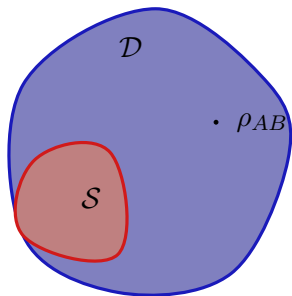# Bound Entanglement

Arne Hansen

ETH Zurich

02/2014

Entanglement

...peculiar, non-classical correlation arising from the tensor product structure in combined quantum systems

Bound Entanglement

Bound Entanglement
non-distillable entanglement

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non-distillable entanglement

Existence Explicite example of a bound entangled state

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Existence Explicite example of a bound entangled state

Use Why should we be concerned about bound entanglement?

Bound Entanglement

Bound Entanglement

non-distillable entanglement

Existence   Explicite example of a bound entangled state

Use   Why should we be concerned about bound entanglement?

Detection   How do we decide for a given state whether it is (bound) entangled?

Roadmap
Showing the existence of BE

[Hor97] construct entangled state with negative partial transpose

[HHH98] NPT states are not distillable

The PPT Criterion
(Peres Criterion)

Definition

The partial transpose of a density matrix $\rho \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_B)$ is defined as

$$\rho^\Gamma := (\mathcal{I}_A \otimes T_B)\rho \qquad (1)$$

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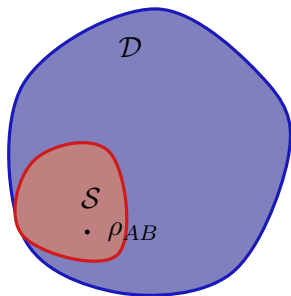
Matrix elements in the tensor basis

In the product basis

$$\left(\rho^\Gamma\right)_{ikjl} = \langle i \otimes k| \, \rho^\Gamma \, |j \otimes l\rangle \tag{2}$$

$$= \langle i \otimes l| \, \rho \, |j \otimes k\rangle = \rho_{iljk} \tag{3}$$
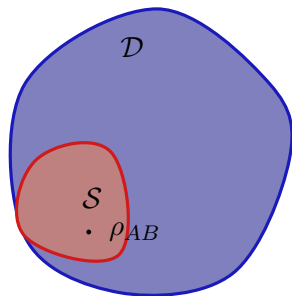
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Entanglement criterion

$$\rho_{AB} \in \mathcal{S} \subset \mathcal{H}_A \otimes \mathcal{H}_B \;\Rightarrow\; \rho_{AB} \in \Gamma \qquad (1)$$

The PPT Criterion

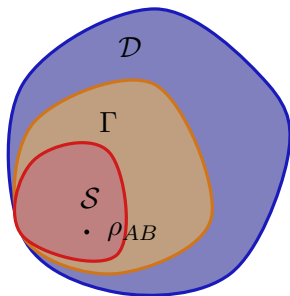(Peres Criterion)

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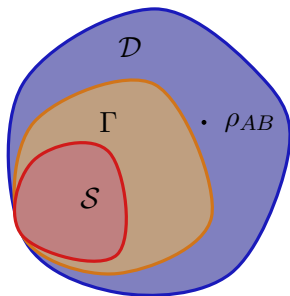
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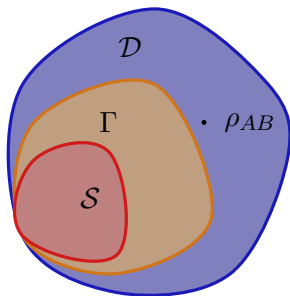### Entanglement criterion

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Accuracy

The criterion is accurate in $2 \otimes 2$ and $2 \otimes 3$. But not in higher dimensions.[HHH96]

The PPT Criterion
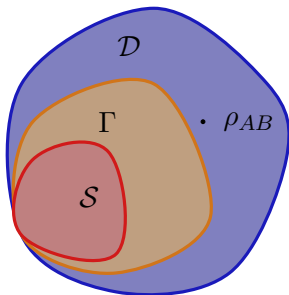(Peres Criterion)

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Group of Entanglement Tests

Any Positive but not Completely Positive Map ('channel') serves as a similar entanglement test. [HHH96]

The Range Criterion [Hor97]

Decomposition of separable states

$$\forall \, \rho \in \mathcal{S} \, :$$

$$\rho = \sum_{i=1}^{n_A} \sum_{j=1}^{n_B} p_{ij} \, |\phi_i\rangle\langle\phi_i| \otimes |\psi_j\rangle\langle\psi_j| \tag{2}$$

$$|\phi_i\rangle \in \mathcal{H}_A, \, |\psi_j\rangle \in \mathcal{H}_B$$

$$n_a \le \dim \mathcal{H}_A, \, n_B \le \dim \mathcal{H}_B$$

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$\rightarrow$ features of finite-dimensional, compact convex sets
(Caratheodories Thm [Car11])

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The range of an operator

$$\mathrm{Ran}\,\rho := \{|\psi\rangle \in \mathcal{H}, \exists \, |\phi\rangle \in \mathcal{H} \; : \; |\psi\rangle = \rho|\phi\rangle\} \qquad (3)$$

The Range Criterion [Hor97]

The range of separable density matrices

Any separable density matrix $\rho \in \mathcal{S} \subset \mathcal{H}_A \otimes \mathcal{H}_B$ can be written as a convex combination of pure products states

$$\rho = \sum_{i,j} p_{ij} \, |\psi_i\rangle\langle\psi_i| \otimes |\phi_j\rangle\langle\phi_j| = \sum_{i,j} p_{ij} \, |\psi_i \otimes \phi_j\rangle\langle\psi_i \otimes \phi_j|$$

$$\rho^\Gamma = (\mathcal{I} \otimes T)\rho = \sum_{i,j} p_{ij} \, |\psi_i\rangle\langle\psi_i| \otimes \underbrace{|\phi_j\rangle\langle\phi_j|^T}_{|\phi_j^*\rangle\langle\phi_j^*|}$$

$$= \sum_{ij} p_{ij} \, |\psi_i \otimes \phi_j^*\rangle\langle\psi_i \otimes \phi_j^*|$$

and the set of product states $\{|\psi_i \otimes \phi_j\rangle\}_{ij}$ ($\{|\psi_i \otimes \phi_j^*\rangle\}_{ij}$) span the range of ρ (ρ^Γ)
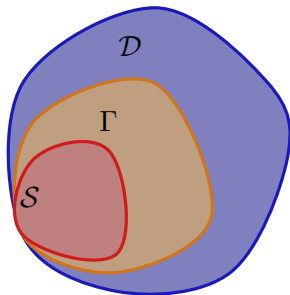
The Range Criterion [Hor97]

Entanglement test

- find product vectors that span the range of ρ
- show that these vectors do not span the range of ρ^{Γ}

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> **Entanglement test**
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> - find product vectors that span the range of ρ
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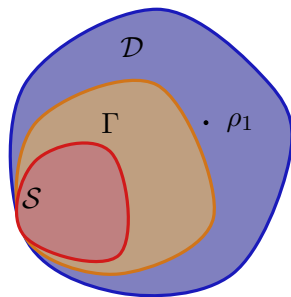$\rightarrow$ "stronger" than PPT criterion

Construction [Hor97]

Components

$$\mathcal{H} = \mathbb{C}^3 \otimes \mathbb{C}^3, \ \{e_i\} \ \text{standard basis}$$

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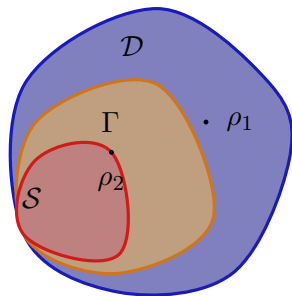
$$P_1 := I \otimes I - \sum_i |e_i\rangle\langle e_i| \otimes |e_i\rangle\langle e_i|$$

$$- |e_3\rangle\langle e_3| \otimes |e_1\rangle\langle e_1|$$

$$P_2 := \frac{1}{3}|\sum_i e_i \otimes e_i\rangle\langle\sum_i e_i \otimes e_i|$$

$$\rho_1 := \frac{1}{8}P_1 + \frac{3}{8}P_2 \quad \text{NPT}$$
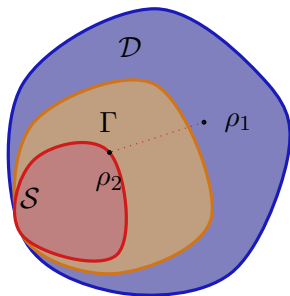
Construction [Hor97]

Components

$$\Psi := e_3 \otimes \left(\sqrt{\frac{1+a}{2}} e_1 + \sqrt{\frac{1-a}{2}} e_3 \right)$$

$$\rho_2 := |\Psi\rangle\langle\Psi| \in \partial\mathcal{S}$$

Construction [Hor97]

Components

$$\rho_a := \frac{8a}{8a+1}\rho_1 + \frac{1}{9a+1}\rho_2$$

The first PPT entangled state [Hor97]

$$\rho_a = \frac{1}{8a+1} \begin{bmatrix} a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1+a}{2} & 0 & \frac{\sqrt{1-a^2}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{1-a^2}}{2} & 0 & \frac{1+a}{2} \end{bmatrix} \qquad a \in (0,1)$$

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Properties of ρ_a

- PPT
- ρ_a and ρ_a^Γ violate the range criterion

Partial Transpose and Distillability

> **Definition**
>
> A density matrix ρ is called distillable if
>
> $$\exists \, \text{LOCC} \quad \Lambda : \mathcal{D}((\mathcal{H}_A \otimes \mathcal{H}_B)^{\otimes n}) \to \mathcal{D}(\mathbb{C}^2) \quad \text{s.t.} :$$
> $$\Lambda(\rho^{\otimes n}) = |\Phi^+\rangle\langle\Phi^+| \quad \text{for some large } n$$

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Cannot distill qubits from PPT density matrices ρ and $\rho^{\otimes N}$

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Non-distillable entanglement

Any PPT entangled state is not distillable.

Distillation of PPT states

Claim

PPT states are not distillable.

Distillation of PPT states

Distilling $\rho^{\otimes n}$

Assume ρ is distillable:

$$\Lambda(\rho^{\otimes n}) = \frac{1}{M} \sum_i (A_i \otimes B_i)\, \rho^{\otimes n}\, (A_i^\dagger \otimes B_i^\dagger) \quad \text{is entangled}$$

$$\Rightarrow \quad \exists\ i_0 \text{ s.t.:}\ \underbrace{(A_{i_0} \otimes B_{i_0})\rho^{\otimes n}(A_{i_0}^\dagger \otimes B_{i_0}^\dagger)}_{=:\rho_{i_0}} \quad \text{is entangled}$$

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$$A_{i_0} = |0\rangle\langle\psi_A| + |1\rangle\langle\phi_A| \quad B_{i_0} = |0\rangle\langle\psi_B| + |1\rangle\langle\phi_B|$$

$$P_A(P_B) := \text{ projector onto } \langle\psi_A, \phi_A\rangle(\langle\psi_B, \phi_B\rangle)$$

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$$\Rightarrow \quad \exists \ \psi \ \text{s.t.:} \ \langle\psi|(\rho')^{\Gamma_B}|\psi\rangle < 0$$

Distillation of PPT states

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$$\Rightarrow \quad \langle \psi | \rho^{\Gamma_B} | \psi \rangle < 0$$

$$\Rightarrow \quad \rho \text{ is NPT}$$

Bound entanglement

a first summary

def entanglement that cannot be distilled

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$\rightarrow$ not a sufficient resource for many quantum
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construction $\quad$ PPT entangled states (any PPT is not distillable)

existence $\quad$ only in higher dimensions (smallest BE systems in
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Bound entanglement
a first summary

def	entanglement that cannot be distilled
	$\rightarrow$ not a sufficient resource for many quantum informational protocols
construction	PPT entangled states (any PPT is not distillable)
existence	only in higher dimensions (smallest BE systems in $3 \otimes 3$ and $4 \otimes 2$)
detection	need a stronger criterion than PPT, e.g. range criterion

Quantum Key Distribution
with bound entangled resources

Secure key from BE

Bound entangled resources can be employed to distill secret key [HHHO05].[1]

---

[1]For further details see [HHHO09]

Quantum Key Distribution
with bound entangled resources

Shielded System with an adversary

$$A \otimes B \otimes \underbrace{A' \otimes B'}_{\text{shield}} \otimes E \tag{4}$$

Quantum Key Distribution
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Assume: Eavesdropper Eve controls the whole environment (purification).

Quantum Key Distribution
with bound entangled resources

Shielded System with an adversary

$$|\psi\rangle\langle\psi| \in \mathcal{D}(A \otimes B \otimes \underbrace{A' \otimes B'}_{\text{shield}} \otimes E) \tag{4}$$

Assume: Eavesdropper Eve controls the whole environment (purification).

Quantum Key Distribution
with bound entangled resources

Security and Key

After measuring in a product basis the systems A, B and tracing out A', B'

$$\rho_{ccq} = \sum_{ij} p_{ij} \, |e_i \otimes f_j\rangle\langle e_i \otimes f_j| \otimes \rho_{ij}^E \qquad (4)$$

Quantum Key Distribution
with bound entangled resources

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is secure

has key if $\{p_{ij}\} = \left\{ \dfrac{1}{d} \delta_{ij} \right\}$

Quantum Key Distribution
with bound entangled resources

Private States

$$\gamma = \frac{1}{d} \sum_{ij} |e_i f_i\rangle\langle e_j f_j|_{AB} \otimes U_i \sigma_{A'B'} U_j^\dagger \tag{4}$$

where U_i's are arbitrary unitary transformations.

Quantum Key Distribution
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Key of private states

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Private states and BE

Private states can be approximated with BE states.

Entanglement Tests

an overview

$\quad\quad$ PPT $\quad$ all NPT states are entangled
$\quad\quad\quad\quad\quad$ $\rightarrow$ does not detect BE

$\quad\quad$ range $\quad$ compare the range of ρ and ρ^{Γ}

$\quad$ extendibility $\quad$ check extendibility of ρ [DPS05] (Semi definite
$\quad\quad\quad\quad\quad$ program $\rightarrow$ computational approaches)

$\quad$ witnesses $\quad$ hyperplanes separating the state from the convex set
$\quad\quad\quad\quad\quad$ of separable states [Ter00]

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witnesses hyperplanes separating the state from the convex set
of separable states [Ter00]

Further: concentration of measure for randomly drawn states $\rightarrow$
conditions for dimension of respective Hilbert spaces to get PPT
states / entangled states [ASY12a] [ASY12b]

Endeavour: Extending entanglement
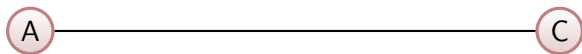
Endeavour: Extending entanglement
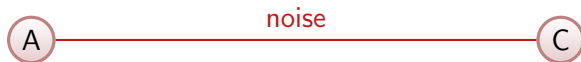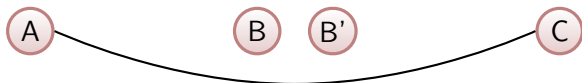
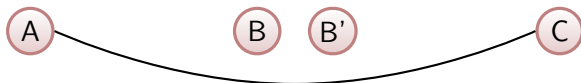# Endeavour: Extending entanglement
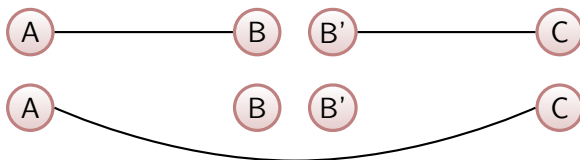
Swap Operator on States

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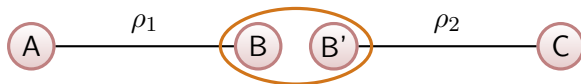
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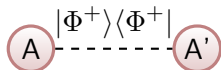

Swap operation

$$\mathrm{swap}(\rho_A, \rho_B) = \frac{1}{N} \mathrm{Tr}_{BB'} \left[\left(|\Phi^+\rangle\langle\Phi^+|_{BB'} \otimes \mathbb{1}_{AC} \right) \left(\rho_1 \otimes \rho_2 \right) \right]$$
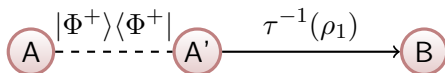
Composing Channels

corresponds to swapping CJ associated states

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Composing Channels

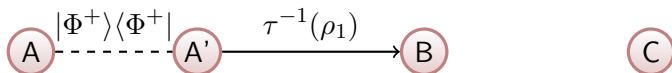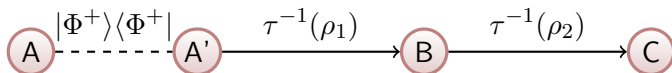corresponds to swapping CJ associated states

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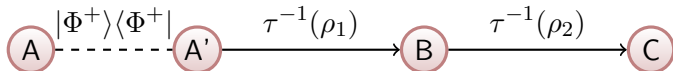
corresponds to swapping CJ associated states

$$A \dashleftarrow{|\Phi^+\rangle\langle\Phi^+|} A' \xrightarrow{\tau^{-1}(\rho_1)} B \xrightarrow{\tau^{-1}(\rho_2)} C$$
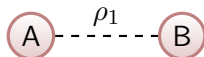
Swap in terms of channels

$$\tau^{-1}(\mathrm{swap}(\rho_1, \rho_2)) = \tau^{-1}(\rho_2) \circ \tau^{-1}(\rho_1)$$

(flipping systems or equivalently transposing channels if necessary, up to normalization)

Hybrid approach

Hybrid approach

Hybrid approach

Hybrid approach

Hybrid formulation

$$\mathrm{swap}(\rho_1, \rho_2) = \left[\mathcal{I}_A \otimes \tau^{-1}(\rho_2)\right](\rho_1) = \left[\mathcal{I}_C \otimes \tau^{-1}(\rho_1)^T\right](\rho_2)$$

(flipping systems or equivalently transposing channels if necessary, up to normalization)

Swapped Bound Entanglement

Entanglement of swapped states from BE resources?

States: Image of the Swap operator

$$\mathrm{swap} : \text{BE states} \times \text{BE states} \rightarrow \mathcal{S}$$

OR

$$\mathrm{swap}(\text{BE states}, \text{BE states}) \cap \mathcal{D} \setminus \mathcal{S} \neq \emptyset$$

---

[1]Defined by their image: $\Lambda_\Gamma(\mathcal{D}) \subset \Gamma$

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Composed Channels

If $\Lambda_1 : \mathcal{D}(A) \to \mathcal{D}(B)$ and $\Lambda_2 : \mathcal{D}(B) \to \mathcal{D}(C)$ are PPT channels[1], is $\Lambda_2 \circ \Lambda_1 : \mathcal{D}(A) \to \mathcal{D}(C)$ separable?

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[1] Defined by their image: $\Lambda_\Gamma(\mathcal{D}) \subset \Gamma$

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https://github.com/EnraH/Presentation_USI