

Vraag 1 (Kwantifiseerders) / Question 1 (Quantifiers)

Gee die detail betekenis van die Eerste-orde logika sin $\forall x P$, waar P enige logiese uitdrukking is. / Give the detail meaning of the First-order logic sentence $\forall x P$, where P is any logical expression. [6]

The expression $\forall x P$ is true in a given model (1 mark) if P is true in all possible extended interpretations (1 mark) constructed from the interpretation (1 mark) given in the model (1 mark), where each extended interpretation (1 mark) specifies a domain element to which x refers (1 mark).

Vraag 2 (Kwantifiseerders) / Question 2 (Quantifiers)

Voltooi die volgende tabel van De Morgan se reëls vir kwantifiseerders. Complete the following table of De Morgan's rules for quantifiers. [4 x 2 = 8]

$\forall x \neg P$	\equiv	(a)
(b)	\equiv	$\exists x \neg P$
$\forall x P$	\equiv	(c)
(d)	\equiv	$\neg \forall x \neg P$

	\equiv	(a) $\neg \exists x P$
(b) $\neg \forall x P$	\equiv	
	\equiv	(c) $\neg \exists x \neg P$
(d) $\exists x P$	\equiv	

Vraag 3 (Die gebruik van Eerste-orde Logika) / Question 3 (Using First-order Logic)

Gee die ses (Peano) aksiomas wat die natuurlike getalle asook die optelling daarvan definieer. / Give the six (Peano) axioms that define natural numbers as well as the addition thereof. [10]

$\text{NatNum}(0)$ (1 mark)

$\forall n \text{NatNum}(n) \Rightarrow \text{NatNum}(S(n))$ (2 marks)

$\forall n 0 \neq S(n)$ (1 mark)

$\forall m, n m \neq n \Rightarrow S(m) \neq S(n)$ (2 marks)

$\forall m \text{NatNum}(m) \Rightarrow +(0, m) = m$ (2 mark)

$\forall m, n \text{NatNum}(m) \wedge \text{NatNum}(n) \rightarrow +(S(m), n) = S(+(m, n))$ or (2 marks)

$\forall m, n \text{NatNum}(m) \wedge \text{NatNum}(n) \Rightarrow (m + 1) + n = (m + n) + 1$

Vraag 4 (Inferensie met Eerste-orde Logika) / Question 4 (Inference in First-order Logic)

Definieer Veralgemeende Modus Ponens. / Define Generalized Modus Ponens. [6]

For atomic sentences p_i, p_i' and q , (1 mark) where there is a substitution θ (1 mark) such that $\text{SUBST}(\theta, p_i') = \text{SUBST}(\theta, p_i)$ for all i (1 mark),

$$\frac{p_1', p_2', \dots, p_n', (p_1 \wedge p_2 \wedge \dots \wedge p_n \Rightarrow q)}{\text{SUBST}(\theta, q)} \quad (3 \text{ marks})$$

Totaal [30] / Total [30]