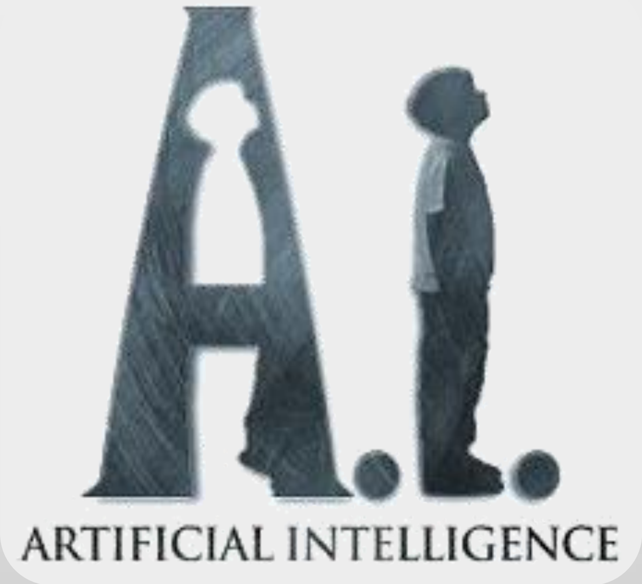
Logical Agents



Chapter 7

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Announcements

- Skip next quiz, BUT
- Test 1
  - 9 September 2021 on Chapter 7
- Test 2
  - Monday, 11 November 2021 on Chapters 8 and 9

Overview of lecture

- Proof procedures
- Monotonicity
- Resolution
- Conjunctive normal form
- A resolution algorithm

Proof procedures

- Standard patterns of inference
  - Called: inference rules
- Modus Ponens

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

$$WumpusAhead \wedge WumpusAlive \Rightarrow Shoot$$

$$WumpusAhead \wedge WumpusAlive$$

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Proof procedures

- And-Elimination $\dfrac{\alpha \wedge \beta}{\alpha}$ or $\dfrac{\alpha \wedge \beta}{\beta}$

$$\frac{WumpusAhead \wedge WumpusAlive}{WumpusAlive}$$

- Modus Ponens and And-Elimination sound

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Proof procedures

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) & \equiv & (\neg\alpha \vee \neg\beta) & \text{de Morgan} \\
\neg(\alpha \vee \beta) & \equiv & (\neg\alpha \wedge \neg\beta) & \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}
$$

Proof procedures

$$\frac{\neg(\alpha \wedge \beta)}{(\neg\alpha \vee \neg\beta)}$$

$$\frac{(\neg\alpha \vee \neg\beta)}{\neg(\alpha \wedge \beta)}$$

Proof procedures

$$R_1 : \neg P_{1,1}$$

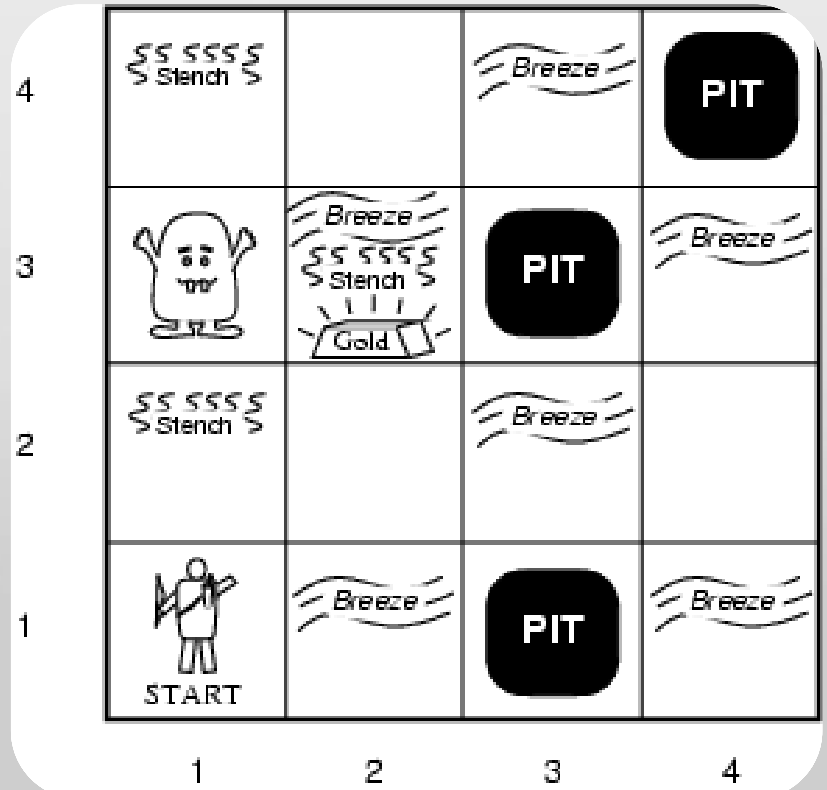
$$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

$$R_4 : \neg B_{1,1}$$

$$R_5 : B_{2,1}$$

$$\neg P_{1,2} \, ?$$



Proof procedures

$$R_2\colon B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$\left[\frac{\alpha \Leftrightarrow \beta}{(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)}\right]$$

[Biconditional elimination]

$$R_6\colon (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

Proof procedures

$$R_6 : (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

$$\left[\text{And - Elimination}\right]$$

$$R_7 : ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

Proof procedures

$$R_7 : ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

$$\left[\frac{(\alpha \Rightarrow \beta)}{(\neg\beta \Rightarrow \neg\alpha)} \right]$$

[Contraposition]

$$R_8 : (\neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1}))$$

Proof procedures

$$R_8 : (\neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1}))$$

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

$$\left[\text{Modus Ponens and } R_4 : \neg B_{1,1}\right]$$

$$R_9 : \neg(P_{1,2} \vee P_{2,1})$$

Proof procedures

$$R_9 : \neg(P_{1,2} \vee P_{2,1})$$

$$\frac{\neg(\alpha \vee \beta)}{(\neg\alpha \wedge \neg\beta)}$$

$$\left[\text{de Morgan}\right]$$

$$R_{10} : \neg P_{1,2} \wedge \neg P_{2,1}$$

Proof procedures

- A sequence of applications of inference rules is called a proof
- Searching for proofs an alternative to enumerating models (truth table)
- Search can go forward from initial knowledge base or
- Search can go backward from goal sentence
- Finding a proof can sometimes be highly efficient

Proof procedures

$$R_1 : \neg P_{1,1}$$

$$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

$$R_4 : \neg B_{1,1}$$

$$R_5 : B_{2,1}$$

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Monotonicity

- The set of entailed sentences can only increase as information is added to the knowledge base
- If KB $\models \alpha$ then KB $\wedge \beta \models \alpha$

Resolution

$$R_1 : \neg P_{1,1}$$

$$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

$$R_4 : \neg B_{1,1}$$

$$R_5 : B_{2,1}$$

$$R_6 : (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

$$R_7 : ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

$$R_8 : (\neg B_{1,1} \Rightarrow \neg (P_{1,2} \vee P_{2,1}))$$

$$R_9 : \neg (P_{1,2} \vee P_{2,1})$$

$$R_{10} : \neg P_{1,2} \wedge \neg P_{2,1}$$

Resolution

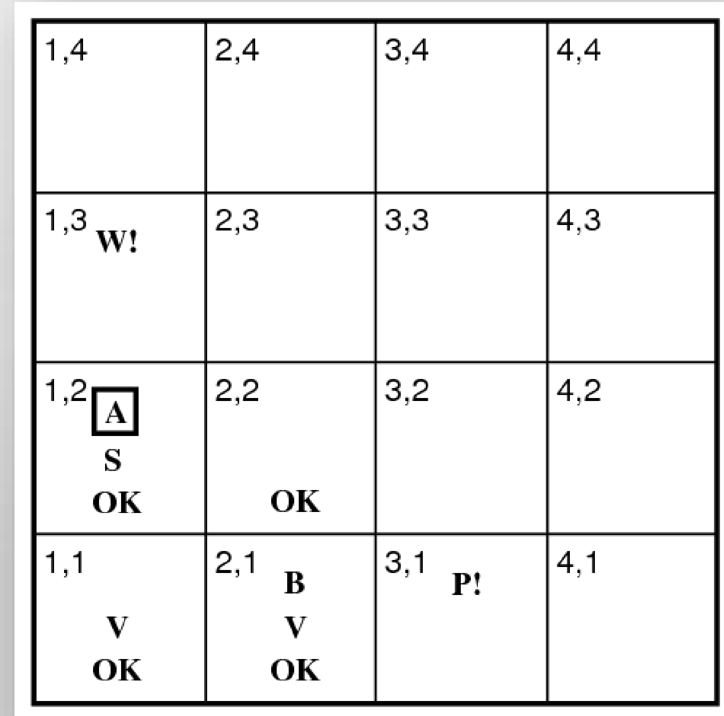
$$R_{11} : \neg B_{1,2}$$

$$R_{12} : B_{1,2} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{1,3})$$

$$R_{13} : \neg P_{2,2}$$

$$R_{14} : \neg P_{1,3}$$

$$R_{15} : P_{1,1} \vee P_{2,2} \vee P_{3,1}$$



Resolution

$$R_{13} : \neg P_{2,2}$$

$$\left[R_{15} : P_{1,1} \vee P_{2,2} \vee P_{3,1}\right]$$

Resolution

$$R_{13} : \neg P_{2,2}$$

$$\left[R_{15} : P_{1,1} \vee P_{2,2} \vee P_{3,1} \right]$$

$$R_{16} : P_{1,1} \vee P_{3,1}$$

Resolution

$$R_1 : \neg P_{1,1}$$

$$\left[R_{16} : P_{1,1} \vee P_{3,1} \right]$$

$$R_{17} : P_{3,1}$$

Resolution

- Clause

$$l_1 \vee \cdots \vee l_k$$

- Unit resolution inference rule

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \quad m}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k}$$

Resolution

- Full resolution rule

$$\frac{l_1 \vee \cdots \vee l_k, \quad m_1 \vee \cdots \vee m_n}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee m_n}$$

- Example

$$\frac{P_{1,1} \vee P_{3,1}, \quad \neg P_{1,1} \vee \neg P_{2,2}}{P_{3,1} \vee \neg P_{2,2}}$$

Resolution

- Resolution rule - factoring

$$\frac{A \vee B, \quad A \vee \neg B}{A \vee A}$$

- Soundness of resolution rule

$$\frac{l_1 \vee \cdots \vee l_k, \quad m_1 \vee \cdots \vee m_n}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee m_n}$$

- Resolution rule is complete

Conjunctive normal form

- Every sentence of propositional logic is logically equivalent to a conjunction of disjunctions of literals (clauses)
- Sentences of this form is called
  - Conjunctive normal form (CNF)
- Convert sentences to CNF
- Consider

$$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

Conjunctive normal form

1. Eliminate $\Leftrightarrow$, replace $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate $\Rightarrow$, replace $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

Conjunctive normal form

3. Move $\neg$ inwards by repeated application of de Morgan's rules and double-negation elimination

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply the distributivity law ($\wedge$ over $\vee$) and simplify

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

A resolution algorithm

- Proof by contradiction
- To show that KB $\models \alpha$, we show that KB $\wedge \neg\alpha$ is unsatisfiable
- Convert KB $\wedge \neg\alpha$ to CNF
- The resolution rule is repeatedly applied to the resulting clauses

A resolution algorithm

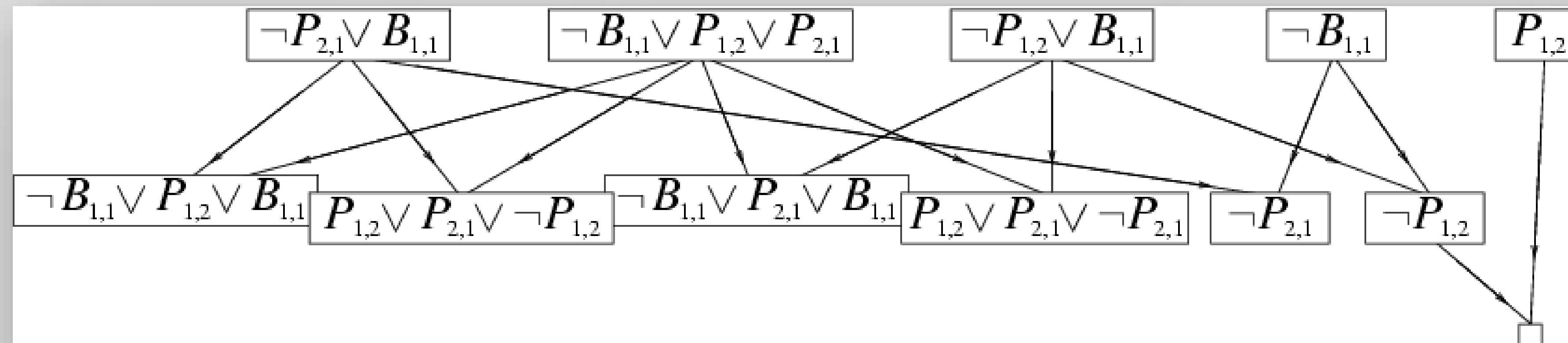
- One of two things happens:
  - There are no new clauses that can be added, in which case KB does not entail α
  - Two clauses resolve to yield the empty clause, in which case KB entails α
- The empty clause (a disjunction of no disjuncts) is equivalent to *False*
- The empty clause arises only from resolving two complementary unit clauses such as P and $\neg$P

Resolution example

$KB = R_2 \wedge R_4$

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$

$\alpha = \neg P_{1,2}$



Assignment

- Study today's work
  - Chapter 7.5 to 7.5.2