EECS101: HOMEWORK #2 SOLUTION

1. According to the problem, we have $\begin{aligned} &Var(N_A) = 3^2 = 9, E(N_A) = 0 \\ &Var(N_p) = S, E(N_p) = 0 \end{aligned}$

The total noise is

$$N_{Total} = N_A + N_P$$

The variance of the total noise is

$$Var(N_{Total}) = Var(N_A + N_p) = (9 + S)$$

$$\frac{\sqrt{Var(N_A)}}{\sqrt{Var(N_{Total})}} = \frac{\sqrt{Var(N_A)}}{\sqrt{Var(N_A + N_P)}} = \frac{3}{\sqrt{9+S}} \le 0.1 \Rightarrow S \ge 891$$

$$SNR = \frac{S}{\sqrt{Var(N_{Total})}} = \frac{S}{\sqrt{9+S}} = \frac{891}{\sqrt{9+891}} = 29.7$$

2.

a) $\frac{1}{10} + \frac{1}{-z} = \frac{1}{8} \Rightarrow z = -40cm$

b) Image plane $3cm \times 3cm \Rightarrow$ Each potential well is

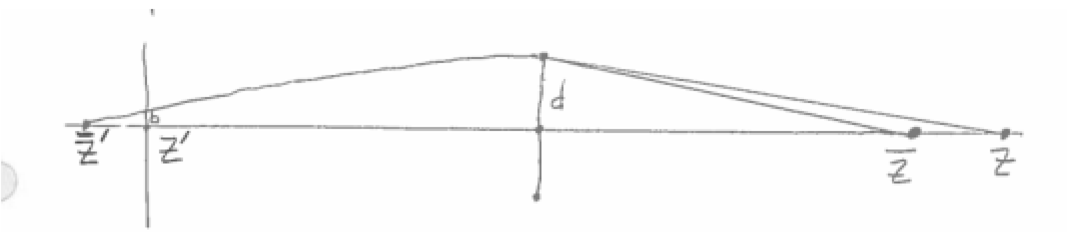
b=0.006cm, d=1cm, z' = 10cm

$\frac{b}{d} = \frac{\overline{z}' - z'}{\overline{z}'} \Rightarrow \frac{0.006}{1} = \frac{\overline{z}' - 10}{\overline{z}'} \Rightarrow \overline{z}' = 10.06036cm$ (or 10.06cm)

$\frac{1}{\overline{z}'} + \frac{1}{-\overline{z}} = \frac{1}{f} \Rightarrow \frac{1}{10.06036cm} + \frac{1}{-\overline{z}} = \frac{1}{8cm} \Rightarrow \overline{z} = -39.06253cm$ (or -39.07cm for 10.06cm)

$\overline{z} - z = 0.937cm$

We can move the point in focus 0.937cm (or 0.93cm) toward the lens before its image extends to more than one potential well



* The question states ‘move toward the lens’. One solution is sufficient to this question. Two reasonable answers are acceptable.

3.

a) $\mu = E((S + N_A + N_P)A + N_Q) = AS + AE(N_A) + AE(N_P) + E(N_Q) = AS$

$$\begin{aligned} V(D) &= V((S + N_A + N_P)A + N_Q) = A^2V(N_A) + A^2V(N_P) + V(N_Q) \\ &= A^2\sigma_A^2 + A^2S + \sigma_Q^2 = Au + A^2\sigma_A^2 + \sigma_Q^2 \end{aligned}$$

b) Image1: $\hat{u} = 49.423, \hat{\sigma}^2 = 15.144$

Image2: $\hat{\mu} = 79.479, \hat{\sigma}^2 = 21.493$

Image3: $\hat{\mu} = 110.721, \hat{\sigma}^2 = 26.886$

Image4: $\hat{\mu} = 160.079, \hat{\sigma}^2 = 35.986$

Least square fit is shown in Figure 1 where dots are the data and line is the fit.

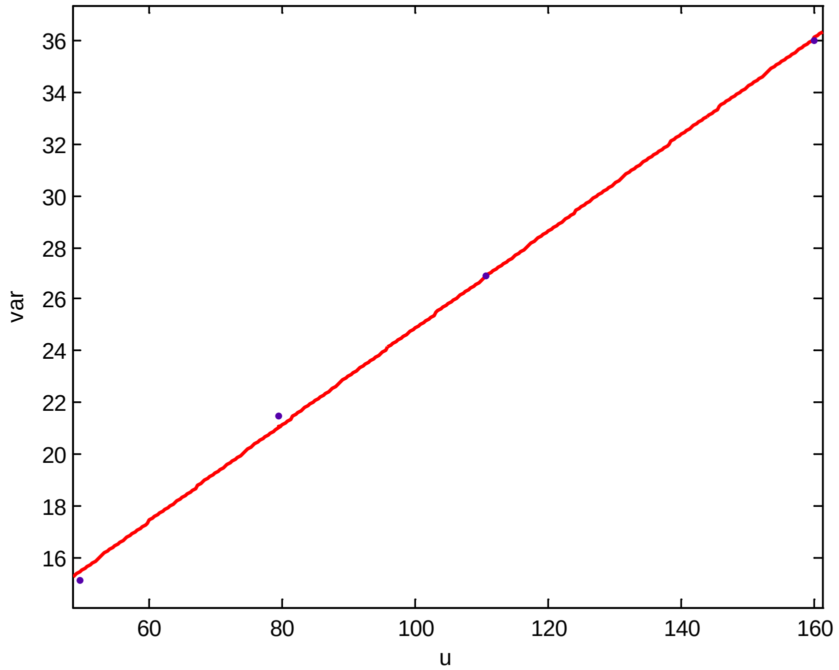


Figure 1

Estimation of A, $\hat{\sigma}_C^2$ are

$\overline{A} = 0.187$

$\overline{\sigma}_C^2 = 6.234$