# EECS101: HOMEWORK #7

**Due: March 3, 2023**

In this assignment, you will write a program to generate images of a sphere under orthographic projection using a reflectance model. Consider a representation for a sphere centered on the optical axis with radius $r$ and center $(0, 0, z_0)$

$$z(x, y) = z_0 + \sqrt{r^2 - (x^2 + y^2)} \qquad (x^2 + y^2) \leq r^2 \tag{1}$$

What is the unit surface normal $\widehat{N}(x, y)$ to the sphere as a function of $x$ and $y$? Turn in this answer with your code.

We will consider only illumination by point sources. Let $\widehat{S}$ denote a unit vector in the direction of the source. We assume that the source is distant relative to the size of the sphere so that for a given source position, the vector $\widehat{S}$ is constant across the surface of the sphere. We let $\widehat{V}$ denote a unit vector in the direction of the camera. We assume that the camera is distant relative to the size of the sphere so that $\widehat{V}$ for this geometry is always $(0, 0, 1)$. The scene radiance $L$ for a Lambertian surface is proportional to $\cos\theta$ where $\theta$ is the angle between $\widehat{S}$ and $\widehat{N}$. For a more general surface, we can write

$$L = aL_l + (1 - a)L_s \tag{2}$$

for a constant $a$ $(0 \leq a \leq 1)$ where $L_l$ is the scene radiance due to Lambertian reflection

$$L_l = \cos\theta \tag{3}$$

and $L_s$ is the scene radiance due to specular reflection. We can model $L_s$ using

$$L_s = e^{-(\alpha/m)^2} \tag{4}$$

where $m$ is a constant that is related to the roughness of the surface and $\alpha$ is defined as follows. Let $\widehat{H}$ be the unit vector that is the angular bisector of $\widehat{V}$ and $\widehat{S}$, i.e.

$$\widehat{H} = \frac{\widehat{V} + \widehat{S}}{|\widehat{V} + \widehat{S}|} \cdot \tag{5}$$

Then $\alpha$ is the angle in radians between $\widehat{N}$ and $\widehat{H}$. $\widehat{H}$ is the hypothetical normal to a surface that would give perfect specular reflection in the direction of the camera $\widehat{V}$. Thus, $\alpha$ measures how much $\widehat{N}$ deviates from this orientation. If $\alpha$ is small, then $L_s$ will be near 1. Otherwise $L_s$ will be small.

Note that the maximum value of $L$ is 1. Your program should generate images of the sphere by evaluating (2) across the surface and scaling the resulting values by 255 to generate numbers that are appropriate for eight bit pixel values. Note that we are using the fact that image irradiance is proportional to scene radiance. Each image will have only one source position $\widehat{S}$. For each image, only the vector $\widehat{N}$ should change as you compute the image since $\widehat{S}$ and $\widehat{V}$ are assumed constant. Your TA will provide values for $\widehat{S}$, $m, a$, and $r$ to generate images to submit. You are encouraged to experiment with different values for these parameters and to try to understand their role in the image formation process. Also submit your code.