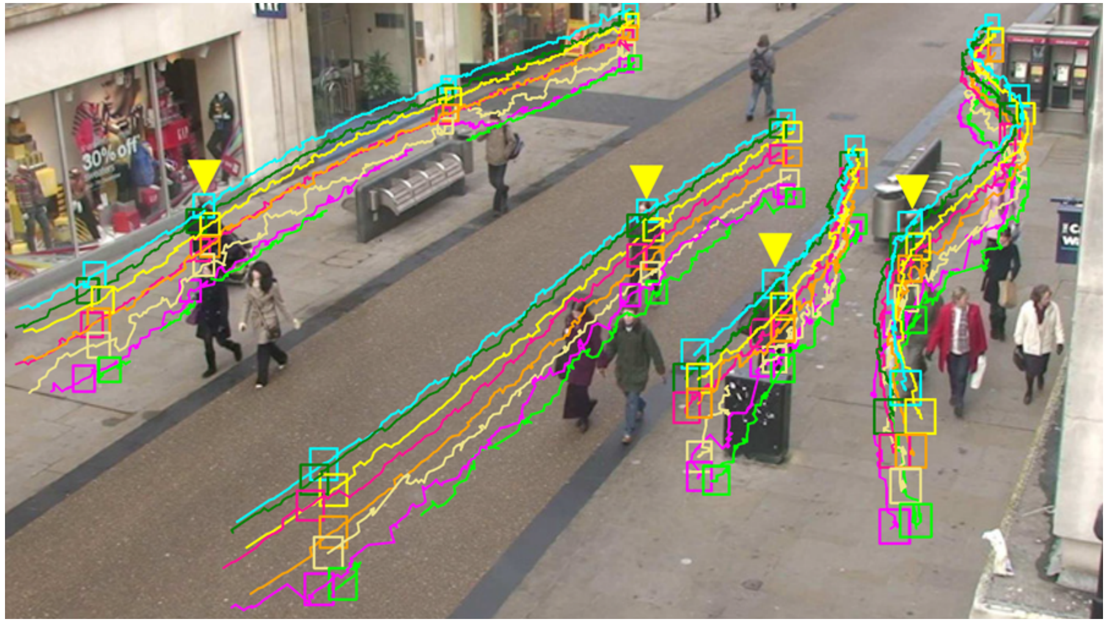
COMP9517

Computer Vision

2024 Term 3 Week 9

Professor Erik Meijering

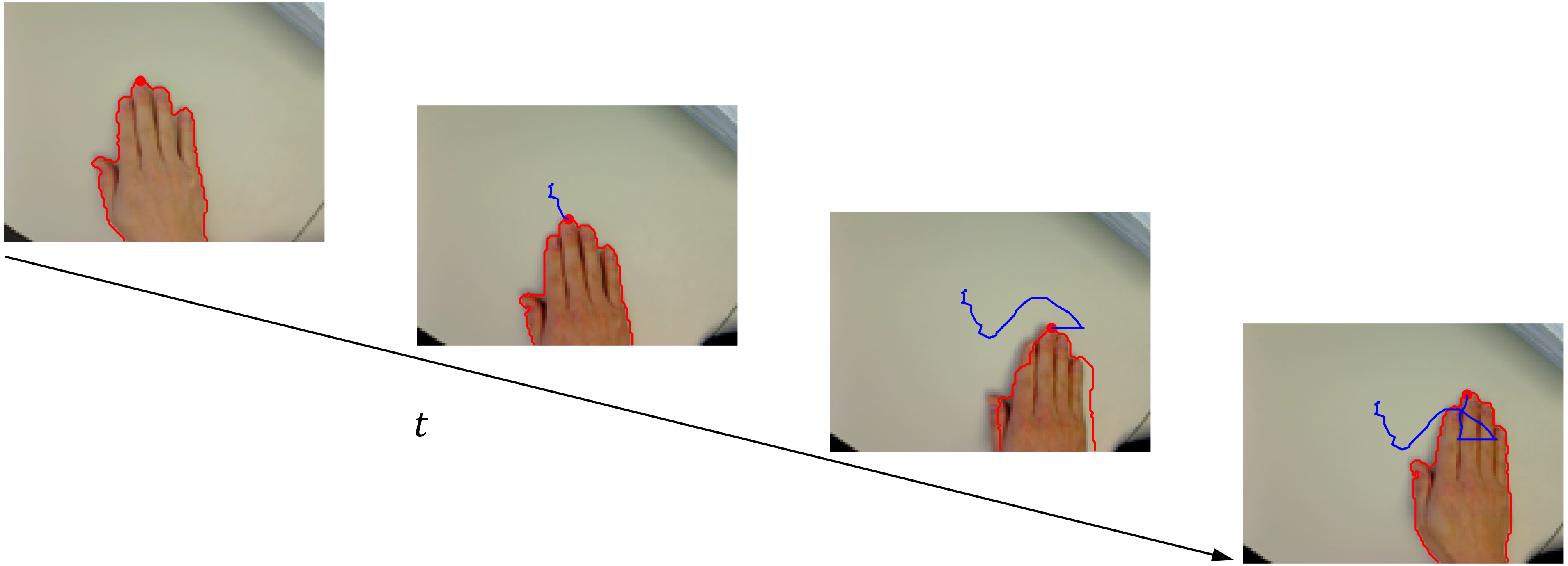


Object Tracking

UNSW SYDNEY

Object tracking aim

- Inferring the motion of an object from a time series of images



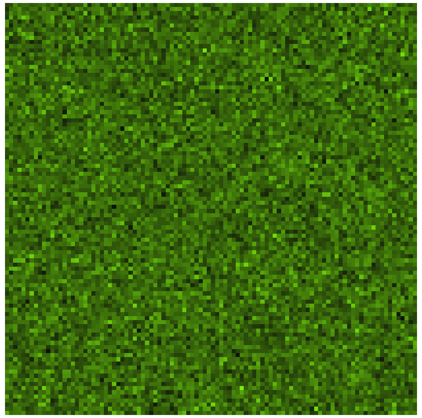
Object tracking applications

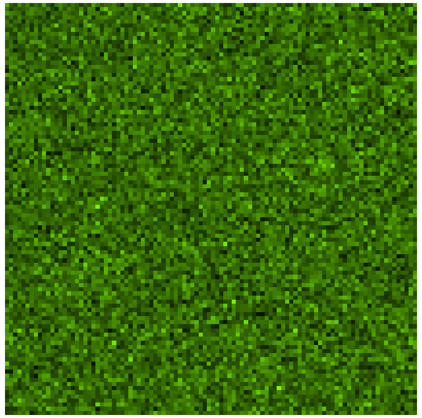
- Motion capture
  - Record motion of people to control cartoon characters in animations
  - Modify the motion record to obtain slightly different behaviours
- Recognition from motion
  - Determine the identity of a moving object
  - Assess what the object is doing
- Video surveillance
  - Detect and track objects in a scene for security
  - Monitor their activities and warn if anything suspicious happens
- Object targeting
  - Decide which objects to target in scene
  - Make sure the objects get hit

Object tracking challenges

- Information loss (3D world projected on a 2D image)
- Noise and other image artifacts
- Complex object motion
- Nonrigid or articulated nature of objects
- Partial and full object occlusions
- Complex object shapes
- Scene illumination changes
- Real-time processing requirements

Example tracking problem





- Single moving microscopic particle
  - Imaged with signal-to-noise ratio (SNR) of 1.5
- Human visual motion perception
  - Not so accurate and reproducible in quantification
  - Good at integrating spatial and temporal information
  - Powerful in making associations and predictions
- Computer vision challenges
  - Integration of spatial and temporal information
  - Modeling and incorporation of prior knowledge
  - Probabilistic rather than deterministic approach
- Bayesian estimation methods…

Motion assumptions

When moving objects do not have unique texture or colour to facilitate identifying them as different entities, the characteristics of the motion itself must be used to connect detected points into trajectories

- Object location changes smoothly over time
- Object Velocity (speed and direction) changes smoothly over time
- Object can be at only one location in space at any given time
- Object cannot be in same location as another object at the same time

Topics

- Bayesian inference

  Using probabilistic models to perform tracking

- Kalman filtering

  Using linear model assumptions for tracking

- Particle filtering

  Using nonlinear models for tracking

Bayesian inference

Problem definition

- A moving object has a **state** which evolves over time

Random variable: X_i

Specific value: x_i

can contain any quantities of interest (position, velocity, acceleration, shape, intensity, colour, …)

- This state is **measured** at each time point

Random variable: Y_i

Specific value: y_i

in computer vision the measurements are typically features computed from the images

- Measurements are combined to estimate the state

Three main steps

- **Prediction:** use the measurements $(y_0, y_1, \dots, y_{i-1})$ up to time $i-1$ to predict the state at time i

$$P(X_i \mid Y_0 = y_0, Y_1 = y_1, \dots, Y_{i-1} = y_{i-1})$$

- **Association**: select the measurements at time i related to the object state
- **Correction**: use the incoming measurement y_i to update the state prediction

$$P(X_i \mid Y_0 = y_0, Y_1 = y_1, \dots, Y_{i-1} = y_{i-1}, Y_i = y_i)$$

Independence assumptions

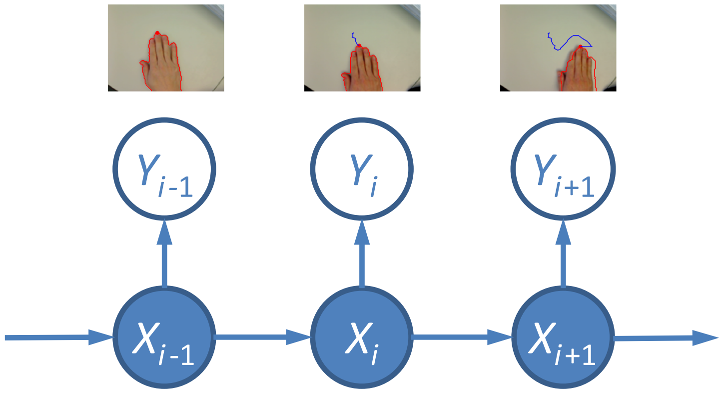
- Current state depends only on the immediate past

$$P(X_i \mid X_0, X_1, \ldots, X_{i-1}) = P(X_i \mid X_{i-1})$$

- Measurements depend only on the current state

$$P(Y_i, Y_j, \ldots, Y_k \mid X_i) = P(Y_i \mid X_i) P(Y_j, \ldots, Y_k \mid X_i)$$

These assumptions imply the tracking problem has the structure of inference on a hidden Markov model



Tracking by Bayesian inference

- Prediction step

$$P(X_i \mid y_0, y_1, \ldots, y_{i-1}) = \int \boxed{P(X_i, X_{i-1} \mid y_0, y_1, \ldots, y_{i-1})}\, dX_{i-1}$$

$$= \int P(X_i \mid X_{i-1}, \cancel{y_0, y_1, \ldots, y_{i-1}}) P(X_{i-1} \mid y_0, y_1, \ldots, y_{i-1})\, dX_{i-1}$$

$$= \int \underbrace{P(X_i \mid X_{i-1})}_{\text{dynamics model}} \underbrace{P(X_{i-1} \mid y_0, y_1, \ldots, y_{i-1})}_{\text{posterior of previous time}}\, dX_{i-1}$$

$$\boxed{\begin{aligned} P(X_i, X_{i-1} \mid y_0, y_1, \ldots, y_{i-1}) &= \frac{P(X_i, X_{i-1}, y_0, y_1, \ldots, y_{i-1})}{P(y_0, y_1, \ldots, y_{i-1})} \\ &= \frac{P(X_i \mid X_{i-1}, y_0, y_1, \ldots, y_{i-1}) P(X_{i-1}, y_0, y_1, \ldots, y_{i-1})}{P(y_0, y_1, \ldots, y_{i-1})} \\ &= P(X_i \mid X_{i-1}, y_0, y_1, \ldots, y_{i-1}) \frac{P(X_{i-1}, y_0, y_1, \ldots, y_{i-1})}{P(y_0, y_1, \ldots, y_{i-1})} \\ &= P(X_i \mid X_{i-1}, y_0, y_1, \ldots, y_{i-1}) P(X_{i-1} \mid y_0, y_1, \ldots, y_{i-1}) \end{aligned}}$$

Tracking by Bayesian inference

- Correction step

$$P(X_i \mid y_0, y_1, \ldots, y_i) = \frac{P(X_i, y_0, y_1, \ldots, y_i)}{P(y_0, y_1, \ldots, y_i)}$$

$$= \frac{P(y_i \mid X_i, \cancel{y_0, y_1, \ldots, y_{i-1}})P(X_i \mid y_0, y_1, \ldots, y_{i-1})P(y_0, y_1, \ldots, y_{i-1})}{P(y_0, y_1, \ldots, y_i)}$$

$$= P(y_i \mid X_i)P(X_i \mid y_0, y_1, \ldots, y_{i-1})\underbrace{\frac{P(y_0, y_1, \ldots, y_{i-1})}{P(y_0, y_1, \ldots, y_i)}}_{\text{constant}}$$

$$\propto \underbrace{P(y_i \mid X_i)}_{\text{measurement model}}\underbrace{P(X_i \mid y_0, y_1, \ldots, y_{i-1})}_{\text{prediction of current state}}$$

Tracking by Bayesian inference

Tracking by Bayesian inference is done by iterative prediction and correction

- Prediction

$$P(X_i \mid Y_{0:i-1}) = \int P(X_i \mid X_{i-1}) \underbrace{P(X_{i-1} \mid Y_{0:i-1})}_{\text{posterior at time } i-1} dX_{i-1}$$

$$\boxed{Y_{0:k} = (Y_0 = y_0, Y_1 = y_1, \ldots, Y_k = y_k)}$$

- Correction

$$\underbrace{P(X_i \mid Y_{0:i})}_{\text{posterior at time } i} \propto P(Y_i \mid X_i) P(X_i \mid Y_{0:i-1})$$

Tracking by Bayesian inference

Two models must be designed to make tracking by Bayesian inference work

- Dynamics model $P(X_i \mid X_{i-1})$
- Measurement model $P(Y_i \mid X_i)$

The specific design choices are application dependent

Tracking by Bayesian inference

Final estimates are computed from the posterior

Example 1: expected a posteriori (EAP)

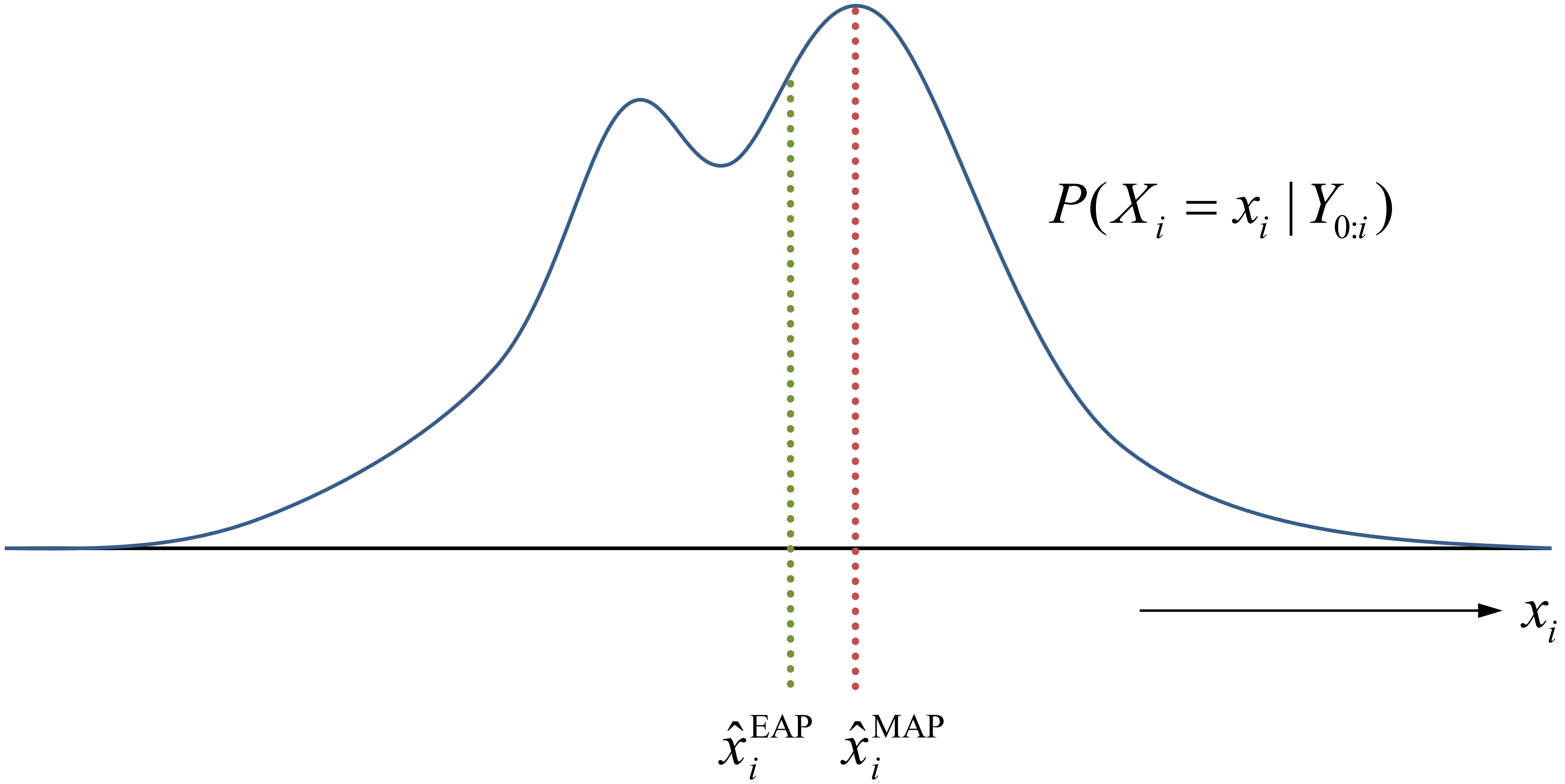
$$\hat{x}_i = \int x_i P(X_i = x_i \mid Y_{0:i}) dx_i$$

Example 2: maximum a posteriori (MAP)

$$\hat{x}_i = \arg\max_{x_i} P(X_i = x_i \mid Y_{0:i})$$

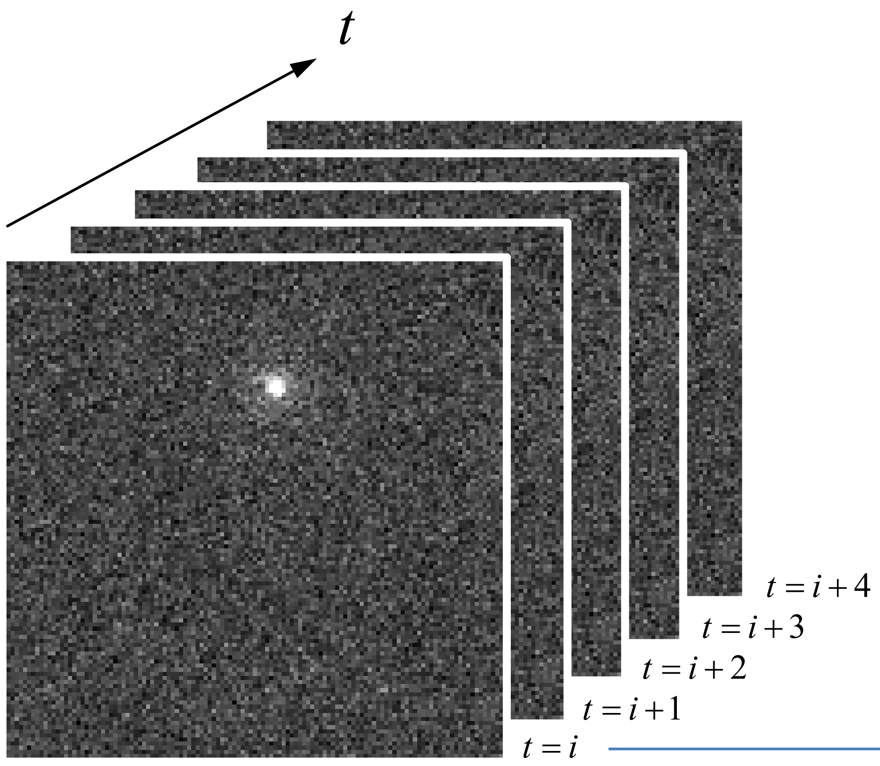
These are the most popular ones but others are possible

Tracking by Bayesian inference

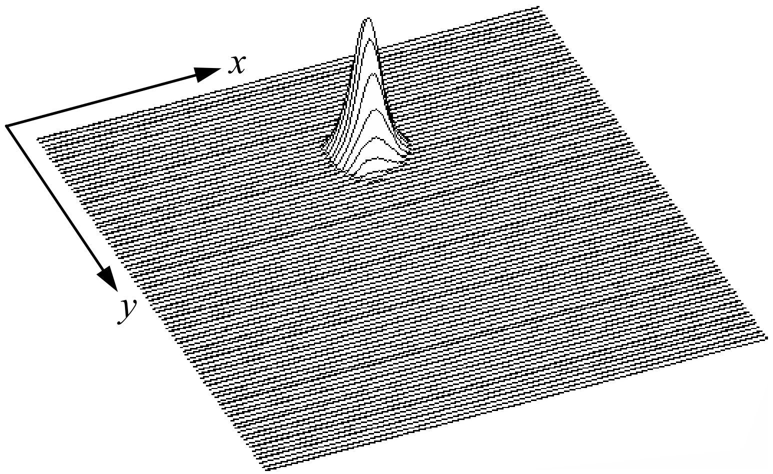


Bayesian tracking example

Estimating the coordinates of a moving particle



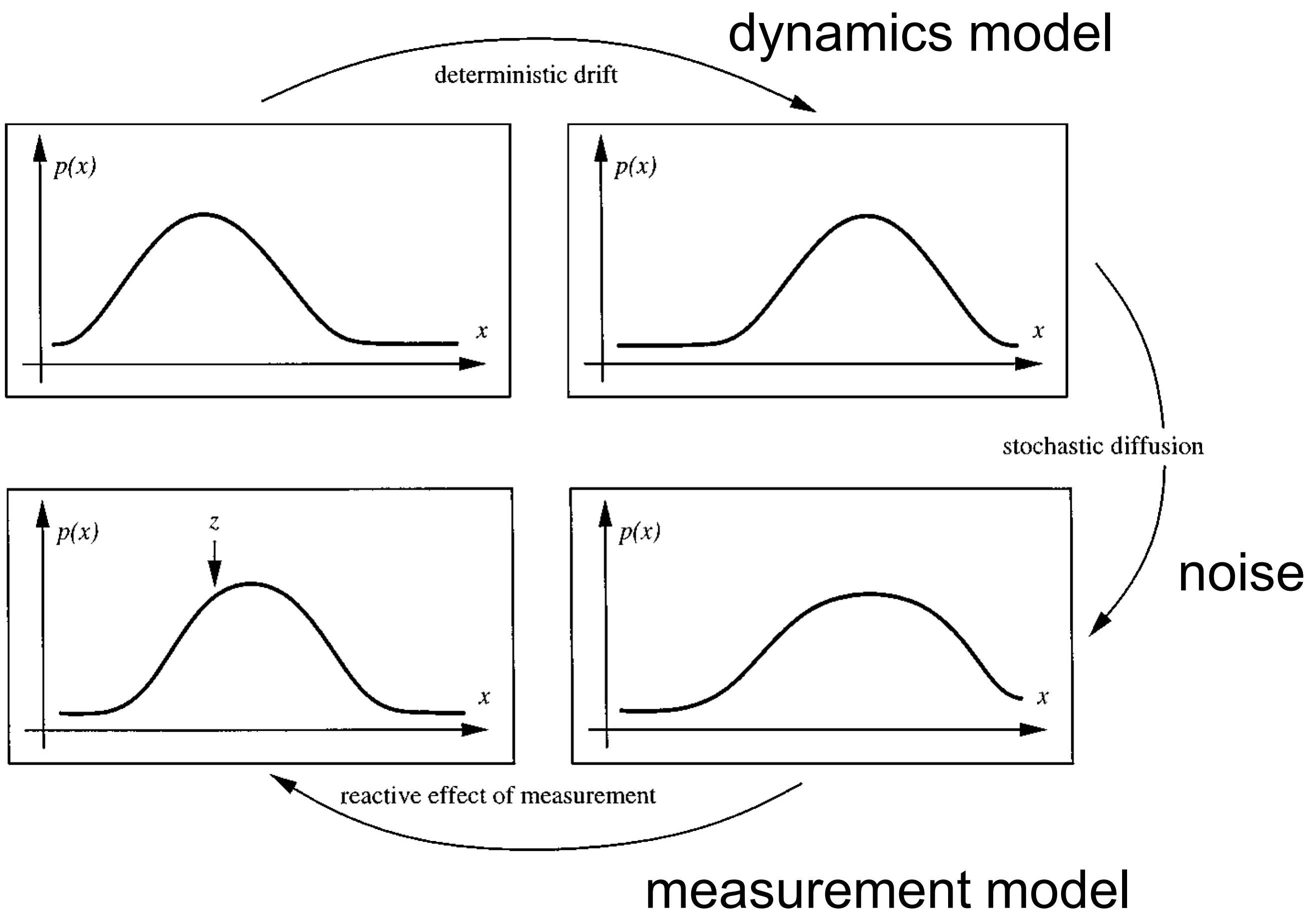
Posterior computed from the image



$P(X_i \mid Y_{0:i})$

Kalman filtering

Probability density propagation



Linearity and Gaussianity assumption

- If we assume the dynamics (state transition) model and the measurement model to be linear, and the noise to be additive Gaussian, then all the probability densities will be Gaussians: $x \sim N(\mu, \Sigma)$
- The state is advanced by multiplying with some known matrix and then adding a zero-mean normal random variable: $x_i = Ax_{i-1} + q_{i-1}$
- The measurement is obtained by multiplying the state by some matrix and then adding a zero-mean normal random variable: $y_i = Hx_i + r_i$

Kalman filtering algorithm

Prediction

1. Predict state

$$x_i^- = Ax_{i-1}$$

2. Predict covariance

$$P_i^- = AP_{i-1}A^T + Q$$

Correction

1. Compute Kalman gain

$$K_i = P_i^- H^T (H P_i^- H^T + R)^{-1}$$

2. Correct state with measurement

$$x_i = x_i^- + K_i (y_i - H x_i^-)$$

3. Correct covariance

$$P_i = (I - K_i H) P_i^-$$

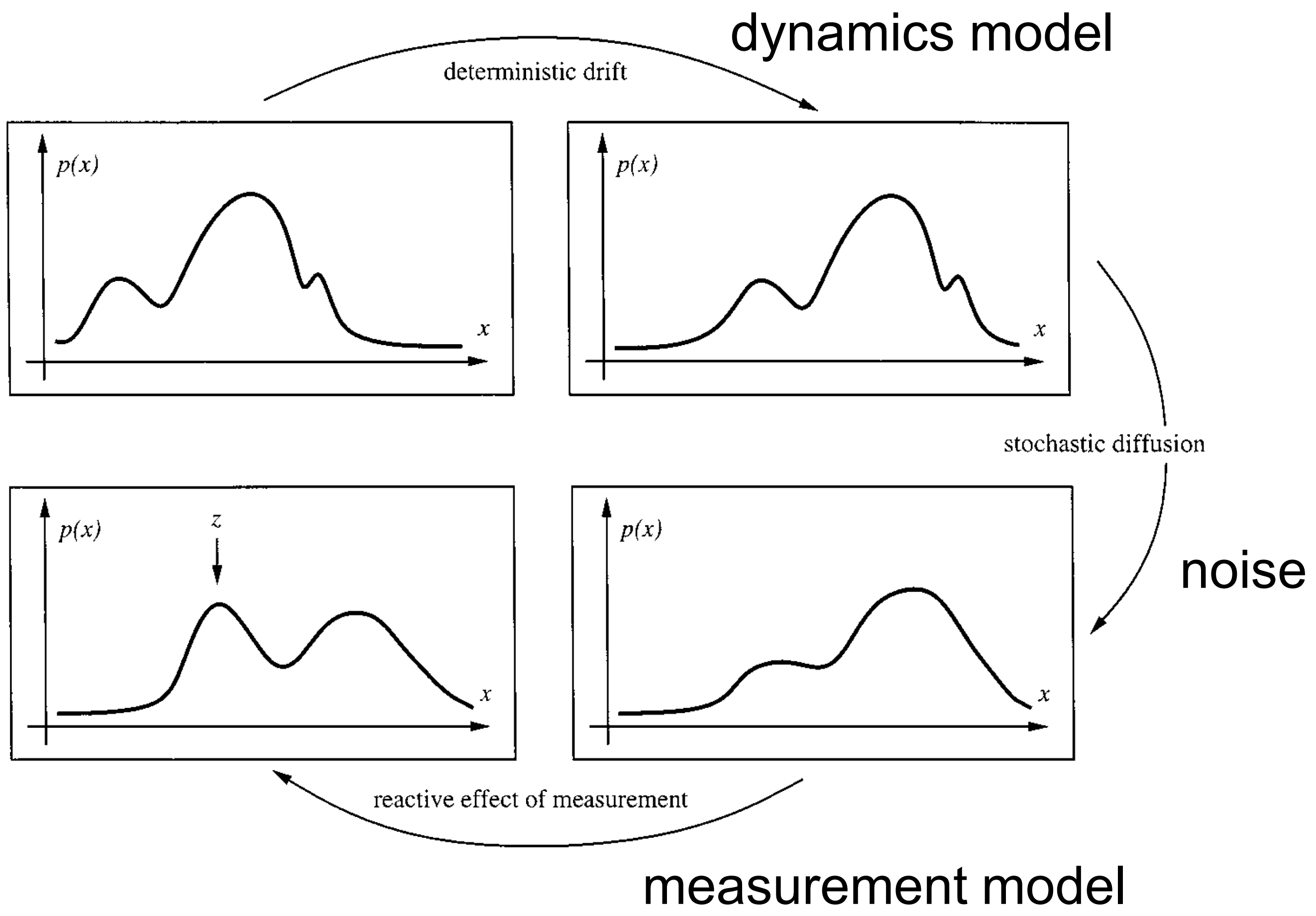
$$i \to i+1$$

$$x_i \sim N(Ax_{i-1}, Q)$$

$$y_i \sim N(Hx_i, R)$$

Particle filtering

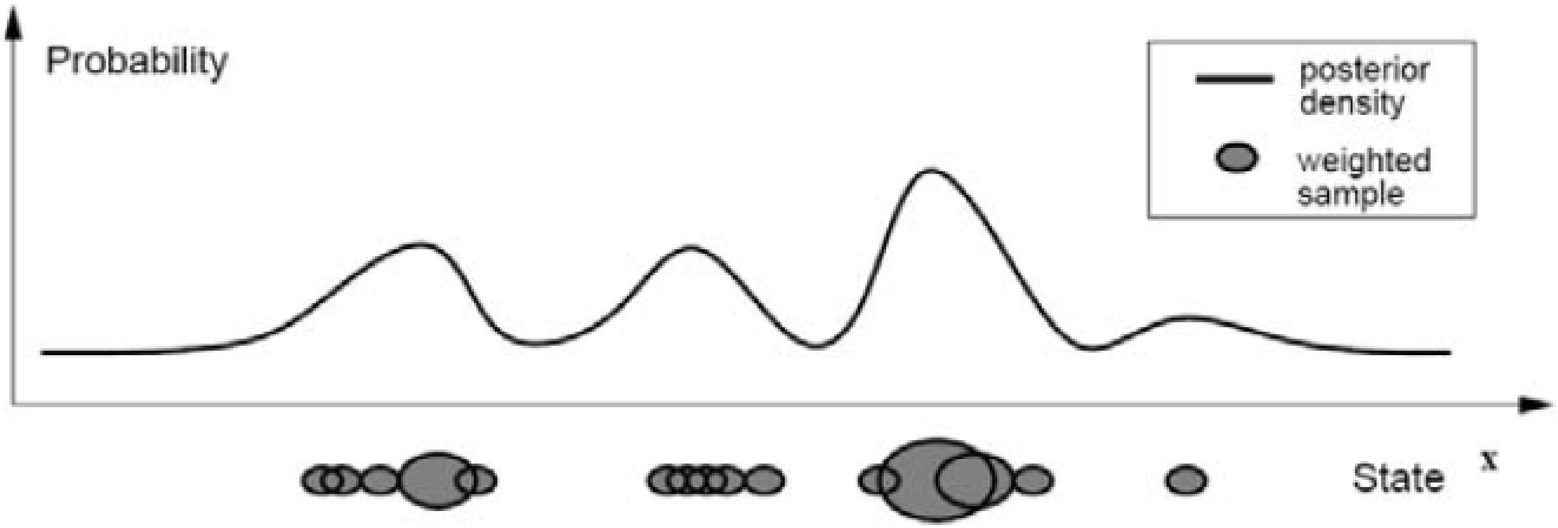
Probability density propagation



Nonlinear and non-Gaussian case

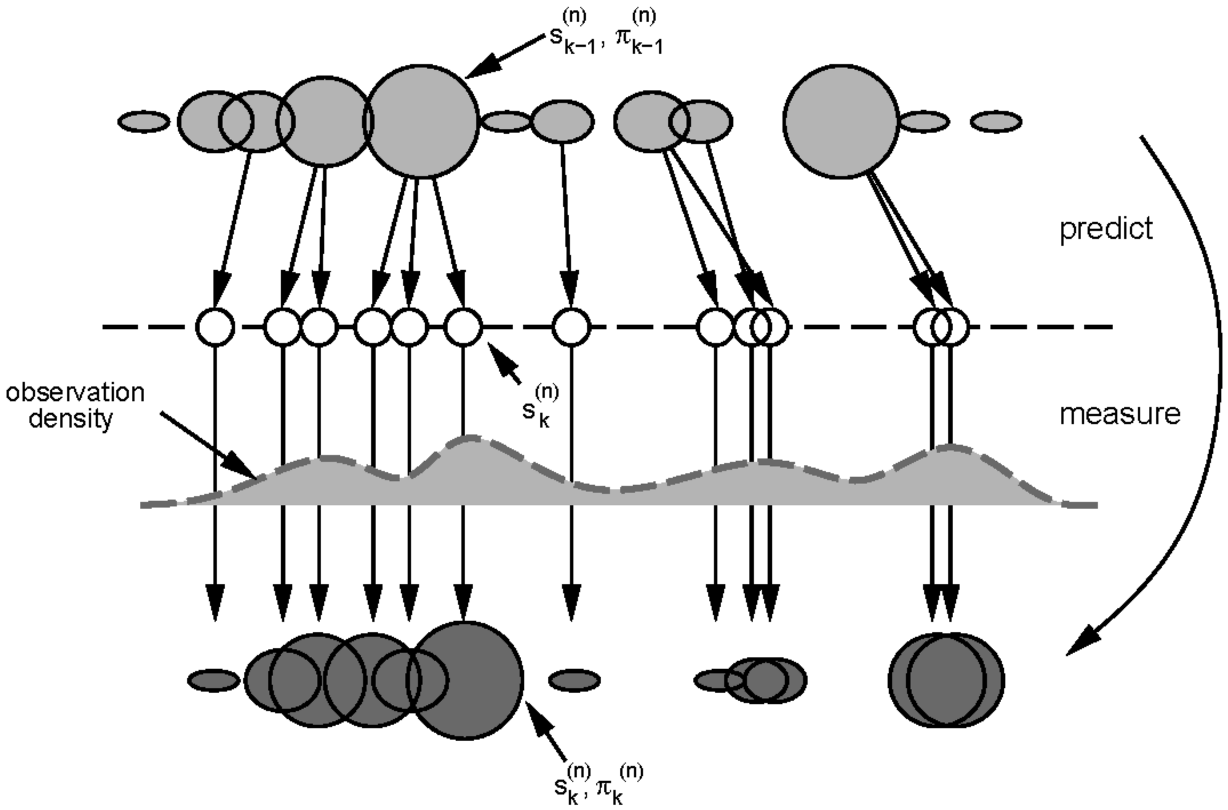
Represent the conditional state density by a set of samples (particles) with corresponding weights (importance)

$$P(X_i \mid Y_{0:i}) \rightarrow \{s_i^{(n)}, \pi_i^{(n)}\}_{n=1}^{N}$$



Particle filtering algorithm

Propagate each sample using the dynamics model and obtain its new weight using the measurement model



Particle filtering algorithm

Iterate

From the "old" sample-set $\{\mathbf{s}_{t-1}^{(n)}, \pi_{t-1}^{(n)}, c_{t-1}^{(n)}, n = 1, \ldots, N\}$ at time-step $t-1$, construct a "new" sample-set $\{\mathbf{s}_t^{(n)}, \pi_t^{(n)}, c_t^{(n)}\}, n = 1, \ldots, N$ for time t.

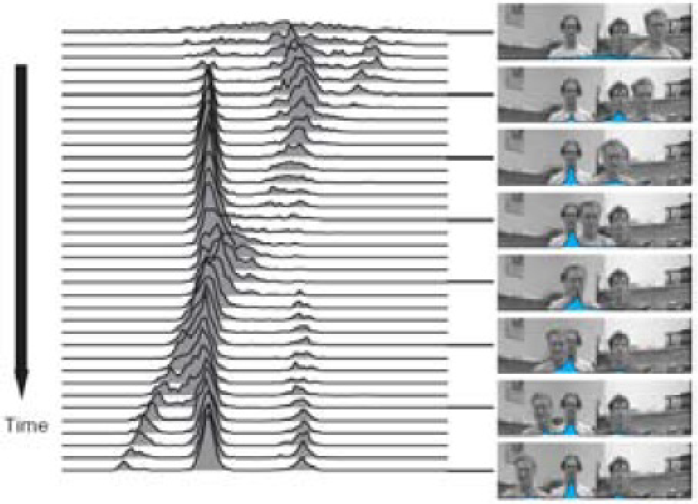
Construct the n^{th} of N new samples as follows:

1. **Select** a sample $\mathbf{s}_t'^{(n)}$ as follows:
   (a) generate a random number $r \in [0, 1]$, uniformly distributed.
   (b) find, by binary subdivision, the smallest j for which $c_{t-1}^{(j)} \geq r$
   (c) set $\mathbf{s}_t'^{(n)} = \mathbf{s}_{t-1}^{(j)}$
2. **Predict** by sampling from
$$p(\mathbf{x}_t | \mathbf{x}_{t-1} = \mathbf{s}'^{(n)}_{t-1})$$
   to choose each $\mathbf{s}_t^{(n)}$.
3. **Measure** and weight the new position in terms of the measured features $\mathbf{z}_t$:
$$\pi_t^{(n)} = p(\mathbf{z}_t | \mathbf{x}_t = \mathbf{s}_t^{(n)})$$
   then normalise so that $\sum_n \pi_t^{(n)} = 1$ and store together with cumulative probability as $(\mathbf{s}_t^{(n)}, \pi_t^{(n)}, c_t^{(n)})$ where
$$\begin{aligned} c_t^{(0)} &= 0, \\ c_t^{(n)} &= c_t^{(n-1)} + \pi_t^{(n)} \quad (n = 1 \ldots N) \end{aligned}$$

Source: NIPS 1996

Example application

Tracking of active contour representations of objects



Source: IJCV 1998

Particle filtering is also known variously as sequential Monte Carlo (SMC) filtering, bootstrap filtering, the CONDENSATION algorithm…

Example application

Tracking of object location in the presence of clutter

Walking pedestrian represented by a state vector consisting of a centre position and a bounding box:

$$s_i = (x, y, w, h)_i$$

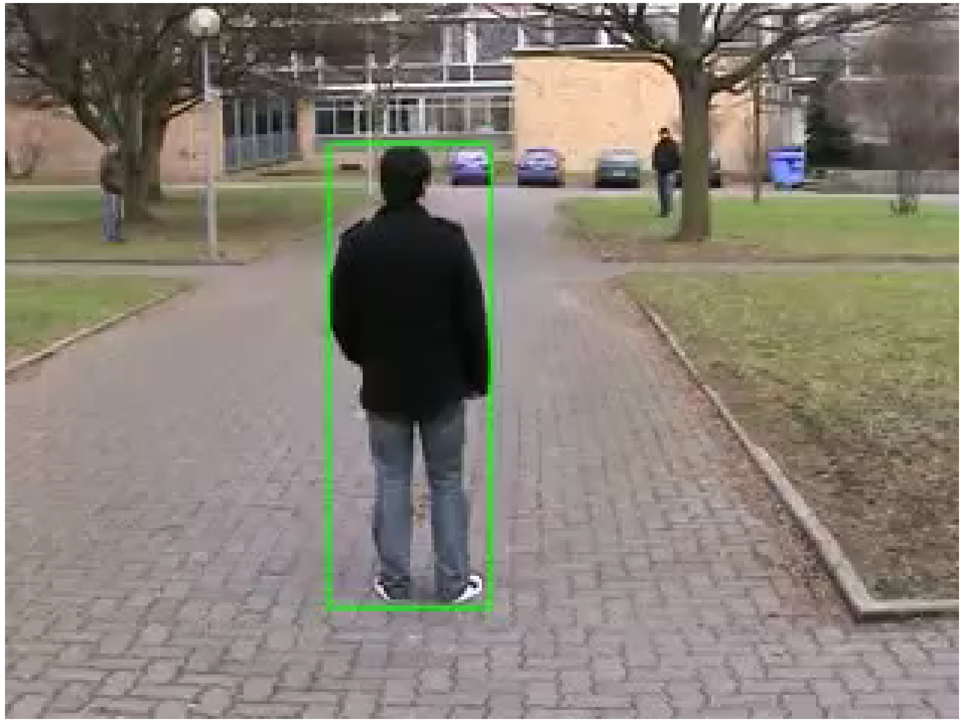
$s_i^{(n)}$ (samples)

$\hat{s}_i$ (estimated)

$\tilde{s}_i$ (truth/annotated)

Example application

Tracking of object location in the presence of clutter



https://www.youtube.com/watch?v=j-duyzShJ_o

Further reading on discussed topics

- Chapters 5 and 8 of Szeliski 2010
- Chapter 18 of Forsyth and Ponce 2011
- Chapter 9 of Shapiro and Stockman 2001
- Isard and Blake 1998 (Available online via the UNSW Library)

Acknowledgement

- Some images drawn from the above references

Example exam question

Which one of the following statements about object tracking is incorrect?

A. The particle filtering method assumes that the dynamics model and the measurement model can be parameterized.

B. The hidden Markov model assumes that the measurements depend only on the current state of the objects.

C. The prediction step of Bayesian inference assumes that the current state of the objects depends only on the previous state.

D. The Kalman filtering method assumes that the dynamics and measurement noise are additive Gaussian.