1. When $t = t_0$, suppose there is E_0, which $E_0 = \cos(y_0 - bt_0) + C$.
After time dt, that is $t_1 = t_0 + dt$. And $E_1 = \cos(y_1 - bt_1) + C = \cos(y_1 - bt_0 - bdt) + C$.
We want to track the optic flow, that is, we want to track the pixel with the same brightness. Solve the equation $E_0 = E_1$ to find the relationship between y_0 and y_1.

$$\cos(y_0 - bt_0) + C = \cos(y_1 - bt_0 - bdt) + C$$

We get $y_0 = y_1 - bdt + 2k\pi$, for the smoothness, $y_0 = y_1 - bdt$.
That is to say, within time dt, the optic flow with the same brightness moves bdt distance in y direction.
The velocity is $v = bdt/dt = b$

a) $u(x, y, t) = 0,\ v(x, y, t) = 1$

b) $E_x u + E_y v + E_t = 0, where\ E_x = \frac{\partial E}{\partial x}, E_y = \frac{\partial E}{\partial y}, E_t = \frac{\partial E}{\partial t}$

c) $E_x = 0, E_y = -\sin(y - t), E_t = \sin(y - t)$

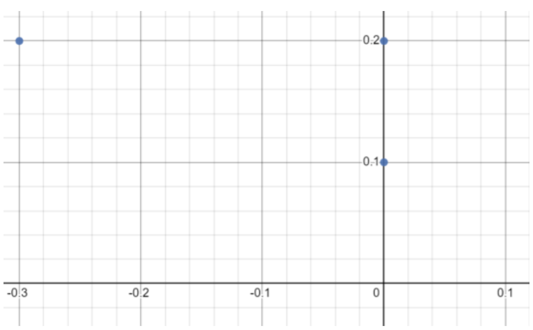
d) $E_x u + E_y v + E_t = 0 \cdot 0 - \sin(y - t) \cdot 1 + \sin(y - t) = 0$. The optical flow constraint equation holds for all x, y, t.

e) $u_x = \frac{\partial u}{\partial x} = 0$. Similarly, $u_y = v_x = v_y = 0$.

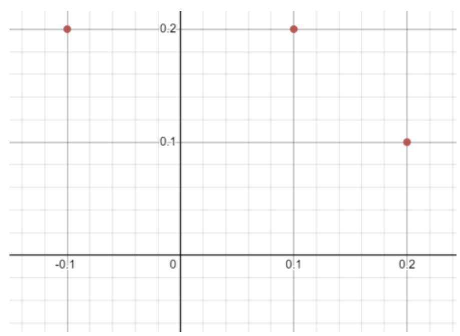
$$e_s(t) = \iint (0 + 0 + 0 + 0)dxdy = 0$$

2.

a) The location of the points are:



left correspondence



right correspondence

Considering the relative locations, the image correspondence for the points:

$$(0, 0.2), (-0.3, 0.2), (0, 0.1)$$
$$(0.1, 0.2), (-0.1, 0.2), (0.2, 0.1)$$

b)

$$x = \frac{b(x_l' + x_r')}{2(x_l' - x_r')}, y = \frac{b(y_l' + y_r')}{2(x_l' - x_r')}, z = \frac{bf}{x_l' - x_r'}$$

The three points are,

$$(-1, -4, -10), (2, -2, -5), (-1, -1, -5)$$

c) Get two vectors, suppose $a = (-1,-4,-10) - (2,-2,-5) = (-3,-2,-5)$; $\mathrm{b} = (-1,-1,-5) - (2,-2,-5) = (-3,1,0)$.

The norm vector is $n = a \times b = (5,15,-9)$. The norm is also $(-p,-q,1)$ and gives $p = 5/9, q = 15/9$

3.

a)

$$P(R,G,B|M_1) = \begin{cases} \dfrac{1}{(100-50)\times(60-30)\times(80-40)} = \dfrac{1}{60000} \\ , when\ 50 \le R \le 100, 30 \le G \le 60, 40 \le B \le 80 \\ 0, \qquad o.w. \end{cases}$$

b)

$$P(R,G,B|M_2) = \begin{cases} \dfrac{1}{(140-80)\times(80-30)\times(100-60)} = \dfrac{1}{120000} \\ , when\ 80 \le R \le 140, 30 \le G \le 80, 60 \le B \le 100 \\ 0, \qquad o.w. \end{cases}$$

c) By the Bayes rule,

$$P(M_1|R,G,B) = \frac{P(M_1)P(R,G,B|M_1)}{P(R,G,B)} = \frac{P(M_1)P(R,G,B|M_1)}{P(R,G,B|M_1)P(M_1) + P(R,G,B|M_2)P(M_2)}$$

(R,G,B) must be in the range $[50,140]\&[30,80]\&[40,100]$ respectively.

i. If $\{R \in [50,80)$ or $B \in [40,60)\}$ and $G \in [30,60)$, $P(M_1|R,G,B) = 1$

ii. If $R \in [80,100)$ and $G \in [30,60)$ and $B \in [60,80)$,

$$P(M_1|R,G,B) = \frac{\frac{2}{5}\times\frac{1}{60000}}{\frac{2}{5}\times\frac{1}{60000} + \frac{3}{5}\frac{1}{120000}} = \frac{4}{7}$$

iii. Otherwise, $P(M_1|R,G,B) = 0$

d)

$$P(M_2|R,G,B) = 1 - P(M_1|R,G,B)$$

$$= \begin{cases} 0, & if\ \{R \in [50,80)\ or\ B \in [40,60)\}\ and\ G \in [30,60) \\ \dfrac{3}{7}, & if\ R \in [80,100)\ and\ G \in [30,60)\ and\ B \in [60,80) \\ 1, & o.w. \end{cases}$$

e) We always pick the higher posterior probability.

$$M = \begin{cases} M_1, & if\ \{R \in [50,80)\ or\ B \in [40,60)\}\ and\ G \in [30,60) \\ M_1, & if\ R \in [80,100)\ and\ G \in [30,60)\ and\ B \in [60,80) \\ M_2, & o.w. \end{cases}$$

f)

$$P(error) = \begin{cases} 0, & if\ \{R \in [50,80)\ or\ B \in [40,60)\}\ and\ G \in [30,60) \\ \dfrac{3}{7}, & if\ R \in [80,100)\ and\ G \in [30,60)\ and\ B \in [60,80) \\ 0, & o.w. \end{cases}$$