

# EECS101: HOMEWORK #2 SOLUTION

2021 Winter

1

According to the problem, we have  $\text{Var}(N_A) = 2, E(N_A) = 0$   
 $\text{Var}(N_p) = S, E(N_p) = 0$

Total noise is

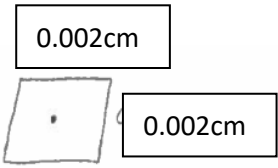
$$\text{Var}(C) = \text{Var}((S + N_A + N_p)A) = A^2 \text{Var}(S + N_A + N_p) = A^2(2 + S)$$

$$\text{SNR}(C) = \frac{E(C)}{\sqrt{\text{Var}(C)}} = \frac{E((S + N_A + N_p)A)}{\sqrt{A^2(1 + S)}} = \frac{AS}{\sqrt{A^2(2 + S)}} = \frac{S}{\sqrt{2 + S}} \geq 50 \Rightarrow S \geq 2502$$

2.

a)  $1/4 = 1/6 + 1/x, \quad x = 12\text{cm}$

b)

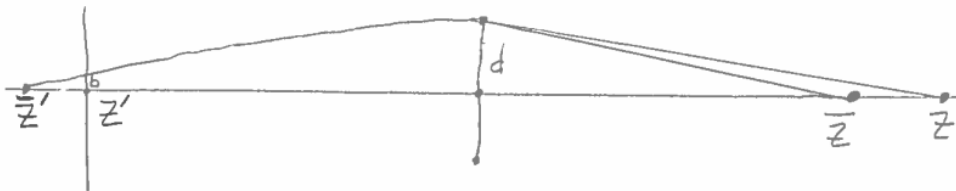
Each potential well is 

$b = 0.002\text{cm}$   $d = 1\text{cm}$   $z' = 6\text{cm}$

$$\frac{b}{d} = \frac{\bar{z}' - z'}{\bar{z}'} \quad \frac{0.002\text{cm}}{1\text{cm}} = \frac{\bar{z}' - 6}{\bar{z}'} \Rightarrow \bar{z}' = 6.012\text{cm}$$

$$\frac{1}{f} = \frac{1}{\bar{z}} + \frac{1}{\bar{z}'} \quad \frac{1}{4} = \frac{1}{\bar{z}} + \frac{1}{6.012} \Rightarrow \bar{z} = 11.952$$

We can move the point in focus  $0.048\text{cm}$  toward the lens before its image extends to more than one potential well.



3.

a)  $\mu = E((S + N_A + N_p)A + N_Q) = AS + AE(N_A) + AE(N_p) + E(N_Q) = AS$

$$V(D) = V((S + N_A + N_p)A + N_Q) = A^2V(N_A) + A^2V(N_p) + V(N_Q)$$

$$A^2\sigma_A^2 + A^2S + \sigma_Q^2 = Au + A^2\sigma_A^2 + \sigma_Q^2$$

b) Image1:  $\hat{u} = 49.423, \hat{\sigma}^2 = 15.144$

Image2:  $\hat{\mu} = 79.479, \hat{\sigma}^2 = 21.493$

Image3:  $\hat{\mu} = 110.721, \hat{\sigma}^2 = 26.886$

Image4:  $\hat{\mu} = 160.079, \hat{\sigma}^2 = 35.986$

Least square fit is shown in Figure 1 where dots are the data and line is the fit.

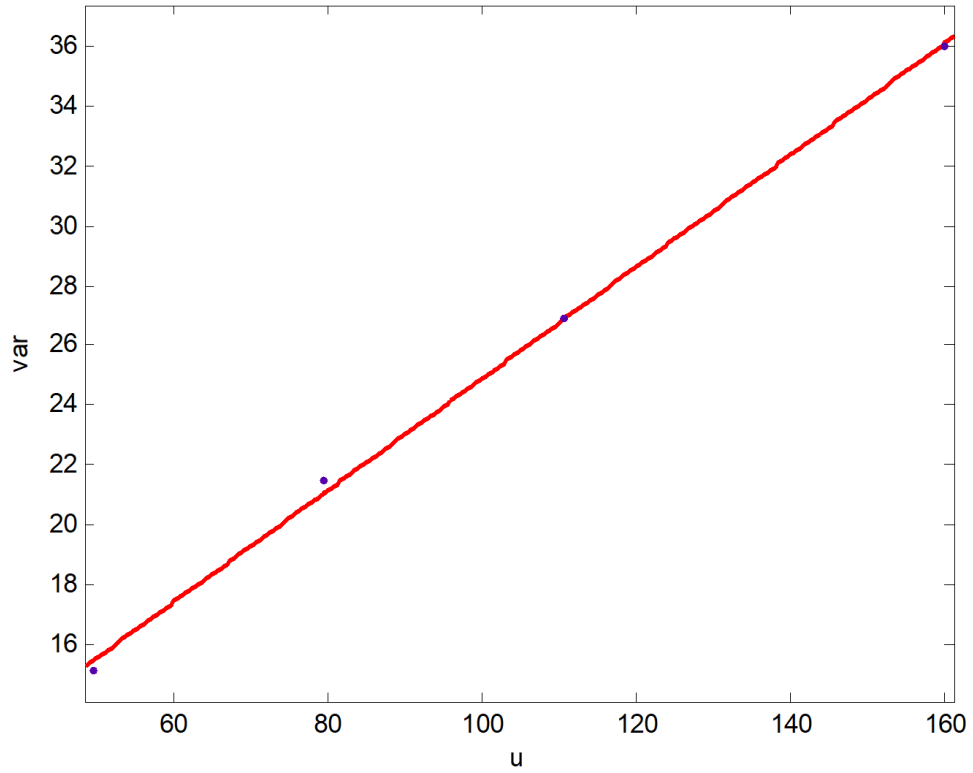


Figure 1

Estimation of  $A$ ,  $\hat{\sigma}_C^2$  are

$$\bar{A} = 0.187$$

$$\bar{\sigma}_C^2 = 6.234$$