

Criticality

The study of Self-organization



Contents

Part 1: Complex Systems, the Study of Emergence

Part 2: Intro to Edge of Chaos and Criticality

Part 3: Criticality in Brains





How did the first person who clapped
convince everyone to do it?



How did the first person who clapped
convince everyone to do it?

Who decided how fast to clap?



How did the first person who clapped
convince everyone to do it?

Who decided how fast to clap?

Who decided to stop?



There is no unit (bird) that has knowledge of the entire system

Yet, a small perturbation to the system can lead to dramatic changes in collective dynamics

Probably all this information is encoded in the Genome right?

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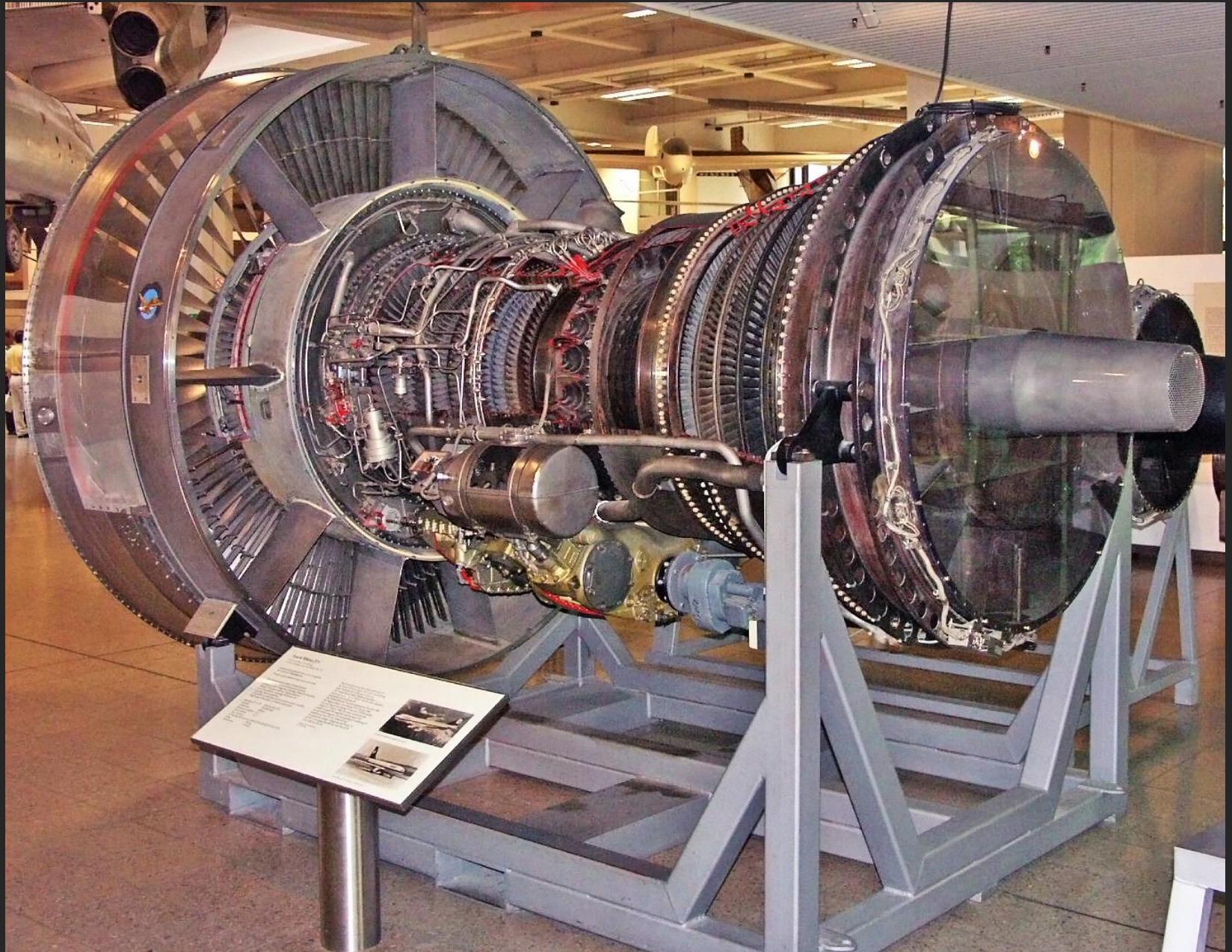
The Human Genome Project



Well, it is not that simple... It's a bit like saying that since we know the human genome we know exactly how ancient Egyptians built the pyramids

There is no encoding in Nature of all possible “functions” that a system can do, functions emerge thanks to the collective effort of many “units” in a system

That is not how
humans do “things”.
Yet, one small little
imprecision can be
catastrophic!



Nature's creations are much more robust than human's

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Reference	Experimental system	Total baseline synaptic change	% synaptic change that is activity / learning-independent	Dvorkin and Ziv, 2016	Glutamatergic synapses in cultured networks of mouse cortical neurons	Partitioned commonly innervated (CI) synapses sharing same axon and dendrite, and non-CI synapses. Quantified covariance in fluorescence change for CI vs non-CI synapses to estimate relative contribution of activity histories to synaptic remodelling	62–64% (plasticity)
Pfeiffer et al., 2018	Adult mouse hippocampus	40% turnover over 4 days	NA				
Loewenstein et al., 2011	Adult mouse auditory cortex	>70% of spines changed size by >50% over 20 days	NA				
Zuo et al., 2005	Adult mouse (barrel, primary motor, frontal) cortex	3–5% turnover over 2 weeks for all regions. $73.9 \pm 2.8\%$ of spines stable over 18 months (barrel cortex)	NA				
Nagaoka et al., 2016	Adult mouse visual cortex	8% turnover per 2 days in visually deprived environment. 15% in visually enriched environment. 7–8% in both environments under pharmacological suppression of spiking.	≈ 50% (turnover)				
Quinn et al., 2019	Glutamatergic synapses, dissociated rat hippocampal culture	$28 \pm 3.7\%$ of synapses formed over 24 hr period. $28.6 \pm 2.3\%$ eliminated. Activity suppression through tetanus neurotoxin -light chain. Plasticity rate unmeasured.	≈ 75% (turnover)	Kasthuri et al., 2015	Adult mouse neocortex (Three-dimensional <i>post mortem</i> reconstruction using electron microscopy).	Data on 124 pairs of 'redundant' synapses sharing a pre/post-synaptic neuron was analysed in Dvorkin and Ziv, 2016. They calculated the correlation coefficient of spine volumes and post-synaptic density sizes between redundant pairs. This should be one if pre/post-synaptic activity history perfectly explains these variables.	77% (post-synaptic density, $r^2 = 0.23$). 66% (spine volume, $r^2 = 0.34$)
Yasumatsu et al., 2008	CA1 pyramidal neurons, primary culture, rat hippocampus	Measured rates of synaptic turnover and spine-head volume change. Baseline conditions vs activity suppression (NMDAR inhibitors). Turnover rates: $32.8 \pm 3.7\%$ generation/elimination per day (control) vs $22.0 \pm 3.6\%$ (NMDAR inhibitor). Rate of spine-head volume change:	≈ $67 \pm 17\%$ (turnover). Size-dependent, but consistently >50% (spine-head volume)	Ziv and Brenner, 2018	Literature review across multiple systems	'Collectively these findings suggest that the contributions of spontaneous processes and specific activity histories to synaptic remodeling are of similar magnitudes'	≈ 50%

Table 1 from "Dhruba V Raman Timothy O'Leary (2021) Optimal plasticity for memory maintenance during ongoing synaptic change eLife 10:e62912."

Nature's creations are much more robust than human's

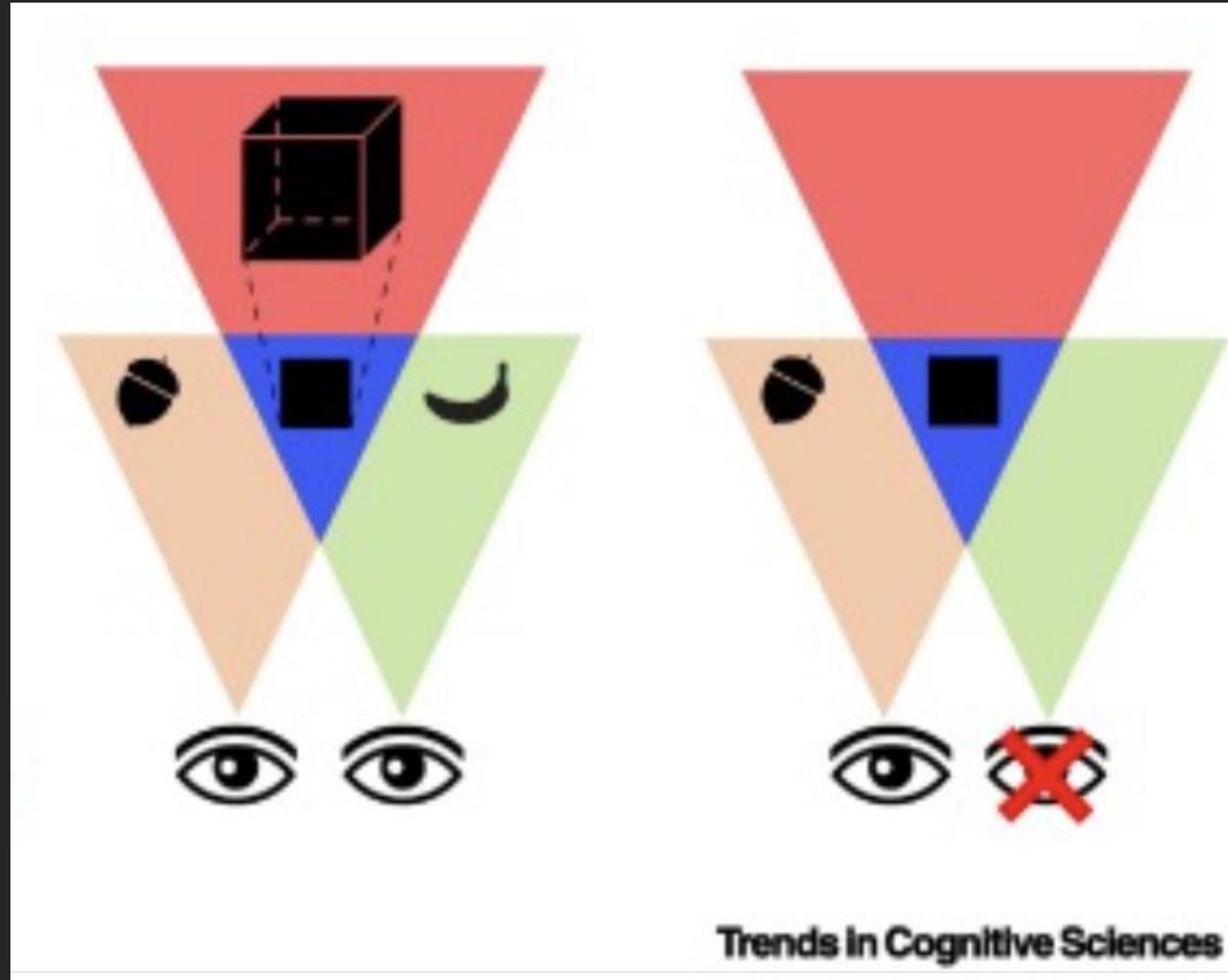
Imagine doing this to the neural network layers of a LLM such as ChatGPT, the answers would be impossible to interpret

Complexity Science is the study of Emergence

Macroscopic features and/or processes of a system that are independent of the underlying units of the system.

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Luppi et al (2024)
Information
decomposition and
the informational
architecture of the
brain

**Put on your philosopher mask and to try to answer this,
what is the difference between a thermite nest and
the Sagrada Família?**



Example taken from Daniel Dennett







Antoni Gaudí



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Part 2

The classical and quantum physicists' secret

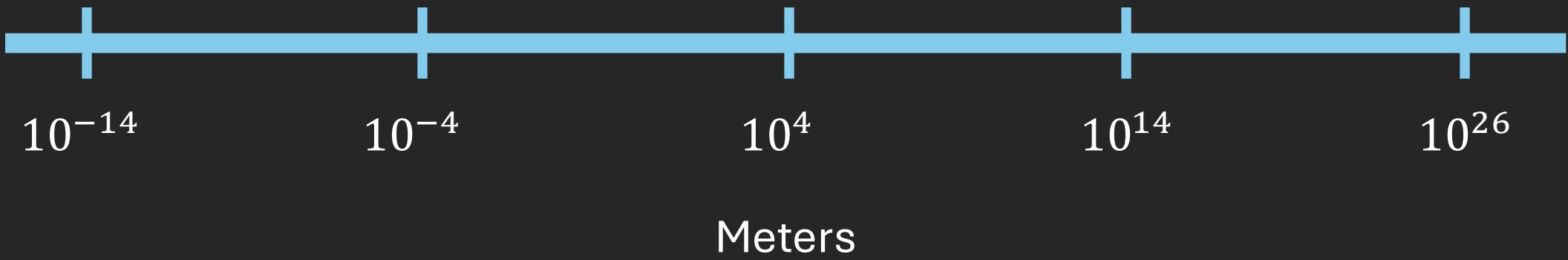
Part 2

The classical and quantum physicists' secret

**Classical and quantum physics are cool ...
Don't get me wrong**

Classical and quantum physics are cool ... Don't get me wrong

$$\begin{aligned} \mathcal{L}_{M\phi} = & -\frac{1}{2}\partial_\mu\phi\partial_\mu\phi - g_s I^{abc}\partial_\mu\phi\partial_\mu\phi - \frac{1}{2}g_s^2 I^{abc}g_s^a g_s^b g_s^c - \partial_\mu W_\mu^+ \partial_\mu W_\mu^- - \\ & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\mu Z_\mu^0 \partial_\mu Z_\mu^0 - \frac{1}{2}M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\mu A_\mu - ig_s(A_\mu W_\mu^+ W_\mu^- - \\ & \bar{A}_\mu \bar{W}_\mu^+ \bar{W}_\mu^-) + \frac{1}{2}Z_\mu^0 \partial_\mu Z_\mu^0 - Z_\mu^0 \partial_\mu W_\mu^+ W_\mu^- - Z_\mu^0 \partial_\mu \bar{W}_\mu^+ \bar{W}_\mu^- - \\ & ig_s c_u A_\mu (W_\mu^+ W_\mu^- - \bar{W}_\mu^+ \bar{W}_\mu^-) + \frac{1}{2}g_s^2 (W_\mu^+ W_\mu^- - \bar{W}_\mu^+ \bar{W}_\mu^-) + \frac{1}{2}g_s^2 (Z_\mu^0 W_\mu^+ Z_\mu^0 W_\mu^- - \\ & W_\mu^+ \bar{W}_\mu^-) - \frac{1}{2}g_s^2 (W_\mu^+ W_\mu^- - \bar{W}_\mu^+ \bar{W}_\mu^-) + \frac{1}{2}g_s^2 (W_\mu^+ W_\mu^- - \bar{W}_\mu^+ \bar{W}_\mu^-) + g_s^2 c_u (A_\mu Z_\mu^0 W_\mu^+ W_\mu^- - \\ & Z_\mu^0 W_\mu^+ W_\mu^-) + g_s^2 c_u (A_\mu W_\mu^+ W_\mu^- - \bar{A}_\mu W_\mu^+ W_\mu^-) + g_s^2 c_u (A_\mu Z_\mu^0 W_\mu^+ W_\mu^- - \\ & Z_\mu^0 W_\mu^+ W_\mu^-) - 2A_\mu (W_\mu^+ W_\mu^-) - \frac{1}{2}g_s H_0 H - 2A_\mu^2 W_\mu^+ W_\mu^- - \partial_\mu A_\mu \partial_\mu \phi - \frac{1}{2}\partial_\mu \phi \partial_\mu \phi - \\ & \partial_\mu (\frac{\partial_\mu \phi}{\partial_\mu \phi}) + \frac{1}{2}\partial_\mu (\partial_\mu \phi \partial_\mu \phi) + \frac{1}{2}\partial_\mu (\partial_\mu \phi \partial_\mu \phi) + \frac{1}{2}\partial_\mu (\partial_\mu \phi \partial_\mu \phi) - \\ & \frac{1}{2}g^2 \alpha_s (H^2 + (\phi^2)^2 + 4(\phi^2)^2 \phi^2 + 4(\phi^2)^2 \phi^2 + 4H(\phi^2)^2 \phi^2 + 2(\phi^2)^2 H^2) - \\ & gM^2 W_\mu^+ W_\mu^- H - \frac{1}{2}\partial_\mu^2 Z_\mu^0 H - \\ & \frac{1}{2}ig (W_\mu^+ (\partial_\mu \phi \partial_\mu \phi - \phi \partial_\mu \phi) - W_\mu^- (\partial_\mu \phi \partial_\mu \phi - \phi \partial_\mu \phi)) + \\ & \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi - \phi \partial_\mu H) + W_\mu^- (H \partial_\mu \phi - \phi \partial_\mu H)) - \frac{1}{2}\partial_\mu^2 (Z_\mu^0 (H \partial_\mu \phi - \phi \partial_\mu H)) + \\ & M (\frac{1}{2}Z_\mu^0 \partial_\mu Z_\mu^0 - \frac{1}{2}Z_\mu^0 \partial_\mu W_\mu^+ W_\mu^- - \frac{1}{2}Z_\mu^0 \partial_\mu \bar{W}_\mu^+ \bar{W}_\mu^-) - \frac{1}{2}g^2 (A_\mu W_\mu^+ W_\mu^- - \bar{A}_\mu \bar{W}_\mu^+ \bar{W}_\mu^-) - \\ & \frac{1}{2}g^2 (W_\mu^+ W_\mu^- (H^2 \phi^2 + 2\phi^2) - \frac{1}{2}g^2 Z_\mu^0 Z_\mu^0 (H^2 \phi^2 + 2\phi^2) - 2(2\phi^2 - 1)^2 \phi^4) - \\ & \frac{1}{2}g^2 \partial_\mu^2 (W_\mu^+ W_\mu^- + W_\mu^+ \bar{W}_\mu^-) - \frac{1}{2}g^2 \partial_\mu^2 Z_\mu^0 Z_\mu^0 (H^2 \phi^2 - W_\mu^+ \phi^2) - \frac{1}{2}g^2 (A_\mu W_\mu^+ W_\mu^- \phi^2 + \\ & W_\mu^+ \phi^2) + \frac{1}{2}g^2 c_u A_\mu H (W_\mu^+ \phi^2 - W_\mu^- \phi^2) - g^2 \partial_\mu (2\phi^2 - 1) Z_\mu^0 A_\mu \phi^2 + \\ & g^2 A_\mu^2 A_\mu \phi^2 \phi^2 - g^2 (c_u^2 + m_{\tilde{q}}^2) (W_\mu^+ \phi^2) \phi^2 - \phi^2 (c_u^2 m_{\tilde{q}}^2) \phi^2 - \phi^2 (m_{\tilde{q}}^2 \phi^2) \phi^2 - g^2 (W_\mu^+ \phi^2) \phi^2 + \\ & g^2 (W_\mu^- \phi^2) \phi^2 - g^2 (c_u^2 + m_{\tilde{q}}^2) (W_\mu^+ \phi^2) \phi^2 - \phi^2 (c_u^2 m_{\tilde{q}}^2) \phi^2 - \phi^2 (m_{\tilde{q}}^2 \phi^2) \phi^2 + \\ & g^2 (Z_\mu^0 \phi^2) \phi^2 - g^2 (c_u^2 + m_{\tilde{q}}^2) (Z_\mu^0 \phi^2) \phi^2 - \phi^2 (c_u^2 m_{\tilde{q}}^2) \phi^2 - \phi^2 (m_{\tilde{q}}^2 \phi^2) \phi^2 + \\ & g^2 (W_\mu^+ \phi^2) \phi^2 + \frac{1}{2}g^2 W_\mu^+ ((\phi^2)^2 \phi^2 + \gamma^2 (\phi^2)^2 \phi^2) + (\phi^2)^2 (1 + \gamma^2) C_\mu^2 \phi^2) + \\ & (\phi^2)^2 (1 + \gamma^2) n) + \frac{1}{2}g^2 W_\mu^- ((\phi^2)^2 \phi^2 + \gamma^2 (\phi^2)^2 \phi^2) + (\phi^2)^2 (1 + \gamma^2) C_\mu^2 \phi^2) + \\ & \frac{1}{2}g^2 W_\mu^+ ((c_u^2 \phi^2 \gamma^2 \phi^2 + \gamma^2 \phi^2 \phi^2) + (C_\mu^2 \phi^2 \gamma^2 \phi^2 + \gamma^2 \phi^2 \phi^2)) + \\ & \frac{1}{2}g^2 W_\mu^- ((c_u^2 \phi^2 \gamma^2 \phi^2 + \gamma^2 \phi^2 \phi^2) + (C_\mu^2 \phi^2 \gamma^2 \phi^2 + \gamma^2 \phi^2 \phi^2)) - \frac{1}{2}g^2 (H \partial_\mu \phi^2) - \\ & \frac{1}{2}g^2 H (\partial_\mu \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (H \partial_\mu \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (H \partial_\mu \phi^2) - \frac{1}{2}g^2 M_{\tilde{q}}^2 (1 - \gamma^2) \phi^2 - \\ & \frac{1}{2}g^2 M_{\tilde{q}}^2 (1 - \gamma^2) \phi^2 - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) + \\ & \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) + \\ & \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) + \\ & \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) + \\ & \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) + \\ & \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) + \\ & \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) - \frac{1}{2}g^2 \partial_\mu^2 (c_u^2 \phi^2 \gamma^2 \phi^2) + \\ & \partial_\mu X^\mu X^\nu) + ig_s A_\mu W_\mu^+ (\partial_\mu X^\mu Y - \partial_\mu Y X^\mu) + ig_s A_\mu W_\mu^- (\partial_\mu X^\mu X^\nu - \\ & \partial_\mu X^\mu X^\nu) + ig_s A_\mu W_\mu^+ (\partial_\mu X^\mu Y - \partial_\mu Y X^\mu) + ig_s A_\mu W_\mu^- (\partial_\mu X^\mu X^\nu - \\ & \partial_\mu X^\mu X^\nu) - \frac{1}{2}g M \left(X^\mu X^\nu H + X^\mu X^\nu H + \frac{1}{2}X^\mu X^\nu H \right) - \frac{1}{2}g M \left(X^\mu X^\nu \phi^2 - X^\mu X^\nu \phi^2 \right) + \\ & \frac{1}{2}g M \left(X^\mu X^\nu \phi^2 - X^\mu X^\nu \phi^2 \right) - g g M S_{\tilde{q}} \left(X^\mu X^\nu \phi^2 - X^\mu X^\nu \phi^2 \right) + \\ & \frac{1}{2}g M \left(X^\mu X^\nu \phi^2 - X^\mu X^\nu \phi^2 \right) \end{aligned}$$





Orion Nebula, Webb Telescope

$$\begin{aligned} \mathcal{L}_{\text{M}} = & -\tfrac{1}{2}\partial_x\partial_y\partial_z\partial_w - g_i I^{\mu\nu}\partial_\mu\partial_\nu\partial_\rho\partial_\sigma - \tfrac{1}{2}I^{\mu\nu}I^{\rho\sigma}\partial_\mu\partial_\nu\partial_\rho\partial_\sigma - \partial_\mu W_\mu^\alpha W_\nu^\beta - \\ & M^\alpha W_\nu^\beta W_\mu^\gamma - \tfrac{1}{2}\partial_\mu Z_\nu^\alpha Z_\mu^\beta - \tfrac{1}{2}\partial_\mu A_\nu A_\mu - ig_c(A_\mu Z_\nu^\alpha W_\nu^\beta W_\mu^\gamma - \\ & \tfrac{1}{2}g_s A_\mu Z_\nu^\alpha Z_\mu^\beta + \tfrac{1}{2}g_s A_\nu Z_\mu^\alpha Z_\mu^\beta - \tfrac{1}{2}g_s A_\mu A_\nu Z_\mu^\alpha Z_\mu^\beta) + \tfrac{1}{2}g_s A_\mu A_\nu Z_\mu^\alpha Z_\mu^\beta - \\ & ig_s A_\mu A_\nu Z_\mu^\alpha Z_\mu^\beta + \tfrac{1}{2}g_s A_\mu A_\nu Z_\mu^\alpha Z_\mu^\beta + \tfrac{1}{2}g_s A_\mu A_\nu Z_\mu^\alpha Z_\mu^\beta + \tfrac{1}{2}g_s A_\mu A_\nu Z_\mu^\alpha Z_\mu^\beta - \\ & W_\mu^\alpha W_\nu^\beta W_\mu^\gamma - \tfrac{1}{2}g_s W_\mu^\alpha W_\nu^\beta W_\mu^\gamma + \tfrac{1}{2}g_s W_\mu^\alpha W_\nu^\beta W_\mu^\gamma + \tfrac{1}{2}g_s W_\mu^\alpha W_\nu^\beta W_\mu^\gamma - \\ & Z_\mu^\alpha Z_\nu^\beta W_\mu^\gamma - g^2 c_s c_a (A_\mu W_\nu^\alpha W_\mu^\gamma - A_\nu W_\mu^\alpha W_\mu^\gamma) + g^2 s_w c_a (A_\mu Z_\nu^\alpha W_\mu^\gamma - \\ & W_\mu^\alpha W_\nu^\beta) - 2 A_\mu W_\nu^\alpha W_\mu^\gamma - \tfrac{1}{2}g_s H_0 H - 2 A_\mu P_0 H^\mu - \partial_\mu \partial_\nu \partial_\rho \partial_\sigma - \tfrac{1}{2}\partial_\mu \partial_\nu \partial_\rho \partial_\sigma - \\ & \partial_\mu (\tfrac{M_\mu}{M_\nu} + \tfrac{M_\mu}{M_\nu}) \partial_\nu \partial_\rho \partial_\sigma + \tfrac{1}{2}(\partial_\mu \partial_\nu \partial_\rho \partial_\sigma)^2 + \tfrac{M_\mu}{M_\nu} \partial_\nu \partial_\rho \partial_\sigma - \\ & \tfrac{1}{2}g^2 \alpha_s (H^2 + (\phi^2)^2 + 2(\phi^2)^2 H + 2(\phi^2)^2 H^2 + \\ & 4(\phi^2)^2 \phi^2 + 4(\phi^2)^2 \phi^2 + 4(\phi^2)^2 \phi^2 + 4(\phi^2)^2 \phi^2 + 2(\phi^2)^2 H^2) - \\ & \tfrac{1}{2}ig (W_+^*(\partial_\mu \phi^+ - \phi^- \partial_\mu \phi^+) - W_-^*(\partial_\mu \phi^+ - \phi^- \partial_\mu \phi^+) + \\ & \tfrac{1}{2}g (W_+^*(H_0 \partial_\mu \phi^+ - \phi^- \partial_\mu H) + W_-^*(H_0 \partial_\mu \phi^+ - \phi^- \partial_\mu H)) + \tfrac{1}{2}g \tfrac{1}{2}Z_0^*(H_0 \partial_\mu \phi^+ - \phi^- \partial_\mu H) + \\ & M \tfrac{1}{2}Z_0^*(\partial_\mu \phi^+ - \phi^- \partial_\mu H) + \tfrac{1}{2}g^2 \tfrac{1}{2}Z_0^*(\partial_\mu \phi^+ - \phi^- \partial_\mu H) + \tfrac{1}{2}g^2 \tfrac{1}{2}Z_0^*(\partial_\mu \phi^+ - \phi^- \partial_\mu H) + M \lambda_s (W_\mu^\alpha \phi^+ - \\ & W_\nu^\alpha \phi^+ - \tfrac{1}{2}g^2 Z_0^*(\partial_\mu \phi^+ - \phi^- \partial_\mu H) + \tfrac{1}{2}g^2 Z_0^*(\partial_\mu \phi^+ - \phi^- \partial_\mu H) + \tfrac{1}{2}g^2 Z_0^*(\partial_\mu \phi^+ - \phi^- \partial_\mu H) - \\ & \tfrac{1}{2}g^2 W_\mu^\alpha W_\nu^\beta (H^2 \phi^+ + W_\mu^\alpha \phi^+ - \tfrac{1}{2}g^2 Z_0^* H W_\mu^\alpha \phi^+ - W_\nu^\beta \phi^+ - \tfrac{1}{2}g^2 A_\mu A_\nu W_\mu^\alpha \phi^+ + \\ & \tfrac{1}{2}g^2 A_\mu^2 W_\mu^\alpha \phi^+ + \tfrac{1}{2}g^2 A_\mu A_\nu H W_\mu^\alpha \phi^+ - W_\nu^\beta \phi^+ - g^2 \tfrac{1}{2}(2d - 1)Z_0^* A_\mu \phi^+ - \\ & g^2 s_w^2 A_\mu A_\nu \phi^+ + \tfrac{1}{2}g_s A_\mu^2 (g^2 s_w^2 \phi^+)^2 - e^2 (g^2 s_w^2 \phi^+)^2 - \nu^2 (\phi^+ + m^2 \phi^+) \nu^2 - \nu^2 (\phi^+ + m^2 \phi^+) \nu^2 - g^2 s_w^2 \phi^+ + \\ & g^2 s_w^2 A_\mu A_\nu \phi^+ + \tfrac{1}{2}g_s A_\mu^2 (g^2 s_w^2 \phi^+)^2 - e^2 (g^2 s_w^2 \phi^+)^2 - \nu^2 (\phi^+ + m^2 \phi^+) \nu^2 - \nu^2 (\phi^+ + m^2 \phi^+) \nu^2 - \tfrac{1}{2}g^2 Z_0^* (g^2 s_w^2 \phi^+)^2 + \tfrac{1}{2}g^2 Z_0^* (g^2 s_w^2 \phi^+)^2 + \\ & (g^2 s_w^2 \phi^+ + \tfrac{1}{2}g^2 Z_0^* (g^2 s_w^2 \phi^+)^2 + \tfrac{1}{2}g^2 W_\mu^\alpha ((g^2 s_w^2 \phi^+ + \tfrac{1}{2}g^2 Z_0^* (g^2 s_w^2 \phi^+)^2 + (g^2 s_w^2 \phi^+ + \tfrac{1}{2}g^2 Z_0^* (g^2 s_w^2 \phi^+)^2) + \\ & \tfrac{1}{2}g^2 W_\mu^\alpha ((g^2 s_w^2 \phi^+ + \tfrac{1}{2}g^2 Z_0^* (g^2 s_w^2 \phi^+)^2 + (g^2 s_w^2 \phi^+ + \tfrac{1}{2}g^2 Z_0^* (g^2 s_w^2 \phi^+)^2) + \\ & \tfrac{1}{2}g^2 \phi^+ (-m^2 (U^\mu_{\alpha\mu} U^\nu_{\alpha\nu} (1 - \nu^2) \nu^2) - m^2 (U^\mu_{\alpha\mu} U^\nu_{\alpha\nu} (1 - \nu^2) \nu^2) + \\ & \tfrac{1}{2}g^2 \phi^+ (m^2 (U^\mu_{\alpha\mu} U^\nu_{\alpha\nu} (1 - \nu^2) \nu^2) - m^2 (U^\mu_{\alpha\mu} U^\nu_{\alpha\nu} (1 - \nu^2) \nu^2) - \tfrac{1}{2}g^2 H (U^\mu_{\alpha\mu} \phi^+) - \\ & \tfrac{1}{2}g^2 H (U^\nu_{\alpha\nu} \phi^+) - \tfrac{1}{2}g^2 Z_0^* (U^\mu_{\alpha\mu} \phi^+) - \tfrac{1}{2}g^2 Z_0^* (U^\nu_{\alpha\nu} \phi^+) - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ - \\ & \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ + \\ & \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ + \\ & \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ + \\ & \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ + \\ & \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ + \\ & \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ + \\ & \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\nu \phi^+ - \tfrac{1}{2}g_s M_s (1 - \eta) \partial_\mu \phi^+ + \\ & \partial_\mu X^\mu X^\nu) + ig_s W_\mu^\alpha (\partial_\mu X^\mu Y - \partial_\nu X^\nu Y) + ig_s W_\mu^\alpha (\partial_\mu X^\mu X^\nu - \\ & \partial_\nu X^\mu X^\nu) + ig_s W_\mu^\alpha (\partial_\mu X^\mu X^\nu - \partial_\nu X^\mu X^\nu) + ig_s W_\mu^\alpha (\partial_\mu X^\mu X^\nu - \\ & \partial_\nu X^\mu X^\nu) - \tfrac{1}{2}g M \left(X^\mu X^\nu H + X^\mu X^\nu H + \tfrac{1}{2}X^\mu X^\nu H \right) + \tfrac{1}{2}g M \left(X^\mu X^\nu \phi^+ - X^\mu X^\nu \phi^+ \right) + \\ & \tfrac{1}{2}g M \left(X^\mu X^\nu \phi^+ - X^\mu X^\nu \phi^+ \right) + g_2 M S_{\mu\nu} \left(X^\mu X^\nu \phi^+ - X^\mu X^\nu \phi^+ \right) + \\ & \tfrac{1}{2}g M \left(X^\mu X^\nu \phi^+ - X^\mu X^\nu \phi^+ \right) \end{aligned}$$



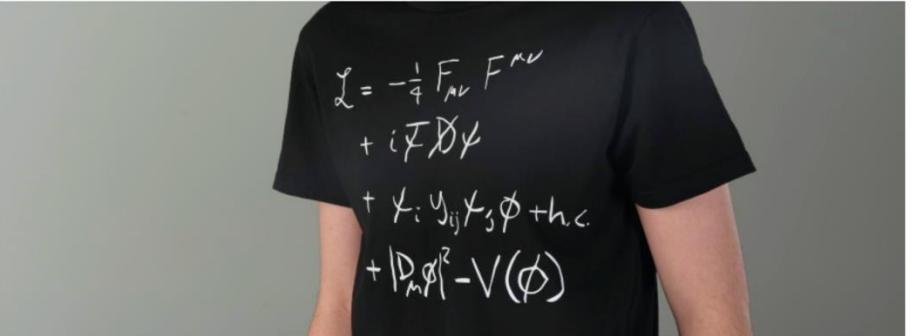
$$\begin{aligned}
\mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_\mu^2 f^{abc} f^{ade} g_\nu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
& M^2 W_\mu^+ W_\mu^- - \frac{1}{2} \partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2} \partial_\mu A_\nu \partial_\mu A_\nu - ig c_w (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - Z_\mu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)) - \\
& igs_w (\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - \\
& W_\nu^- \partial_\nu W_\mu^+)) - \frac{1}{2} g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2} g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - \\
& Z_\mu^0 Z_\mu^0 W_\nu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\mu W_\nu^+ W_\nu^-) + g^2 s_w c_w (A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - 2 A_\mu Z_\mu^0 W_\nu^+ W_\nu^-) - \frac{1}{2} \partial_\mu H \partial_\mu H - 2 M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2} \partial_\mu \phi^0 \partial_\mu \phi^0 - \\
& \beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^4}{g^2} \alpha_h - \\
& g \alpha_h M (H^3 + H \phi^0 \phi^0 + 2H \phi^+ \phi^-) - \\
& \frac{1}{8} g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
& g M W_\mu^+ W_\mu^- H - \frac{1}{2} g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \\
& \frac{1}{2} i g (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
& \frac{1}{2} g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2} g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
& M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + igs_w M A_\mu (W_\mu^+ \phi^- - \\
& W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + igs_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
& \frac{1}{4} g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{8} g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
& \frac{1}{2} g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2} i g^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2} g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
& W_\mu^- \phi^+) + \frac{1}{2} i g^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
& g^2 s_w^2 A_\mu \phi^+ \phi^- + \frac{1}{2} i g_s \lambda_{ij}^a (\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda (\gamma \partial + m_\nu^\lambda) \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + \\
& m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + m_d^\lambda) d_j^\lambda + igs_w A_\mu (-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3} (\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3} (\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)) + \\
& \frac{ig}{4c_w} Z_\mu^0 \{(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) d_j^\lambda) + \\
& (\bar{u}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 + \gamma^5) u_j^\lambda)\} + \frac{ig}{2\sqrt{2}} W_\mu^+ ((\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) U^{lep}{}_{\lambda\kappa} e^\kappa) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)) + \\
& \frac{ig}{2\sqrt{2}} W_\mu^- ((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda} \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\kappa\lambda}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda)) + \\
& \frac{ig}{2M\sqrt{2}} \phi^+ (-m_e^\kappa (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 - \gamma^5) e^\kappa) + m_\nu^\lambda (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 + \gamma^5) e^\kappa) + \\
& \frac{ig}{2M\sqrt{2}} \phi^- (m_e^\lambda (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1 + \gamma^5) \nu^\kappa) - m_\nu^\kappa (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1 - \gamma^5) \nu^\kappa) - \frac{g}{2} \frac{m_\lambda}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
& \frac{g}{2} \frac{m_\lambda}{M} H (\bar{e}^\lambda e^\lambda) + \frac{ig}{2} \frac{m_\lambda}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \frac{ig}{2} \frac{m_\lambda}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \hat{\nu}_\kappa - \\
& \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \hat{\nu}_\kappa + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_d^\kappa (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa)) + \\
& \frac{ig}{2M\sqrt{2}} \phi^- (m_d^\lambda (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^\kappa) - \frac{g}{2} \frac{m_u}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \\
& \frac{g}{2} \frac{m_d}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_d}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_d}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c + \\
& \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + ig c_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\
& \partial_\mu \bar{X}^+ X^0) + igs_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + ig c_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
& \partial_\mu \bar{X}^0 X^+) + igs_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + ig c_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^- - \\
& \partial_\mu \bar{X}^- X^-) + igs_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
& \partial_\mu \bar{X}^- X^-) - \frac{1}{2} g M \left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H \right) + \frac{1-2c_w^2}{2c_w} ig M (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\
& \frac{1}{2c_w} ig M (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + ig M s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \\
& \frac{1}{2} ig M (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0) .
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
& M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - ig c_w (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - Z_\mu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)) - \\
& ig s_w (\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - \\
& W_\nu^- \partial_\nu W_\mu^+)) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - \\
& Z_\mu^0 Z_\mu^0 W_\nu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\mu W_\nu^+ W_\nu^-) + g^2 s_w c_w (A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
& \beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^4}{g^2} \alpha_h - \\
& g \alpha_h M (H^3 + H \phi^0 \phi^0 + 2H \phi^+ \phi^-) - \\
& \frac{1}{8}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
& g M W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \\
& \frac{1}{2}ig (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
& \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
& M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - \\
& W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
& \frac{1}{4}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{8}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
& \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
& W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
& g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \frac{1}{2}ig s_w \lambda_{ij}^a (\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda (\gamma \partial + m_\nu^\lambda) \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + \\
& m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + m_d^\lambda) d_j^\lambda + ig s_w A_\mu (-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)) + \\
& \frac{ig}{4c_w} Z_\mu^0 \{(\bar{\nu}_\mu \gamma^\mu \nu_\mu) - (\bar{e}_\mu \gamma^\mu e_\mu) + (\bar{u}_\mu \gamma^\mu u_\mu) - (\bar{d}_\mu \gamma^\mu d_\mu)\} + \\
& (\bar{u}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2) \bar{d}_j^\lambda) + \bar{e}_j^\lambda \bar{\nu}_\lambda \frac{1}{2M} \phi^- \left(\frac{ig}{2M\sqrt{2}} \phi^- \left(\frac{g m_\lambda}{2 M} H \left(\frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \frac{ig}{2M\sqrt{2}} \phi^- \right) \right) \right) \right) + \\
& \bar{X}^+ (\partial^2 - M^2) \partial_\mu \partial_\nu
\end{aligned}$$

$$\begin{aligned}
& \partial_\mu \bar{X}^- X^-) - \frac{1}{2}! \frac{1}{2c_w} i \\
& \partial_\mu \bar{X}^- X^-) - \frac{1}{2}!
\end{aligned}$$

$$\frac{1}{2c_w} i$$

Standard Model formula T-Shirt



"Lagrangian of the Standard Model of particle physics"
 $\frac{1}{2}ig M (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0)$.

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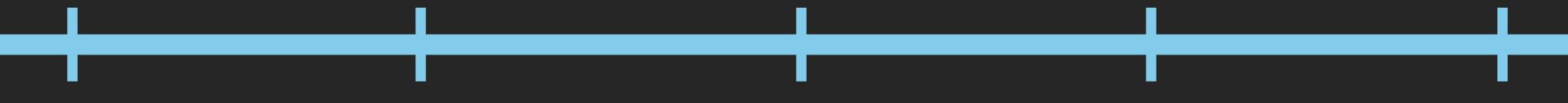
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$$\begin{aligned}
\mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_\mu^2 f^{abc} f^{ade} g_\nu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
& M^2 W_\mu^+ W_\mu^- - \frac{1}{2} \partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2} \partial_\mu A_\nu \partial_\mu A_\nu - ig c_w (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - Z_\mu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)) - \\
& igs_w (\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - \\
& W_\nu^- \partial_\nu W_\mu^+)) - \frac{1}{2} g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2} g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - \\
& Z_\mu^0 Z_\mu^0 W_\nu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\mu W_\nu^+ W_\nu^-) + g^2 s_w c_w (A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - 2 A_\mu Z_\mu^0 W_\nu^+ W_\nu^-) - \frac{1}{2} \partial_\mu H \partial_\mu H - 2 M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2} \partial_\mu \phi^0 \partial_\mu \phi^0 - \\
& \beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^4}{g^2} \alpha_h - \\
& g \alpha_h M (H^3 + H \phi^0 \phi^0 + 2H \phi^+ \phi^-) - \\
& \frac{1}{8} g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
& g M W_\mu^+ W_\mu^- H - \frac{1}{2} g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \\
& \frac{1}{2} i g (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
& \frac{1}{2} g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2} g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
& M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + igs_w M A_\mu (W_\mu^+ \phi^- - \\
& W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + igs_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
& \frac{1}{4} g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{8} g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
& \frac{1}{2} g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2} i g^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2} g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
& W_\mu^- \phi^+) + \frac{1}{2} i g^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
& g^2 s_w^2 A_\mu \phi^+ \phi^- + \frac{1}{2} i g_s \lambda_{ij}^a (\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda (\gamma \partial + m_\nu^\lambda) \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + \\
& m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + m_d^\lambda) d_j^\lambda + igs_w A_\mu (-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3} (\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3} (\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)) + \\
& \frac{ig}{4c_w} Z_\mu^0 \{(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) d_j^\lambda) + \\
& (\bar{u}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 + \gamma^5) u_j^\lambda)\} + \frac{ig}{2\sqrt{2}} W_\mu^+ ((\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) U^{lep}{}_{\lambda\kappa} e^\kappa) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)) + \\
& \frac{ig}{2\sqrt{2}} W_\mu^- ((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda} \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\kappa\lambda}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda)) + \\
& \frac{ig}{2M\sqrt{2}} \phi^+ (-m_e^\kappa (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 - \gamma^5) e^\kappa) + m_\nu^\lambda (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 + \gamma^5) e^\kappa) + \\
& \frac{ig}{2M\sqrt{2}} \phi^- (m_e^\lambda (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1 + \gamma^5) \nu^\kappa) - m_\nu^\kappa (\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa} (1 - \gamma^5) \nu^\kappa) - \frac{g}{2} \frac{m_\lambda}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
& \frac{g}{2} \frac{m_\lambda}{M} H (\bar{e}^\lambda e^\lambda) + \frac{ig}{2} \frac{m_\lambda}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \frac{ig}{2} \frac{m_\lambda}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \hat{\nu}_\kappa - \\
& \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \hat{\nu}_\kappa + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_d^\kappa (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa)) + \\
& \frac{ig}{2M\sqrt{2}} \phi^- (m_d^\lambda (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^\kappa) - \frac{g}{2} \frac{m_u}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \\
& \frac{g}{2} \frac{m_d}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_d}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_d}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c + \\
& \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + ig c_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\
& \partial_\mu \bar{X}^+ X^0) + igs_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + ig c_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
& \partial_\mu \bar{X}^0 X^+) + igs_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + ig c_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^- - \\
& \partial_\mu \bar{X}^- X^-) + igs_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
& \partial_\mu \bar{X}^- X^-) - \frac{1}{2} g M \left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H \right) + \frac{1-2c_w^2}{2c_w} ig M (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\
& \frac{1}{2c_w} ig M (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + ig M s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \\
& \frac{1}{2} ig M (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0) .
\end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{int}} = & -\frac{1}{2}\partial_{\mu}\partial_{\nu}\partial_{\lambda}\partial_{\mu} - g_{\mu\nu}\partial_{\lambda}\partial_{\mu}\partial_{\lambda}\partial_{\mu} - \frac{1}{4}\partial_{\mu}^2f^{\mu\nu}\partial_{\nu}g_{\mu\nu}g_{\mu}^{\nu} - \partial_{\mu}W_{\mu}^{\alpha}W_{\nu}^{-\alpha} - \\ & M^2W_{\mu}^{\alpha}W_{\nu}^{-\alpha} - \frac{1}{2}\partial_{\mu}Z_{\mu}^{\nu}\partial_{\nu}Z_{\nu}^{-\mu} - \frac{1}{2M}M^2Z_{\mu}^{\nu}Z_{\nu}^{-\mu} - \frac{1}{2}[\partial_{\mu}A_{\nu}\partial_{\nu}A_{\mu} - ig_{\nu\mu}c_s(A_{\nu}^2W_{\mu}^{\alpha}W_{\nu}^{-\alpha} - \\ & - f_{\nu\mu}A_{\nu}Z_{\nu}^{\mu}Z_{\nu}^{-\mu})] - \frac{1}{2}W_{\mu}^{\alpha}W_{\nu}^{-\alpha}[Z_{\mu}^{\nu}Z_{\nu}^{-\mu} + (W_{\mu}^{\alpha}W_{\nu}^{-\alpha} + W_{\nu}^{-\alpha}W_{\mu}^{\alpha})] - \\ & ig_{\nu\mu}(\frac{1}{2}(W_{\mu}^{\alpha}W_{\nu}^{-\alpha} + W_{\nu}^{-\alpha}W_{\mu}^{\alpha})) + ig_{\nu\mu}c_s(A_{\nu}^2W_{\mu}^{\alpha}W_{\nu}^{-\alpha} - \\ & - W_{\mu}^{\alpha}W_{\nu}^{-\alpha}) - \frac{1}{2}g^2W_{\mu}^{\alpha}W_{\nu}^{-\alpha} - \frac{1}{2}g^2W_{\mu}^{\alpha}W_{\nu}^{-\alpha} + i\sqrt{2}(Z_{\mu}^{\nu}W_{\nu}^{\alpha}Z_{\nu}^{-\mu} - \\ & Z_{\mu}^{\mu}W_{\nu}^{\alpha}Z_{\nu}^{-\mu}) + g^2c_s(A_{\nu}W_{\mu}^{\alpha}W_{\nu}^{-\alpha} - A_{\mu}A_{\nu}W_{\mu}^{\alpha}W_{\nu}^{-\alpha}) + g^2s_c(A_{\nu}Z_{\mu}^{\nu}W_{\mu}^{\alpha}W_{\nu}^{-\alpha} - \\ & W_{\mu}^{\alpha}W_{\nu}^{-\alpha}) - 2A_{\mu}c_s(W_{\mu}^{\alpha}W_{\nu}^{-\alpha}) - \frac{1}{2}gH\partial_{\mu}H - 2A_{\mu}F\partial_{\mu}H^2 - \partial_{\mu}A_{\mu} - \frac{1}{2}\partial_{\mu}\partial_{\mu}\partial_{\mu}\partial_{\mu} - \\ & \partial_{\mu}(\frac{M^2}{2}(W_{\mu}^{\alpha}W_{\nu}^{-\alpha} + W_{\nu}^{-\alpha}W_{\mu}^{\alpha})) + M^2\alpha_{\mu} - \\ & ig^2\alpha_{\mu}(H^2 + (\phi^2)^2 + 2H\phi\partial^{\mu}\phi) + 2H\phi\partial^{\mu}\phi + 2(\phi\partial^{\mu})^2H^2 - \\ & \frac{1}{2}g^2\alpha_{\mu}(H^2 + (\phi^2)^2 + 4(\phi\partial^{\mu}\phi)^2 + 4(\phi\partial^{\mu}\phi)^2 + 4H\phi\partial^{\mu}\phi + 2(\phi\partial^{\mu})^2H^2 - \\ & gM^2W_{\mu}^{\alpha}W_{\nu}^{-\alpha}H - \frac{1}{2}\partial_{\mu}Z_{\mu}^{\nu}H - \\ & \frac{1}{2}ig(W_{\mu}^{\alpha}(\partial_{\mu}\phi^{\nu} - \phi^{\nu}\partial_{\mu}\phi) - W_{\nu}^{\alpha}(\partial_{\mu}\phi^{\nu} - \phi^{\nu}\partial_{\mu}\phi)) + \\ & \frac{1}{2}g(W_{\mu}^{\alpha}(\partial_{\mu}\phi^{\nu} - \phi^{\nu}\partial_{\mu}\phi) + W_{\nu}^{\alpha}(\partial_{\mu}\phi^{\nu} - \phi^{\nu}\partial_{\mu}\phi) - \frac{1}{2}g^2\frac{1}{2}(\partial_{\mu}^2(H\partial_{\mu}\phi^{\nu} - \phi^{\nu}\partial_{\mu}\phi)) + \\ & M(\frac{1}{2}\partial_{\mu}^2\phi^{\nu} - \frac{1}{2}\partial_{\mu}^2W_{\mu}^{\alpha}W_{\nu}^{-\alpha}) - \frac{1}{2}g^2(\partial_{\mu}^2W_{\mu}^{\alpha}W_{\nu}^{-\alpha} + \partial_{\mu}^2W_{\nu}^{\alpha}W_{\mu}^{-\alpha}) + M(\partial_{\mu}W_{\mu}^{\alpha}\partial_{\mu}W_{\nu}^{-\alpha} - \\ & W_{\mu}^{-\alpha}\partial_{\mu}W_{\nu}^{\alpha}) - \frac{1}{2}g^2(\partial_{\mu}^2Z_{\mu}^{\nu}W_{\nu}^{-\alpha} - \partial_{\mu}^2Z_{\mu}^{\nu}W_{\nu}^{-\alpha} - 2(2x^2 - 1)\partial^{\mu}\phi^{\nu}\partial^{\nu}\phi^{\mu}) - \\ & \frac{1}{2}g^2W_{\mu}^{\alpha}W_{\nu}^{-\alpha}(H^2 + (\phi^2)^2 + 2\phi^{\mu}\partial^{\nu}\phi^{\mu} - \\ & - \frac{1}{2}g^2Z_{\mu}^{\nu}Z_{\nu}^{-\mu}(H^2\phi^{\nu} + W_{\nu}^{-\alpha}) - \frac{1}{2}g^2Z_{\mu}^{\nu}H(W_{\nu}^{\alpha} - W_{\nu}^{-\alpha}) + \frac{1}{2}g^2A_{\mu}A_{\nu}W_{\mu}^{\alpha}W_{\nu}^{-\alpha} + \\ & - \frac{1}{2}g^2c_s^2A_{\mu}A_{\nu}\phi^{\mu}\phi^{\nu} + \frac{1}{2}g^2s_c^2A_{\mu}A_{\nu}W_{\mu}^{\alpha}W_{\nu}^{-\alpha} - \varepsilon^{\mu\nu}(\phi^{\mu}\phi^{\nu})^2 - \varepsilon^{\mu\nu}(\phi^{\mu}\phi^{\nu})^2 + m^2\phi^{\mu}\phi^{\nu} - \varepsilon^{\mu\nu}(\phi^{\mu}\phi^{\nu})^2 - \\ & g^2s_c^2A_{\mu}A_{\nu}\phi^{\mu}\phi^{\nu} + \frac{1}{2}g^2s_c^2A_{\mu}A_{\nu}W_{\mu}^{\alpha}W_{\nu}^{-\alpha} + \varepsilon^{\mu\nu}(\phi^{\mu}\phi^{\nu})^2 + \varepsilon^{\mu\nu}(\phi^{\mu}\phi^{\nu})^2 - m^2\phi^{\mu}\phi^{\nu} - \varepsilon^{\mu\nu}(\phi^{\mu}\phi^{\nu})^2 + \\ & g^2c_s^2A_{\mu}A_{\nu}W_{\mu}^{\alpha}W_{\nu}^{-\alpha} + \frac{1}{2}g^2c_s^2A_{\mu}A_{\nu}W_{\mu}^{\alpha}W_{\nu}^{-\alpha} + \varepsilon^{\mu\nu}(\phi^{\mu}\phi^{\nu})^2 + \varepsilon^{\mu\nu}(\phi^{\mu}\phi^{\nu})^2 + \frac{1}{2}g^2(\partial_{\mu}W_{\mu}^{\alpha}(W_{\nu}^{\alpha} - W_{\nu}^{-\alpha}) + W_{\mu}^{\alpha}\partial_{\mu}W_{\nu}^{\alpha} - \partial_{\mu}W_{\mu}^{\alpha}\partial_{\mu}W_{\nu}^{\alpha}) + \\ & (s_c^2 + r^2)m^2 + \frac{1}{2}g^2W_{\mu}^{\alpha}(W_{\mu}^{\alpha}s_c^2 + r^2r^2m^2) + \frac{1}{2}g^2W_{\nu}^{\alpha}(W_{\nu}^{\alpha}s_c^2 + r^2r^2m^2) + \frac{1}{2}g^2W_{\nu}^{\alpha}W_{\mu}^{\alpha}(s_c^2s_c^2 + r^2r^2m^2) + \\ & + \frac{1}{2}g^2W_{\nu}^{\alpha}W_{\mu}^{\alpha}(r^2s_c^2s_c^2 + r^2r^2m^2) - m^2(\partial_{\mu}W_{\mu}^{\alpha}s_c^2 + r^2r^2m^2) + \\ & - \frac{u^2}{2M}g^2\phi^{\mu}(\tau\eta C_{\mu}^{\nu\alpha} + \tau^2s_c^2r^2m^2) - m^2(\partial_{\mu}W_{\mu}^{\alpha}s_c^2)(1 - r^2)s_c^2 - \frac{1}{2}g^2(H\partial^{\mu}\phi^{\nu}) - \\ & - \frac{1}{2}g^2H(\partial^{\mu}\phi^{\nu}) - \frac{1}{2}g^2\partial^{\mu}(\phi^{\nu}\partial^{\mu}\phi^{\nu}) - \frac{1}{2}g^2\partial^{\mu}(\phi^{\nu}\partial^{\mu}\phi^{\nu}) - \partial_{\mu}M_{\mu}^2(1 - r^2)\partial_{\nu} - \\ & \frac{1}{2}\partial_{\mu}M_{\mu}^2(1 - r^2)\partial_{\nu} - \frac{1}{2}g^2\partial^{\mu}C_{\mu}^{\nu\alpha} - m^2(\partial_{\mu}C_{\mu}^{\nu\alpha}(1 - r^2)s_c^2) + \\ & - \frac{3}{2}g^2\partial_{\mu}C_{\mu}^{\nu\alpha}(1 - r^2)s_c^2) - \frac{1}{2}g^2H(\partial_{\mu}\phi^{\nu}) + \\ & \frac{3}{2}g^2(\partial_{\mu}C_{\mu}^{\nu\alpha}(1 - r^2)s_c^2) - \frac{1}{2}g^2\partial_{\mu}(\phi^{\nu}\partial^{\mu}\phi^{\nu}) + G^{\mu\nu}P\phi^{\nu} + f^{\mu\nu}\partial_{\mu}\phi^{\nu}G^{\nu\mu}\phi^{\mu} + \\ & X^{\mu}\partial^{\nu}X^{\mu} + X^{\nu}\partial^{\mu}X^{\mu} + X^{\mu}\partial^{\nu}X^{\mu} - \frac{M^2}{2}X^{\mu} + Y^{\mu}\partial^{\nu}Y^{\mu} + ig_sW_{\nu}^{\mu}(\partial_{\mu}X^{\nu} - \partial_{\nu}X^{\mu}) + ig_sW_{\nu}^{\mu}(\partial_{\mu}Y^{\nu} - \partial_{\nu}Y^{\mu}) + \\ & \partial_{\mu}X^{\nu}X^{\mu} + ig_sW_{\nu}^{\mu}(\partial_{\mu}X^{\nu} - \partial_{\nu}X^{\mu}) + ig_sA_{\mu}A_{\nu}X^{\mu}X^{\nu} - \\ & \partial_{\mu}X^{\nu}X^{\mu} - \frac{1}{2}gM(X^{\mu}X^{\nu}H + X^{\nu}X^{\mu}H + \frac{1}{2}X^{\mu}X^{\nu}H) + \frac{1}{2}g^2igM(X^{\mu}X^{\nu}\phi^{\mu} - X^{\nu}X^{\mu}\phi^{\mu}) + \\ & \frac{1}{2}gM(X^{\mu}X^{\nu}\phi^{\mu} - X^{\nu}X^{\mu}\phi^{\mu}) + ig_2Ms_{\mu}(X^{\mu}X^{\nu}\phi^{\mu} - X^{\nu}X^{\mu}\phi^{\mu}) + \\ & \frac{1}{2}gM(X^{\mu}X^{\nu}\phi^{\mu} - X^{\nu}X^{\mu}\phi^{\mu}) \end{aligned}$$



Meters

Perturbation Theory

Taylor Series

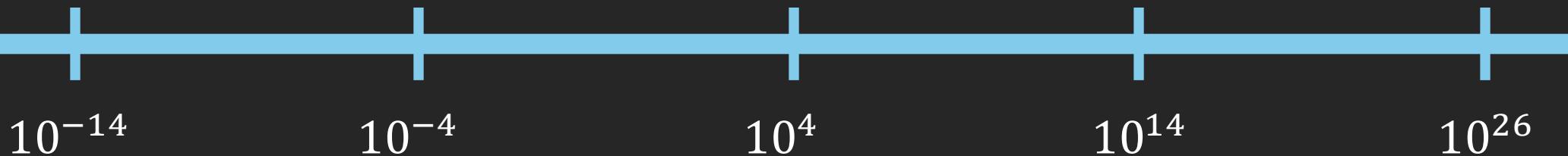
$$f(x \pm h) = f(x) \pm h \frac{df}{dx} + \frac{1}{2!} h^2 \frac{d^2 f}{dx^2} \pm \frac{1}{3!} h^3 \frac{d^3 f}{dx^3} + O(h^4)$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots$$

$$|n\rangle = |n^{(0)}\rangle + \lambda |n^{(1)}\rangle + \lambda^2 |n^{(2)}\rangle + \dots$$

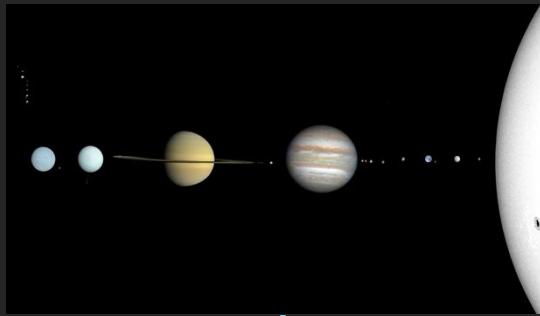
Renormalization and other Dimensionality Reduction Methods

$$\begin{aligned} \mathcal{L}_{eff} = & -\tfrac{1}{2}\partial_\mu\phi^\dagger\partial_\mu\phi - g_s I^{abc}\partial_\mu\phi^\dagger\partial_\mu\phi - \tfrac{1}{4}g_1^2 I^{abc}I^{def}\phi^\dagger\phi^\dagger\phi^\dagger\phi - \partial_\mu W_\mu^+ \partial_\mu W_\mu^- - \\ & M^2 W_\mu^+ W_\mu^- - \tfrac{1}{2}\partial_\mu Z_\mu^+ \partial_\mu Z_\mu^- - \tfrac{1}{2}\partial_\mu^2 Z_\mu^+ Z_\mu^- - \tfrac{1}{2}\partial_\mu A_\mu^+ A_\mu^- - ig_{sc}(\partial_\mu Z_\mu^+ W_\mu^+ W_\mu^- - \\ & W_\mu^+ \partial_\mu Z_\mu^+ - W_\mu^- \partial_\mu Z_\mu^- - W_\mu^+ W_\mu^- Z_\mu^+ Z_\mu^- - Z_\mu^+ W_\mu^+ W_\mu^- - W_\mu^+ \partial_\mu Z_\mu^+ W_\mu^- - \\ & W_\mu^- \partial_\mu Z_\mu^- W_\mu^+ - W_\mu^+ W_\mu^- W_\mu^+ - W_\mu^+ W_\mu^- W_\mu^- + i\sqrt{2}(Z_\mu^+ W_\mu^+ Z_\mu^- W_\mu^- - \\ & Z_\mu^+ Z_\mu^- W_\mu^+ W_\mu^-) + g_{sc}^2(A_\mu^+ W_\mu^+ A_\mu^- - A_\mu^+ W_\mu^- W_\mu^+ - g_{sc}^2 c_{sc}(A_\mu^+ Z_\mu^+ W_\mu^- - \\ & W_\mu^+ W_\mu^- - 2A_\mu^+ W_\mu^- W_\mu^+) - \tfrac{1}{2}g_s H_0 H - 2g_s^2 H^2 - \phi \cdot \partial_\mu \phi - \tfrac{1}{2}g^2 \phi \partial_\mu \phi - \\ & \partial_\mu (\tfrac{M^2}{2}W_\mu^+ W_\mu^- + \tfrac{1}{2}g^2 Z_\mu^+ Z_\mu^- + \tfrac{1}{2}g_{sc}^2 Z_\mu^+ W_\mu^- + \tfrac{1}{2}g_{sc}^2 W_\mu^+ Z_\mu^-) + \tfrac{ig}{2}m_{sc} - \\ & \tfrac{1}{2}g^2 m_{sc}^2 (H^2 + (\phi^2)^2 + 4(\phi^2)^2 + 4(\phi^2)^2 + 4(\phi^2)^2 + 4(\phi^2)^2 + 2(\phi^2)^2 H^2) - \\ & \tfrac{1}{2}g^2 m_{sc}^2 (H^2 + (\phi^2)^2 + 4(\phi^2)^2 + 4(\phi^2)^2 + 4(\phi^2)^2 + 4(\phi^2)^2 + 2(\phi^2)^2 H^2) - \\ & \tfrac{1}{2}ig (W_\mu^+(\phi^2 \partial_\mu \phi - \phi \cdot \partial_\mu \phi) - W_\mu^-(\phi^2 \partial_\mu \phi - \phi \cdot \partial_\mu \phi)) + \\ & \tfrac{1}{2}g (W_\mu^+(\partial_\mu^2 \phi^2 - \phi^2 \partial_\mu^2 \phi) + W_\mu^-(\partial_\mu^2 \phi^2 - \phi^2 \partial_\mu^2 \phi)) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(H_0 \phi^2 - \phi^2 \partial_\mu H) + \\ & M (\tfrac{1}{2}Z_\mu^+ Z_\mu^-) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(H_0 \phi^2 - \phi^2 \partial_\mu H) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^-(H_0 \phi^2 - \phi^2 \partial_\mu H) + \\ & M (\tfrac{1}{2}Z_\mu^+ Z_\mu^-) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(H_0 \phi^2 - \phi^2 \partial_\mu H) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^-(H_0 \phi^2 - \phi^2 \partial_\mu H) + \\ & \tfrac{1}{2}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^2)^2 + 2\phi \cdot \phi) - \tfrac{1}{2}g^2 Z_\mu^+ Z_\mu^- (H^2 + (\phi^2)^2 + 2(2\phi - 1)\phi^2 \phi) - \\ & \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(W_\mu^+ \phi^2 + W_\mu^- \phi^2) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(H_0 W_\mu^+ \phi^2 - W_\mu^- \phi^2) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(H_0 W_\mu^- \phi^2 - W_\mu^+ \phi^2) - \\ & g^2 \tfrac{1}{2}Z_\mu^+(A_\mu^+ \phi^2 + A_\mu^- \phi^2) + \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(W_\mu^+ \phi^2 + W_\mu^- \phi^2) - g^2 \tfrac{1}{2}Z_\mu^+(2\phi - 1)Z_\mu^+ A_\mu^+ \phi^2 + \\ & g^2 \tfrac{1}{2}Z_\mu^+(A_\mu^+ \phi^2 + A_\mu^- \phi^2) + \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(W_\mu^+ \phi^2 + W_\mu^- \phi^2) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(2\phi - 1)Z_\mu^+ A_\mu^+ \phi^2 + \\ & g^2 \tfrac{1}{2}Z_\mu^+(A_\mu^+ \phi^2 + A_\mu^- \phi^2) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(W_\mu^+ \phi^2 + W_\mu^- \phi^2) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(2\phi - 1)Z_\mu^+ A_\mu^+ \phi^2 + \\ & g^2 \tfrac{1}{2}Z_\mu^+(A_\mu^+ \phi^2 + A_\mu^- \phi^2) + \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(W_\mu^+ \phi^2 + W_\mu^- \phi^2) - \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(2\phi - 1)Z_\mu^+ A_\mu^+ \phi^2 + \\ & g^2 \tfrac{1}{2}Z_\mu^+(A_\mu^+ \phi^2 + A_\mu^- \phi^2) + \tfrac{1}{2}g^2 \tfrac{1}{2}Z_\mu^+(W_\mu^+ \phi^2 + W_\mu^- \phi^2) + (s_{sc}^2(1 + \gamma^2)C_\mu^2) + \\ & (s_{sc}^2(1 + \gamma^2)n_C) + \tfrac{1}{2}g_s W_\mu^+ ((\phi^2 \partial_\mu \phi + \gamma^2 \partial_\mu^2 \phi) + (\phi^2 C_\mu^2 \partial_\mu \phi + \gamma^2 C_\mu^2)) + \\ & \tfrac{1}{2}g_s W_\mu^- ((\phi^2 \partial_\mu \phi + \gamma^2 \partial_\mu^2 \phi) + (\phi^2 C_\mu^2 \partial_\mu \phi + \gamma^2 C_\mu^2)) + \\ & \tfrac{1}{2}g_s \phi^2 (-m_{sc}^2(U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) - m_{sc}^2(U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) + \\ & \tfrac{1}{2}g_s \phi^2 (m_{sc}^2(U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) - m_{sc}^2(U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) - \tfrac{1}{2}g_s^2 H_0 \phi^2 \phi^2) - \\ & \tfrac{1}{2}g_s^2 H_0 \phi^2 \phi^2 - \tfrac{1}{2}g_s^2 (U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) - \tfrac{1}{2}g_s^2 (U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) - \\ & \tfrac{1}{2}g_s^2 M_{sc}^2 (1 - \gamma^2) \partial_\mu \phi + \tfrac{1}{2}g_s^2 (U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) - m_{sc}^2(C_\mu^2 \partial_\mu \phi + (1 + \gamma^2) \partial_\mu \phi) + \\ & \tfrac{1}{2}g_s^2 M_{sc}^2 (1 - \gamma^2) \partial_\mu \phi - \tfrac{1}{2}g_s^2 (U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) - m_{sc}^2(C_\mu^2 \partial_\mu \phi + (1 - \gamma^2) \partial_\mu \phi) + \\ & \tfrac{1}{2}g_s^2 M_{sc}^2 (1 - \gamma^2) \partial_\mu \phi - \tfrac{1}{2}g_s^2 (U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) - m_{sc}^2(C_\mu^2 \partial_\mu \phi + (1 - \gamma^2) \partial_\mu \phi) + \\ & \tfrac{1}{2}g_s^2 M_{sc}^2 (1 - \gamma^2) \partial_\mu \phi - \tfrac{1}{2}g_s^2 (U_\mu^+ \partial_\mu \phi + U_\mu^- \partial_\mu \phi) + G^2 P^2 C_\mu^2 + g^2 f^2 \partial_\mu \phi \partial_\mu C_\mu^2 + \\ & \partial_\mu X^+ X^- + X^- (\partial_\mu^2 - M^2) X^+ + X^0 X^- + Y_\mu^+ Y_\mu^- + ig_s W_\mu^+ (\partial_\mu X^+ X^- - \\ & \partial_\mu X^+ X^-) + ig_s W_\mu^- (\partial_\mu X^+ Y - \partial_\mu Y X^- + \partial_\mu X^+ X^- - \\ & \partial_\mu X^+ X^-) + ig_s A_\mu^+ (\partial_\mu X^+ X^- - \\ & \partial_\mu X^+ X^-) + ig_s A_\mu^- (\partial_\mu X^+ X^- - \\ & \partial_\mu X^+ X^-) - \tfrac{1}{2}g M \left(X^+ X^+ H + X^+ X^- H + \tfrac{1}{2}X^0 X^+ H\right) - \tfrac{1}{2}g M \left(X^+ X^+ \phi^2 - X^+ X^0 \phi^2\right) + \\ & \tfrac{1}{2}g M \left(X^+ X^+ \phi^2 - X^+ X^0 \phi^2\right) - g g S_{sc} (X^+ X^- \phi^2 - X^0 X^- \phi^2) + \\ & \tfrac{1}{2}g M \left(X^+ X^- \phi^2 - X^0 X^- \phi^2\right)$$



Meters

$$\begin{aligned} \mathcal{L}_{EM} = & -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi - g_s I^{mn}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}Z_1^2 I^{mn}\partial_\mu\phi\partial_\nu\phi - \partial_\mu W_+^a \partial_\nu W_-^a - \\ & M^2 W_+^a W_-^a - \frac{1}{2}\partial_\mu Z_2^a \partial_\nu Z_2^a - \frac{1}{8}\epsilon^m_{abc}M^2 Z_2^c Z_2^a - \frac{1}{8}\epsilon^a_{abc}\partial_\mu A_b \partial_\nu A_c - ig_s c_s (W_+^a W_-^b - \\ & W_+^b W_-^a) Z_2^c Z_2^a Z_2^b + g_s Z_2^c Z_2^a Z_2^b W_+^a W_-^b - g_s Z_2^c Z_2^a Z_2^b W_+^b W_-^a - \\ & ig_s c_s (A_b W_+^a W_-^b - A_b W_-^a W_+^b) + \frac{1}{2}\epsilon^m_{abc}\epsilon^{kl}W_+^a W_-^b W_+^c W_-^l + \frac{1}{2}\epsilon^a_{abc}\epsilon^{kl}W_+^a W_-^b W_+^c W_-^l - \\ & W_+^a W_-^b W_+^c W_-^l - \frac{1}{2}\epsilon^m_{abc}\epsilon^{kl}W_+^a W_-^b W_+^c W_-^l + \frac{1}{2}\epsilon^a_{abc}\epsilon^{kl}W_+^a W_-^b W_+^c W_-^l + \epsilon^a_{abc}(\partial_\mu Z_2^c W_+^a W_-^b - \\ & Z_2^c W_+^a W_-^b) + \epsilon^m_{abc}(A_b W_+^a W_-^b - A_b W_-^a W_+^b) + g_s^2 c_s c_a (A_b Z_2^c W_+^a W_-^b - \\ & Z_2^c W_+^a W_-^b) - 2A_b Z_2^c W_+^a W_-^b - \frac{1}{2}\partial_\mu H_0 \partial_\nu H - 2A_b T_0^a \partial_\nu H^a - \partial_\mu g_s \partial_\nu g_s - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \\ & \partial_\mu (\frac{\partial_\mu H_0}{H_0} + \partial_\mu T_0^a H^a) \partial_\nu (\frac{\partial_\nu H_0}{H_0} + \partial_\nu T_0^a H^a) + \frac{\partial_\mu H_0}{H_0} \partial_\nu T_0^a H^a + \\ & \frac{1}{2}g^2 \alpha_s (H^2 + (\phi^2)^2 + 4(g/\phi)^2\phi^2 + 4(g/\phi)^2\phi^2 + 4H(\phi^2)H^2 + 2(\phi^2)^2H^2) - \\ & \frac{1}{2}ig (W_+^a (\partial_\mu\phi, \phi - \partial_\mu\phi) - W_-^a (\partial_\mu\phi, \phi - \partial_\mu\phi)) + \\ & \frac{1}{2}g (W_+^a (\partial_\mu H_0, \phi - \partial_\mu H) + W_-^a (\partial_\mu H_0, \phi - \partial_\mu H)) + \frac{1}{2}\frac{1}{2}\epsilon^a_{abc}Z_2^c (H_0 \partial_\mu\phi - \phi\partial_\mu H) + \\ & M (\frac{1}{2}Z_2^2 \partial_\mu\phi^2 + \frac{1}{2}\epsilon^a_{abc}Z_2^c (\partial_\mu A_b + \partial_\mu A_c) + \frac{1}{2}\epsilon^m_{abc}Z_2^c (\partial_\mu A_b + \partial_\mu A_c) + M M_b A_b W_+^a \phi - \\ & M_b A_b (\partial_\mu W_+^a) - \frac{1}{2}\epsilon^a_{abc}Z_2^c (\partial_\mu A_b + \partial_\mu A_c)) + \frac{1}{2}\epsilon^a_{abc}Z_2^c (\partial_\mu A_b + \partial_\mu A_c) - \\ & \frac{1}{2}g^2 W_+^a W_-^b (H^2 + (\phi^2)^2 + 2g^2\phi^2) - \frac{1}{2}g^2 Z_2^a Z_2^b (H^2 + (\phi^2)^2 + 2(2g^2 - 1)\phi^2\phi^2) - \\ & \frac{1}{2}g^2 \frac{\partial_\mu H_0}{H_0} W_+^a W_-^b (W_+^b \phi + W_-^a \phi^2) - \frac{1}{2}g^2 Z_2^a Z_2^b H_0 W_+^b \phi - W_+^a \phi^2 + \frac{1}{2}g^2 s_a A_b (\partial_\mu W_+^a \phi + \\ & W_+^a \phi^2) + \frac{1}{2}g^2 s_a A_b H W_+^b \phi^2 - W_+^b \phi^2 - g^2 b_a (2g^2 - 1)Z_2^a A_b \phi^2 + \\ & g^2 s_a^2 A_b A_c \phi^2 + \frac{1}{2}g^2 s_a^2 A_b (\eta^{bc}\eta^{ad}W_+^c \phi^2 - \epsilon^a(\eta^{bc}\eta^{ad})^2 - \epsilon^a(\eta^{ad}\eta^{bc})^2 - \epsilon^b(\eta^{ad}\eta^{bc})^2 + \\ & m_a^2) - g^2(\eta^{ab} + m_a^2)(\eta^{cd} + m_d^2)(\eta^{ac} + m_c^2)(\eta^{bd} + m_b^2) + \frac{1}{2}(\epsilon^a(\eta^{ad}\eta^{bc})^2 + \\ & \frac{1}{2}g^2 s_a^2 W_+^a (\eta^{cd}\eta^{ad}W_+^c \phi^2 + \eta^{cd}\eta^{ad}W_+^c \phi^2 + (\eta^{cd}\eta^{ad}(\eta^{bc}\eta^{ac})^2)) + \\ & (g_s^2)^2 ((X^a + Y^a)(\eta^{cd}\eta^{ad}W_+^c \phi^2 + \eta^{cd}\eta^{ad}W_+^c \phi^2 + (\eta^{cd}\eta^{ad}(\eta^{bc}\eta^{ac})^2)) + \\ & \frac{1}{2}W_+^a W_-^b (\epsilon^a(\eta^{cd}\eta^{ad}W_+^c \phi^2 + \eta^{cd}\eta^{ad}W_+^c \phi^2 + (\eta^{cd}\eta^{ad}(\eta^{bc}\eta^{ac})^2)) + \\ & \frac{1}{2}W_+^a W_-^b (-mc^2(\eta^{cd}\eta^{ad}W_+^c \phi^2 + \eta^{cd}\eta^{ad}W_+^c \phi^2 + (\eta^{cd}\eta^{ad}(\eta^{bc}\eta^{ac})^2)) + \\ & \frac{1}{2}W_+^a W_-^b (mc^2(\eta^{cd}\eta^{ad}W_+^c \phi^2 + \eta^{cd}\eta^{ad}W_+^c \phi^2 + (\eta^{cd}\eta^{ad}(\eta^{bc}\eta^{ac})^2)) - \frac{1}{2}W_+^a W_-^b (H^2 \phi^2) - \\ & \frac{1}{2}g^2 H (H^2 \phi^2) - \frac{1}{2}\frac{2}{3}\epsilon^a_{abc}(\phi\partial_\mu a^b + a^b\partial_\mu\phi) - \frac{1}{2}\frac{2}{3}\epsilon^a_{abc}(\phi\partial_\mu a^b + a^b\partial_\mu\phi) - \\ & \frac{1}{2}g^2 M_b^2 (1 - \eta^b\eta^c) - \frac{1}{2}\frac{2}{3}\epsilon^a_{abc}(-m_2^2(\eta^c\eta^d\eta^a - \eta^a\eta^d\eta^c) - m_2^2(\eta^c\eta^d\eta^a - \eta^a\eta^d\eta^c) + \\ & \frac{2}{3}\eta^2\eta^2\eta^2\eta^2) + (m_2^2(\eta^d\eta^a\eta^c + \eta^a\eta^c\eta^d) - m_2^2(\eta^d\eta^a\eta^c + \eta^a\eta^c\eta^d) - \frac{2}{3}\eta^2\eta^2\eta^2\eta^2) + \\ & \frac{1}{2}\frac{2}{3}\epsilon^a_{abc}(d\eta^d C_{ab}^c + d\eta^c C_{ab}^d) + \frac{1}{2}\frac{2}{3}\epsilon^a_{abc}(d\eta^d C_{ab}^c + d\eta^c C_{ab}^d) + \frac{1}{2}g^2 \partial_\mu\phi C^d \phi^2 + \\ & \frac{1}{2}\frac{2}{3}g^2 (d\eta^d C_{ab}^c + d\eta^c C_{ab}^d) + \frac{1}{2}\frac{2}{3}\eta^2\eta^2\eta^2\eta^2 + \frac{1}{2}g^2 g_s f_a^f f_a^f g_s C^d \phi^2 + \\ & g_s^2 M_a^2 X^a X^a + X^a (F^a - M^a X^a + X^a X^a) + \frac{M_a^2}{M^a} X^a + Y^a P^a + ig_s W_+^a (\partial_a X^a X^a - \\ & \partial_a X^a X^a) + ig_s W_+^a (\partial_a X^a Y - \partial_a Y^a X) + ig_s W_+^a (\partial_a X^a X^a - \\ & \partial_a X^a X^a) + ig_s W_+^a (\partial_a X^a Y - \partial_a Y^a X) + ig_s W_+^a (\partial_a X^a X^a - \\ & \partial_a X^a X^a) - \frac{1}{2}g M (\bar{X}^a X^a H + \bar{Y}^a Y^a H + \bar{X}^a Y^a H) - \frac{1}{2}\frac{2}{3}ig M (\bar{X}^a X^a \phi^2 - \bar{X}^a X^a \phi^2) + \\ & \frac{1}{2}\frac{2}{3}ig M (\bar{X}^a X^a \phi^2 - \bar{X}^a X^a \phi^2) + ig_s M S_{ab} (\bar{X}^a X^a \phi^2 - \bar{X}^a X^a \phi^2) + \\ & \frac{1}{2}g M (\bar{X}^a X^a \phi^2 - \bar{X}^a X^a \phi^2) \end{aligned}$$



10^{-14}

10^{-4}

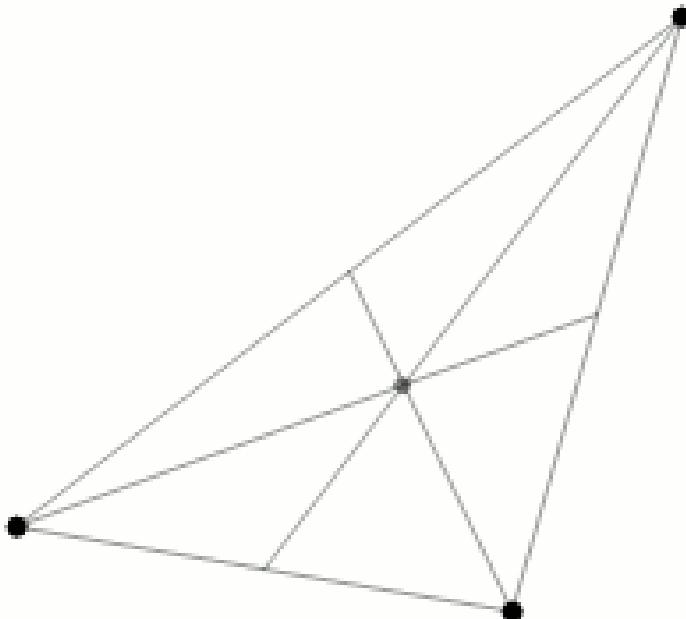
10^4

10^{14}

10^{26}

Meters

But can you solve this?



Three-body problem equations (classical)

$$\begin{aligned}\ddot{\mathbf{r}}_1 &= -Gm_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - Gm_3 \frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3}, \\ \ddot{\mathbf{r}}_2 &= -Gm_3 \frac{\mathbf{r}_2 - \mathbf{r}_3}{|\mathbf{r}_2 - \mathbf{r}_3|^3} - Gm_1 \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3}, \\ \ddot{\mathbf{r}}_3 &= -Gm_1 \frac{\mathbf{r}_3 - \mathbf{r}_1}{|\mathbf{r}_3 - \mathbf{r}_1|^3} - Gm_2 \frac{\mathbf{r}_3 - \mathbf{r}_2}{|\mathbf{r}_3 - \mathbf{r}_2|^3}.\end{aligned}$$

Three-body problem

Article Talk

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This article is about the physics theory. For other uses, see [Three-body problem](#)



The accessibility of this article is in question. Relevant discussions may be found on the [talk page](#). (June 2024)

In physics, specifically [classical mechanics](#), the **three-body problem** involves taking the initial positions and velocities (or [momenta](#)) of three [point masses](#) that orbit each other in space and calculating their subsequent trajectories using [Newton's laws of motion](#) and [Newton's law of universal gravitation](#).^[1]

Unlike the [two-body problem](#), the three-body problem has no general [closed-form](#) solution.^[1] When three bodies orbit each other, the resulting [dynamical system](#) is chaotic for most [initial conditions](#), and the only way to predict the motions of the bodies is to calculate them using [numerical methods](#).

Three-body problem equations (classical)

$$Gm_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - Gm_3 \frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3},$$
$$Gm_3 \frac{\mathbf{r}_2 - \mathbf{r}_3}{|\mathbf{r}_2 - \mathbf{r}_3|^3} - Gm_1 \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3},$$
$$Gm_1 \frac{\mathbf{r}_3 - \mathbf{r}_1}{|\mathbf{r}_3 - \mathbf{r}_1|^3} - Gm_2 \frac{\mathbf{r}_3 - \mathbf{r}_2}{|\mathbf{r}_3 - \mathbf{r}_2|^3}.$$

Three-body problem

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3 Body Problem

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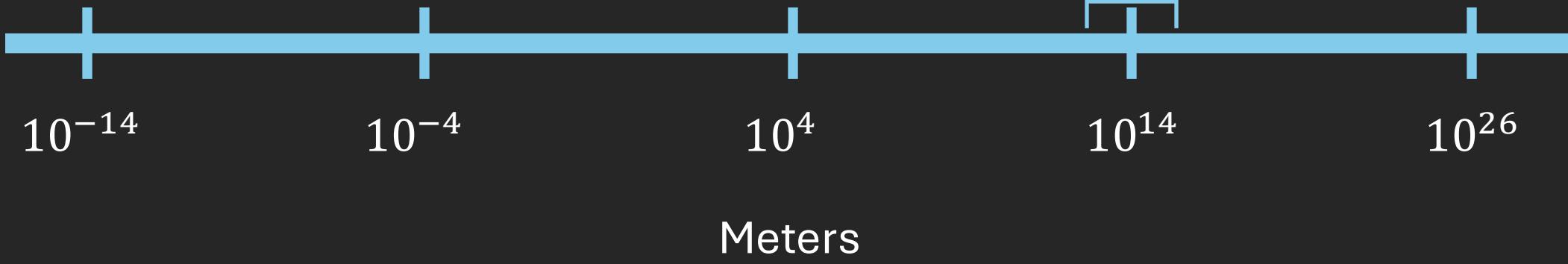
In physics, specifically classical mechanics, the three-body problem is the problem of predicting the motion of three masses interacting via gravitation. Given three bodies in space, taking the initial positions and velocities of all three into account, it is a challenge to predict their future positions. The three bodies may orbit each other in space and time.

Newton's laws of motion and universal gravitation allow the solution of the two-body problem, which is the problem of predicting the motion of two bodies interacting via gravitation. Unlike the two-body problem, there is no general solution for the three-body problem. When three bodies interact via gravitation, the motion is chaotic for most initial conditions, and the only way to predict the motions of the bodies is to calculate them using numerical methods.



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$$\begin{aligned} \mathcal{L}_{\text{ext}} = & -\frac{1}{2}\partial_x\phi\partial_y\phi - g_i I^{i\infty}\partial_y\phi\partial_y\phi - \frac{1}{2}Z^2 I^{\infty i}g_i\partial_y\phi\partial_y\phi - \partial_yW_i\partial_yW_i - \\ & M^2 W_i^*W_i - \frac{1}{2}\partial_z\phi\partial_z\phi - \frac{1}{2}M^2 Z^2 Z^2 - \frac{1}{2}\partial_xA_i\partial_xA_i - ig_{\infty}(A_i\partial_x^2(W_i^*W_i - \\ & W_iW_i^*) + Z_i^2\partial_x^2(W_i^*W_i) + Z_i^2\partial_x^2(W_iW_i^*)) - \frac{1}{2}\partial_x^2(A_i\partial_xW_i^* - A_i\partial_xW_i) - \\ & ig_{\infty}(A_i\partial_x^2(W_i^*W_i) + A_i\partial_x^2(W_iW_i^*)) - \frac{1}{2}\partial_x^2(A_i\partial_xW_i^* - A_i\partial_xW_i) - \\ & W_i^*W_i^* - 2g_i^2W_i^*W_i^* - 2g_i^2W_iW_i^* - ig_i^2W_i^*W_iW_i^*W_i + ig_i^2(Z_i^2W_i^*Z_i^2W_i^* - \\ & Z_i^2Z_i^2W_i^*W_i^*) + g_i^2(A_iW_i^*A_iW_i^* - A_iA_iW_i^*W_i^*) + g_i^2c_0(A_iZ_i^2W_i^*W_i^* - \\ & W_i^*W_i^* - 2A_iA_iW_i^*W_i^*) - \frac{1}{2}g_iH_0H - 2A_iT_0T_0H^* - \partial_y\phi\partial_y\phi - \frac{1}{2}\partial_y\phi\partial_y\phi - \\ & \partial_y(\frac{\partial_x^2\phi}{\partial_x^2\phi} + \frac{1}{2}\partial_x^2(\phi\partial_y\phi)) + \partial_y(\frac{\partial_x^2\phi}{\partial_x^2\phi} + \frac{1}{2}\partial_x^2(\phi\partial_y\phi)) + \frac{1}{2}\partial_y\alpha_i(H_i + (\phi^2 + \phi\partial_y\phi)^2 + 4(\phi\partial_y\phi)^2 + 4(\phi\partial_y\phi)^2 + 4(\phi\partial_y\phi)^2 + 4(\phi\partial_y\phi)^2) + \\ & \frac{1}{2}g_i\alpha_i(H_i + (\phi^2 + \phi\partial_y\phi)^2 + 4(\phi\partial_y\phi)^2 + 4(\phi\partial_y\phi)^2 + 4(\phi\partial_y\phi)^2 + 4(\phi\partial_y\phi)^2) - \\ & \frac{1}{2}ig(W_i^*(\phi\partial_y\phi - \phi\partial_y\phi) - W_i^*(\phi\partial_y\phi - \phi\partial_y\phi)) + \\ & \frac{1}{2}g(W_i^*(H_0\phi - \phi\partial_yH) + W_i^*(H_0\phi - \phi\partial_yH)) + \frac{1}{2}\frac{1}{2}(Z_i^2(H_0\phi - \phi\partial_yH) + \\ & M(\frac{1}{2}Z^2Z^2 - \frac{1}{2}Z^2Z^2)(H_0\phi - \phi\partial_yH) + \frac{1}{2}g^2(A_i\partial_xW_i^* - A_i\partial_xW_i)(H_0\phi - \phi\partial_yH) - \\ & W_i^*W_i^* - ig_i^2Z_i^2Z_i^2)(H_0\phi - \phi\partial_yH) + \frac{1}{2}g^2(A_i\partial_xW_i^* - A_i\partial_xW_i)(H_0\phi - \phi\partial_yH) - \\ & \frac{1}{2}g^2W_i^*W_i^*(H^2\phi^2 + 2\phi\phi^2) - \frac{1}{2}g^2Z_i^2Z_i^2(H^2\phi^2 + 2\phi\phi^2) - 2(2\phi^2 - 1)\phi^2\phi^2 - \\ & \frac{1}{2}g^2\frac{1}{2}Z_i^2Z_i^2(W_i^*\phi^2 + W_i\phi^2) - \frac{1}{2}g^2\frac{1}{2}Z_i^2Z_i^2(H^2W_i^*\phi^2 - W_i\phi^2) + \frac{1}{2}g^2A_i\partial_xW_i^*(W_i^*\phi^2 + \\ & W_i\phi^2) + ig_i^2A_iH(W_i^*\phi^2 - W_i\phi^2) - g^2\frac{1}{2}(2\phi^2 - 1)2A_i\phi^2\phi^2 - \\ & g^2A_i^2A_i\phi^2\phi^2 - g_i^2(\phi^2 + m_0^2)(W_i^*\phi^2 + W_i\phi^2) - e^2(\phi^2m_0^2\phi^2) - \phi^2(\phi^2 + m_0^2)\phi^2 - \phi^2(\phi^2 + \\ & m_0^2) - g_i^2(\phi^2 + m_0^2) + ig_{\infty}A_i(-(\phi^2m_0^2\phi^2) - \phi^2(\phi^2 + m_0^2)\phi^2) - \frac{1}{2}g^2(Y_i^2\phi^2) + \\ & \frac{1}{2}g^2(X_i^2\phi^2) - \frac{1}{2}g^2(Y_i^2\phi^2) + \frac{1}{2}g^2(X_i^2\phi^2) + g^2(\phi^2 + m_0^2)\phi^2 + g^2(\phi^2 + m_0^2) - \\ & (g_i^2Y_i^2\phi^2 + X_i^2\phi^2) + \frac{1}{2}g_i^2W_i^*((\phi^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2) + (X_i^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2)) + \\ & \frac{1}{2}g_i^2W_i^*((\phi^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2) + (X_i^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2)) + \\ & \frac{1}{2}g_i^2\phi^2(-m_0^2(\phi^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2) - m_0^2(\phi^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2) + \\ & \frac{1}{2}g_i^2\phi^2(m_0^2(\phi^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2) - m_0^2(\phi^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2) - \frac{1}{2}g_i^2H(\phi^2m_0^2\phi^2) - \\ & \frac{1}{2}g_i^2H((\phi^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2) - \frac{1}{2}g_i^2H(\phi^2m_0^2\phi^2) - \frac{1}{2}g_i^2H((\phi^2m_0^2\phi^2) - \frac{1}{2}g_i^2H(\phi^2m_0^2\phi^2) - \\ & \frac{1}{2}g_i^2H(\phi^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2) - \frac{1}{2}g_i^2H(\phi^2m_0^2\phi^2) - \frac{1}{2}g_i^2H((\phi^2m_0^2\phi^2) - \frac{1}{2}g_i^2H(\phi^2m_0^2\phi^2) + \\ & \frac{1}{2}g_i^2H((\phi^2m_0^2\phi^2 + Y_i^2m_0^2\phi^2) - \frac{1}{2}g_i^2H((\phi^2m_0^2\phi^2) + G^2P^2G^2 + g^2f^2\partial_x^2G^2\phi^2 + \\ & X^2(\partial_x^2X^2 + X^2(\partial_x^2X^2 + X^2X^2) + \frac{1}{2}X^2\partial_x^2X^2 + Y_i^2\partial_x^2Y_i^2 + ig_{\infty}W_i^*(\partial_x^2X^2 - \\ & \partial_x^2X^2) + ig_{\infty}W_i^*(\partial_x^2Y_i^2 - \partial_x^2Y_i^2) + ig_{\infty}W_i^*(\partial_x^2X^2 + \\ & \partial_x^2X^2) + ig_{\infty}W_i^*(\partial_x^2Y_i^2 + \partial_x^2Y_i^2) + ig_{\infty}W_i^*(\partial_x^2X^2 - X^2X^2) - \\ & \frac{1}{2}gM(X^2X^2H + X^2X^2H + \frac{1}{2}X^2X^2H) + \frac{1}{2}gM(X^2X^2\phi^2 - X^2X^2\phi^2) + \\ & \frac{1}{2}gM(X^2X^2\phi^2 - X^2X^2\phi^2) + igMS_{\infty}(X^2X^2\phi^2 - X^2X^2\phi^2) + \\ & \frac{1}{2}gM(X^2X^2\phi^2 - X^2X^2\phi^2)$$



$$\begin{aligned} \mathcal{L}_{SM} = & -\tfrac{1}{2}\partial_\mu g_{\nu}^{\alpha}\partial_\mu g_{\nu}^{\beta} - g_{\nu} I^{ab} \partial_\mu g_{\nu}^{\alpha}g_{\mu}^{\beta} - \tfrac{1}{2}g_{\nu}^2 I^{ab} I^{cd} g_{\mu}^{\alpha}g_{\mu}^{\beta} - \partial_\mu W_{\nu}^a \partial_\mu W_{\nu}^c - \\ & M^2 W_{\nu}^a W_{\nu}^c - \tfrac{1}{2}\partial_\nu Z_{\mu}^a \partial_\mu Z_{\nu}^c - \tfrac{1}{2}M^2 Z_{\mu}^a Z_{\nu}^c - \tfrac{1}{2}\partial_\mu A_{\nu}^a \partial_\nu A_{\mu}^c - ig_{\nu} c_s A_{\nu}^a W_{\nu}^c - \\ & \tfrac{1}{2}g_{\nu} s_w A_{\nu}^a Z_{\nu}^c + Z_{\mu}^a \partial_\mu W_{\nu}^c - Z_{\nu}^a \partial_\nu W_{\mu}^c - \tfrac{1}{2}g_{\mu} s_w A_{\mu}^a Z_{\nu}^c - g_{\mu} c_s A_{\mu}^a W_{\nu}^c - \\ & ig_{\nu} s_w A_{\nu}^a (W_{\nu}^c - W_{\nu}^a) - \tfrac{1}{2}g_{\mu} s_w A_{\mu}^a (W_{\nu}^c - W_{\nu}^a) + \tfrac{1}{2}g_{\mu} c_s A_{\mu}^a (W_{\nu}^c - W_{\nu}^a) - \\ & W_{\nu}^a (W_{\nu}^c - W_{\nu}^a) - \tfrac{1}{2}g_{\mu}^2 W_{\mu}^a W_{\nu}^c + \tfrac{1}{2}g_{\nu}^2 W_{\nu}^a W_{\mu}^c + \tfrac{1}{2}g_{\mu}^2 Z_{\mu}^a Z_{\nu}^c - \\ & Z_{\mu}^a Z_{\nu}^c W_{\mu}^c) + g_{\mu}^2 c_s c_a (A_{\mu}^a W_{\nu}^c - A_{\nu}^a W_{\mu}^c) + g_{\mu}^2 s_w c_a (A_{\mu}^a Z_{\nu}^c - A_{\nu}^a Z_{\mu}^c) - \\ & W_{\nu}^a W_{\nu}^c - 2A_{\nu}^a W_{\nu}^c - \tfrac{1}{2}g_{\mu} H_0 H - 2A_{\mu}^a H_0 H^2 - \partial_\nu \partial_\mu g_{\nu}^{\alpha} - \tfrac{1}{2}\partial_\nu \partial_\mu g_{\nu}^{\beta} - \\ & \partial_\mu (\tfrac{g_{\mu}^2}{2}g_{\nu}^{\alpha}g_{\nu}^{\beta} + \tfrac{1}{2}(H_0^2 + g_{\mu}^2)(g_{\nu}^{\alpha}g_{\nu}^{\beta} + g_{\mu}^2)) + \tfrac{ig_{\mu}}{2}m_{\alpha} - \\ & ig_{\mu} m_{\alpha} (H_0^2 + (g_{\mu}^2)^2 + 4(g_{\mu}^2)^2 + 4(g_{\mu}^2)^2 + 4H_0^2 + 2(g_{\mu}^2)^2 H^2) - \\ & \tfrac{1}{2}g_{\mu}^2 W_{\mu}^a W_{\nu}^c H - \tfrac{1}{2}g_{\mu}^2 Z_{\mu}^a H - \\ & \tfrac{1}{2}ig_{\mu} (W_{\nu}^a (\partial_\mu \phi^a - \phi^a \partial_\mu) - W_{\nu}^c (\partial_\mu \phi^a - \phi^a \partial_\mu)) + \\ & \tfrac{1}{2}ig_{\mu} (W_{\nu}^a (\partial_\mu \phi^c - \phi^c \partial_\mu) + W_{\nu}^c (\partial_\mu \phi^c - \phi^c \partial_\mu)) + \tfrac{1}{2}g_{\mu}^2 Z_{\mu}^a (H \partial_\mu \phi^a - \phi^a \partial_\mu H) + \\ & M (\tfrac{1}{2}Z_{\mu}^a Z_{\nu}^c - \tfrac{1}{2}Z_{\mu}^a Z_{\nu}^a) + \tfrac{1}{2}g_{\mu}^2 Z_{\mu}^a (H \partial_\mu \phi^c - \phi^c \partial_\mu H) + \tfrac{1}{2}g_{\mu}^2 Z_{\mu}^c (H \partial_\mu \phi^a - \phi^a \partial_\mu H) + \\ & W_{\nu}^c (W_{\nu}^c - W_{\nu}^a) - \tfrac{1}{2}g_{\mu}^2 Z_{\mu}^c (H^2 - (g_{\mu}^2)^2 - 2(2g_{\mu}^2 - 1)^2 \phi^a \phi^c) - \\ & \tfrac{1}{2}g_{\mu}^2 W_{\mu}^a W_{\nu}^c (H^2 \phi^a - \phi^a H^2) - \tfrac{1}{2}g_{\mu}^2 Z_{\mu}^a H (H^2 \phi^c - \phi^c H^2) + \tfrac{1}{2}g_{\mu}^2 A_{\nu}^a W_{\mu}^c \phi^a + \\ & \tfrac{1}{2}g_{\mu}^2 Z_{\mu}^a W_{\nu}^c \phi^c + \tfrac{1}{2}g_{\mu}^2 W_{\mu}^a H (W_{\nu}^c - W_{\nu}^a) - g_{\mu}^2 \delta_{\mu}^2 (2g_{\mu}^2 - 1) Z_{\mu}^a \phi^a + \\ & g_{\mu}^2 A_{\nu}^a A_{\nu}^c \phi^c + \tfrac{1}{2}g_{\mu}^2 A_{\nu}^a (g_{\mu}^2 \phi^c - \phi^c \partial_\mu) + \tfrac{1}{2}g_{\mu}^2 A_{\nu}^c (g_{\mu}^2 \phi^a - \phi^a \partial_\mu) + m_{\mu}^2 \phi^a \phi^c - \\ & g_{\mu}^2 A_{\nu}^a A_{\nu}^c \phi^c - \tfrac{1}{2}g_{\mu}^2 \delta_{\mu}^2 (g_{\mu}^2 \phi^c - \phi^c \partial_\mu) + \tfrac{1}{2}g_{\mu}^2 A_{\nu}^a (g_{\mu}^2 \phi^a - \phi^a \partial_\mu) - \tfrac{1}{2}g_{\mu}^2 A_{\nu}^c (g_{\mu}^2 \phi^a - \phi^a \partial_\mu) + \\ & \tfrac{1}{2}g_{\mu}^2 Z_{\mu}^a (W_{\nu}^c - W_{\nu}^a) + \tfrac{1}{2}g_{\mu}^2 W_{\mu}^a (W_{\nu}^c - W_{\nu}^a) + (g_{\mu}^2 \phi^a \phi^c + g_{\mu}^2 \phi^a (1 + \gamma^a) C_a \phi^c) + \\ & (g_{\mu}^2 \phi^c \phi^a + g_{\mu}^2 \phi^c (1 + \gamma^c) C_c \phi^a) + \tfrac{1}{2}g_{\mu}^2 W_{\mu}^a ((\phi^a \gamma^a + \gamma^a \phi^a) \phi^c + (\phi^c \gamma^a + \gamma^a \phi^c) \phi^a) + \\ & \tfrac{1}{2}g_{\mu}^2 W_{\mu}^c ((\phi^a \gamma^a + \gamma^a \phi^a) \phi^c + (\phi^c \gamma^a + \gamma^a \phi^c) \phi^a) + \\ & \tfrac{1}{2}g_{\mu}^2 \phi^a (-m_{\mu}^2 (U_{\mu}^a U_{\mu}^a (1 - \gamma^a) \phi^a) - m_{\mu}^2 (U_{\mu}^a U_{\mu}^a \phi^a + \gamma^a \phi^a) + \\ & \tfrac{1}{2}g_{\mu}^2 \phi^c (-m_{\mu}^2 (U_{\mu}^c U_{\mu}^c (1 - \gamma^c) \phi^c) - m_{\mu}^2 (U_{\mu}^c U_{\mu}^c \phi^c + \gamma^c \phi^c) - \tfrac{1}{2}g_{\mu}^2 H (H \partial_\mu \phi^a) - \\ & \tfrac{1}{2}g_{\mu}^2 H (H \partial_\mu \phi^c) - \tfrac{1}{2}g_{\mu}^2 \delta_{\mu}^2 (g_{\mu}^2 \phi^a \phi^c) - \tfrac{1}{2}g_{\mu}^2 \delta_{\mu}^2 (g_{\mu}^2 \phi^c \phi^a) - \tfrac{1}{2}g_{\mu} M_{\mu} (1 - \gamma^a) \phi^a + \\ & \tfrac{1}{2}g_{\mu} M_{\mu} (1 - \gamma^c) \phi^c - \tfrac{1}{2}g_{\mu}^2 \delta_{\mu}^2 (g_{\mu}^2 \phi^a \phi^c) - m_{\mu}^2 (g_{\mu}^2 C_a (1 - \gamma^a) \phi^a) + \\ & \tfrac{1}{2}g_{\mu}^2 \delta_{\mu}^2 (g_{\mu}^2 \phi^a \phi^c) - m_{\mu}^2 (g_{\mu}^2 C_c (1 - \gamma^c) \phi^c) - \tfrac{1}{2}g_{\mu}^2 H (g_{\mu}^2 \phi^a) + \\ & \tfrac{1}{2}g_{\mu}^2 H (g_{\mu}^2 \phi^c) - \tfrac{1}{2}g_{\mu}^2 \delta_{\mu}^2 (g_{\mu}^2 \phi^a \phi^c) - \tfrac{1}{2}g_{\mu}^2 \delta_{\mu}^2 (g_{\mu}^2 \phi^c \phi^a) + G^{\mu \nu} \partial_\mu \phi^a + f^{\mu \nu} \partial_\mu \phi^c + G^{\mu \nu} \partial_\mu \phi^c + f^{\mu \nu} \partial_\mu \phi^a + \\ & \tfrac{1}{2}g_{\mu}^2 (d_{\mu}^2 d_{\nu}^2 (1 + \gamma^a) \phi^c) - m_{\mu}^2 (d_{\mu}^2 d_{\nu}^2 (1 - \gamma^a) \phi^a) - \tfrac{1}{2}g_{\mu}^2 H (g_{\mu}^2 \phi^a \phi^c + \\ & g_{\mu}^2 \phi^c \phi^a) + X_{\mu}^a (F_{\mu}^2 - M^2) X_{\mu}^a + X_{\mu}^c (F_{\mu}^2 - M^2) X_{\mu}^c + Y_{\mu}^a (P_{\mu}^2 - M^2) Y_{\mu}^a + Y_{\mu}^c (P_{\mu}^2 - M^2) Y_{\mu}^c - \\ & \partial_\mu X_{\mu}^a X_{\mu}^a + ig_{\mu} W_{\mu}^a (\partial_\mu X_{\mu}^a - \partial_\mu Y_{\mu}^a) + ig_{\mu} W_{\mu}^c (\partial_\mu X_{\mu}^c - \partial_\mu Y_{\mu}^c) - \\ & \partial_\mu X_{\mu}^c X_{\mu}^c + ig_{\mu} W_{\mu}^a (\partial_\mu X_{\mu}^c - \partial_\mu Y_{\mu}^c) + ig_{\mu} W_{\mu}^c (\partial_\mu X_{\mu}^a - \partial_\mu Y_{\mu}^a) - \\ & \partial_\mu X_{\mu}^a X_{\mu}^c + \tfrac{1}{2}g M \left(X^a X^b H + X^b X^a H + \tfrac{1}{2}X^a X^b H \right) - \tfrac{1}{2}g M \left(X^c X^b \phi^a - X^a X^b \phi^c \right) + \\ & \tfrac{1}{2}g M \left(X^a X^c \phi^b - X^c X^a \phi^b \right) + g_2 M S_{\mu} \left(X^a X^b \phi^c - X^c X^b \phi^a \right) + \\ & \tfrac{1}{2}g M \left(X^a X^c \phi^b - X^c X^a \phi^b \right) \end{aligned}$$

10^{-14}

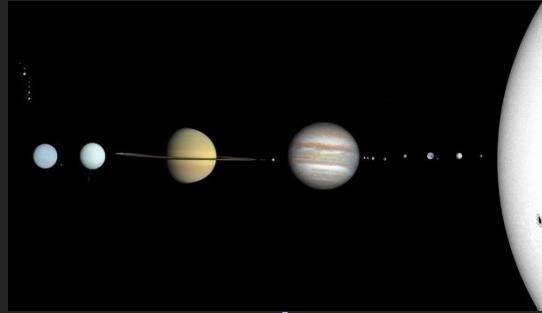
10^{-4}

10^4

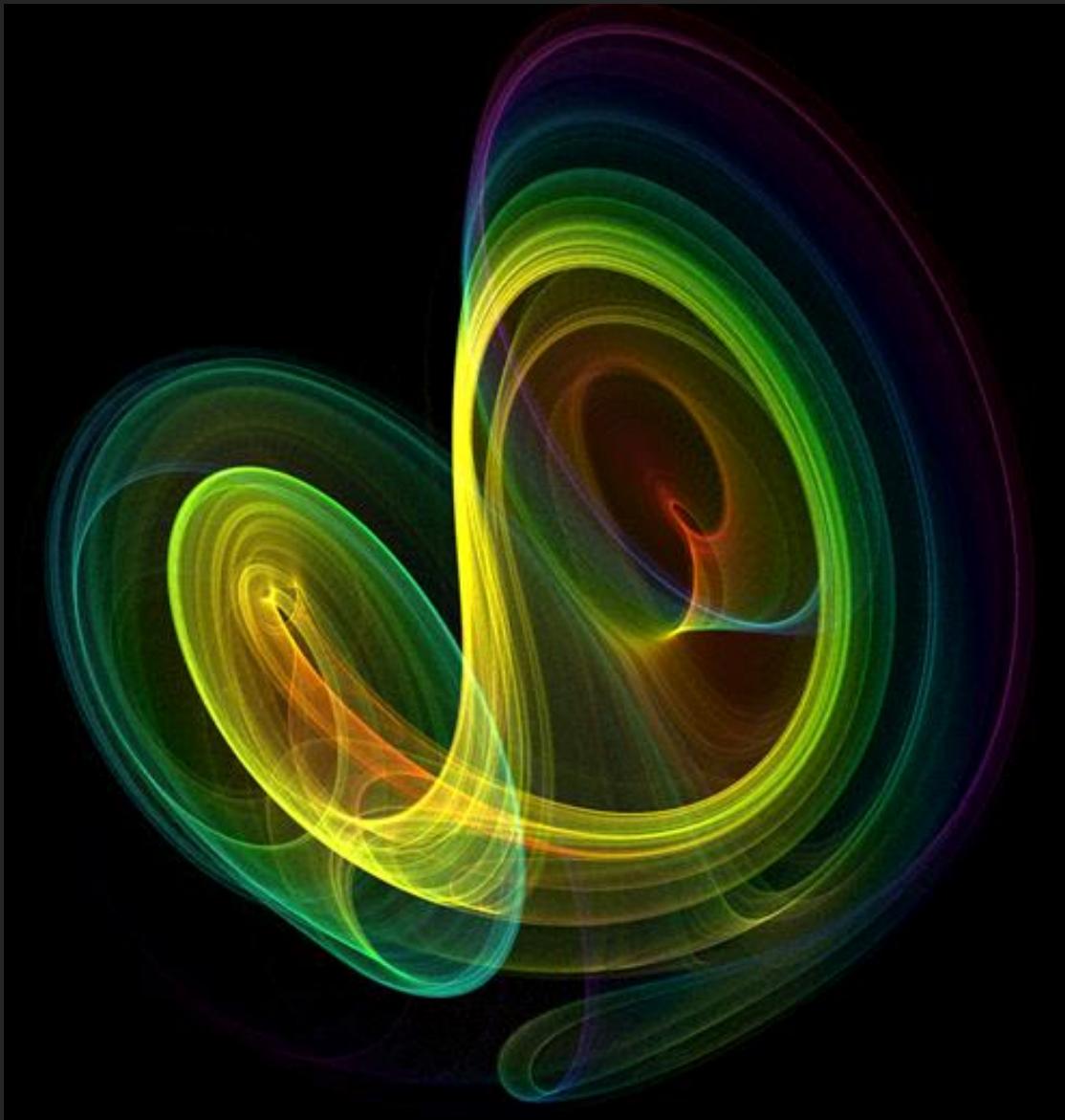
10^{14}

10^{26}

Meters



Or this?



**Atmospheric Convection
Lorentz System**

$$\frac{dx}{dt} = \sigma(y - x),$$

$$\frac{dy}{dt} = x(\rho - z) - y,$$

$$\frac{dz}{dt} = xy - \beta z.$$

What do I mean by “we can’t solve these equations”, I’ve just shown some pretty graphs right?

(You can do that too, just download python and ask ChatGPT to write the code for you)

$$\begin{aligned}\ddot{\mathbf{r}}_1 &= -Gm_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - Gm_3 \frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3}, \\ \ddot{\mathbf{r}}_2 &= -Gm_3 \frac{\mathbf{r}_2 - \mathbf{r}_3}{|\mathbf{r}_2 - \mathbf{r}_3|^3} - Gm_1 \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3}, \\ \ddot{\mathbf{r}}_3 &= -Gm_1 \frac{\mathbf{r}_3 - \mathbf{r}_1}{|\mathbf{r}_3 - \mathbf{r}_1|^3} - Gm_2 \frac{\mathbf{r}_3 - \mathbf{r}_2}{|\mathbf{r}_3 - \mathbf{r}_2|^3}.\end{aligned}$$

$$\begin{aligned}\frac{dx}{dt} &= \sigma(y - x), \\ \frac{dy}{dt} &= x(\rho - z) - y, \\ \frac{dz}{dt} &= xy - \beta z.\end{aligned}$$



Linear

Non-linear

But every simulation is completely unique,
change the initial parameters just a tiny bit,
and everything breaks down

In other words, you have to restart your simulation all over again

Solvable System: Projectile motion

Given some initial condition (v_0)



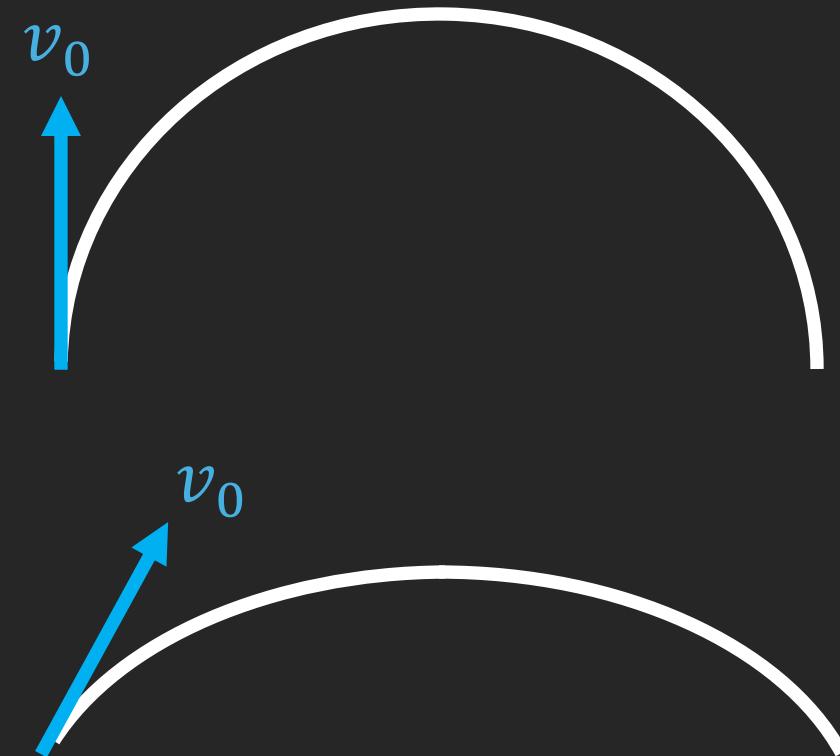
Solvable System: Projectile motion

Given some initial condition (v_0), we can solve the equations of motion and calculate where the cannon ball will be at each point in time

Closed form solution

$$x = v_0 t \cos(\theta),$$

$$y = v_0 t \sin(\theta) - \frac{1}{2} g t^2.$$

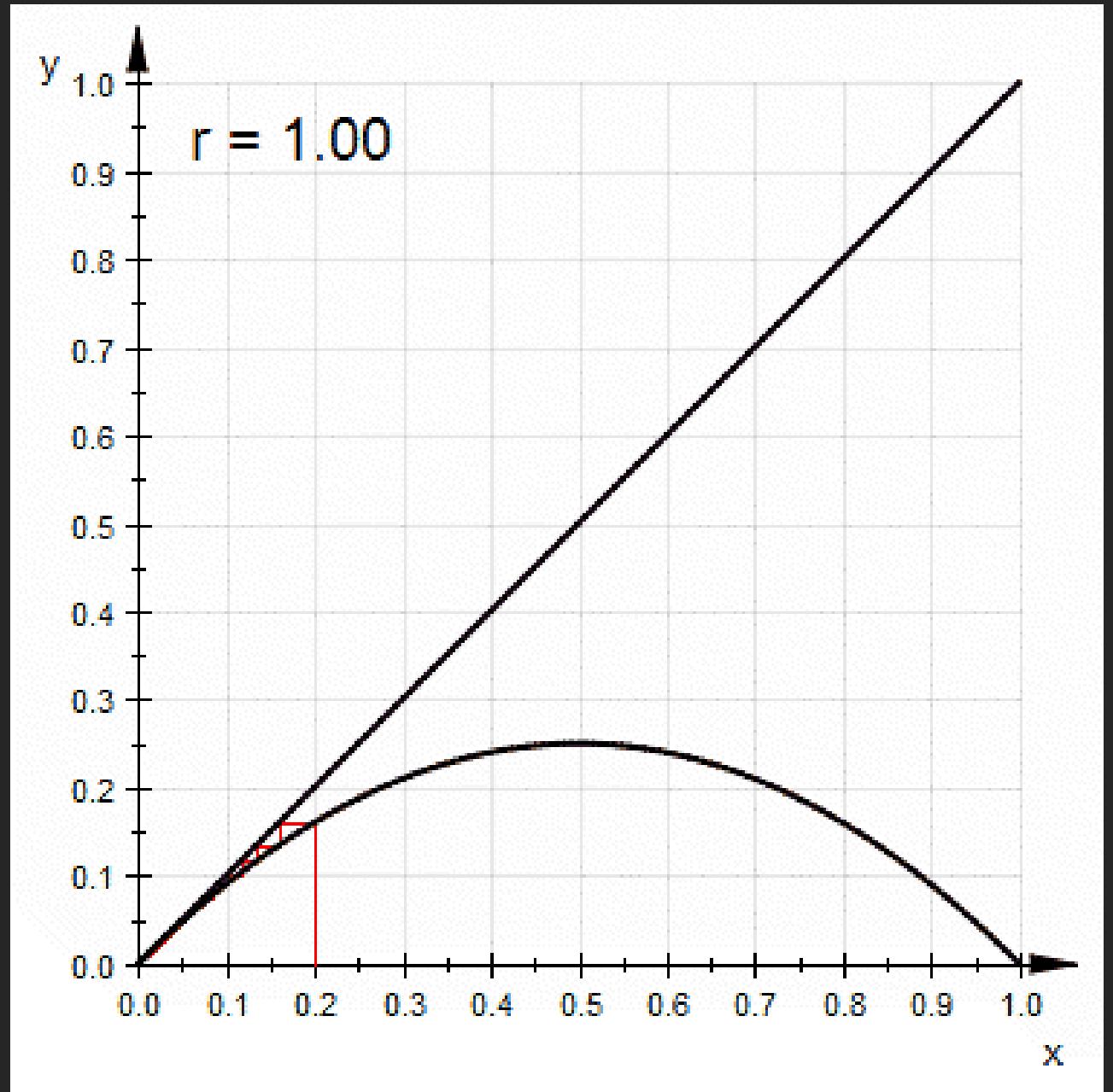


Non solvable system: Logistic map

We can't even solve the
simplest complex system!

$$x_{n+1} = rx_n(1 - x_n),$$

Start with any value of x, if r is large enough every
Single simulation will be completely different!

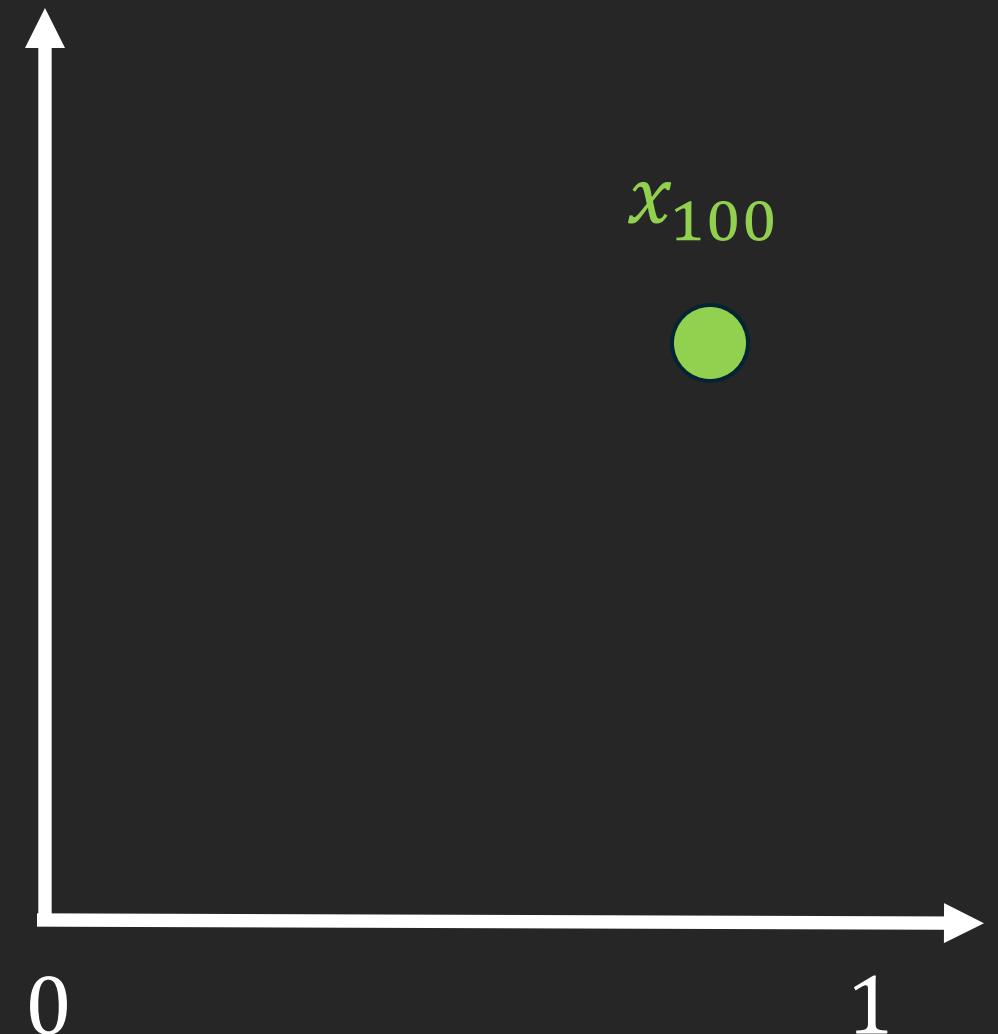


Non solvable system: Logistic map

$$x_{n+1} = rx_n(1 - x_n),$$

Start with any value of x , if r is large enough every
Single simulation will be completely different!

$$x_0 = 0.2 \rightarrow x_{100} = 0.9$$



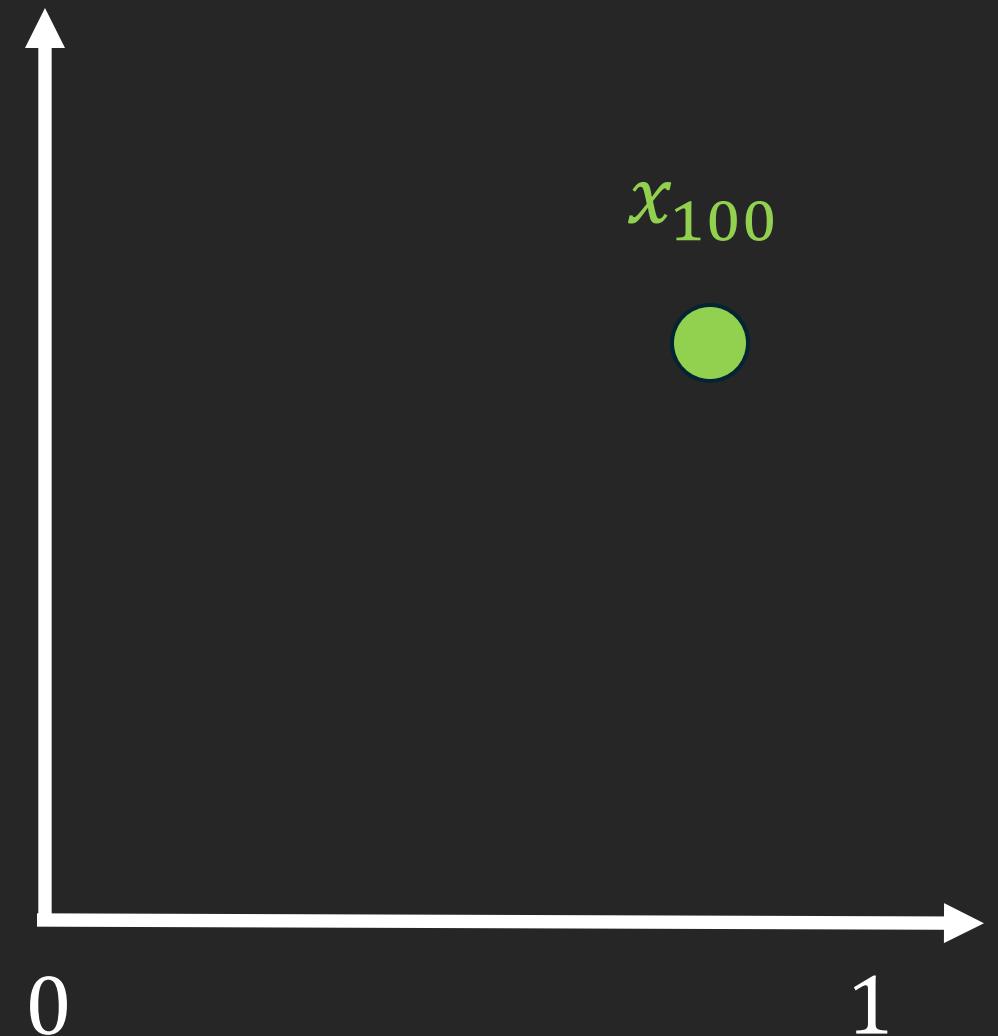
Non solvable system: Logistic map

$$x_{n+1} = rx_n(1 - x_n),$$

Start with any value of x , if r is large enough every
Single simulation will be completely different!

$$x_0 = 0.2 \rightarrow x_{100} = 0.9$$

$$x_0 = 0.200000001 \rightarrow$$



Non solvable system: Logistic map

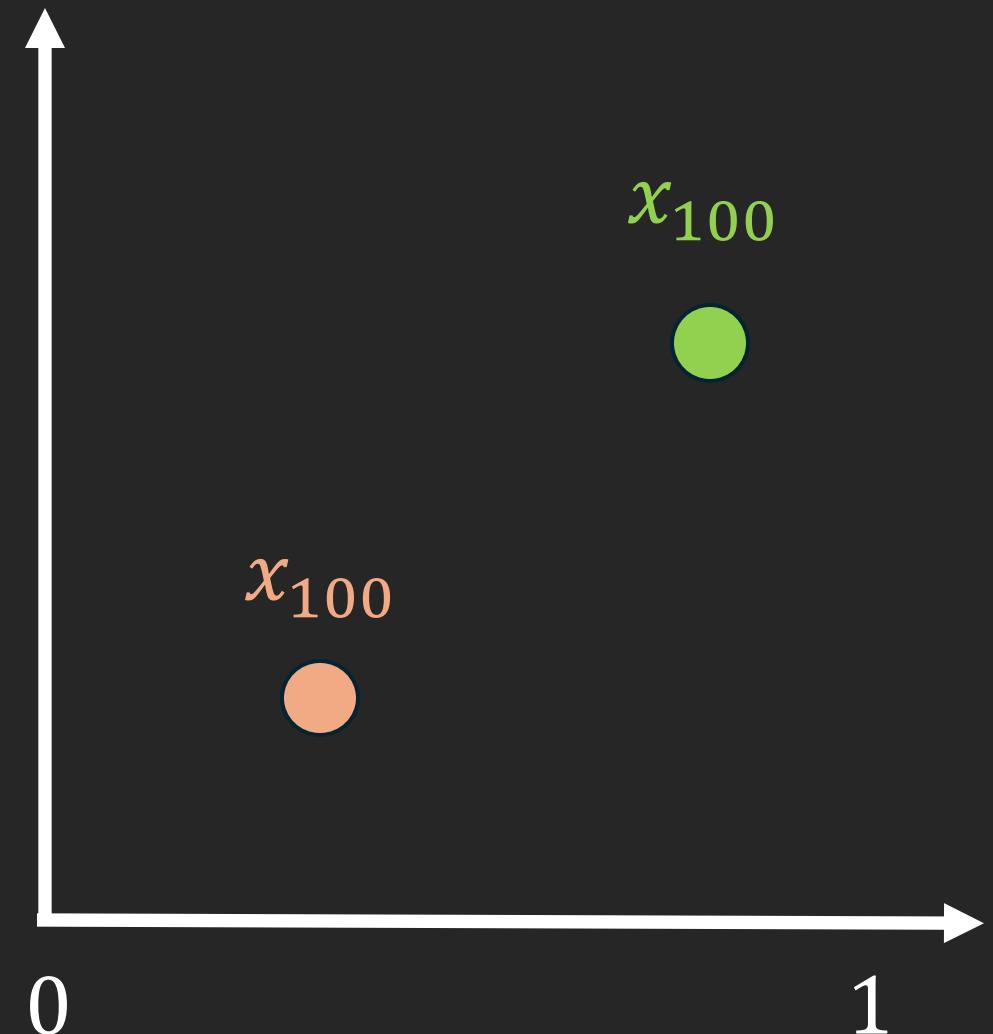
$$x_{n+1} = rx_n(1 - x_n),$$

Start with any value of x , if r is large enough every
Single simulation will be completely different!

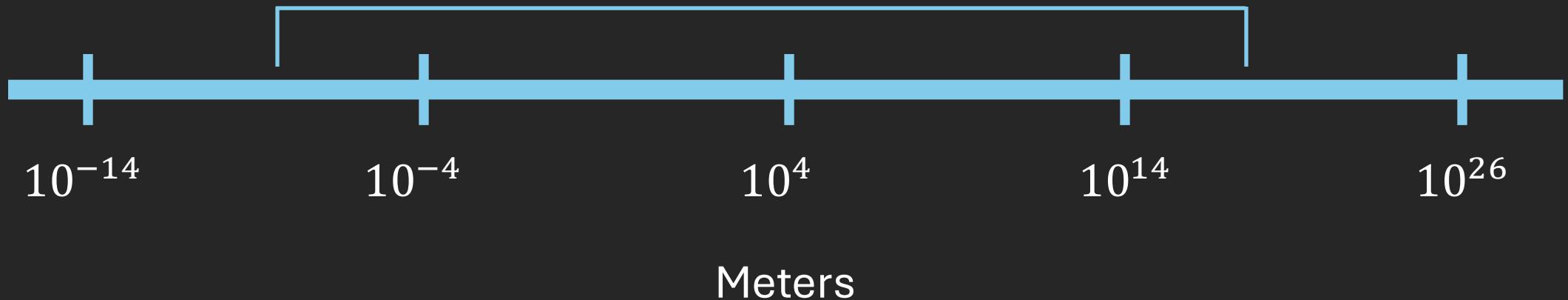
Now you can say you actually understand
what is known as the “butterfly effect”

$$x_0 = 0.2 \rightarrow x_{100} = 0.9$$

$$x_0 = 0.200000001 \rightarrow x_{100} = 0.3$$



But pretty much everything around us,
“at our scale”,
is described by nonlinear equations



Nature deals with nonlinear equations pretty well



Even though every single seed is slightly different, it evolves into a functioning plant

Nature deals with nonlinear equations pretty well



Nature deals with nonlinear equations pretty well



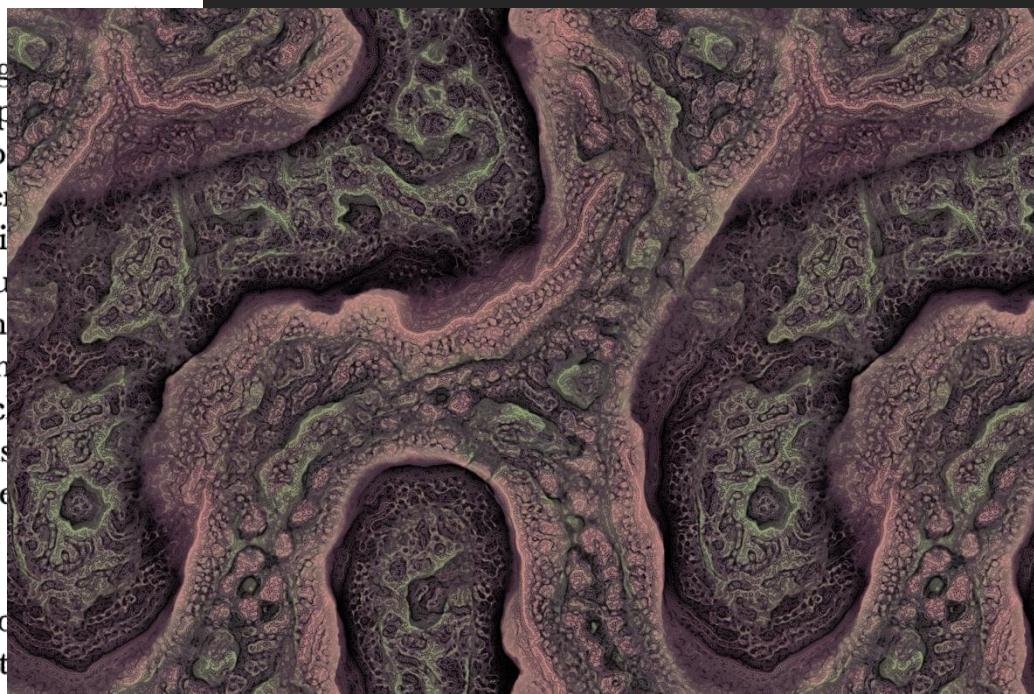
THE CHEMICAL BASIS OF MORPHOGENESIS

By A. M. TURING, F.R.S. *University of Manchester*

(Received 9 November 1951—Revised 15 March 1952)

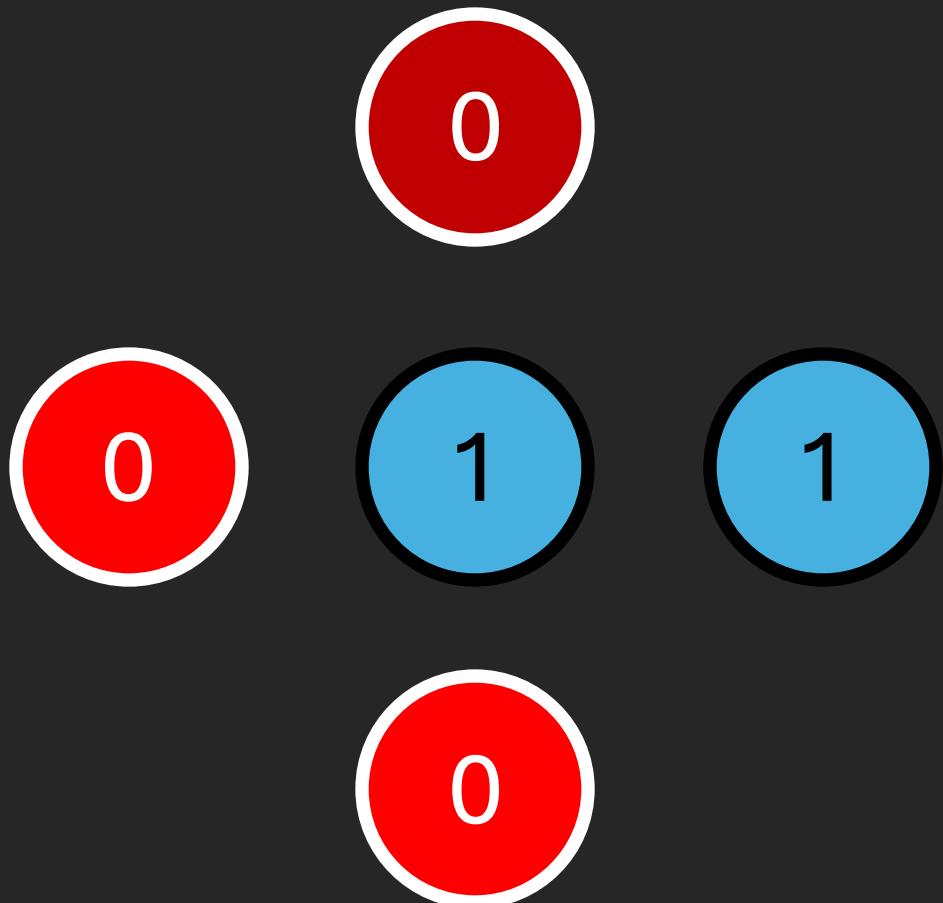
It is suggested that a system of chemical substances, called morphogens, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis. Such a system, although it may originally be quite homogeneous, may later develop regions of different concentrations and properties, forming stable or unstable structures of great complexity depending on conditions which may be internal or external to the system. It is suggested that such reaction-diffusion systems are considered in some detail in order to understand the onset of instability. It is found that there are two distinct forms of instability, corresponding to the two types of pattern formation on an isolated ring of cells, a mathematically convenient, though biologically unusual, model for the investigation. These are, in fact, the two types of pattern formation on a sphere. The investigation is chiefly concerned with the onset of instability. It is found that there are two distinct forms which this may take. In the most interesting form stationary waves appear on the ring. It is suggested that this might account, for instance, for the tentacles on *Hydra* and for whorled leaves. A system of reactions and diffusion on a sphere is also considered. Such a system appears to account for gastrulation. Another reaction system of two dimensions gives rise to patterns reminiscent of dappling. It is also suggested that these waves in two dimensions could account for the phenomena of phyllotaxis.

The purpose of this paper is to discuss a possible mechanism by which the genes of an organism may determine the anatomical structure of the resulting organism. The theory does not involve new hypotheses; it merely suggests that certain well-known physical laws are sufficient to account for many of the facts. The full understanding of the paper requires a good knowledge of mathematics, some biology, and some elementary chemistry. Since readers cannot be expected to be experts in all of these subjects, a number of elementary facts are explained, which can be found in text-books, but whose omission would make the paper difficult reading.



The Ising model

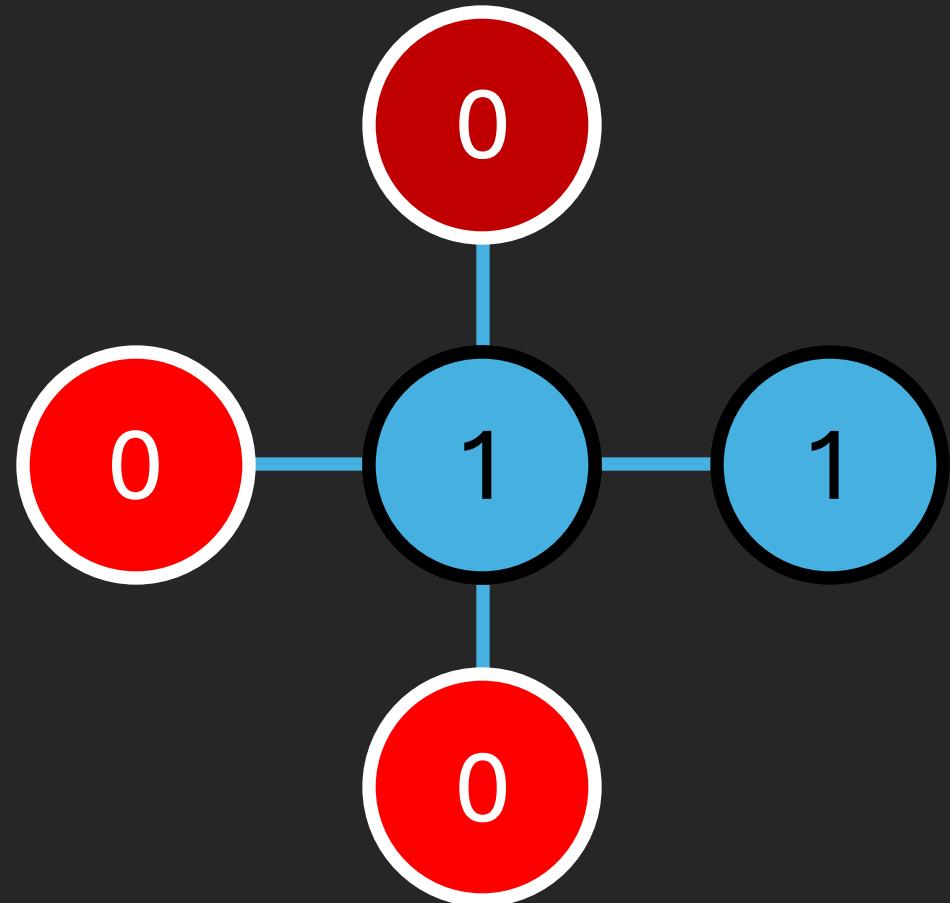
Consider a very simple object that can only take two values, either $\sigma = 0$ or 1



The Ising model

Consider a very simple object that can only take two values, either $\sigma = 0$ or 1

Now make them interact

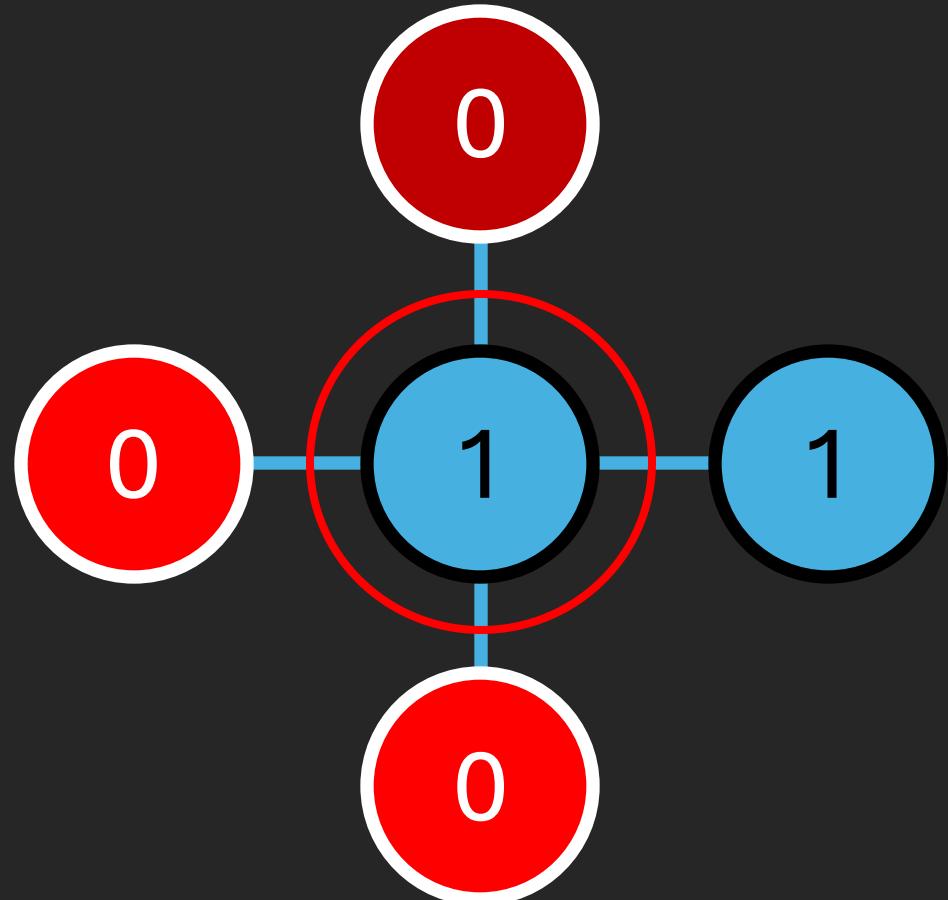


The Ising model

Consider a very simple object that can only take two values, either $\sigma = 0$ or 1

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Finally, choose a “temperature”
(basically, a parameter that controls how easily an object changes its value).

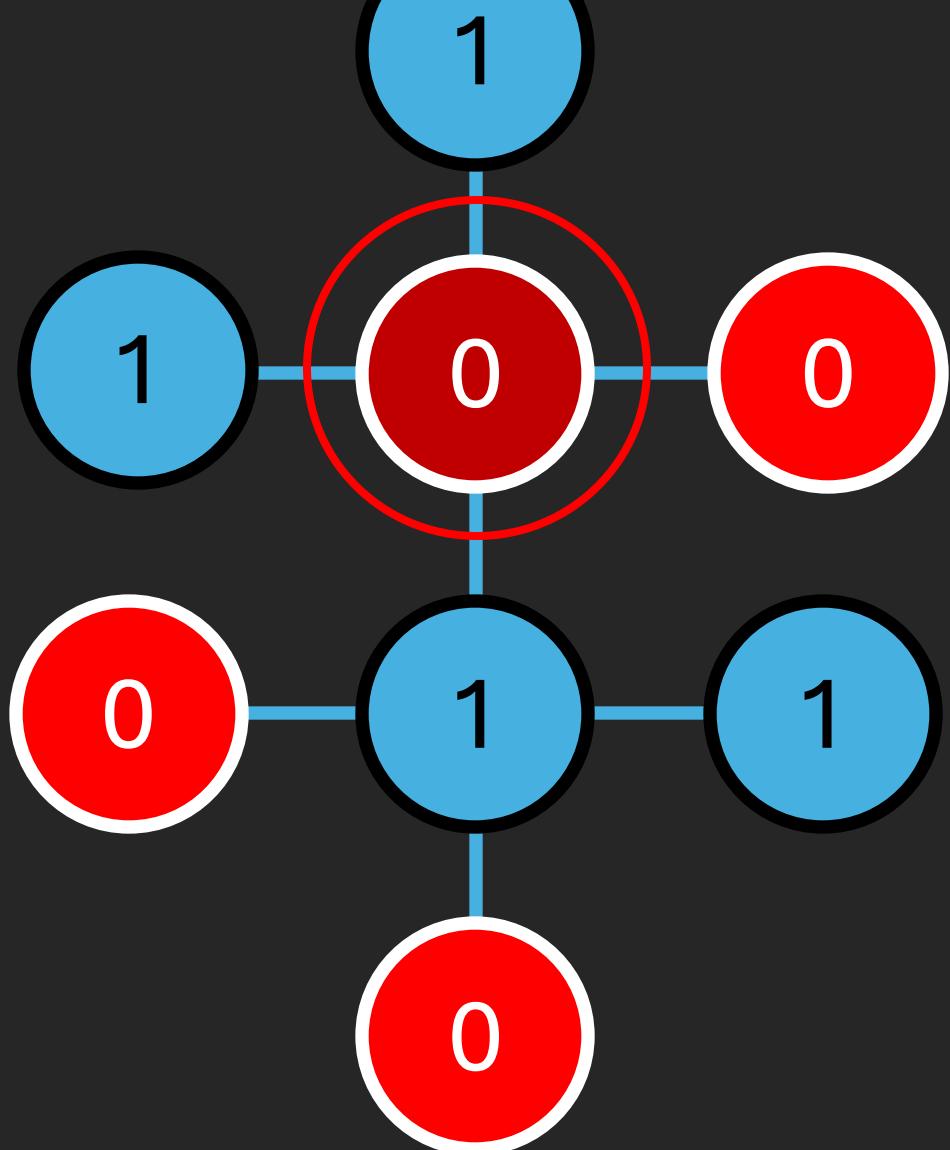


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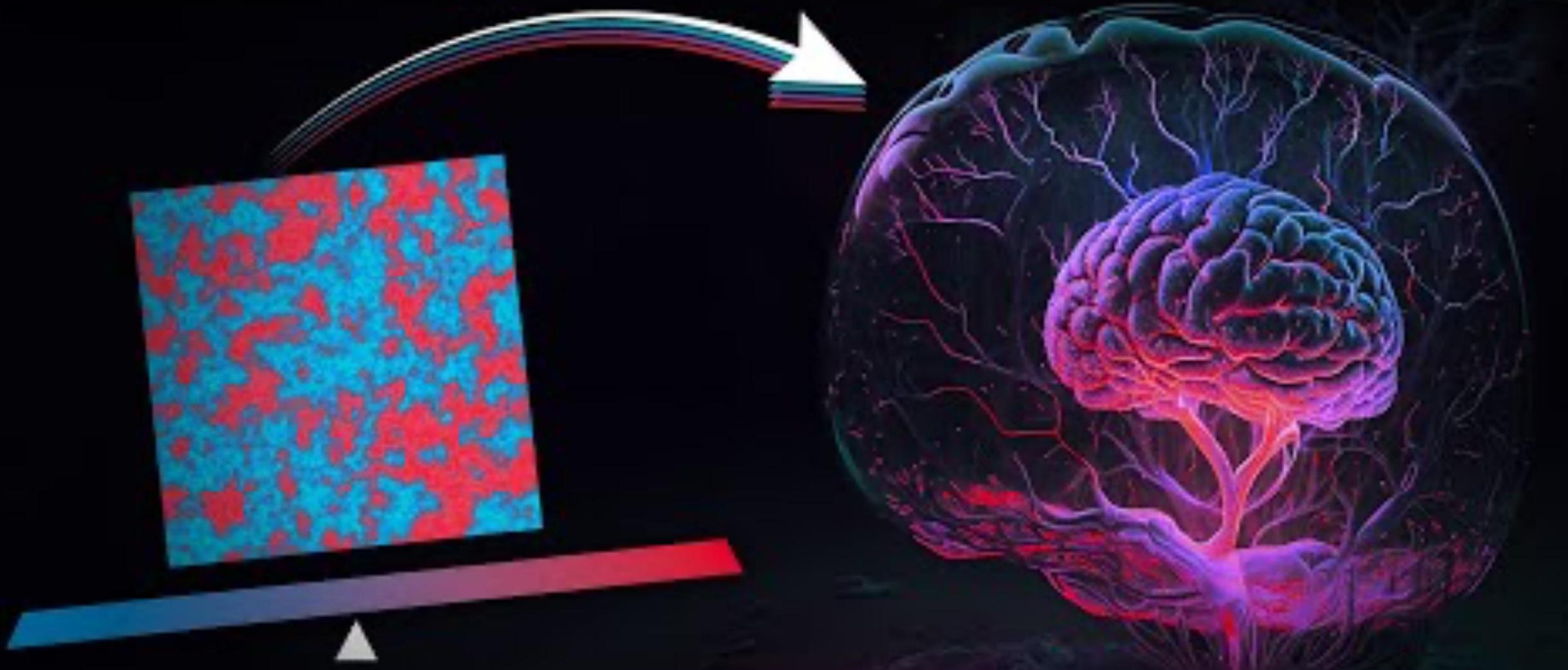
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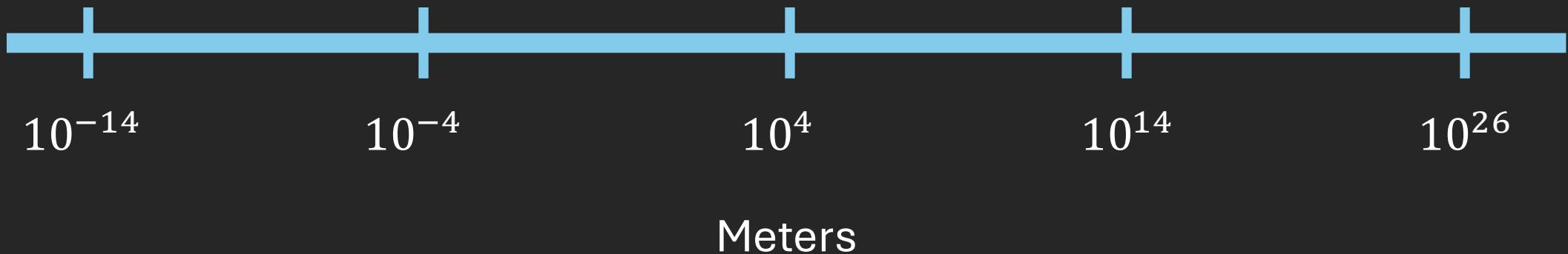
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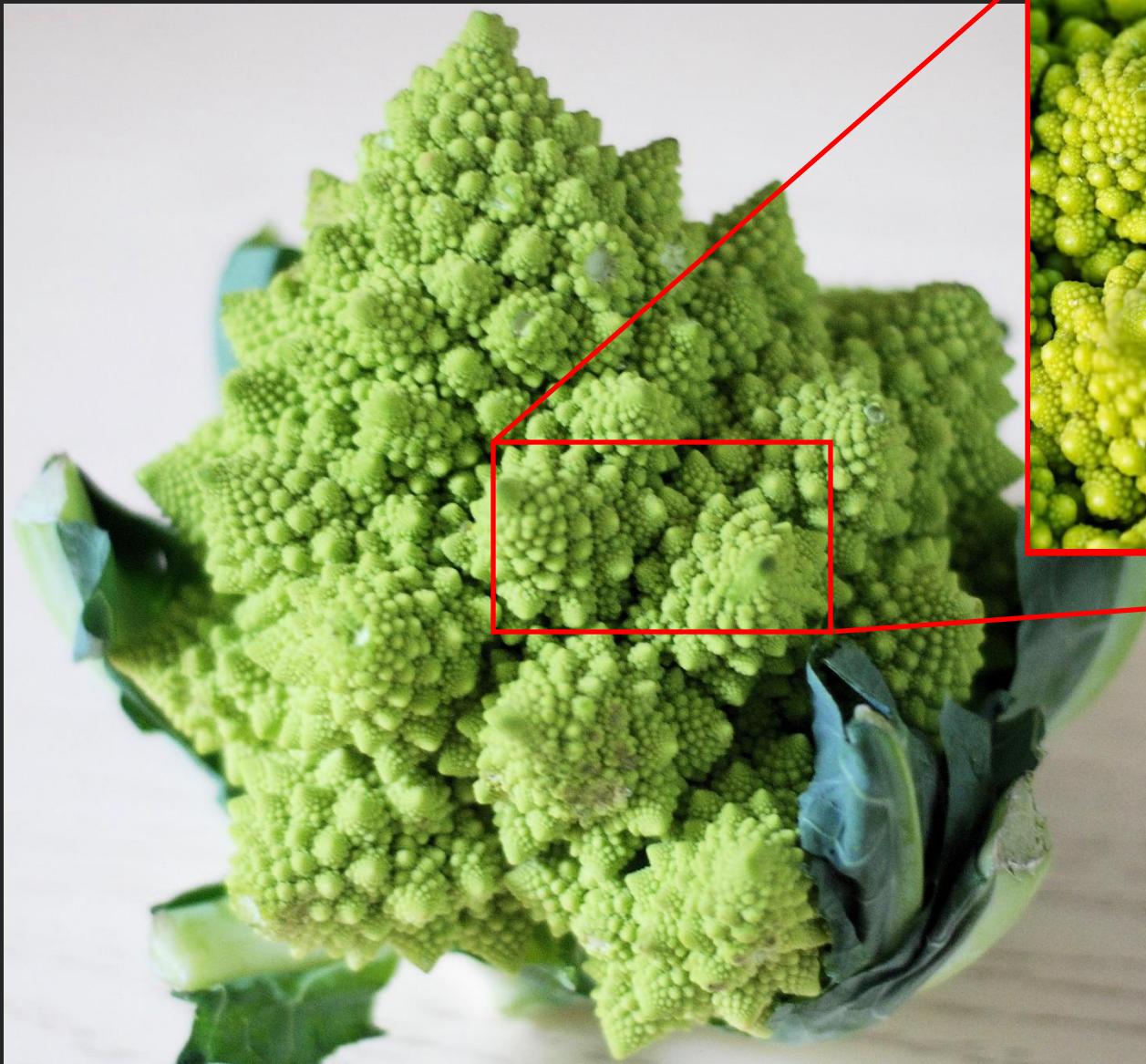
Phase transition?



At the critical point, we don't care about scales anymore!



But how does nature do it??



Things in Nature seem to self-organize at a critical point, in which hierarchies emerge and a “characteristic scale” ceases to exist

Self-organized criticality: An explanation of the $1/f$ noise

Per Bak, Chao Tang, and Kurt Wiesenfeld

Phys. Rev. Lett. **59**, 381 – Published 27 July 1987

Article

References

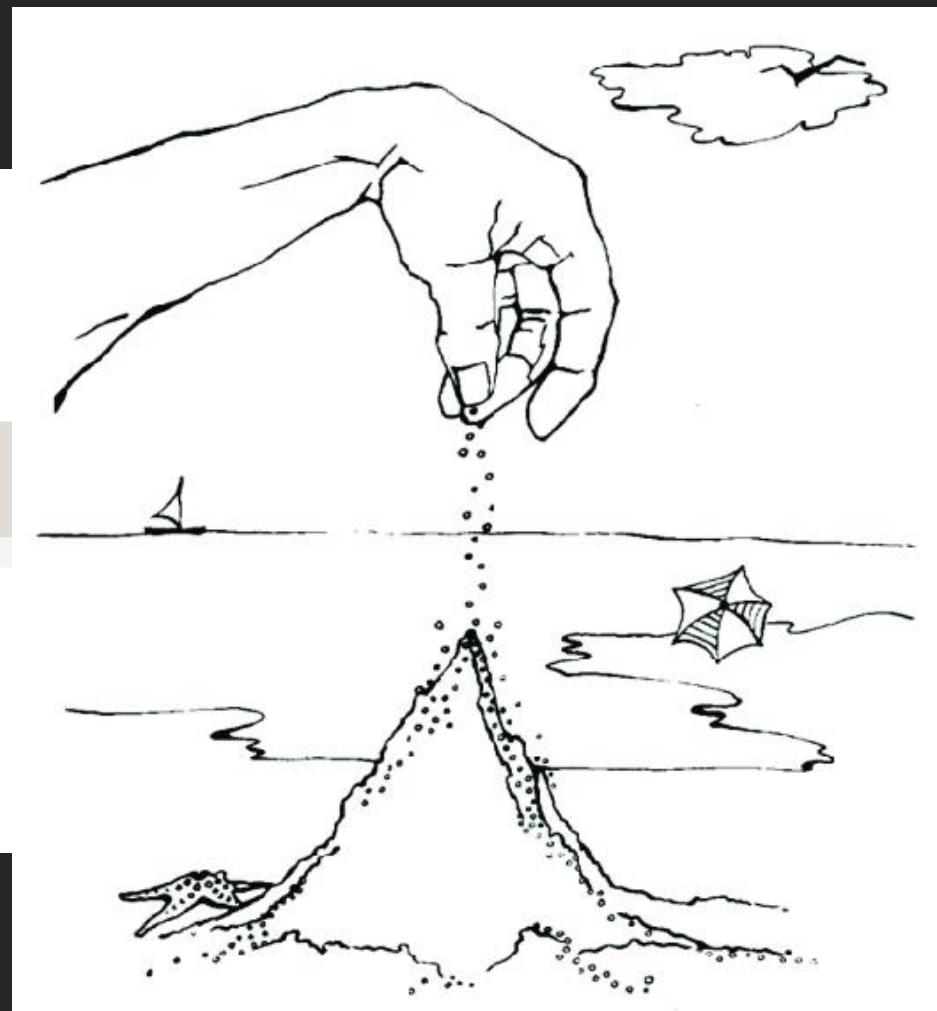
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ABSTRACT

We show that dynamical systems with spatial degrees of freedom naturally evolve into a self-organized critical point. Flicker noise, or $1/f$ noise, can be identified with the dynamics of the critical state. This picture also yields insight into the origin of fractal objects.



Self-Organized Criticality

Systems made of many degrees of freedom
(that interact non-linearly) which **self-organize** into a critical point in which **scale invariant** phenomena occur

Systems made of many degrees of freedom (**that interact non-linearly**) which **self-organize** into a critical point in which **scale invariant** phenomena occur

Part 3: Criticality in Brains

Last Example, I promise

Systems made of many degrees of freedom (**that interact non-linearly**) which **self-organize** into a critical point in which **scale invariant** phenomena occur

Part 3: Criticality in Brains

Last Example, I promise

Everyday you wake up and you are still the same person!



Systems made of many degrees of freedom (**that interact non-linearly**) which **self-organize** into a critical point in which **scale invariant** phenomena occur

Part 3: Criticality in Brains

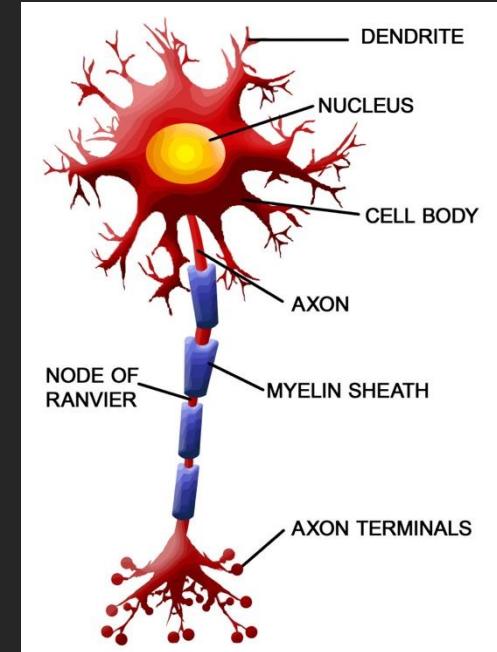
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Let's look at the brain as a system:

90 billion neurons



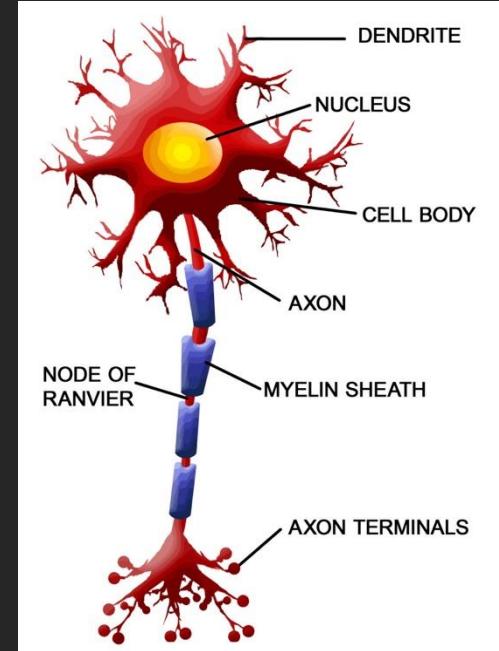
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90 billion neurons

Trillions of synapses



$$I = C_m \frac{dV_m}{dt} + \bar{g}_K n^4 (V_m - V_K) + \bar{g}_{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l),$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1 - n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1 - m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1 - h) - \beta_h(V_m)h$$

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Many degrees of freedom

Trillions of synapses

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Many degrees of freedom

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Interact non-linearly

Scale invariance?

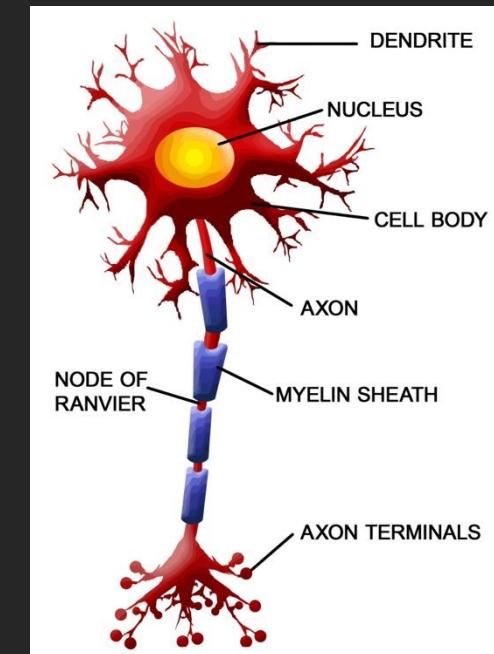
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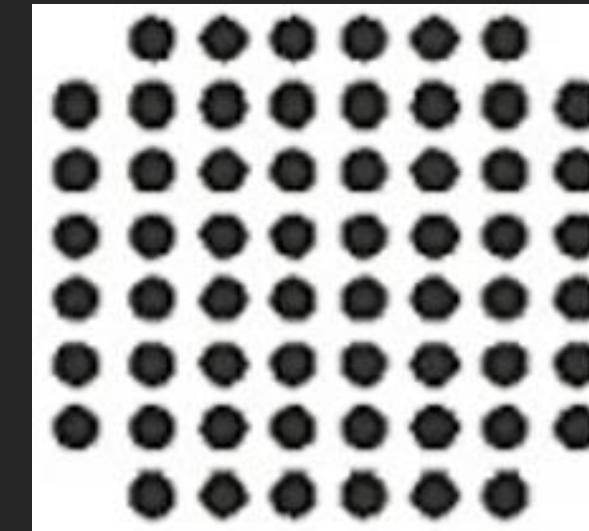
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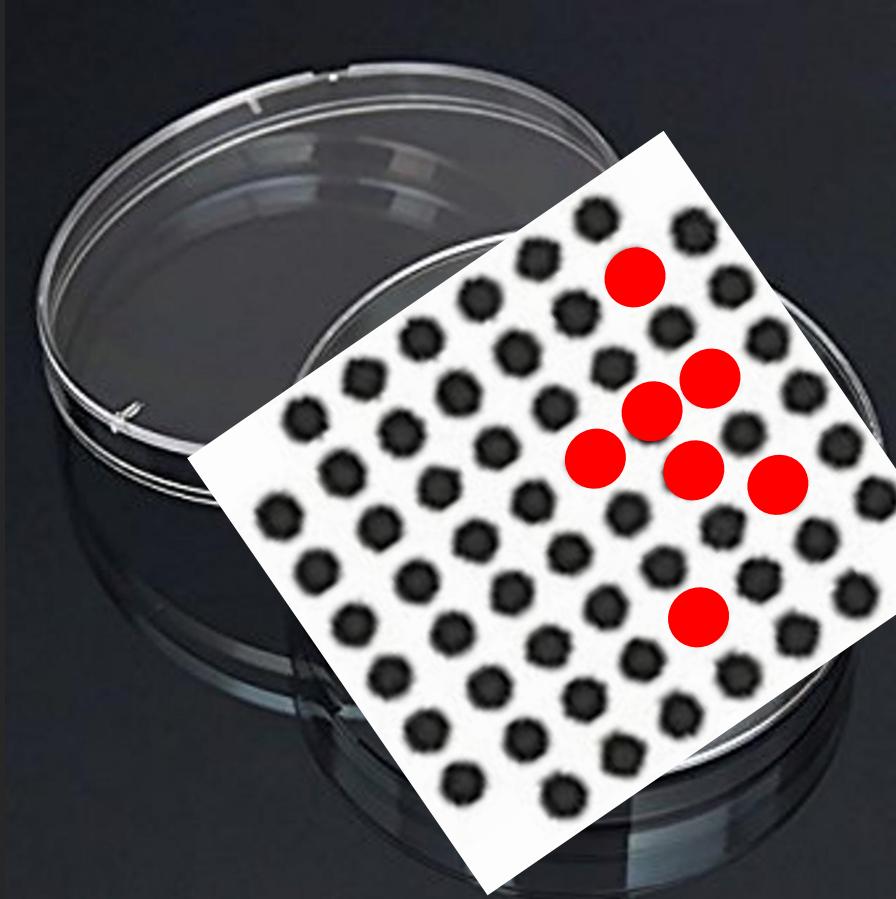
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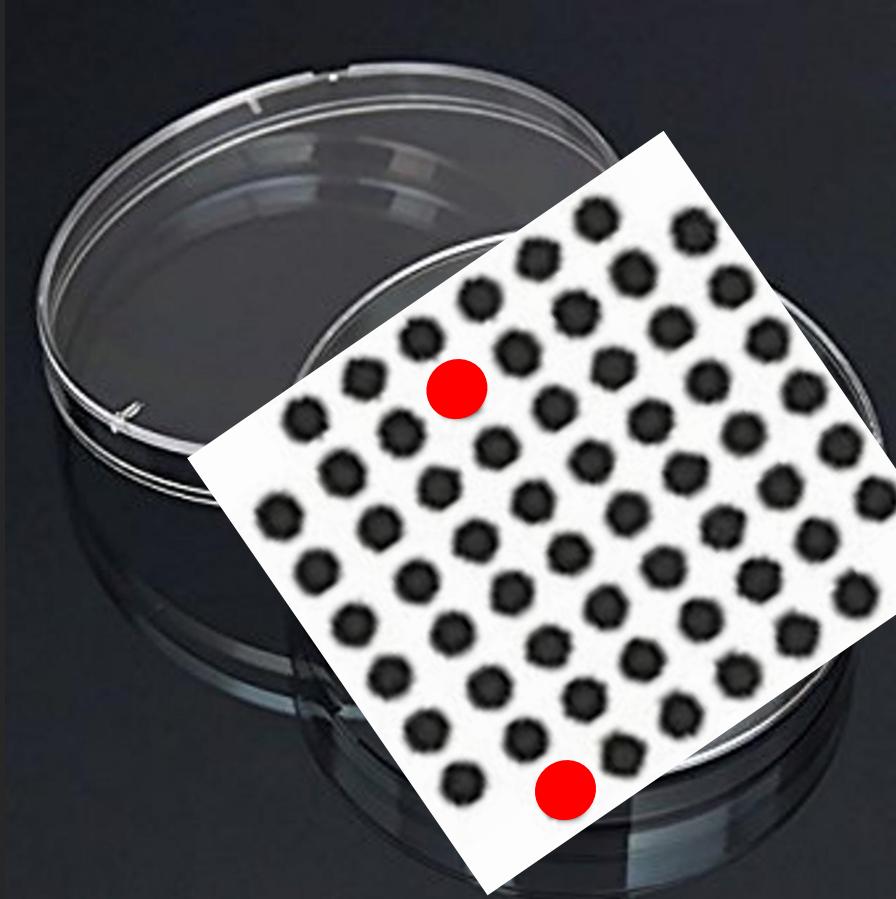
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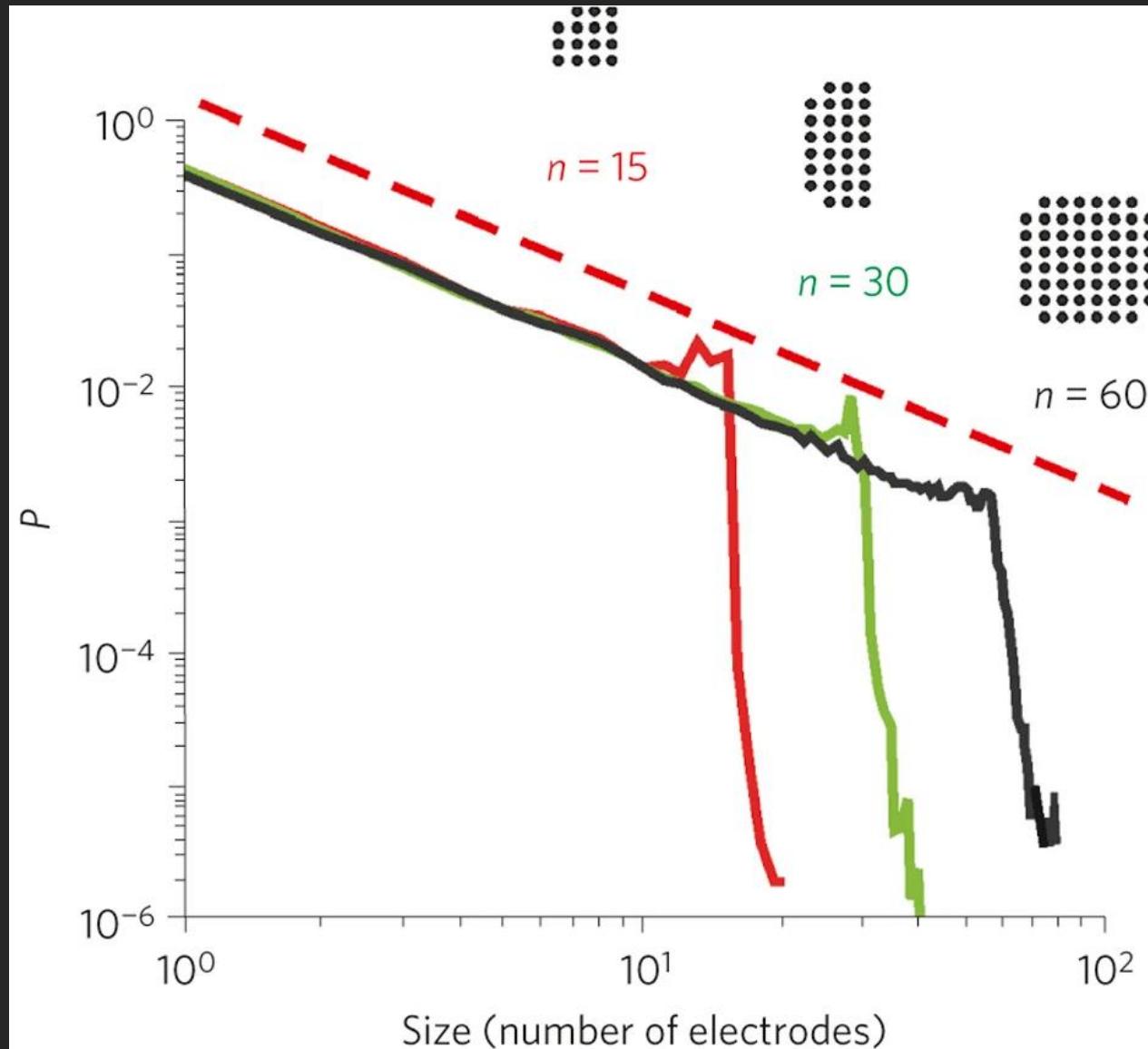


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Review Article | Published: 01 October 2010

Emergent complex neural dynamics

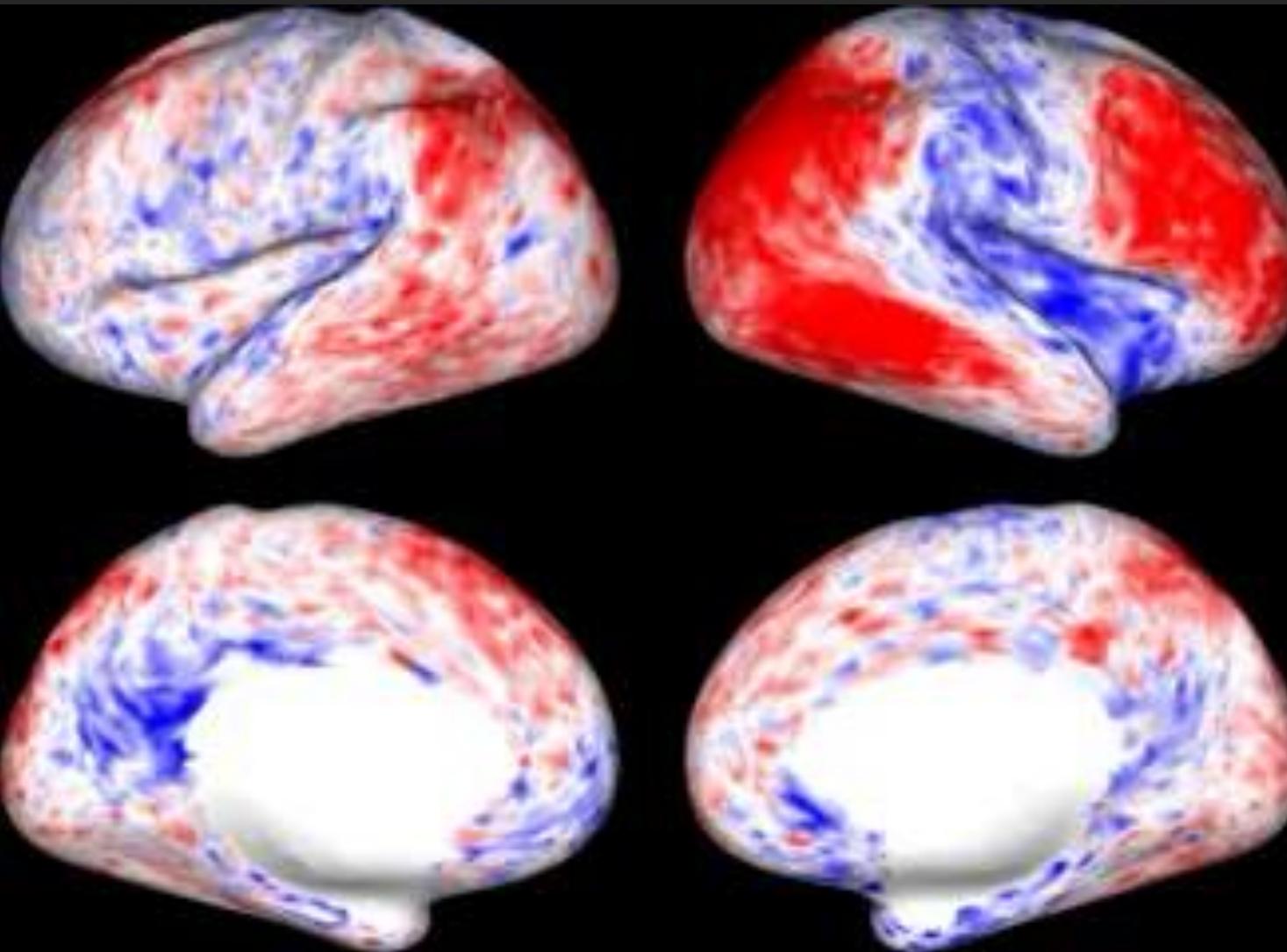
[Dante R. Chialvo](#)

[Nature Physics](#) 6, 744–750 (2010) | [Cite this article](#)

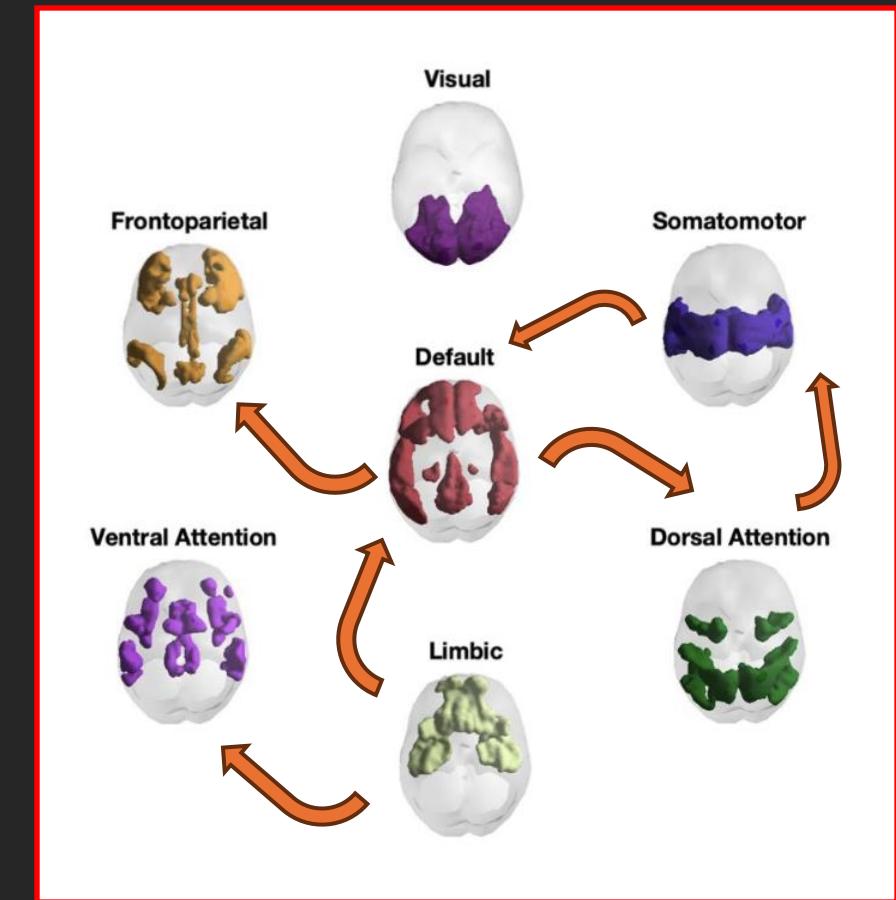
13k Accesses | 681 Citations | 66 Altmetric | [Metrics](#)

Okay, so I guess neurons display critical behaviours, so what?

Brain Activity at Rest



2x speed
Real time brain
activity via fMRI

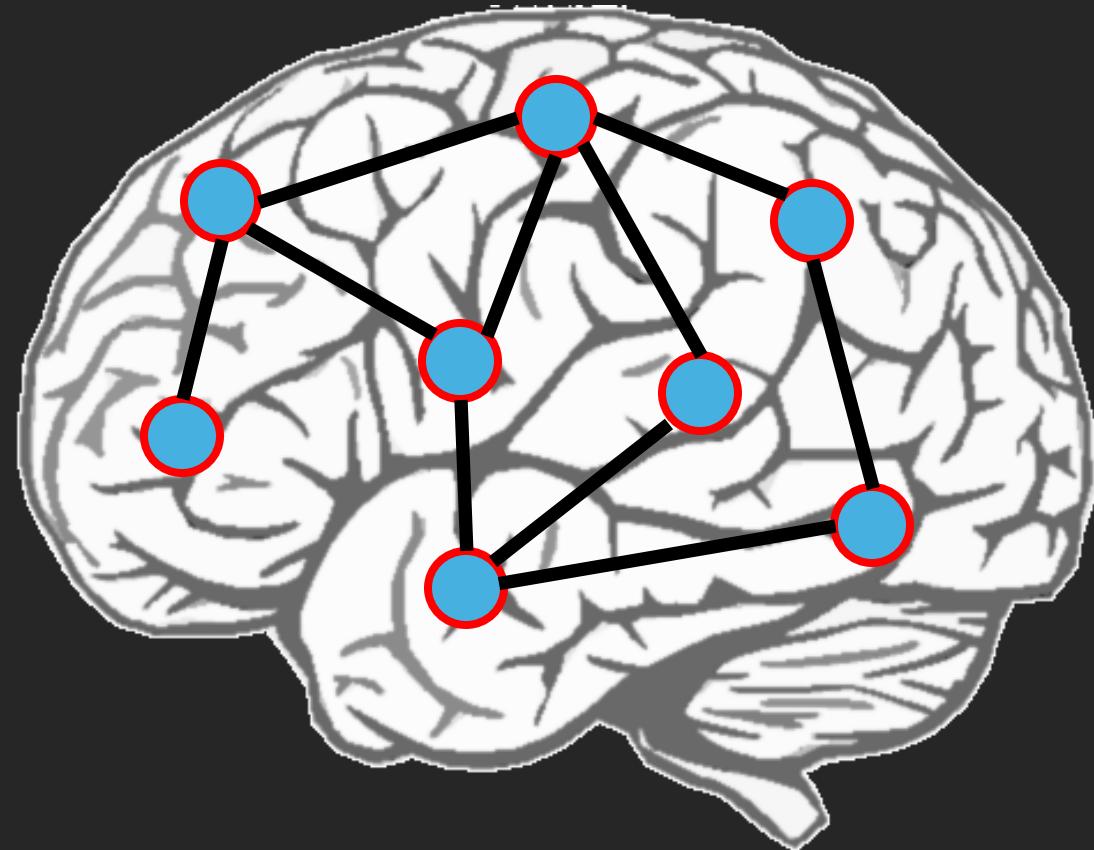


J. Vohryzek et al, "Ghost Attractors in Spontaneous Brain Activity: Recurrent Excursions Into Functionally-Relevant BOLD Phase-Locking States," *Frontiers in Systems Neuroscience*

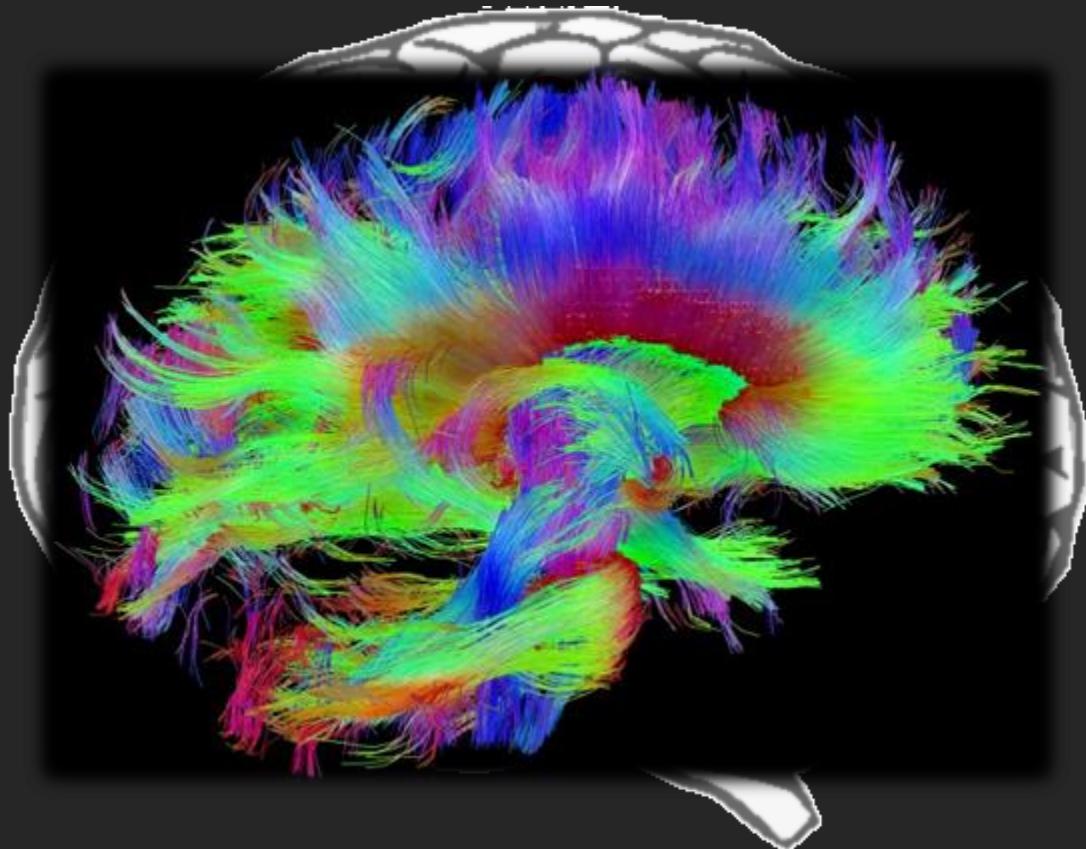
How to model this? Is it critical?



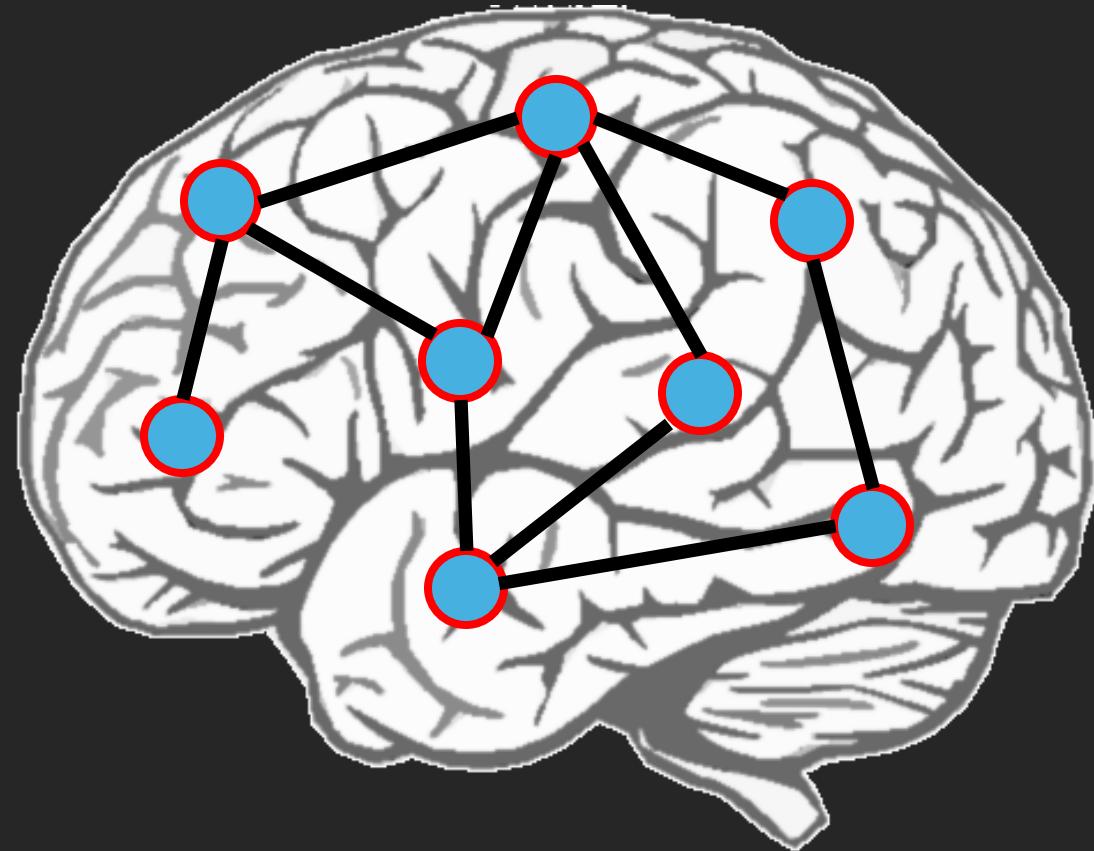
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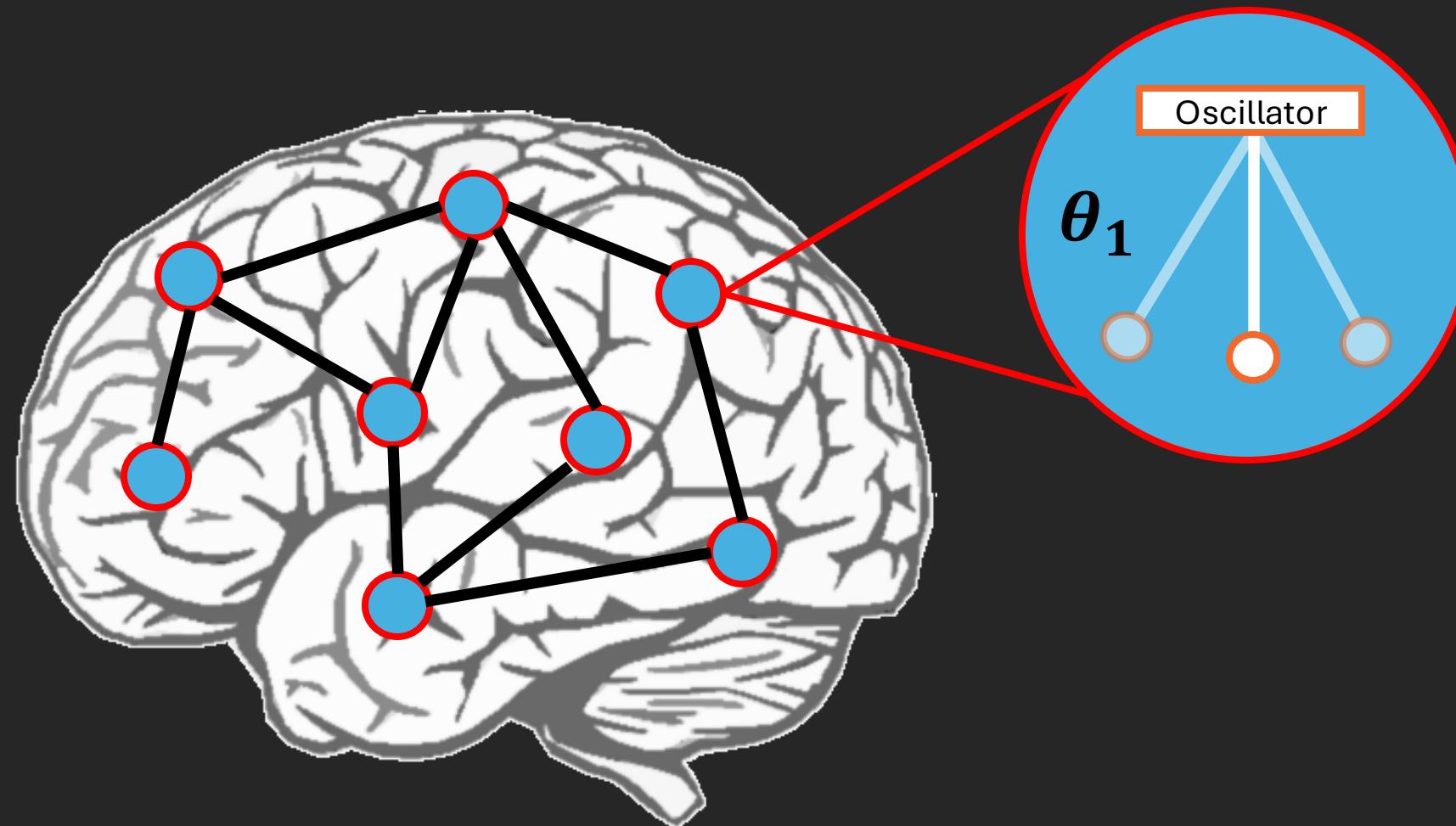
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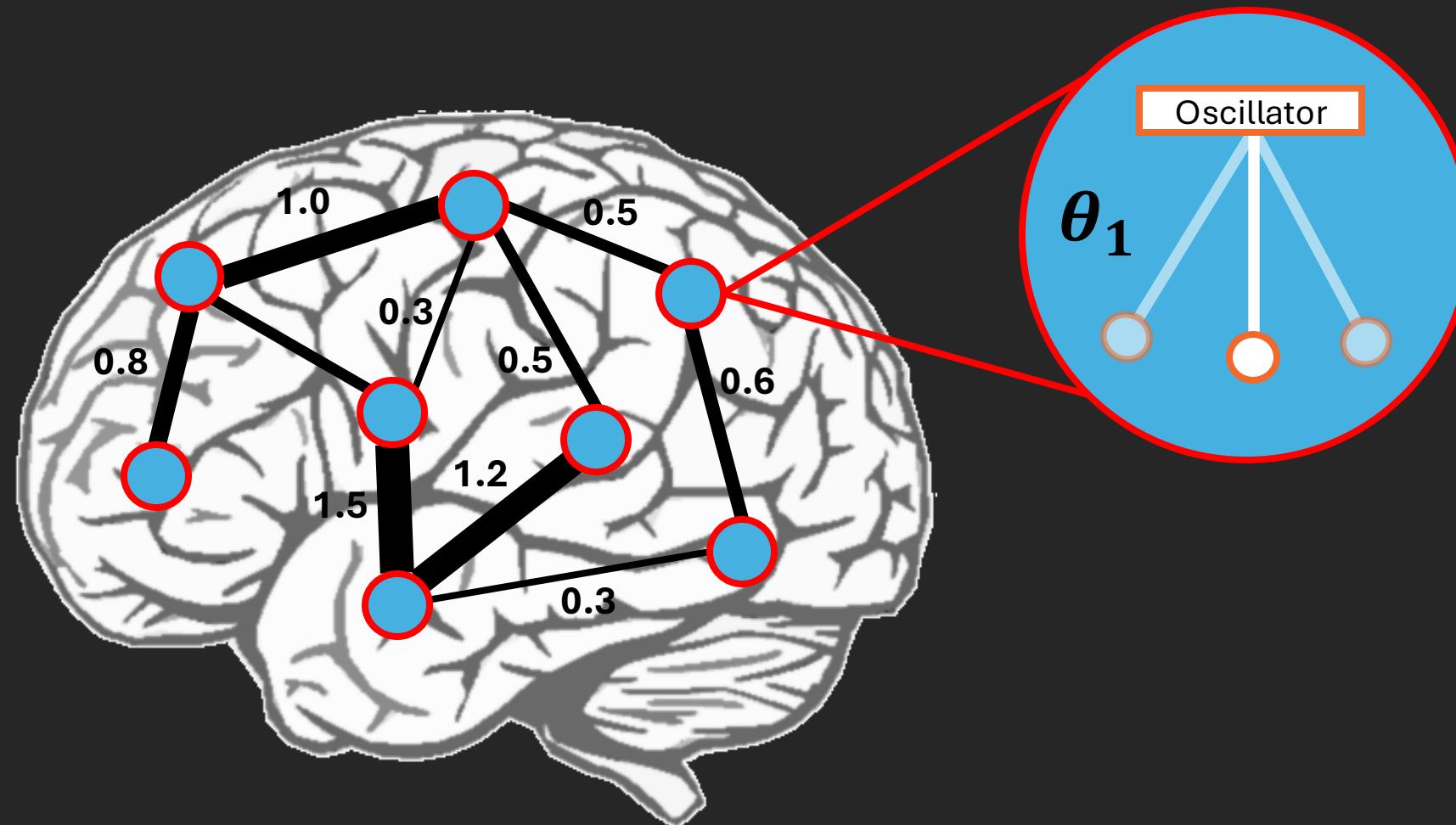
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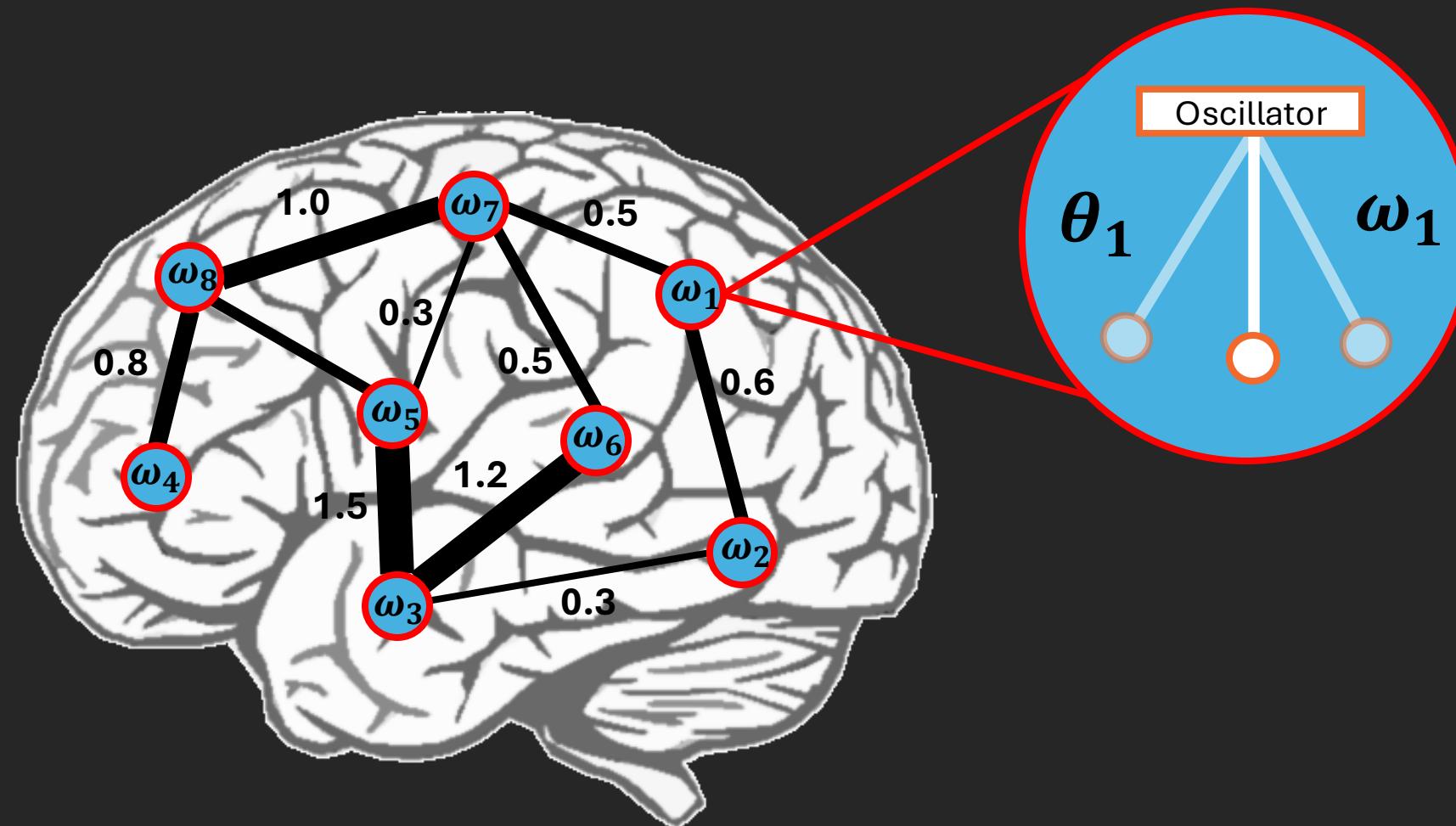
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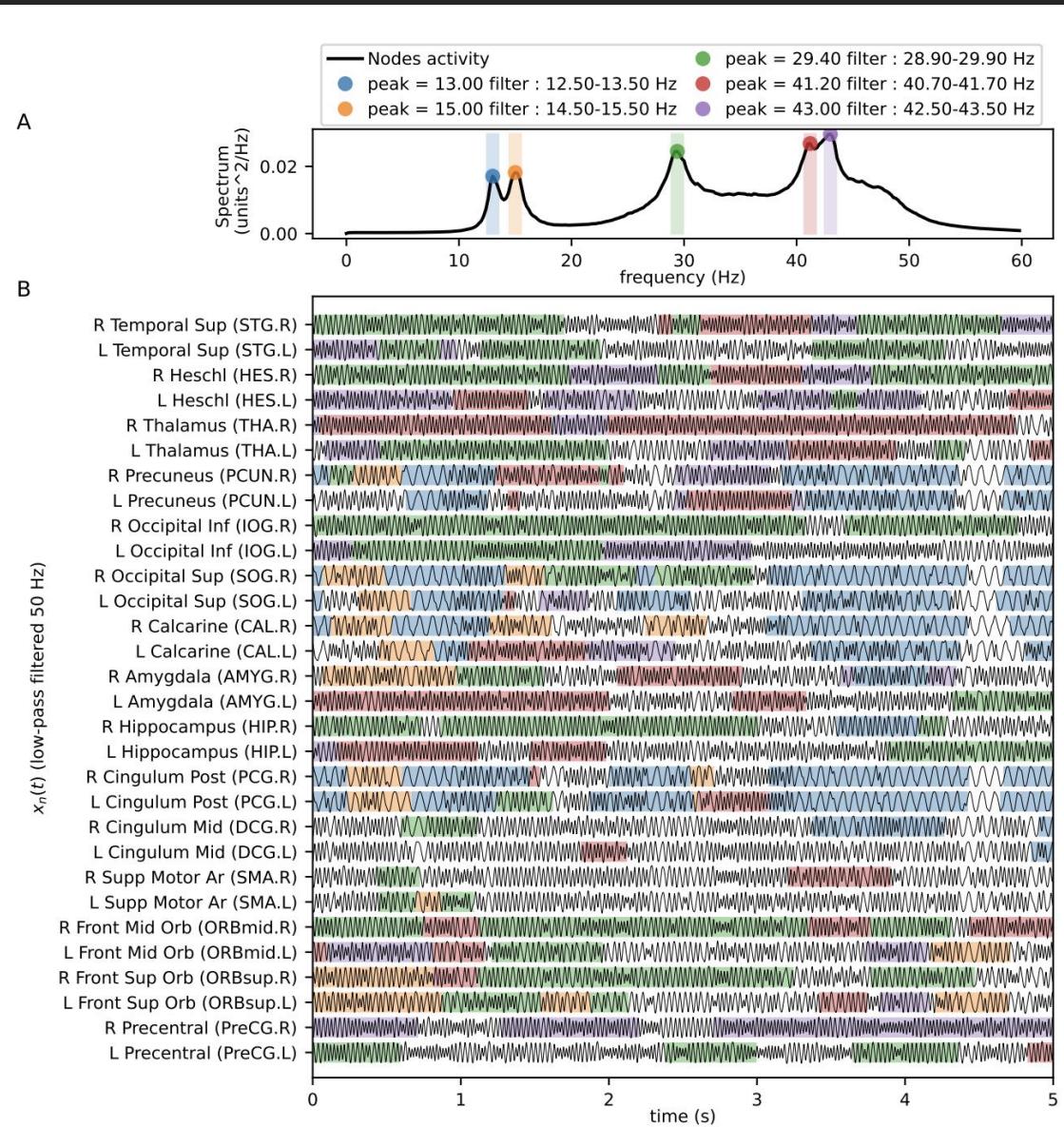


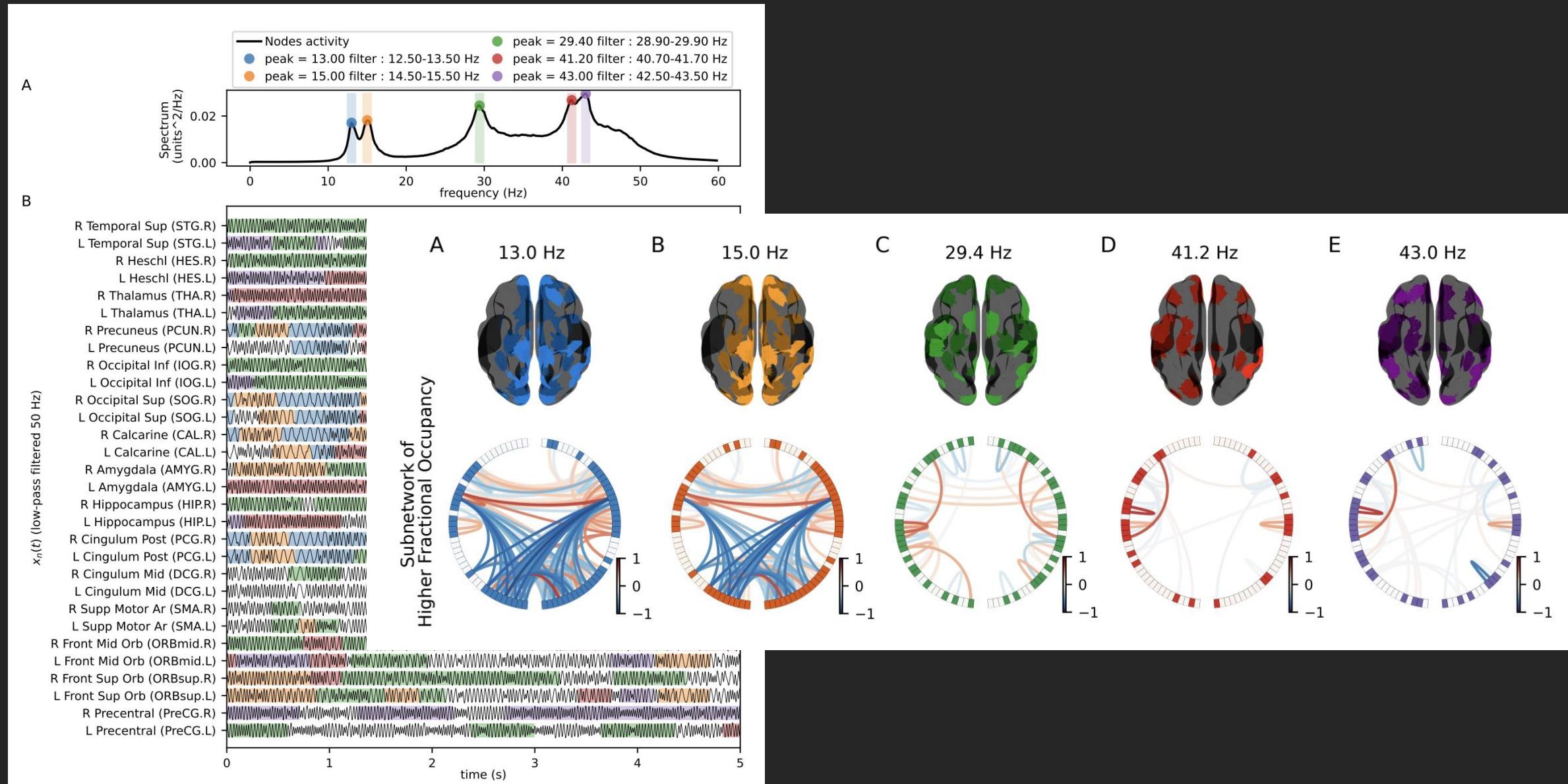
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Conclusions

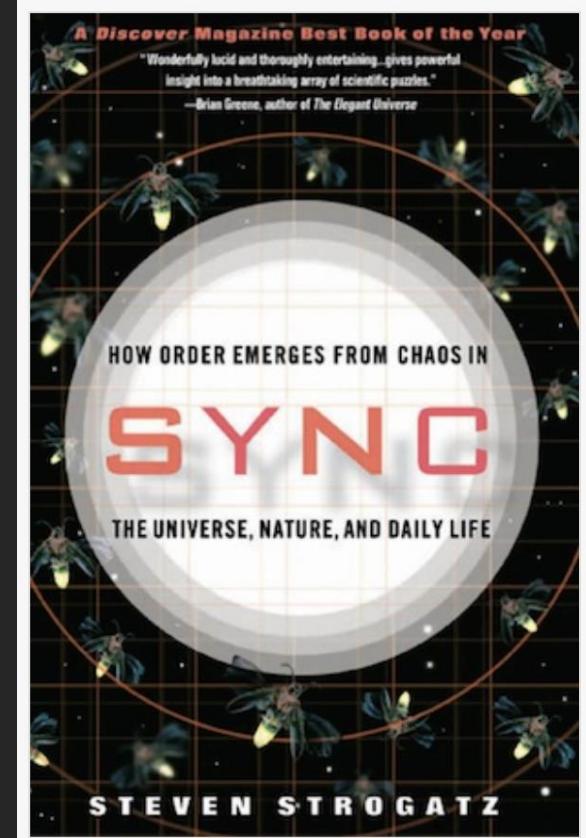
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William Bialek^{a,1}, Andrea Cavagna^{b,c}, Irene Giardina^{b,c}, Thierry Mora^d, Oliver Pohl^{b,c,2}, Edmondo Silvestri^{b,c}, Massimiliano Viale^{b,c}, and Aleksandra M. Walczak^e



No. 4486 October 22, 1955

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The presence of uranium in the meteorite should result in the production of the 2.33-day

Magnitude and Energy of Earthquakes

Criticality and scaling in evolutionary ecology

Ricard V. Solé ^a✉, Susanna C. Manrubia ^b✉, Michael Benton ^c✉, Stuart Kauffman ^d✉, Per Bak ^e✉

Conclusions

Nature speaks the language of nonlinear equations, and nonlinear equations are really hard to solve

Adaptive systems of many units interacting nonlinearly often self-organize into a dynamical regime very close to the critical point