EECS101: HOMEWORK #1

Due: January 20, 2023

For this assignment, your TA will supply a C program to help generate image files that you will display. For each case you only need to change the indicated two assignment statements for the image coordinates *xprime* and *yprime*. You will also need to add declarations for any other variables that you may need, for example, to define constants. The supplied function plot_logical_point maps the (x', y') image plane over the range $-4 \leq x' \leq 4$ and $-4 \leq y' \leq 4$ to image pixel coordinates.

Consider the imaging system of Figure 2-1 in the Horn textbook (page 20) having a pinhole at the origin with the z-axis horizontal (positive z to the left), the y-axis vertical (positive y up) and the x-axis forward (positive x out). The image plane is located at $z = f' \ \ (f' > 0)$. All events in 3-D are assumed to take place in the space $z < 0$. The coordinates (x', y') define points in the image plane and the coordinates (x, y, z) define points in the world. Curves in space can be defined parametrically using the parameter t.

1. For a line in 3-D the parametric equations are:

$$\begin{aligned} x &= x_0 + ta \\ y &= y_0 + tb \\ z &= z_0 + tc \end{aligned}$$

for constants x_0, y_0, z_0, a, b, and c. As t takes on values in the range $(-\infty, \infty)$ we get a line.

a) Write expressions for the image coordinates x' and y' in terms of these constants and f' and t for both the perspective and orthographic projection of the line.

b) Is the perspective projection of the line also a line in the image? Explain.

c) Is the orthographic projection of the line also a line in the image? Explain.

d) Using the constant settings:

$$\begin{aligned} x_0 &= .5 \\ y_0 &= -1 \\ z_0 &= 0 \\ a &= 0 \\ b &= 1 \\ c &= -1 \\ f' &= 1 \end{aligned}$$

use a C program to generate images of both the perspective and orthographic projections of the line. Since $\pm\infty$ is difficult to represent in the computer we can use values of t for a smaller range like between 0.01 and 10000. The sample C program will only display regions in the range $-4 \leq x' \leq 4, -4 \leq y' \leq 4$ so you may want to further limit the range of t so that the values of x' and y' will fall within this range.

e) What happens in the perspective projection case as t goes to ∞? Is this consistent with the image that your program generates?

2. For two parallel lines in a plane that is parallel to the image plane the value for z is a constant z_0. The parametric equations for these lines are

$$\begin{aligned} x &= x_1 + ta \\ \hat{x} &= x_2 + ta \\ y &= y_1 + tb \\ \hat{y} &= y_2 + tb \\ z &= z_0 \\ \hat{z} &= z_0 \end{aligned}$$

where Line 1 is defined by (x, y, z) and Line 2 is defined by $(\hat{x}, \hat{y}, \hat{z})$ and t is in the range $(-\infty, \infty)$.

a) Write expressions for the image coordinates x' and y' in terms of the constants above and f' and t for both the perspective and orthographic projection of the lines.

b) Using the following constant settings:

$$\begin{aligned} x_1 &= 0.5 \\ x_2 &= -0.5 \\ y_1 &= -1 \\ y_2 &= -1 \\ z_0 &= -1, -2, -3 \\ a &= 1 \\ b &= 1 \\ f' &= 1 \end{aligned}$$

use a C program to generate images of both the perspective and orthographic projections of the lines by letting t range from 0.01 to 10000.

c) Will the projections of the lines in the image be parallel for i) perspective projection and ii) orthographic projection? Explain.

d) Is your answer for part c) consistent with the images that your program generates?

e) Is orthographic projection a good approximation to perspective projection for the geometry for this problem? Explain.

f) What occurs if $z_0 = |f'|$?

3. Consider two parallel lines that lie in a plane that is not parallel to the image plane. Assume that these lines are defined by

$$\begin{aligned} x &= x_1 \\ \hat{x} &= x_2 \\ y &= y_0 + tb \\ \hat{y} &= y_0 + tb \\ z &= z_0 + tc \\ \hat{z} &= z_0 + tc \end{aligned}$$

a) Write expressions for the image coordinates x' and y' in terms of the constants above and f' and t for both the perspective and orthographic projection of the lines.

b) Using the following constant settings,

$$\begin{aligned} x_1 &= -1 \\ x_2 &= 1 \\ y_0 &= -1 \\ z_0 &= 0 \\ b &= 0, 1, -1 \\ c &= 1, -1 \\ f' &= 1 \end{aligned}$$

use a C program to generate images of both the perspective and orthographic projections of the lines by letting t range from 0.01 to 10000.

c) Will the projections of the lines be parallel for i) perspective projection and 2) orthographic projection? Explain.

d) Is your answer for part c) consistent with the images that your program generates?

e) Is orthographic projection a good approximation to perspective projection for the geometry for this problem? Explain.

f) What happens in the perspective projection as t goes to ∞? Is this consistent with the images that your program generates for this case?

What to turn in: Submit the displayable images that you generate and your code. You are also required to demonstrate your code to your TA during lab.