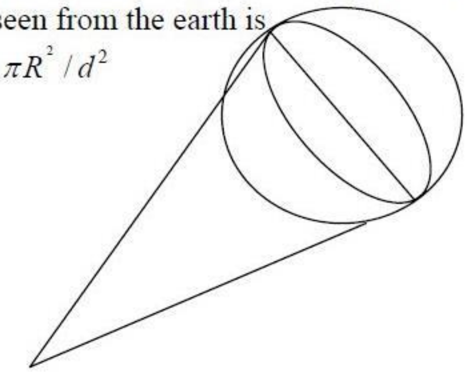
1. The moon is a sphere with radius R at a distance d. The disc that is seen from the earth has an area of πR^2 and has a normal along the viewing axis. Thus we have $\theta = 0$. So the solid angle of the moon as seen from the earth is

$$\Omega = \pi R^2 / d^2$$



For a circular plate, the angle θ ranges between 0 and 90 degrees. Therefore the range of possible solid angles is 0 to $\frac{\pi R^2}{d^2}$.

2. Suppose the room is in a 3D coordinate system. The center of the ceiling is at $(50, 50, 100)$.
a) Suppose the corner of the room on the floor is at $(100, 0, 0)$
The norm vector of the square patch is $N = (0,0,-1)$
Then the vector between the center of the ceiling and the corner on the floor is $V = (50, -50, -100)$

$$cos\theta = \frac{N \cdot V}{|N \cdot V|} = \frac{\sqrt{6}}{3}$$

The foreshortened area is,

$$Acos\theta = 1 \times \frac{\sqrt{6}}{3} \approx 0.817ft^2$$

b) The square distance is $d^2 = |V| = 1.5 \times 10^4$
The solid angle is,

$$\Omega = \frac{Acos\theta}{d^2} = \frac{\sqrt{6}}{45000} \approx 5.44 \times 10^{-5}$$

c) Suppose the corner of the room on the ceiling is at $(100, 0, 100)$
Then the vector between the center of the ceiling and the corner on the floor is $V = (-50,50,0)$

$$cos\theta = \frac{N \cdot V}{|N \cdot V|} = 0$$

Then the solid angle is 0.

3. a)

$$z = -7x - \sqrt{50}y - 2$$

$$p = \frac{\partial z}{\partial x} = -7, q = \frac{\partial z}{\partial y} = -\sqrt{50} = -5\sqrt{2}$$

b) To get the reflected radiance from P in the direction of (0, 0, 0) as large as possible, we should place the light source on the direction of the normal vector

The norm vector is $(-p, -q, 1) = (7, \sqrt{50}, 1)$

The location for the light source is $(0,0,-2) + k(7, \sqrt{50}, 1)$,

where k satisfies $\sqrt{k^2(7^2 + 50 + 1)} = 20 => k = \pm 2$

The source and $(7, \sqrt{50}, 1)$ should on the same side, $k = 2$

The location of source is $(0,0,-2) + 2 \times \left(7, \sqrt{50}, 1\right) = \left(14, 2\sqrt{50}, 0\right) = (14,10\sqrt{2}, 0)$