NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOMKAMPUS

Benodigdhede vir hierdie vraestel/Requirements for this paper:					
Multikeusekaarte/ Multi-choice cards:		Nie-programmeerbare sakrekenaar/ Non-programmable calculator:	X	Oopboek-eksamen/ Open book examination?	NEE/ NO
Grafiekpapier/ Graph paper:		Draagbare Rekenaar/ Laptop:			

EKSAMEN/TOETS EXAMINATION/TEST:	Semestertoets / Semester Test	KWALIFIKASIE/ QUALIFICATION:	Honns. B.Sc., M.Sc.
MODULEKODE/ MODULE CODE:	ITRI626	TYDSDUUR/ DURATION:	90 minute/*minutes*
MODULEBESKRYWING/ MODULE DESCRIPTION:	Kunsmatige Intelligensie / Artificial Intelligence	MAKS/ MAX:	62
EKSAMINATOR(E)/ EXAMINER(S):	DR. J. V. DU TOIT	DATUM/ DATE:	06/09/2016
		TYD/TIME:	07:30
MODERATOR:	DR. J. V. DU TOIT		

Vraag 1 (Logiese Agente) / *Question 1 (Logical Agents)*

1.1 Gee drie definisies vir logiese gevolgtrekking ($\alpha \models \beta$).

Give three definitions of logical entailment ($\alpha \models \beta$). [3 x 3 = 9]

Answer:

a) For any sentences α and β, $\alpha \models \beta$ if and only if $M(\alpha) \subseteq M(\beta)$ [3 marks].
b) For any sentences α and β, $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid [3 marks].
c) For any sentences α and β, $\alpha \models \beta$ if and only if $(\alpha \wedge \neg\beta)$ is unsatisfiable [3 marks].

1.2 Teken die volgende waarheidstabel oor en voltooi dit.

Draw the following truth table and complete it. [5]

P	Q	¬P	P ∧ Q	P ∨ Q	P ⇒ Q	P ⇔ Q
T	T	?	?	?	?	?
T	F	?	?	?	?	?
F	T	?	?	?	?	?
F	F	?	?	?	?	?

Answer:

P	Q	¬P	P ∧ Q	P ∨ Q	P ⇒ Q	P ⇔ Q
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Each column counts 1 mark.

1.3 Voltooi die volgende tabel met logiese ekwivalensies.

Complete the following table with logical equivalences. [5 x 2 = 10]

(α ⇒ β)	≡	1.3 (a)
(α ⇒ β)	≡	1.3 (b)
(α ⇔ β)	≡	1.3 (c)
1.3 (d)	≡	(¬α ∧ ¬β)
1.3 (e)	≡	((α ∧ β) ∨ (α ∧ γ))

Answer:

(α ⇒ β)	≡	(¬β ⇒ ¬α) [1.3 (a)]	2 marks
(α ⇒ β)	≡	(¬α ∨ β) [1.3 (b)]	2 marks
(α ⇔ β)	≡	((α ⇒ β) ∧ (β ⇒ α)) [1.3 (c)]	2 marks
¬(α ∨ β) [1.3 (d)]	≡	(¬α ∧ ¬β)	2 marks
(α ∧ (β ∨ γ)) [1.3 (e)]	≡	((α ∧ β) ∨ (α ∧ γ))	2 marks

1.4 Bewys dat P ⇒ P ∨ Q altyd waar is deur van 'n bewys en ekwivalensies gebruik te maak. Wys jou redenasies en stappe volledig.

Show that P ⇒ P ∨ Q is always true by using a proof and equivalences. Show all your reasoning and steps clearly. [10]

Answer:

P ⇒ P ∨ Q

≡ ¬P ∨ (P ∨ Q)	Implication elimination
≡ (¬P ∨ P) ∨ Q	Association
≡ (P ∨ ¬P) ∨ Q	Commutative
≡ (True ∨ Q)	Negation
≡ (Q ∨ True)	Commutative
≡ True	

The left side counts 5 marks and the right side counts 5 marks. The exact steps and number of them can differ.

1.5 Toon aan dat ((P ∧ Q) ∨ R) ∧ (R ⇒ S) ⊨ (P ∨ S) deur van die Resolusie algoritme gebruik te maak. Wys jou redenasies en stappe volledig.

Show that ((P ∧ Q) ∨ R) ∧ (R ⇒ S) ⊨ (P ∨ S) by using the Resolution algorithm. Show all your reasoning and steps clearly. [28]

Let KB = ((P ∧ Q) ∨ R) ∧ (R ⇒ S) and α = (P ∨ S)

∴ Proof that KB ∧ ¬α is unsatisfiable.

Convert KB ∧ ¬α to conjunctive normal form (CNF):

((P ∧ Q) ∨ R) ∧ (R ⇒ S) ∧ ¬(P ∨ S)

≡ ((P ∧ Q) ∨ R) ∧ (R ⇒ S) ∧ (¬P ∧ ¬S)	De Morgan
≡ ((P ∧ Q) ∨ R) ∧ (¬R ∨ S) ∧ (¬P) ∧ (¬S)	Implication elimination
≡ (R ∨ (P ∧ Q)) ∧ (¬R ∨ S) ∧ (¬P) ∧ (¬S)	Association
≡ (R ∨ P) ∧ (R ∨ Q) ∧ (¬R ∨ S) ∧ (¬P) ∧ (¬S)	Association

2 mark for each correct clause produced = 10 marks. 5 marks for the conversion and 5 marks for the explanations on the right = 10 marks.

Number the clauses:

1) $(R \vee P)$
2) $(R \vee Q)$
3) $(\neg R \vee S)$
4) $(\neg P)$
5) $(\neg S)$

1 mark.

Perform resolution:

Resolution between (5) and (3) gives:

6) $(\neg R)$

Resolution between (6) and (1) gives:

7) (P)

Resolution between (7) and (4) gives:

8) $\square$

6 marks for the application of the resolution rule.

$\therefore$ Since the empty clause was found, $KB \vDash \alpha$

$\therefore ((P \wedge Q) \vee R) \wedge (R \Rightarrow S) \vDash (P \vee S)$

1 mark for the final answer.

TOTAAL/TOTAL: 62