Assignment

Homelab HL4

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Musical Acoustics

POLITECNICO
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Brass instrument simulation

This study aims at evaluating the input impedance for the various components of a trumpet. This is achieved by simulating the acoustic behaviour of the simplified model in *COMSOL Multiphysics*, which relies on the *Finite Element Method* for computing the simulation.

Model Implementation

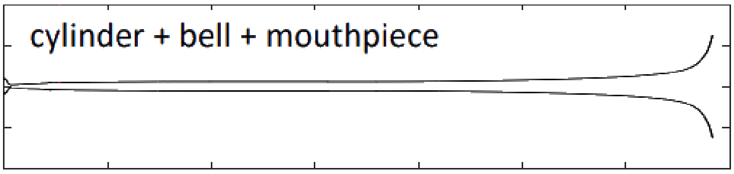

Figure 1: Simplified trumpet model section.

The simplified trumpet for the simulation is represented in the figure above and is composed of a cylindrical tube, an exponential bell and the mouthpiece. All these components will gradually be added to the geometry of the model and the frequency study will be computed for each configuration in order to evaluate the input impedance of the instrument.

- Geometry

The shape of the trumpet in analysis depicted in figure (1) allows to employ the 2D-axisymmetric geometry builder which constructs the three-dimensional model through a complete rotation around the main symmetry axis. This way, not only designing the geometry will be much easier, but also the computational cost will benefit from this feature. Apart for defining the geometry of the configuration being examined, it is also needed to set up the propagation domain. In this case this will correspond to a sphere of air surrounded by a *perfectly matched layer* which will act as a perfect absorber, efficiently avoiding reflections. The resulting geometry for the tube configuration, treated in section (1), can be seen in figure (3). All the parameters employed for designing the simulation are listed in the figure below:

Name	Expression	Value	Description
rT	0.6[cm]	0.006 m	Tube radius
rS	2[m]	2 m	Air sphere radius
N	255	255	Number of frequencies
Lt	1.37[m]	1.37 m	Tube length
Lspace	20[mm]	0.02 m	Length empty space
lambdaMax	c0/fMax	0.08575 m	Wave length max freq
fMin	50[Hz]	50 Hz	Min freq
fMax	4000[Hz]	4000 Hz	Max freq
c0	343[m/s]	343 m/s	Wave speed
Lh	0.2[m]	0.2 m	Bell length
m	28	28	Bell exponent
St	rT^2*pi	1.131E-4 m²	Surface tube
rM	rT+0.3[cm]	0.009 m	Mouthpiece radius
Lm	10[cm]	0.1 m	Mouthpiece length
step	(fMax-fMin)/N	15.49 1/s	
endR	sqrt(rM^2-(rT/8)^2)	0.0089687 m	

Figure 2: Simulation parameters.

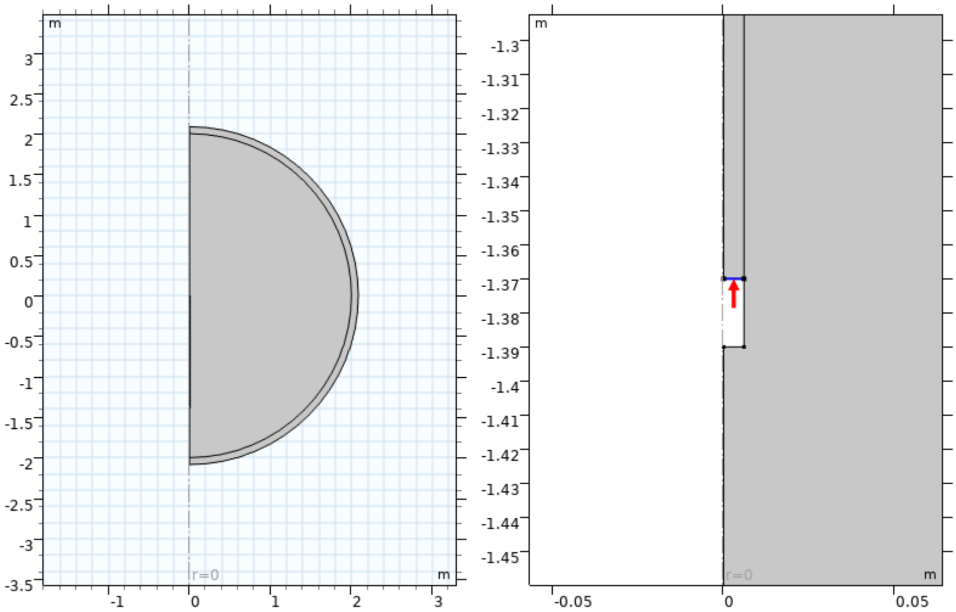

Figure 3: Snapshots for the entire geometry (left) and detail of the tube input port (right).

- Mesh

The mesh is built using the *Free triangular* one. In particular the mesh is calibrated for fluid dynamics, allowing *COMSOL* to take care of the element size distribution over the tube, bell, mouthpiece and propagation domain. Furthermore, the maximum mesh element size is set to be $lambdaMax/5$ in order to satisfy the required 5 points per wavelength condition. The PML mesh, on the other hand, is defined using the *mapped* node, as recommended by *COMSOL* PML documentation.

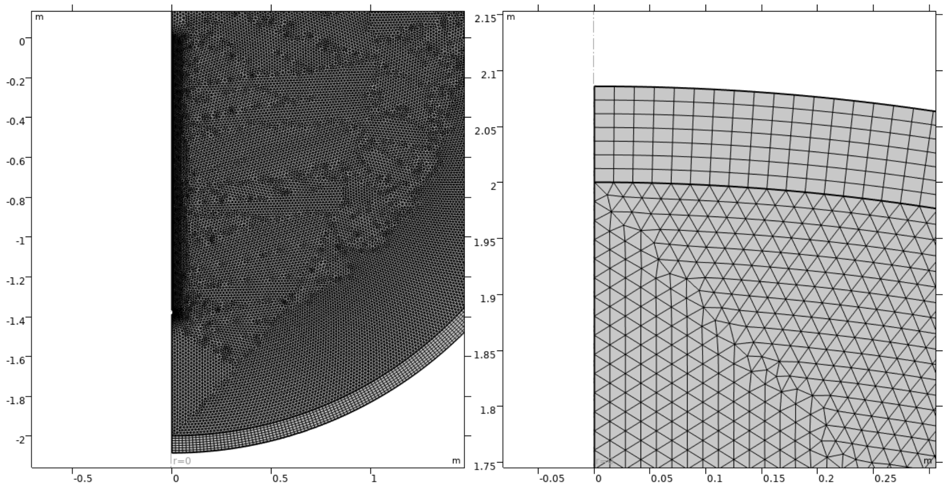

Figure 4: Mesh details for tube distribution and PML distribution.

- Physics

The physics component employed in the simulation is the *Pressure Acoustic*. More specifically, the simulation will impose an *Interior Sound Hard Boundary (Wall)* at the tube, bell or mouthpiece edge, as to confine the propagation of the sound inside the instrument. Within the focus of this study, *viscous boundary layers* can be neglected. In addition to this, the input pressure has to be set up by employing the *Port boundary condition*, which is used to excite and absorb pressure waves that enter or leave waveguide structures. The amplitude of the pressure wave is set accordingly to the requirements to 1.1 Pa. In order to avoid reflections in the propagation domain a PML has to be set up. The *perfectly matched layer (PML)* is a domain or layer that is added to an acoustic model to mimic an open and nonreflecting infinite domain. It sets up a perfectly absorbing domain as an alternative to nonreflecting boundary conditions. The physical thickness of the layers is not important in frequency domain models. If the PML is located close to a radiating source, evanescent wave components can interact with the PML and generate unphysical reflections. In order to avoid this the PML should be placed at least $\lambda/8$ away from the source, condition which is satisfied in this case of study. Furthermore, *COMSOL* documentation recommends the PML to be meshed with at least 5 or 6 mesh layers, for this reason the thickness of the PML is set to $lambdaMax$. Accordingly to the 5 points per wavelength condition, this thickness will guarantee the PML mesh to respect this rule. In order to further ensure this condition is satisfied, the mesh of the PML has been set to mapped as previously discussed, fixing the number of elements to seven. Finally it is needed to set up a *Parametrized curve 3D* node, in order to evaluate the Sound Pressure Level at the internal boundary of the PML. This will be needed in plotting the radiation pattern of the various models, after normalizing it with respect to its maximum value.

- Study

In order to evaluate the input impedance for the required frequency range, a *frequency domain study* has to be employed. In particular, the study requires to evaluate the results from $f_{min} = 50\ Hz$ to $f_{max} = 4000\ Hz$ with $N = 255$ frequency steps in between. This means that each frequency step will result as:

$$step = \frac{f_{max} - f_{min}}{N} = 15.49\ Hz \tag{1}$$

The pressure and velocity are integrated over the surface of the tube entrance and saved as variables. These values are then converted in the desired input impedance by means of the relation:

$$Z_{in} = 10\ log_{10}\left(\left|\frac{P_{in}}{U_{in}}\right|\right) \tag{2}$$

a) Exercise 1 - Impedance of a tube

Once the simulation has been run, the following results are obtained.

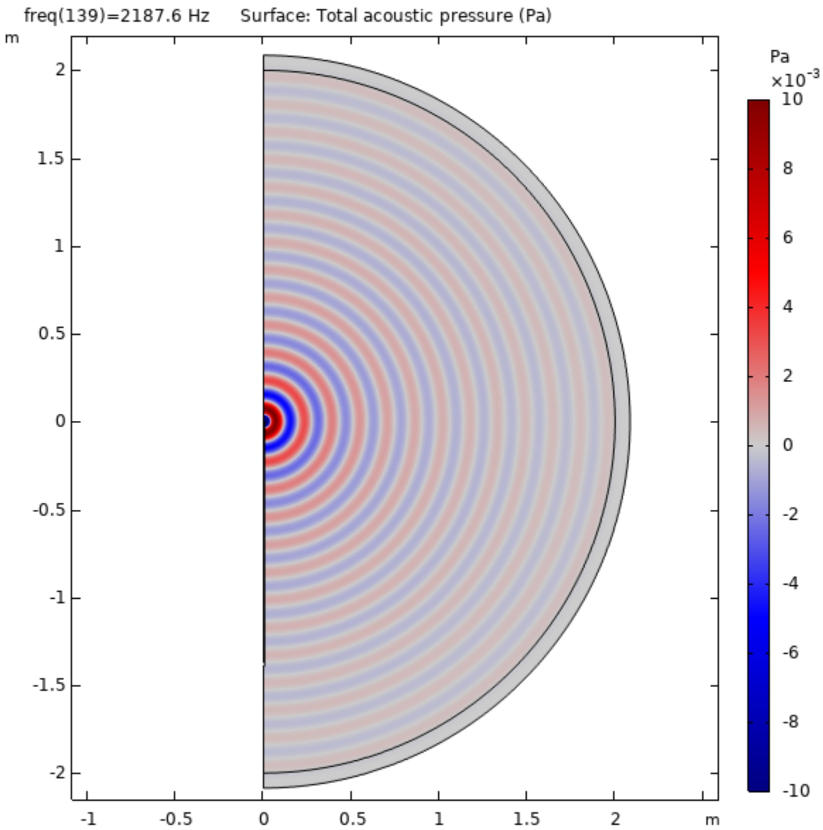

Figure 5: Pressure distribution for maximum of the impedance at 2187.6 Hz.

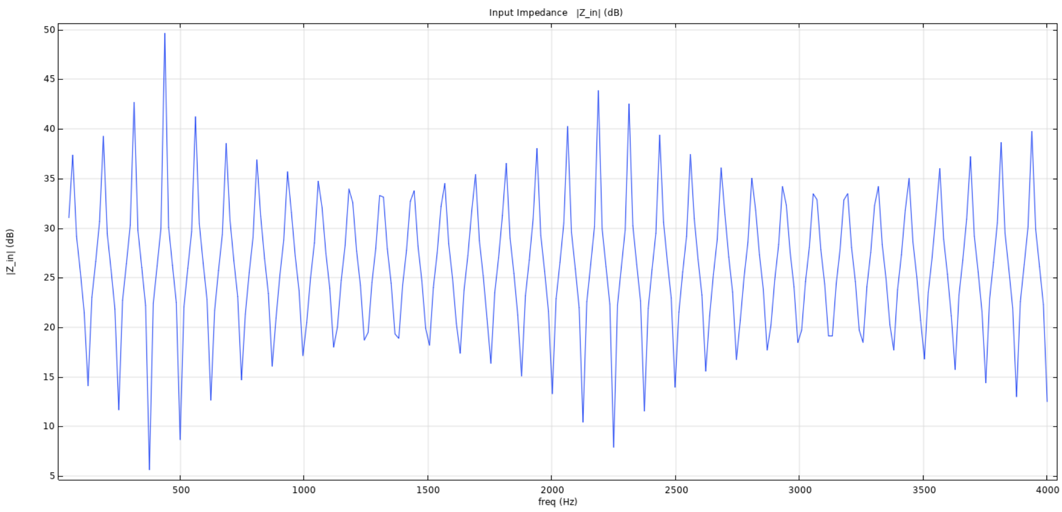

Figure 6: Module of the input impedance of the tube.

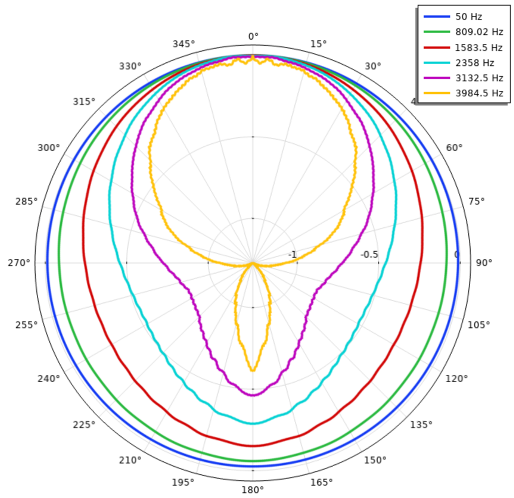

Figure 7: Polar plot of the directivity pattern, tube configuration (dB).

The results from the simulation are perfectly coherent with theory. In particular minima are positioned at even harmonics, following the rule:

$$f_{min} = \frac{nc}{2L} \tag{3}$$

The same goes for the maxima which are positioned at frequencies which undergo the relation:

$$f_{max} = \frac{(2n-1)c}{4L} \tag{4}$$

The simulated data for the first five input impedance maxima and minima, are compared to the theoretical one in the following table:

	Simulation minima	Numerical minima	Simulation maxima	Numerical maxima
1	127.45	125.18	65.49	62.59
2	251.37	250.37	189.41	187.77
3	375.29	375.55	313.33	312.95
4	499.22	500.73	437.25	438.14
5	623.14	625.91	561.17	563.32

Table 1: Simulated and numerical impedance maxima and minima frequencies.

For what concerns the polar plot, figure (7), it is clear that the tube only configuration is a bad radiator due to its omnidirectionality. At higher frequencies a more prominent directivity appears, but still with a small variation ($1\ dB$) with respect to the frontal lobe.

b) Exercise 2 - Tube with bell

Let's now evaluate the effect of a bell attached to the tube radiating bore. The shape of the exponential bell is determined by the following relation:

$$r = \sqrt{\frac{St}{\pi} e^{mz}} \tag{5}$$

where St is the surface of the tube, m is the exponential parameter of the bell and z is the vertical coordinate. This shape can be implemented in *COMSOL* by employing the *parametric curve*. A detail of the geometry for this configuration can be seen in the figure on the right. The rest of the geometry components i.e. the tube and the propagation domain, remain the same as for section (1). Once the geometry has been set up, the same study conducted in the previous section can be computed. The results are reported below.

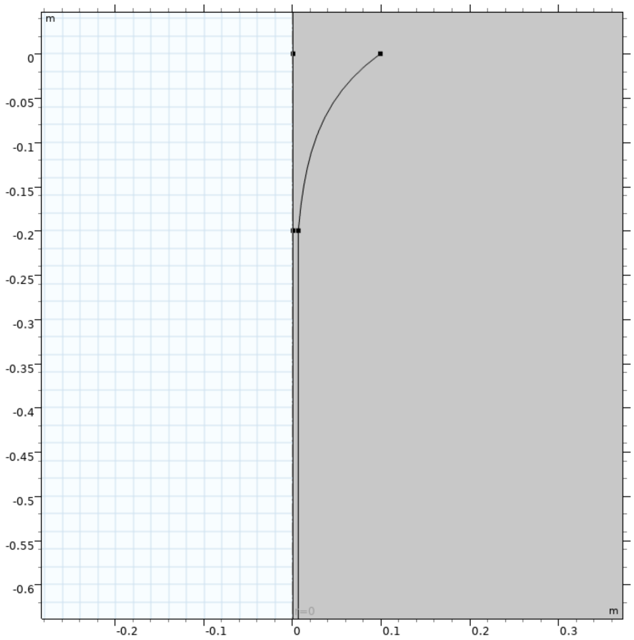

Figure 8: Detail of the bell geometry.

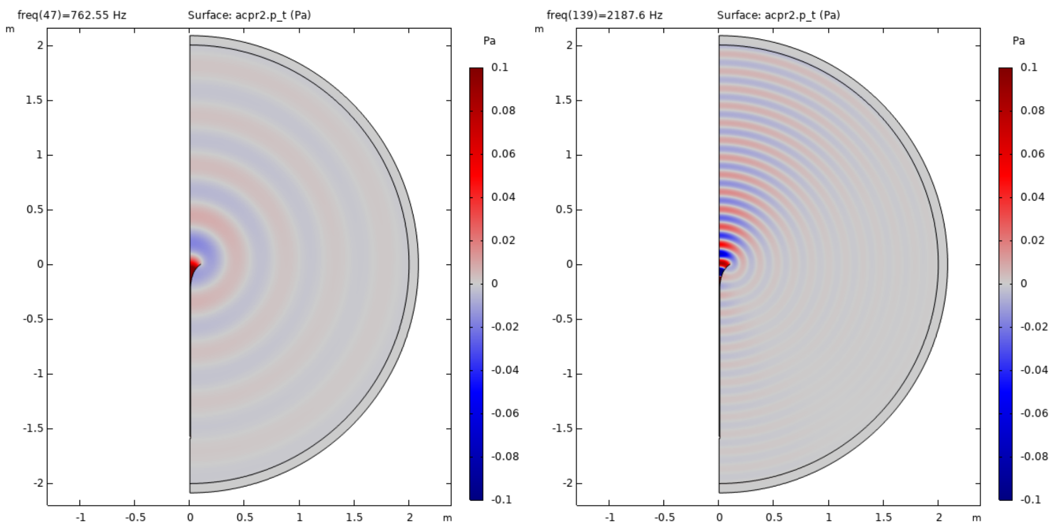

Figure 9: Pressure distribution at 762.55 Hz (left) and at 2187.6 Hz (right).

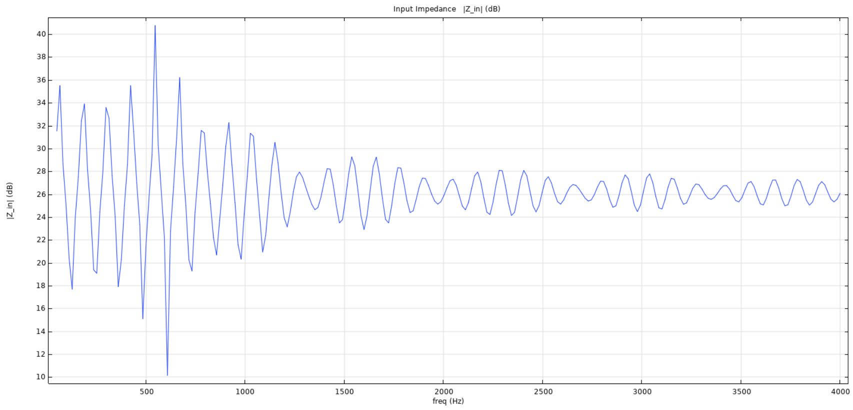

Figure 10: Module of the input impedance of the tube with flaring bell configuration.

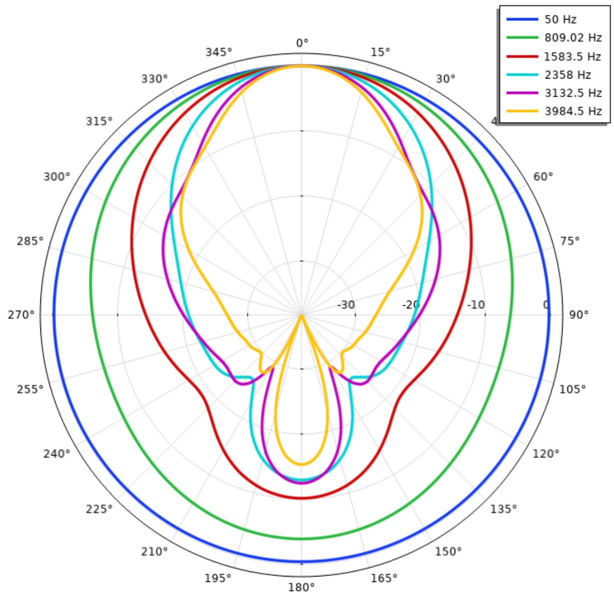

Figure 11: Polar plot of the directivity pattern, tube with flaring bell configuration (dB).

The effect of adding the bell is clear both in the pressure distribution and in the input impedance trend. In particular the impedance adaptation exerted by the bell allows for a bigger amplitude of the radiated pressure. This is noticeable comparing the color scale of figure (9) with the one for the tube only configuration, figure (5). The same graph highlights also how, for higher frequencies, the bell introduces a more prominent directivity with respect to the previous case. This is also clear from the directivity pattern in figure (11). For what concerns the the input impedance, the bell imposes an attenuation of the curve at higher frequencies which results smoother. Also a slight shift of resonances towards lower frequencies is introduced.

c) Exercise 3 - Mouthpiece

In this section a simplified version of the mouthpiece will be analyzed. The mouthpiece geometry, which can be seen in the figure on the right, is designed as a hemisphere attached to a cone. In particular, in the *2D-axisymmetric* context, this geometry is achieved defining a line and employing the *circular arc* node, in which the starting point, end point and radius are specified. The geometry of the air volume and PML remains the same as for previous sections, but the gap at the input of the mouthpiece, and consequently the port at which the system is excited, must be adapted to the new section of the mouthpiece hole. Furthermore, the propagation domain radius has been halved as to adapt it to the smaller dimension of the mouthpiece. Once the simulation has been run, the following results are obtained.

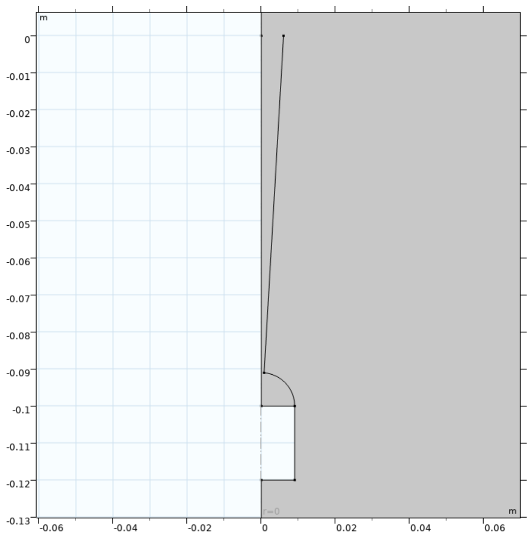

Figure 12: Mouthpiece geometry.

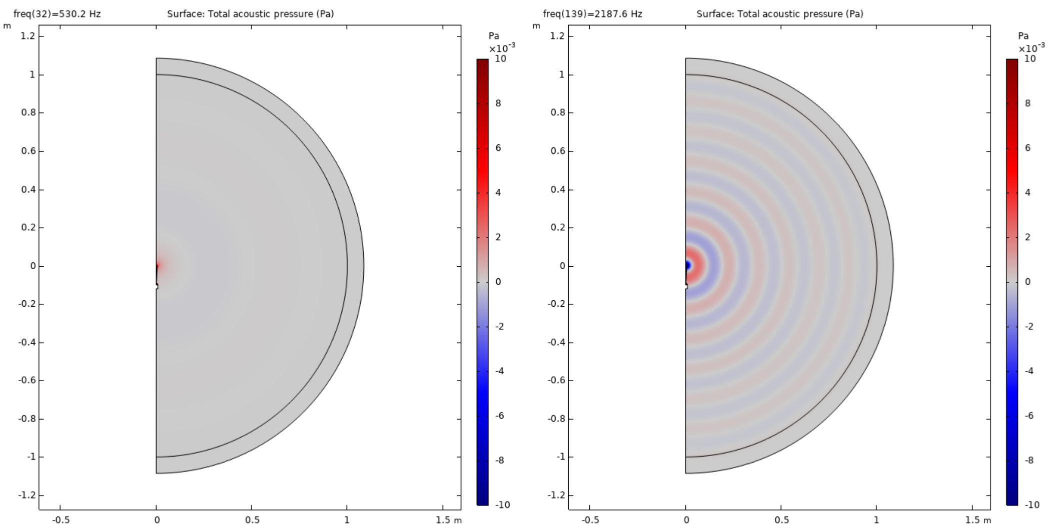

Figure 13: Pressure distribution for maximum of the input impedance at 530.2 Hz (left) and at 2187.6 Hz (right).

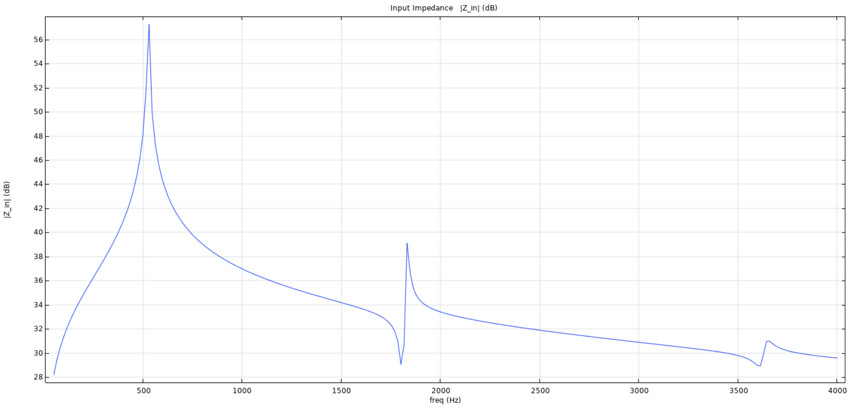

Figure 14: Module of the input impedance of the mouthpiece.

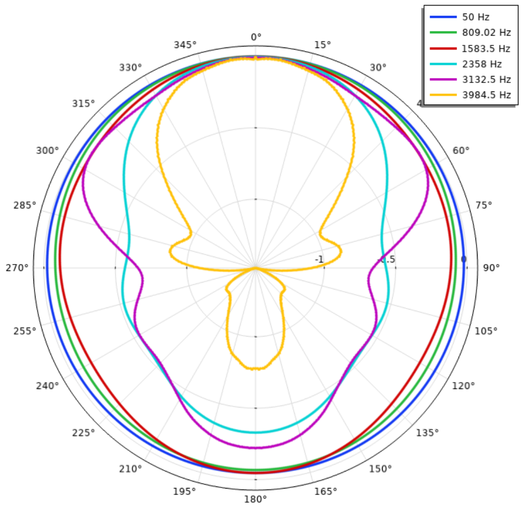

Figure 15: Polar plot of the directivity pattern for the mouthpiece (dB).

As expected from theory, the mouthpiece presents a single major resonance at $530.2\ Hz$. In order to verify this result, the mouthpiece can be approximated as an Helmoltz resonator having resonance frequency:

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{S_c}{V l_c}} \tag{6}$$

where V is the volume of the cup, S_c the cross-section of the constriction and l_c the length of the constriction. By evaluating the formula with the data of this study a value of $590.95\ Hz$ is obtained for the main resonance of the mouthpiece. Under the approximation conducted, this results verifies the simulation. Furthermore, observing figure (13) it is clear how the maximum of the input impedance does not correspond to the best radiating condition, this not being the actual aim of the mouthpiece. On the other hand, by changing the geometry of the mouthpiece its resonance frequency can be tuned, allowing to control the tonal range of the trumpet.

d) Exercise 4 - Complete model

In this final section all the elements treated in the previous sections are combined into the final simplified trumpet. The geometry of the instrument is easily derived from the previous ones as well as the propagation domain which is unchanged. The study is also the same as before in order to keep consistency between the results of the various configurations.

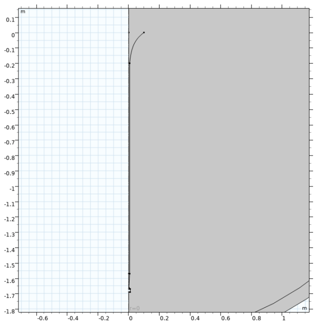

Figure 16: Geometry of the simplified complete trumpet.

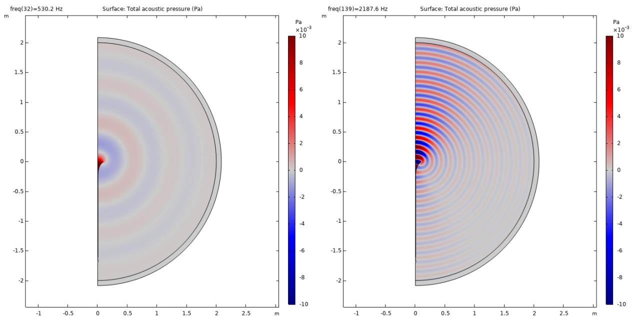

Figure 17: Pressure distribution for maximum of the input impedance at 530.2 Hz (left) and at 2187.6 Hz (right).

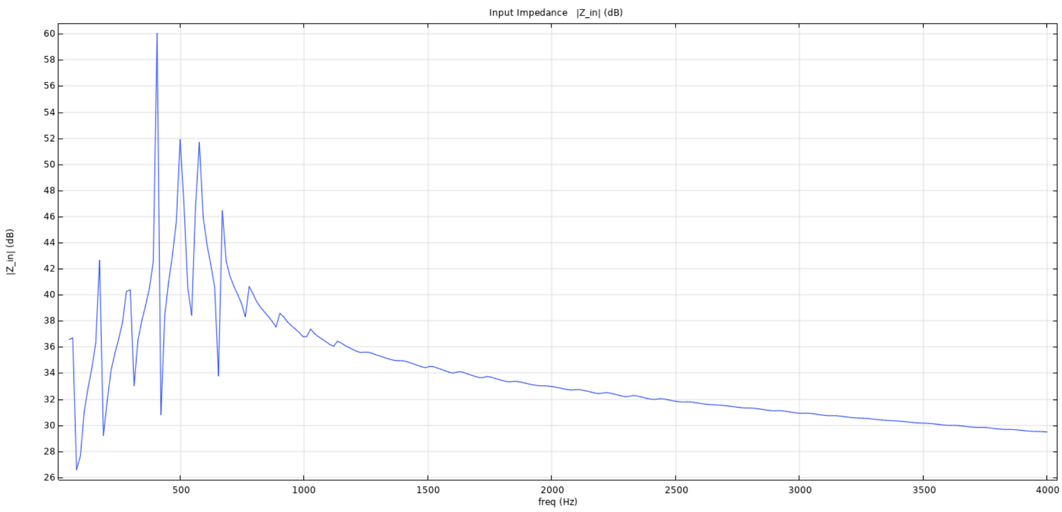

Figure 18: Module of the input impedance of the complete trumpet.

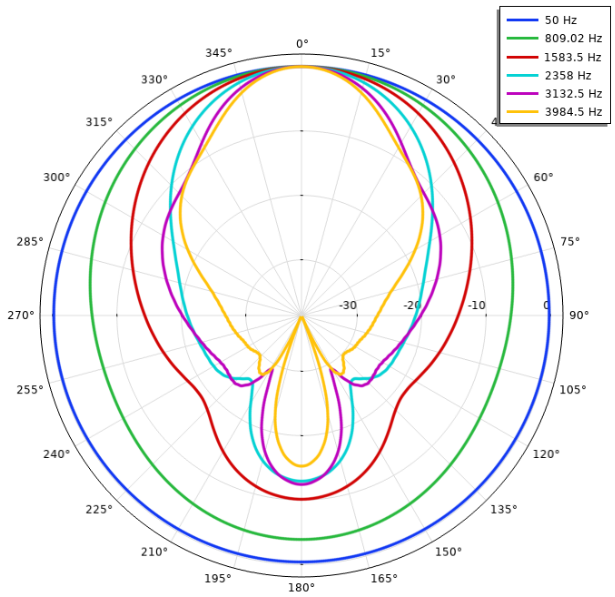

Figure 19: Polar plot of the directivity pattern, complete trumpet configuration (dB).

As it's clear from the figures reported above, the complete model maintains the magnitude and directivity characteristic of the tube with bell configuration, with increasing directivity towards the frontal lobe at high frequencies and a more omnidirectional behaviour in the low register. The effect of the mouthpiece is much more visible in the input impedance rather than in the pressure field. The mouthpiece contribution, summarized in figure (20), determines an improvement of the response in the mid frequency range, smoothing and attenuating lower and higher frequencies. It is in fact important to tune the mouthpiece as desired, so that its resonance emphasizes impedance peaks in the desired playing range.

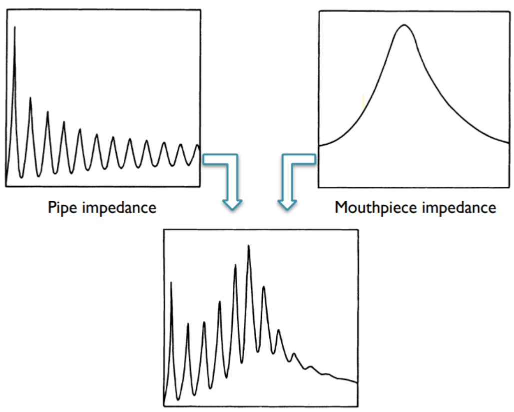

Figure 20: Effect of the mouthpiece on the input impedance.