COMP3411 Tutorial - Week 5
Logic

Question 1 - Propositional Logic

Decide whether each of the following sentences is valid, satisfiable, or unsatisfiable. Verify your decisions using truth tables or logical equivalence and inference rules. For those that are satisfiable, list all the models that satisfy them.

a. Smoke ⇒ Smoke
b. Smoke ⇒ Fire
c. (Smoke ⇒ Fire) ⇒ (¬ Smoke ⇒ ¬ Fire)
d. Smoke ∨ Fire ∨ ¬ Fire
e. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
f. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)
g. Big ∨ Dumb ∨ (Big ⇒ Dumb)
h. (Big ∧ Dumb) ∨ ¬ Dumb

Question 2 - Tautologies

Determine whether the following sentences are valid (i.e. tautologies) using truth tables.

(i) $((P \vee Q) \wedge \neg P) \rightarrow Q$
(ii) $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$
(iii) $\neg(\neg P \wedge P) \wedge P$
(iv) $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

Question 3 - Entailment

Show using the truth table method that the corresponding inferences are valid.

(i) $P \rightarrow Q, \neg Q \models \neg P$
(ii) $P \rightarrow Q \models \neg Q \rightarrow \neg P$
(iii) $P \rightarrow Q, Q \rightarrow R \models P \rightarrow R$

Question 4 - Inference Rules

Consider the following Knowledge Base of facts:

"If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is mortal and a mammal. If the unicorn in either immortal or a mammal, then it is horned. The unicorn is magical if it is horned."

1. Translate the above statements into Propositional Logic, using the symbols:

Myth	The unicorn is Mythical
Mortal	The unicorn is Mortal
Mammal	The unicorn is a Mammal
Horned	The unicorn is Horned
Magic	The unicorn is Magical

2. Convert this Knowledge Base into Conjunctive Normal Form.
3. Use a series of resolutions to prove that the unicorn is Horned.
4. Give all models that satisfy the Knowledge Base. Can you prove that the unicorn is Mythical? How about Magical?

Question 5 - First Order Logic

Represent the following sentences in first-order logic, using a consistent vocabulary.

a. Some students studied French in 2015.
b. Only one student studied Greek in 2014.
c. The highest score in Greek is always higher than the highest score in French.
d. Every person who buys a policy is smart.
e. No person buys an expensive policy.
f. There is a barber who shaves all men in town who do not shave themselves.
g. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time. (Use *Fool*(*p, x, t*) to mean that *p* fools *x* at time *t*).