Question 3

You are given a simple directed graph with n vertices and m edges. Each vertex v has an associated 'starting cost' s_v and 'finishing cost' f_v, and each edge e has a corresponding weight w_e. All values w_e, s_v and f_v are positive integers.

For this problem, we define the *score* of a path from vertex a to vertex b as the sum of its starting cost s_a, all edge weights on the path, and its finishing cost f_b. Your goal is to determine the smallest score of any path in the graph.

A path may consist of zero or more edges. The endpoints of a path do not need to be distinct.

3.1 **[8 marks]** Design an algorithm which runs in $O((n+m)\log(n+m))$ time and achieves the goal.

An algorithm running in $\Theta((n+m)^2\log(n+m))$ time will be eligible for up to 6 marks.

Answer:

First , choose any one vertex as the start, and use the dijkstra's algorithm to find the length to other vertex. record the least length with other vertexes.
Repeat to do the action for each vertex and we can ignore the vertex which had record.

3.2 **[6 marks]** Now consider only paths consisting of *at least one edge*.

Design an algorithm which runs in $O((n+m)\log(n+m))$ time and achieves the goal.

Your answer here.

3.3 **[6 marks]** Now consider only paths consisting of *at least three edges*.

Design an algorithm which runs in $O((n+m)\log(n+m))$ time and achieves the goal.

Your answer here.