COMP3411 Tutorial - Week 10
Probability and Grammars

1. Probability

1.1. Conditional Probability

Suppose you are give the following information:

- Mumps causes fever 75% of the time
- The chance of a patient having mumps is $\frac{1}{15000}$
- The chance of a patient having fever is $\frac{1}{1000}$

Determine the conditional probability of a patient suffering from mumps given that they have don't have a fever, i.e. $P(Mumps \mid \neg Fever)$.

1.2.Consider the following statements:

Headaches and blurred vision may be the result of sitting too close to a monitor. Headaches may also be caused by bad posture. Headaches and blurred vision may cause nausea. Headaches may also lead to blurred vision.

(i) Represent the causal links in a Bayesian network. Let H stand for "headache", B for "blurred vision", S for "sitting too close to a monitor", P for "bad posture" and N for "nausea". In terms of conditional probabilities, write a formula for the event that all five variables are true, i.e. $P\ (H \wedge B \wedge S \wedge P \wedge N\ )$.

(ii) Suppose the following probabilities are given:

$P(H \mid S, P) = 0.8$ $\quad$ $P(H \mid \neg S, P) = 0.4$

$P(H \mid S, \neg P) = 0.6$ $\quad$ $P(H \mid \neg S, \neg P) = 0.02$

$P(B \mid S, H) = 0.4$ $\quad$ $P(B \mid \neg S, H) = 0.3$

$P(B \mid S, \neg H) = 0.2$ $\quad$ $P(B \mid \neg S, \neg H) = 0.01$

$P(S) = 0.1$

$P(P) = 0.2$

$P(N \mid H, B) = 0.9$ $\quad$ $P(N \mid \neg H, B) = 0.3$

$P(N \mid H, \neg B) = 0.5$ $\quad$ $P(N \mid \neg H, \neg B) = 0.7$

Furthermore, assume that some patient is suffering from headaches but not from nausea. Calculate joint probabilities for the 8 possibilities, according to whether S, B, P are true or false.

$P(H \wedge B \wedge S \wedge P \wedge \neg N) = \ldots$
$P(H \wedge \neg B \wedge S \wedge P \wedge \neg N) = \ldots$
$P(H \wedge B \wedge \neg S \wedge P \wedge \neg N) = \ldots$
$P(H \wedge \neg B \wedge \neg S \wedge P \wedge \neg N) = \ldots$
$P(H \wedge B \wedge S \wedge \neg P \wedge \neg N) = \ldots$
$P(H \wedge \neg B \wedge S \wedge \neg P \wedge \neg N) = \ldots$
$P(H \wedge B \wedge \neg S \wedge \neg P \wedge \neg N) = \ldots$

(iii) What is the probability that the patient suffers from bad posture given that they are suffering from headaches but not from nausea?

1.3 Consider the "burglar alarm" Bayesian network from the lectures.

Derive, using Bayes' Rule, an expression for P(Burglary|Alarm) in terms of the conditional probabilities represented in the network. Then calculate the value of this probability.

Is this number what you expected? Explain what is going on.

2. Definite Clause Grammars

The following definite clause grammar parses a very small subset of English:

```
sentence -->
      noun_phrase,
      verb_phrase.

noun_phrase -->
      determiner,
      noun.

verb_phrase -->
      verb,
      noun_phrase.

determiner --> [a].
determiner --> [the].

noun --> [cat].
noun --> [cats].
noun --> [mouse].
noun --> [mice].

verb --> [scares].
verb --> [hates].
verb --> [hate].
```

2.1. Extend the grammar to handle number agreement. That is a query like:

```
?- sentence([the, cat, hate, the mouse], []).
```

should fail.

2.2.Further extend the grammar so that it constructs a parse tree, like:

```
?- sentence(X, [the, cat, scares, the, mouse], Y).

X = sentence(noun_phrase(determiner(the), noun(cat)),
verb_phrase(verb(scares), noun_phrase(determiner(the),
noun(mouse))))
```

The output represents the parse tree: