NORTH-WEST UNIVERSITY
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Benodigdhede vir hierdie vraestel/Requirements for this paper:			
Antwoordskrifte/ Answer scripts:	X	Multikeusekaarte (A5)/ Multi-choice cards (A5):	
Presensiestrokies (Invulvraestel)/ Attendance slips (Fill-in paper):		Multikeusekaarte (A4)/ Multi-choice cards (A4):	
Rofwerkpapier/ Scrap paper:		Grafiekpapier/ Graph paper:	

Sakrekenaars/Calculators:	Nee/No
Ander hulpmiddels/Other resources:	

Tipe Assessering/ Type of Assessment:	**Eksamen 1e geleentheid** **Exam 1st opportunity** **Vraestel/Paper 1**	Kwalifikasie/ Qualification:	**B.Sc. Honns**
Modulekode/ Module code:	**ITRI626**	Tydsduur/ Duration:	**3 uur** **3 hour**
Module beskrywing/ Module description:	**Kunsmatige Intelligensie / Artificial Intelligence**	Maks/ Max:	**100**
Eksaminator(e)/ Examiner(s):	**Prof. J. V. (Tiny) du Toit**	Datum/ Date:	**14/11/2019**
Interne/Internal Moderator(s):	**Dr. A. (Annette) van der Merwe**	Tyd/ Time:	**09:00**

Inhandiging van antwoordskrifte/Submission of answer scripts: **Gewoon/Ordinary**

Vraag 1 (Proposisielogika) / *Question 1 (Propositional Logic)*

1.1 Skakel die volgende logiese uitdrukking om in konjunkte normaalvorm (KNV). Toon al u redenasiestappe aan.

Convert the following logical expression into conjunctive normal form (CNF). Show all your reasoning steps. [12]

$((Q \Rightarrow R) \Rightarrow P) \wedge \neg R$

$\therefore ((\neg Q \vee R) \Rightarrow P) \wedge \neg R$	Eliminating ⇒
$\therefore (\neg(\neg Q \vee R) \vee P) \wedge \neg R$	Eliminating ⇒
$\therefore ((Q \wedge \neg R) \vee P) \wedge \neg R$	De Morgan
$\therefore (P \vee Q) \wedge (P \vee \neg R) \wedge (\neg R)$	Distributivity law

Trying something: (4 x ✓).

Performing the conversion to CNF without naming the steps: (6 x ✓).

Performing the conversion to CNF and naming the steps: (12 x ✓).

1.2 Bepaal of elkeen van die volgende proposisielogika sinne bevredigbaar, onbevredigbaar of geldig is. Toon all u redenasiestappe aan. / *Determine whether each of the following propositional logic sentences is satisfiable, unsatisfiable, or valid. Show all your reasoning steps.*

a) $\neg[(Q \Rightarrow R) \Rightarrow \neg P]$ [6]

P	Q	R	¬P	Q ⇒ R	(Q ⇒ R) ⇒ ¬P	¬[(Q ⇒ R) ⇒ ¬P]
T	T	T	F	T	F	T
T	T	F	F	F	T	F
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	F
F	T	F	T	F	T	F
F	F	T	T	T	T	F
F	F	F	T	T	T	F

Answer: Satisfiable. (✓) Truth table (5 x ✓)

b) ¬((Q ⇒ P) ∨ (R ⇒ Q)) [6]

P	*Q*	*R*	*Q* ⇒ *P*	*R* ⇒ *Q*	(*Q* ⇒ *P*) ∨ (*R* ⇒ *Q*)	¬((*Q* ⇒ *P*) ∨ (*R* ⇒ *Q*))
T	T	T	T	T	T	F
T	T	F	T	T	T	F
T	F	T	T	F	T	F
T	F	F	T	T	T	F
F	T	T	F	T	T	F
F	T	F	F	T	T	F
F	F	T	T	F	T	F
F	F	F	T	T	T	F

Answer: Unsatisfiable. (✓) Truth table (5 x ✓)

c) R ∨ (¬Q ⇒ ¬(P ∧ R)) [6]

P	*Q*	*R*	¬*Q*	*P* ∧ *R*	¬(*P* ∧ *R*)	¬*Q* ⇒ ¬(*P* ∧ *R*)	*R* ∨ (¬*Q* ⇒ ¬(*P* ∧ *R*))
T	T	T	F	T	F	T	T
T	T	F	F	F	T	T	T
T	F	T	T	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

Answer: Valid. (✓) Truth table (5 x ✓)

1.3 Beskou die volgende logiese raaisel. Toe sy gevra is oor die ouderdomme van haar drie kinders het Mev. Baker geantwoord dat Alice haar jongste kind is as Bill nie haar jongste kind is nie en dat Alice nie haar jongste kind is as Carl nie haar jongste kind is nie. Stel 'n kennisbasis op wat hierdie raaisel voorstel asook die nodige agtergrondkennis dat slegs een van die kinders haar jongste kind kan wees. Toon aan deur van resolusie gebruik te maak dat Bill haar jongste kind is. / *Consider the following logical puzzle. When asked for the ages of her three children, Mrs. Baker says that Alice is her youngest child if Bill is not her youngest child, and that Alice is not her youngest child if Carl is not her youngest child. Write down a knowledge base that describes this riddle and the necessary background knowledge that only one of the three children can be her youngest child. Show with resolution that Bill is her youngest child.* [20]

Answer:

Let the propositions A, B and C denote that Mrs. Baker's youngest child is Alice, Bill and Carl, respectively (✓). We then have the following knowledge base (KB):

1. $A \vee B \vee C$ (One child has to be the youngest) (✓)
2. $A \Rightarrow \neg B$ (Alice and Bill cannot both be the youngest) (✓)
3. $A \Rightarrow \neg C$ (Alice and Carl cannot both be the youngest) (✓)
4. $B \Rightarrow \neg C$ (Bill and Carl cannot both be the youngest.) (✓)
5. $\neg B \Rightarrow A$ (Alice is her youngest child if Bill is not her youngest child) (✓)
6. $\neg C \Rightarrow \neg A$ (Alice is not her youngest child if Carl is not her youngest child) (✓)

The query (goal) Q is: B (✓)

To prove $KB \models Q$, show that $KB \wedge \neg Q$ is unsatisfiable (✓).

Add $\neg Q$ to the knowledge base:

7. $\neg B$

Convert $KB \wedge \neg Q$ to conjunctive normal form (7 x ✓):

1. $A \vee B \vee C$
2. $\neg A \vee \neg B$
3. $\neg A \vee \neg C$
4. $\neg B \vee \neg C$
5. $A \vee B$
6. $\neg A \vee C$
7. $\neg B$

Resolution between 5. and 7. gives

8. A

Resolution between 3. and 6. gives

9. $\neg A$

Resolution between 8. and 9. gives (2 x ✓)

10. □

Consequently, $KB \wedge \neg Q$ is unsatisfiable (✓) and $KB \models Q$. So Bill is the youngest child. (✓)

Vraag 2 (Eerste-orde Logika) / *Question 2 (First-Order Logic)*

2.1 Skryf De Morgan se reëls vir gekwantifiseerde en ongekwantifiseerde sinne neer. / *Write De Morgan's rules for quantified and unquantified sentences down.*

$\forall x\ P \equiv \neg\exists x\ \neg P$ (2½ x ✓)
$\forall x\ \neg P \equiv \neg\exists x\ P$ (2½ x ✓)
$\exists x\ P \equiv \neg\forall x\ \neg P$ (2½ x ✓)
$\exists x\ \neg P \equiv \neg\forall x\ P$ (2½ x ✓)

or

$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$ (2½ x ✓)
$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ (2½ x ✓)
$P \wedge Q \equiv \neg(\neg P \vee \neg Q)$ (2½ x ✓)
$P \vee Q \equiv \neg(\neg P \wedge \neg Q)$ (2½ x ✓) [10]

2.2 Skryf die volgende Engelse sinne oor in eerste-orde logika. / *Rewrite the following English sentences into first-order logic.*

a) None of my friends are perfect. [5]

$\neg\exists x\ (Friend(x) \wedge Perfect(x))$ (5 x ✓)

b) Every teacher is liked by some student. [5]

$\forall x\ Teacher(x) \Rightarrow \exists y\ [Student(y) \wedge Likes(y, x)]$ (5 x ✓)

c) Some boys in the class are taller than all the girls. [5]

$\exists x\ Boy(x) \wedge \forall y\ [Girl(y) \Rightarrow Taller(x, y)]$ (5 x ✓)

d) Nobody in the Calculus class is smarter than everyone in the AI class. [5]

$\neg[\exists x\ TakesCalculus(x) \wedge (\forall y\ TakesAI(y) \Rightarrow SmarterThan(x, y))]$ (5 x ✓)

Vraag 3 (Inferensie met Eerste-orde Logika) / *Question 3 (Inference in First-Order Logic)*

Beskou die volgende kennisbasis: / *Consider the following knowledge base*:

Engels / *English*	Eerste-orde logika / *First order logic*
1. Marcus was a man.	1. Man(Marcus)
2. Marcus was a Roman.	2. Roman(Marcus)
3. All men are people.	3. $\forall x$ Man(x) $\Rightarrow$ Person(x)
4. Caesar was a ruler.	4. Ruler(Caesar)
5. All Romans were either loyal to Caesar or hated him (or both).	5. $\forall x$ Roman(x) $\Rightarrow$ Loyal(x, Caesar) $\vee$ Hate(x, Caesar)
6. Everyone is loyal to someone.	6. $\forall x \exists y$ Loyal(x, y)
7. People only try to assassinate rulers they are not loyal to.	7. $\forall x \forall y$ Person(x) $\wedge$ Ruler(y) $\wedge$ Tryassasin(x, y) $\Rightarrow$ $\neg$Loyal(x, y)
8. Marcus tried to assassinate Caesar.	8. Tryassasin(Marcus, Caesar)

Navraag / *Query*:

Did someone hate Caesar?	$\exists x$ Hate(x, Caesar)

Deur gebruik te maak van resolusie bewys dat die navraag logies volg uit die kennisbasis. Toon al u stappe en redenasies duidelik aan. / *By using resolution prove that the query follows logically from the knowledge base. Show all your steps and reasoning clearly.* [20]

a) Om te bewys dat KB ⊨ Navraag, bewys dat KB ∧ ¬Navraag onbevredigbaar is. / To prove that KB ⊨ Query, prove that KB ∧ ¬Query is unsatisfiable (✓✓).

b) Voeg die ontkenning van die navraag by die kennisbasis. / *Add the negation of the query to the knowledge base.* (✓)

¬∃x Hate(x, Caesar)

1. Man(Marcus)
2. Roman(Marcus)
3. ∀x Man(x) ⇒ Person(x)
4. Ruler(Caesar)
5. ∀x Roman(x) ⇒ Loyal(x, Caesar) ∨ Hate(x, Caesar)
6. ∀x∃y Loyal(x, y)
7. ∀x∀y Person(x) ∧ Ruler(y) ∧ Tryassasin(x, y) ⇒ ¬Loyal(x, y)
8. Tryassasin(Marcus, Caesar)
9. ¬∃x Hate(x, Caesar)

c) Skakel hierdie kennisbasis om na konjunkte normaalvorm en standaardiseer die veranderlikes apart. / *Convert this knowledge base to conjunctive normal form and standardise the variables apart.*

1. Man(Marcus) (✓)
2. Roman(Marcus) (✓)
3. ¬Man(x) ∨ Person(x) (✓)
4. Ruler(Caesar) (✓)
5. ¬Roman(y) ∨ Loyal(y, Caesar) ∨ Hate(y, Caesar) (✓)
6. Loyal(z, F(z)) (✓)
7. ¬Person(q) ∨ ¬Ruler(p) ∨ ¬Tryassasin(q, p) ∨ ¬Loyal(q, p) (✓)
8. Tryassasin(Marcus, Caesar) (✓)
9. ¬Hate(t, Caesar) (✓)

d) Pas die resolusie algoritme toe op die kennisbasis in (b). / *Apply the resolution algorithm on the knowledge base in (b).*

Resolution between (5.) and (9.) with θ = {t / y} gives (✓)

10. ¬Roman(y) ∨ Loyal(y, Caesar) (✓)

Resolution between (10.) and (2.) with θ = {y / Marcus} gives (✓)

11. Loyal(Marcus, Caesar) (✓)

Resolution between (11.) and (7.) with θ = {q / Marcus, p / Caesar} gives (✓)

12. ¬Person(Marcus) ∨ ¬Ruler(Caesar) ∨ ¬Tryassasin(Marcus, Caesar) (✓)

Resolution between (12.) and (3.) with θ = {x / Marcus} gives

13. ¬Man(Marcus) ∨ ¬Ruler(Caesar) ∨ ¬Tryassasin(Marcus, Caesar)

Resolution between (13.) and (1.) gives

14. ¬Ruler(Caesar) ∨ ¬Tryassasin(Marcus, Caesar)

Resolution between (14.) and (4.) gives

15. ¬Tryassasin(Marcus, Caesar)

Resolution between (15.) and (8.) gives

□

Gevolglik het daar iemand bestaan wat Caesar haat (Marcus). / *Consequently, there existed someone who hated Caesar (Marcus).* (✓✓)

TOTAAL/TOTAL: 100

Verwysingsnommer: 8.1.7.2.2