· U C ·

Complex Systems

2020/2021

Exercise Sheet 3

Ernesto Costa
Nuno Lourenço

3

Chaos

3.1 Exercises

Exercise 3.1 **E**

Consider the logistic equation, $x_{n+1} = rx_n(1 - x_n)$. For each of the r values listed, plot the final-state diagram. Use the simcx framework to plot this type of diagram (*FinalStateDigram* class). Before creating the final-state diagram, try to determine from the orbit what the final states will be. Assume that $x_0 = 0.9$.

1. $r = 0.5$
2. $r = 1.5$
3. $r = 2.8$
4. $r = 3.3$
5. $r = 3.5$
6. $r = 3.56$
7. $r = 3.835$
8. $r = 4.0$

■

Exercise 3.2 **M**

Considering the logistic equation with $r = 4.0$.

1. Plot the orbit for the first thirty iterates with $x_0 = 0.1$.
2. Plot the orbit for the first thirty iterates with $x_0 = 0.11$.
3. Do the two orbits differ significantly? If so, at what iterate does the difference become noticeable?
4. Do the same for $x_0 = 0.1001$.

■

Exercise 3.3 **M**

Using the simcx framework, implement a program that, for a given function, and two different seeds, plots the difference of the orbits between these two seeds.

■

Exercise 3.4 **M**

Using the class implemented in the previous exercise, test the logistic equation using different values of r, and determine if they are Sensitively Dependent on Initial Conditions (SDIC).

■

Exercise 3.5 **M**

Using the *BifurcationDiagram* visual and *FinalStateIterator* simulator from SimCX, find r values that yield orbits with the following properties. Once you found the r value, check that it is behaving as you expected by ploting its orbit.

1. Period 4
2. Period 6 (Hint: Look near period 3.)
3. Chaotic behaviour for some r different than 4.0.
4. Period 5 (Hint: Look between 3.7 and 3.8.)
5. Periodic behaviour of some other period that is not a multiple of 2.

■

Exercise 3.6 **M**

Using the tools already at hand, analyse the following functions for chaotic behaviour:

1. $f(x) = rx^2(1-x)$
2. $f(x) = r\sin(\frac{\pi x}{2})$

■

Exercise 3.7 **M**

Using the bifurcation diagram for the logistic equation, visually determine the δ_n for the period doubling region near $r = 3.83$.

■

Further Reading

✔ David P. Feldman, *Chaos and Fractals – An Elementary Introduction.*