1. $E(x,y,t) = \cos(y-t) + 1 \qquad 0 \leq x \leq 2\pi, \quad 0 \leq y \leq 2\pi, \quad t \geq 0$

a) This is a traveling cosine wave along the y-axis with unit velocity. $u(x,y,t)=0$, $v(x,y,t)=1$

b) The optical flow constraint equation

$$E_x u + E_y v + E_t = 0 \quad \left(E_x = \frac{\partial E}{\partial x}, E_y = \frac{\partial E}{\partial y}, E_t = \frac{\partial E}{\partial t}\right)$$

c) $E_x = \frac{\partial E}{\partial x} = 0$, $E_y = \frac{\partial E}{\partial y} = -\sin(y-t)$

$E_t = (-1) \cdot (-\sin(y-t)) = \sin(y-t)$

d) Because $u(x,y,t)=0$ and $v(x,y,t)=1$

$$0 \cdot u + (-\sin(y-t)) \cdot 1 + \sin(y-t) = 0$$

Combining what we get, we can prove the correctness of the equation. Hence the values obtained satisfy the constraint equation.

e) $u_x = \frac{\partial u}{\partial x} = u_y = v_x = v_y = 0 \qquad \iint ((u_x^2 + u_y^2) + (v_x^2 + v_y^2))\,dx\,dy = 0$

Because the optical flow doesn't change, the motion is perfectly smooth.

2.

a)

image 1

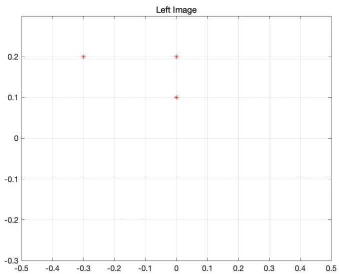
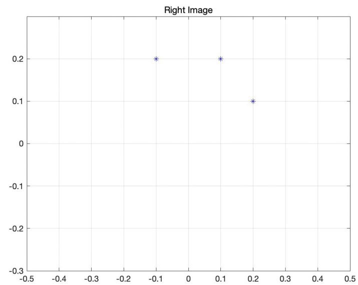


image 2



b)

$$\frac{x_l'}{f} = \frac{x+\frac{b}{2}}{z} \qquad \frac{x_r'}{f} = \frac{x-\frac{b}{2}}{z} \qquad \frac{y_l'}{f} = \frac{y_r'}{f} = \frac{y}{z}$$

To find (x, y, z) given (x_l', y_l') and (x_r', y_r')

$$x = \frac{b(x_l'+x_r')}{2(x_l'-x_r')} \qquad y = \frac{b(y_l'+y_r')}{2(x_l'-x_r')} \qquad z = \frac{bf}{x_l'-x_r'}$$

left Image $(-0.3, 0.2)$ $(0, 0.2)$ $(0, 0.1)$

Right Image

$(-0.1, 0.2)$ $(0.1, 0.2)$ $(0.2, 0.1)$

$b=2$ $f=0.5$

For the three points are

$(2, -2, -5)$, $(-1, -4, -10)$, $(-1, -1, -5)$

c)

$\vec{P_1}(2, -2, -5)$ $\vec{P_2}(-1, -4, -10)$ $\vec{P_3}(-1, -1, -5)$

$$\vec{n} = (\vec{P_2} - \vec{P_1}) \times (\vec{P_3} - \vec{P_1})$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$(-3, -2, -5) \qquad (-3, 1, 0)$$

$$\begin{vmatrix} i & j & k \\ -3 & -2 & -5 \\ -3 & 1 & 0 \end{vmatrix}$$

$$\to 0i + 15j - 3k - 6k + 0j + 5i$$

$$\Rightarrow 5i + 15j - 9k$$

$$\vec{n} = (-p, -q, 1)$$

$$\vec{n} = \left(-\frac{5}{9}, \frac{-15}{9}, 1\right)$$

Ans: $p = \frac{5}{9}$, $q = \frac{15}{9}$

3. $P(M_1)=\frac{2}{5}$ $P(M_2)=\frac{3}{5}$

Probabilites from Uniform pdf of each edge of cube for M_1

$P(R|M_1)=\frac{1}{50}$ $P(G|M_1)=\frac{1}{30}$ $P(B|M_1)=\frac{1}{40}$ in M_1 cube

Otherwise it's 0.

probabilites from Uniform pdf of each edge of cube for M_2.

$P(R|M2)=\frac{1}{60}$ $P(G|M_2)=\frac{1}{50}$ $P(B|M_2)=\frac{1}{40}$ in M_2 cube.

Otherwise it's 0.

Becanse. R, G, B are independent.

a)

$$P(R,G,B|M_1) = P(R|M_1)\,P(G|M_1)\,P(B|M_1)$$

$$= (\frac{1}{50})^*(\frac{1}{30})^*(\frac{1}{40}) = \frac{1}{60000} \text{ in } M_1 \text{ cube.}$$

otherwise it's 0.

b) $P(R,G,B|M_2) = P(R|M_2)\,P(G|M_2)\,P(B|M_2)$

$= \frac{1}{120000}$ in M_2 cube, otherwise it's 0.

c) $$P(M_1|R,G,B) = \frac{P(R,G,B|M_1)^* P(M_1)}{P(R,G,B|M_1)^* P(M_1) + P(R,G,B|M_2)^* P(M_2)}$$

$$= \frac{\frac{1}{60000} * \frac{2}{5}}{\frac{1}{60000} * \frac{2}{5} + \frac{1}{120000} * \frac{3}{5}} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{2} * \frac{3}{5}} = \frac{2}{2+1.5} = \frac{4}{7}$$

$$\therefore P(M_1 \mid R, G, B) = \begin{cases} \frac{4}{7} & \text{for overlapping region} \\ 1 & M_1 \text{ region only} \\ 0 & \text{otherwise.} \end{cases}$$

d) $$P(M_2 \mid R, G, B) = \frac{P(R,G,B \mid M_2) * P(M_2)}{P(R,G,B \mid M_1) * P(M_1) + P(R,G,B \mid M_2) * P(M_2)}$$

$$= \frac{\frac{1}{120000} * \frac{3}{5}}{\frac{1}{60000} * \frac{2}{5} + \frac{1}{120000} * \frac{3}{5}}$$

$$\therefore P(M_2 \mid R, G, B) = \begin{cases} \frac{3}{7} & \text{for overlapping region} \\ 1 & M_2 \text{ region only} \\ 0 & \text{otherwise.} \end{cases}$$

e) what is the best guess for what material we are looking at as a function of (R, G, B)?

For m_1 Region Only: m_1 ; m_2 Region only: m_2;

Overlapping Region: m_1.

f) what is the probability of error as a function of (R, G, B) if we take the guess in part e)?

The error for (m_1 Region Only) 0.

(m_2 Region Only) 0

(Overlapping Region) $\frac{3}{7}$.