COMP3411 Tutorial- Week 3
Constraint Satisfaction

2023 version 1.1

Question 1 - Cryptarithmetic

Cryptarithmetic is a type of mathematical puzzle where the numbers have been replaced with letters, or other symbols.

Solve the famous Cryptarithmetic problem and Provide not just the final answer, but also explain your reasoning along the way.

$$\begin{array}{cccccc} & S & E & N & D \\ + & M & O & R & E \\ \hline M & O & N & E & Y \end{array}$$

Variables:	Constraints:
D E M N O R S Y	$M \neq 0, S \neq 0$ (unary constraints)
Domains:	$Y = D + E$ *or* $Y = D + E - 10$, etc.
{0,1,2,3,4,5,6,7,8,9}	$D \neq E, D \neq M, D \neq N$, etc.

a) Can you identify any backtracking heuristics or enhancements that you may have (unknowingly) used when you solved the problem?

b) Are there any backtracking heuristics or enhancements that you would now use to solve the problem more efficiently?

What heuristics and strategies did you use along the way?

The sum of two 4-digit numbers cannot exceed 1998, so M=1.

10+O = S+1 or S+1+1, i.e. S = O+9 or O+8, but 1 has already been used, so O=0.

Therefore S=9, because there is no possibility of a carry from E+O.

We then have E+1 = N and 10+E = N+R or N+R+1, so R = 8 or 9, but 9 has already been assigned, so R=8.
(Note how Minimum Remaining Values has been used at each step.)

The puzzle now looks like this:

9END + 108E

10NEY

This gives us the two constraints: E+1 = N

D+E = 10+Y

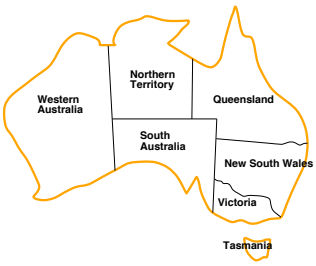
The remaining values are 2,3,4,5,6,7.

We have D+E <= 6+7 = 13, so Y = 2 or 3. (Note: MRV again).

But if Y=3 (Most Constraining Value) then all three variables D,E,N would need to take values 6 or 7, which is impossible (Constraint Propagation).

Therefore Y=2, E=5, N=6 and D=7.

Question 2 - Map Colouring



(Refer to lectures for week 2)

Use Forward Checking to show that the Australia map-colouring problem has no solution when we assign WA=green, V=Red, NT=Red. If we apply Arc Consistency as well, can the inevitable failure be detected further up the tree?

Present your answer to this question and discuss with others in the tutorial group.

Solution:

	WA	NT	Q	NSW	V	SA	T
initial domains	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Green	G	R B	R G B	R G B	R G B	R B	R G B
V = Red	G	R B	R G B	G B	R	B	R G B
NT = Red	G	R	G B	G B	R	B	R G B
SA = Blue	G	R	G	G	R	B	R G B
Q = Green	G	R	G		R	B	R G B

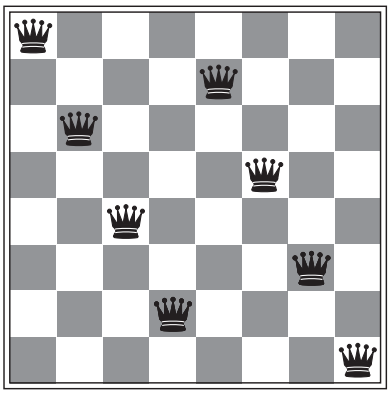
No options remain for NSW, so there is no solution.

If we also apply Arc Consistency, the question can be resolved further up the tree (but with extra computation at each node) as follows:

	WA	NT	Q	NSW	V	SA	T
initial domains	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Green	G	R B	R G B	R G B	R G B	R B	R G B
V = Red	G	R B	R G B	G B	R	B	R G B
NT→SA, Q→SA, NSW→SA	G	R	R G	G	R	B	R G B
NT → Q	G	R	G	G	R	B	R G B
NSW → Q	G	R		G	R	B	R G B

Question 3 - 8-queens problem

Formulate the 8-Queens problem as a constraint satisfaction problem with 8 variables (one for each column) whose domain is the set of possible row positions. Then trace forward checking and domain splitting with arc consistency.



Let Q_i be variables whose domain is $\{1, \cdots, 8\}$. The value assigned to Q_i is the row of the Queen in column i.

Backtracking search starts by assigning 1 to Q_1 (assuming variables and values are chosen in order). Forward checking removes 1 and i from the domains of each other Q_i. Then Q_2 is assigned 3, and forward checking removes 3 from the domains of $Q_3, \cdots, Q_8$, 2 and 4 from Q_3, 5 from Q_4, 6 from Q_5, 7 from Q_6 and 8 from Q_7. Then Q_3 is assigned 5, etc.

Domain splitting is similar. At the first step, the domain of Q_1 is split into $\{1, 2, 3, 4\}$ and $\{5,, 6, 7, 8\}$. These are both arc consistent. The first domain is split again into $\{1, 2\}$ and $\{3, 4\}$. Again these subproblems are arc consistent. The first domain is split again, into $\{1\}$ and $\{2\}$, and now arc consistency proceeds much as forward checking. However, this depends on the order of variable and problem selection.