FUNDAMENTALS OF VIBRATION ANALYSIS AND VIBROACOUSTICS MODULE 1: FUNDAMENTALS OF VIBRATION ANALYSIS (PROF. STEFANO ALFI)

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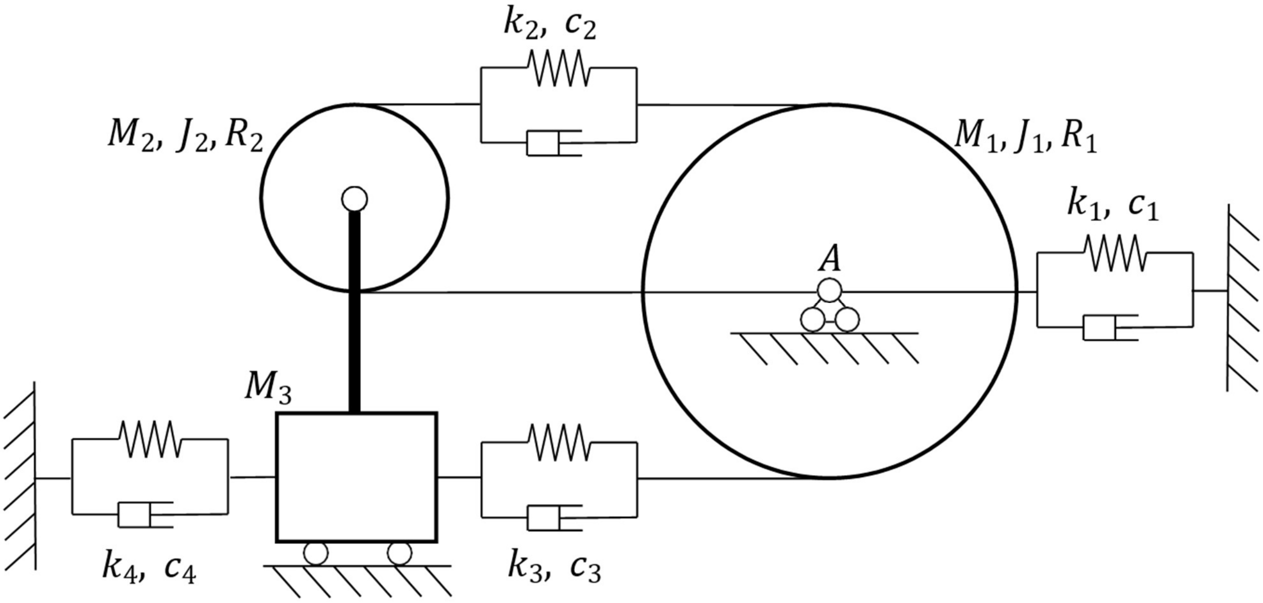
Assignment 2:* *Dynamics of n-dof systems

The system to be studied consists of 3 rigid bodies. A disk (M_2, J_2, R_2) is constrained through a hinge to the extremity of a mass (M_3) that slides in in the horizontal plane. An inextensible string rolls up on the disk with mass M_2: one extremity is rigidly connected to the centre of another disk (M_1, J_1, R_1) while the other is elastically connected to the surface of disk with mass M_1. The vertical motion of disk M_1 is constrained by means of a roller. Springs (k_1, k_2, k_3, k_4) and dampers (c_1, c_2, c_3, c_4) realize together with the rigid bodies the mechanical system depicted below.



$M_1\ [kg]$	$J_1\ [kg\ m^2]$	$R_1\ [m]$	$M_2 [kg]$	$J_2\ [kg\ m^2]$	$R_2\ [m]$	$M_3 [kg]$	
5	2.5	1	1.25	0.16	0.5	10	
$k_1\ [N/m]$	$c_1\ [Ns/m]$	$k_2\ [N/m]$	$c_2\ [Ns/m]$	$k_3\ [N/m]$	$c_3\ [Ns/m]$	$k_4\ [N/m]$	$c_4\ [Ns/m]$
1000	0.5	100	0.5	560	1	800	4
$A_1\ [N]$	$A_2\ [N]$	$f_1\ [Hz]$	$f_2\ [Hz]$	$f_0\ [Hz]$			
15	7	1,5	3.5	0.75			
$x_{3,0}\ [m]$	$\theta_{1,0}\ [rad]$	$\theta_{2,0}\ [rad]$	$\dot{x_{3,0}}\ [m/s]$	$\dot{\theta_{1,,0}}\ [rad/s]$	$\dot{\theta_{2,0}}\ [rad/s]$		
0.1	$\pi/12$	$-\pi/12$	1	0.5	2		

According to the data defined in the above table, it is requested to:

1) **Equations of motion and system matrices:**
   a. Write the equations of motion for small vibrations about the represented configuration considering that the system is in its static equilibrium position.
   b. Evaluate the eigenfrequencies and corresponding eigenvectors in case of <u>undamped</u> and <u>damped</u> system.
   c. Assuming Rayleigh damping, evaluate α and β to approximate the generalized damping matrix [C*] through the Rayleigh proportional damping formula α[M]+β[K].

2) **Free motion of the system (considering the Rayleigh damping as in 1.c)**
   a. Plot and comment the free motion of the system starting from the initial conditions reported in the table, being $x3$, $\theta1$, $\theta2$ the independent variables describing the motion of the system.
   b. Impose a set of particular initial conditions so that only one mode contributes to the free motion of the system.
3) **Forced motion of the system (considering the Rayleigh damping as in 1.c)**
   a. Plot and comment the elements of the Frequency Response Matrix H(Ω).
   b. Plot the co-located FRF of point A at the centre of the disk 1.
   c. Plot the co-located FRF between the rotation of the disk or radius $R2$ and a generic torque applied on the disk itself.
   d. Starting from the initial condition defined in (2.a), evaluate the complete time response of the system for the three degrees of freedom to a horizontal force applied in A, considering that the force is expressed by:

$$F(t) = A_1\, cos(2\pi f_1\, t) + \; A_2\, cos(2\pi f_2\, t)$$

   Compare the time histories of the force with both the steady-state response and the complete response.
   e. Evaluate the steady-state response of the horizontal displacement of point A to the horizontal force applied in A considering that the force is a periodic triangular wave, with fundamental frequency $f0$, of the form:

$$F(t) = \frac{8}{\pi^2}\sum_{k=0}^{4}(-1)^k \frac{\sin\big((2k+1)\, 2\pi f_0\, t\big)}{(2k+1)^2}$$

4) **Modal approach (considering the Rayleigh damping as in 1.c)**
   a. Derive the equations of motion of the system in modal coordinates and plot the elements of the corresponding Frequency Response Matrix $H_q(\Omega)$.
   b. Reconstruct the co-located FRF of point A employing a modal approach and compare with the one obtained using physical coordinates in (3.b).
   c. Reconstruct the co-located FRF between the rotation of the disk or radius $R2$ and the torque applied at the disk 2 employing a modal approach and compare with the one obtained using physical coordinates in (3.c).
   d. Compute the steady state amplitude of response for the three degrees of freedom when excited by a horizontal force applied in A. Compare the complete system response with the one obtained considering only the first mode of vibration, for the two following cases:
      - a harmonic force $F(t) = A_1\, cos(2\pi f_1\, t)$;
      - a harmonic force $F(t) = A_2\, cos(2\pi f_2\, t)$.

OPTIONAL

i) Give a qualitative graphical representation of the mode shapes.

ii) Represent one element (of your choice) of the Frequency Response Matrix H(Ω) in (3.a) as superposition of the modal frequency response functions comparing them to the original diagram of (3.a) but considering different types of diagrams: real and imaginary parts / Nyquist plot.