Bound Entanglement

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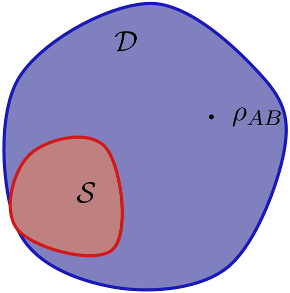
02/2014

Entanglement



. . . peculiar,
non-classical correlation arising from the tensor product structure in combined quantum systems

Bound Entanglement

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non-distillable entanglement

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Existence Explicite example of a bound entangled state

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Existence: Explicite example of a bound entangled state

Use: Why should we be concerned about bound entanglement?

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- **Existence** Explicite example of a bound entangled state
- **Use** Why should we be concerned about bound entanglement?
- **Detection** How do we decide for a given state whether it is (bound) entangled?

Roadmap

Showing the existence of BE

[Hor97] construct entangled state with negative partial transpose

[HHH98] NPT states are not distillable

The PPT Criterion

(Peres Criterion)

Definition

The partial transpose of a density matrix $\rho \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_B)$ is defined as

$$\rho^\Gamma := (\mathcal{I}_A \otimes T_B)\rho \tag{1}$$

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Matrix elements in the tensor basis

In the product basis

$$\begin{aligned}\left(\rho^\Gamma\right)_{ikjl} &= \langle i \otimes k|\ \rho^\Gamma\ |j \otimes l\rangle & (2)\\ &= \langle i \otimes l|\ \rho\ |j \otimes k\rangle = \rho_{iljk} & (3)\end{aligned}$$

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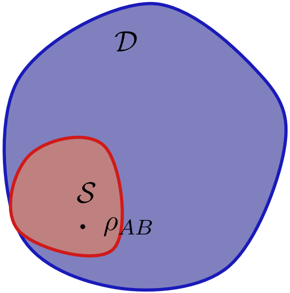

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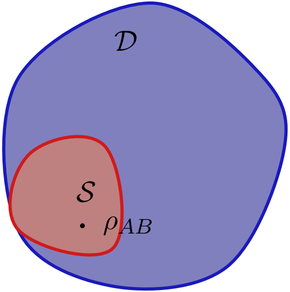

Entanglement criterion

$$\rho_{AB} \in \mathcal{S} \subset \mathcal{H}_A \otimes \mathcal{H}_B \;\Rightarrow\; \rho_{AB} \in \Gamma \tag{1}$$

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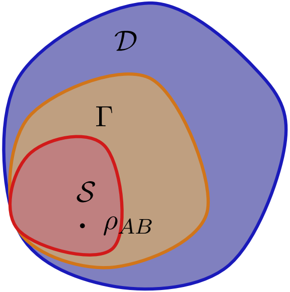

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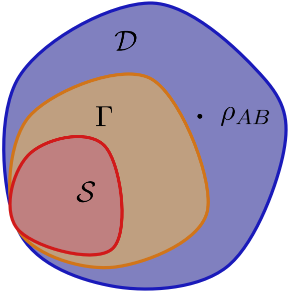

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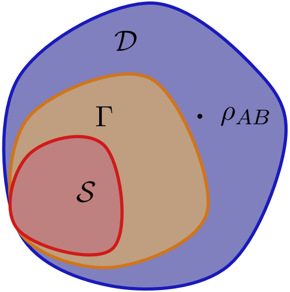

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Accuracy

The criterion is accurate in $2 \otimes 2$ and $2 \otimes 3$.
But not in higher dimensions.[HHH96]

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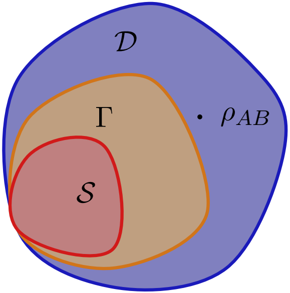

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Group of Entanglement Tests

Any Positive but not Completely Positive Map ('channel') serves as a similar entanglement test. [HHH96]

The Range Criterion [Hor97]

Decomposition of separable states

$$\forall\ \rho \in \mathcal{S}\ :$$

$$\begin{aligned}
&\rho = \sum_{i=1}^{n_A} \sum_{j=1}^{n_B} p_{ij}\ |\phi_i\rangle\langle\phi_i| \otimes |\psi_j\rangle\langle\psi_j| \\
&|\phi_i\rangle \in \mathcal{H}_A,\ |\psi_j\rangle \in \mathcal{H}_B \\
&n_a \leq \dim \mathcal{H}_A,\ n_B \leq \dim \mathcal{H}_B
\end{aligned} \tag{2}$$

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$\rightarrow$ features of finite-dimensional, compact convex sets (Caratheodories Thm [Car11])

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The range of an operator

$$\operatorname{Ran} \rho := \{|\psi\rangle \in \mathcal{H}, \exists\ |\phi\rangle \in \mathcal{H}\ :\ |\psi\rangle = \rho|\phi\rangle\} \tag{3}$$

The Range Criterion [Hor97]

The range of separable density matrices

Any separable density matrix $\rho \in \mathcal{S} \subset \mathcal{H}_A \otimes \mathcal{H}_B$ can be written as a convex combination of pure products states

$$\rho = \sum_{i,j} p_{ij} \; |\psi_i\rangle\langle\psi_i| \otimes |\phi_j\rangle\langle\phi_j| = \sum_{i,j} p_{ij} \; |\psi_i \otimes \phi_j\rangle\langle\psi_i \otimes \phi_j|$$

$$\rho^\Gamma = (\mathcal{I} \otimes T)\rho = \sum_{i,j} p_{ij} \; |\psi_i\rangle\langle\psi_i| \otimes \underbrace{|\phi_j\rangle\langle\phi_j|^T}_{|\phi_j^*\rangle\langle\phi_j^*|}$$

$$= \sum_{ij} p_{ij} \; |\psi_i \otimes \phi_j^*\rangle\langle\psi_i \otimes \phi_j^*|$$

and the set of product states $\{|\psi_i \otimes \phi_j\rangle\}_{ij}$ ($\{|\psi_i \otimes \phi_j^*\rangle\}_{ij}$) span the range of ρ (ρ^Γ)

The Range Criterion [Hor97]

Entanglement test

- find product vectors that span the range of ρ
- show that these vectors do not span the range of ρ^Γ

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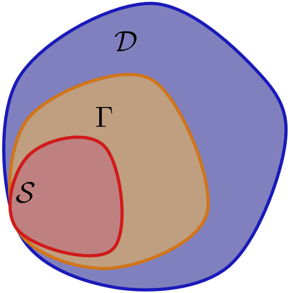
$\rightarrow$ "stronger" than PPT criterion

Construction [Hor97]



Components

$$\mathcal{H} = \mathbb{C}^3 \otimes \mathbb{C}^3,\ \{e_i\}\ \text{standard basis}$$

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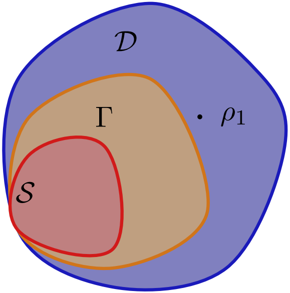

Components

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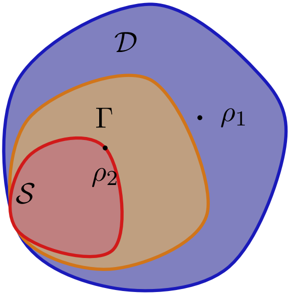
$$P_1 := I \otimes I - \sum_i |e_i\rangle\langle e_i| \otimes |e_i\rangle\langle e_i| - |e_3\rangle\langle e_3| \otimes |e_1\rangle\langle e_1|$$

$$P_2 := \frac{1}{3} |\sum_i e_i \otimes e_i\rangle\langle\sum_i e_i \otimes e_i|$$

$$\rho_1 := \frac{1}{8} P_1 + \frac{3}{8} P_2 \quad \text{NPT}$$

Construction [Hor97]

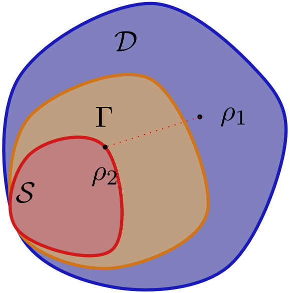


Components

$$\Psi := e_3 \otimes \left(\sqrt{\frac{1+a}{2}} e_1 + \sqrt{\frac{1-a}{2}} e_3 \right)$$

$$\rho_2 := |\Psi\rangle\langle\Psi| \in \partial\mathcal{S}$$

Construction [Hor97]



Components

$$\rho_a := \frac{8a}{8a+1}\rho_1 + \frac{1}{9a+1}\rho_2$$

The first PPT entangled state [Hor97]

$$\rho_a = \frac{1}{8a+1} \begin{bmatrix} a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1+a}{2} & 0 & \frac{\sqrt{1-a^2}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{1-a^2}}{2} & 0 & \frac{1+a}{2} \end{bmatrix} \qquad a \in (0,1)$$

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Properties of ρ_a

- PPT
- ρ_a and ρ_a^Γ violate the range criterion

Partial Transpose and Distillability

Definition

A density matrix ρ is called distillable if

$$\exists \text{ LOCC } \quad \Lambda : \mathcal{D}((\mathcal{H}_A \otimes \mathcal{H}_B)^{\otimes n}) \to \mathcal{D}(\mathbb{C}^2) \quad \text{s.t. :}$$
$$\Lambda(\rho^{\otimes n}) = |\Phi^+\rangle\langle\Phi^+| \quad \text{for some large } n$$

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Distillation of PPT states

Cannot distill qubits from PPT density matrices ρ and $\rho^{\otimes N}$

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Non-distillable entanglement

Any PPT entangled state is not distillable.

Distillation of PPT states

Claim

PPT states are not distillable.

Distillation of PPT states

Distilling $\rho^{\otimes n}$

Assume ρ is distillable:

$$\Lambda(\rho^{\otimes n}) = \frac{1}{M} \sum_i (A_i \otimes B_i)\ \rho^{\otimes n}\ (A_i^\dagger \otimes B_i^\dagger) \quad \text{is entangled}$$

$$\Rightarrow \quad \exists\ i_0 \text{ s.t.: } \underbrace{(A_{i_0} \otimes B_{i_0})\rho^{\otimes n}(A_{i_0}^\dagger \otimes B_{i_0}^\dagger)}_{=:\rho_{i_0}} \quad \text{is entangled}$$

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$$A_{i_0} = |0\rangle\langle\psi_A| + |1\rangle\langle\phi_A| \quad B_{i_0} = |0\rangle\langle\psi_B| + |1\rangle\langle\phi_B|$$

$$P_A(P_B) := \text{ projector onto } \langle\psi_A, \phi_A\rangle(\langle\psi_B, \phi_B\rangle)$$

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$$\Rightarrow \quad \exists\ \psi \text{ s.t.: } \quad \langle\psi|(\rho')^{\Gamma_B}|\psi\rangle < 0$$

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$$\Rightarrow \quad \langle\psi|\rho^{\Gamma_B}|\psi\rangle < 0$$

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a first summary

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existence	only in higher dimensions (smallest BE systems in $3 \otimes 3$ and $4 \otimes 2$)

Bound entanglement

a first summary

def entanglement that cannot be distilled
$\rightarrow$ not a sufficient resource for many quantum informational protocols

construction PPT entangled states (any PPT is not distillable)

existence only in higher dimensions (smallest BE systems in $3 \otimes 3$ and $4 \otimes 2$)

detection need a stronger criterion than PPT, e.g. range criterion

Quantum Key Distribution

with bound entangled resources

Secure key from BE

Bound entangled resources can be employed to distill secret key [HHHO05].[1]

[1] For further details see [HHHO09]

Quantum Key Distribution

with bound entangled resources

Shielded System with an adversary

$$A \otimes B \otimes \underbrace{A' \otimes B'}_{\text{shield}} \otimes E \tag{4}$$

Quantum Key Distribution

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Assume: Eavesdropper Eve controles the whole environment (purification).

Quantum Key Distribution

with bound entangled resources

Shielded System with an adversary

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Quantum Key Distribution

with bound entangled resources

Security and Key

After measuring in a product basis the systems A, B and tracing out A', B'

$$\rho_{ccq} = \sum_{ij} p_{ij} \; |e_i \otimes f_j\rangle\langle e_i \otimes f_j| \otimes \rho_{ij}^{E} \tag{4}$$

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Quantum Key Distribution

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$$\rho_{ccq} = \sum_{ij} p_{ij} \, |e_i \otimes f_j\rangle\langle e_i \otimes f_j| \otimes \rho^E \tag{4}$$

is secure and has key if $\{p_{ij}\} = \left\{ \frac{1}{d} \delta_{ij} \right\}$

Quantum Key Distribution

with bound entangled resources

Private States

$$\gamma = \frac{1}{d} \sum_{ij} |e_i f_i\rangle\langle e_j f_j|_{AB} \otimes U_i \sigma_{A'B'} U_j^\dagger \tag{4}$$

where U_i's are arbitrary unitary transformations.

Quantum Key Distribution

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Key of private states

ρ is a private state $\Leftrightarrow$ ρ has key

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Private states and BE

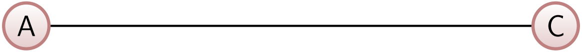
Private states can be approximated with BE states.

Entanglement Tests

- **PPT**: all NPT states are entangled
  $\rightarrow$ does not detect BE
- **range**: compare the range of ρ and ρ^{Γ}
- **extendibility**: check extendibility of ρ [DPS05] (Semi definite program)

Further: ranges for randomly drawn states (concentration of measure) [ASY12a] [ASY12b]

Endeavour: Extending entanglement



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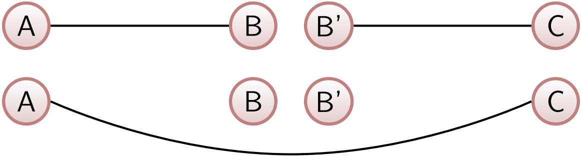
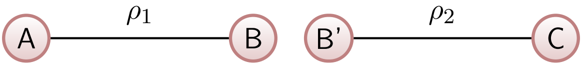


Swap Operator on States



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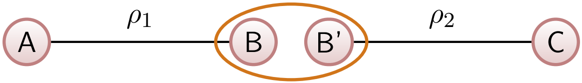

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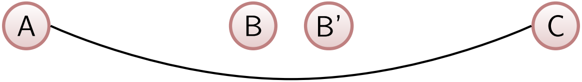

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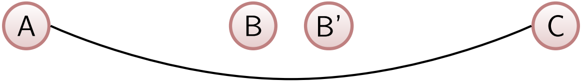
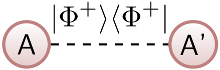


Swap operation

$$\mathrm{swap}(\rho_A, \rho_B) = \frac{1}{N}\,\mathrm{Tr}_{BB'}\left[\ \left(|\Phi^+\rangle\langle\Phi^+|_{BB'} \otimes \mathbb{1}_{AC}\right)\ \left(\rho_1 \otimes \rho_2\right)\ \right]$$

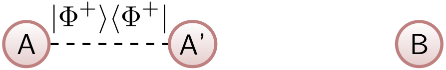
Composing Channels

corresponds to swapping CJ associated states



Composing Channels

corresponds to swapping CJ associated states



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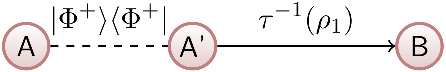

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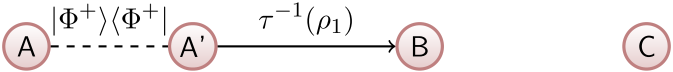

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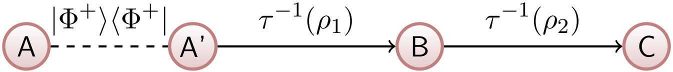


Composing Channels

corresponds to swapping CJ associated states

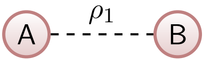
$$\begin{tikzcd} \text{A} \arrow[r, dash, dashed, "|\Phi^+\rangle\langle\Phi^+|"] & \text{A'} \arrow[r, "\tau^{-1}(\rho_1)"] & \text{B} \arrow[r, "\tau^{-1}(\rho_2)"] & \text{C} \end{tikzcd}$$

Swap in terms of channels

$$\tau^{-1}(\mathrm{swap}(\rho_1, \rho_2)) = \tau^{-1}(\rho_2) \circ \tau^{-1}(\rho_1)$$

(flipping systems or equivalently transposing channels if necessary, up to normalization)

Hybrid approach



Hybrid approach

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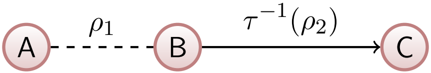
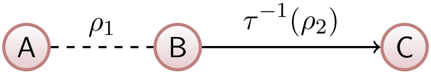


Hybrid approach



Hybrid formulation

$$\mathrm{swap}(\rho_1, \rho_2) = \left[\mathcal{I}_A \otimes \tau^{-1}(\rho_2)\right](\rho_1) = \left[\mathcal{I}_C \otimes \tau^{-1}(\rho_1)^T\right](\rho_2)$$

(flipping systems or equivalently transposing channels if necessary, up to normalization)

Swapped Bound Entanglement

Entanglement of swapped states from BE resources?

States: Image of the Swap operator

$$\mathrm{swap} : \text{BE states} \times \text{BE states} \to \mathcal{S}$$

OR

$$\mathrm{swap}(\text{BE states}, \text{BE states}) \cap \mathcal{D} \setminus \mathcal{S} \neq \emptyset$$

[1] Defined by their image: $\Lambda_\Gamma(\mathcal{D}) \subset \Gamma$

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Composed Channels

If $\Lambda_1 : \mathcal{D}(A) \to \mathcal{D}(B)$ and $\Lambda_2 : \mathcal{D}(B) \to \mathcal{D}(C)$ are PPT channels[1], is $\Lambda_2 \circ \Lambda_1 : \mathcal{D}(A) \to \mathcal{D}(C)$ separable?

[1] Defined by their image: $\Lambda_\Gamma(\mathcal{D}) \subset \Gamma$

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