Question 4

There are $2n$ players who have signed up to a chess tournament. For all $1 \leq i \leq 2n$, the ith player has a known skill level of s_i, which is a non-negative integer. Let $S = \sum_{i=1}^{2n} s_i$, the total skill level of all players.

In the tournament, there will be n matches. Each match is between two players, and each player will play in exactly one match. The *imbalance* of a match is the absolute difference between the skill levels of the two players. That is, if a match is played between the ith player and the jth player, its imbalance is $|s_i - s_j|$. The *total imbalance* of the tournament is the sum of imbalances of each match.

The organisers have provided you with a value m which they consider to be the ideal total imbalance of the tournament.

Design an algorithm which runs in $O(n^2 S)$ time and determines whether or not it is possible to arrange the matches in order to achieve a total imbalance of m, assuming:

4.1 [**4 marks**] all s_i are either 0 or 1;

Answer:

According to the topic, when s_i only have the possibility 0 and 1, it means that There are S players with level 1 and $2n - S$ players with level 0 players.
The least possibility of total tournament imbalance:

The least probability of total tournament means battle between as many of the same levels as possible.
As $2n$ is even, If S is even, it means level 0 players can divide to 2 parts and battle with each other, $2n - S$ will also be even, and it also can divide to 2 parts. Therefore, the least total imbalance of tournament is $T_{min} = 0$.
If S is odd, the $2n - S$ will also be odd, both of them cannot divide to two parts. There must be 1 pair players with different level. the least total imbalance of tournament is $T_{min} = 1$.

The largest possibility of total tournament imbalance:

The largest probability of total tournament means battle between as many of the different levels as possible and the best situation will be all of smaller number of players between S and $2n - S$ battle with different level players, the other $|2n - 2S|$ players battle with each other. As $2n$ and $2S$ are even, therefore, the other players can divide to two part.
If $S \geq 2S - n$, it means level 0 have less people. The total imbalance of the tournament is $T_{max} = (2S - n) \times 1 + |2S - 2n| \times 0 = 2S - n$.
If $S \leq 2S - n$, it means level 0 have less people. The total imbalance of the tournament is $T_{max} = S \times 1 + |2S - 2n| \times 0 = S$.
According to the topic, the maximum of total tournament imbalance is the less number players between level 1 and 0.

According to the topic, when finish the calculating of S, we can get the result.
if $m = 0$, check if S is odd, the ideal total imbalance of tournament cannot reach.
if $m > min\{S, 2S - n\}$, according to the above, it means m is larger than the maximum total imbalance of tournament in this situation, the ideal total imbalance of tournament cannot reach.
If else, the ideal total imbalance of tournament can reach.

4.2 [**16 marks**] the s_i are distinct non-negative integers.

Answer: