

Fourth-order Runge-Kutta Method:

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| $\frac{dy'}{dx} = f(x, y, y')$ | $\frac{dy}{dx} = y'$ |
| $y'_{i+1} = y_i + \frac{1}{6}(f_1 + 2f_2 + 2f_3 + f_4)h$ | $y_{i+1} = y_i + \frac{1}{6}(F_1 + 2F_2 + 2F_3 + F_4)h$ |
| $f_1 = f(x_i, y_i, y'_i)$ | $F_1 = y'_i$ |
| $f_2 = f\left(x_i + \frac{1}{2}h, y_i + F_1\frac{1}{2}h, y'_i + f_1\frac{1}{2}h\right)$ | $F_2 = y'_i + f_1\frac{1}{2}h$ |
| $f_3 = f\left(x_i + \frac{1}{2}h, y_i + F_2\frac{1}{2}h, y'_i + f_2\frac{1}{2}h\right)$ | $F_3 = y'_i + f_2\frac{1}{2}h$ |
| $f_4 = f(x_i + h, y_i + F_3h, y'_i + f_3h)$ | $F_4 = y'_i + f_3h$ |
| $y'_{i+1} = y_i + \frac{1}{6}(f_1 + 2f_2 + 2f_3 + f_4)h$ | $y_{i+1} = y_i + y'_ih + \frac{1}{6}\{f_1 + f_2 + f_3\}h^2$ |
| $f_1 = f(x_i, y_i, y'_i)$ | |
| $f_2 = f\left(x_i + \frac{1}{2}h, y_i + y'_i\frac{1}{2}h, y'_i + f_1\frac{1}{2}h\right)$ | |
| $f_3 = f\left(x_i + \frac{1}{2}h, y_i + y'_i\frac{1}{2}h + f_1\frac{1}{4}h^2, y'_i + f_2\frac{1}{2}h\right)$ | |
| $f_4 = f\left(x_i + h, y_i + y'_ih + f_2\frac{1}{2}h, y'_i + f_3h\right)$ | |