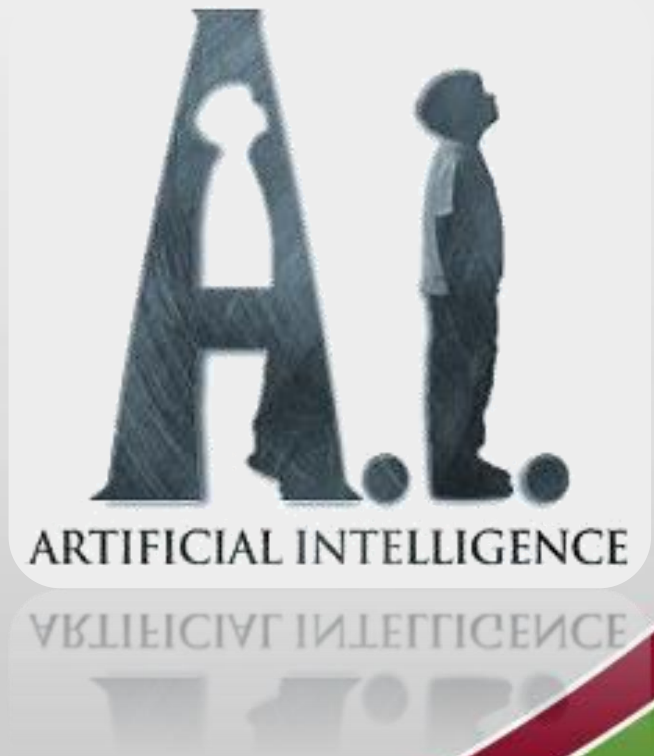


# Inference in First-Order Logic

## Chapter 9



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# Overview of lecture

- Forward chaining
- Backward chaining



# Forward chaining

- Forward chaining algorithm for propositional definite clauses in Chapter 7
- Start with atomic sentences in knowledge base
- Apply Modus Ponens in forward direction
- Add new atomic sentences until no new inferences can be made
- First-order case

# First-order definite clauses

- Disjunctions of atomic sentences with exactly one atomic sentence that is positive
- Definite clause is
  - Atomic sentence or
  - Implication with condition a conjunction of positive atomic sentences and conclusion a single positive atomic sentence

# First-order definite clauses

- Example of first-order definite clauses:

$\text{King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$

$\text{King}(\text{John})$

$\text{Greedy}(\text{John})$

- Can contain variables (implicitly universal quantified, not written)
- Definite clauses normal form for Generalized Modus Ponens

# First-order definite clauses

- The (American) law says that it is a crime for an American to sell weapons to hostile nations
- The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American
- Prove that Colonel West is a criminal



# First-order definite clauses

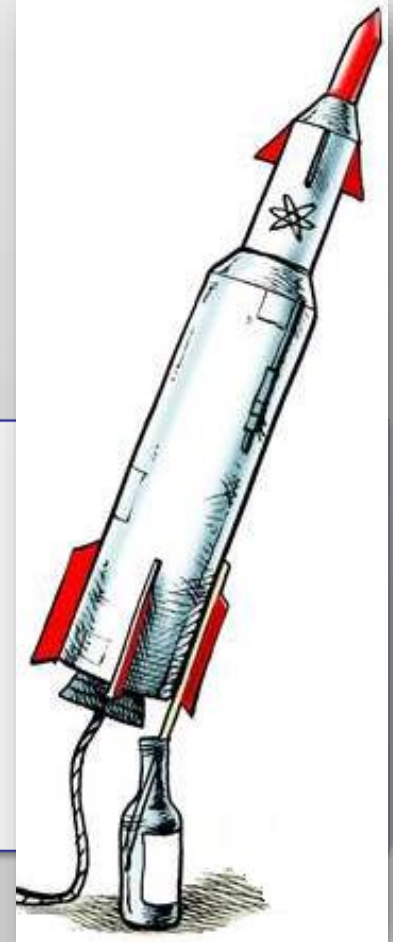
“it is a crime for an American to sell weapons to hostile nations”


$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \wedge \text{Hostile}(z) \Rightarrow \text{Criminal}(x)$$

# First-order definite clauses

“Nono ... has some missiles”

$\exists x \text{ Owns}(\text{Nono}, x) \wedge \text{Missile}(x)$

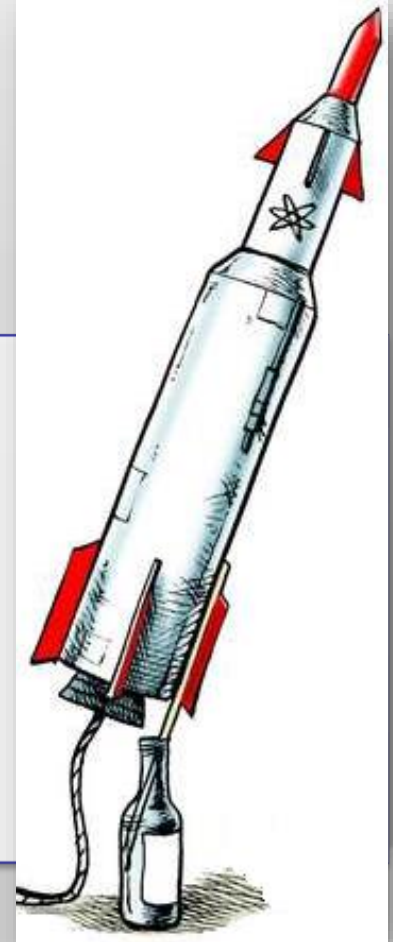




# First-order definite clauses

“Nono ... has some missiles”

$\text{Owns}(\text{Nono}, M_1)$   
 $\text{Missile}(M_1)$



# First-order definite clauses

“all of its missiles were sold  
to it by Colonel West”

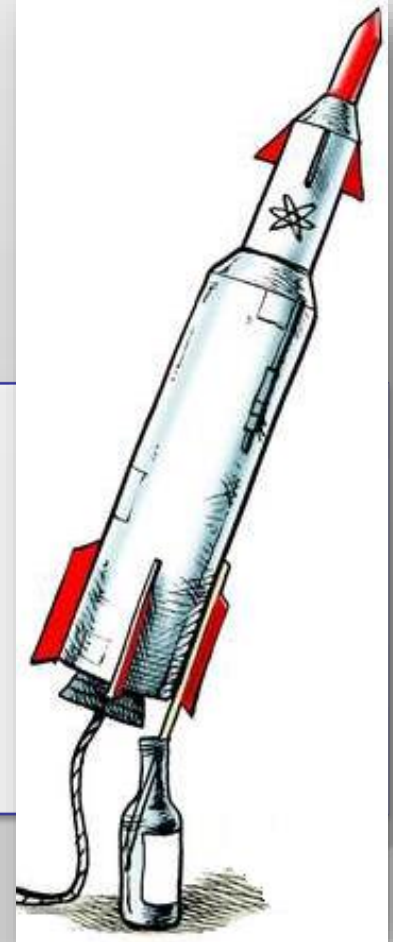
$\text{Missile}(x) \wedge \text{Owns}(\text{Nono}, x) \Rightarrow$   
 $\text{Sells}(\text{West}, x, \text{Nono})$



# First-order definite clauses

“Missiles are weapons”

$\text{Missile}(x) \Rightarrow \text{Weapon}(x)$



# First-order definite clauses

“An enemy of America counts as hostile”

$\text{Enemy}(x, \text{America}) \Rightarrow \text{Hostile}(x)$



# First-order definite clauses

“West, who is American”

American(West)



# First-order definite clauses

“The country Nono, an enemy of America”

Enemy(Nono, America)



# First-order definite clauses

$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \wedge$   
 $\text{Hostile}(z) \Rightarrow \text{Criminal}(x) \quad (9.3)$

$\text{Owns}(\text{Nono}, M_1) \quad (9.4)$

$\text{Missile}(M_1) \quad (9.5)$

$\text{Missile}(x) \wedge \text{Owns}(\text{Nono}, x) \Rightarrow$   
 $\text{Sells}(\text{West}, x, \text{Nono}) \quad (9.6)$

$\text{Missile}(x) \Rightarrow \text{Weapon}(x) \quad (9.7)$

$\text{Enemy}(x, \text{America}) \Rightarrow \text{Hostile}(x) \quad (9.8)$

$\text{American}(\text{West}) \quad (9.9)$

$\text{Enemy}(\text{Nono}, \text{America}) \quad (9.10)$



# First-order definite clauses

*American(West)*

(9.9)

*Missile(M1)*

(9.5)

*Owns(Nono,M1)*

(9.4)

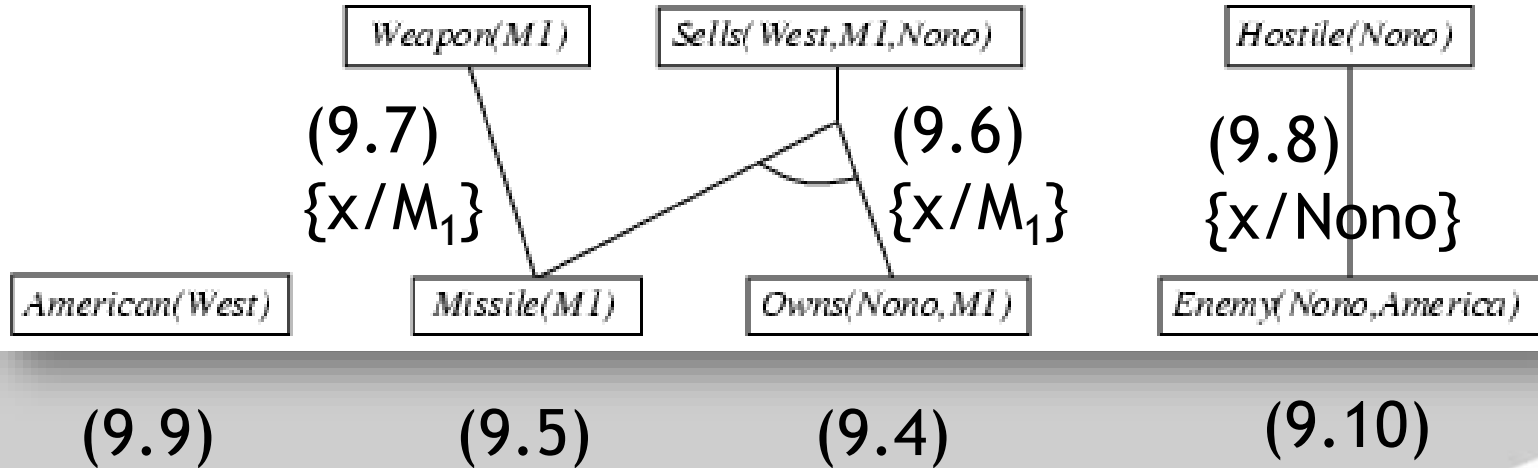
*Enemy(Nono,America)*

(9.10)

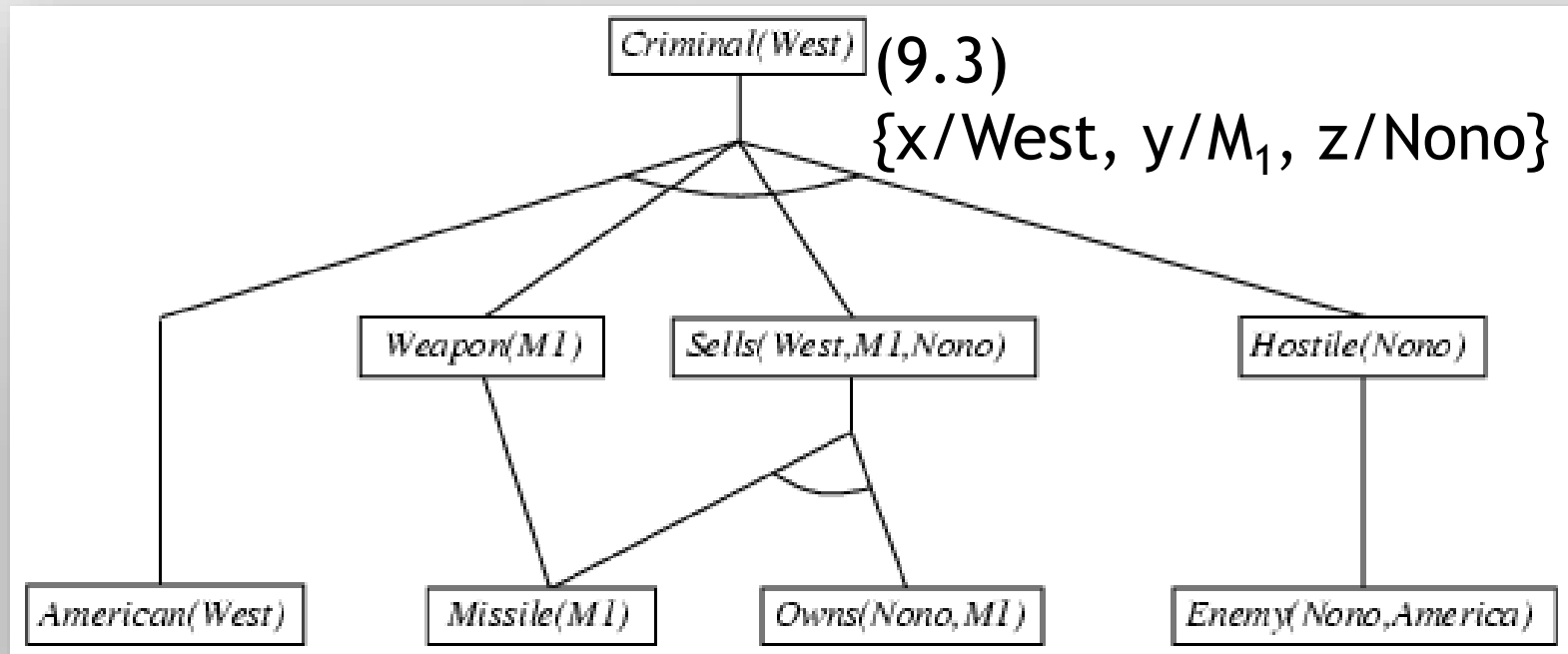


# First-order definite clauses

~~(9.3)~~



# First-order definite clauses



# First-order definite clauses

- Datalog knowledge base
  - First-order definite clauses with no function symbols
- No new inferences possible - fixed point
- Forward chaining is sound
  - Application of Modus Ponens
- Forward chaining is complete
  - For definite clauses knowledge base

# First-order definite clauses

- Ignore section on efficient forward chaining

# Backward chaining

- Algorithm works backwards from goal, through the rules, to obtain known facts to support the proof

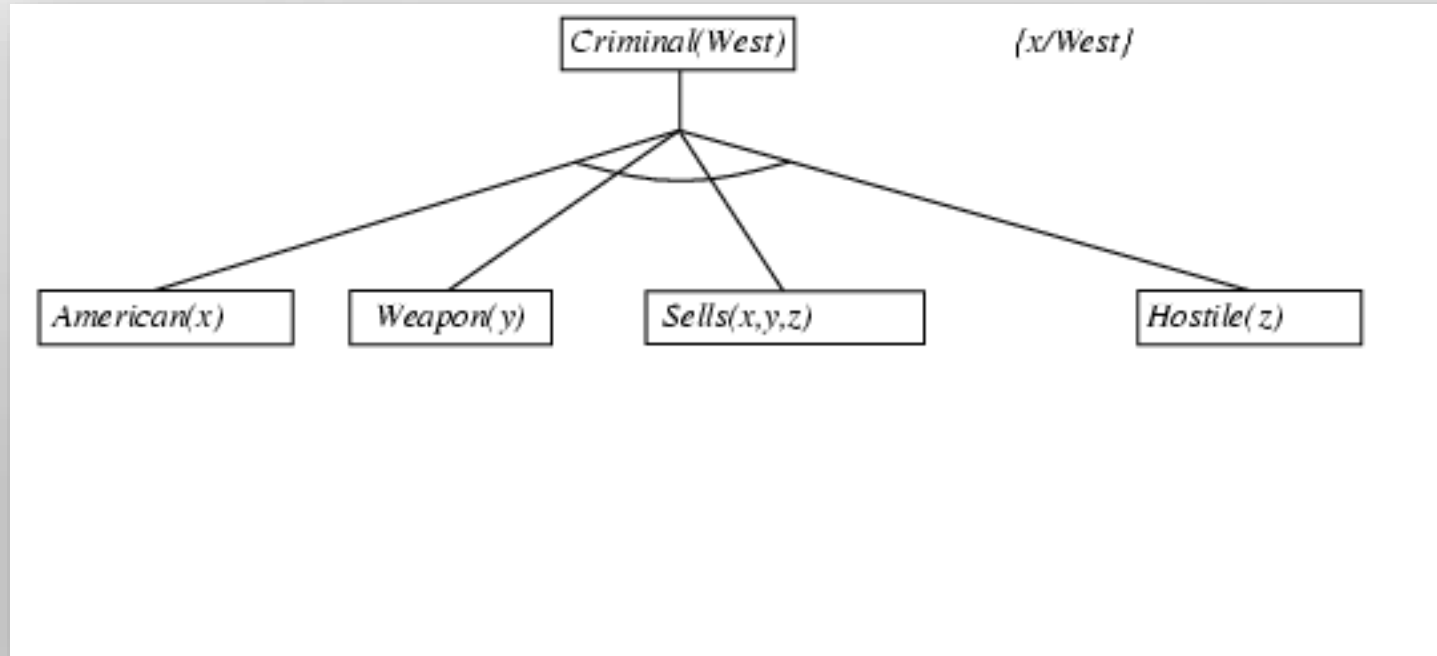
# Backward chaining

$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \wedge \text{Hostile}(z) \Rightarrow \text{Criminal}(x) \quad (9.3)$$

*Criminal(West)*

# Backward chaining

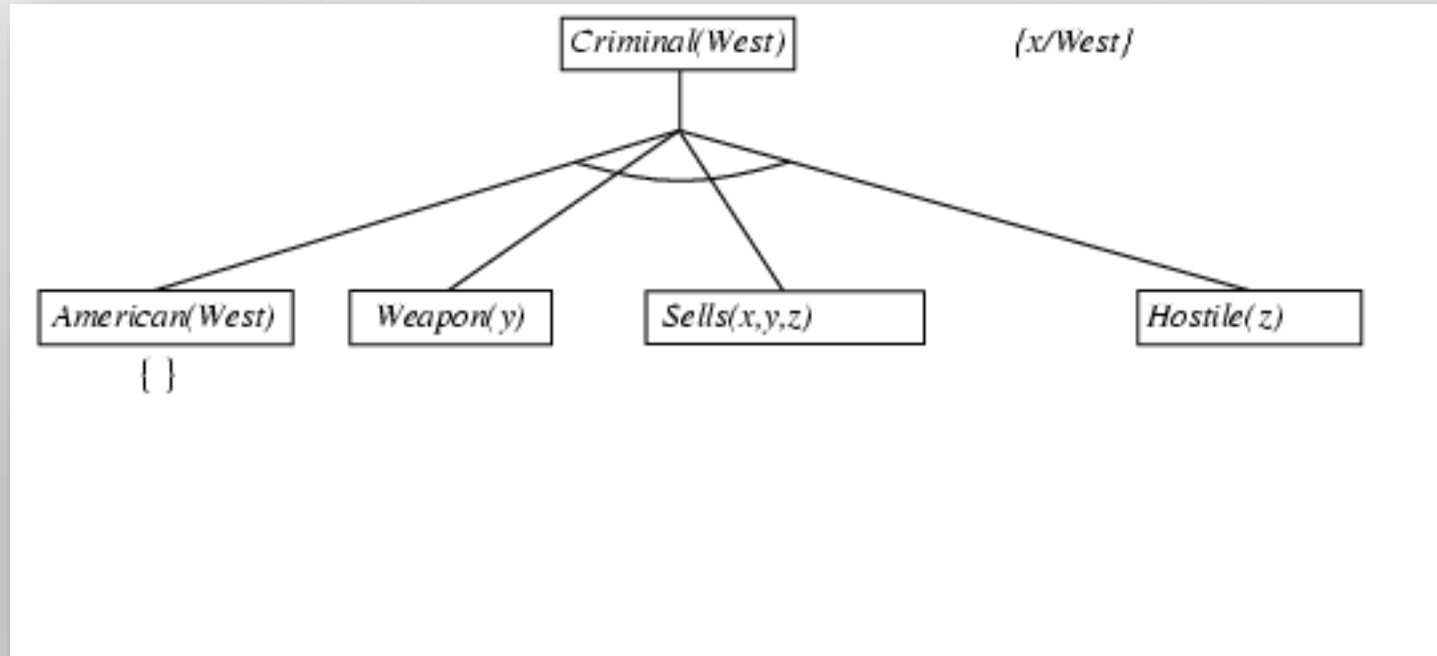
$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \wedge \text{Hostile}(z) \Rightarrow \text{Criminal}(x) \quad (9.3)$$



# Backward chaining

American(West)

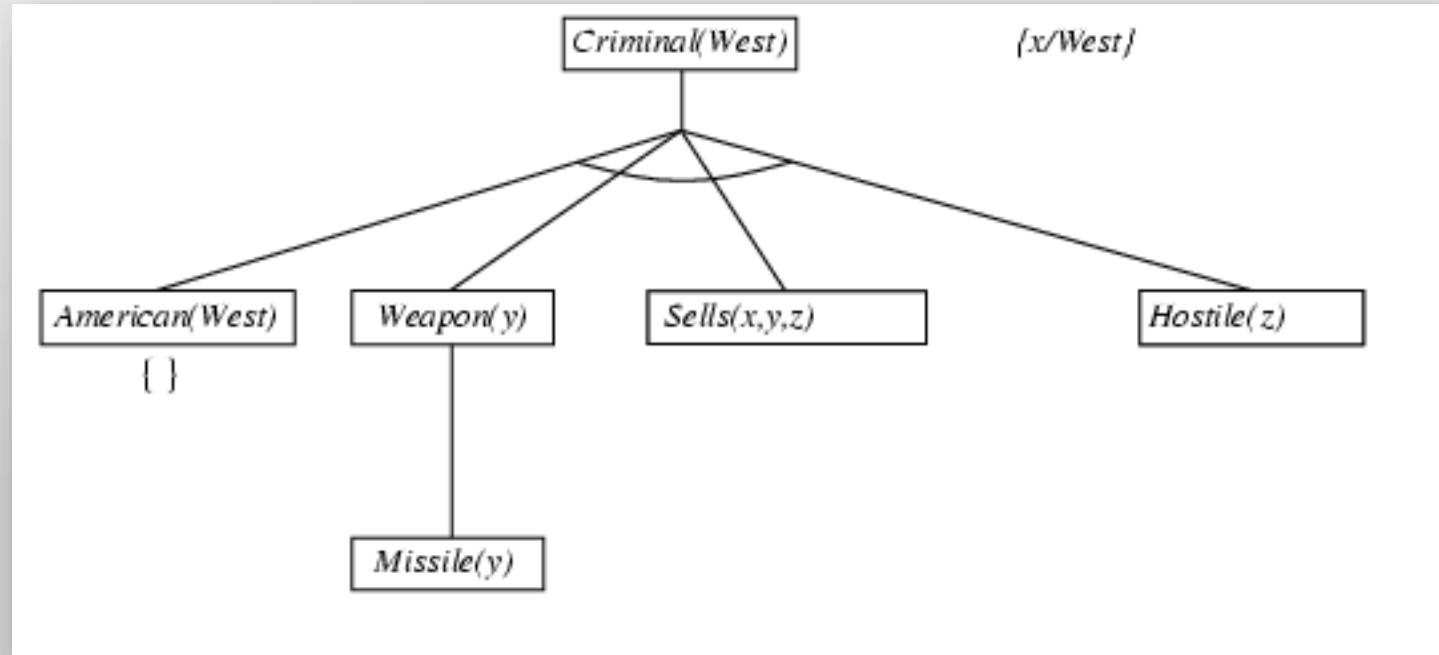
(9.9)





# Backward chaining

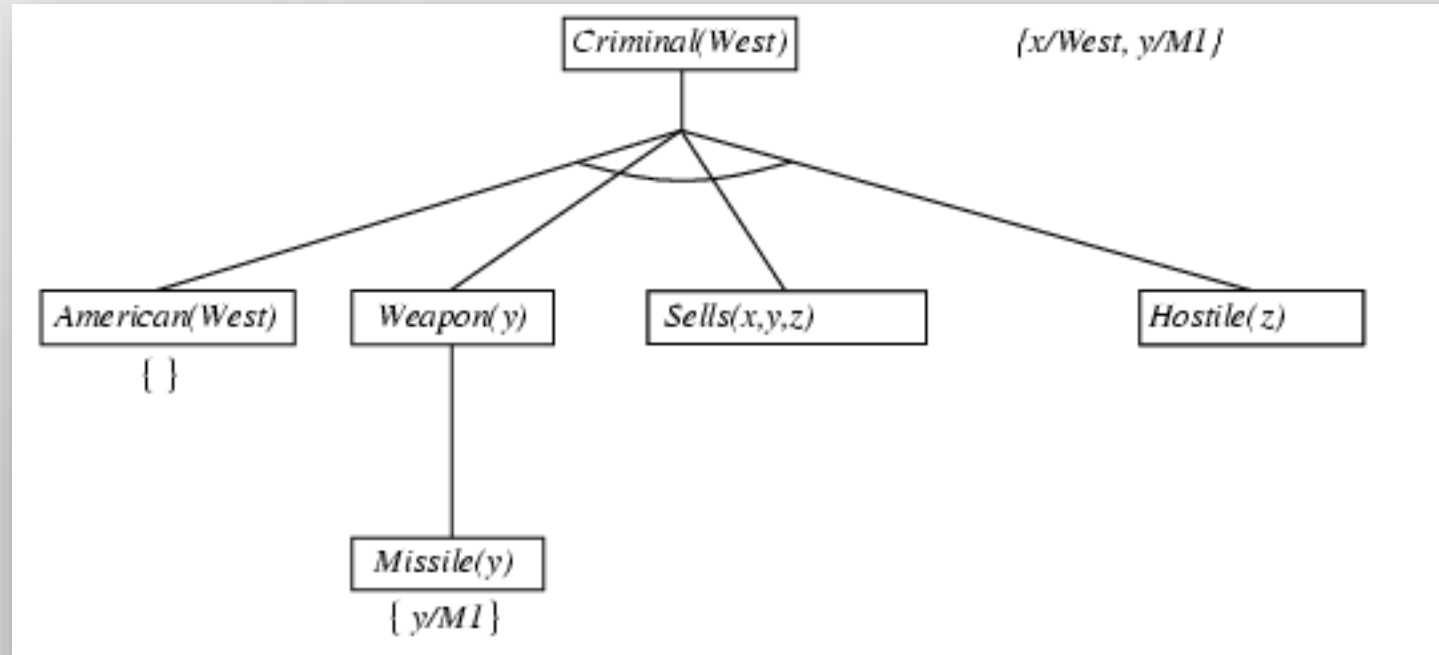
$\text{Missile}(x) \Rightarrow \text{Weapon}(x)$  (9.7)



# Backward chaining

$\text{Missile}(x) \Rightarrow \text{Weapon}(x)$  (9.7)

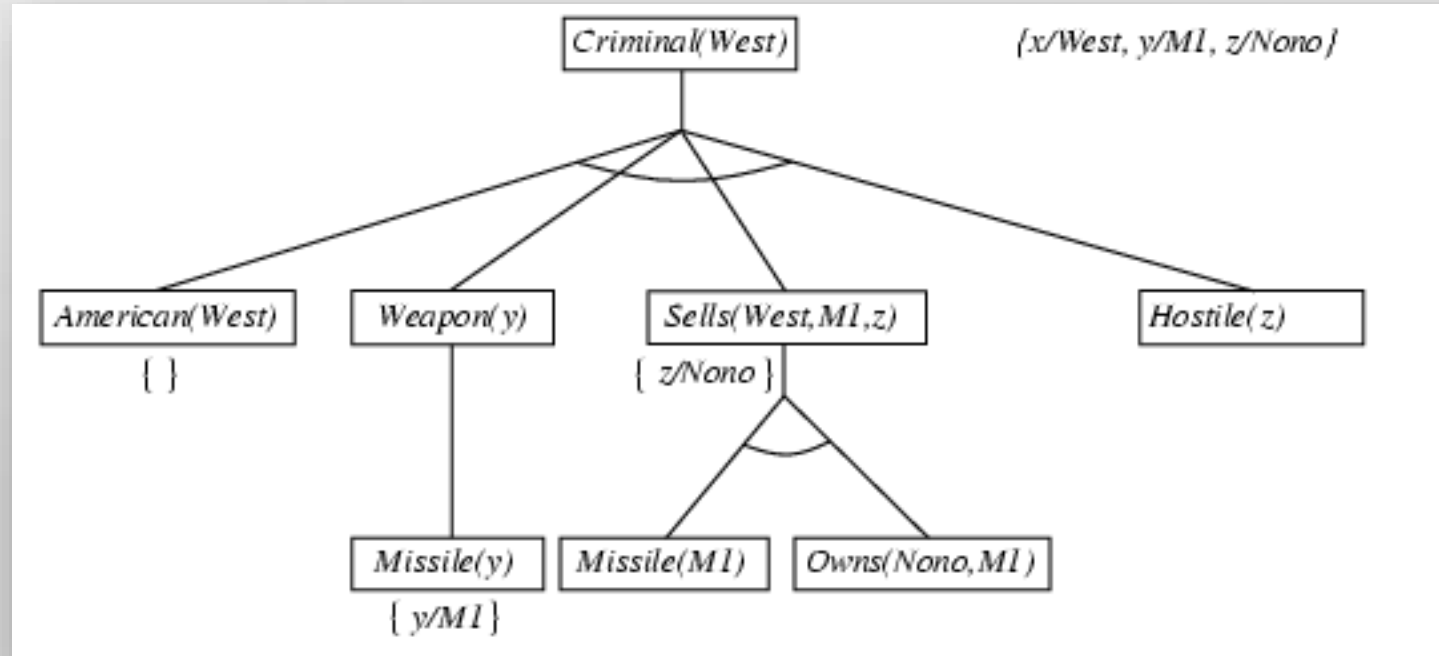
$\text{Missile}(M_1)$  (9.5)



# Backward chaining

$\text{Missile}(x) \wedge \text{Owns}(\text{Nono}, x) \Rightarrow \text{Sells}(\text{West}, x, \text{Nono})$  (9.6)

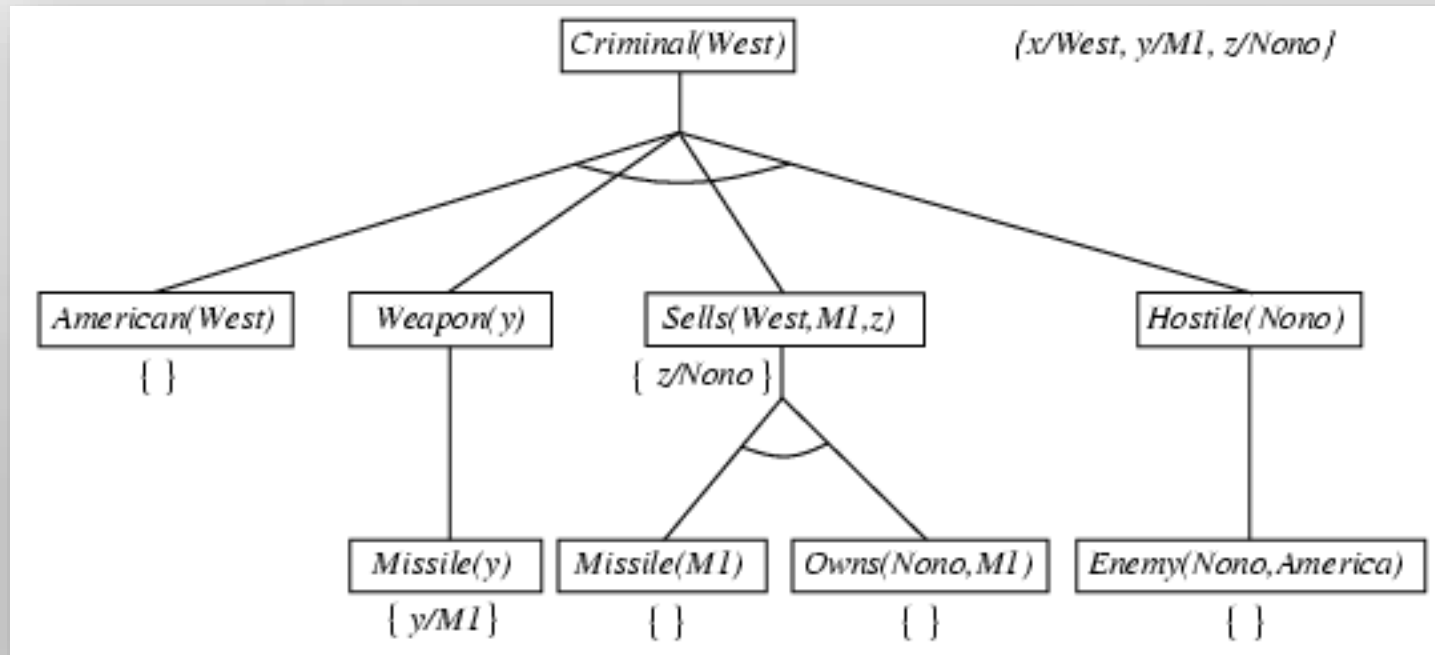
$\text{Owns}(\text{Nono}, M_1)$  (9.4) and  $\text{Missile}(M_1)$  (9.5)



# Backward chaining

Enemy(Nono, America)

(9.10)



# Backward chaining

- Ignore section from “Logic programming” to before “9.5 Resolution”

# Announcements

- Theory quiz 7
  - Thursday, 28 October 2021
  - Forward and backward chaining
- Please remember to do Python practical assignment 5 (R05) on eFundi
  - The due date for the assignments is Thursday, 4 November 2021 at 18:00

