VIBRATION ANALYSIS AND VIBROACOUSTICS

MODULE 1: VIBRATION ANALYSIS (PROF. STEFANO ALFI)

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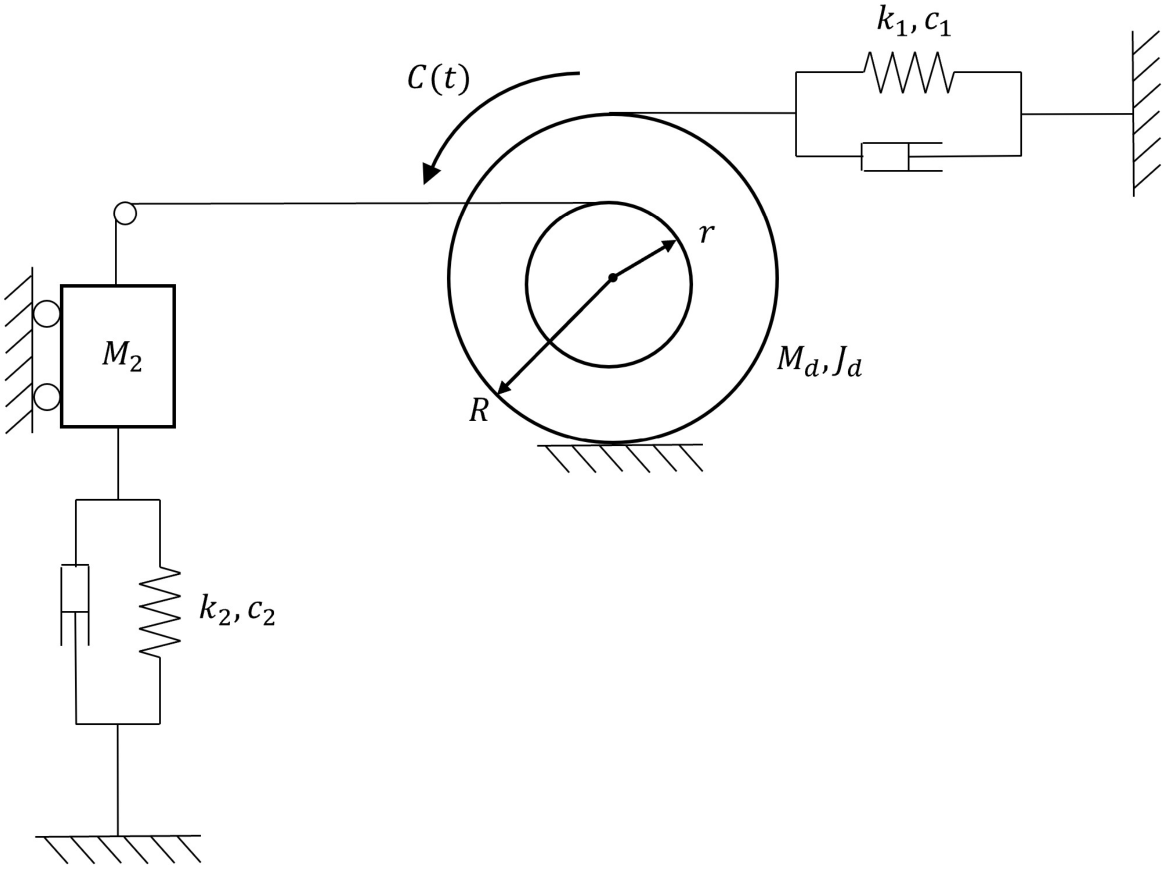
<u>Assignment 1</u>: Dynamics of 1-dof systems

The mechanical system in figure lies on the vertical plane and it is represented in its static equilibrium position. The system consists of 2 rigid bodies. The first one (M_d, J_d) is composed by two disks welded together (of radius R and r respectively). It can rotate on the ground. Horizontal spring and damper (k_1, c_1) connect the disk to the ground in correspondence of the external circumference of radius R. Moreover, an inextensible string connects the disk to a mass (M_2) whose motion is only allowed in the vertical direction. Finally, the mass M_2 is connected to the ground by a vertical spring and damper (k_2, c_2), realizing the mechanical system depicted below.



M_d $[kg]$	J_d $[kg\ m^2]$	r $[m]$	R $[m]$	$M_2[kg]$
2	0.1	0.2	0.5	5
k_1 $[N/m]$	c_1 $[Ns/m]$	k_2 $[N/m]$	c_2 $[Ns/m]$	
155	1.5	50	2	
A $[N]$	φ $[rad]$	f_1 $[Hz]$	f_2 $[Hz]$	
2.5	$\pi/3$	0.35	10	
B_1 $[N]$	β_1 $[rad]$	f_1 $[Hz]$		
1.2	$\pi/4$	0.35		
B_2 $[N]$	β_2 $[rad]$	f_2 $[Hz]$		
0.5	$\pi/5$	2.5		
B_3 $[N]$	β_3 $[rad]$	f_3 $[Hz]$		
5	$\pi/6$	10		

According to the data defined in the above table, it is requested to compute:

1) Equations of motion:
   a. Write the equation of motion for small vibrations about the represented configuration considering that the system is in its static equilibrium position.
   b. Evaluate the **natural frequency** of the system.
   c. Compute the **adimensional damping ratio** and the **damped frequency**.

2) Free motion of the system:
   a. Plot and comment the free motion of the system starting from generic non null initial conditions.
   b. Considering the same initial conditions, evaluate and comment the free motion of the system when the system is characterised by an adimensional damping ratio 4 times the one considered in 2.a.
   c. Considering the same initial conditions, evaluate and comment the free motion of the system when the system is characterised by an adimensional damping ratio 25 times the one considered in 2.a.

3) Forced motion of the system:
   a. Plot and comment the Frequency Response Function *H(Ω)* diagrams (magnitude and phase) resulting from conditions reported in (2.a), (2.b) and (2.c).
   b. Starting from the general initial conditions and the system characteristics defined in (2.a), evaluate the complete time response of the system to the torque applied to the disk, considering that the torque is harmonic of the form:
   $$C(t) = A\,cos(2\pi f_1\,t + \varphi)$$
   c. Considering the harmonic torque:
   $$C(t) = A\,cos(2\pi f_i\,t + \varphi)$$
   for the two cases $f_i = f_1$ and $f_i = f_2$:
   - compute the amplitude of the response when the force is dynamically applied,
   - compute the amplitude of the response when the force is considered statically applied,
   - compare the time trend of the steady-state response w.r.t the one of the torque.
   d. Evaluate the steady-state response of the system to a periodic torque applied to the disk, considering that the periodic torque consists of the superposition of three harmonic contributions of the form:
   $$C(t) = \sum_{k=1}^{3} B_k\,cos(2\pi f_k\,t + \beta_k)$$
   For both the periodic torque and the steady-state response of the system, plot:
   - the time histories of the signals
   - the spectra of the signals (magnitude and phase)