range of possible solid angles is 0 to $\frac{\pi R^2}{d^2}$.



2. The equation for a right circular cylinder with center at $(0,0,z_0)$ is given by

$$(z - z_0)^2 + x^2 = r^2$$
$$\Rightarrow z = z_0 + \sqrt{r^2 - x^2}$$

The surface normal is given by $(-p,-q,1)$ where

$$p = \frac{\partial z}{\partial x} = \frac{-x}{\sqrt{r^2 - x^2}} = \frac{-x}{z - z_0}$$

$$q = \frac{\partial z}{\partial y} = 0$$

Since we assume illumination from a collimated light source far off from the object, the direction of incident light is the same at all points on the cylinder as shown in the figure. Also, as the light source is at the same position as the viewer (along the +ve z axis), we have $(p_s, q_s) = (0,0)$. For a lambertian surface illuminated by a collimated source, the reflectance map is given by

$$R(p,q) = \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1}\sqrt{p_s^{\ 2} + q_s^{\ 2} + 1}}$$
$$= \frac{1}{\sqrt{p^2 + q^2 + 1}}$$

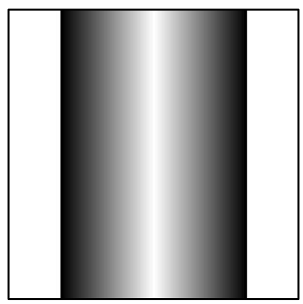
Putting in the values of p and q we get

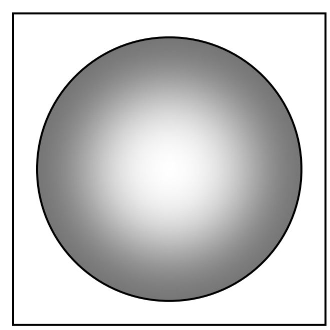
$$R(p,q) = \frac{z - z_0}{\sqrt{x^2 + (z - z_0)^2}} = \frac{z - z_0}{r}$$

Since E(x,y) = R(p,q),

$$E(x,y) = \frac{z - z_0}{r} = \sqrt{1 - \left(\frac{x}{r}\right)^2}$$

Note that R(p,q) is a function of x,y and z whereas E(x,y) is a function of only x and y. The corresponding image of the cylinder would look like the following.





3. The image would look like the one shown below. We have $(p_s, q_s) = (0,0)$. Thus,

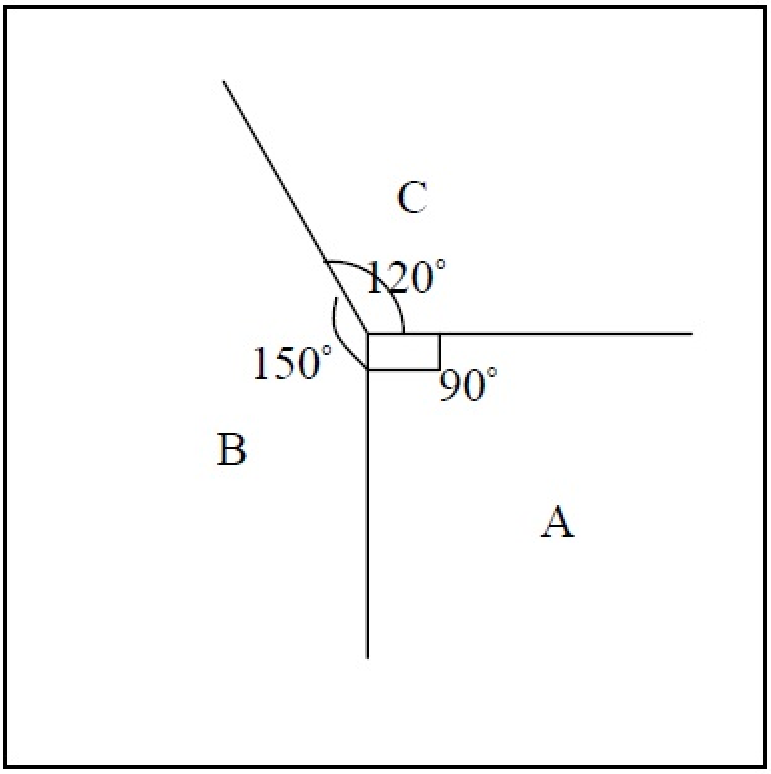
$$R(p,q) = \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1}\sqrt{p_s^{\ 2} + q_s^{\ 2} + 1}}$$
$$= \frac{1}{\sqrt{p^2 + q^2 + 1}}$$
$$= E(x,y)$$

We have only one equation and two variables p and q. The only possible value of E(x,y) for which both p and q can be uniquely determined is E(x,y) = 1. For this case, p=q=0. This corresponds to the center bright spot in the image of the sphere. For E(x,y) = 0.5, we have,

$$\frac{1}{\sqrt{p^2 + q^2 + 1}} = 0.5$$
$$\Rightarrow p^2 + q^2 = 3$$

Thus, the possible values of p and q form a locus of points lying on a circle of radius $\sqrt{3}$ centered at origin in the p-q space.

5. For a lambertian polyhedron illuminated by a collimated source, we know that the gradients for intersecting planes lie on a line in the gradient space that is perpendicular to the line of intersection of the planes in the image plane. For the three planes A, B and C, let the corresponding gradients be $(p_A, q_A), (p_B, q_B)$ and (p_C, q_C). We have the following:



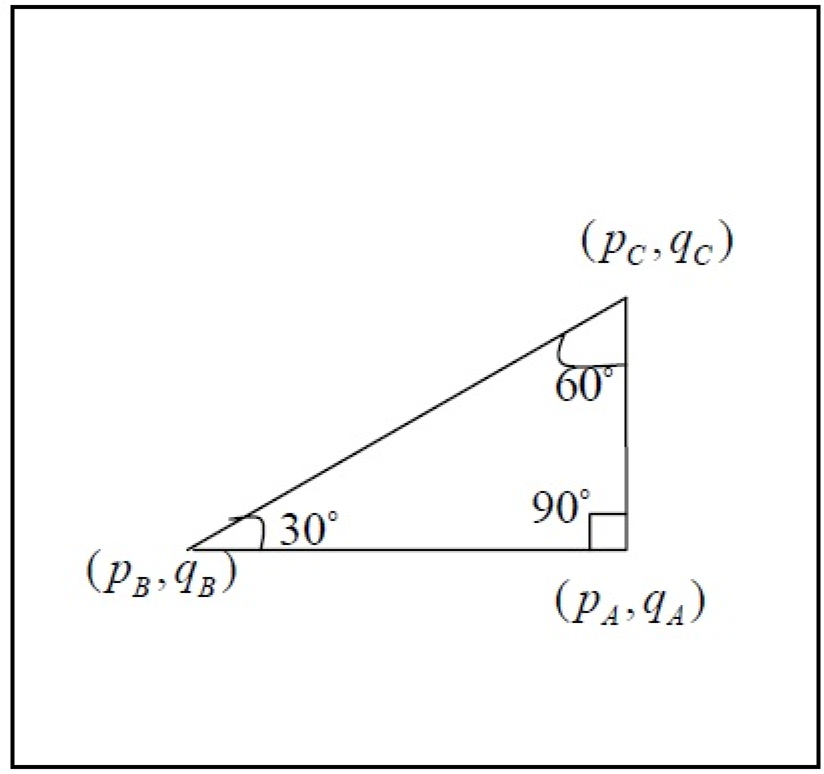


Image plane

Gradient (p,q) space

Since the light source is at the same position as the viewer, $(p_s, q_s) = (0,0)$. This gives us

$$E_\alpha(x, y) = \frac{1}{\sqrt{p_\alpha^2 + q_\alpha^2 + 1}} \qquad (1)$$

where α can be A, B or C. It is given that $E_A(x, y) = 0.918$ and $E_B(x, y) = 1$. Using equation (1) for plane B, we obtain (p_B, q_B) = (0,0). This gives us $q_A = 0$ (see the gradient space image). Using equation (1) again for plane A, we get

$$\frac{1}{\sqrt{p_A^2 + 1}} = 0.918 \Rightarrow p_A = 0.432 \qquad (2)$$

From the gradient space image, $p_A = p_C$ and $q_C = p_A \tan(30^\circ)$=0.249 . Putting these values in (1), we have $E_C = 0.895$. Given this image, we can have two possible values of p_A from (2), one being the negative of the other. We can discard the negative value as the surface normal should be in the same direction as the light for the viewer to be able to see the surface. Thus, the possible set of orientations for the planes given this image is unique.