NORTH-WEST UNIVERSITY ®
YUNIBESITI YA BOKONE-BOPHIRIMA
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Benodigdhede vir hierdie vraestel/Requirements for this paper:			
Antwoordskrifte/ Answer scripts:	X	Multikeusekaarte (A5)/ Multi-choice cards (A5):	
Presensiestrokies (Invulvraestel)/ Attendance slips (Fill-in paper):		Multikeusekaarte (A4)/ Multi-choice cards (A4):	
Rofwerkpapier/ Scrap paper:		Grafiekpapier/ Graph paper:	

Sakrekenaars/Calculators: **Nee/No**

Ander hulpmiddels/Other resources:

Tipe Assessering/ Type of Assessment:	**Assesseringstoets Assessment test Vraestel/Paper 1**	Kwalifikasie/ Qualification:	**Honns. B.Sc.**
Modulekode/ Module code:	**ITRI626**	Tydsduur/ Duration:	**1 uur / 1 hour**
Module beskrywing/ Module description:	**Kunsmatige Intelligensie / Artificial Intelligence**	Maks/ Max:	**60**
Eksaminator(e)/ Examiner(s):	**Prof. J. V. (Tiny) du Toit**	Datum/ Date:	**09/10/2018**
Interne/Internal Moderator(s):	**Geen**	Tyd/ Time:	**08:00**

Inhandiging van antwoordskrifte/Submission of answer scripts: **Gewoon/Ordinary**

Vraag 1 (Proposisielogika) / *Question 1 (Propositional Logic)*

1.1 Definieer die volgende terme: / *Define the following terms:*

a) Logiese gevolgtrekking (in terme van versamelings) / *Logical entailment (in terms of sets)* [3]

α ⊨ β (1) if and only if M(α) (1) ⊂ M(β) (1)

b) Die deduksie stelling / *The deduction theorem* [4]

For any sentences α and β (1), α ⊨ β (1) if and only if the sentence (α ⇒ β) (1) is valid (1).

c) ’n Bewys deur ’n teenstrydigheid / *A proof by a contradiction* [4]

For any sentences α and β (1), α ⊨ β (1) if and only if the sentence (α ∧ ¬β) (1) is unsatisfiable (1).

d) Die volle resolusie reël / *The full resolution rule* [4]

$$\frac{l_1 \vee \cdots \vee l_k, \quad m_1 \vee \cdots \vee m_n}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee m_n}$$

1.2 Deur van ‘n waarheidstabel en die deduksie stelling gebruik te maak, bewys die volgende. Toon al jou stappe duidelik aan./ *By using a truth table and the deduction theorem prove the following. Show all your steps clearly.* [10]

$P \wedge Q \wedge R \vDash \neg R \Rightarrow (P \vee Q)$

According to the deduction theorem $P \wedge Q \wedge R \vDash \neg R \Rightarrow (P \vee Q)$ if and only if $(P \wedge Q \wedge R) \Rightarrow (\neg R \Rightarrow (P \vee Q))$ is valid.

P	Q	R	¬R	P ∨ Q	P ∧ Q ∧ R	¬R ⇒ (P ∨ Q)	(P ∧ Q ∧ R) ⇒ (¬R ⇒ (P ∨ Q))
T	T	T	F	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	F	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	F	F	F	T	T
F	F	F	T	F	F	F	T

8 marks = 1 mark for each column in the truth table.

From the last column in the truth table it can be seen that $(P \wedge Q \wedge R) \Rightarrow (\neg R \Rightarrow (P \vee Q))$ is valid, thus $P \wedge Q \wedge R \vDash \neg R \Rightarrow (P \vee Q)$. (2)

1.3 Gegee ‘n kennisbasis en ‘n doelwit, maak van resolusie gebruik om te bewys die doelwit is waar. Toon al jou stappe duidelik aan. / *Given a knowledge base and a goal, use resolution to proof the goal is true. Show all your steps clearly.* [16]

Kennisbasis (KB): / *Knowledgebase (KB):*

1. $Q \vee (P \wedge R)$
2. $R \Rightarrow (P \vee Q)$
3. $R \wedge \neg(P \wedge Q)$
4. $\neg(P \vee Q \vee R)$

Doelwit: / *Goal:*

$P \vee (Q \wedge \neg R)$

Convert the sentences to conjunctive normal form (CNF):

Sentence	CNF	
Q ∨ (P ∧ R)	(P ∨ Q) ∧ (Q ∨ R)	(2 marks)
R ⇒ (P ∨ Q)	(P ∨ Q ∨ ¬R)	(2 marks)
R ∧ ¬(P ∧ Q)	(R) ∧ (¬P ∨ ¬Q)	(2 marks)
¬(P ∨ Q ∨ R)	(¬P) ∧ (¬Q) ∧ (¬R)	(2 marks)

(8)

The goal is $P \vee (Q \wedge \neg R)$, so we add the CNF of $\neg(P \vee (Q \wedge \neg R))$ to the list of facts, the new set is: (2)

1. $(P \vee Q) \wedge (Q \vee R)$
2. $(P \vee Q \vee \neg R)$
3. $(R) \wedge (\neg P \vee \neg Q)$
4. $(\neg P) \wedge (\neg Q) \wedge (\neg R)$
5. $(\neg P) \wedge (R \vee \neg Q)$

Resolution between (3.) (R) and (4.) (¬R) gives the empty clause (□) and thus. KB ⊨ goal. (6)

<u>Vraag 2 (Eerste-orde Logika) / *Question 2 (First-order Logic)*</u>

2.1 Gee die detail beskrywing van die werking van eksistensiële kwantifisering (∀). / *Give the detail description of the functioning of existencial quantification (*∀). [6]

∃x P is true in a given model (1) if P is true (1) in at least one (1) extended interpretation (1) that assigns x (1) to a domain element (1).

2.2 Gee die formele betekenis van ‘n term. / *Give the formal semantics of a term.* [4]

Consider a term $f(t_1,\ldots, t_n)$ (1). The function symbol f refers to some function in the model (call it F) (1); the argument terms refer to objects in the domain (call them $d_1,\ldots, d_n$) (1); and the term as a whole refers to the object that is the value of the function F applied to $d_1,\ldots, d_n$ (1).

2.3 Vertaal elkeen van die volgende Engelse sinne na die taal van standaard Eerste-orde Logika deur die bedoelde interpretasie te gebruik vir elke predikaat, funksie of konstante wat jy gebruik: / *Translate each of the following English sentences into the language of standard First-order Logic by using the intended interpretation for every predicate, function or constant that you use:*

a) There are at least two peaks in South Africa. [3]
b) There is exactly one letter in the mailbox. [3]
c) There are exactly two letters in the mailbox. [3]

a) $\exists x, y\ Peak(x) \land Peak(y) \land InSouthAfrica(x) \land InSouthAfrica(y) \land x \neq y$
b) $\exists x\ Letter(x) \land InMailbox(x) \land \forall y\ (Letter(y) \land InMailbox(y) \Rightarrow x = y)$
c) $\exists x, y\ Letter(x) \land InMailbox(x) \land Letter(y) \land InMailbox(y) \land x \neq y \land \forall z\ (Letter(z) \land InMailbox(z) \Rightarrow (z = x \lor z = y)$

TOTAAL/TOTAL: 60

Verwysingsnommer: 8.1.7.2.2