1.

a) $Var(x) = mean(x^2) - mean^2(x)$

$Var(c) = A^2 Var(S+N_A+N_P) = A^2(2+S)$

$Var(S+N_A+N_P)$

$\boxed{E(S+N_A+N_P) = S}$

$= E[(S+N_A+N_P)^2] - E^2(S+N_A+N_P)$

$= E(S^2+N_A^2+N_P^2+2SN_A+2SN_P+2N_AN_P) - S^2$

$= E(N_A^2) + E(N_P^2) + 2SE(N_A) + 2SE(N_P) + 2E(N_AN_P)$

$= 2+S+2E(N_A)\cdot E(N_P) = 2+S$

b)

signal-to-noise for the measurement c : $\dfrac{E(C)}{\sqrt{Var(C)}} = \dfrac{S}{\sqrt{2+S}}$

c)

$\dfrac{S}{\sqrt{2+S}} > 50$

$\dfrac{S^2}{2+S} > 2500$

$S = 2502$

The minimal value of S will be 2502 that exceeds 50.

2.

a) $\frac{1}{z'} + \frac{1}{-z} = \frac{1}{f}$ $\quad z'$: image distance.

$$\frac{1}{6} + \frac{1}{-z} = \frac{1}{4}$$

$$-z = 12\text{cm}$$

$$z = -12\text{cm}$$

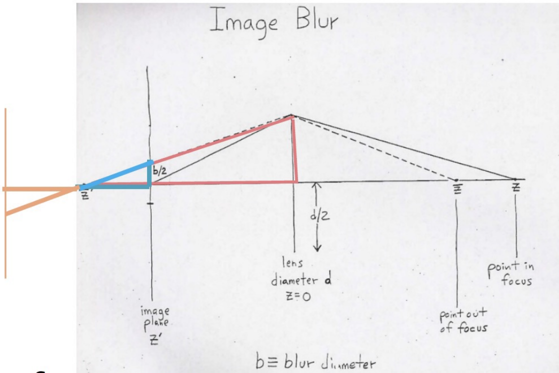
we need to place a point 12cm away from the lens.

b) Image plane 2cm × 2cm ⟹ Each pixel is

$b = \frac{2\text{cm}}{1000} = 2\times10^{-3}\text{cm}$ $\quad d = 1\text{cm}$ $\quad z' = 6\text{cm}$

$$\frac{b}{d} = \frac{\bar{z}' - z'}{\bar{z}'} = \frac{2\times10^{-3}\text{cm}}{1\text{cm}} = \frac{\bar{z}' - 6\text{cm}}{\bar{z}'}$$

$$\bar{z}' = 6.012024\text{cm}.$$



$$\frac{1}{\bar{z}'} + \frac{1}{-\bar{z}} = \frac{1}{f} \Rightarrow \frac{1}{6.012024\,cm} + \frac{1}{-\bar{z}} = \frac{1}{4}$$

$$-\bar{z} = 11.9522\,cm.$$

$$\bar{z} - z = -11.9522\,cm - (-12\,cm) = 0.0478\,cm.$$

3.

a) $\mu = E((S+N_A+N_P)A+N_Q)$

$= E(S+N_A+N_P)\cdot A + E(N_Q)$

$= AE(S) + AE(N_A) + AE(N_P) + E(N_Q) = AS.$

$\because E(N_A), E(N_P), E(N_Q)$ are zero-mean

$V(D) = A^2 V(S+N_A+N_P) + V(N_Q)$

$= A^2 V(S) + A^2 V(N_A) + A^2 (N_P) + V(N_Q)$

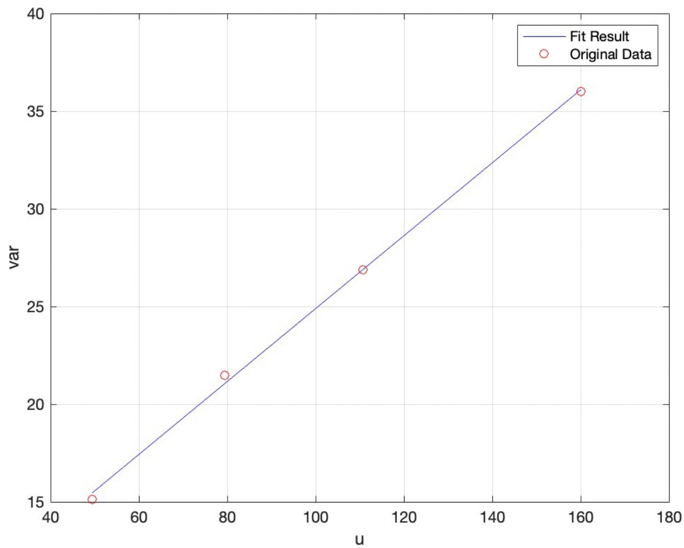
$= A^2\sigma_A^2 + A^2 S + \sigma_Q^2 = A\mu + A^2\sigma_A^2 + \sigma_Q^2 = \sigma_C^2$

$\therefore \sigma_D^2 = A\mu + \sigma_C^2.$

b)

image1.raw: {u=49.422600, var=15.144367}
image2.raw: {u=79.478500, var=21.492956}
image3.raw: {u=110.721100, var=26.886707}
image4.raw: {u=160.079193, var=35.986778}

c)



$A = 0.18657$ $\sigma_0^2 = 6.23444$