Question 4

You are given a flow network G with n vertices, including a source s and sink t, and m directed edges with integer capacities.

Your friend will ask you several queries of the form: "If an edge were to be added to G, going from vertex a to vertex b with capacity c, what would the maximum flow of the modified network be?" Note that each query considers adding an edge to the original flow network, so the modified network has $m + 1$ edges.

Before asking any queries, your friend gives you some time to prepare, which you can spend on precomputation.

Design an algorithm which performs any required precomputation and then answers each query in constant time, subject to the following restrictions:

4.1 **[5 marks]** $O(nm^2)$ precomputation; $b = t$ and $c = 1$ in all queries.

Answer:

According to the meaning of the question, this situation means that a link with capacity 1 is added between a node a and the sink node. Use the Edmonds-Karp algorithm, we only need to find whether there is an augmented path through a. If there is, and it is valid, and if there is not, we can conduct the next search. The time complexity of Edmonds-Karp is $O(v * E^2)$, in this question $v = n$ and $E = m$, the time complexity is $O(nm^2)$.

4.2 **[6 marks]** $O(nm^2)$ precomputation; $c = 1$ in all queries.

Your answer here.

4.3 **[7 marks]** $O(n^2m^2)$ precomputation; $b = t$ in all queries.

Your answer here.

4.4 **[2 marks]** $O(n^2m^2)$ precomputation; queries unrestricted.

Your answer here.