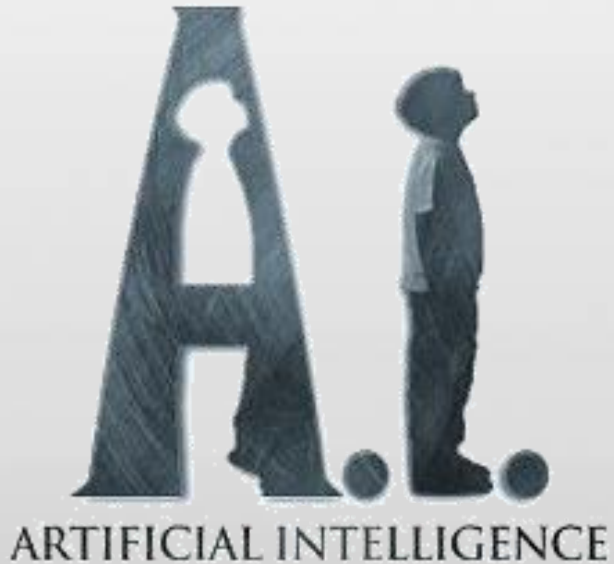
Logical Agents



Chapter 7

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
INSTITUTIONAL OFFICE

Announcements

- Practical assignment 1 (eFundi)
  - Thursday, 19 August 2021 before 23:55
- Practical assignment 2 (eFundi)
  - Thursday, 26 August 2021 before 23:55
- Theory quiz 2
  - Monday, 23 August 2021
  - Chapter 7.4

Lecture outline

- Propositional Logic
  - Syntax
  - Semantics
  - A simple knowledge base
  - Inference
  - Equivalence
  - Validity
  - Satisfiability

Propositional logic: Syntax

- Propositional logic the simplest
  - Illustrates basic ideas
- The proposition symbols $P_{1,1}$, $P_{2,2}$ is sentences
  - If S is a sentence, then $\neg S$ is a sentence (negation)
  - If S_1 and S_2 are sentences, then $S_1 \wedge S_2$ is a sentence (conjunction)
  - If S_1 and S_2 are sentences, then $S_1 \vee S_2$ is a sentence (disjunction)

Propositional logic: Syntax

- If S_1 and S_2 are sentences, then $S_1 \Rightarrow S_2$ is a sentence (implication)
- If S_1 and S_2 are sentences, then $S_1 \Leftrightarrow S_2$ is a sentence (if and only if) (biconditional)

Exercise

R

S

Exercise

1) It is either raining or snowing
2) It is both raining and snowing
3) It is raining, but it is not snowing
4) It is not both raining and snowing
5) If it is not raining, then it is snowing
6) It is raining if and only if it is not snowing

Exercise

1) $R \lor S$

2) $R \land S$

3) $R \land \neg S$

4) $\neg(R \land S)$

5) $\neg R \Rightarrow S$

6) $R \Leftrightarrow \neg S$

Propositional logic: Semantic

- Each model specifies the true or false value of each proposition symbol
- Consider possible model m_1:
  - $P_{1,2}$ is false
  - $P_{2,2}$ is true
  - $P_{3,1}$ is false
- With these three symbols, ? models can be automatically enumerated?

Propositional logic: Semantic

- Rules to evaluate the truth with respect to a model *m*
  - $\neg S$ is true if and only if S is false
  - $S_1 \wedge S_2$ is true if and only if S_1 is true and S_2 is true
  - $S_1 \vee S_2$ is true if and only if S_1 is true or S_2 is true
  - $S_1 \Rightarrow S_2$ is true if and only if S_1 is false or S_2 is true
  - $S_1 \Rightarrow S_2$ is false if and only if S_1 is true and S_2 is false

Propositional logic: Semantic

- Rules to evaluate the truth with respect to a model *m* (continued)

  $S_1 \Leftrightarrow S_2$ is true if and only if $S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true

- Simple recursive process evaluate an arbitrary sentence, e.g.

  $\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1})$

  $= true \wedge (true \vee false)$

  $= true \wedge true$

  $= true$

Truth table for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$
$false$	$false$	$true$	$false$	$false$
$false$	$true$	$true$	$false$	$true$
$true$	$false$	$false$	$false$	$true$
$true$	$true$	$false$	$true$	$true$

Truth table for connectives

P	Q	$P \Rightarrow Q$
$false$	$false$	$true$
$false$	$true$	$true$
$true$	$false$	$false$
$true$	$true$	$true$

Truth table for connectives

P = The sky is blue

Q = America does not experience a recession

P	Q	$P \Rightarrow Q$
$false$	$false$	$true$
$false$	$true$	$true$
$true$	$false$	$false$
$true$	$true$	$true$

Truth table for connectives

P = Ten is an uneven number
Q = It rains outside

P	Q	$P \Rightarrow Q$
$false$	$false$	$true$
$false$	$true$	$true$
$true$	$false$	$false$
$true$	$true$	$true$

Truth table for connectives

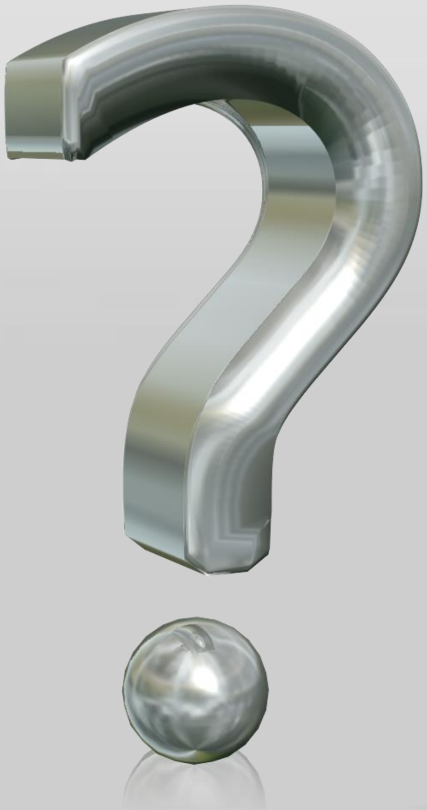
P	Q	$P \Rightarrow Q$
$false$	$false$	$true$
$false$	$true$	$true$
$true$	$false$	$false$
$true$	$true$	$true$

Truth table for connectives

P	Q	$P \Leftrightarrow Q$
$false$	$false$	$true$
$false$	$true$	$false$
$true$	$false$	$false$
$true$	$true$	$true$

Exercise

- Construct a truth table for the formula $\neg P \vee Q$



Exercise

- Construct a truth table for the formula $\neg P \vee Q$

P	Q	$\neg P$	$\neg P \vee Q$

Exercise

- Construct a truth table for the formula $\neg P \lor Q$

P	Q	$\neg P$	$\neg P \lor Q$
T	T		
T	F		
F	T		
F	F		

Exercise

- Construct a truth table for the formula $\neg P \vee Q$

P	Q	¬P	¬P V Q
T	T	F	
T	F	F	
F	T	T	
F	F	T	

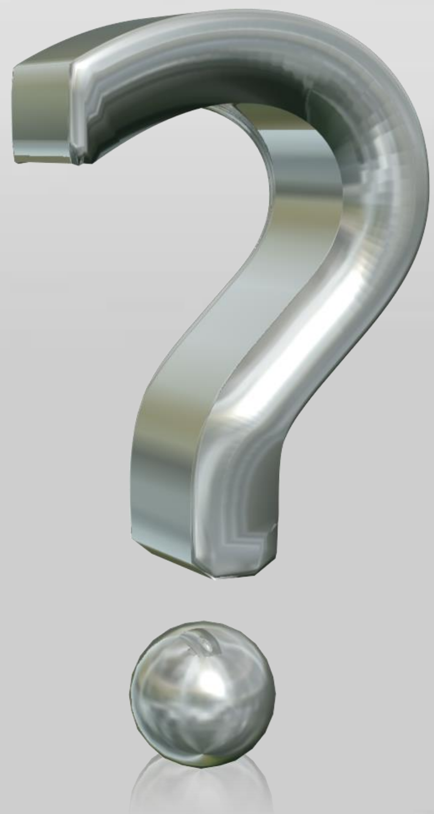
Exercise

- Construct a truth table for the formula $\neg P \vee Q$

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Exercise

- Construct a truth table for the formula $(P \vee Q) \wedge \neg(P \wedge Q)$



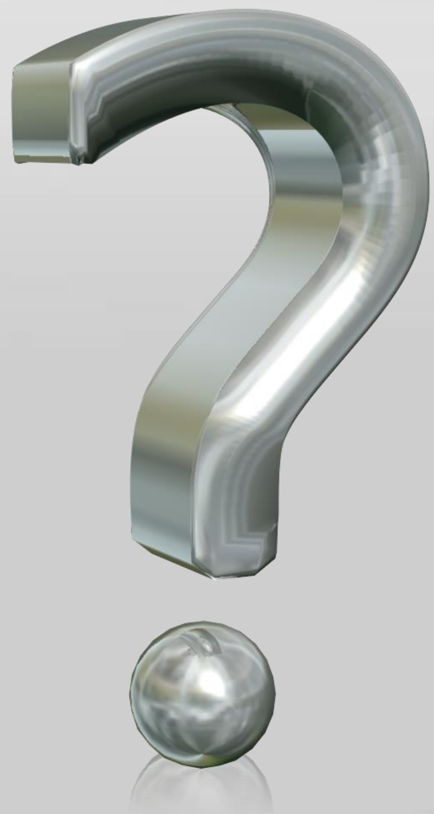
Exercise

- Construct a truth table for the formula
  $(P \vee Q) \wedge \neg(P \wedge Q)$

P	Q	$P \vee Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$(P \vee Q) \wedge \neg(P \wedge Q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Exercise

- Construct a truth table for the formula $P \Rightarrow (Q \vee \neg R)$



Exercise

- Construct a truth table for the formula
  $P \Rightarrow (Q \vee \neg R)$

P	Q	R	$\neg R$	$Q \vee \neg R$	$P \Rightarrow (Q \vee \neg R)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	T	T

Wumpus world sentences

- Let $P_{i,j}$ be true if there is a pit in [i, j]
- Let $B_{i,j}$ be true if there is a breeze in [i,j]

R_1: $\neg P_{1,1}$

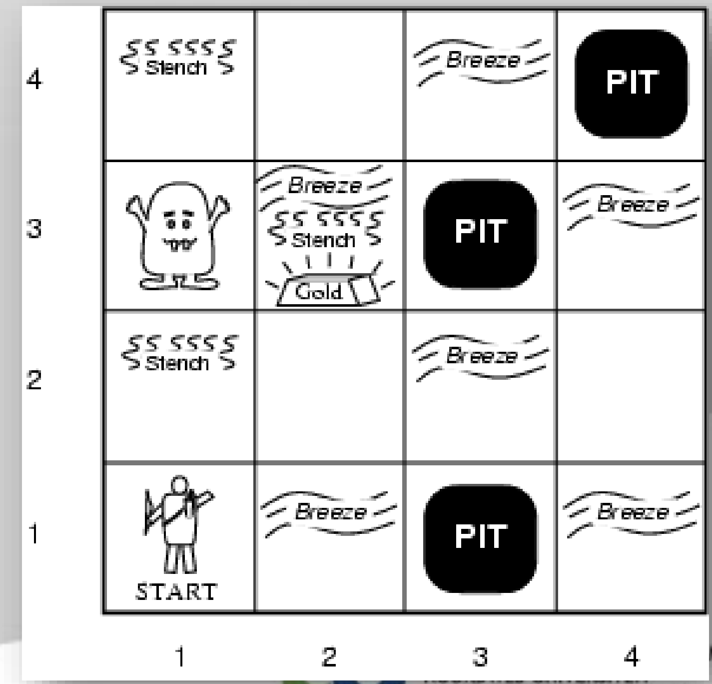
R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

$KB = R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$



Truth table for inference

KB ⊨ α_1 where α_1 = "[1,2] has no pit"

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
false	*false*	*false*	*false*	*false*	*false*	*false*	*false*	*true*
false	*false*	*false*	*false*	*false*	*false*	*true*	*false*	*true*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*	*true*
false	*true*	*false*	*false*	*false*	*false*	*true*	<u>*true*</u>	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*false*	<u>*true*</u>	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*true*	<u>*true*</u>	<u>*true*</u>
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*	*true*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*false*

Logical Equivalence

- Two sentences are logically equivalent if and only if they are true in the same set of models
- $\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

Important!

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) & \equiv & (\neg\alpha \vee \neg\beta) & \text{de Morgan} \\
\neg(\alpha \vee \beta) & \equiv & (\neg\alpha \wedge \neg\beta) & \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}
$$

Validity and Satisfiability

- A sentence is valid if it is true in all models
  - For example: *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$
- Validity is connected to inference by the Deduction Theorem:
  - $\alpha \models \beta$ if and only if $(\alpha \Rightarrow \beta)$ is valid
- A sentence is satisfiable if it is true in a model
  - For example: $R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$

Validity and Satisfiability

- A sentence is unsatisfiable if it is true in no models
  - For example: $A \wedge \neg A$
- Satisfiability is connected to inference by the following:
  - $\alpha \models \beta$ if and only if $(\alpha \wedge \neg \beta)$ is unsatisfiable

ARTIFICIAL INTELLIGENCE

Assignment

- Please study today's work
  - Chapter 7.4

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