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ROTATING BLACK HOLES: LOCALLY NONROTATING FRAMES, ENERGY EXTRACTION, AND SCALAR SYNCHROTRON RADIATION*

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ABSTRACT

This paper outlines and applies a technique for analyzing physical processes around rotating black holes. The technique is based on the orthonormal frames of "locally nonrotating observers." As one application of the technique, it is shown that the extraction of the rotational energy of a black hole, although possible in principle (e.g., the "Penrose-Christodoulou" process), is unlikely in any astrophysically plausible context. As another application, it is shown that, in order to emit "scalar synchrotron radiation," a particle must be highly relativistic as seen in the locally non-rotating frame—and can therefore not move along an astrophysically reasonable orbit. The paper includes a number of useful formulae for particle orbits in the Kerr metric, many of which have not been published previously.

I. INTRODUCTION

Although there is as yet no certain observational identification of a black hole, the study of the properties of black holes and their interactions with surrounding matter is astrophysically important. Black-hole astrophysics is important for the following reasons. (i) At least some stars of mass $\gtrsim 2\ M_{\odot}$ probably fail to shed sufficient matter, when they die, to become white dwarfs or neutron stars, and instead collapse to form black holes. (ii) At least one irregularly pulsating X-ray source, Cygnus X-1, has been identified with a binary system which has a massive, invisible component; this might well be a black hole emitting X-rays as it accretes matter from its companion (for observations, see, e.g., Schreier *et al.* 1971 and Wade and Hjellming 1972). (iii) A black hole of 10^4–$10^8\ M_{\odot}$ might lie at the center of the Galaxy and be responsible for radio and infrared phenomena observed there (Lynden-Bell and Rees 1971). (iv) Gravitational waves seem to have been detected coming from the direction of the galactic center with such intensity (Weber 1971 and references cited therein) that black-hole processes are the least unreasonable source. We are faced with a double mystery: first, puzzling observations; second, a poor theoretical understanding of what processes *might* occur near a black hole. Both sides of the mystery call for further theoretical work.

Most interactions of a black hole with its surroundings can be treated accurately by perturbation techniques, where the dynamics of matter, electromagnetic and gravitational waves takes place in the fixed background geometry generated by the

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In these and all subsequent formulae, the upper sign refers to direct orbits (i.e., corotating with $L > 0$), while the lower sign refers to retrograde orbits (counter-rotating with $L < 0$). For an extreme-rotating black hole, $a = M$, equations (2.12) and (2.13) simplify somewhat,

$$E/\mu = \frac{r \pm M^{1/2}r^{1/2} - M}{r^{3/4}(r^{1/2} \pm 2M^{1/2})^{1/2}}, \quad \text{for } a = M; \tag{2.14}$$

$$L/\mu = \frac{\pm M(r^{3/2} \pm M^{1/2}r + Mr^{1/2} \mp M^{3/2})}{r^{3/4}(r^{1/2} \pm 2M^{1/2})^{1/2}}, \quad \text{for } a = M. \tag{2.15}$$

The coordinate angular velocity of a circular orbit is

$$\Omega \equiv d\varphi/dt = \pm M^{1/2}/(r^{3/2} \pm aM^{1/2}). \tag{2.16}$$

Circular orbits do not exist for all values of r. The denominator of equations (2.12) and (2.13) is real only if

$$r^{3/2} - 3Mr^{1/2} \pm 2aM^{1/2} \geq 0. \tag{2.17}$$

The limiting case of equality gives an orbit with infinite energy per unit rest mass, i.e., a photon orbit. This photon orbit is the innermost boundary of the circular orbits for particles; it occurs at the root of (2.17),

$$r = r_{\text{ph}} \equiv 2M\{1 + \cos[\tfrac{2}{3}\cos^{-1}(\mp a/M)]\}. \tag{2.18}$$

For $a = 0$, $r_{\text{ph}} = 3M$, while for $a = M$, $r_{\text{ph}} = M$ (direct) or $4M$ (retrograde).

For $r > r_{\text{ph}}$ not all circular orbits are bound. An unbound circular orbit is one with $E/\mu > 1$. Given an infinitesimal outward perturbation, a particle in such an orbit will escape to infinity on an asymptotically hyperbolic trajectory. The unbound circular orbits are circular in geometry but hyperbolic in energetics, and they are all unstable. Bound circular orbits exist for $r > r_{\text{mb}}$, where r_{mb} is the radius of the marginally bound ("parabolic") circular orbit with $E/\mu = 1$,

$$r_{\text{mb}} = 2M \mp a + 2M^{1/2}(M \mp a)^{1/2}. \tag{2.19}$$

Note also that r_{mb} is the minimum perihelion of all parabolic ($E/\mu = 1$) orbits. In astrophysical problems, particle infall from infinity is very nearly parabolic, since the velocities of matter at infinity satisfy $v \ll c$. Any parabolic trajectory which penetrates to $r < r_{\text{mb}}$ must plunge directly into the black hole. For $a = 0$, $r_{\text{mb}} = 4M$; for $a = M$, $r_{\text{mb}} = M$ (direct) or $5.83M$ (retrograde).

Even the bound circular orbits are not all stable. Stability requires that $V_r''(r) \leq 0$, which yields the three equivalent conditions

$$1 - (E/\mu)^2 \geq \tfrac{2}{3}(M/r),$$

$$r^2 - 6Mr \pm 8aM^{1/2}r^{1/2} - 3a^2 \geq 0,$$

or

$$r \geq r_{\text{ms}}, \tag{2.20}$$

where r_{ms} is the radius of the marginally stable orbit,

$$r_{\text{ms}} = M\{3 + Z_2 \mp [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}\},$$

$$Z_1 \equiv 1 + (1 - a^2/M^2)^{1/3}[(1 + a/M)^{1/3} + (1 - a/M)^{1/3}],$$

$$Z_2 \equiv (3a^2/M^2 + Z_1{}^2)^{1/2}. \tag{2.21}$$

For $a = 0$, $r_{\text{ms}} = 6M$; for $a = M$, $r_{\text{ms}} = M$ (direct) or $9M$ (retrograde). Figure 1