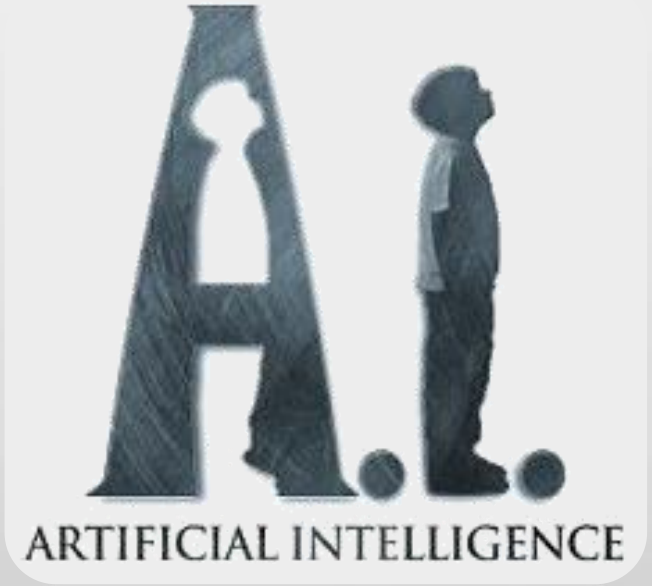
First-order Logic



Chapter 8

NORTH-WEST UNIVERSITY
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Announcements

- Theory quiz 4 on today's work at Thursday, 23 September 2021
- Please complete Practical assignment 4 for Wednesday, 22 September 2021

ARTIFICIAL INTELLIGENCE

Overview of lecture

- Representation
- Syntax and semantics of First-order Logic (FOL)

Representation

- Programming languages like C++ and Java most in common use
  - Represent computational processes
  - Data structures represent facts
  - Lacks general mechanism for deriving facts from other facts
  - Cannot handle partial information in an easy way
  - Procedural approach

Representation

- Propositional Logic
  - Declarative language
  - Semantics based on truth value relation between sentences and possible worlds
  - Can deal with partial information
  - Has property of compositionality $B_{1,1}$, $B_{2,2}$ $(B_{1,1} \vee B_{2,2})$
  - Meaning independent of context
  - Cannot describe environment with many objects concisely

Representation

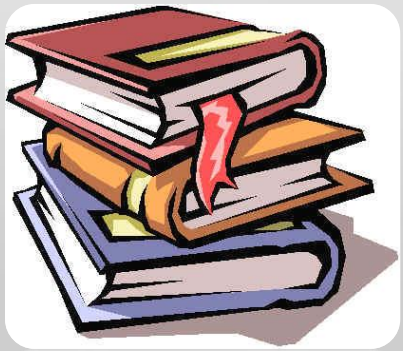
- Natural languages like English, Afrikaans, Spanish, Tswana
  - Very expressive
  - Textbook in English
  - Medium for communication rather than pure representation
  - Context important
  - Does not have property of compositionality
  - Ambiguity

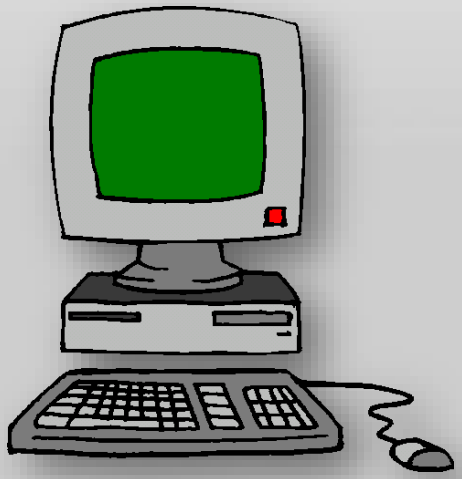
First-order Logic

- Adopt the foundation of propositional logic
  - Declarative
  - Compositional
  - Context independent
  - Unambiguous
- Use positive properties of natural language
  - Expressive

First-order Logic

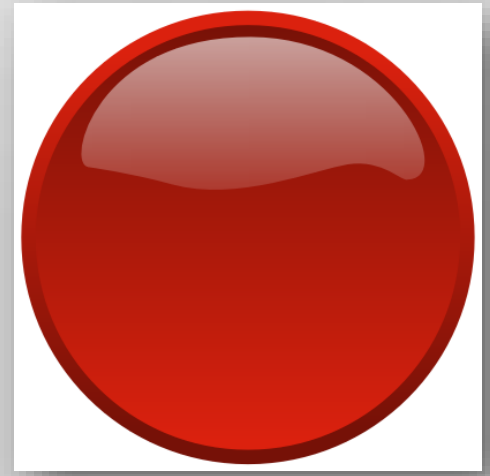
- Natural language elements
  - Objects





First-order Logic

- Natural language elements (continued)
  - Relationships

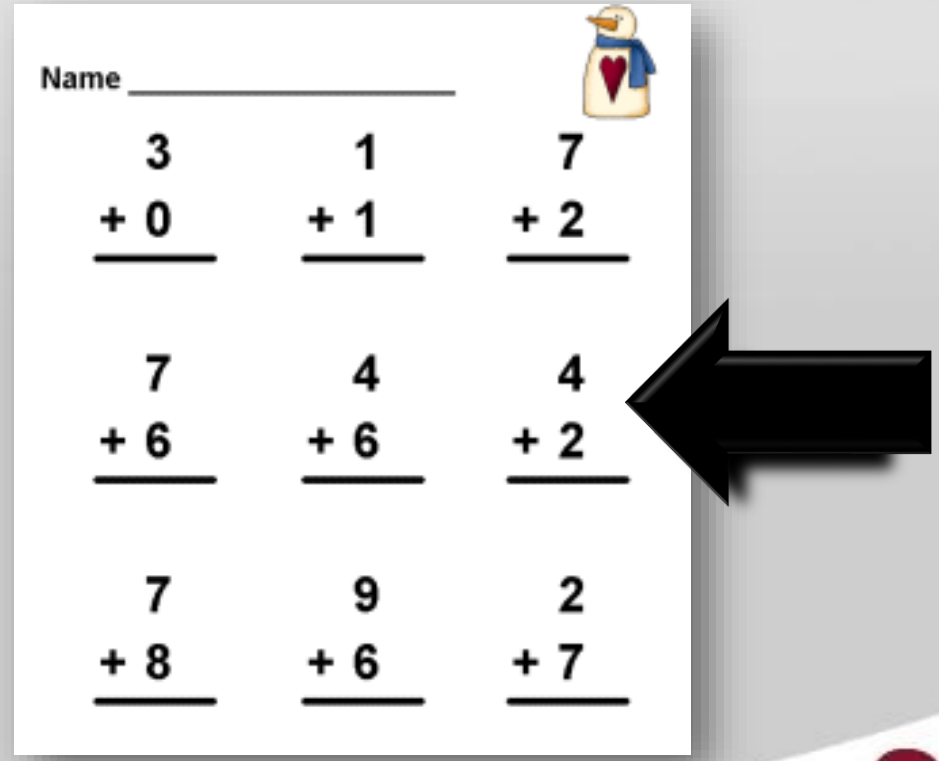


First-order Logic

- Natural language elements (continued)
  - Functions

First-order Logic

- Assertions refer to objects, properties, and functions



First-order Logic

- First-order logic build around objects and relations
- Facts about some or all the objects in the world
- General laws or rules
- Difference between logics
  - Ontological commitment
  - What is assumed about the nature of reality

First-order Logic

- Propositional logic
  - Facts hold or do not hold in the world
  - Each fact can be in one of two states: true or false
- First-order logic
  - The world consists of objects with certain relations that do or do not hold
- Temporal logic
  - Facts hold at particular times
  - Times are ordered

First-order Logic

- Higher-order logic
  - Relations and functions are objects
- Epistemological commitments
  - The possible states of knowledge that it allows with respect to each fact
  - Propositional logic and First-order logic: True, false, or has no opinion
  - Probability theory: degree of belief ranging from 0 (total disbelief) to 1 (total belief)

FOL Syntax and Semantics

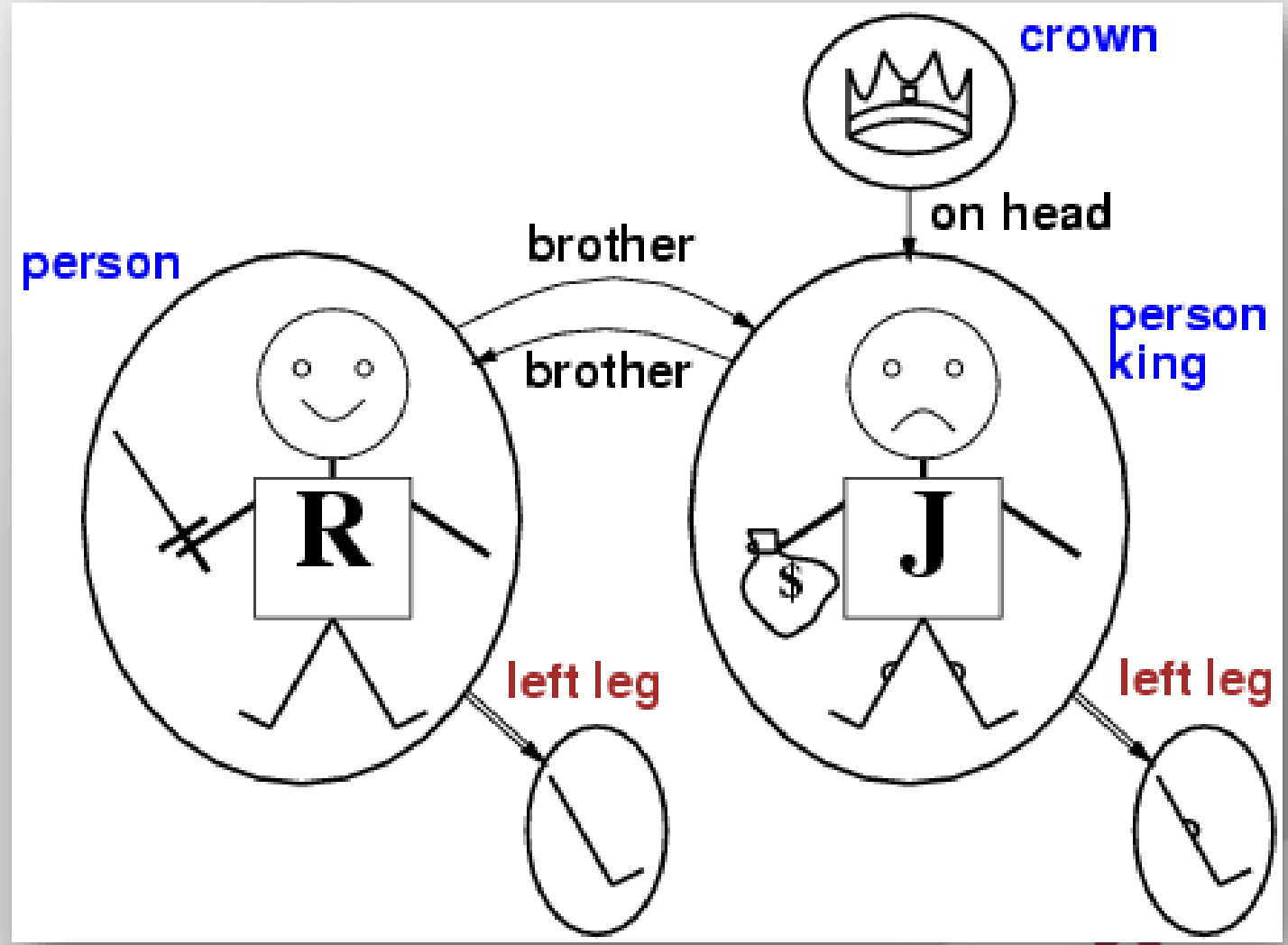
- Models for First-order logic
  - Models of logical languages are formal structures that constitute the possible worlds under consideration
  - Models for propositional logic
  - Models for First-order Logic contains objects
  - Domain (elements) of models is set of objects it contains

FOL Syntax and Semantics

1189 tot 1199

1199 tot 1215

FOL Syntax and Semantics



FOL Syntax and Semantics

- Models for First-order Logic (continues)
- Relation is set of tuples of objects
  - Brotherhood relation: { <Richard the Lionheart, King John>, <King John, Richard the Lionheart> }
  - “On head” relation: { <the crown, King John> }
  - Properties: “person”, “king”, “crown”
  - Certain relationships best considered as functions

FOL Syntax and Semantics

- Models for First-Order Logic (continues)
  - Function example: LeftLeg

  <Richard the Lionheart> → Richard's left leg

  <King John> → John's left leg
- Symbols and Interpretations (Syntax)
  - Symbols for objects: constant symbols
  - Symbols for relations: predicate symbols
  - Symbols for functions

FOL Syntax and Semantics

- Symbols and Interpretations (continued)
  - Symbols for objects, relationships and functions start with capital letters
  - Examples: *Richard*, *John*, *Brother*, *OnHead*, *Person*, *King*, *Crown*, *LeftLeg*
  - Interpretation specifies which objects, relations and functions are referred to by symbols
  - A possible interpretation: the intended interpretation
  - Other interpretations

FOL Syntax and Semantics

- Symbols and Interpretations (continued)
  - Truth of sentence determined by a model and a interpretation for a sentence's symbols
  - Logical entailment, validity, etc. defined in terms of all possible models and all possible interpretations
  - Domain (elements) of model can be infinitely large
  - Enumeration of models impossible

FOL Syntax and Semantics

- Terms
  - Logical expression that refers to an object
  - Constant symbols are terms
  - Function used instead of constants, e.g. LeftLeg(John)
  - Complex term is $f(t_1, t_2, \ldots, t_n)$
  - Complex term not subroutine that returns a value
  - Formal semantics of term $f(t_1, t_2, \ldots, t_n)$

FOL Syntax and Semantics

- Atomic Sentences
  - Predicate symbol with terms in brackets
  - Example: Brother(Richard, John)
  - Complex terms as arguments: Married(Father(Richard), Mother(John))
  - Atomic sentence is true in given model, under given interpretation, if (predicate) relation holds among objects referred to by the arguments

FOL Syntax and Semantics

- Complex Sentences
  - Logical connectives construct complex sentences
  - Semantics of complex sentences the same as propositional logic

$$\neg Brother(LeftLeg(Richard), John)$$

$$Brother(Richard, John) \wedge Brother(John, Richard)$$

$$King(Richard) \vee King(John)$$

$$\neg King(Richard) \Rightarrow King(John)$$

Assignment

- Study today's work
  - Sections 8.1 to 8.2.5
- Read 8.1.1 - the language of thought, with attention