NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT

Benodigdhede vir hierdie vraestel/Requirements for this paper:			
Antwoordskrifte/ Answer scripts:	X	Multikeusekaarte (A5)/ Multi-choice cards (A5):	
Presensiestrokies (Invulvraestel)/ Attendance slips (Fill-in paper):		Multikeusekaarte (A4)/ Multi-choice cards (A4):	
Rofwerkpapier/ Scrap paper:		Grafiekpapier/ Graph paper:	

Sakrekenaars/Calculators: **Nee/No**

Ander hulpmiddels/Other resources:

Tipe Assessering/ Type of Assessment:	**Eksamen 1e geleentheid** **Exam 1st opportunity** **Vraestel/Paper 1**	Kwalifikasie/ Qualification:	**B.Sc. Honns**
Modulekode/ Module code:	**ITRI626**	Tydsduur/ Duration:	**3 uur** **3 hour**
Module beskrywing/ Module description:	**Kunsmatige Intelligensie / Artificial Intelligence**	Maks/ Max:	**100**
Eksaminator(e)/ Examiner(s):	**Prof. J. V. (Tiny) du Toit**	Datum/ Date:	**02/11/2018**
Interne/Internal Moderator(s):	**Mnr. H. (Henry) Foulds**	Tyd/ Time:	**09:00**

Inhandiging van antwoordskrifte/Submission of answer scripts: **Gewoon/Ordinary**

Vraag 1 (Proposisielogika) / *Question 1 (Propositional Logic)*

1.1 Gee al die stappe van die resolusie algoritme.

Give all the steps of the resolution algorithm. [10]

To show that KB $\models \alpha$, we show that KB $\wedge \neg\alpha$ is unsatisfiable.

Step 1. Convert KB $\wedge \neg\alpha$ to conjunctive normal form (CNF). (2 marks)

Step 2. The resolution rule is repeatedly applied to the resulting clauses. (2 marks)

Step 3. One of two things happens (2 mark):

- There are no new clauses that can be added, in which case KB does not entail α. (2 marks)
- Two clauses resolve to yield the empty clause, in which case KB entails α. (2 marks)

1.2 Skakel die volgende logiese uitdrukking om in konjunkte normaalvorm (KNV). Toon al jou redenasiestappe aan.

Convert the following logical expression into conjunctive normal form (CNF). Show all you reasoning steps. [12]

$(\neg R \Rightarrow (P \wedge Q)) \Rightarrow R$

$(\neg R \Rightarrow (P \land Q)) \Rightarrow R$

$\therefore \neg[(\neg R \Rightarrow (P \land Q)] \lor R)$	(Eliminate ⇒)
$\therefore \neg[\neg(\neg R) \lor (P \land Q)] \lor R)$	(Eliminate ⇒)
$\therefore \neg[(R) \lor (P \land Q)] \lor R)$	(Double negation)
$\therefore \neg[(R \lor P) \land (R \lor Q)] \lor R$	(De Morgan)
$\therefore [\neg(R \lor P) \lor \neg(R \lor Q)] \lor R$	(De Morgan)
$\therefore [(\neg R \land \neg P) \lor (\neg R \land \neg Q)] \lor R$	(De Morgan)
$\therefore [R \lor (\neg R \land \neg P)] \lor [R \lor (\neg R \land \neg Q)]$	(Distributive law)
$\therefore [(R \lor \neg R) \land (R \lor \neg P)] \lor [(R \lor \neg R) \land (R \lor \neg Q)]$	(Distributive law)
$\therefore [True \land (R \lor \neg P)] \lor [True \land (R \lor \neg Q)]$	(Simplify)
$\therefore (R \lor \neg P) \lor (R \lor \neg Q)$	(Simplify)
$\therefore (R \lor \neg P \lor \neg Q)$	(Simplify)

Trying something: 6 marks.
Performing the conversion to CNF steps without naming the steps: 8 marks.
Performing the conversion to CNF steps and naming the steps: 12 marks.

1.3 Bepaal of elkeen van die volgende proposisielogika sinne bevredigbaar, onbevredigbaar of geldig is. Toon all jou redenasiestappe aan.

Determine whether each of the following propositional logic sentences is satisfiable, unsatisfiable, or valid. Show all your reasoning steps.

a) $Q \land R \land \neg(P \lor Q)$ [6]

P	Q	R	P ∨ Q	¬(P ∨ Q)	Q ∧ R ∧ ¬(P ∨ Q)
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	T	F

Truth table: (3).

Unsatisfiable (1) since there is no model with a true value (2).

b) $P \Rightarrow (P \lor Q \lor (Q \land R))$ [6]

P	Q	R	Q ∧ R	P ∨ Q ∨ (Q ∧ R)	P ⇒ (P ∨ Q ∨ (Q ∧ R))
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T
F	F	F	F	F	T

Truth table: (3).

Valid (1) since all the models are true (2).

c) (¬P ∨ ¬Q) ⇒ R [6]

P	Q	R	¬P	¬Q	¬P ∨ ¬Q	(¬P ∨ ¬Q) ⇒ R
T	T	T	F	F	F	T
T	T	F	F	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	T	T	T	T
F	F	F	T	T	T	F

Truth table: (3).

Satisfiable (1) since there are at least one model with a true value (2).

1.4 Gegee ‘n kennisbasis en ‘n doelwit, maak van resolusie gebruik om te bewys die doelwit is waar. Toon al jou stappe duidelik aan. / *Given a knowledge base and a goal, use resolution to proof the goal is true. Show all your steps clearly.* [20]

Kennisbasis (KB): / *Knowledge base* (KB):

1. Q ⋀ (P ⇒ R)
2. ¬P ⇒ (Q ⋁ R)
3. Q ⇒ (¬P ⇒ R)
4. (¬P ⇒ R) ⇒ Q

Doelwit: / *Goal:*

(P ⋀ Q) ⇒ R

Convert the sentences to conjunctive normal form (CNF):

Sentence	CNF	
Q ⋀ (P ⇒ R)	(Q) ⋀ (¬P ⋁ R)	(3 marks)
¬P ⇒ (Q ⋁ R)	P ⋁ Q ⋁ R	(3 marks)
Q ⇒ (¬P ⇒ R)	P ⋁ R ⋁ ¬Q	(3 marks)
(¬P ⇒ R) ⇒ Q	(Q ⋁ ¬P) ⋀ (Q ⋁ ¬R)	(3 marks)

(12)

The goal is (P ⋀ Q) ⇒ R, so we add the CNF of ¬((P ⋀ Q) ⇒ R) to the list of facts, the new set is: (1)

1. (Q) ⋀ (¬P ⋁ R)
2. P ⋁ Q ⋁ R
3. P ⋁ R ⋁ ¬Q
4. (Q ⋁ ¬P) ⋀ (Q ⋁ ¬R)
5. (P) ⋀ (Q) ⋀ (¬R) (1)

Resolution between (5.) (P) and (1.) (¬P) gives 6. (R).

6. R

Resolution between (6.) (R) and (5.) (¬R) gives the empty clause (□) and thus KB ⊨ goal. (6)

Vraag 2 (Eerste-orde Logika) / *Question 2 (First-Order Logic)*

2.1 Skryf die volgende Engelse sinne oor in eerste-orde logika. / *Rewrite the following English sentences into first-order logic.*

a) Every student loves some student. [4]

$\forall x\ (Student(x) \Rightarrow \exists y\ (Student(y) \land Loves(x,y)))$

b) Every student loves some other student. [4]

$\forall x\ (Student(x) \Rightarrow \exists y\ (Student(y) \land \neg(x = y) \land Loves(x,y)))$

c) There is a student who is loved by every other student. [4]

$\exists x\ (Student(x) \land \forall y\ (Student(y) \land \neg(x = y) \Rightarrow Loves(y,x)))$

d) Bill takes either Programming or AI (but not both). [4]

$Takes(Bill, Programming) \Leftrightarrow \neg Takes(Bill, AI)$

e) Only one student failed Security. [4]

$\exists x\ (Student(x) \land Failed(x, Security) \land \forall y\ (Student(y) \land Failed(y, Security) \Rightarrow x = y))$

Vraag 3 (Inferensie met Eerste-orde Logika) / *Question 3 (Inference in First-Order Logic)*

3.1 Beskou die volgende kennisbasis: / *Consider the following knowledge base*:

A pig is faster than a slug. A buffalo is faster than a pig. If Y is faster than Z and X is faster than Y then X is faster than Z. Bob is a buffalo, Pat is a pig and Steve is a slug.

Deur van voorwaardse skakeling gebruik te maak, bewys dat Bob vinniger as Steve is. Toon al jou redenasiestappe aan.

By using forward chaining, prove that Bob is faster than Steve. Show all your reasoning steps. [20]

Convert the KB and goal query to First-Order Logic sentences:

1. $\forall x,y\ Buffalo(x) \land Pig(y) \Rightarrow Faster(x, y)$
2. $\forall y,z\ Pig(y) \land Slug(z) \Rightarrow Faster(y, z)$
3. $\forall x,y,z\ Faster(x, y) \land Faster(y, z) \Rightarrow Faster(x, z)$
4. Buffalo(Bob)
5. Pig(Pat)
6. Slug(Steve)

Goal query: Faster(Bob, Steve) (12)

OR

Convert the KB and goal query to First-Order definite clauses:

1) $Buffalo(x) \land Pig(y) \Rightarrow Faster(x, y)$
2) $Pig(y) \land Slug(z) \Rightarrow Faster(y, z)$
3) $Faster(x, y) \land Faster(y, z) \Rightarrow Faster(x, z)$
4) Buffalo(Bob)
5) Pig(Pat)
6) Slug(Steve)

Goal query: Faster(Bob,Steve) (12)

Perform forward chaining:

Modus ponens applied to (1), (4) and (5) with $\theta = \{x / Bob, y / Pat\}$ gives

7) Faster(Bob, Pat)

Modus ponens applied to (2), (5) and (6) with θ = {y / Pat, z / Steve} gives

8) Faster(Pat, Steve)

Modus ponens applied to (3), (7) and (8) with θ = {x / Bob, y / Pat, z / Steve} gives

9) Faster(Bob, Steve) (6)

This proves that Bob is faster than Steve.

If First-order definite clauses was used: 2 marks.

TOTAAL/TOTAL: 100

Verwysingsnommer: 8.1.7.2.2