Question 3

You are given an array A containing each integer from 1 to n exactly once. Your task is to compute $f(A)$, the sum of $\max(A[\ell..r])$ over all pairs of indices (ℓ, r) such that $\ell \leq r$.

3.1 **[2 marks]** Suppose $n = 3$ and the array is $A = [2, 1, 3]$. Determine the value of $f(A)$.

Answer: $f(A) = 14$
Proof:

According to the topic, the subarray of A is

$$[2],\ [1],\ [3],\ [2,1],\ [2,3],\ [1,3],\ [2,1,3]$$

But $A[\ell..r]$ means that the element must be continued. Therefore, $f(A)$ will find between

$$[2],\ [1],\ [3],\ [2,1],\ [2,3],\ [2,1,3]$$

$f(A) = 2 + 1 + 3 + 2 + 3 + 3 = 14$.

For 3.2 and 3.3, suppose i and j are indices such that $1 \leq i \leq j \leq n$, and let $g(i, j)$ be the number of subarrays $A[\ell..r]$ where $r > j$ and the maximum value is $A[i]$.

3.2 **[4 marks]** For a given pair of indices (i, j), under what conditions is $g(i, j)$ nonzero? In other words, what is the criterion for $A[i]$ to be the maximum of some subarray with its right endpoint at an index greater than j?

Answer:

According to th topic, as long as it is not sorted from small to large, there is a value larger than the adjacent to the right of the value. It

3.3 **[6 marks]** Given an index j, design an algorithm which runs in $O(n)$ time and determines the values of $g(i, j)$ for all $i < j$.

Answer:

3.4 **[8 marks]** Design an algorithm which runs in $O(n \log n)$ time and determines the value of $f(A)$.

Your answer here.