Mobile Robotics

Enna Sachdeva

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1 Bernstein basis polynomial method to determine the trajectory of the robot

Let $f(x)$ be a real-valued function defined and bounded on the interval [0,1], then $B_n(f)$ is the polynomial on [0,1].

$$\mathbf{B_n(f(x);t) = \sum_{i=1}^{n} f(\frac{i}{n})^n C_i t^i (1-t)^{n-i}} \tag{1}$$

If function f(x) is continuous on [0,1],then the Bernstein polynomial $B_n(f(x))$ tends uniformly to f as $n \to \infty$. For the given initial(start) t_0 and final(end) time t_f of the trajectory of a nonholonomic systems, the functions x and $\tan\theta$ can be approximated as a linear combination of Bernstein basis polynomials.

$$x(t) \approx B_n(x(t)) = B_x(\mu(t)) = \sum_{i=1}^{5} W_{x_i} B_i(\mu(t)), \tag{2}$$

Similarly,

$$\tan\theta(t) = k(t) \approx B_n(k(t)) = B_k(\mu(t)) = \sum_{i=1}^{5} W_{k_i} B_i(\mu(t)) \tag{3}$$

where,
$B_i(\mu(t)) =^n C_i (1-\mu)^i (\mu)^{n-i}$
$\mu(t) = \frac{t-t_0}{t_f-t_0}$
Differentiating equation 5 w.r.t time gives

$$\dot{x}(t) = \dot{B}_x(\mu(t)) = \sum_{i=1}^{5} W_{x_i} \dot{B}_i(\mu(t) \tag{4}$$

using above 2 equations, equation 2 can be rewritten as-

$$y = y_0 + \int_{t_0}^{t_f} (\sum_{i=1}^{5} W_{x_i} \dot{B}_i(\mu(t))(\sum_{i=1}^{5} W_{k_i} B_i(\mu(t)) \tag{5}$$

2 Assignment:

As per the Assignment question, we have used position and velocity constraints of the state of a non-holonomic robot, in a 2D environment, at time $t_0 = 0$,$t_c = 5$ and $t_f = 10$ seconds to determine the weights W_{X_i} for i=0...5. To put strict constraints on the initial and the final positions of the robot, the optimization has been applied only to solve for weights W_{X_1} to W_{X_4} while keeping the weight parameters W_{X_0} and W_{X_5} fixed as X_0 and X_f, respectively. This is achieved by solving equations corresponding to X_0 and X_f, separately using substitution method, rather than solving along with other constraints (as $A = BW$).

3 Plots and Analysis:

This sections shows the plots obtained after applying bernstein basis polynomial method to determine the trajectory of the robot, with the given constraints (as discussed in the previous section) for under, over and critically constrained systems, along with the analysis and observations.
Since, there were only 6 constraints on X and $\dot{X}$ in total, *Underconstrained* and *Critically constrained* situations were formulated by changing the number of constraint equations on X and $\dot{X}$ while keeping the no of constraints for determining the values of $W_{k_i}s$ exactly equal to 6(critically constrained). For *Over constrained* formulation, the number of constrained used for solving W_{k_0} to W_{k_5} was 7 in total.($K_0 = 0, K_f = 0, \dot{K}_0 = 1, \dot{K}_f = 1, Y_{t_0} = 10, Y_{t_c} = 25$ and $Y_{t_f} = 50$).

3.1 Under Constrained cases

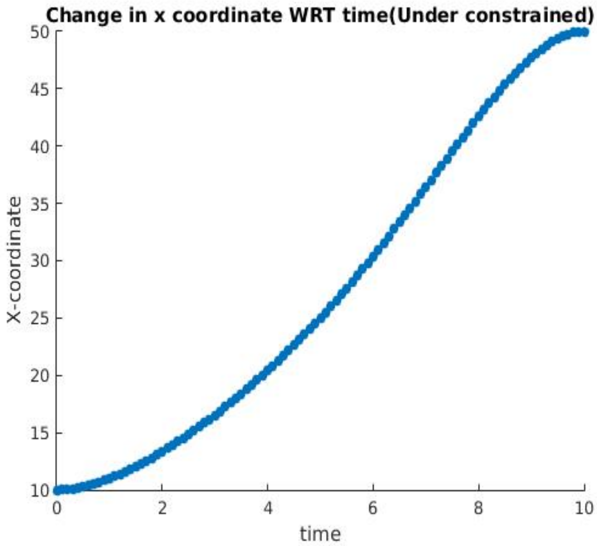


Figure 1:

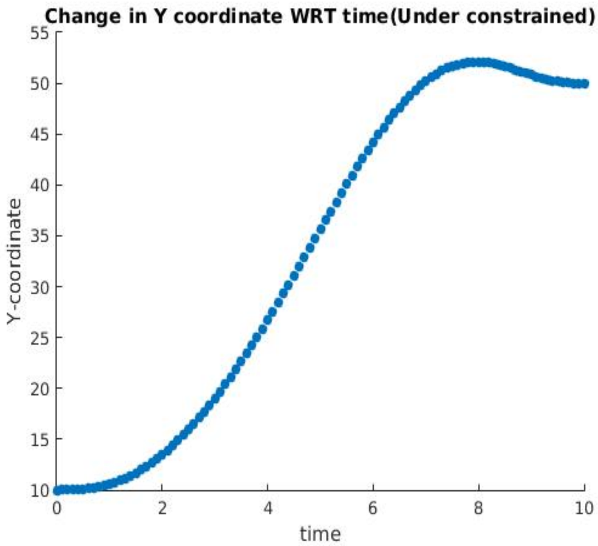


Figure 2:

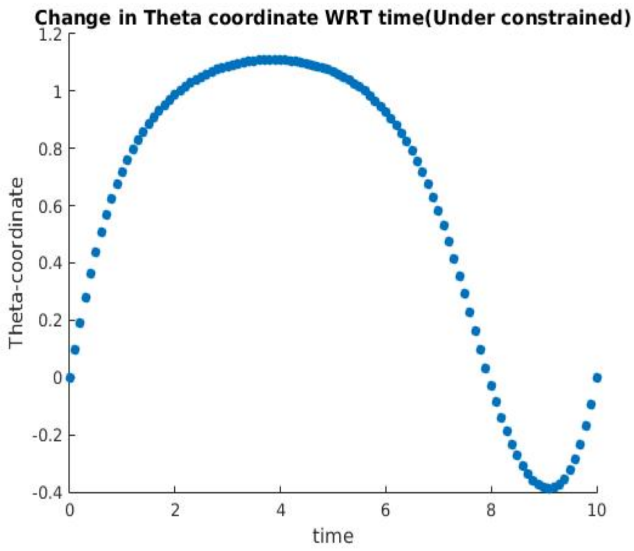


Figure 3:

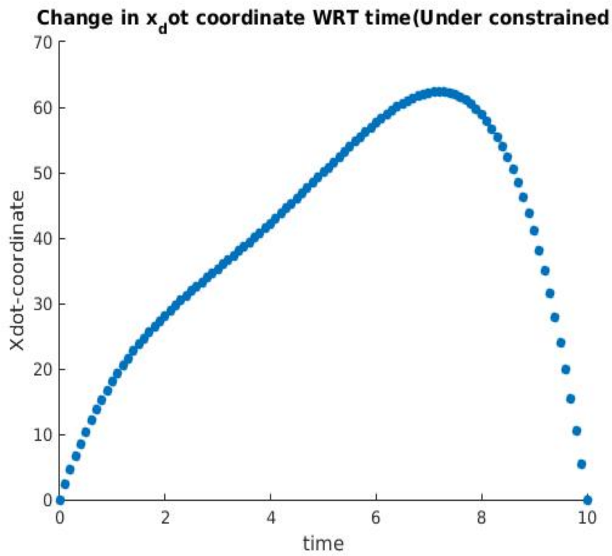


Figure 4:

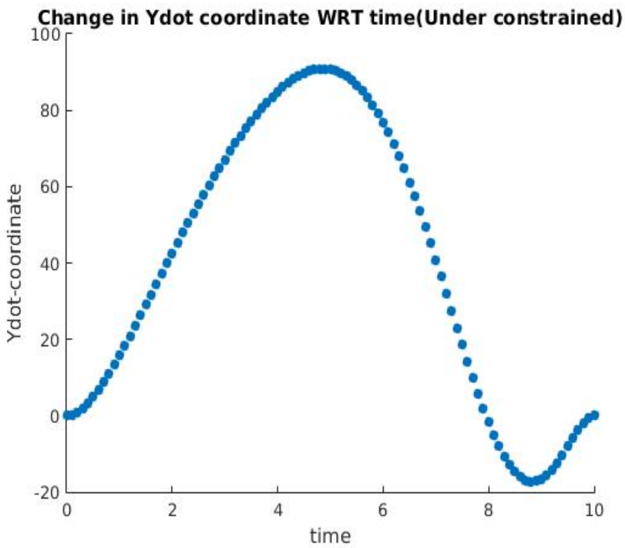


Figure 5:

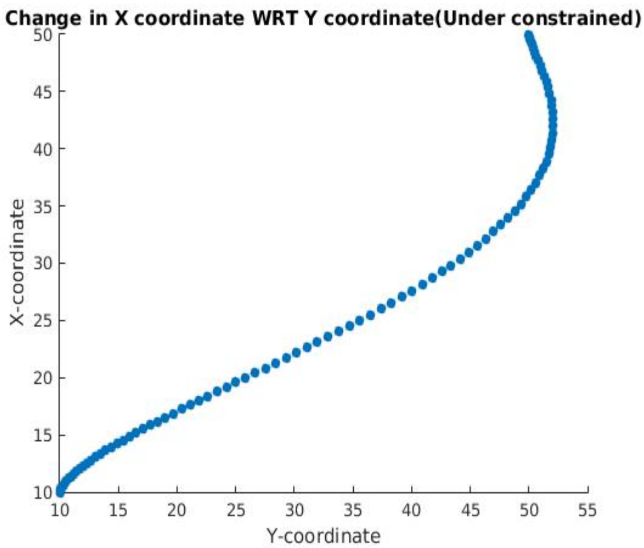
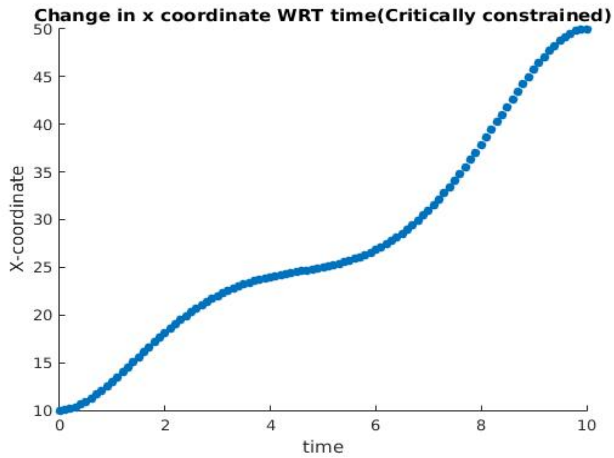


Figure 6:

Considering only initial, final position and velocity and mid points'(X_{t_c}, Y_{t_c}) position as constraints (the velocity constraint in the middle$(\dot{X_{t_c}}, \dot{Y_{t_c}})$ are not being considered), the plots of x,y and θ w.r.t are shown in $Figures$ 1,2,3, respectively. From $Figures$ 4and 5, it can be seen that velocity at 5th second is not zero(since we have not included constraint $((\dot{X}_{t_c}), \dot{Y}_{t_c} = (0,0))$in the under constraint case). Also, ignoring this constraint reflects the change in the $XvsY$ trajectory when we compare it with critically constrained and over constrained cases.

3.2 Critically Constrained cases



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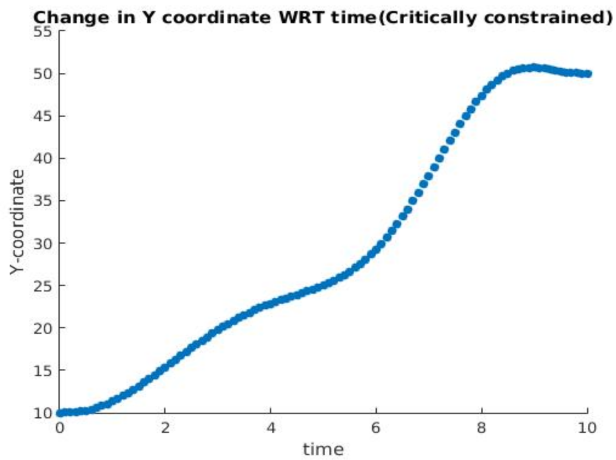
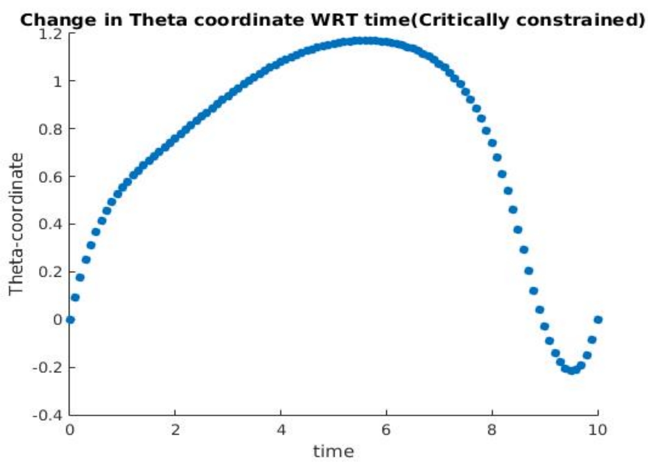


Figure 8:



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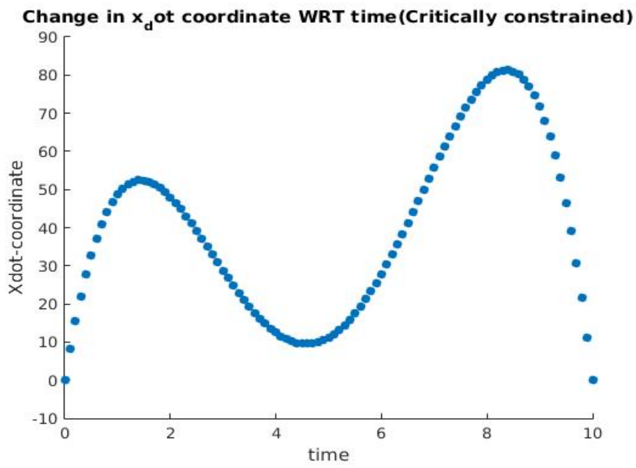

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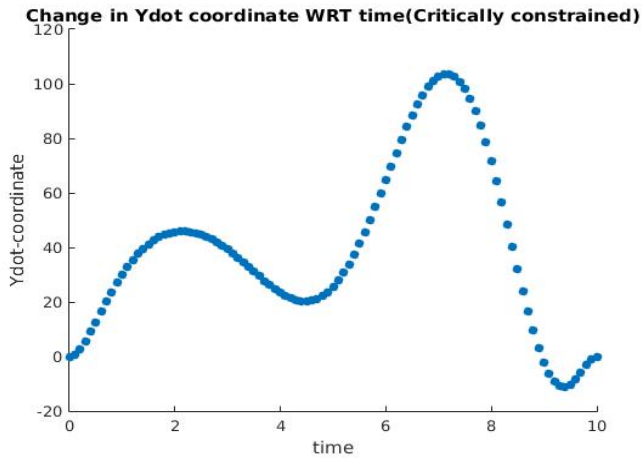

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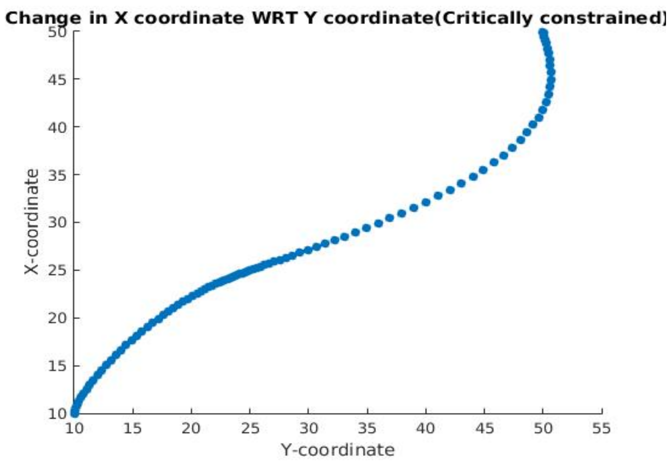


Figure 12:

The effect of adding another constraint (making the matrix full rank) can be seen in the trajectory of $Figure$ 12 . The effect of adding the velocity constraint (only in $\dot{X_{t_c}}$) at X_{t_c} can be seen in the velocity profiles of both x and y in $Figures$ 10 and 11, respectively. Since the constraint didn't consider any y component of the velocity at t_c, we can see that although x-component almost touches zero at t_c, y component fails to do so.

3.3 Over Constrained cases

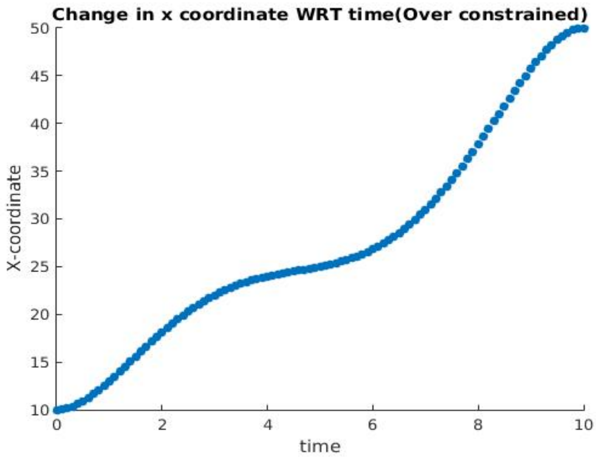


Figure 13:

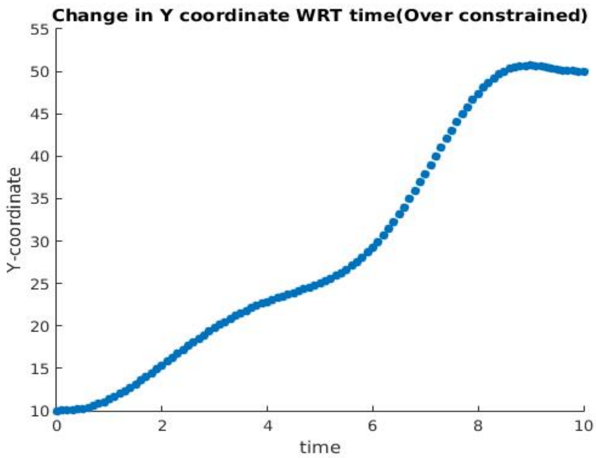
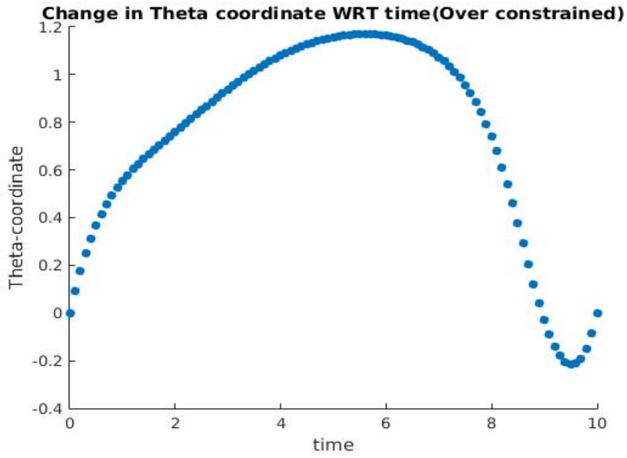


Figure 14:



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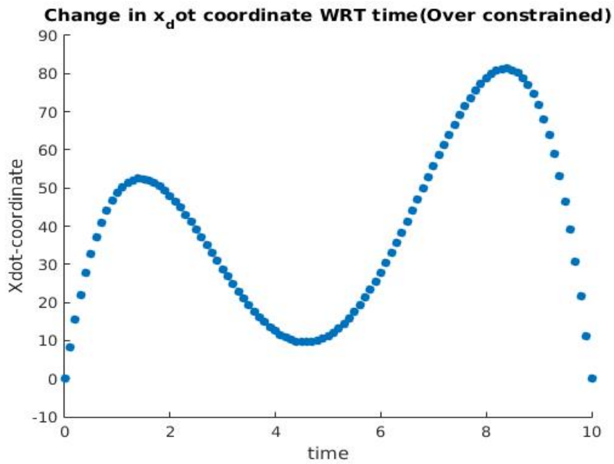

Figure 16:

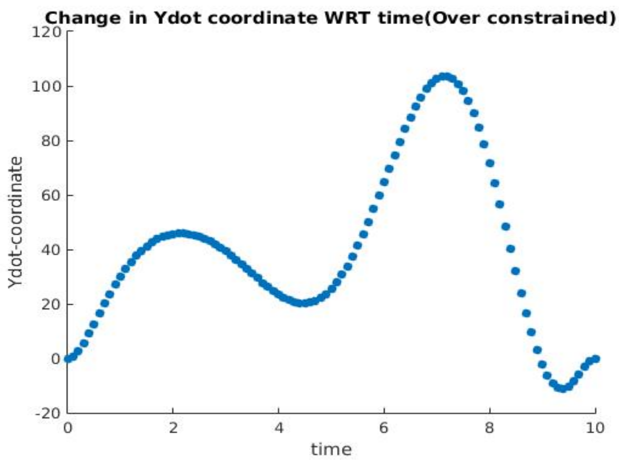

Figure 17:

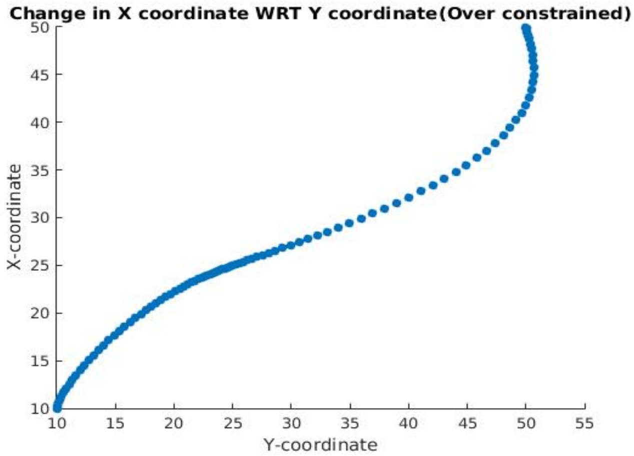

Figure 18:

From the graphs obtained , there is not much visible difference between critically constrained and over constrained cases.