· U  C ·

Complex Systems

2020/2021

Exercise Sheet 5

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Cellular Automata

9.1 Exercises

Exercise 9.1 E

Consider Wolfram's elementary cellular automata. An example of the first 15 iterations of rule 30, when starting with a single cell set at 1 in the middle, is shown in figure 9.1. An example implementation is provided in the SimCX framework. Test and inspect the code of that implementation. Use this implementation to complete the next exercises.

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Exercise 9.2 M

Find rules of elementary cellular automata that belong to each of the four classes defined by Wolfram (fixed-point, periodic, chaotic, complex).

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Exercise 9.3 M

Consider rule 184 elementary CA. Run several tests with different starting conditions. What type of system do you believe can be simulated by this rule? Hint: Try different densities of 1's in the starting condition.

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Exercise 9.4 M

Using the example implementation as a base, create a program that allows the visualization of every one of the 256 rules in the same screen.

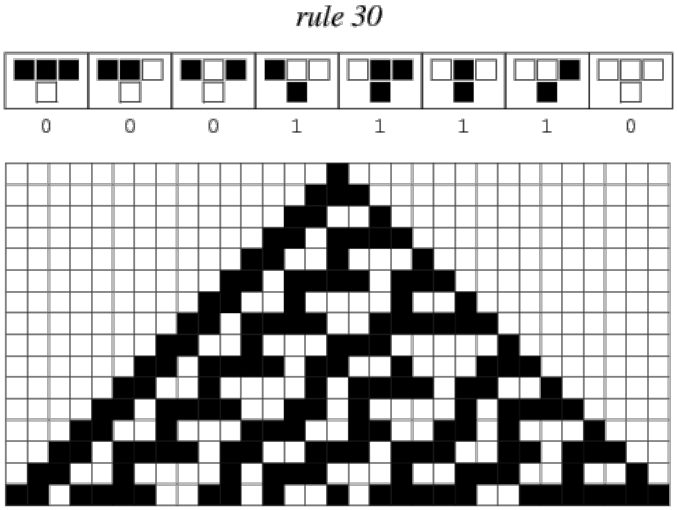

Figure 9.1: The first 15 iterations of Rule 30. (Weisstein, Eric W. "Rule 30." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/Rule30.html

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Exercise 9.5 E

Using the example implementation as a base, create a program that allows the visualization of a given set of different initialization conditions for a given rule.

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Exercise 9.6 H

Consider rule 30 elementary CA. Implement a pseudo random number generator using this CA. Try different generators using the CA. Do a simple test of the generator by plotting the frequency of each generated value. Note that this simple test is certainly not enough to avail the actual randomness of the generator.

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Exercise 9.7 E

Using the example of the Game of Life provided in the SimCX framework, find and test different patterns that exhibit interesting behaviour. For

example, the glider (a pattern that moves on the grid) is already provided in the example.

Exercise 9.8 M

Implement a probabilistic 2D CA. Use the provided example as a base. Test it by implementing the Gossip Model.

Exercise 9.9 M

Implement a historical 2D CA. Use the provided Game of Life example as a base. Experiment with using different colors to visualize each cell based on its history.

Exercise 9.10 H

Consider a 2D CA with the following configuration. Each cell has an integer state. Whenever one cell's value is equal or greater than 4, its value is decreased by 4, and each of its four neighbours' (von-neumann neighbourhood) values are increased by 1. Implement such a CA and test its behaviour.

Exercise 9.11 VH

A feature commonly found in complex systems is that of nesting or layering. That is, a set of components of a system interact in such a way as to form higher level structures, which in turn, interact to also form other higher level structures. Implement and test a CA where each cell is itself a CA.

Further Reading

- ✔ Stephen Wolfram, *A New Kind of Science.*
- ✔ David P. Feldman, *Chaos and Fractals – An Elementary Introduction.*
- ✔ Random.org, *Statistical Analysis.* (https://www.random.org/analysis/)