ASP Final Project

r11942131 馬振洋

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1 The details of these beamformers

Assume uniform linear array (ULA) with N isotropic antenna and inter-element spacing $\frac{2}{\lambda}$.

1.1 The beamformer with uniform weights

1.1.1 Uniform weightings

$$\mathrm{w} = \frac{1}{N}$$

1.1.2 Beamformer output

$$y(t) = \mathrm{w}^H \mathrm{x}(t) = \mathrm{w}^H(\mathrm{a}(\theta)s_1(t) + \mathrm{n}(t)) = (\mathrm{w}^H \mathrm{a}(\theta))s_1(t) + (\mathrm{w}^H \mathrm{n}(t))$$

1.1.3 Beampattern

$$B_\theta(\theta) = \mathrm{w}^H \mathrm{a}(\theta) = e^{j\frac{N-1}{2}\pi \sin\theta} \times \frac{1}{N} \times \frac{\sin(\frac{N}{2}\pi\sin\theta)}{\sin(\frac{1}{2}\pi\sin\theta)}$$

Uniform weighting and DOA $\theta_1 = 0$: N-times SNR enhancement.
Uniform weighting and DOA $\theta_1 = \theta$: SNR depending on $B_\theta(\theta)$.
It can be shown that the maximum of $|B_\theta(\theta)|$ occurs at $\theta = 0°$.

1.2 The beamformer with array steering

1.2.1 Weight vector

$$\mathrm{w} = \frac{1}{N}\mathrm{a}(\theta_s)$$

1.2.2 Beampattern

$$B_\theta(\theta) = \mathrm{w}^H \mathrm{a}(\theta) = e^{j\frac{N-1}{2}(\pi\sin\theta - \pi\sin\theta_s)} \times \frac{1}{N} \times \frac{\sin[\frac{N}{2}(\pi\sin\theta - \pi\sin\theta_s)]}{\sin[\frac{1}{2}(\pi\sin\theta - \pi\sin\theta_s)]}$$

It can be shown that the maximum of $|B_\theta(\theta)|$ occurs at $\theta = \theta_s$.

1.3 The MVDR beamformer

1.3.1 Weight vector (for the minimum variance distortionless response beamformer is the solution to the following optimization problem)

$$\mathrm{w}_{MVDR} = arg\ \underset{w}{min}\, \mathbb{E}[|y(t)|^2] = \mathrm{w}^H \mathrm{R}_x \mathrm{w}$$

$$\text{subject to}$$
$$\mathrm{w}^H \mathrm{a}(\theta_s) = 1,\ y(t) = \mathrm{w}^H \mathrm{x}(t).$$

1.3.2 MVDR beamformer weight vector

$$\mathrm{w}_{MVDR} = \frac{\mathrm{R}^{-1}\mathrm{a}(\theta_s)}{\mathrm{a}^H(\theta_s)\mathrm{R}^{-1}\mathrm{a}(\theta_s)}$$

We assume that x(t) is WSS. The correlation matrix $\mathrm{R} \triangleq \mathbb{E}[\mathrm{x}(t)\mathrm{x}^H(t)]$.

1.4 The LCMV beamformer

1.4.1 Weight vector (for the linearly constrained minimum variance beamformer is the solution to the following optimization problem)

$$\mathrm{w}_{LCMV} = arg\ \underset{w}{min}\, \mathbb{E}[|y(t)|^2] = \mathrm{w}^H \mathrm{R}_x \mathrm{w}$$

$$\text{subject to}$$
$$\mathrm{C}^H \mathrm{w} = \mathrm{g},\ y(t) = \mathrm{w}^H \mathrm{x}(t).$$

If $\mathrm{C} = \mathrm{a}(\theta_s)$ and $\mathrm{g} = 1$, then LCMV is equivalent to MVDR.

1.4.2 LCMV beamformer weight vector

$$\mathrm{w}_{LCMV} = \mathrm{R}^{-1}\mathrm{C}(\mathrm{C}^H\mathrm{R}^{-1}\mathrm{C})^{-1}\mathrm{g},$$

where the correlation matrix $\mathrm{R} \triangleq \mathbb{E}[\mathrm{x}(t)\mathrm{x}^H(t)]$.

Assume the matrix inverses exit.

2 Design an algorithm to denoise $\tilde{\theta}_s(t)$ and $\tilde{\theta}_i(t)$. The denoised results are denoted by $\hat{\theta}_s(t)$ and $\hat{\theta}_i(t)$.

2.1 Describe the details of my algorithm using mathematical equations.

Step 1: Refer to RLS algorithm, I design a gain vector $\mathbf{k}(t)$ update with a designed update vector $\mathbf{P}(t)$.

$$\mathrm{k}(t) = \frac{\lambda^{-1}\mathrm{P}(t-1)}{1+\lambda^{-1}\mathrm{P}(t-1)}$$

$$\mathrm{P}(t) = (\lambda^{-1}\mathrm{P}(t-1) - \lambda^{-1}\mathrm{k}(t)\mathrm{P}(t-1))/2 + \mathrm{P}(t-1)\mathrm{k}(t)$$

Step 2: Find the local maximum and local minimum through the noisy DOAs.

Using conditions to find the local maximum and local minimum.

Set local maximum as $\tilde{\theta}_{peak}$, local minimum as $\tilde{\theta}_{dip}$.

For local maximum's condition: $\tilde{\theta}(t) > \tilde{\theta}(t-1)$ && $\tilde{\theta}(t) > \tilde{\theta}(t+1)$

For local minimum's condition: $\tilde{\theta}(t) < \tilde{\theta}(t-1)$ && $\tilde{\theta}(t) < \tilde{\theta}(t+1)$

Step 3: Choose the local maximum and local minimum from step 1 and step 2.

From the conditions, choose $\tilde{\theta}_{peak}$, $\tilde{\theta}_{dip}$ equals to the noisy DOAs, and add a gain vector to revise $\tilde{\theta}_{peak}$, $\tilde{\theta}_{dip}$.

For local maximum's condition: $\tilde{\theta}_{peak} = \tilde{\theta}(t)/(\mathrm{k}^2(t))$

For local minimum's condition: $\tilde{\theta}_{dip} = \tilde{\theta}(t) * (\mathrm{k}^2(t))$

Set first and last of local maximums and local minimums:

$$\tilde{\theta}_{peak} = \tilde{\theta}(t),\ \tilde{\theta}_{dip} = \tilde{\theta}(t)$$

Step 4: Choose the denoised DOAs from step 3.

Set all of local maximums as $\tilde{\theta}_{all\ peaks}$, all of local minimums as $\tilde{\theta}_{all\ dips}$.

Using spline function to construct $\tilde{\theta}_{all\ peaks}$ through $\tilde{\theta}_{peak}$.

Using spline function to construct $\tilde{\theta}_{all\ dips}$ through $\tilde{\theta}_{dip}$.

Set denoised DOAs $\hat{\theta}(t) = (\tilde{\theta}_{all\ peaks} + \tilde{\theta}_{all\ dips})/2$

2.2 Summarize my algorithm similar to the format of that in Page 27 of 10_LS_RLS.pdf.

My algorithm:Summary

Algorithm 1 Denoised algorithm of DOAs

Require:The noisy DOAs. The parameters $0 < \lambda \leq 1$, time = L, and $\delta > 0$.

q=linspace(0,L,L);
P(0) = δ^{-1}*ones(L);
for all t = 1,2,3,...L **do**
  $\mathrm{k}(t) = \frac{\lambda^{-1}\mathrm{P}(t-1)}{1+\lambda^{-1}\mathrm{P}(t-1)}$;
  if $t = 1$ **then**
    t_{peak} = q(t);
    $\tilde{\theta}_{peak} = \tilde{\theta}(t)$;
    t_{dip} = q(t);
    $\tilde{\theta}_{dip} = \tilde{\theta}(t)$;
  end if
  if $t = L$ **then**
    t_{peak} = q(t);
    $\tilde{\theta}_{peak} = \tilde{\theta}(t)$;
    t_{dip} = q(t);
    $\tilde{\theta}_{dip} = \tilde{\theta}(t)$;
  end if
  if $t \neq 1$ && $t \neq L$ **then**
    if $\tilde{\theta}(t) > \tilde{\theta}(t-1)$ && $\tilde{\theta}(t) > \tilde{\theta}(t+1)$ **then**
      t_{peak} = q(t);
      $\tilde{\theta}_{peak} = \tilde{\theta}(t)/(\mathrm{k}^2(t))$;
    end if
    if $\tilde{\theta}(t) < \tilde{\theta}(t-1)$ && $\tilde{\theta}(t) < \tilde{\theta}(t+1)$ **then**
      t_{dip} = q(t);
      $\tilde{\theta}_{dip} = \tilde{\theta}(t) * (\mathrm{k}^2(t))$;
    end if
  end if
  $\mathrm{P}(t) = (\lambda^{-1}\mathrm{P}(t-1) - \lambda^{-1}\mathrm{k}(t)\mathrm{P}(t-1))/2 + \mathrm{P}(t-1)\mathrm{k}(t)$;
end for
$\tilde{\theta}_{all\ peaks}$ = spline(t_{peak}, $\tilde{\theta}_{peak}$, q);
$\tilde{\theta}_{all\ dips}$ = spline(t_{dip}, $\tilde{\theta}_{dip}$, q);
$\hat{\theta}(t) = (\tilde{\theta}_{all\ peaks} + \tilde{\theta}_{all\ dips})/2$;

2.3 What are the advantages of my algorithm?

My algorithm advantages :

(1) It can let the over exceeding value of the noisy DOAs to be found.

(2) It can make the over exceeding value of the noisy DOAs to be less suppressed through the gain vector.

$$\text{if } \tilde{\theta}(t) > \tilde{\theta}(t-1) \ \&\& \ \tilde{\theta}(t) > \tilde{\theta}(t+1), \text{ then } \mathrm{t}_{peak} = \mathrm{q(t)}, \ \tilde{\theta}_{peak} = \tilde{\theta}(t)/(\mathrm{k}^2(t)).$$

$$\text{if } \tilde{\theta}(t) < \tilde{\theta}(t-1) \ \&\& \ \tilde{\theta}(t) < \tilde{\theta}(t+1), \text{ then } \mathrm{t}_{dip} = \mathrm{q(t)}, \ \tilde{\theta}_{dip} = \tilde{\theta}(t) * (\mathrm{k}^2(t)).$$

(3) It can let the finding value be constructed that will not having too much distortion through the spline function.

$$\tilde{\theta}_{all\ peaks} = \mathrm{spline}(\mathrm{t}_{peak}, \tilde{\theta}_{peak}, \mathrm{q})$$

$$\tilde{\theta}_{all\ dips} = \mathrm{spline}(\mathrm{t}_{dip}, \tilde{\theta}_{dip}, \mathrm{q})$$

3 Plot the estimated DOAs $\hat{\theta}_s(t)$ and $\hat{\theta}_i(t)$ over the time index t, using my estimator in Item 2.

3.1 Please create only one plot in this figure. There are two curves for $\hat{\theta}_s(t)$ and $\hat{\theta}_i(t)$ with proper legends.

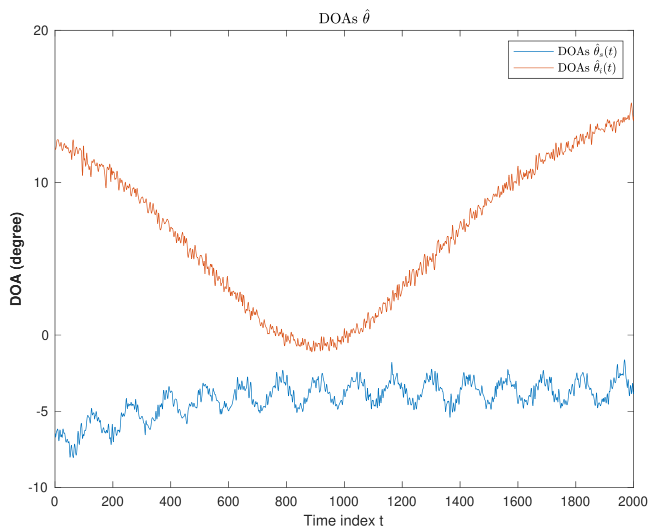

Figure 1: Estimated DOAs $\hat{\theta}$

3.2 Save your figure to ASP_Final_DOA.fig.

4 Design a beamformer that extracts the source signal in (1). Beamformer should be different from those in Item 1. Two beamformers are different if the weight vectors are different.

4.1 Elaborate on the details of my beamformer by using mathematical equations.

Step 1: Refer to LCMV beamformer, I design a sample correlation matrix $\hat{\mathrm{R}}$ that will change the weight vectors to calculate the estimated source signal $\hat{s}(t)$ to construct my designed beamformer.

$$\text{Designed } \hat{\mathrm{R}} = \frac{1}{N}\sum_{t=1}^{L} \frac{\tilde{\mathrm{x}}(t)\tilde{\mathrm{x}}^H(t)}{(\sqrt{\sum_{t=1}^{L}\tilde{\mathrm{x}}(t)^2}\sqrt{\sum_{t=1}^{L}\tilde{\mathrm{x}}^H(t)^2})^2} + \delta * \lambda^L \mathrm{I}$$

Step 2: Calculate the steering vectors through the denoised DOAs.

Set designed source DOA as $a(\theta_s)$, local minimum as $a(\theta_i)$.

$$\text{steering vector } a(\theta) = [1 \;\; e^(j\pi\sin\theta) \;\; ... \;\; e^(j(N-1)\pi\sin\theta)]^T$$

Step 3: Choose the constraint matrix C and constraint vector g from step 2.

$$\text{constraint matrix C} = [a(\theta_s) \;\; a(\theta_i)]$$

$$\text{Set constraint vector g} = [1 \;\; 0]^T$$

Step 4: Calculate the weight from step 1 to step 3.

$$\text{weight } \mathrm{w}_{designed} \ = \ \hat{\mathrm{R}}^{-1}\mathrm{C}(\mathrm{C}^{H}\hat{\mathrm{R}}^{-1}\mathrm{C})^{-1}\mathrm{g}$$

Step 5: Calculate the estimated source signal $\hat{s}(t)$ from step 1 to step 4.

The estimated source signal is as closed as the output of the beamformer.

$$\text{the estimated source signal } \hat{s}(t) \ = \ \mathrm{w}^{H}_{designed}\mathrm{x}(t)$$

4.2 Summarize my beamformer similar to the format of that in Page 27 of 10 LS RLS.pdf.

My designed beamformer:Summary

Algorithm 2 Designed beamformer (for changing weight by designing sample correlation matrix)

Require:The designed DOAs. The parameters $0 < \lambda \leq 1$, time = L, signal's quantities = N, noisy array measurement = x, and $\delta > 0$.

$\mathrm{g} = [1 \;\; 0]^T$;
$\hat{\mathrm{R}}$=zeros(N);
for all t = 1,2,3,...L **do**
    $\hat{\mathrm{R}} = \hat{\mathrm{R}} + \frac{1}{N}\sum_{t=1}^{L} \frac{\tilde{\mathrm{x}}(t)\tilde{\mathrm{x}}^H(t)}{(\sqrt{\sum_{t=1}^{L}\tilde{\mathrm{x}}(t)^2}\sqrt{\sum_{t=1}^{L}\tilde{\mathrm{x}}^H(t)^2})^2}$;
end for
$\hat{\mathrm{R}} = \hat{\mathrm{R}} + \delta * \lambda^L \mathrm{I}$;
$a(\theta_s)$ = zeros(N,L);
$a(\theta_i)$ = zeros(N,L);
for all t = 1,2,3...L **do**
    for all n = 1,2,3,...N **do**
        $a(\theta_s)(n,t) = exp(j(N-1)\pi \sin(\hat{\theta}_s(t) * \pi/180))$;
        $a(\theta_i)(n,t) = exp(j(N-1)\pi \sin(\hat{\theta}_i(t) * \pi/180))$;
    end for
    $\mathrm{C} = [a(\theta_s) \;\; a(\theta_i)]$;
end for
$\mathrm{w}_{designed}$ = zeros(N,L);
for all t = 1,2,3,...L **do**
    $\mathrm{w}_{designed} = \hat{\mathrm{R}}^{-1}\mathrm{C}(\mathrm{C}^H\hat{\mathrm{R}}^{-1}\mathrm{C})^{-1}\mathrm{g}$
end for
$\hat{s}(t)$ = zeros(1,L);
for all t = 1,2,3,...L **do**
    $\hat{s}(t) = \mathrm{w}_{designed}^H \mathrm{x}(t)$
end for

4.3 What are the advantages of my beamformer?

My beamformer advantage:

(1) It can make the weight being well through the designed sample correlation matrix $\hat{\mathrm{R}}$.

(2) It can let the bad scale of the recursive implementation of sample correlation matrix $\hat{\mathrm{R}}$ less through the adding of exponentially and diagonally-loaded

$$\hat{\mathrm{R}} = \frac{1}{N}\sum_{t=1}^{L} \frac{\tilde{\mathrm{x}}(t)\tilde{\mathrm{x}}^{H}(t)}{(\sqrt{\sum_{t=1}^{L}\tilde{\mathrm{x}}(t)^{2}}\sqrt{\sum_{t=1}^{L}\tilde{\mathrm{x}}^{H}(t)^{2}})^{2}} + \delta * \lambda^{L}\mathrm{I}$$

5 Plot the real and imaginary parts of estimated source signal $\hat{s}(t)$ over the time index t, using your beamformer in Item 4.

5.1 Please create two subplots in one figure. One subplot for the real part and the other for the imaginary part.

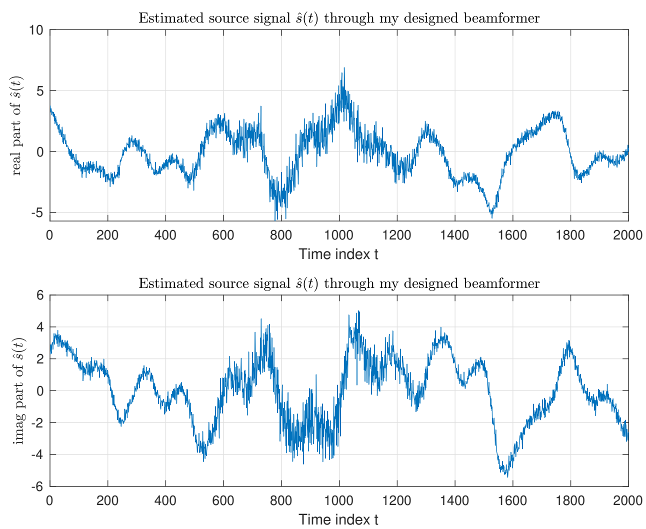

Figure 2: Estimated source signal $\hat{s}(t)$

5.2 Save your figure to ASP_Final_Source.fig.

6 Plot the real and imaginary parts of MVDR, LCMV source signal $\hat{s}(t)$ over the time index t,using MVDR, LCMV beamformer.

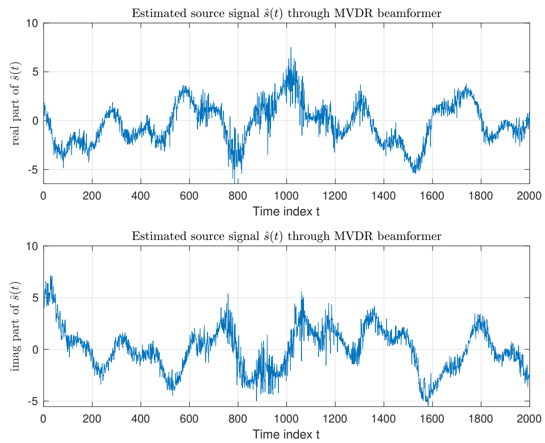

Figure 3: MVDR source signal $\hat{s}(t)$

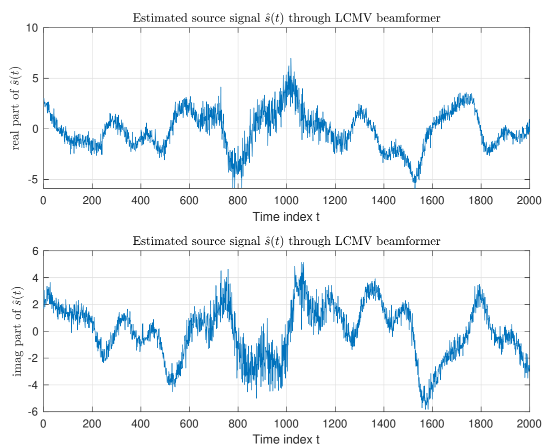

Figure 4: LCMV source signal $\hat{s}(t)$