Comparison of $k - \epsilon$ STD and RNG turbulence models

Del Negro Marta, Greppi Enrico, Mensa Jacopo

Contents

1. Abstract

The paper compares $k - \epsilon$ standard with wall functions and $k - \epsilon$ RNG with wall functions calculations of vortex-shedding flow past a square cylinder at $Re = 22000$. Results are even compared with various LES, DNS and experimental data, highlighting the strengths and weaknesses of the two turbulence models in the flow simulation.

2. Introduction

In this work, we used the Ansys-Fluent software to build the computational simulation model of a vortex shedding flow past a square cylinder at $Re = 22000$.
Regarding the computational grid, we considered two subdomains: one is characterized by significant gradients of flow variables, the other is necessary to avoid the boundary conditions effects on the solution. For the convective terms we used the interpolation scheme QUICK on the first subdomain, discretized with a quadrangular grid, while on latter, discretized with a triangular grid, we used the LUDS scheme. For the diffusive terms the interpolation scheme used is the CDS scheme. Regarding the time progress, we utilized the second order implicit Euler scheme.

First we considered the $k - \epsilon$ RNG turbulent model with wall functions and then, with the same flow conditions and computational setup, we changed the turbulence model into the $k - \epsilon$ standard (STD) with wall functions. We collected and analyzed the data extrapolated from the two models, comparing them with bibliographic data. We searched a physical data interpretation, emphasizing the differences between the two turbulence models and looking for their mathematical and computational justification.

3. Description of turbulence models

In $k-\epsilon$ models the Reynolds tensor, which represents the correlations between the various components of turbulent velocity, is reduced to a scalar, the so called eddy viscosity ν_T. ν_T is expressed by means of turbulent kinetic energy k and energy dissipation rate ϵ, with the formula $\nu_T = C_\mu \frac{k^2}{\epsilon}$. Therefore two transport equations, for k and ϵ are needed.

3.1. $k-\epsilon$ STD model

The $k-\epsilon$ STD model is derived from RANS equations. Some assumptions are made to construct the standard model, the most relevant are: isotropy of the phenomenon, ergodicity and a closure hypothesis [1].
The k equation is derived in exact form from RANS equations:

$$\frac{\partial k}{\partial t} + \frac{\partial \bar{u_j} k}{\partial x_j} = \frac{\partial}{\partial x_j}\big[\big(\frac{\nu_t}{\sigma_k} + \nu\big)\frac{\partial k}{\partial x_j}\big] + P_k - \epsilon. \tag{1}$$

Instead the ϵ equation, in the STD model, is not exact but it's built by analogy to that of k:

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial \bar{u_j} \epsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\big[\big(\frac{\nu_t}{\sigma_\epsilon} + \nu\big)\frac{\partial \epsilon}{\partial x_j}\big] + \frac{\epsilon}{k} C_{\epsilon 1} P_k - \frac{\epsilon}{k} C_{\epsilon 2}\epsilon. \tag{2}$$

All the constants are determined via experimental data or computational optimization. One of the weaknesses of the STD model is an inappropriate use of constants, extracted from a standard particular case but taken as universal. This often leads to errors in the prediction of the most complex phenomena.

3.2. $k-\epsilon$ RNG model

RNG turbulence model is based on renormalization group method and it's derived from the instantaneous N-S equations [2]. This statistical procedure, which uses dynamic scaling and invariance, together with iterated perturbation methods, allows to evaluate transport coefficients and transport equations for the large scales modes.
Using the analogy with equilibrium statistical mechanics, the Navier-Stokes equations are replaced by a more general, but equivalent form:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) v = \Gamma - \frac{1}{\rho}\nabla p + \nu_0 \nabla^2 u \tag{3}$$

with the presence of a random force $\Gamma(x, t)$ (noise) chosen to generate the velocity field described by the Kolmogorov energy spectrum. This correspondence principle is the basis of the RNG method.

It's very important to highlight that RNG theory does not include any experimentally adjustable parameters, contrary to the STD model. Thus the RNG turbulence model is not influenced by errors on the constants determination.
In the STD method the evolution equation model for ϵ is constructed by analogy to that of k, and therefore can easily be imprecise. In the RNG model the renormalization group method is applied to derive the equation governing the turbulent kinetic energy k and also its dissipation ϵ. Thanks to the RNG differential recursion relations, the construction by analogy is left apart, together with its problems.

4. Physical consideration and computational justifications

In this section we will give a physical interpretation of the collected data. We highlight now that the results of the RNG model simulation are consistent with the bibliographic data, which we know to be well-trusted. They include DNS 3D, LES 3D and experimental data by Lee, Lyn, Durao, Bearman and Ohtsuki. We will focus on the role of the turbulence model used, equal to flow conditions and computational model, and we will analyze differences between RNG and STD models, showing that the first is better than the latter. In all graphs below we used a continuous red line for the RNG model, named in legend as 'DIMAT', and a continuous black one for the STD.

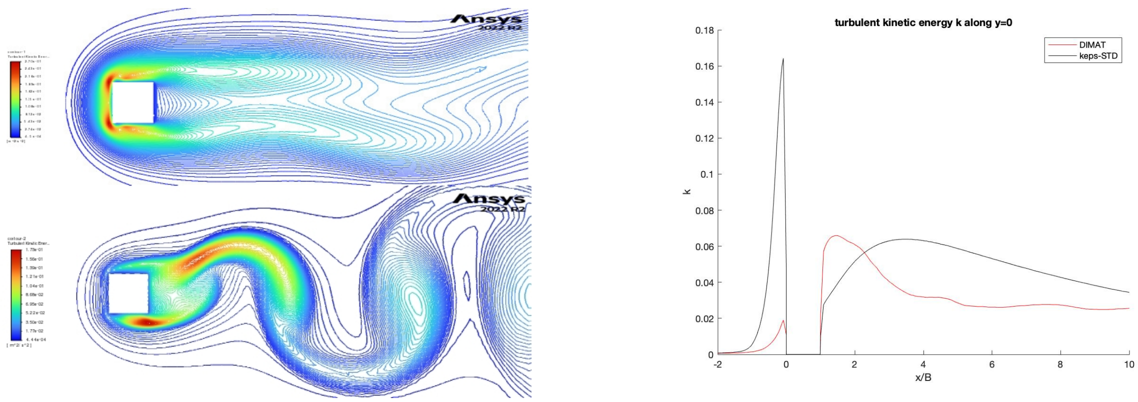

Figure 1: *Left panel:* production of turbulent kinetic energy in the STD model (top image) and in the RNG model (bottom image). *Right panel:* development of the turbulent kinetic energy k along $y = 0$.

4.1. Turbulent kinetic energy

From the turbulent kinetic energy graphs (Fig. 1) we can see that the STD model gives us quite poor results. The turbulent kinetic energy is wrongly overestimated in the frontal area and assumes too low values in the wake zone, where it should be much higher. In this zone we also notice that the turbulent kinetic energy's peak of the STD simulation seems to be shifted downstream in comparison to the RNG's one. Recalling [3] by Rodi et al., this is accounted as a consequence of the too large separation length predicted by the STD model.

The anomalously large production of turbulent kinetic energy in stagnation points (Fig. 1) is the main weakness of the $k - \epsilon$ STD model. This spurious behavior upsets the rest of the flow computation. The common explanation provided is that the eddy-viscosity formula gives an erroneous normal stress difference. Some computations suggest an additional consideration: as the stagnation point is approached, the turbulent time scale became very large. In the ϵ-equation this causes a too small production of ϵ, consequently causing too high turbulent kinetic energy [4]. The unjustified production of k in the stagnation zone, where we should have:

$$u', v' \to 0 \qquad \Rightarrow \qquad k \to 0 \tag{4}$$

is due to the fact that:

$$lim_{k\to 0}\left(-\frac{\epsilon^2}{k}C_{\epsilon 2}\right) = \infty \qquad \text{(also if } \epsilon \ \to 0). \tag{5}$$

This cause an increase of the dissipation of ϵ, to which follows a decrease of energy dissipation rate ϵ and so an increase of k and ν_t.

The RNG model shows a better behaviour (Fig. 1). The production of k is now more realistic: low in the frontal zone and a little bit higher downstream with the peak located roughly correctly.

From a mathematical point of view, in low-Reynolds-number flow regions, where $k \to 0$, RNG differential recursion relations are solved. Thanks to this, $C_{\epsilon 2}$ goes from constant to variable, as a function of the strain rate, therefore the limit (5) does not diverge.

However, in the wake region k predicted by the RNG model is still strongly underestimated in comparison to the LES and experiments results reported in [3]. The reason is that 2D RANS-based models are not able to detect well the low frequencies fluctuations, whose energetic turbulent contributes are great since they lies in the so called energetic layer. Because of their 3D nature, these fluctuations are well caught only by 3D models like the 3D LES simulations, as illustrated by Rodi in [3].

4.2. Drag coefficient C_D

Comparing the values of C_D (Tab. 2), we notice that the RNG's drag coefficient is bigger than the $k - \epsilon$ STD's one, which has very low periodical fluctuations, almost zero. In fact the simulation of the vortex formation and their velocity gradients are not good enough. These complex phenomena are not adequately captured by the STD model, causing an underestimation of C_D. The STD model simulates a non-stationary flow as a stationary one (topic that we will discuss better in transient section) and this results in lower C_D fluctuations.

Turbulence_Model	St	mean_Cl	Cl_amplitude	mean_Cd	Cd_amplitude
'k-eps RNG'	0.13812	-0.0019539	1.3971	1.902	0.058963
'k-eps STD'	0.083472	-4.6647e-05	0.050823	1.5421	0.00037239

Figure 2: table that shows the values of the Strouhal number, the lift coefficient and the drag coefficient obtained using the RNG model (first row) and the STD model (second row).

The causes of the C_D underestimation can be various and pertaining to the basal area. One of them is the too long separation region in the STD model, which can be seen in the mean velocity graph and will be discussed later. Another one is the larger base pressure which comes from the STD model and reduces the drag force acting on the cylinder. Moreover the lower vorticity ($\omega = C\frac{\epsilon}{k}$) of the STD model is a further reason of C_D underestimation. Our results totally agree with Rodi [3].

4.3. Lift coefficient C_L

The lift coefficient C_L calculated with the STD model is way smaller than the one obtained with the RNG model (see Tab. 2). This happens because the STD simulates less pronounced eddies, influencing the coefficient's behaviour. The asymmetry in the basal area in STD model, because of the less vorticity, results in a lower C_L.
The frequency of eddies detachment, called Strouhal number, is equal to the oscillation frequency of the lift coefficient. So we can extract it from the C_L data. We notice that there's a huge difference between the RNG lift period (T=7,24 s) and the STD one (T = 12 s), and therefore an underestimation of the Strouhal number in the STD approach (Tab. 2). This could be due to the fact that the statistical correction in the RNG model allows us to simulate a turbulence behaviour with high frequency of vortex shedding, which is closer to the experiments, much better than the STD. The latter predicts a lower frequency, it's less accurate and it shows many issues approaching a non-stationary phenomena.

4.4. Pressure coefficient C_p

The development of the mean pressure coefficient can be seen in Fig. 3, where we notice that on the frontal face of the cylinder the overall behaviour of the STD C_p is closer to the experimental data, DNS, LES and RNG simulations, while there's a huge overestimation of the base pressure. The fact that the STD prediction gets worse, as soon as we pass the separation point, reveals that STD is not suitable if the flow starts separating and exhibits a pronounced turbulent behaviour. This dissimilarity is due to the different behavior of the boundary layer in the two models. In the STD model the lower velocity outside the boundary layer, at the point of separation, causes an increase in pressure on the basal surface, whose effects on the underestimation of C_D has already been explained.

The standard deviation of C_p in the STD model is approximately zero, way lower than the RNG's one. This is a consequence of the fact that STD model poorly simulates non-stationary flows, approximating them as almost-stationary. This results in less deviation from the average value.

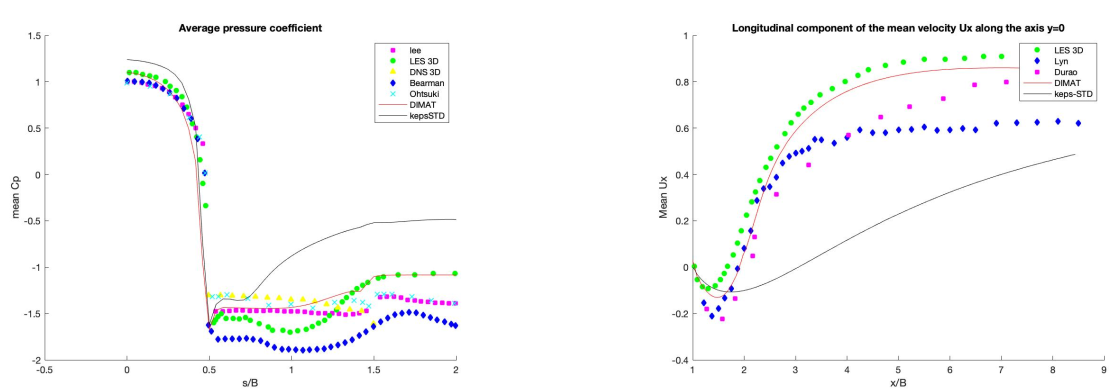

Figure 3: *Left panel:* development of the average pressure coefficient. *Right panel:* distribution of the longitudinal component of the mean velocity $\bar{u}_x$ along the axis $y = 0$.

4.5. Transient

We define transient the time spent by the flow to pass the non-stationary regime and enter in the periodic stationary one. The flow overcomes the transient as soon as its first and second statistical moments become invariant. A noteworthy aspect is that the STD's transient is longer than the RNG's one. The RNG simulation suffices 140 flow time to reach the almost-stationary behaviour, while the STD, requires something like 1600 flow time. This can also be related to the STD's lower Strouhal number: the oscillation period is longer and the flow needs a greater time to became periodic stationary.

The STD transient appears to be smooth, nearly periodic with oscillations increasing but even decreasing in amplitude, while the RNG's one is rugged and monotonously increasing in amplitude. We have tried to give a possible explanation for these results. The phenomenon under consideration is non-stationary but at the basis of the STD model there is the hypothesis of ergodicity [1], which requires the assumption of stationarity of the phenomenon. Therefore, the model takes longer to enter in the periodic regime and this is also reflected later in the behavior of vortex shedding, which is simulated as almost-stationary.
On the contrary, the RNG model doesn't require the ergodicity hypothesis and therefore better simulates non-stationary phenomena.
Furthermore, the effect of the high turbulent diffusion, characterizing the $k-\epsilon$ STD model, is another cause of the simulation of all fluxes as almost-stationary even if they are not.

4.6. Vortex shedding flow

While the RNG model predicts correctly the geometric separation at the corners of the buff body, in the STD's one the separation is "delayed" due to the lower adverse pressure gradient. This explains why, even visually, we notice that RNG model simulates much sharper wake eddies, with higher frequency of detachment than the STD model, which shows stretched vortex with a low frequency shedding dynamic (Fig. 4). This is consistent with the fact that the STD model's lower Strouhal number ($St_{STD} < St_{RNG}$). As the vorticity ω can be expressed in function of ϵ and k as $\omega = \frac{\epsilon}{k}C$, an underestimation of ϵ causes the underestimation of ω. This means lower deformation rate with the resulting less sharp eddies.

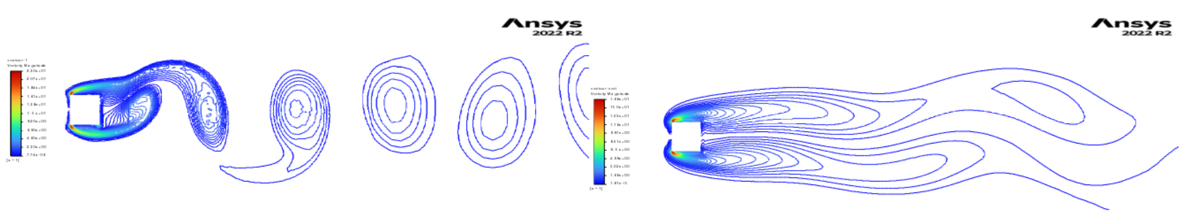

Figure 4: *Left image:* vortex detachment obtained with the $k-\epsilon$ RNG model. *Right image:* vortex detachment obtained with the $k-\epsilon$ STD model.

4.7. Mean and instantaneous velocities

The STD model underestimates the longitudinal component of the mean velocity $\bar{u}_x$ along the axis $y = 0$ in the wake region (Fig. 3). The main reason lies in the turbulent kinetic energy's overestimation. In fact, k feeds itself by subtracting energy from the mean flow field, so from $\bar{u}_x$ itself. From a purely mathematical point of view this aspect can be explained considering the balance equations of kinetic energy of the mean motion and of turbulent motion. In the first equation there is a dissipative term, related to Reynolds tensions, $< u_i' u_j' > \frac{\partial \bar{u}_i}{\partial x_j}$ (always negative), while in the second one the same term appears with the opposite sign, as an energy source. In fact, big eddies extract kinetic energy from the mean motion and transfer it to the turbulent motion.
According to the graph, the mean velocity returns to grow further downstream in the STD than others, revealing that the STD predicts a too long separation region. This reflects on a poor simulation of the velocity in the wake region, as shown by Rodi [3]. The critical issues we just talked about are fixed in the RNG model. It gives us a better approximation, closer to the experiments and LES' data. Looking at the instantaneous distribution of the velocity's longitudinal component along the line x/B=1 (Fig. 5, at a fixed phase, here phase 1), we notice that both the STD and the RNG model don't fit well

enough Lyn's data. They are pretty close where velocity is positive but, approaching along the vertical to the base face, some significant differences emerge. The STD model simulates poorly the vortex shedding dynamic and the relative zone of retrograde flow. The RNG model is better: it detects at least the vortex detaching from the floor, but not the one from the roof, while the STD simulation ignores both. This means that at the fixed time in which the flow is observed the STD model wrongly takes over no detachment of vortex. This agrees with the STD's lower Strouhal number. That's another proof of the poor results that come from the $k - \epsilon$ STD model, not suitable for simulating an high turbulence flow with several retrograde flow zones. Instantaneous velocity graphs (Fig. 5) perfectly reflect vortex shedding considerations: the most pronounced oscillations in the graphs of the RNG simulation are related with the sharper vortex profile, as well as the more regular graphs related to the STD model are consistent with the STD eddies simulation. Similar considerations can be made about the transversal component of the instantaneous velocity along $y = 0$. The results obtained with the STD model are extremely smooth and have symmetric shape, like a perfect sine wave with very low gradient of velocity: a totally unrealistic behaviour. This is the effect of the high turbulent diffusion ($\frac{\nu_T}{\nu} >> 1$), which tends to soften the velocity contours and simulate all fluxes as almost-stationary.

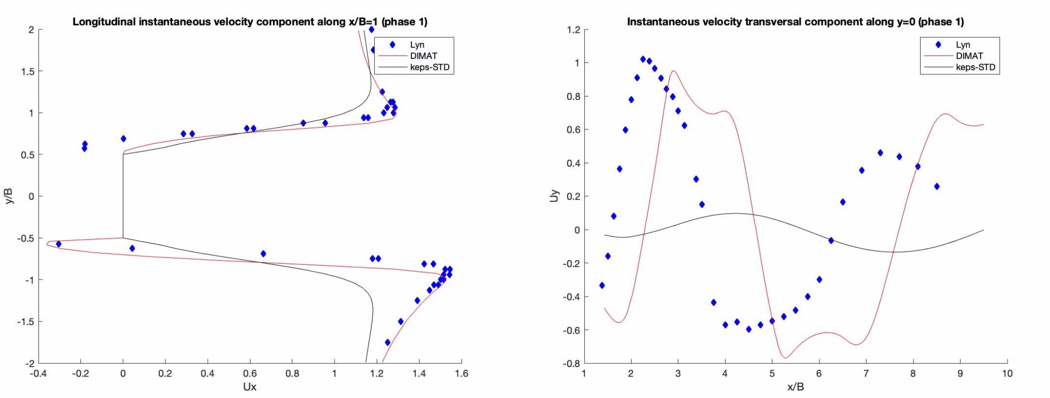

Figure 5: *Left panel:* phase 1 longitudinal instantaneous velocity component along $x/B = 1$. *Right panel:* phase 1 transverse instantaneous velocity component along $y = 0$.

5. Conclusion

In conclusion, we can say that the STD model has some critical issues. First of all the overestimation of the turbulent kinetic energy production, which generates too high turbulent diffusion on the mean flow. This has serious effects on the simulation, which is not sufficiently precise and provides a too approximate picture of the flow. We can conclude that a non-stationary flow with high Reynolds number, characterized by large separation zones and areas of retrograde flow, approached with the STD model gives poor results. In these cases, like the one just investigated, the periodic motion resulting is enormously underestimated. Since, in our case, the computational time of the RNG simulation is similar to the STD one and its results are consistent with the bibliographic data, the RNG approach is definitely a better choice. In fact, the different derivation of this method allows to correct the excessive production of turbulent kinetic energy in the stagnation points, which is the main problem of the $k - \epsilon$ STD model. So, although the RNG model still exhibit some critical aspects, typical of a 2D $k - \epsilon$ model, this approach gives way better results.

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