Strong scaling comparison of three viscous tensor operator classes

D. T. Graves

May 13, 2024

1 The Point of All This

This document is meant to compare the performance of Proto-based and Fortran-based viscous tensor operators. These operators solve the viscous tensor equation using the Martin-Cartwright algorithm as implemented in AMRMultigrid. For the purposes of this document, we well identify these operators as follows.

- ChF Viscous is the fortran-based Chombo operator that lives in `Chombo/lib/src/AMRElliptic`. The class name is `ViscousTensorOp`.
- Proto Slow VTO is the naive Proto implmentatation (called `Proto_Viscous_Tensor_Op`).
- Proto Fast VTO is a smarter Proto implmentatation (called `Proto_FastVTO`). Because Proto coarse-fine interpolation includes corners, we could avoid the whole intermediate cell-centerd gradient calculation and do everything in one grand stencil that rules them all and in darkness binds them.

2 Important Caveats

In trying to come up with a fair comparison between these operators, I found myself making decisions that, at the very least, need to be disclosed. The explanations are not mandatory but they may be self-exculpatory.

- All three are using the slowGSRB relaxation algorithm which properly calculates the residual everywhere on both red and black sweeps. I realize this leaves performance on the table but the best option here (evaluation of the stencil in place on colors with weird skippy loops) is not available in Proto.
- The fortran operator showed much worse multigrid convergence, especially in three dimensions.
- I do not know why this poor convergence is happening and I do not want to spend my life debugging the multigrid convergence of a decades-old operator scheduled for the scrap heap.

- The bottom line is that I had to over smooth the fortran-based codes to get convergence, especially in three dimension. This may result in sub-optimal performance in two dimensional examples (two and three dimensional runs use the same input files).
- There are a few cases that still did not converge. I removed them from the data.

3 Summary of data marked 5-10-2024

In two dimensions, the fortran-based operator was consistently faster overall. I think some work needs to be done to get the different representations to stop at the same level of convergence. So, in two dimensions, this may look somewhat worse for the Proto operators than is warranted. At the very least, this warrants fiddling with the example input files a bit more.

In three dimensions, the Proto operators are quite competive and often even faster than the fortran implementation.

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	2	0	1	1.783595e-11	6	1.0994e+00
ChF Viscous Op	2	0	2	1.783662e-11	6	6.1174e-01
ChF Viscous Op	2	0	4	1.783507e-11	6	3.8586e-01
Proto Slow VTO	2	0	1	7.096546e-13	13	7.0531e+00
Proto Slow VTO	2	0	2	7.096546e-13	13	4.3746e+00
Proto Slow VTO	2	0	4	7.096546e-13	13	2.7630e+00
Proto Fast VTO	2	0	1	7.315260e-13	13	5.9470e+00
Proto Fast VTO	2	0	2	7.315260e-13	13	3.4919e+00
Proto Fast VTO	2	0	4	7.315260e-13	13	2.0155e+00

Table 1: dim = 2, case = 0

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	2	1	1	3.920064e-11	9	1.4934e+00
ChF Viscous Op	2	1	2	3.919354e-11	9	8.2743e-01
ChF Viscous Op	2	1	4	3.918998e-11	9	5.1241e-01
Proto Slow VTO	2	1	1	7.078782e-13	13	7.2282e+00
Proto Slow VTO	2	1	2	7.078782e-13	13	4.5135e+00
Proto Slow VTO	2	1	4	7.078782e-13	13	2.8712e+00
Proto Fast VTO	2	1	1	7.345236e-13	21	8.2631e+00
Proto Fast VTO	2	1	2	7.345236e-13	21	4.8452e+00
Proto Fast VTO	2	1	4	7.345236e-13	21	2.7983e+00

Table 2: dim = 2, case = 1

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	2	2	1	6.054535e-11	9	9.3746e-01
ChF Viscous Op	2	2	2	6.054535e-11	9	5.5041e-01
ChF Viscous Op	2	2	4	6.054535e-11	9	3.8586e-01
Proto Fast VTO	2	2	1	7.009948e-13	21	8.4273e+00
Proto Fast VTO	2	2	2	7.009948e-13	21	5.0224e+00
Proto Fast VTO	2	2	4	7.009948e-13	21	2.9132e+00

Table 3: dim = 2, case = 2

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	2	3	1	4.163159e-11	21	4.5645e-01
ChF Viscous Op	2	3	2	4.163070e-11	21	2.9438e-01
ChF Viscous Op	2	3	4	4.163070e-11	21	2.6304e-01
Proto Slow VTO	2	3	1	4.942713e-13	13	9.9005e-01
Proto Slow VTO	2	3	2	4.942713e-13	13	6.5703e-01
Proto Slow VTO	2	3	4	4.942713e-13	13	4.7014e-01
Proto Fast VTO	2	3	1	5.233591e-13	20	1.3334e+00
Proto Fast VTO	2	3	2	5.233591e-13	20	8.3537e-01
Proto Fast VTO	2	3	4	5.233591e-13	20	5.5627e-01

Table 4: dim = 2, case = 3

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	2	4	1	9.378631e-11	21	4.1673e-01
ChF Viscous Op	2	4	2	9.379342e-11	21	2.8667e-01
ChF Viscous Op	2	4	4	9.379342e-11	21	2.5349e-01
Proto Slow VTO	2	4	1	6.419310e-13	13	1.0299e+00
Proto Slow VTO	2	4	2	6.419310e-13	13	7.0574e-01
Proto Slow VTO	2	4	4	6.419310e-13	13	5.2431e-01
Proto Fast VTO	2	4	1	5.297984e-13	20	1.4037e+00
Proto Fast VTO	2	4	2	7.334133e-13	20	8.9978e-01
Proto Fast VTO	2	4	4	7.334133e-13	20	6.4096e-01

Table 5: dim = 2, case = 4

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	3	0	1	9.853118e-11	12	3.1528e+02
ChF Viscous Op	3	0	2	9.853340e-11	12	1.7701e+02
ChF Viscous Op	3	0	4	9.853207e-11	12	7.9786e+01
ChF Viscous Op	3	0	8	9.853163e-11	12	4.5044e+01
Proto Slow VTO	3	0	1	9.592327e-13	15	3.4601e+02
Proto Slow VTO	3	0	2	9.592327e-13	15	1.9920e+02
Proto Slow VTO	3	0	4	9.592327e-13	15	1.1311e+02
Proto Slow VTO	3	0	8	9.592327e-13	15	6.0518e+01
Proto Fast VTO	3	0	1	9.388046e-13	16	3.0504e+02
Proto Fast VTO	3	0	2	9.063861e-13	16	1.6882e+02
Proto Fast VTO	3	0	4	9.727774e-13	16	8.9513e+01
Proto Fast VTO	3	0	8	9.388046e-13	16	4.8091e+01

Table 6: dim = 3, case = 0

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	3	1	1	9.880896e-11	14	3.1190e+02
ChF Viscous Op	3	1	2	9.880985e-11	14	2.4343e+02
ChF Viscous Op	3	1	4	9.880896e-11	14	9.8596e+01
ChF Viscous Op	3	1	8	9.880874e-11	14	5.4087e+01
Proto Slow VTO	3	1	1	9.011680e-13	15	3.8495e+02
Proto Slow VTO	3	1	2	9.011680e-13	15	2.1375e+02
Proto Slow VTO	3	1	4	9.011680e-13	15	1.1683e+02
Proto Slow VTO	3	1	8	9.011680e-13	15	6.5328e+01
Proto Fast VTO	3	1	1	8.935075e-13	22	4.3738e+02
Proto Fast VTO	3	1	2	8.907319e-13	22	2.4601e+02
Proto Fast VTO	3	1	4	8.940626e-13	22	1.2037e+02
Proto Fast VTO	3	1	8	9.184875e-13	22	6.7538e+01

Table 7: dim = 3, case = 1

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	3	2	1	1.807132e-11	15	4.3311e+02
ChF Viscous Op	3	2	2	1.807132e-11	15	2.8374e+02
ChF Viscous Op	3	2	4	1.807132e-11	15	1.1176e+02
ChF Viscous Op	3	2	8	1.807132e-11	15	6.9411e+01
Proto Slow VTO	3	2	1	9.403589e-13	15	2.1097e+02
Proto Slow VTO	3	2	2	9.403589e-13	15	1.2835e+02
Proto Slow VTO	3	2	4	9.403589e-13	15	5.7464e+01
Proto Slow VTO	3	2	8	9.403589e-13	15	3.2924e+01
Proto Fast VTO	3	2	1	9.003909e-13	22	4.3444e+02
Proto Fast VTO	3	2	2	8.890666e-13	22	2.4232e+02
Proto Fast VTO	3	2	4	9.006129e-13	22	1.2046e+02
Proto Fast VTO	3	2	8	9.131584e-13	22	6.8197e+01

Table 8: dim = 3, case = 2

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	3	3	1	7.822121e-11	98	1.0312e+02
ChF Viscous Op	3	3	2	7.821943e-11	98	6.7036e+01
ChF Viscous Op	3	3	4	7.822099e-11	98	2.9249e+01
ChF Viscous Op	3	3	8	7.822099e-11	98	1.7595e+01
Proto Slow VTO	3	3	1	4.052314e-13	14	3.7960e+01
Proto Slow VTO	3	3	2	4.052314e-13	14	2.3696e+01
Proto Slow VTO	3	3	4	4.052314e-13	14	1.2425e+01
Proto Slow VTO	3	3	8	4.052314e-13	14	7.4764e+00
Proto Fast VTO	3	3	1	8.129053e-13	21	5.0020e+01
Proto Fast VTO	3	3	2	8.129053e-13	21	3.0519e+01
Proto Fast VTO	3	3	4	8.129053e-13	21	1.6132e+01
Proto Fast VTO	3	3	8	8.129053e-13	21	9.5662e+00

Table 9: dim = 3, case = 3

Op	D	Case	N_p	Final $\|R\|$	Iter	Main Time
ChF Viscous Op	3	4	1	9.568346e-11	98	1.0456e+02
ChF Viscous Op	3	4	2	9.568613e-11	98	6.7511e+01
ChF Viscous Op	3	4	4	9.568368e-11	98	3.0511e+01
ChF Viscous Op	3	4	8	9.568368e-11	98	1.9083e+01
Proto Slow VTO	3	4	1	5.471179e-13	14	4.1786e+01
Proto Slow VTO	3	4	2	5.471179e-13	14	2.4630e+01
Proto Slow VTO	3	4	4	4.778400e-13	14	1.3277e+01
Proto Slow VTO	3	4	8	4.778400e-13	14	8.3531e+00
Proto Fast VTO	3	4	1	8.057999e-13	21	5.1915e+01
Proto Fast VTO	3	4	2	8.057999e-13	21	3.1450e+01
Proto Fast VTO	3	4	4	8.057999e-13	21	1.7320e+01
Proto Fast VTO	3	4	8	8.057999e-13	21	1.0817e+01

Table 10: dim = 3, case = 4