1.a) perspective projection:

$$x' = \frac{f'x}{z} = \frac{f'(x_0+at)}{z_0+ct} = \frac{-1}{2t}$$

$$y' = \frac{f'y}{z} = \frac{f'(y_0+bt)}{z_0+ct} = \frac{1-t}{t}$$

orthographic projection:

$$x' = x = x_0 + at = 0.5$$

$$y' = y = y_0 + bt = -1+t$$

1. b) The perspective projection of the line is a line.

let $p_1(x_1, y_1, z_1)$ $p_2(x_2, y_2, z_2)$ in the original line;

while $p_1'(x_1', y_1', z_1')$ $p_2'(x_2', y_2', z_2')$ are two points in the projection line.

$$\begin{cases} x_1 = x_0 + at_1 \\ y_1 = y_0 + bt_1 \\ z_1 = z_0 + ct_1 \end{cases} \Longleftrightarrow \begin{cases} x_1' = \dfrac{f'(x_0+at_1)}{z_0+ct_1} \\ y_1' = \dfrac{f'(y_0+bt_1)}{z_0+ct_1} \\ z_1' = f' \end{cases}$$

$$\begin{cases} x_2 = x_0 + at_2 \\ y_2 = y_0 + bt_2 \\ z_2 = z_0 + ct_2 \end{cases} \Longleftrightarrow \begin{cases} x_2' = \dfrac{f'(x_0+at_2)}{z_0+ct_2} \\ y_2' = \dfrac{f'(y_0+bt_2)}{z_0+ct_2} \\ z_2' = f' \end{cases}$$

The slope of the $p_1'p_2'$

$$k = \frac{y_2' - y_1'}{x_2' - x_1'} = \frac{\dfrac{y_0+bt_2}{z_0+ct_2} - \dfrac{y_0+bt_1}{z_0+ct_1}}{\dfrac{x_0+at_2}{z_0+ct_2} - \dfrac{x_0+at_1}{z_0+ct_1}} = \frac{-(z_0y_0+bt_1z_0+ct_2y_0+bct_1t_2)}{y_0z_0+cy_0t_1+bt_2z_0+bct_1t_2} \ldots$$

$$= \frac{y_0 c - z_0 b}{x_0 c - z_0 a}$$

The intercept $b = y' - x'k = \dfrac{f'(x_0 b - y_0 a)}{x_0 c - z_0 a}$

Because of for any two nodes P_1, P_2 on the projection line.

The k and b of the line P_1P_2 are independent of t.

Besides the parameter on the projection line is the same..

$\therefore$ The perspective projection of the line is also a line.

1.c) The orthographic projection of the line is also a line.

Let $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$ in the original line;

while $P_1'(x_1' y_1' z_1')$ $P_2(x_2' y_2' z_2')$ are two points in the projection line.

$$\begin{cases} x_1 = x_0 + at_1 \\ y_1 = y_0 + bt_1 \\ z_1 = z_0 + ct_1 \end{cases} \Longleftrightarrow \begin{cases} x_1' = x_0 + at_1 \\ y_1' = y_0 + bt_1 \\ z_1' = f' \end{cases}$$

$$k = \frac{y_2' - y_1'}{x_2' - x_1'} = \frac{b}{a}$$

$$\begin{cases} x_2 = x_0 + at_2 \\ y_2 = y_0 + bt_2 \\ z_2 = z_0 + ct_2 \end{cases} \Longleftrightarrow \begin{cases} x_2' = x_0 + at_2 \\ y_2' = y_0 + bt_2 \\ z_2' = f' \end{cases}$$

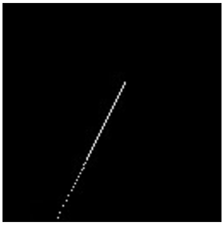
The intercept b

$$b = y_1' - x_1' \cdot \frac{b}{a} = (y_0 + bt_1) - \frac{b}{a}(x_0 + at_1) = y_0 - \frac{b}{a} x_0.$$

k and b (intercept) are independent of t. And the parametre is fixed. So the orthographic projection is line.

1.d)

perspective image



orthographic. image

1.e)

$$\lim_{t\to\infty} x' = \lim_{t\to\infty} \frac{f'(\frac{x_0}{t}+a)}{\frac{z_0}{t}+c} = \frac{af'}{c} = 0$$

$$\lim_{t\to\infty} y' = \lim_{t\to\infty} \frac{f'(\frac{y_0}{t}+b)}{\frac{z_0}{t}+c} = \frac{f'b}{c} = -1$$

for perspetive projection, as $t \to \infty$, the projection point will be close to $(0,-1)$

And It is consistent with the image that my program generates. Those points close to $(0,-1)$ are brighter.

2.a)

if $z_0 = -1$

Line 1 (perspetive) $x = -0.5 - t$ $y = 1 - t$

Line 2 (perspective) $x = 0.5 - t$ $y = 1 - t$

Line 1 (orthographic) $x = 0.5 + t$ $y = -1 + t$

Line 2 (orthographic) $x = -0.5 + t$ $y = -1 + t$

if $z_0 = -2$

Line 1 (perspective) $x = \frac{-0.5 - t}{2}$ $y = \frac{1 - t}{2}$

Line 2 (perspective) $x = \frac{0.5 - t}{2}$ $y = \frac{1 - t}{2}$

Line 1 (orthographic) $x = 0.5 + t$ $y = -1 + t$

Line 2 (orthographic) $x = -0.5 + t$ $y = -1 + t$

if $z_0 = -3$

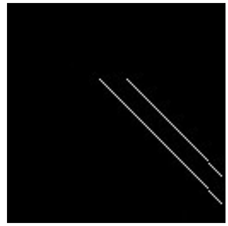
Line 1 (perspective) $x = \frac{-0.5 - t}{3}$ $y = \frac{1 - t}{3}$

Line 2 (perspective) $x = \frac{0.5 - t}{3}$ $y = \frac{1 - t}{3}$

Line 1 (orthographic) $x = 0.5 + t$ $y = -1 + t$

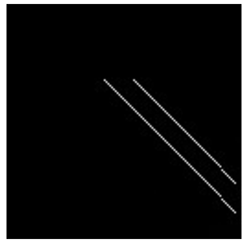
Line 2 (orthographic) $x = -0.5 + t$ $y = -1 + t$

2.b)



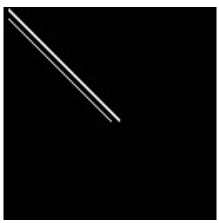
$z_0 = -1$ perspective projection

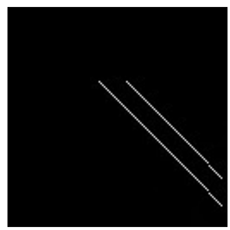
orthographic projection



$z_0 = -2$ perspective projection

orthographic projection





$z_0 = -3$ perspective projection

orthographic projection.

2.c) The projections of the lines are parallel for both perspective and orthographic projection

$$k_1 (\text{perspective}, L_1) = \frac{y_1 c - z_0 b}{x_1 c - z_0 a} = 1 \ \& \ k_2 (\text{perspective}, L_2) = \frac{y_2 c - z_0 b}{x_2 c - z_0 a} = 1$$

$$k_1 = k_2$$

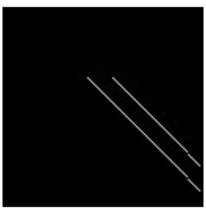
$k_1(\text{orthographic}, L_1) = \frac{b}{a}$ $k_2(\text{orthographic}, L_2) = \frac{b}{a}$

$\therefore$ The two lines in one image are parallel.

2.d) The answer for part c) is consistent with the images that my program generates.

2.e) In this case, orthographic projection is a good approximation because in the course of perspective projection, x, y are equally magnified

$z_0 = |f'|$

2.f) 

perspective

orthographic

when $z_0 = |f'|$, there is no magnification effect. The projections of both perspective and orthographic are the same in the picture.

3. a)

$b=0$ $c=1$

Line1 (perspective) $x = \frac{-1}{t}$ $y = \frac{-1}{t}$

Line2 (perspective) $x = \frac{1}{t}$ $y = \frac{-1}{t}$

Line1 (orthographic) $x=-1$ $y=-1$

Line2 (orthographic) $x=1$ $y=-1$

$b=0$ $c=-1$

Line1 (perspective) $x = \frac{1}{t}$ $y = \frac{1}{t}$

Line2 (perspective) $x = \frac{-1}{t}$ $y = \frac{1}{t}$

Line1 (orthographic) $x=-1$ $y=-1$

Line2 (orthographic) $x=1$ $y=-1$

$b=-1$ $c=1$

Line1 (perspective) $x = \frac{-1}{t}$ $y = \frac{-1-t}{t}$

Line2 (perspective) $x = \frac{1}{t}$ $y = \frac{-1-t}{t}$

Line1 (orthographic) $x=-1$ $y=-1-t$

Line2 (orthographic) $x=1$ $y=-1-t$

$b=-1$ $c=-1$

Line1 (perspective)

$x = \frac{1}{t}$ $y = \frac{1+t}{t}$

Line2 (Perspective)

$x = \frac{-1}{t}$ $y = \frac{1+t}{t}$

Line1 (orthographic)

$x=-1$ $y=-1-t$

Line2 (orthographic)

$x=1$ $y=-1-t$

$b=1$ $c=1$

Line 1 (perspective)

$x=\frac{-1}{t}$ $y=\frac{t-1}{t}$

Line 2 (perspective)

$x=\frac{1}{t}$ $y=\frac{t-1}{t}$

Line 1 (orthographic)

$x=-1$ $y=t-1$

Line 2 (orthographic)

$x=1$ $y=t-1$

$b=1$ $c=-1$

Line 1 (perspective)

$x=\frac{1}{t}$ $y=\frac{1-t}{t}$

Line 2 (perspective)

$x=\frac{-1}{t}$ $y=\frac{1-t}{t}$

Line 1 (orthographic)

$x=-1$ $y=t-1$

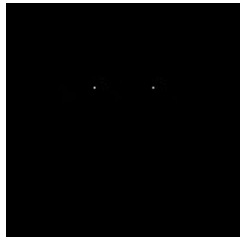
Line 2 (orthographic)

$x=1$ $y=t-1$

3.b)

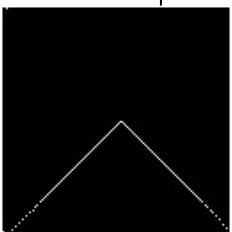
$b=0 \quad c=1$

perspective

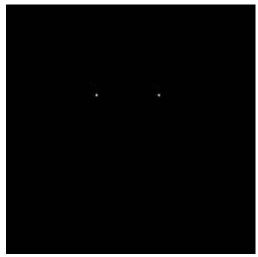


orthographic

$b=0 \quad c=-1$

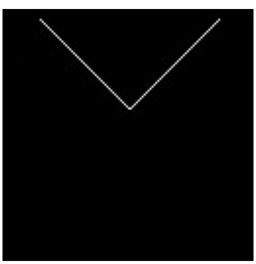


perspective

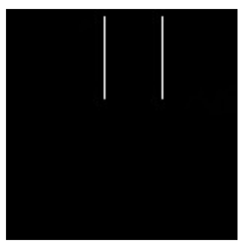


orthographic

$b=-1 \quad c=1$

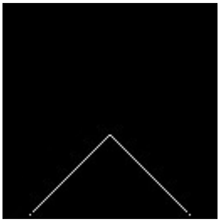


perspective



orthographic

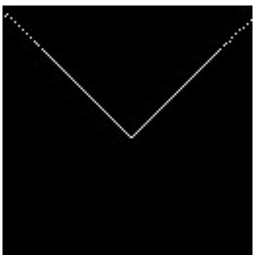
$b=-1 \quad c=-1$



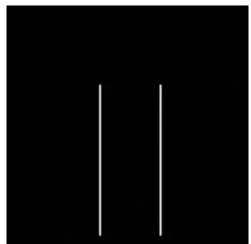
perspective

orthographic

$b=1 \quad c=1$

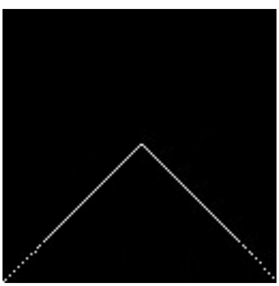


perspective

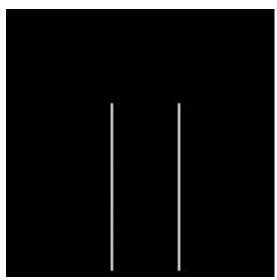


orthographic

$b=1 \quad c=-1$



perspective



orthographic

3.c)

The projections of the lines aren't parallel for the perspective projection.

$$k_1(\text{perspective}, L_1) = \frac{y_0 c - z_0 b}{x_1 c - z_0 a} = \frac{y_0 c - z_0 b}{x_1 c}$$

$$k_2(\text{perspective}, L_2) = \frac{y_0 c - z_0 b}{x_2 c - z_0 a} = \frac{y_0 c - z_0 b}{x_2 c}$$

$\because k_1 \neq k_2$

$\therefore$ the projections of the lines aren't parallel for perspective.

The projections for orthographic projection are parallel.

Because they are $x = -1$ and $x = 1$. And when $b = 0$, they are just two points in the orthographic projection.

3.d) Consistent.

3.e) In this question, the orthographic projection isn't a good approximation to perspective projection.

Because, in the perspective projection, the two lines aren't parallel; in the orthographic projection, the two lines are parallel.

3.f) $x' = \frac{f'x}{z} = \frac{f'(x_0+at)}{z_0+ct} = \frac{x_0 \, (x_1/x_2)}{ct}$

$y' = \frac{f'y}{z} = \frac{f'(y_0+bt)}{z_0+ct} = \frac{-1+bt}{tc}$

$$\lim_{t\to\infty} x' = \lim_{t\to\infty} \frac{f'x_1}{z_0+ct} = 0$$

$$\lim_{t\to\infty} y' = \lim_{t\to\infty} \frac{f'(y_0+tb)}{z_0+ct} = \frac{f'b}{c}$$

For (b, c)

$\frac{f'b}{c}$ table:

c\b	0	1	-1
1	0	1	-1
-1	0	-1	1

In the perspective projection, as t goes to ∞, the projection gets closer to point $(0, \frac{f'b}{c})$

∴ the brighter the line is, the larger t is.

And they are consistent with my images.