Vraag 1 / *Question 1*

Maak gebruik van logiese ekwivalensies en 'n bewys om aan te toon dat $\neg(P \vee \neg(P \wedge Q)) \models \text{False}$. / *Use logical equivalences and a proof to show that* $\neg(P \vee \neg(P \wedge Q)) \models \text{False}$. [6]

$\neg(P \vee \neg(P \wedge Q))$
$\therefore \neg P \wedge \neg(\neg(P \wedge Q))$ [De Morgan's Law]
$\therefore \neg P \wedge (P \wedge Q)$ [Double Negation Law]
$\therefore (\neg P \wedge P) \wedge Q$ [Associative Law]
$\therefore \text{False} \wedge Q$ [Contradiction]
$\therefore \text{False}$

Naming the laws: 3 marks, applying the laws: 3 marks.

Vraag 2 / *Question 2*

Maak gebruik van die resolusie algoritme om aan te toon dat $(P \Rightarrow Q) \wedge (R \Rightarrow S) \models (P \vee R \Rightarrow Q \vee S)$. / *Use the resolution algorithm to show that* $(P \Rightarrow Q) \wedge (R \Rightarrow S) \models (P \vee R \Rightarrow Q \vee S)$. [12]

Let KB = $(P \Rightarrow Q) \wedge (R \Rightarrow S)$
and $\alpha = (P \vee R \Rightarrow Q \vee S)$

Thus, show that KB $\models \alpha$

Convert KB $\wedge \neg\alpha$ to conjunctive normal form.

1 mark.

$\therefore$ Convert $(P \Rightarrow Q) \wedge (R \Rightarrow S) \wedge \neg(P \vee R \Rightarrow Q \vee S)$ to conjunctive normal form.

$(P \Rightarrow Q) \wedge (R \Rightarrow S) \wedge \neg(P \vee R \Rightarrow Q \vee S)$
$\therefore (\neg P \vee Q) \wedge (\neg R \vee S) \wedge \neg(P \vee R \Rightarrow Q \vee S)$ [Implication elimination]
$\therefore (\neg P \vee Q) \wedge (\neg R \vee S) \wedge \neg((P \vee R) \Rightarrow (Q \vee S))$
$\therefore (\neg P \vee Q) \wedge (\neg R \vee S) \wedge \neg(\neg(P \vee R) \vee (Q \vee S))$ [Implication elimination]
$\therefore (\neg P \vee Q) \wedge (\neg R \vee S) \wedge ((P \vee R) \wedge \neg(Q \vee S))$ [De Morgan]
$\therefore (\neg P \vee Q) \wedge (\neg R \vee S) \wedge (P \vee R) \wedge \neg Q \wedge \neg S$ [De Morgan]

5 marks (1 mark for each clause in the correct form).

Let

R1: $(\neg P \vee Q)$
R2: $(\neg R \vee S)$
R3: $(P \vee R)$
R4: $\neg Q$
R5: $\neg S$

Resolution between R1 and R4:

R6: $\neg P$

Resolution between R2 and R5:

R7: $\neg R$

Resolution between R3 and R6:

R8: R

Resolution between R7 and R8:

R9: $\square$

5 marks for applications of resolution rule. There can be more or less applications as shown here.

Thus, $(P \Rightarrow Q) \land (R \Rightarrow S) \models (P \lor R \Rightarrow Q \lor S)$.

1 mark for the conclusion.

Totaal [18] / *Total [18]*