

# Introduction to Structural Equation Modeling

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University of Trento*



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Lectures for the *PhD course in Cognitive Science*  
Department of Psychology and Cognitive Science  
University of Trento  
January 11 / 12 / 18 / 19 / 26, 2024



# Who am I?

## *Academic*

**Research Fellow (RTD-a)**

Work and Organizational Psychology (M-PSI/06)

*July 1 2021 – ongoing*

**Postdoctoral Researcher – University of Trento**

Work and Organizational Psychology (M-PSI/06)

*Sept 15, 2018 – June 30, 2021*

## *Education*

**Ph.D. – Sapienza University of Rome**

Personality and Organizational Psychology

*Feb 2018*

**M.S. – University of Bologna**

Clinical Psychology

*Feb 2013*

**B.S. – University of L'Aquila**

Psychology

*Oct 2010*

# Who am I?

## *Research Areas*

1. Organizational Psychology, Personality Psychology, Psychometrics
2. Educational Psychology and Individual Differences

## *Research Topics*

1. Social desirability scales
2. Individual differences in personality from childhood (moral disengagement, effortful control) to Adulthood (big five, positivity)
3. Methodological and theoretical issues in the study of self-esteem
4. Personality factors and organizational outcomes (e.g., Well-Being at work)
5. Methodological issues in organizational psychology and management
6. Non-cognitive skills in junior high school students

# Who am I?

## *Research Projects and Practical Applications*

1. Guardia di Finanza (Sapienza University of Rome)
2. Mobbing (Autonomous Province of Trento and Univ of Trento)
3. Nudging and Unemployment (Autonomous Province of Trento and Univ of Trento)
4. Non-cognitive skills in Junior high school students (Fondazione Caritro and Univ of Trento)
5. Member of the WebeWo LAB -> Interdisciplinary lab (Law, Psychology, Sociology) that aims to study interventions, antecedents, and outcomes of well-being at work
6. PRIN PNRR 2022 – P.I. of the project «*Loneliness, non-cognitive skills, and academic achievement in Junior High School: Assessment and intervention based on Big Data, longitudinal, and intensive methods*»

*More info at*

<https://webapps.unitn.it/du/it/Persona/PER0209265/Curriculum>

# Who Are You?

*....please, introduce your*

- *name*
- *background*
- *skills*
- *acquired knowledge*
- *expectations about this course..... and the PhD in general*

*This is useful to start creating a collaborative, positive, and multidisciplinary organizational climate...*



# Schedule

- 11 January 2024 – Aula 1
- 12 “ “ – “
- 18 “ “ – “
- 19 “ “ – “
- 26 “ “ – Aula 11 (*Project Presentation*)

Your project for this PhD course will involve one of the following options:

- A theoretical or practical argument focusing on a Structural Equation Modeling (SEM) technique (e.g., mediation with SEM, Longitudinal SEM, Latent Growth Modeling, etc.)
- The examination and discussion of one or more articles that report SEM findings.
- Data analysis using *Mplus* or R. You have the flexibility to choose whether to analyze your own dataset, an open-access dataset, or a simulated dataset.
- Hypothesize a SEM pipeline – from Model Formulation to Model Evaluation – accompanied by hypothetical codes (in R or *Mplus*)

Let's start our journey...



# The need for Latent Variable Models

## Problems with *Measurement*

The focus of Psychology is the study of human behaviors and human cognitive processes. Thus, it is difficult to think at tools capable to assess them.



# The need for Latent Variable Models

## Problems with *Measurement*

In 1940 a committee established by the British Association for the Advancement of Science concluded that measurements taken in psychological methods could not be considered “scientific measurements”.



# The need for Latent Variable Models

Solutions for improving *Measurement*

Stevens (1946)

***MEASUREMENT***  
is defined as the  
assignment of numerals  
to objects or events  
according to rules



## On the Theory of Scales of Measurement

S. S. Stevens  
*Director, Psycho-Acoustic Laboratory, Harvard University*

FOR SEVEN YEARS A COMMITTEE of the British Association for the Advancement of Science debated the problem of measurement. Appointed in 1932 to represent Section A (Mathematical and Physical Sciences) and Section J (Psychology), the committee was instructed to consider and report upon the possibility of "quantitative estimates of sensory events"—meaning simply: Is it possible to measure human sensation? Deliberation led only to disagreement, mainly about what is meant by the term measurement. An interim report in 1938

by the formal (mathematical) properties of the scales. Furthermore—and this is of great concern to several of the sciences—the statistical manipulations that can legitimately be applied to empirical data depend upon the type of scale against which the data are ordered.

**A CLASSIFICATION OF SCALES OF MEASUREMENT**  
Paraphrasing N. R. Campbell (Final Report, p. 340), we may say that measurement, in the broadest sense, is defined as the assignment of numerals to objects or events according to rules. The fact that



# The need for Latent Variable Models

## Solutions for improving *Measurement*

Stevens (1946)

### SCALES

*Nominal*

*Ordinal*

*Interval*

*Ratio*

TABLE 1

Scale	Basic Empirical Operations	Mathematical Group Structure	Permissible Statistics (invariantive)
NOMINAL	Determination of equality	<i>Permutation group</i> $x' = f(x)$ $f(x)$ means any one-to-one substitution	Number of cases Mode Contingency correlation
ORDINAL	Determination of greater or less	<i>Isotonic group</i> $x' = f(x)$ $f(x)$ means any monotonic increasing function	Median Percentiles
INTERVAL	Determination of equality of intervals or differences	<i>General linear group</i> $x' = ax + b$	Mean Standard deviation Rank-order correlation Product-moment correlation
RATIO	Determination of equality of ratios	<i>Similarity group</i> $x' = ax$	Coefficient of variation



# The need for Latent Variable Models

Solutions for improving *Measurement*

Self-report questionnaires, Other-report questionnaires, Objective measures, ecc.

Two pivotal concepts:

RELIABILITY → Precision of a measurement ( $X_i = T_i + E_i$ )

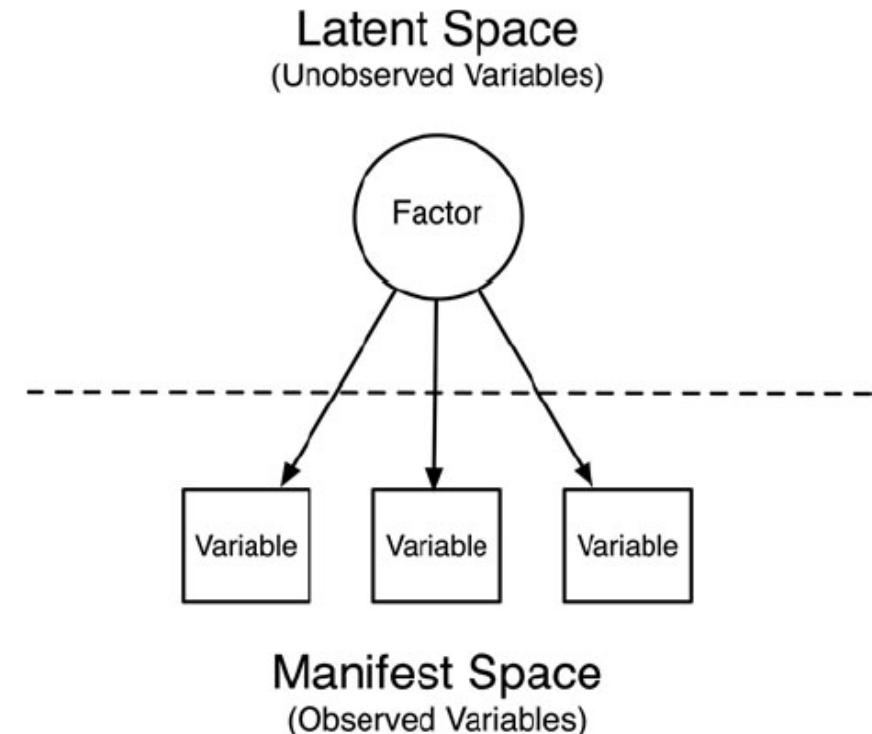
VALIDITY → The instrument is a proper tool for measuring what it aims to measure.

An instrument may be reliable, but not valid (e.g., I cannot use a “reliable” ruler for measuring intelligence)

# The need for Latent Variable Models

## Solutions for improving *Measurement*

The focus on statistical methods has been pivotal for the advancement of the concept of measurement in Psychology



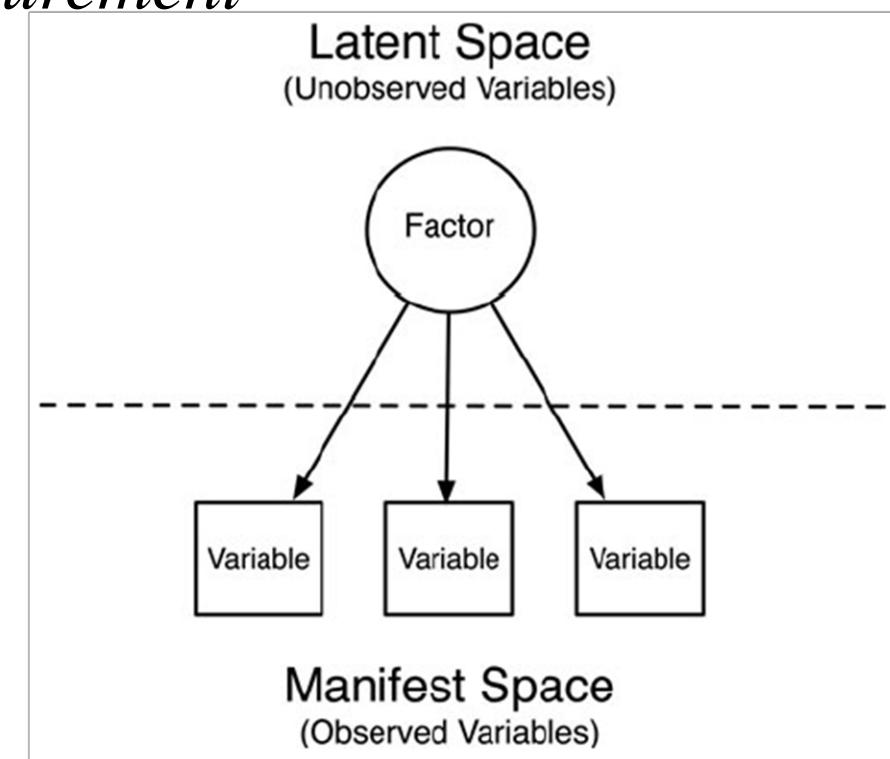
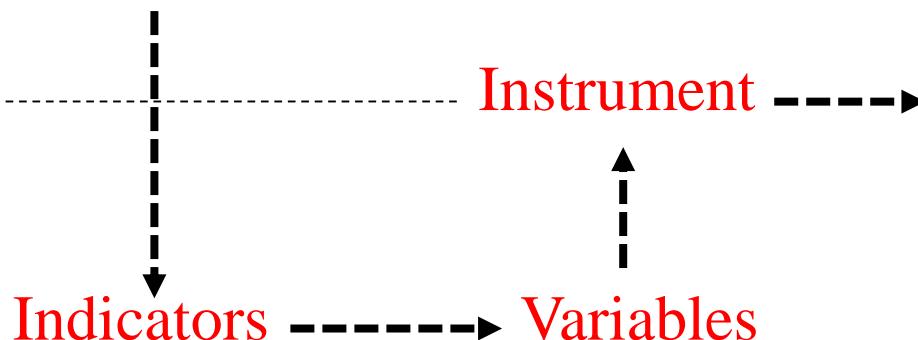
From Nesselroade & Molenaar (2016)

# The need for Latent Variable Models

Solutions for improving *Measurement*

**Operationalization Process**

**Construct**



## Operationalization Process: An Example

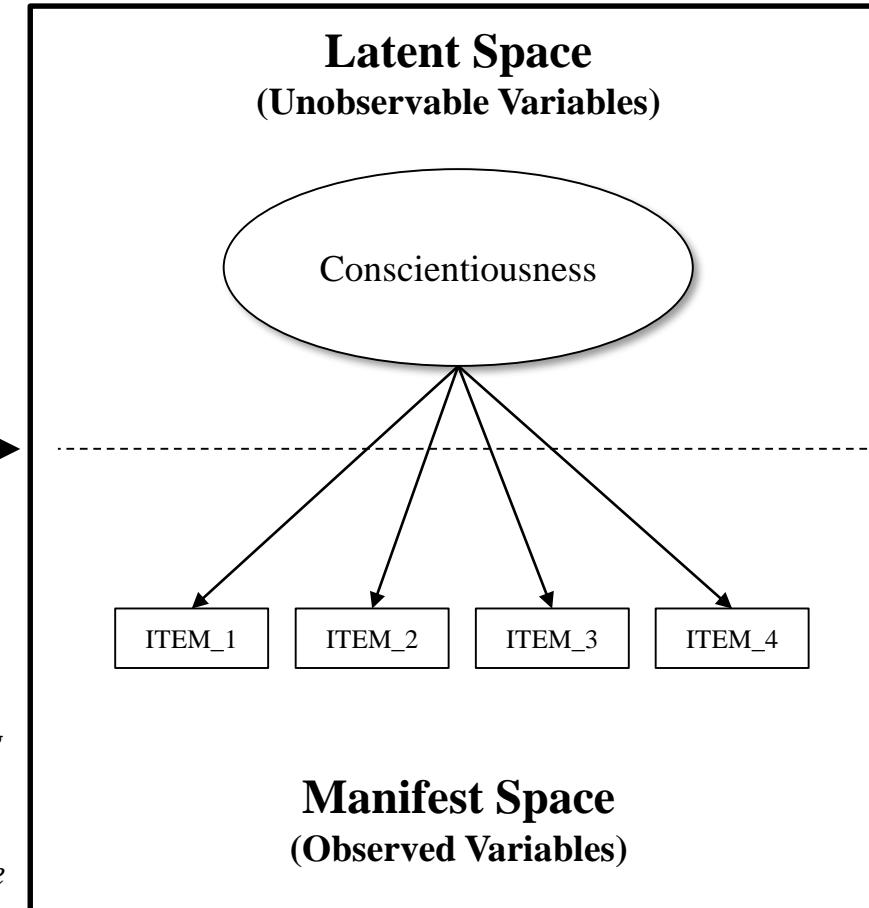
### Conscientiousness

“Scrupulousness”  
(orderliness and precision) and  
“Perseverance”  
(capability of fulfilling one’s own tasks and commitments)

### Big Five Questionnaire

(only selected items that refer to conscientiousness)

- ITEM\_1 *I tend to be very thoughtful*
- ITEM\_2 *Before completing a job I spend a lot of time revising it*
- ITEM\_3 *It's difficult for me to give up an activity I've undertaken*
- ITEM\_4 *If I fail in a task, I keep trying until I succeed*



# The need for Latent Variable Models

## Reliability: Consistency (precision) of a measure

### - Reliability (General formula)

$\rho_{xx} = [Var(T) / Var(T + E)]$ , with values ranging from 0 to 1

### - Cronbach's Alpha

$$X = Y_1 + Y_2 + Y_3 + \dots + Y_k$$

$$\alpha = \frac{K}{K-1} \left( 1 - \frac{\sum_{i=1}^K \sigma_{Y_i}^2}{\sigma_X^2} \right)$$

$K$  = number of components (e.g., items)

$\sigma_{Y_i}^2$  = variance of the component  $i$

$\sigma_X^2$  = variance of the observed total score

### - Omega total

$$\omega_{Total} = \frac{\left( \sum_{i=1}^k \lambda_i \right)^2}{\left( \sum_{i=1}^k \lambda_i \right)^2 + \sum_{i=1}^k \theta_{ii}}$$

$\lambda_i$  = factor loading of the component  $i$ ;  
a factor loading represents the strength of the relationship between an indicator and its latent variable

$\theta_{ii}$  = Residual variance of the component  $i$

### - AVE (Average Variance Extracted)

$$AVE = \frac{\left( \sum_{i=1}^k \lambda_i^2 \right)}{\left( \sum_{i=1}^k \lambda_i^2 \right) + \left( \sum_{i=1}^k \theta_{ii} \right)}$$

> .50



# The need for Latent Variable Models

**Validity: A test is valid if it measures what it purports to measure”** (Borsboom et al., 2004, p. 1061)

“The problem of validity cannot be solved by psychometric techniques or models alone. On the contrary, it must be addressed by substantive theory”

(Borsboom et al., 2004, p. 1062).

Psychological Review  
2004, Vol. 111, No. 4, 1061–1071

Copyright 2004 by the American Psychological Association  
0033-295X/04/\$12.00 DOI: 10.1037/0033-295X.111.4.1061

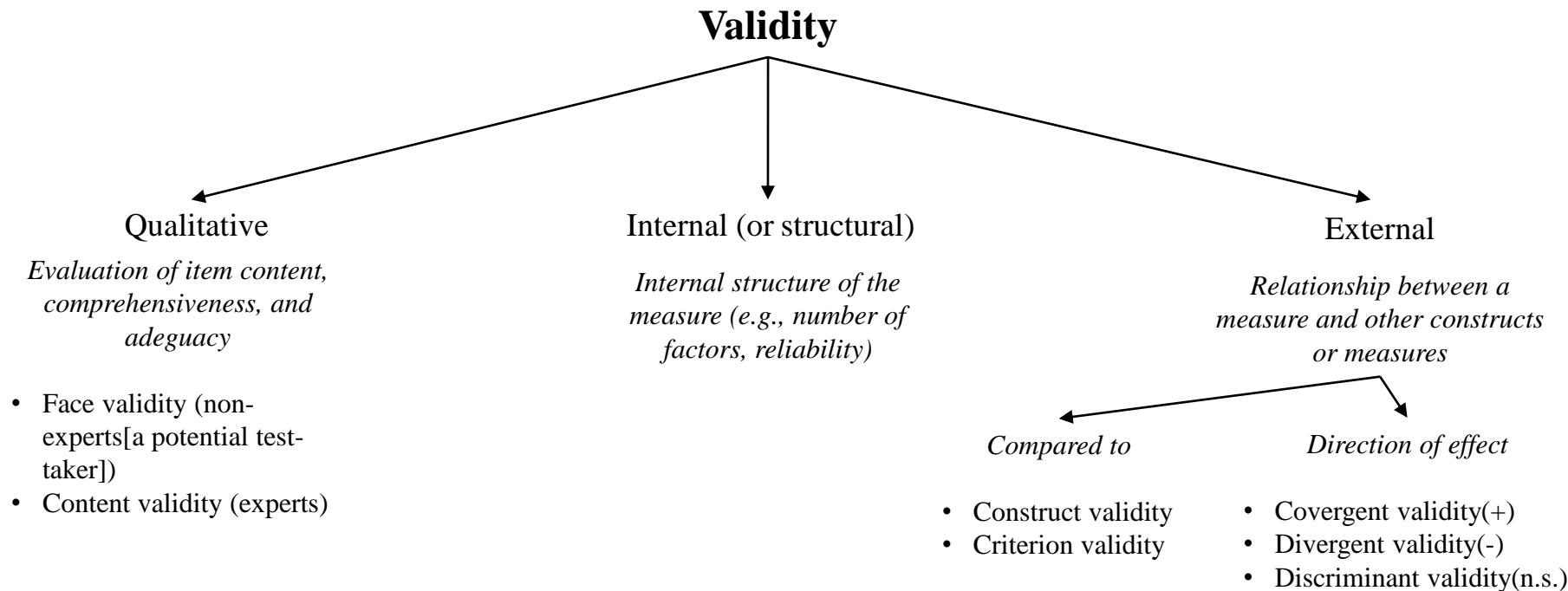
## The Concept of Validity

Denny Borsboom and Gideon J. Mellenbergh  
University of Amsterdam

Jaap van Heerden  
Maastricht University

# The need for Latent Variable Models

**Validity:** “A test is valid if it measures what it purports to measure” (Borsboom et al., 2004, p. 1061)



# The need for Latent Variable Models

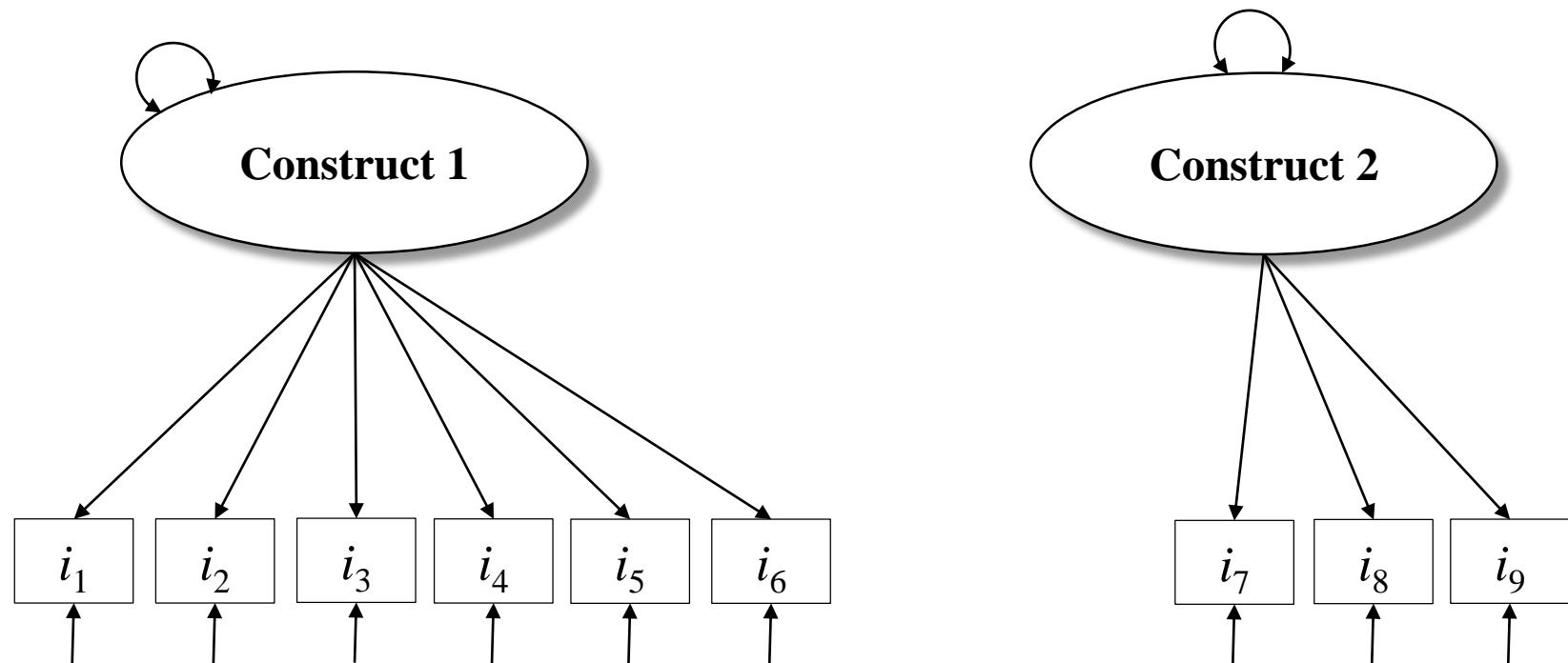
**Researcher's main tasks in Personality Psychology**

**Measuring constructs**  
(Exploratory Factorial Analysis; EFA)

**Connecting constructs**  
(Regression Models)

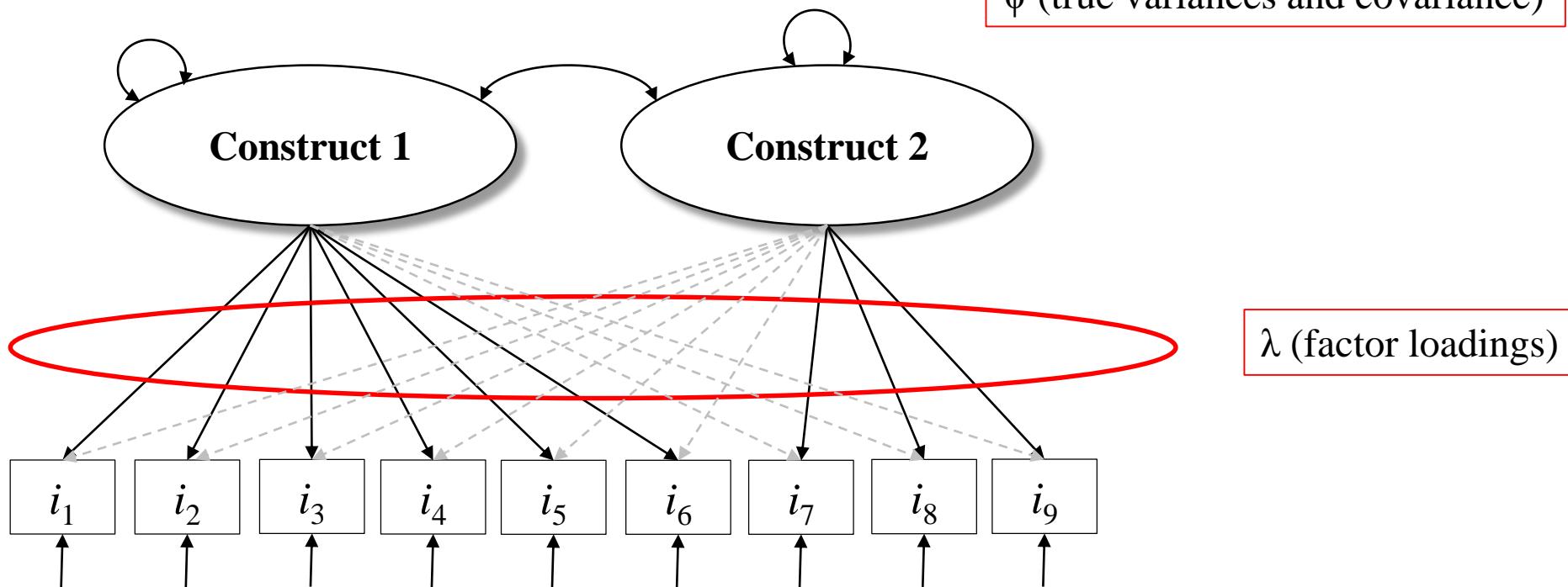
# The need for Latent Variable Models

## Measuring Constructs



# The need for Latent Variable Models

## Measuring Constructs (EFA)



Seeking Resources

1. I ask other feedback on my performance
2. I ask colleagues for advice [I ask other educational superintendents for advice]
3. I ask my supervisor for advice [I ask other managers for advice (not educational superintendents)]
4. I try to learn new things at work
5. I contacted other people from work (e.g., colleagues, supervisors) to get the necessary information for completing my tasks
6. When I have difficulties or problems at my work, I discuss them with people from my work environment

Seeking Challenges

7. I ask for more tasks if I finish my work [I look for other projects to carry out, as soon as I conclude those planned]
8. I ask for more responsibilities
9. I ask for more odd jobs [I try to engage in projects/assignments that test myself]

The Likert scale ranged from 1 = *never* to 5 = *often*.

Hypothesized Construct	Item	1	2
Seeking Resources ( $\alpha = .71$ )	1	.34***	.19***
	2	.59***	-.10***
	3	.53***	.18***
	4	.46***	.13***
	5	.38***	-.09***
	6	.76***	-.06**
Seeking Challenges ( $\alpha = .83$ )	7	-.05*	.61***
	8	.09**	.79***
	9	.02 n.s.	.82***

*N* = 1060 superintendents

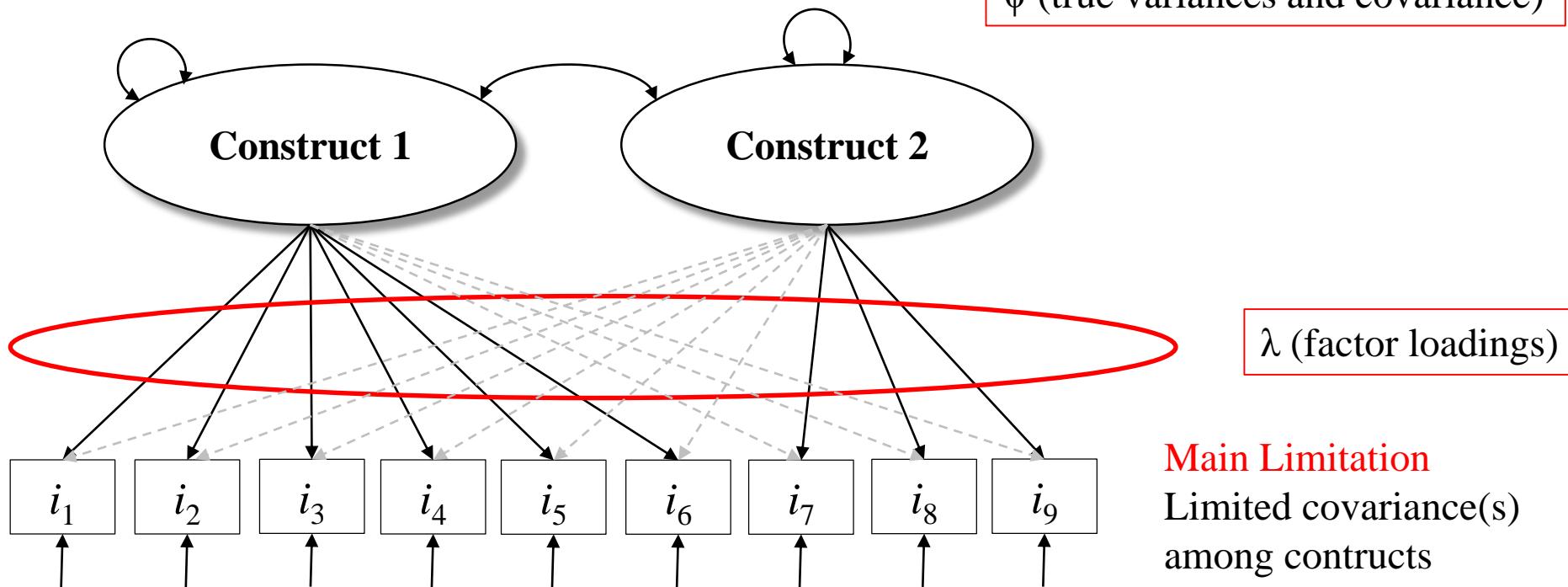
Latent Correlation = .40\*\*\*

Recommended thresholds:

- $|\text{Target } \lambda| > .30$
- $|\text{Non-target } \lambda| < .30$

# The need for Latent Variable Models

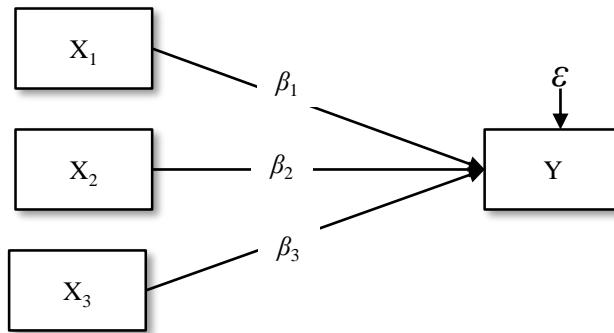
## Measuring Constructs (EFA)



# The need for Latent Variable Models

## Connecting Constructs (Regression)

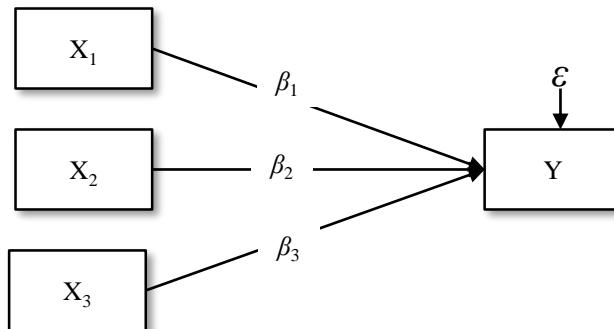
$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i$$



# The need for Latent Variable Models

## Connecting Constructs (Regression)

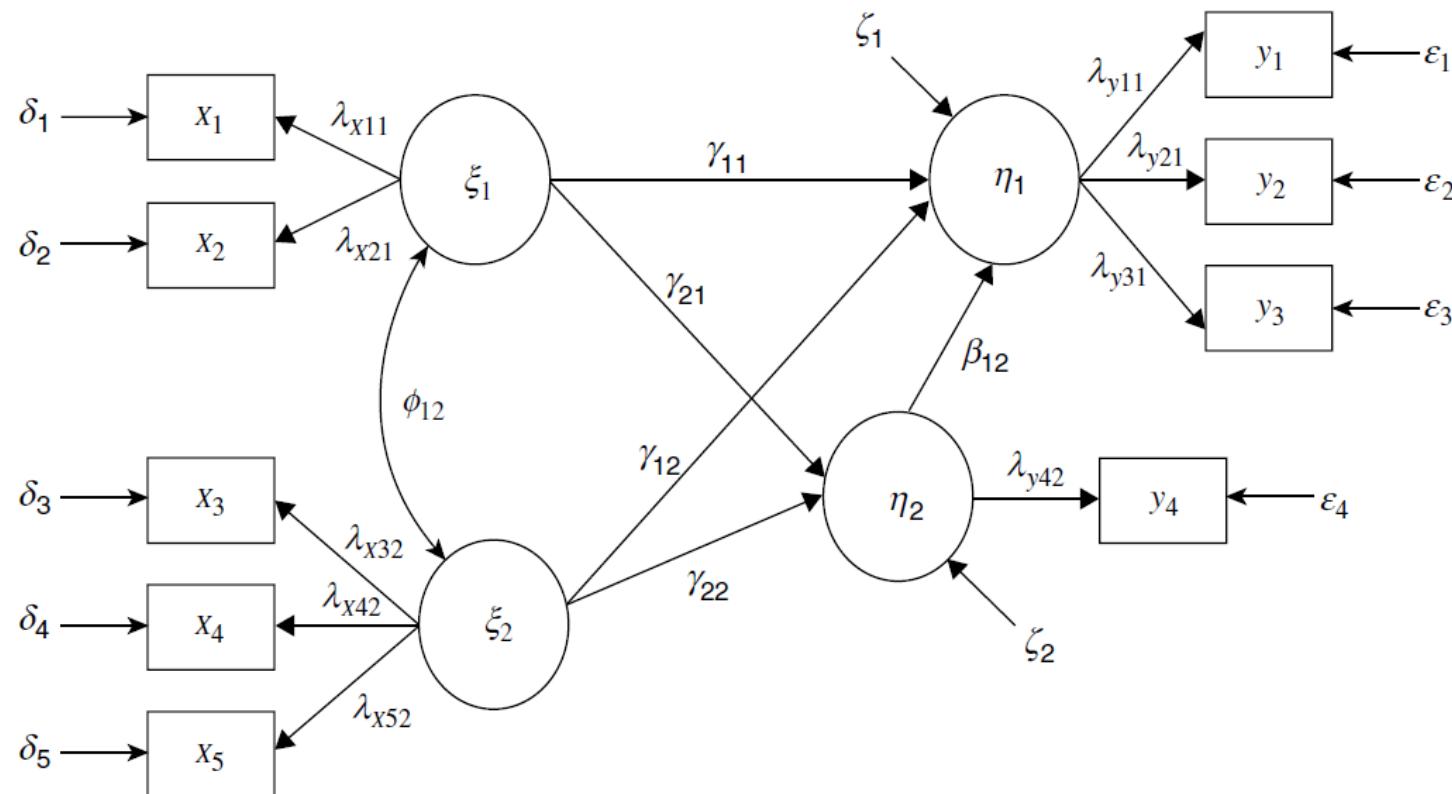
$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i$$



### Main Limitations

- 1) Don't take into account measurement error
- 2) One Dependent Variable

# Structural Equation Modeling (SEM)



**Figure 1.1** A hypothesized general structural equation model.

# History of SEM

SEM (also called **covariance structure analysis**, **covariance structure modeling**, or **analysis of covariance structures**) does not designate a single statistical technique but instead refers to a family of related procedures (Kline, 2016, p. 9).

In SEM, unobservable latent variables (constructs or factors) are estimated from observed indicator variables, and the focus is on estimation of the relations among the latent variables free of the influence of measurement errors.

**Measurement (Factor Analysis)**  
(Spearman, 1904; Tucker, 1955)

**Simultaneous Equations (Path Analysis)**  
(Wright, 1918, 1921, 1934)

**SEM**  
(Jöreskog, 1967, 1969, 1973; Keesling, 1972; Wiley, 1973)

**JKW Model**

Matsueda, R. L. (2012). Key advances in the history of Structural Equation Modeling. In R. H. Hoyle (Eds.), *Handbook of Structural Equation Modeling* (pp. 17-42). New York, NY: Guilford



# History of SEM

Matsueda (2012) argued that we can track **4 stages across SEM history**, each of which characterized by a strong reliance on insights from statistics

- (1) early disciplinary-specific developments of ***path analysis*** first from **genetics** and later **sociology**, ***factor analysis*** from **psychology**, and simultaneous equation models in **economics**;
- (2) cross-disciplinary fertilization between economics, sociology, and psychology, leading to an **explosion of empirical applications** of SEM;
- (3) a period of developing methods for handling **discrete, ordinal, and limited dependent variables**;
- (4) a recent period of incorporating **statistical advances** into the SEM framework, including generalized linear models, mixed effects models, **mixture** regression models, **Bayesian** methods, graphical models, and methods for identifying **causal effects**.

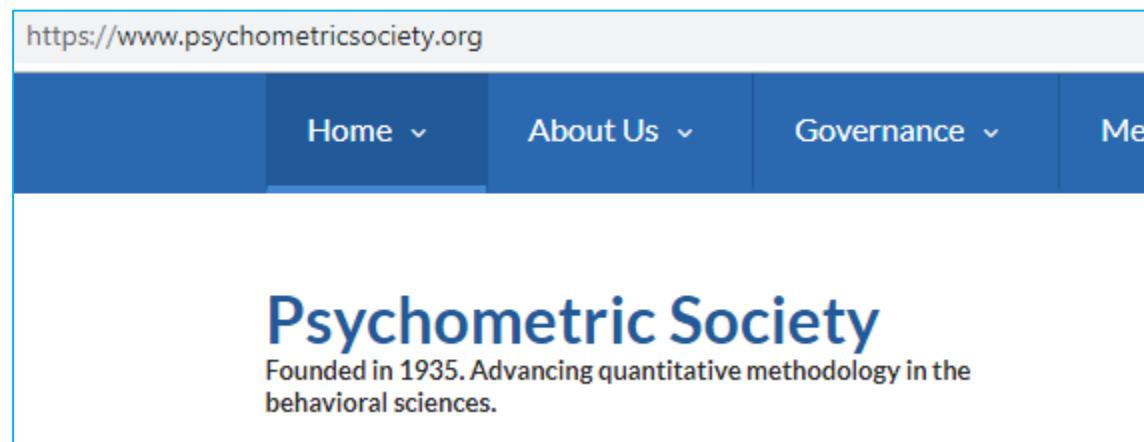
We are now approaching a fifth stage (e.g., Jacobucci & Grimm, 2020):

- (5) Latent variables ↔ data science, artificial intelligence (machine learning)



# SEM and Psychometrics

The study of quantitative measurement in psychology



## *Assessment*

### *INSTRUMENTS*

Self-reports, Other-reports, Objective measures

### *DESIGNS*

Cross-sectional, Longitudinal, Hierarchical

## *Data Analytic Strategy*

General Linear Models, Factor Analysis, **Structural Equation Modeling**, Cluster analysis

## *Simulation*

Monte Carlo Methods, Performance of estimators, Optimal sample size



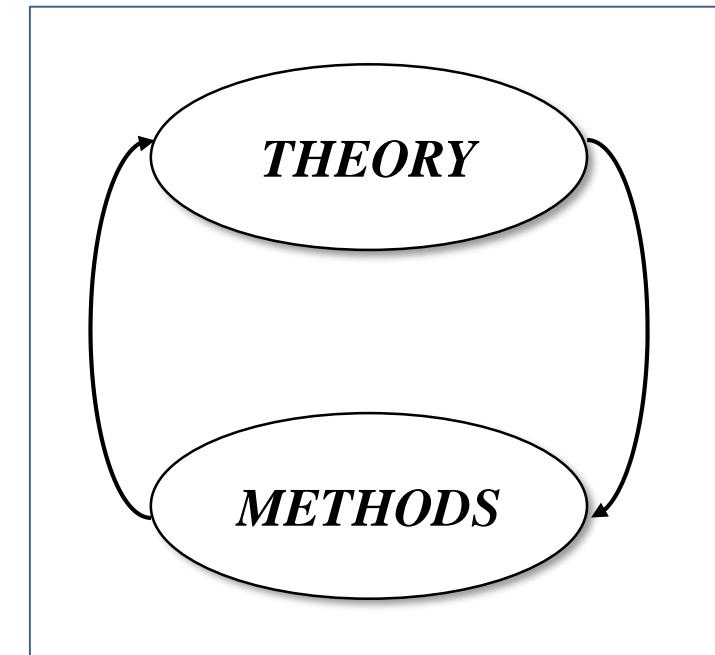
PSYCHOMETRIKA—VOL. 71, NO. 3, 425–440  
SEPTEMBER 2006  
DOI: 10.1007/s11336-006-1447-6

## THE ATTACK OF THE PSYCHOMETRICIANS

DENNY BORSBOOM

UNIVERSITY OF AMSTERDAM

This paper analyzes the theoretical, pragmatic, and substantive factors that have hampered the integration between psychology and psychometrics. Theoretical factors include the operationalist mode of thinking which is common throughout psychology, the dominance of classical test theory, and the use of “construct validity” as a catch-all category for a range of challenging psychometric problems. Pragmatic factors include the lack of interest in mathematically precise thinking in psychology, inadequate representation of psychometric modeling in major statistics programs, and insufficient mathematical training in the psychological curriculum. Substantive factors relate to the absence of psychological theories that are sufficiently strong to motivate the structure of psychometric models. Following the identification of these problems, a number of promising recent developments are discussed, and suggestions are made to further the integration of psychology and psychometrics.



“The founding fathers of the Psychometric Society—scholars such as Thurstone, Thorndike, Guilford, and Kelley—**were substantive psychologists as much as they were psychometricians**.

Contemporary psychometricians do not always display a comparable interest with respect to the substantive field that lends them their credibility.

It is perhaps worthwhile to emphasize that, even though psychometrics has benefited greatly from the input of mathematicians, **psychometrics is not a pure mathematical discipline but an applied one**. If one strips the application from an applied science one is not left with very much that is interesting; and **psychometrics without the “psycho” is not, in my view, an overly exciting discipline**. It is therefore **essential that a psychometrician keeps up to date with the developments in one or more subdisciplines of psychology**.” (Borsboom, 2006, p. 436)



# Structural Equation Modeling

## Steps

1. **Model formulation** Specifying the SEM model that the researcher wants to test. The model may be formulated on the basis of theory or empirical findings. A general SEM model is composed of two parts: the measurement model and the structural model.
2. **Model identification** It determines whether there is a unique solution for all the free parameters in the specified model.
3. **Model estimation** Estimate model parameters and generate fitting function. Various estimation methods are available for SEM (e.g., ML, WLS, Bayes, ecc.).
4. **Model evaluation** Assess whether the model fits the data.
5. **Model modification** If the model does not fit the data, re-specification or modification of the model is needed. In this instance, the researcher makes a decision regarding how to delete, add, or modify parameters in the model.

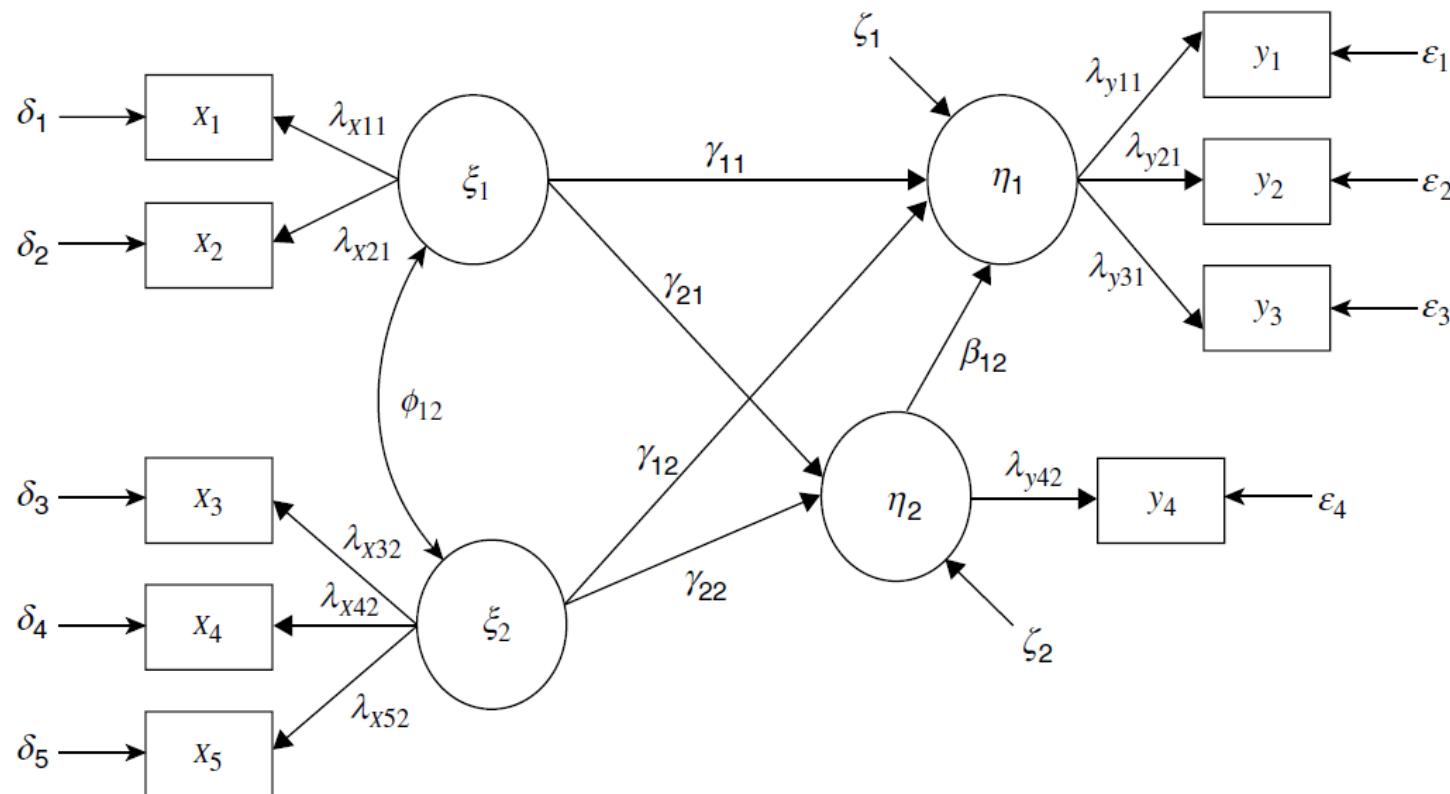


# Model Formulation

## Steps

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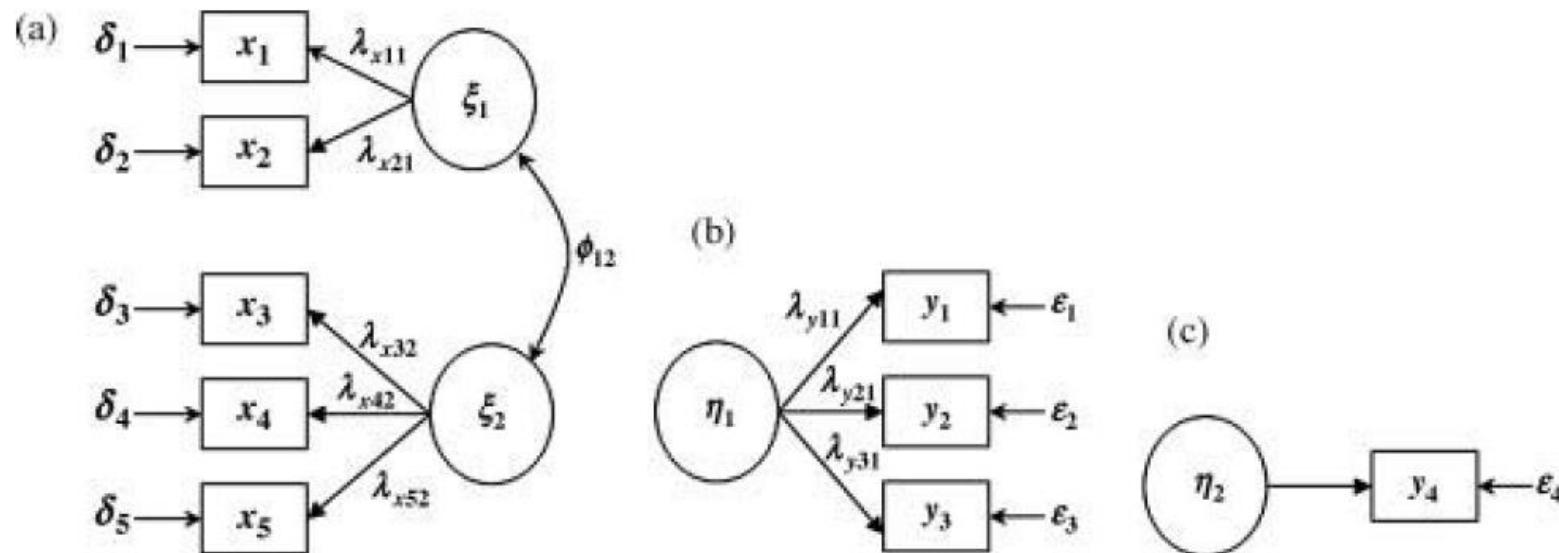
# Model Formulation



**Figure 1.1** A hypothesized general structural equation model.

# Model Formulation

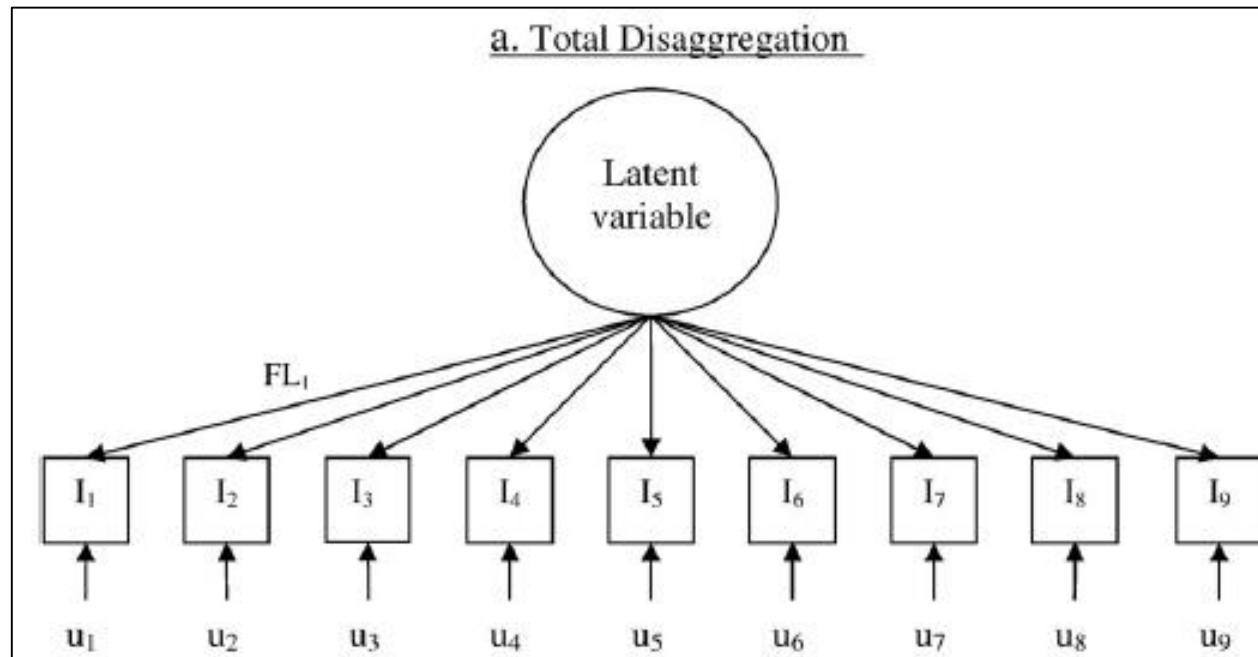
## *Measurement Models*



**Figure 1.2** (a) Measurement model 1. (b) Measurement model 2. (c) Measurement model 3.

# Model Formulation

## *Types of Measurement Models*

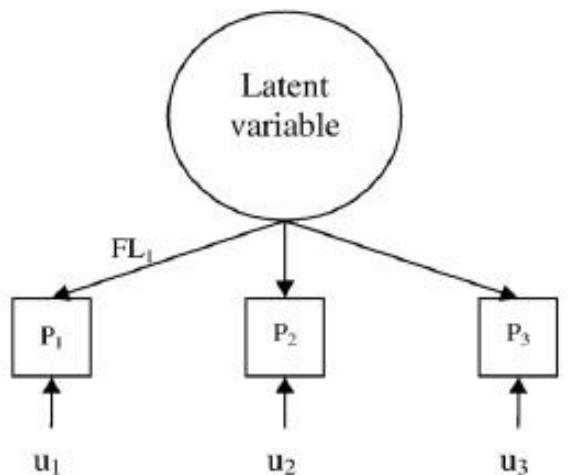


- *Single items*

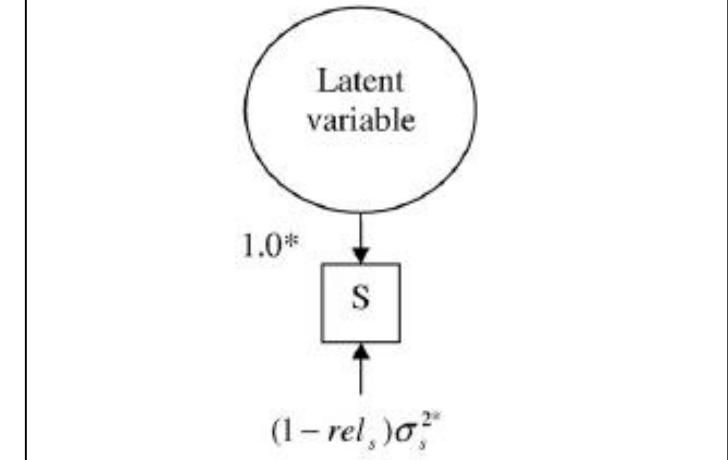
# Model Formulation

## *Types of Measurement Models*

b. Partial Disaggregation



c. Total Aggregation with Reliability Correction



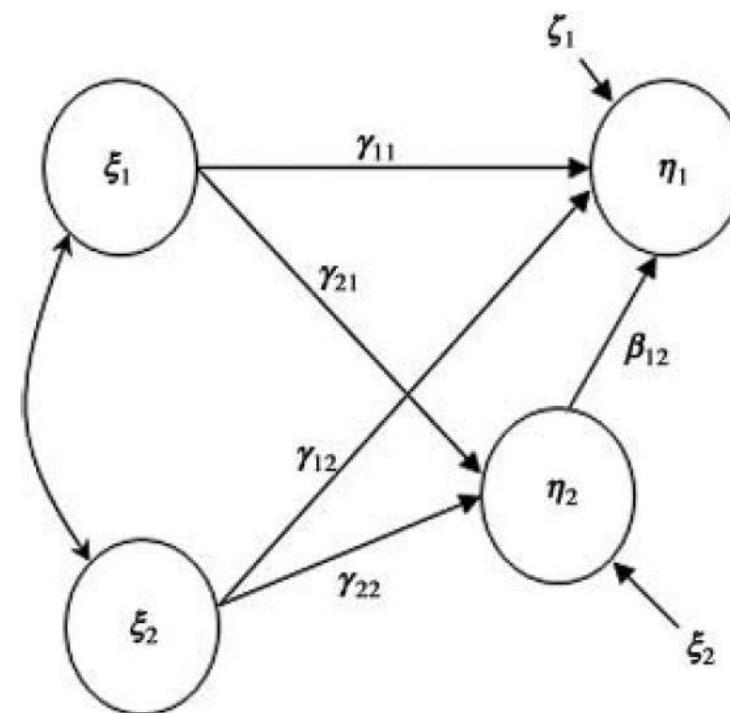
- *Parcel approach*
- *Domain representative approach*

- *Composite score*

# Model Formulation

## *Structural Model*

The term "structural" stands for the assumption that the parameters are not just descriptive measures of association but rather that they reveal an invariant "causal" relation.  
(Bollen, 1989, p. 4)



**Figure 1.3** Structural model.



# Model Formulation

## Variables

Variable	Definition	Dimension
$\eta$ (eta)	Latent endogenous variable	$m \times 1$
$\xi$ (xi)	Latent exogenous variable	$n \times 1$
$\zeta$ (zeta)	Residual term in equations	$m \times 1$
$y$	Endogenous indicators	$p \times 1$
$x$	Exogenous indicators	$q \times 1$
$\varepsilon$ (epsilon)	Measurement errors of $y$	$p \times 1$
$\delta$ (delta)	Measurement errors of $x$	$q \times 1$

*exogenous* = independent

*endogenous* = dependent (but can also be predictors!!)

$n$  = latent exogenous variables ( $\xi$ )  
 $m$  = latent endogenous variables ( $\eta$ )  
 $q$  = exogenous indicators ( $x$ )  
 $p$  = endogenous indicators ( $y$ )



# Model Formulation

## *Matrices*

Matrix	Definition	Dimension
Coefficient matrices		
$\Lambda_y$ (lambda y)	Factor loadings relating $y$ to $\eta$	$p \times m$
$\Lambda_x$ (lambda x)	Factor loadings relating $x$ to $\xi$	$q \times n$
$B$ (beta)	Coefficient matrix relating $\eta$ to $\eta$	$m \times m$
$\Gamma$ (gamma)	Coefficient matrix relating $\xi$ to $\eta$	$m \times n$
Variance/covariance matrices		
$\Phi$ (phi)	Variance/covariance matrices of $\xi$	$n \times n$
$\Psi$ (psi)	Variance/covariance matrices of $\zeta$	$m \times m$
$\Theta_\epsilon$ (theta-epsilon)	Variance/covariance matrices of $\epsilon$	$p \times p$
$\Theta_\delta$ (theta-delta)	Variance/covariance matrices of $\delta$	$q \times q$

# Model Formulation

## *Equations*

Structural Model

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}$$

Measurement Models

$$\mathbf{Y} = \boldsymbol{\Lambda}_y \boldsymbol{\eta} + \boldsymbol{\varepsilon}$$

$$\mathbf{X} = \boldsymbol{\Lambda}_x \boldsymbol{\xi} + \boldsymbol{\delta}$$



# Model Formulation

## *Equations*

Structural Model

$$\eta = B\eta + \Gamma\xi + \zeta$$

Measurement Models

$$Y = \Lambda_y\eta + \epsilon$$

$$X = \Lambda_x\xi + \delta$$

$$\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} 0 & \beta_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}$$

$$\eta_{1i} = \beta_{12} \eta_{2i} + \gamma_{11} \xi_{1i} + \gamma_{12} \xi_{2i} + \zeta_{1i}$$

$$\eta_{2i} = \gamma_{21} \xi_{1i} + \gamma_{22} \xi_{2i} + \zeta_{2i}$$



# Model Formulation

## *Equations*

Structural Model

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}$$

Measurement Models

$$\mathbf{Y} = \boldsymbol{\Lambda}_y \boldsymbol{\eta} + \boldsymbol{\varepsilon}$$

$$\mathbf{X} = \boldsymbol{\Lambda}_x \boldsymbol{\xi} + \boldsymbol{\delta}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_{y21} & 0 \\ \lambda_{y31} & 0 \\ 0 & \lambda_{y42} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}$$

$$y_{1i} = \eta_{1i} + \varepsilon_{1i}$$

$$y_{2i} = \lambda_{y21} \eta_{1i} + \varepsilon_{2i}$$

$$y_{3i} = \lambda_{y31} \eta_{1i} + \varepsilon_{3i}$$

$$y_{4i} = \lambda_{y42} \eta_{2i} + \varepsilon_{4i}$$

(Wang & Wang, 2012)

# Model Formulation

## *Equations*

Structural Model

$$\eta = B\eta + \Gamma\xi + \zeta$$

Measurement Models

$$Y = \Lambda_y\eta + \epsilon$$

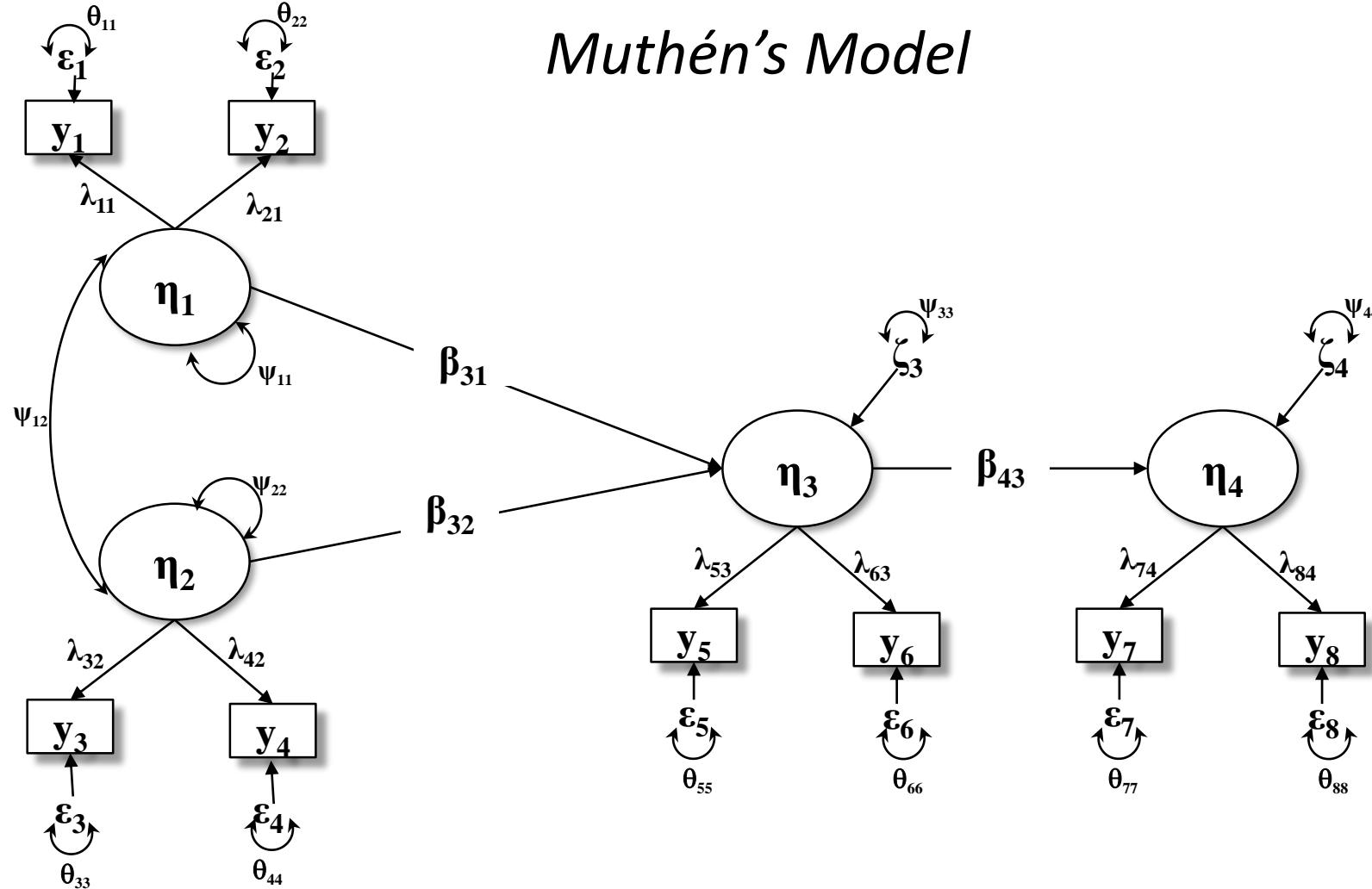
$$X = \Lambda_x\xi + \delta$$

$$\begin{aligned}x_{1i} &= \xi_{1i} + \delta_{1i} \\x_{2i} &= \lambda_{x21} \xi_{1i} + \delta_{2i} \\x_{3i} &= \xi_{1i} + \delta_{3i} \\x_{4i} &= \lambda_{x42} \xi_{2i} + \delta_{4i} \\x_{5i} &= \lambda_{x52} \xi_{2i} + \delta_{5i}\end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & & & \\ \lambda_{x21} & 0 & & & \\ 0 & 1 & & & \\ 0 & & \lambda_{x42} & & \\ 0 & & & \lambda_{x52} & \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{bmatrix}$$

# Model Formulation

*Muthén's Model*





# Model Formulation

## *Muthén's Model*

Structural Model

$$\eta = B\eta + F\xi + \zeta$$

Measurement Models

$$Y = \Lambda_y \eta + \epsilon$$

$$X = A_x \xi + \delta$$

Structural Model

$$\eta = B\eta + \zeta$$

Measurement Model

$$Y = \Lambda\eta + \epsilon$$

By adding intercepts/means

Structural Model

$$\eta = v + B\eta + \zeta$$

Measurement Models

$$Y = a + \Lambda\eta + \epsilon$$



# Model Formulation

## *Exercise*

- Draw a SEM or find it in a publication
- Specify which are the measurement models
- Specify the “type/s” of measurement model (i.e., total disaggregation, partial disaggregation, total aggregation with reliability correction)
- Specify what is the structural model

# Model Identification

## Steps

1. **Model formulation** Specifying the SEM model that the researcher wants to test. The model may be formulated on the basis of theory or empirical findings. A general SEM model is composed of two parts: the measurement model and the structural model.
2. **Model identification** It determines whether there is a unique solution for all the free parameters in the specified model.
3. **Model estimation** Estimate model parameters and generate fitting function. Various estimation methods are available for SEM (e.g., ML, WLS, Bayes, ecc.).
4. **Model evaluation** Assess whether the model fits the data.
5. **Model modification** If the model does not fit the data, re-specification or modification of the model is needed. In this instance, the researcher makes a decision regarding how to delete, add, or modify parameters in the model.



# Model Identification

## *Parameters*

Model identification concerns whether a unique value for each and every unknown parameter can be estimated from the observed data (Wang & Wang, 2012, p. 11).

Parameters can be:

1. FIXED: forced to assume a specific value, such as 0 or 1
2. CONSTRAINED: to be equal to another parameter
3. FREE: estimated by the model



# Model Identification

## *Degrees of Freedom*

$$df = [p(p + 1)]/2 - \text{NFP} > 0$$

$p$  = number of observed variables, NFP = Number of free parameters

	1.	2.	3.	4.	5.	6.	7.	8.
1. $y_1$	Var( $y_1$ )							
2. $y_2$	Cov( $y_1 y_2$ )	Var( $y_2$ )						
3. $y_3$	Cov( $y_1 y_3$ )	Cov( $y_2 y_3$ )	Var( $y_3$ )					
4. $y_4$	Cov( $y_1 y_4$ )	Cov( $y_2 y_4$ )	Cov( $y_3 y_4$ )	Var( $y_4$ )				
5. $y_5$	Cov( $y_1 y_5$ )	Cov( $y_2 y_5$ )	Cov( $y_3 y_5$ )	Cov( $y_4 y_5$ )	Var( $y_5$ )			
6. $y_6$	Cov( $y_1 y_6$ )	Cov( $y_2 y_6$ )	Cov( $y_3 y_6$ )	Cov( $y_4 y_6$ )	Cov( $y_5 y_6$ )	Var( $y_6$ )		
7. $y_7$	Cov( $y_1 y_7$ )	Cov( $y_2 y_7$ )	Cov( $y_3 y_7$ )	Cov( $y_4 y_7$ )	Cov( $y_5 y_7$ )	Cov( $y_6 y_7$ )	Var( $y_7$ )	
8. $y_8$	Cov( $y_1 y_8$ )	Cov( $y_2 y_8$ )	Cov( $y_3 y_8$ )	Cov( $y_4 y_8$ )	Cov( $y_5 y_8$ )	Cov( $y_6 y_8$ )	Cov( $y_7 y_8$ )	Var( $y_8$ )

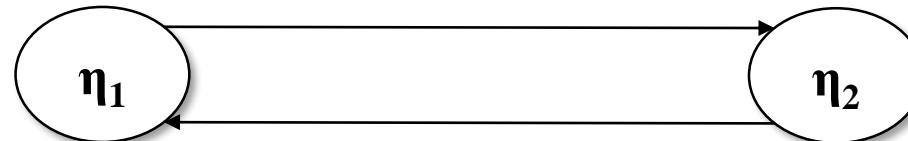
e.g. with 8 observed variables ( $p$ ) we have  $(8*9)/2 = 36$  non-redundant elements. Thus, NFP could be up to 35.

# Model Identification

## *Other assumptions*

There are a number of other assumptions for ensuring identification (see Bollen, 1989), but the most important is that the model must be **recursive**.

An example of **nonrecursive** model is a model with loops (Rigdon, 1995):





# Model Identification

## *Exercise*

On your previous SEM (or another one)

- Compute the maximum number of possible free parameters
- Identify some fixed and free parameters (and constrained, if there are)



# Model Estimation

## Steps

1. **Model formulation** Specifying the SEM model that the researcher wants to test. The model may be formulated on the basis of theory or empirical findings. A general SEM model is composed of two parts: the measurement model and the structural model.
2. **Model identification** It determines whether there is a unique solution for all the free parameters in the specified model.
3. **Model estimation** Estimate model parameters and generate fitting function. Various estimation methods are available for SEM (e.g., ML, WLS, Bayes, ecc.).
4. **Model evaluation** Assess whether the model fits the data.
5. **Model modification** If the model does not fit the data, re-specification or modification of the model is needed. In this instance, the researcher makes a decision regarding how to delete, add, or modify parameters in the model.

# Model Estimation

**Estimation of SEM models is different from that of multiple regressions.**

Instead of minimizing the discrepancies between the fitted and observed values of the response variable [i.e.,  $\Sigma(y - \hat{y})$ ], SEM estimation procedures minimize the residuals that are differences between the **sample variances/covariances** and the **variances/covariances** estimated from the model. (Wang & Wang, 2012, p. 14).

$$\Sigma = \Sigma(\theta)$$

Given that the **population variances/covariances matrix ( $\Sigma$ )** is unknown, it is replaced by the **sample variances/covariances matrix ( $S$ )**, so that the null hypothesis is

$$S = \Sigma(\theta)$$



# Model Estimation

There are several estimators, but the most common procedure is the use of **Maximum Likelihood function**

$$F_{ML} = \log|\Sigma(\theta)| + \text{tr}(\mathbf{S}\Sigma^{-1}(\theta)) - \log|\mathbf{S}| - (p + q) \quad (4.67)$$

(Bollen, 1989, p. 107)

$(N - 1)F_{ML}$  is chi-squared distributed with  $df = \{[p(p + 1)]/2 - \text{NFP}\}$ , where  $N$  is the sample size and  $F_{ML}$  is the value of the fitting function evaluated at the final estimates (Bollen, 1989, p. 110).

Recall that the null hypothesis of this chi-square test is  $H_0: \mathbf{S} = \Sigma(\theta)$ . Thus, a good model should show a **non-significant chi-square test**, because we expect **non-significant differences** between  $\mathbf{S}$  and  $\Sigma(\theta)$



# Model Estimation

$$\log L_0 = - \frac{N-1}{2} \{ \log |\hat{\Sigma}| + \text{tr}(\hat{\Sigma}^{-1} \mathbf{S}) \} \quad (7.43)$$

$$\begin{aligned} \log L_1 &= - \frac{(N-1)}{2} \{ \log |\mathbf{S}| + \text{tr}(\mathbf{S}^{-1} \mathbf{S}) \} \\ &= - \frac{(N-1)}{2} \{ \log |\mathbf{S}| + q \} \end{aligned} \quad (7.47)$$

$$-2 \log \left( \frac{L_0}{L_1} \right) = -2 \log L_0 + 2 \log L_1$$

# Model Estimation

*Example from an Mplus Output*

```
THE MODEL ESTIMATION TERMINATED NORMALLY
```

```
MODEL FIT INFORMATION
```

```
Number of Free Parameters
```

116

```
Loglikelihood
```

H0 Value

-5249.015

Log-Likelihood for my specified model ( $\log L_0$ )

H1 Value

-4935.988

```
Information Criteria
```

Akaike (AIC)

10730.030

Bayesian (BIC)

11167.155

Sample-Size Adjusted BIC

$(n^* = (n + 2) / 24)$

10799.223

Log-Likelihood for the saturated model ( $\log L_1$ ),  
namely for the model in which  $S = \Sigma(\theta)$

```
Chi-Square Test of Model Fit
```

Value

626.054

```
> (-2*-5249.015)+(2*(-4935.988))  
[1] 626.054
```

Degrees of Freedom

397

P-Value

0.0000

Model from Perinelli, Alessandri, Cepale, & Fraccaroli (2022)



# Model Estimation

## *Exercise*

Compute both chi-square statistics and degrees of freedom using the following information. **Pay attention!** Add the vector of means (for simplicity, again *p*) to the computation of degrees of freedom, that is  $\{ \{[p * (p + 1)] / 2\} + p \} - NFP$

Number of observed variables	9
Number of Free Parameters	41

Loglikelihood

H0 Value	-1406.522
H1 Value	-1397.180



# Model Estimation

## *Exercise*

Compute both chi-square statistics and degrees of freedom using the following information. **Pay attention!** Add the vector of means (for simplicity, again *p*) to the computation of degrees of freedom, that is  $\{ \{[p*(p + 1)]/2\} + p \} - NFP$

Number of observed variables	9
Number of Free Parameters	41

Loglikelihood

H0 Value	-1406.522
H1 Value	-1397.180

## Solution

Chi-square =  $(-2*-1406.522) + (2*-1397.180) = 18.684$   
df =  $((9*10)/2) + 9 - 41 = 13$



# Model Evaluation

## Steps

1. **Model formulation** Specifying the SEM model that the researcher wants to test. The model may be formulated on the basis of theory or empirical findings. A general SEM model is composed of two parts: the measurement model and the structural model.
2. **Model identification** It determines whether there is a unique solution for all the free parameters in the specified model.
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4. **Model evaluation** Assess whether the model fits the data.
5. **Model modification** If the model does not fit the data, re-specification or modification of the model is needed. In this instance, the researcher makes a decision regarding how to delete, add, or modify parameters in the model.



# Model Evaluation

$\chi^2$  test is highly sensitive to sample size and to the number of variables included in the model.

“As such, the significance of the  $\chi^2$  test should not be a reason by itself to reject a model” (Wang & Wang, 2012, p. 18).

# Model Evaluation

## *Other Common Indices of Fit*

$$CFI = \frac{d_{null} - d_{specified}}{d_{null}} > .90-.95$$

$d$  = noncentrality parameter =  $\chi^2 - df$   
 $null$  = the worst model (all cov. are zero)

$$TLI = \frac{\left( \frac{\chi^2_{null}}{df_{null}} - \frac{\chi^2_{specified}}{df_{specified}} \right)}{\left( \frac{\chi^2_{null}}{df_{null}} - 1 \right)} > .90-.95$$

$$RMSEA = \sqrt{\frac{\chi^2 - df}{(n-1)df}} < .08 - .05$$

$$SRMR = \sqrt{\frac{\sum_{i=1}^p \sum_{j=1}^i \left( \frac{s_{ij} - \hat{s}_{ij}}{s_{ii}s_{jj}} \right)^2}{p(p+1)/2}} = \sqrt{\frac{\text{sum of squared std residuals}}{\text{size var-covar matrix}}} < .08 - .05$$



# Model Evaluation

## *Nested models*

Definition:

- Models “are *nested* [when] one model is produced by changing the status of one or more parameters in the other” (Hoyle, 2012, p. 11).
- “*Nested* models are those for which the parameter space associated with one model is a subset of the parameter space associated with the other model” (Merkle, You, & Preacher, 2016, p. 152-153)
- “Two models are nested if they are of the same form, but the free parameters of one model are a subset of the free parameters of the other model” (Hoyle, 2012, p. 141)

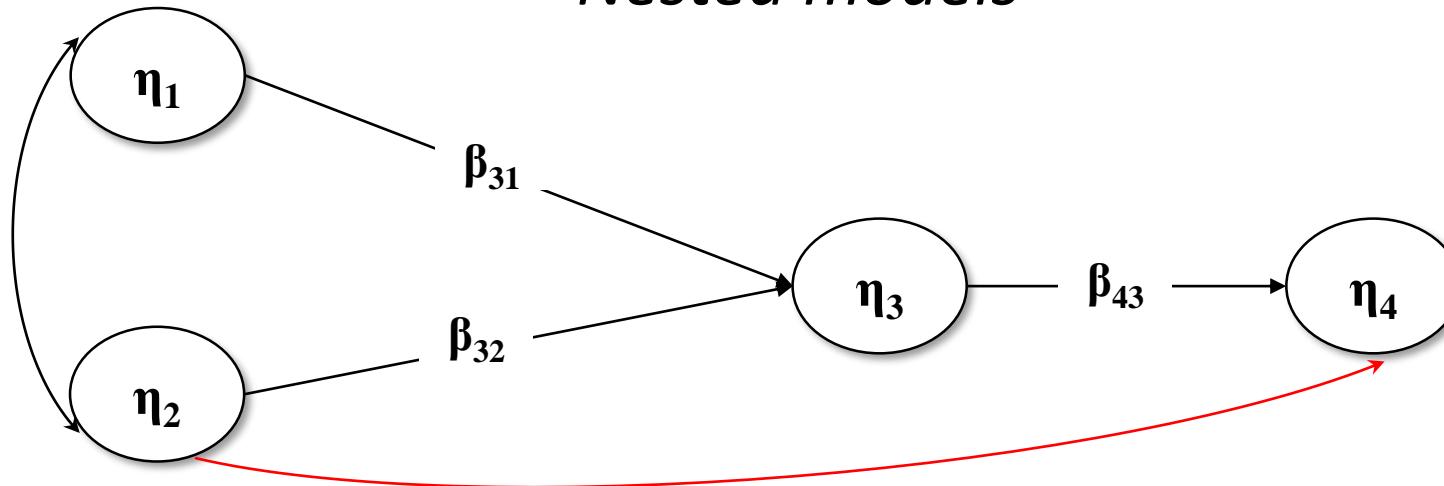
Nested models can be compared by means of  $\Delta\chi^2$  statistics, distributed with  $\Delta df$ .

The resulting  $p$ -value will attest

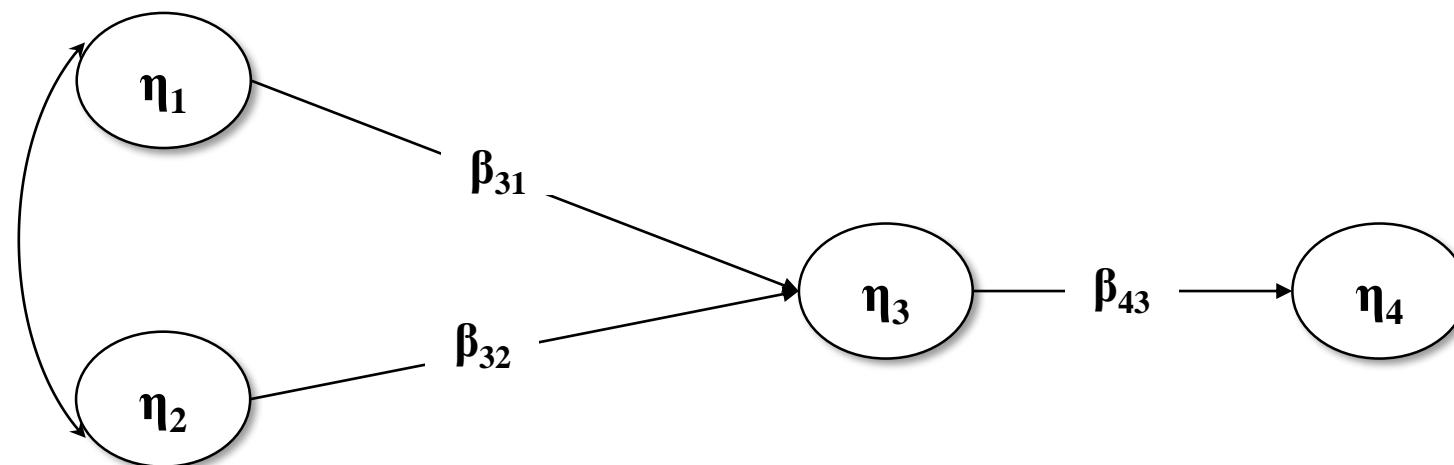
- whether removing 1 or more parameters significantly worsen (or not) the fit of the model
- whether adding 1 or more parameters significantly improve the fit of the model

# Model Evaluation

*Nested models*



$\Delta df = 1$



# Model Evaluation

## *Non-Nested models*

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MERKLE, YOU, AND PREACHER

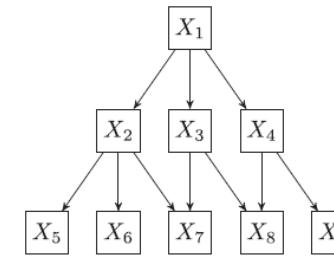
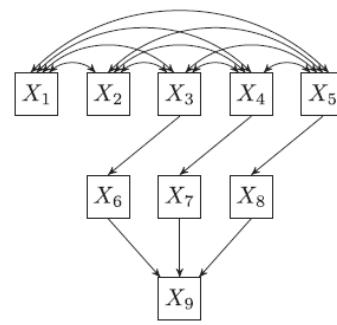
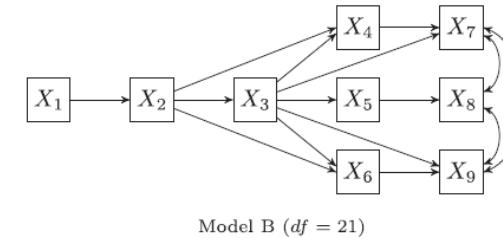
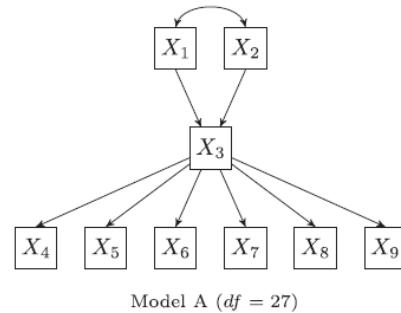


Figure 2. Path diagrams reflecting the models used in Simulation 2.



# Model Evaluation

## *Information Criteria Indices*

Useful for comparing **non-nested model**

$$AIC = -2\ln(L) + 2q$$

$$BIC = -2\ln(L) + q\ln(N)$$

$$ABIC = -2\ln(L) + q\ln[(N+2)/24]$$

$\ln(L)$  = log-likelihood value

$q$  = number of free parameters

$N$  = sample size

The model with the **smaller values** in these indices is preferred.

**IMPORTANT: YOU CANNOT USE THESE INDICES WHEN THE  $\log L_1$  IS DIFFERENT ACROSS MODELS**

# Model Evaluation

## *Exercise: Fit and Nested Model Comparison*

**Table 2.** Model fits of the predicted and alternative model tests.

	$\chi^2$	df	$p$	CFI	TLI	RMSEA	SRMR	AIC	MC	$\Delta\chi^2$	$\Delta df$	$p$
M1: No constraint	12.794	9	.172	0.991	0.967	0.066	0.022	2897.154				
M2: CS → OI fixed to be zero	13.819	11	.243	0.993	0.980	0.052	0.027	2894.179	M2 Vs M1	1.025	2	0.599
<b>M3: PD → OI fixed to be zero</b>	<b>18.684</b>	<b>13</b>	<b>.133</b>	<b>0.986</b>	<b>0.966</b>	<b>0.067</b>	<b>0.041</b>	<b>2895.044</b>	<b>M3 Vs M2</b>	<b>4.865</b>	<b>2</b>	<b>0.088</b>
M4 PD → CS fixed to be zero	29.094	15	.016	0.966	0.926	0.099	0.073	2901.454	M4 Vs M3	10.410	2	0.005

Note: CS = Colleague Support; OI = Organizational Identification, PD = Psychological Distress. CFI = Comparative Fit Index; TLI = Tucker-Lewis Index; RMSEA = Root Mean Square Error of Approximation; SRMR = Standardized Root Mean Square Residual; MC = Model Comparison. M1 = no constraint were imposed; M2 = the paths from colleague support T1 to organizational identification T2 and from colleague support T2 to organizational identification T3 were fixed to be zero; M3 = the paths from psychological distress T1 to organizational identification T2 and from psychological distress T2 to organizational identification T3 were fixed to be zero; M4 = the paths from psychological distress T1 to colleague support T2 and from psychological distress T2 to colleague support T3 were fixed to be zero.

# Model Evaluation

## *Exercise: Fit and Non-Nested Model Comparison*

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**Table 3** Indices of hypothesized models (see Fig. 2) and alternative models ( $N = 109$ )

Model	Free Parameters	$\chi^2(df)$	CFI	TLI	RMSEA[90% CI]	SRMR	AIC
Hyp. mod.	21	10.10 (6) <sup>n.s.</sup>	.974	.934	.080[.000, .163]	.037	3085.58
Alt. mod. 1	23	6.55 (4) <sup>n.s.</sup>	.984	.939	.077[.000, .179]	.026	3086.03
Alt. mod. 2	22	8.98 (5) <sup>n.s.</sup>	.975	.924	.086[.000, .176]	.038	3086.47
Alt. mod. 3	21	12.9 (6) <sup>*</sup>	.955	.888	.104[.017, .183]	.048	3088.45

See paragraph “Alternative Models” in Results section for the description of each alternative model

All the models had the same maximum log-likelihood value for the unrestricted (H1) model, thus they are comparable with AIC

*Hyp. mod.* hypothesized model (see Fig. 2), *Alt. mod.* alternative model, *df* degrees of freedom, *CFI* Comparative Fit Index, *TLI* Tucker-Lewis Index, *RMSEA* Root Mean Square Error of Approximation, *CI* Confidence interval, *SRMR* Standardized Root Mean Square Residual, *AIC* Akaike Information Criterion

<sup>n.s.</sup>  $p > .10$ , <sup>\*</sup>  $p < .05$



# Model Modification

## Steps

1. **Model formulation** Specifying the SEM model that the researcher wants to test. The model may be formulated on the basis of theory or empirical findings. A general SEM model is composed of two parts: the measurement model and the structural model.
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# Model Modification

“If the model does not fit the data, re-specification or modification of the model is needed. In this instance, the researcher makes a decision regarding how to delete, add, or modify parameters in the model” (Wang & Wang, 2012, p. 2).

**Modification Indices (MI):** Change in  $\chi^2$  (i.e.,  $\Delta\chi^2$ ) after freeing a parameter

Usually, MI are provided in the output.

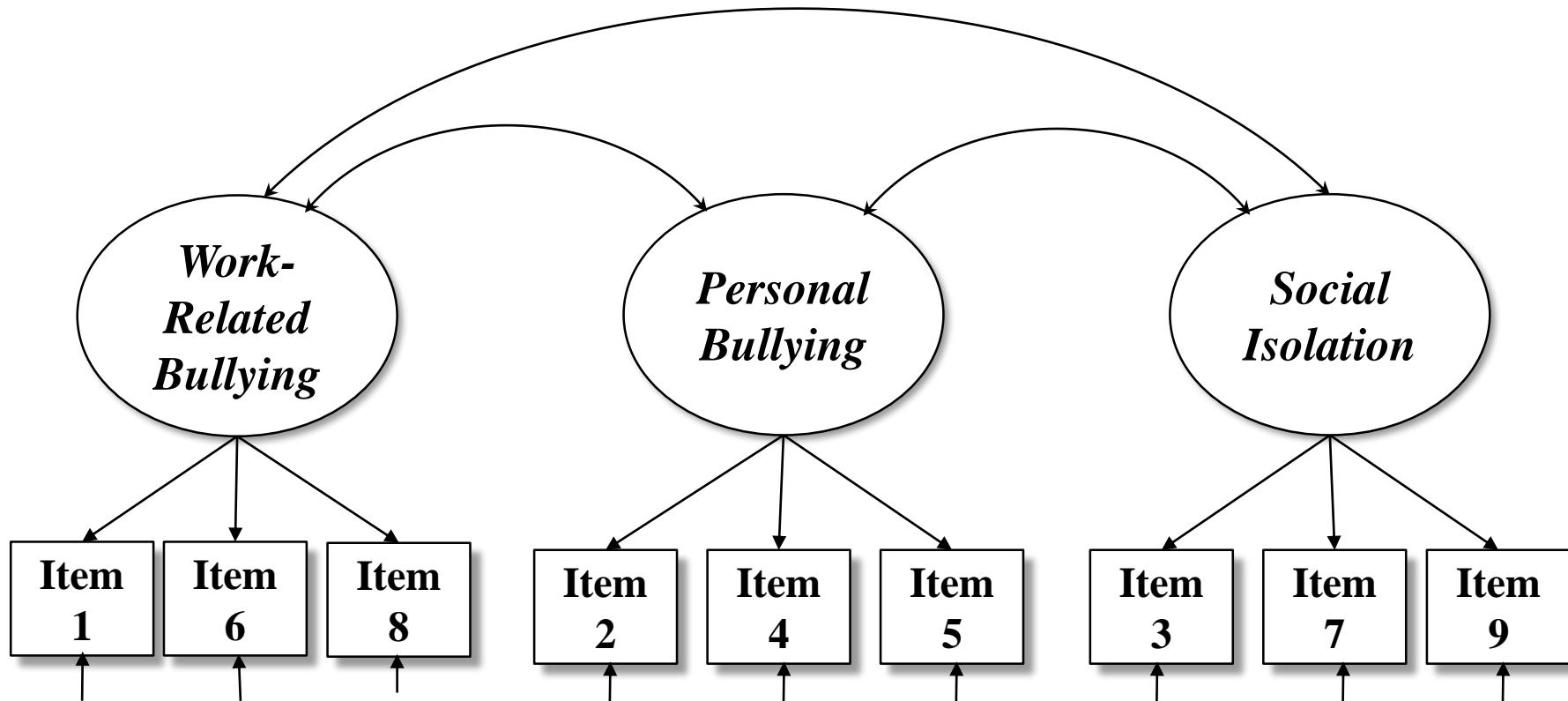
Use MI with caution: They must be **justified** from a theoretical and methodological point of view



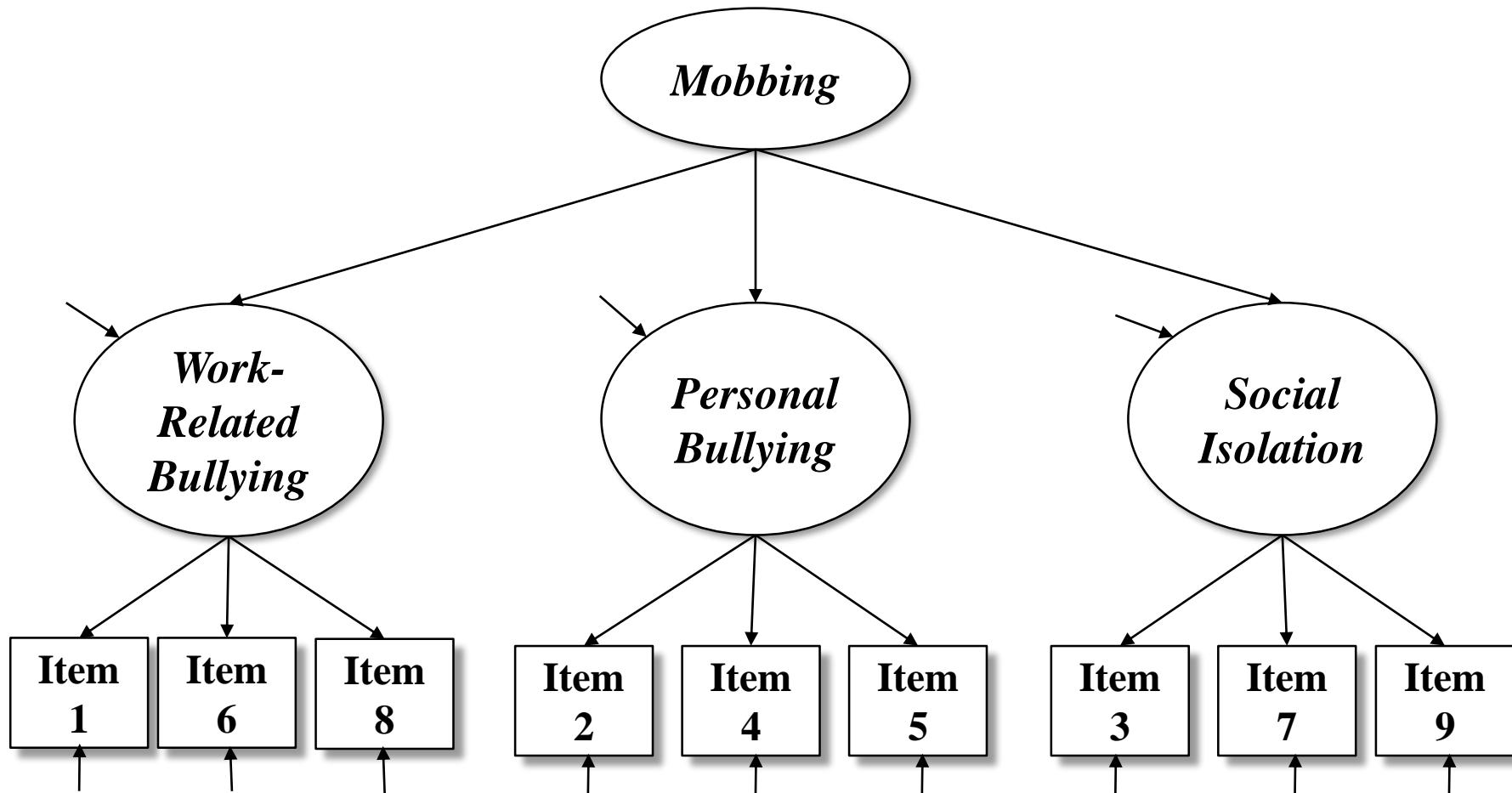
# Structural Equation Modeling

*Some Types and Applications*

# Confirmatory Factor Analysis (CFA)

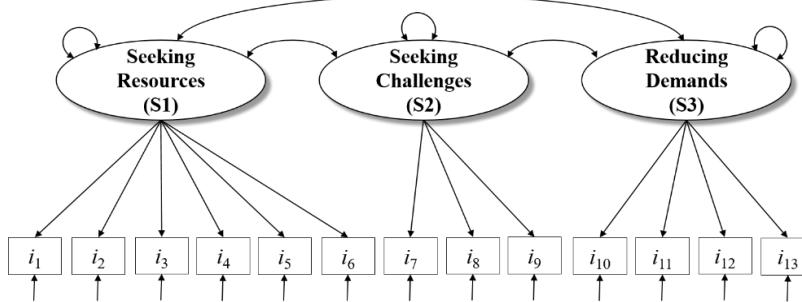


# Higher-order (or Second-order) CFA

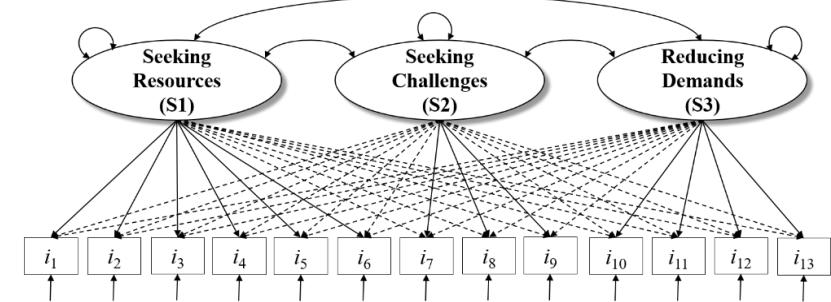


# Multidimensional Structure

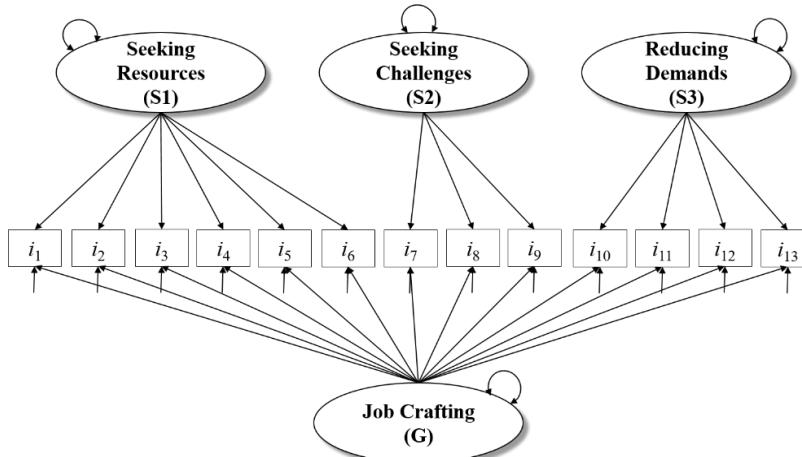
ICM-CFA



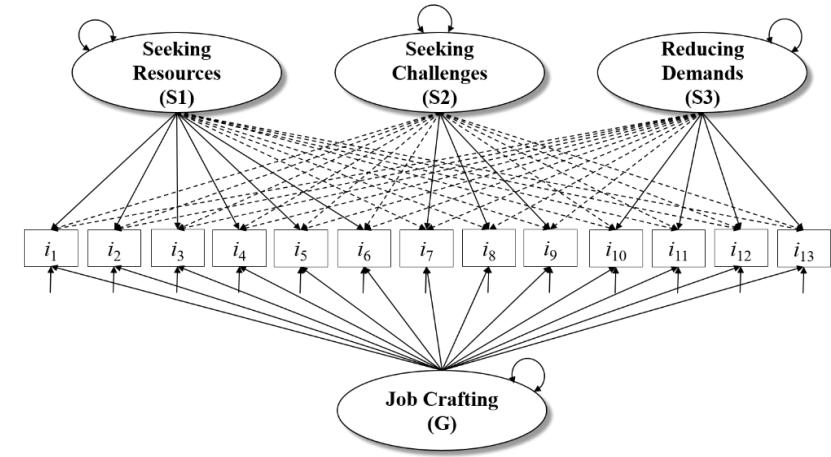
ESEM



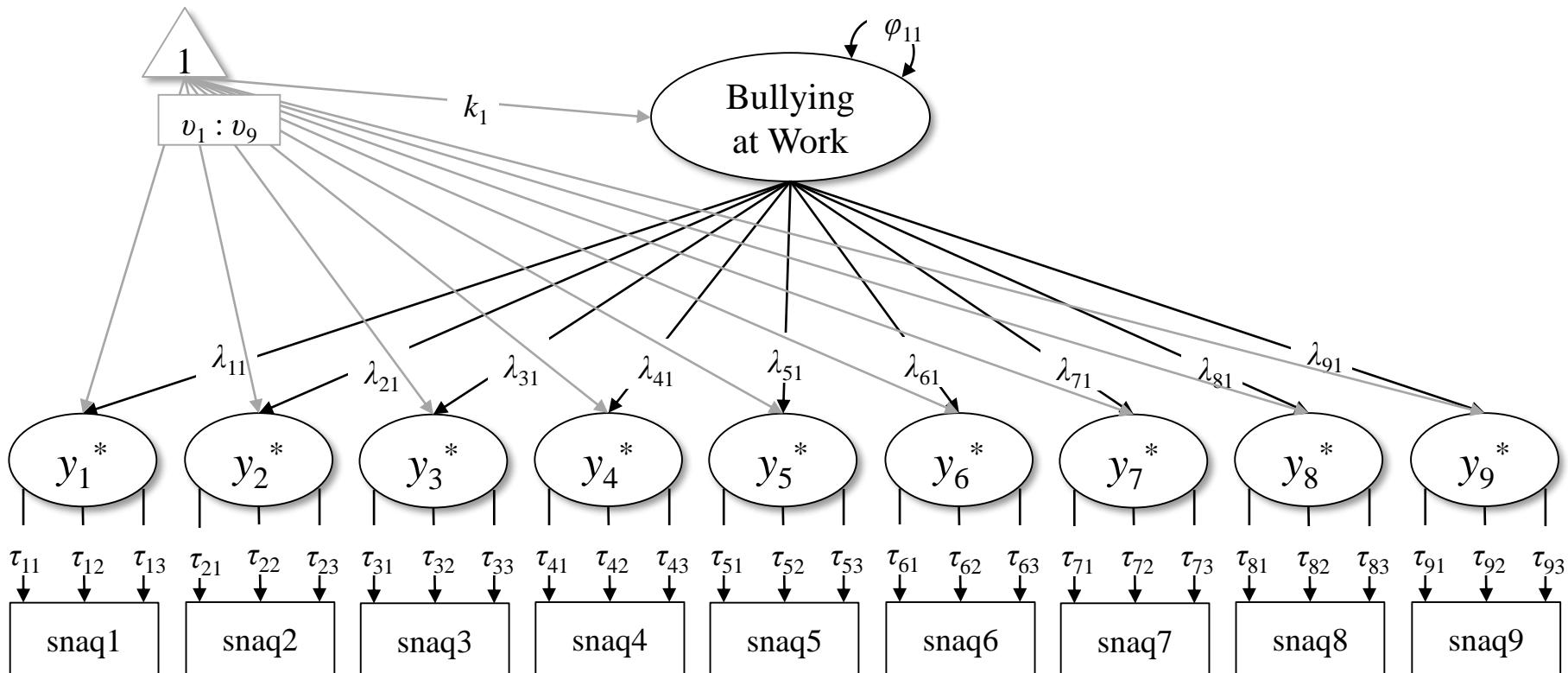
B-CFA



B-ESEM



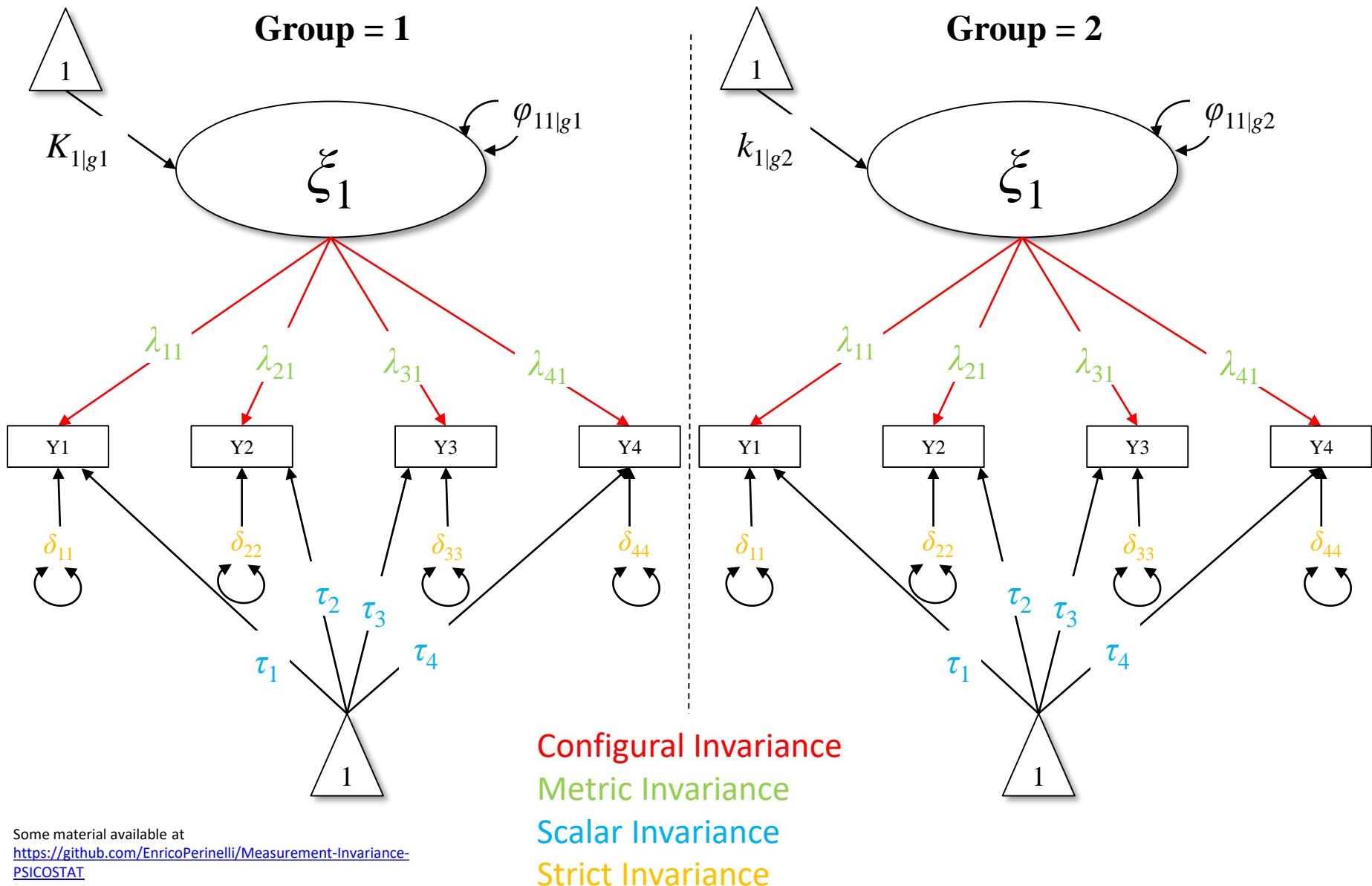
# CFA with Ordinal Data



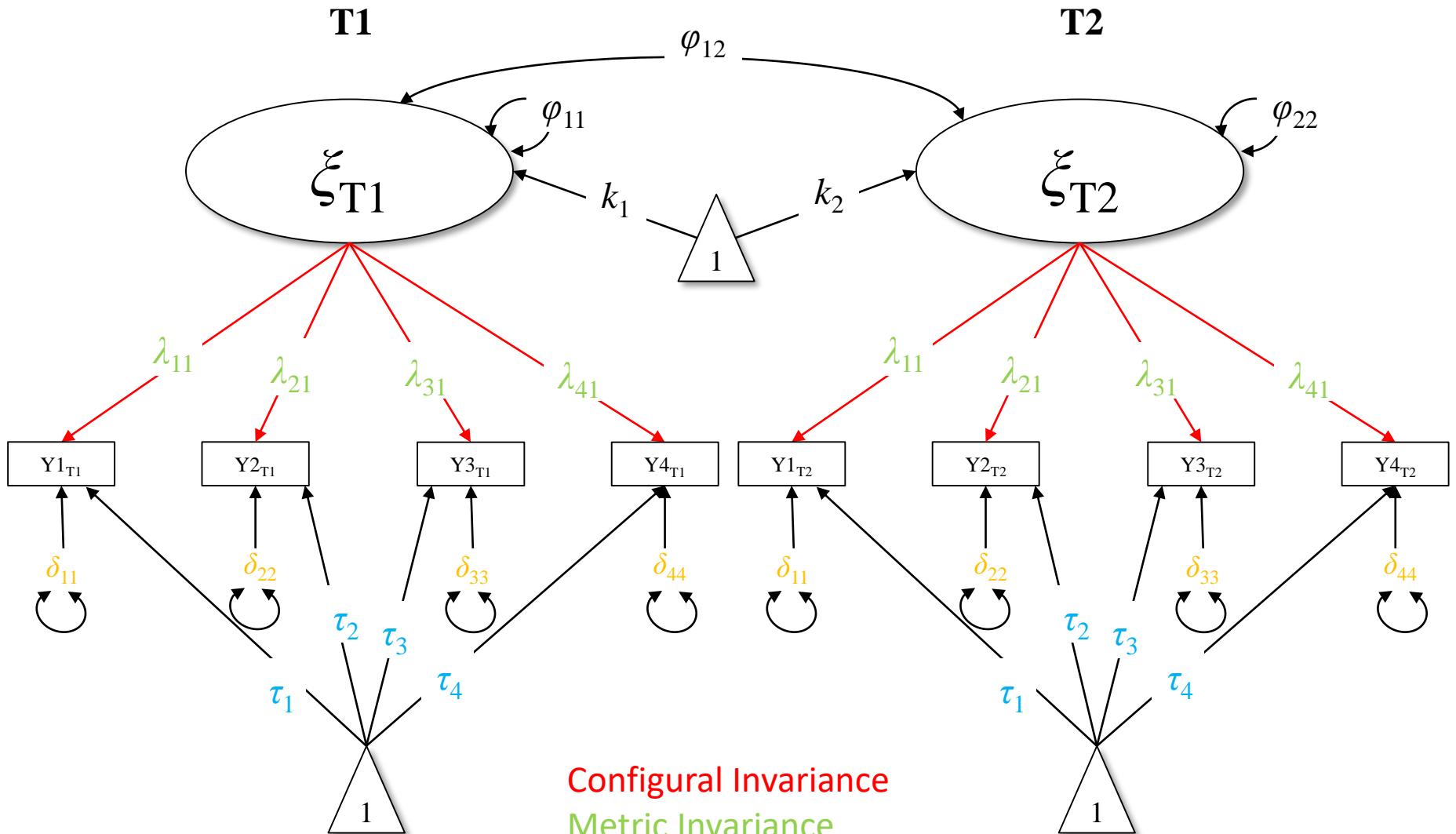
From Perinelli, Balducci, & Fraccaroli (2023, *Current Psychology*)

R codes are available here [https://psicostat.dpsu.psy.unipd.it/files/2022-06-17\\_perinelli.html](https://psicostat.dpsu.psy.unipd.it/files/2022-06-17_perinelli.html), while R codes and Mplus syntaxes are available here <https://doi.org/10.1007/s12144-022-03741-4> under "Supplementary Information"

# Measurement Invariance (Multiple-group)



# Measurement Invariance (Longitudinal)

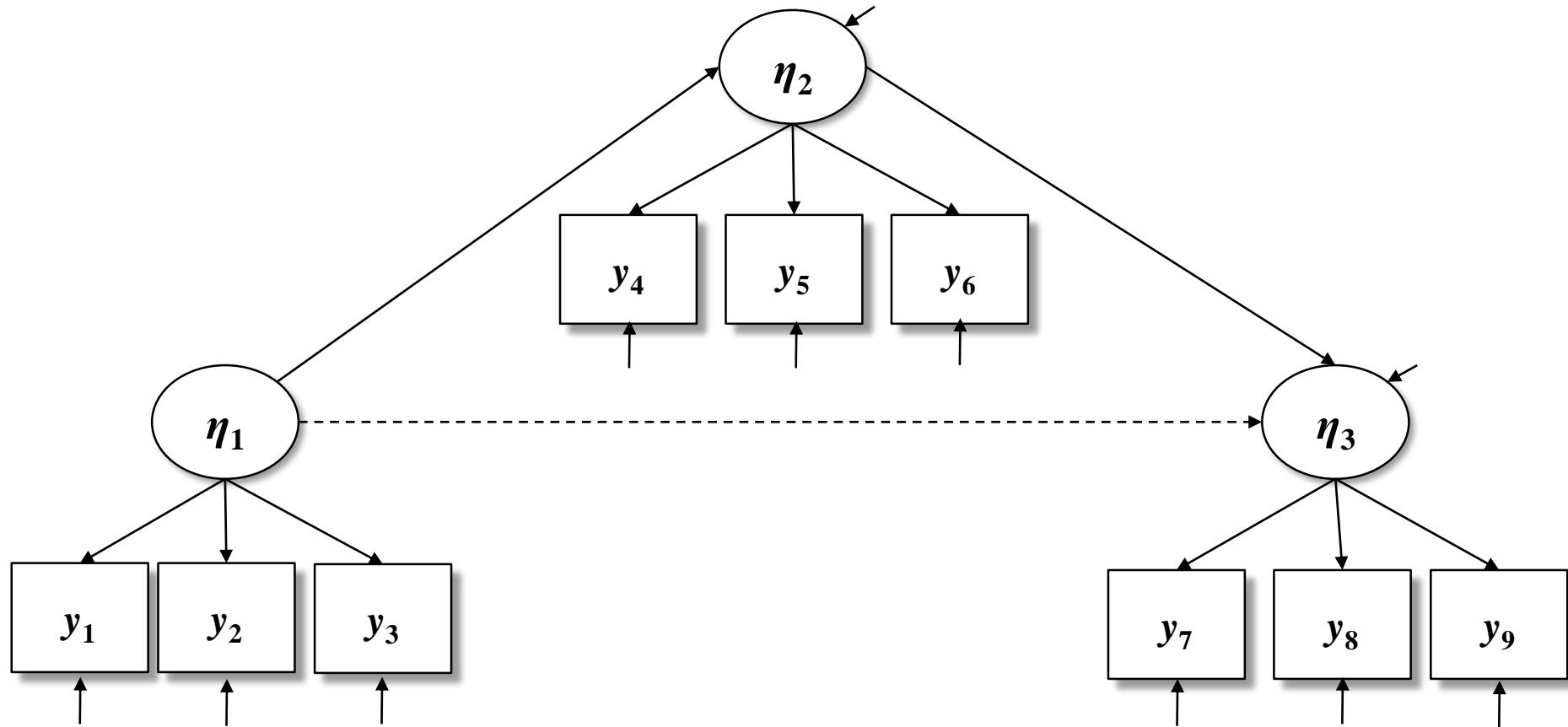


Configural Invariance  
 Metric Invariance  
 Scalar Invariance  
 Strict Invariance

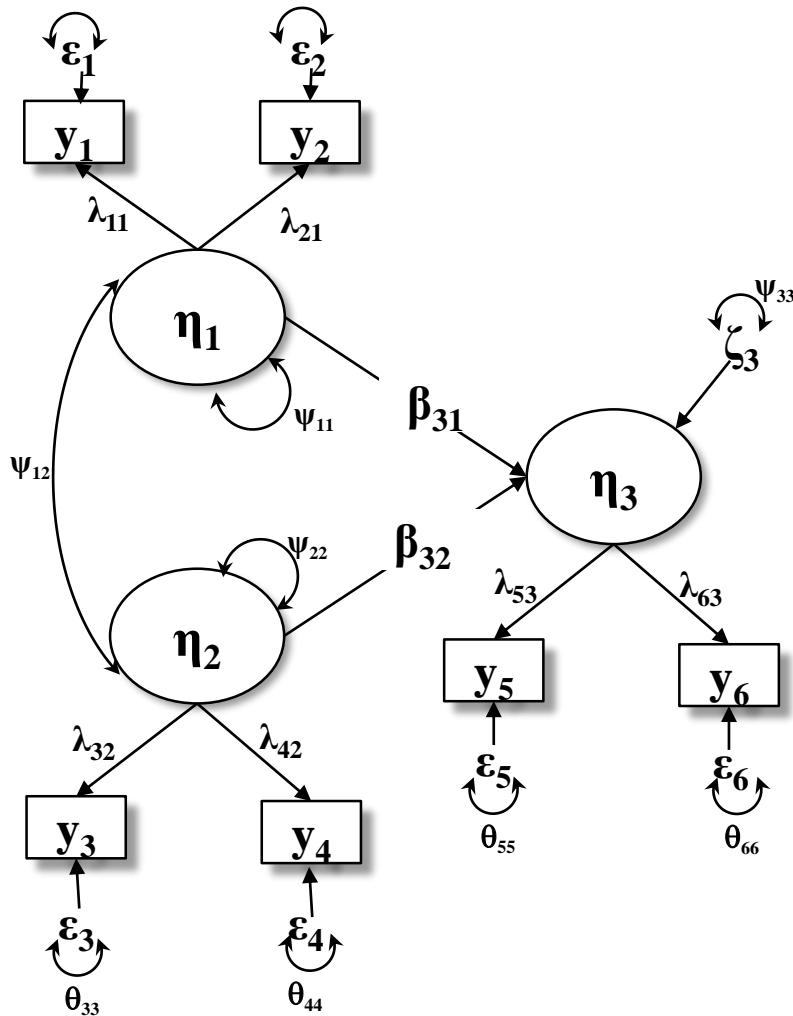
Note. Covariances between uniqueness are omitted for the sake of clarity

Some material available at  
<https://github.com/EnricoPerinelli/Measurement-Invariance-PSICOSTAT>

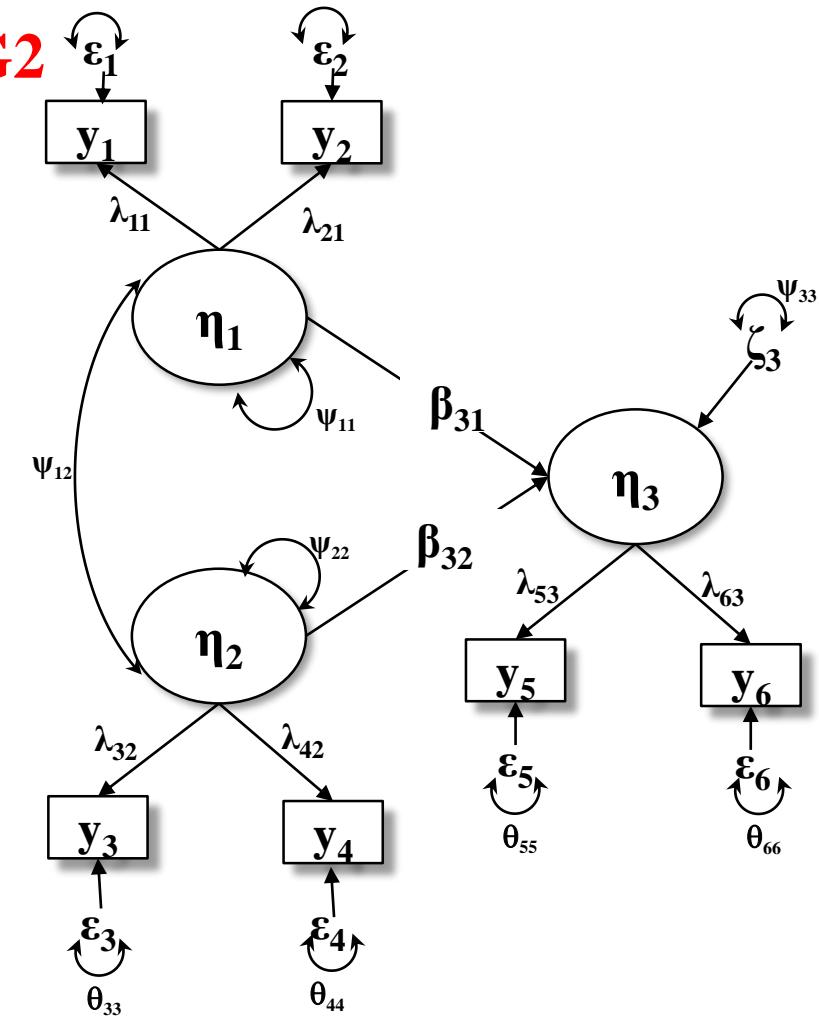
# Cross-sectional Mediation with Latent Variables



# Multiple-group SEM (Jöreskog, *Psychometrika*, 1971)

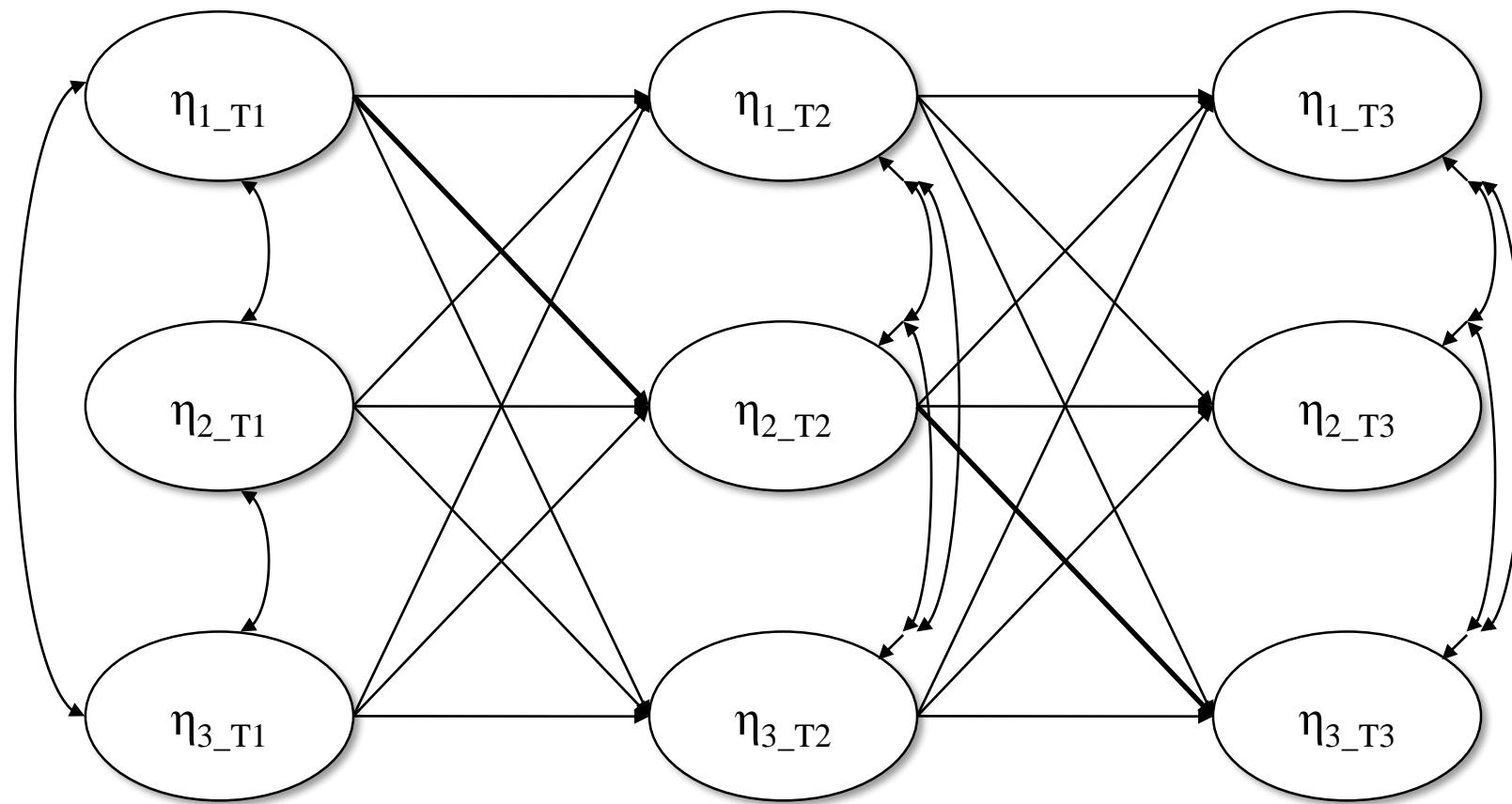


G1



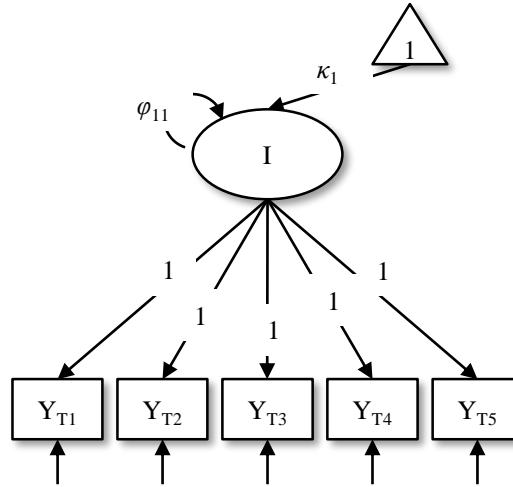
G2

# Autoregressive Cross-Lagged Panel Model

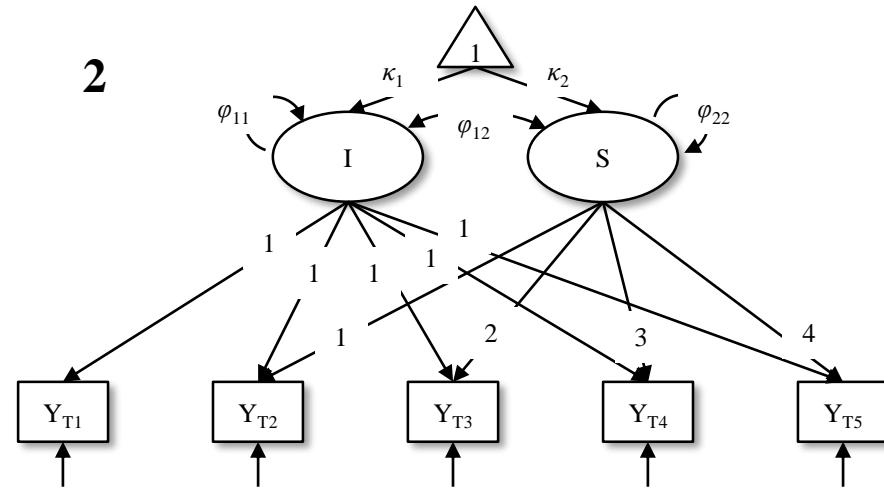


# Latent Growth Models (Meredith & Tisak, *Psychometrika*, 1990)

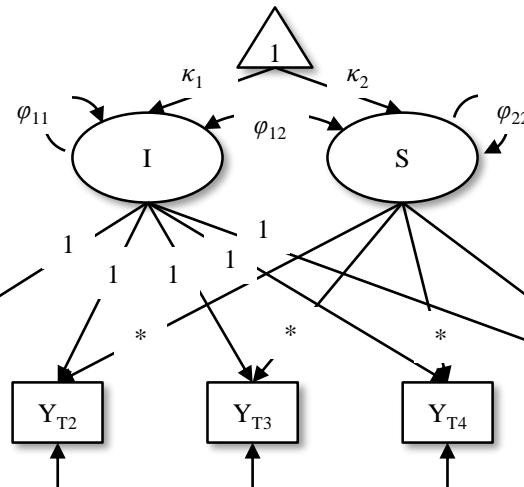
1



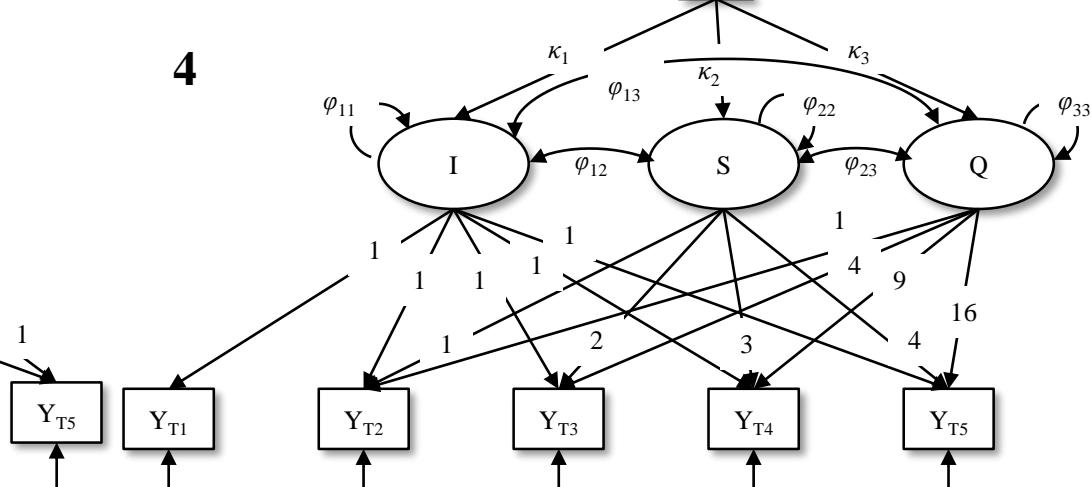
2



3



4

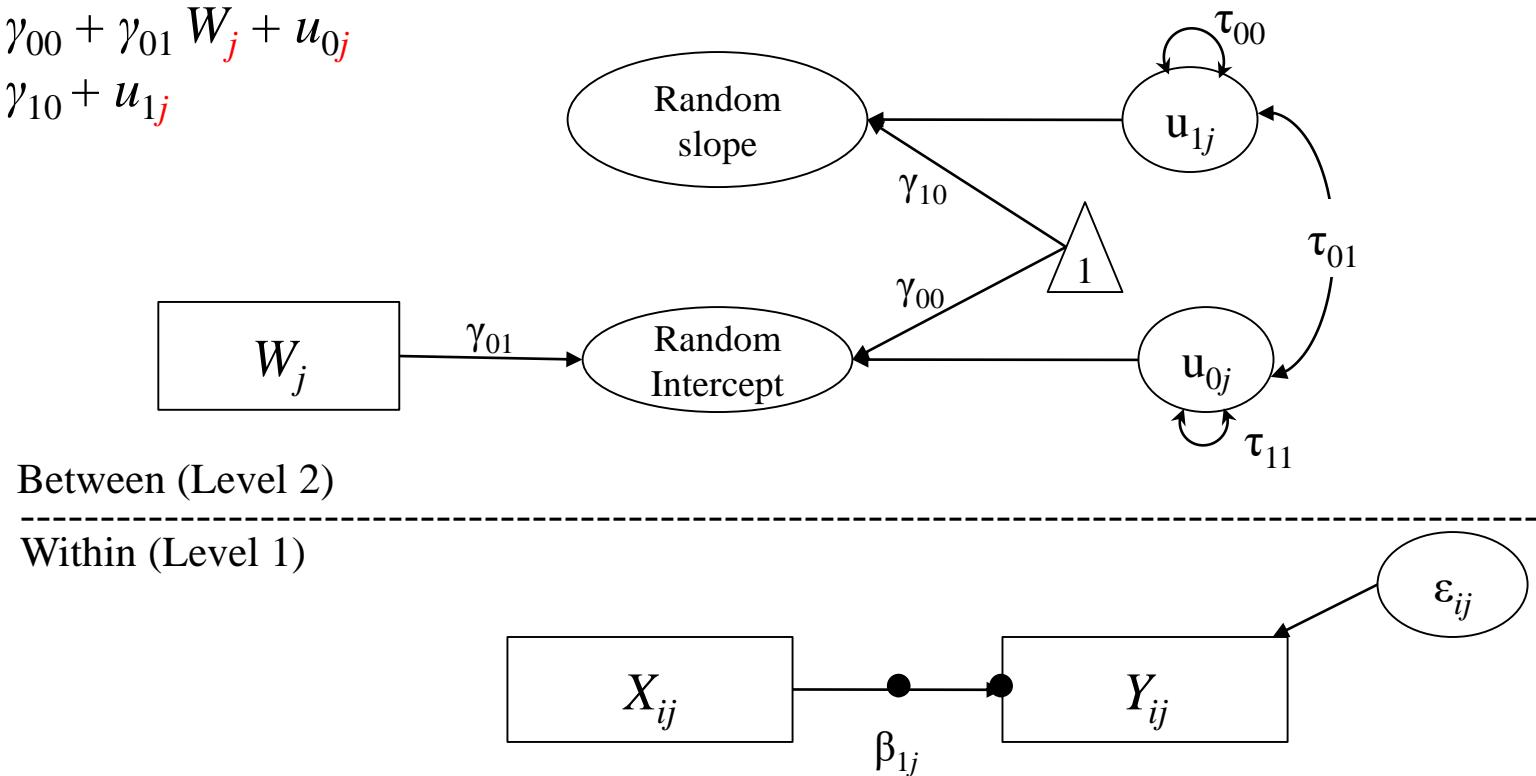


# Multilevel SEM

$$\text{Level 1: } Y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + \varepsilon_{ij}$$

$$\text{Level 2: } \beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j}$$

$$\text{Level 2: } \beta_{1j} = \gamma_{10} + u_{1j}$$

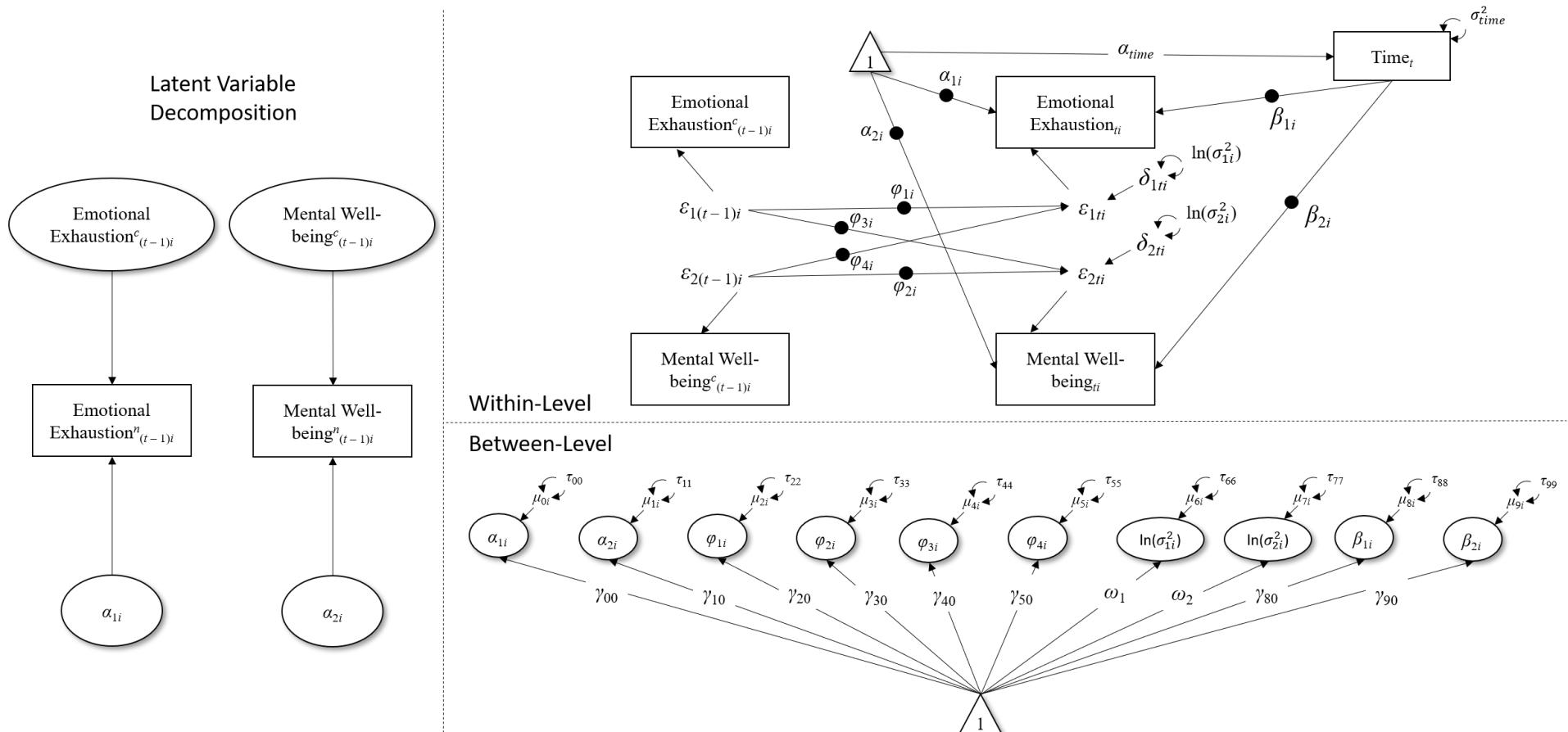


For sake of clarity, Formula and Figure refer to a simple multilevel regression.

The use of latent variables, or adding more than one dependent variable, would make this model a Multilevel SEM.

# Dynamic SEM

(Combination of Time Series Modeling, Multilevel Modeling, and SEM)

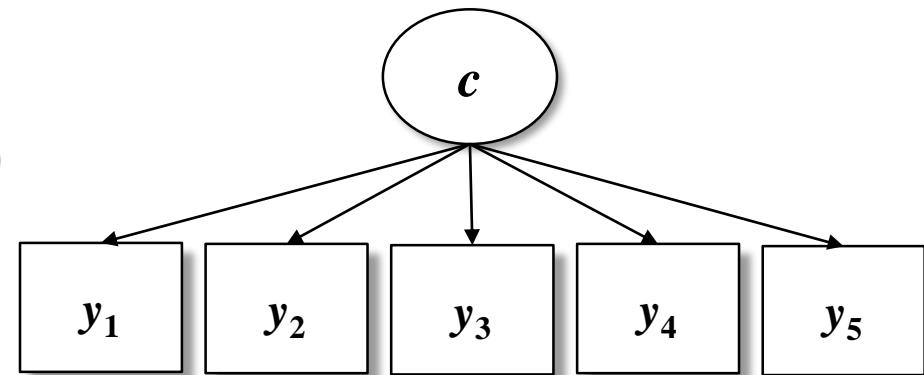


# Mixture Models

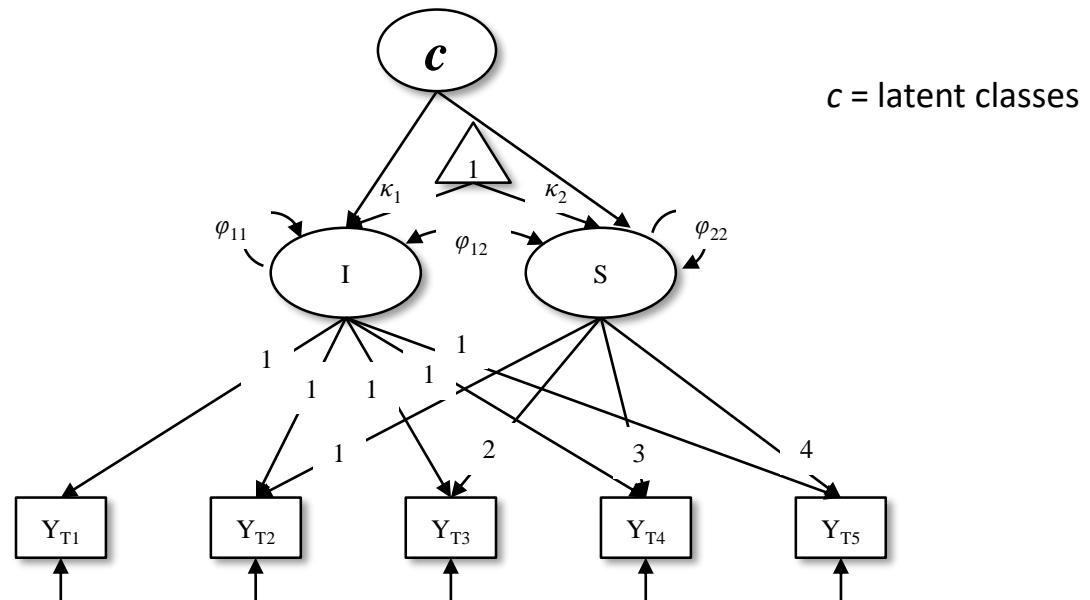
(Muthén, *Psychometrika*, 1989)

Latent Class Analysis (LCA; if  $y$  is dichotomous)

Latent Profile Analysis (LPA; if  $y$  is continuous)



Growth Mixture Model (GMM)





# Advancing General Linear Models



METHODS  
published: 02 March 2017  
doi: 10.3389/fpsyg.2017.00223



## Evaluating Intervention Programs with a Pretest-Posttest Design: A Structural Equation Modeling Approach

Guido Alessandri<sup>1</sup>\*, Antonio Zuffianò<sup>2</sup> and Enrico Perinelli<sup>1</sup>

<sup>1</sup> Department of Psychology, Sapienza University of Rome, Rome, Italy, <sup>2</sup> Department of Psychology, Liverpool Hope University, Liverpool, UK

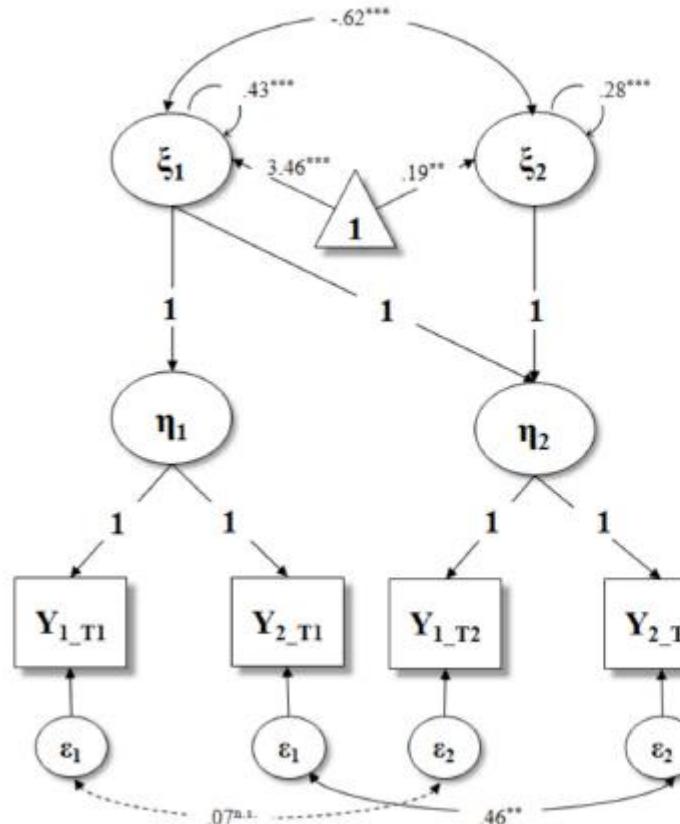
A common situation in the evaluation of intervention programs is the researcher's possibility to rely on two waves of data only (i.e., pretest and posttest), which profoundly impacts on his/her choice about the possible statistical analyses to be conducted. Indeed, the evaluation of intervention programs based on a pretest-posttest design has been usually carried out by using classic statistical tests, such as family-wise ANOVA analyses, which are strongly limited by exclusively analyzing the intervention effects at

Alessandri, G., Zuffianò, A., & Perinelli, E. (2017). Evaluating intervention programs with a pretest-posttest design: A Structural Equation Modeling approach. *Frontiers in Psychology: Quantitative Psychology and Measurement*, 8, Article 223.  
<https://doi.org/10.3389/fpsyg.2017.00223>

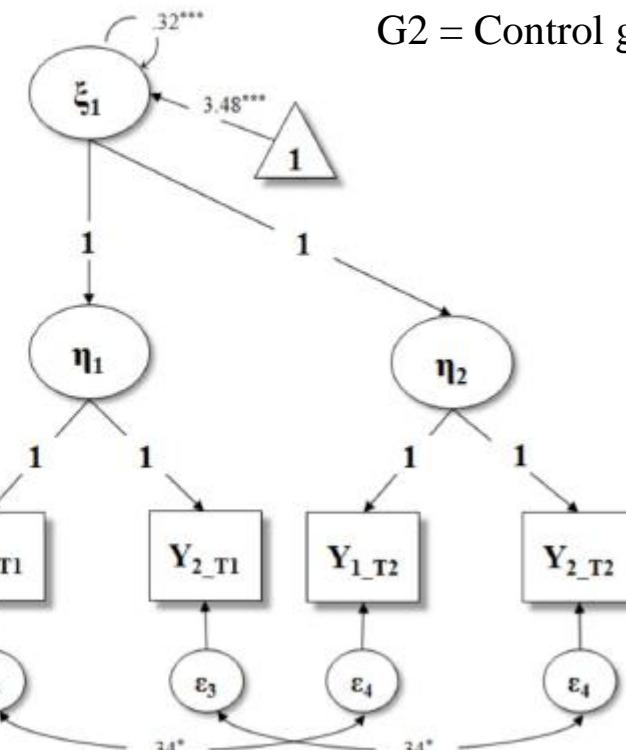
# Advancing General Linear Models

## *Second-Order Multiple-Group Latent Curve Model*

Model for G1 (Latent Change Model)



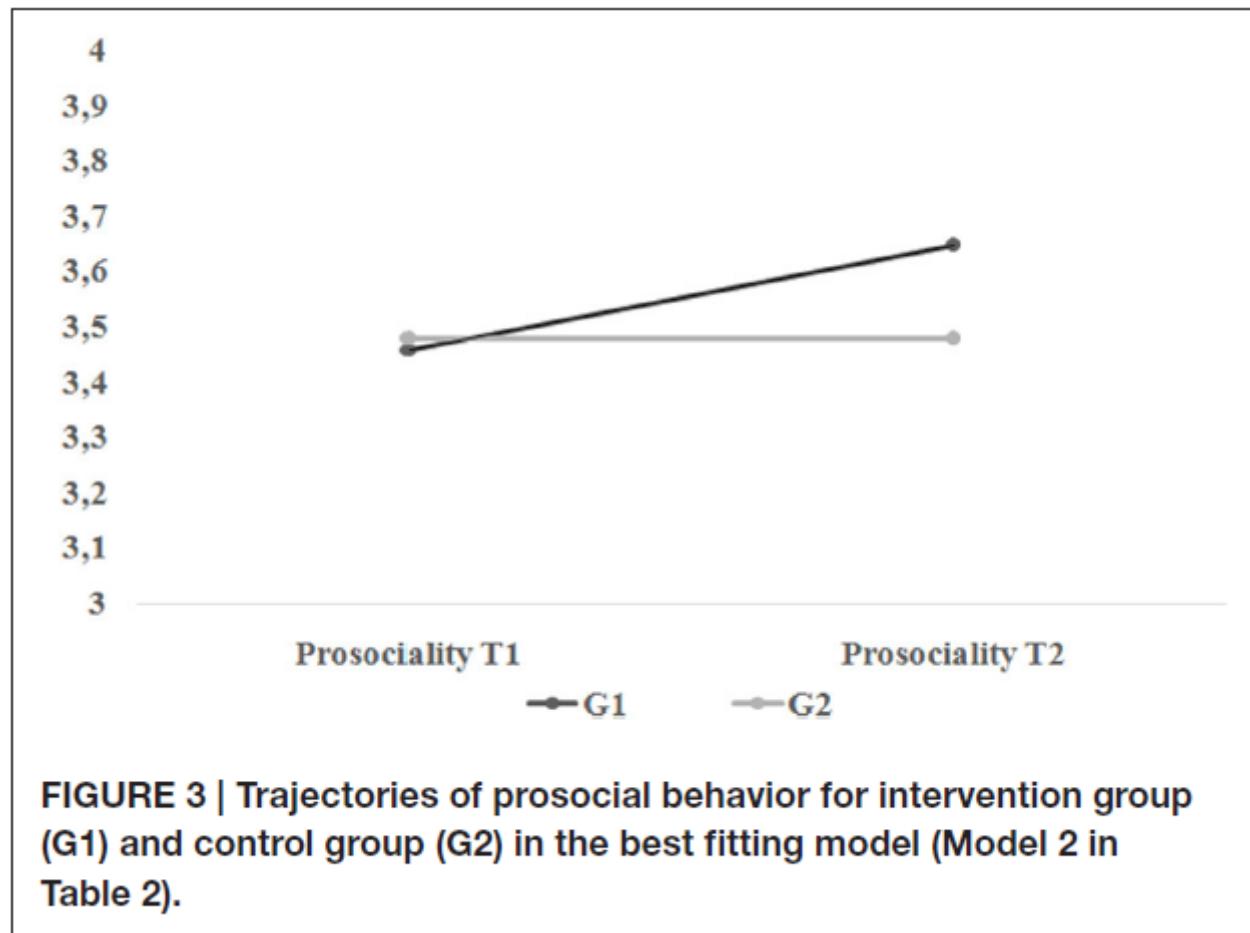
Model for G2 (no-Change Model)



G1 = Intervention group  
G2 = Control group

# Advancing Mixed Models

## *Second-Order Multiple-Group Latent Curve Model*



# Other Types

- 1) Latent Transition Analysis
- 2) Latent Change (Difference) Score
- 3) Latent State-Trait (LST) Models

- 
- 
- 
- 
- 

PhD Thesis

**On the Validity of the State-Like Component of Global Self-Esteem:  
Relationships with Implicit Self-Esteem and Work-Related Variables  
Using Different Latent State-Trait Models**

Supervisor

Professor Guido Alessandri

PhD Candidate

Enrico Perinelli

February 2018

*and much else....*



# Introduction to *Mplus*

# Mplus

*Mplus* is currently one of the most used software for latent variables analyses.

The logo for Mplus, featuring the word "Mplus" in a large, bold, blue serif font. The "M" is a dark blue, and the "plus" is a lighter blue with a shadow effect.

Copyright © 1998-2017 Muthén & Muthén  
Program Copyright © 1998-2017 Muthén & Muthén  
Version 8  
April 2017

Muthén & Muthén  
3463 Stoner Avenue  
Los Angeles, CA 90066  
Tel: (310) 391-9971  
Fax: (310) 391-8971  
Web: [www.StatModel.com](http://www.StatModel.com)  
Support@StatModel.com



# Mplus

Its popularity is mainly due to:  
a user-friendly syntax

Syntax	Action
ON	Regressed on (defines structural paths)
BY	Measured by (defines factor loadings)
WITH	Covaried with
IND	Indirect effect
[ ]	Mean structure and intercept
( )	Assigns a label
@	Fixes a parameter to a specific value
*	Parameter freed/Specifies a starting value for a parameter
MODEL INDIRECT	Specifies the indirect effect to be computed
MODEL CONSTRAINT	Specifies linear and nonlinear constraints

# Mplus

**Its popularity is mainly due to:**

The availability of a web-site (<https://www.statmodel.com/>) that provides:

1. information on the use of the software through a full-detailed user's guide ([https://www.statmodel.com/download/usersguide/MplusUserGuideVer\\_8.pdf](https://www.statmodel.com/download/usersguide/MplusUserGuideVer_8.pdf))
2. A forum in which users can deal (by questioning, by answering or simply by reading) with a broad variety of topics (<https://www.statmodel.com/cgi-bin/discus/discus.cgi>)
3. A plethora of real data examples (<https://www.statmodel.com/ug excerpts.shtml>)
4. Videos and Handouts for Short Courses  
([https://www.statmodel.com/course\\_materials.shtml](https://www.statmodel.com/course_materials.shtml))



# Mplus

**Its popularity is mainly due to:**

The availability of several estimators.

<i>Syntax</i>	<i>Estimator</i>
ML	Maximum Likelihood
MLM	Satorra-Bentler chi-square
MLMV	ML with mean- and variance-adjusted
MLR	Yuan-Bentler chi-square
MLF	ML with standard errors approximated by first-order derivatives
MUML	Muthén's limited information parameter estimates
WLS	Weighted least square
WLSM	Weighted least square mean-adjusted
WLSMV	Weighted least square mean- and variance-adjusted
ULS	Unweighted least squares
ULSMV	ULS mean- and variance-adjusted
GLS	Generalized Least Square
BAYES	Bayesian posterior parameter estimates with credibility intervals and posterior predictive checking



# Mplus

Its popularity is mainly due to:

Constantly updated software

*Current version (Feb 2023): 8.8*

Version 8	April 2017
Version 7	September 2012
Version 6	April 2010
Version 5	November 2007
Version 4	February 2006
Version 3	March 2004
Version 2	February 2001
Version 1	November 1998



# Mplus

**Its popularity is mainly due to:**  
The availability of several books

- Byrne, B. M. (2013). *Structural equation modeling with Mplus: Basic concepts, applications, and programming*. New York and London: Routledge.
- Geiser, C. (2013). *Data analysis with Mplus*. New York: Guilford Press.
- Grimm, K. J., Ram, N., & Estabrook, R. (2017). *Growth Modeling: Structural equation and multilevel modeling approaches*. New York: Guilford Press.
- Heck, R. H., & Thomas, S. L. (2015). *An introduction to multilevel modeling techniques: MLM and SEM approaches using Mplus*. New York and London: Routledge.
- Muthén, B., Muthén, L. K., Asparouhov , T. (2016). *Regression and mediation analysis using Mplus*. Los Angeles, CA: Muthén & Muthén.
- Wang, J., & Wang, X. (2012). *Structural equation modeling: Applications using Mplus*. Chichester, UK: John Wiley & Sons.
- Wickrama, K. A. S., Lee, T. K., O'Neal, C. W., & Lorenz, F. O. (2016). *Higher order growth curves and mixture modeling with Mplus: A practical guide*. New York, NY: Routledge.



# *Mplus*

**Important steps for preparing dataset to be used in *Mplus*:**

- 1) Coding missing values (I suggest -99 if you work in SPSS; for R, see the subsequent slide)
- 2) Create an ASCII data file and eventually convert commas with dots (in Notepad; for R this is not necessary)
- 3) Commands and Basic analysis (in *Mplus*)
- 4) Output for Basic analysis (in *Mplus*)

# Mplus

## Important steps for preparing dataset to be used in **Mplus**:

In order to create a dataset for **Mplus**, for **R** use the subsequent syntax

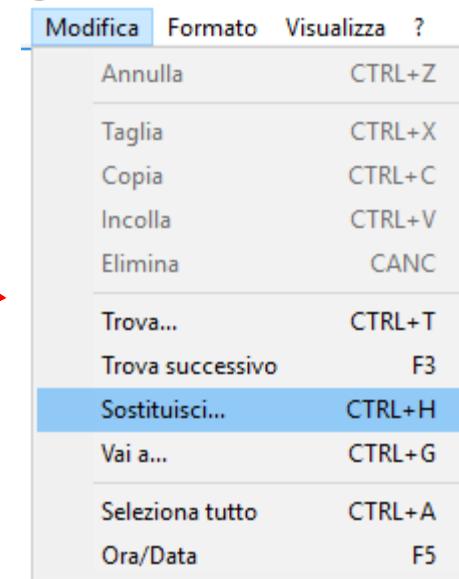
```
# Initial setup -----  
  
rm(list = ls())  
setwd("C:/Users/Enrico Perinelli/Desktop") # set your working directory or use a .Rproj file)  
  
# Load or extract dataset -----  
  
library(psych)  
my_bfi <- bfi  
  
# Load `MplusAutomation` package -----  
  
library(MplusAutomation)  
  
# Use the `prepareMplusData` function -----  
  
prepareMplusData(  
  my_bfi,  
  filename = "bfi.dat"  
)
```

# Mplus

big\_five - Blocco note

File	Modifica	Formato	Visualizza	?
404	3,50	3,63	4,38	4,50 1,13
311	4,00	4,75	4,75	4,13 2,38
379	2,88	3,50	3,75	3,88 1,38
338	4,38	3,75	4,00	3,00 1,13
209	3,75	4,38	4,13	4,25 1,63
141	-99,00	5,00	5,00	5,00 4,50
5	4,00	4,00	4,00	3,88 1,50
83	3,13	3,13	4,25	3,75 1,25
106	3,88	2,63	3,88	4,00 1,38
409	3,75	4,13	-99,00	-99,00 1,00
414	3,63	4,00	4,13	4,00 1,25
312	3,38	3,13	4,00	3,13 1,75
54	4,00	3,38	4,88	4,75 -99,00
296	3,00	3,00	3,00	3,00 3,00
411	4,38	4,00	4,88	4,50 1,00
343	4,25	4,13	5,00	4,00 1,00
287	2,88	3,00	3,00	3,00 2,63
369	4,00	-99,00	4,63	4,63 1,25
65	-99,00	4,63	4,50	4,50 1,00
89	3,50	4,63	4,13	3,38 1,25
347	3,13	3,50	-99,00	4,25 2,50
96	3,63	2,00	4,25	4,13 1,00
206	4,75	3,50	5,00	4,88 -99,00
142	1,00	1,00	1,00	1,00 1,50
144	4,00	5,00	5,00	5,00 3,50
143	4,00	4,00	4,00	4,00 3,75
324	3,63	3,38	4,50	4,00 1,13

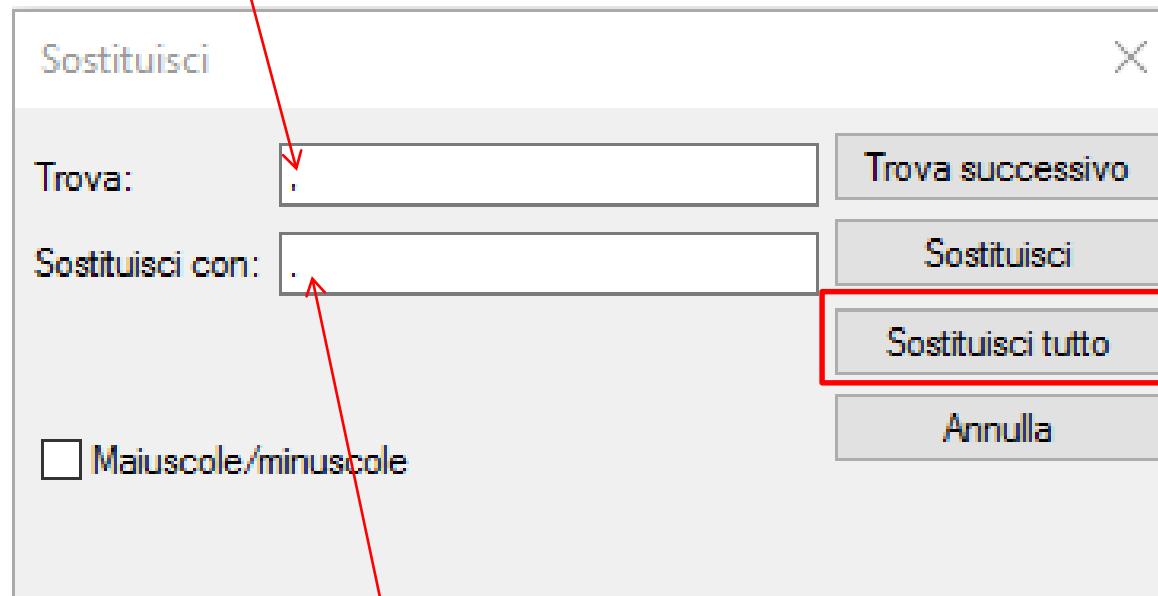
For **SPSS**, instead, you must be sure that commas are replaced with dots (1)



# Mplus

, (comma)

For **SPSS**, instead, you must  
be sure that commas are  
replaced with dots (2)



. (dot)



# Mplus

big_five - Blocco note						
File	Modifica	Formato	Visualizza	?		
404	3.50	3.63	4.38	4.50	1.13	
311	4.00	4.75	4.75	4.13	2.38	
379	2.88	3.50	3.75	3.88	1.38	
338	4.38	3.75	4.00	3.00	1.13	
209	3.75	4.38	4.13	4.25	1.63	
141	-99.00	5.00	5.00	5.00	4.50	
5	4.00	4.00	4.00	3.88	1.50	
83	3.13	3.13	4.25	3.75	1.25	
106	3.88	2.63	3.88	4.00	1.38	
409	3.75	4.13	-99.00	-99.00	1.00	
414	3.63	4.00	4.13	4.00	1.25	
312	3.38	3.13	4.00	3.13	1.75	
54	4.00	3.38	4.88	4.75	-99.00	
296	3.00	3.00	3.00	3.00	3.00	
411	4.38	4.00	4.88	4.50	1.00	
343	4.25	4.13	5.00	4.00	1.00	
287	2.88	3.00	3.00	3.00	2.63	
369	4.00	-99.00	4.63	4.63	1.25	
65	-99.00	4.63	4.50	4.50	1.00	
89	3.50	4.63	4.13	3.38	1.25	
347	3.13	3.50	-99.00	4.25	2.50	
96	3.63	2.00	4.25	4.13	1.00	
206	4.75	3.50	5.00	4.88	-99.00	
142	1.00	1.00	1.00	1.00	1.50	
144	4.00	5.00	5.00	5.00	3.50	
143	4.00	4.00	4.00	4.00	3.75	

For **SPSS**, instead, you must  
be sure that commas are  
replaced with dots (3)

NOW, ALL THE  
COMMAS ARE  
DOTS



# Mplus

Syntax	Function
<b>TITLE</b>	Provide a title for the analysis
<b>DATA</b>	Provide information about the data set to be analyzed
<b>VARIABLE</b>	Provide information about the variables in the data set
<b>DEFINE</b>	Transform existing variables and create new variables
<b>ANALYSIS</b>	Describe the technical details of the analysis (e.g., Estimator, type)
<b>MODEL</b>	Describe the model to be estimated
<b>OUTPUT</b>	Request additional output not included as the default
<b>SAVEDATA</b>	Save the analysis data (e.g., Auxiliary data)
<b>PLOT</b>	Request graphical displays of analysis results
<b>MONTECARLO</b>	Specify the details of a monte carlo simulation study

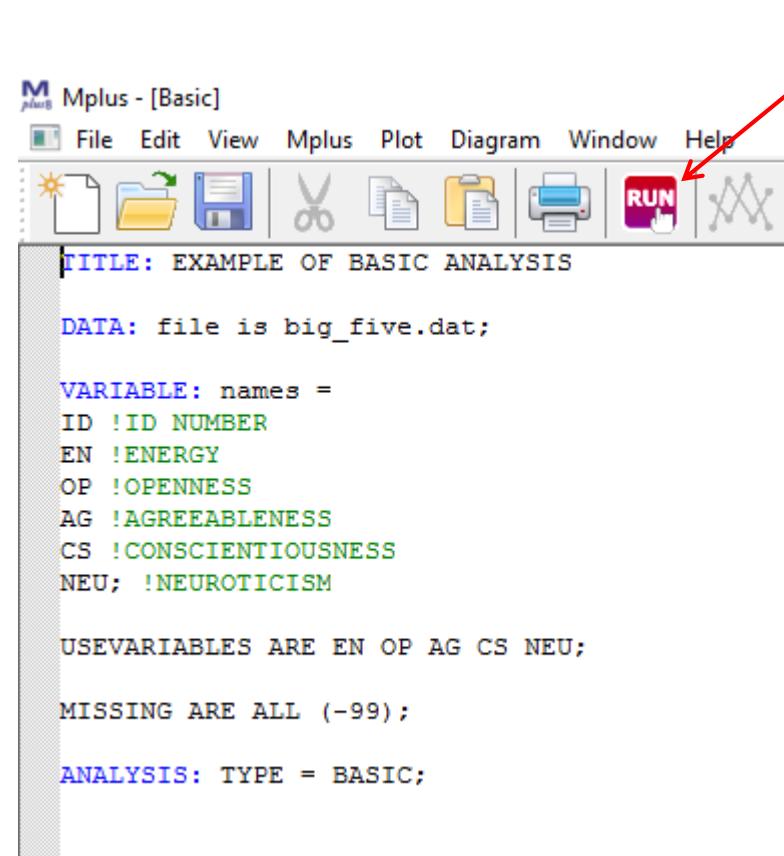
! This symbol is used to take notes

; This symbol must be placed at the end of each command line



# Mplus

CLICK RUN



```
Mplus - [Basic]
File Edit View Mplus Plot Diagram Window Help
TITLE: EXAMPLE OF BASIC ANALYSIS

DATA: file is big_five.dat;

VARIABLE: names =
ID !ID NUMBER
EN !ENERGY
OP !OPENNESS
AG !AGREEABLENESS
CS !CONSCIENTIOUSNESS
NEU; !NEUROTICISM

USEVARIABLES ARE EN OP AG CS NEU;

MISSING ARE ALL (-99);

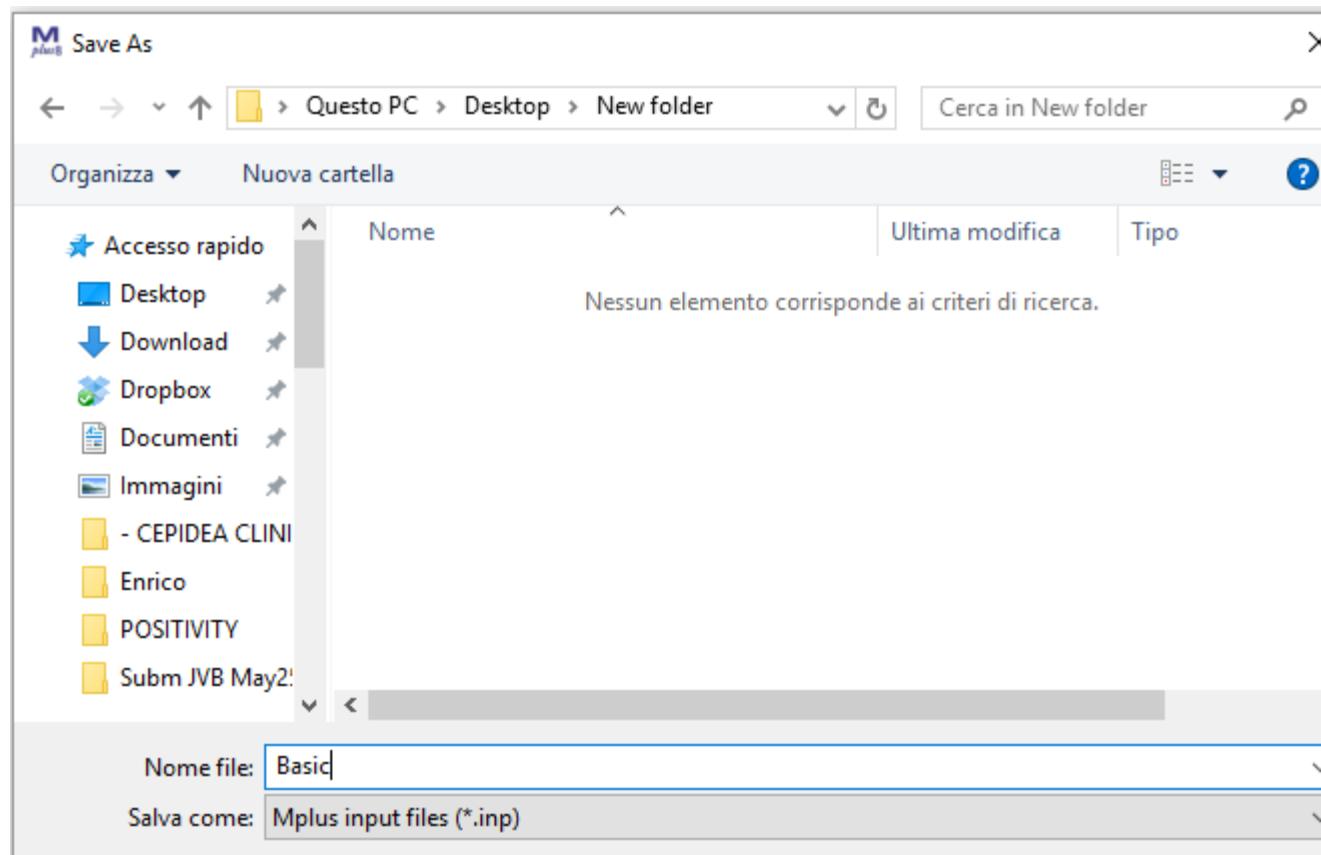
ANALYSIS: TYPE = BASIC;
```

Remember that  
variable names cannot  
be longer than  
8 characters



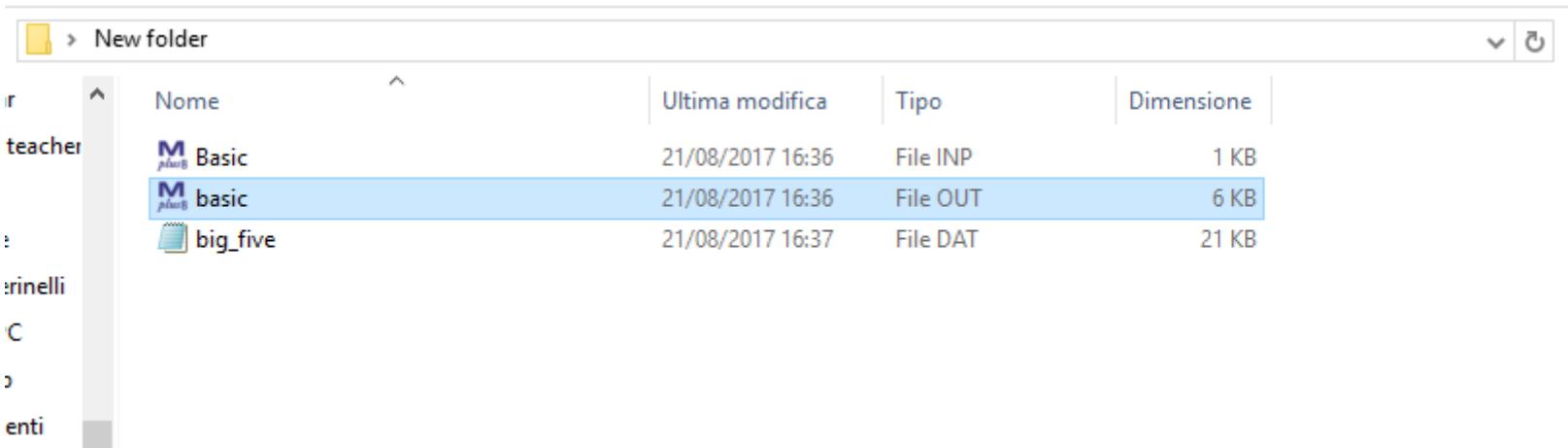
# Mplus

Save the input file in the same folder of the .dat file



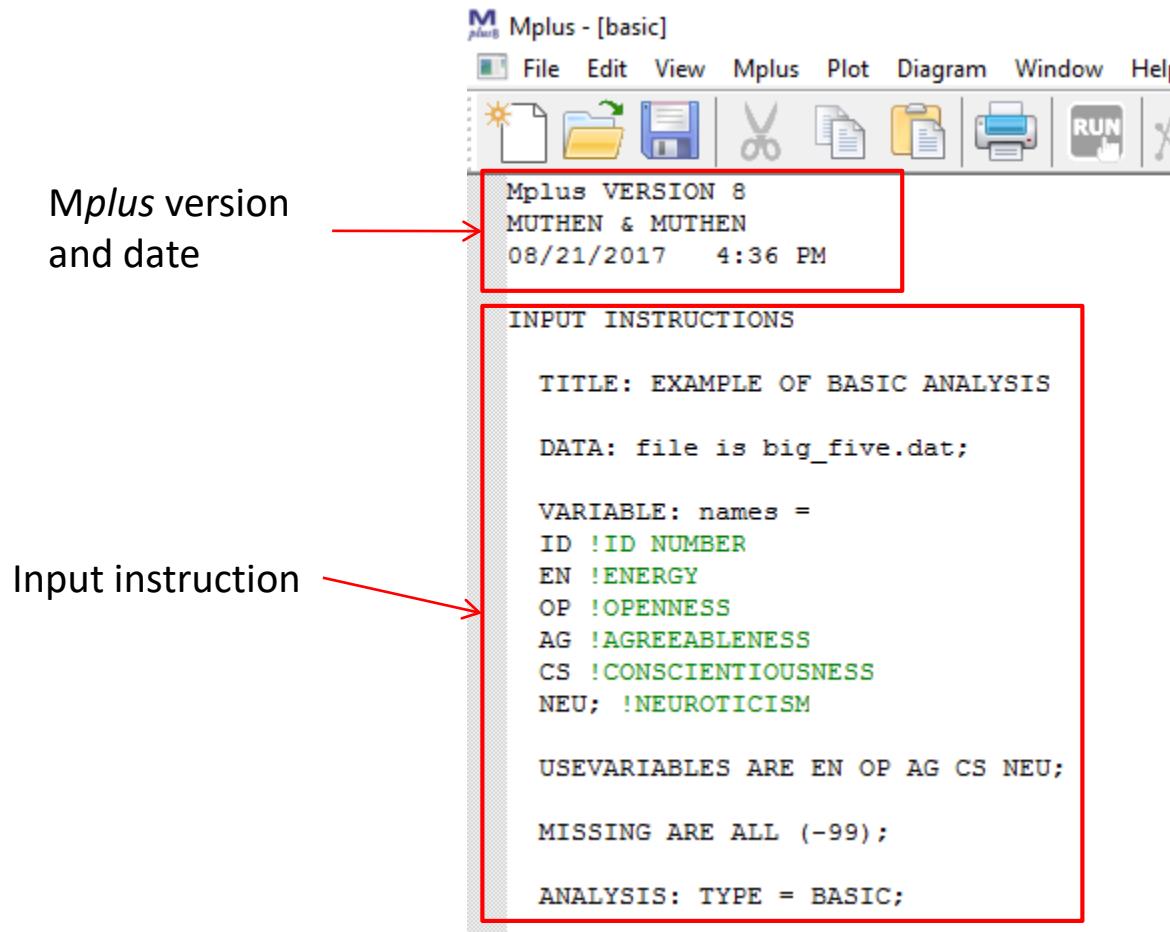


# Mplus



New folder				
	Nome	Ultima modifica	Tipo	Dimensione
teacher	Mplus Basic	21/08/2017 16:36	File INP	1 KB
	Mplus basic	21/08/2017 16:36	File OUT	6 KB
	big_five	21/08/2017 16:37	File DAT	21 KB

# Mplus



# Mplus

The analysis has gone well!!!

Title and information on the number of variables. Here we have only continuous variables, but this part of the output may displays also the number of latent variables

By default in most of analyses the estimator is ML (in Multilevel modeling the default estimator is MLR)

INPUT READING TERMINATED NORMALLY

## EXAMPLE OF BASIC ANALYSIS

### SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	416
Number of dependent variables	5
Number of independent variables	0
Number of continuous latent variables	0

### Observed dependent variables

Continuous	EN	OP	AG	CS	NEU
------------	----	----	----	----	-----

### Estimator

Information matrix	ML	OBSERVED
Maximum number of iterations	1000	
Convergence criterion	0.500D-04	
Maximum number of steepest descent iterations	20	
Maximum number of iterations for H1	2000	
Convergence criterion for H1	0.100D-03	

Input data file(s)  
big\_five.dat

Input data format FREE

# Mplus

## COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value 0.100

## PROPORTION OF DATA PRESENT

Covariance Coverage				
	EN	OP	AG	CS
EN	0.988			
OP	0.954	0.966		
AG	0.957	0.935	0.969	
CS	0.966	0.950	0.952	0.978
NEU	0.962	0.945	0.945	0.952
				0.974



This matrix shows the percentage of covariance coverage for each pair of observed variables. When dataset does not have missing data, each pair of observed variables has a covariance coverage of 100%.



# Mplus

## RESULTS FOR BASIC ANALYSIS

### ESTIMATED SAMPLE STATISTICS

#### Means

	EN	OP	AG	CS	NEU
	3.812	3.811	4.463	4.097	1.382

#### Covariances

	EN	OP	AG	CS	NEU
EN	0.279				
OP	0.129	0.386			
AG	0.112	0.131	0.204		
CS	0.089	0.132	0.123	0.264	
NEU	-0.024	-0.004	-0.068	-0.012	0.264

#### Correlations

	EN	OP	AG	CS	NEU
EN	1.000				
OP	0.392	1.000			
AG	0.470	0.469	1.000		
CS	0.330	0.412	0.531	1.000	
NEU	-0.087	-0.012	-0.294	-0.046	1.000

Here, information concerning Means, Covariances and Correlations are reported.



# Mplus

## UNIVARIATE SAMPLE STATISTICS

### UNIVARIATE HIGHER-ORDER MOMENT DESCRIPTIVE STATISTICS

Variable/ Sample Size	Mean/ Variance	Skewness/ Kurtosis	Minimum/ % with		Percentiles		
			Maximum	Min/Max	20%/60%	40%/80%	Median
EN 411.000	3.810 0.279	-0.390 1.442	1.000 5.000	0.24% 1.70%	3.380 4.000	3.630 4.250	3.880
OP 402.000	3.809 0.386	-0.407 0.654	1.000 5.000	0.25% 2.49%	3.250 4.000	3.750 4.380	3.880
AG 403.000	4.465 0.205	-1.496 7.433	1.000 5.000	0.25% 14.89%	4.000 4.630	4.380 4.880	4.500
CS 407.000	4.098 0.264	-0.806 2.712	1.000 5.000	0.25% 4.91%	3.630 4.250	4.000 4.500	4.130
NEU 405.000	1.382 0.264	2.439 8.499	1.000 4.500	29.63% 0.49%	1.000 1.250	1.130 1.750	1.130

Here, information concerning univariate sample statistics are reported.

A VERY IMPORTANT STEP IS TO CHECK THE TRUSTWORTHINESS OF THESE VALUES!!!!  
WHEN THE DATASET IS LARGE, IT IS EASY TO WRONG SPECIFY VARIABLE NAMES, OR FORGET TO  
SPECIFY THE MASSING DATA VALUE.



# Mplus User's guide

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Intro

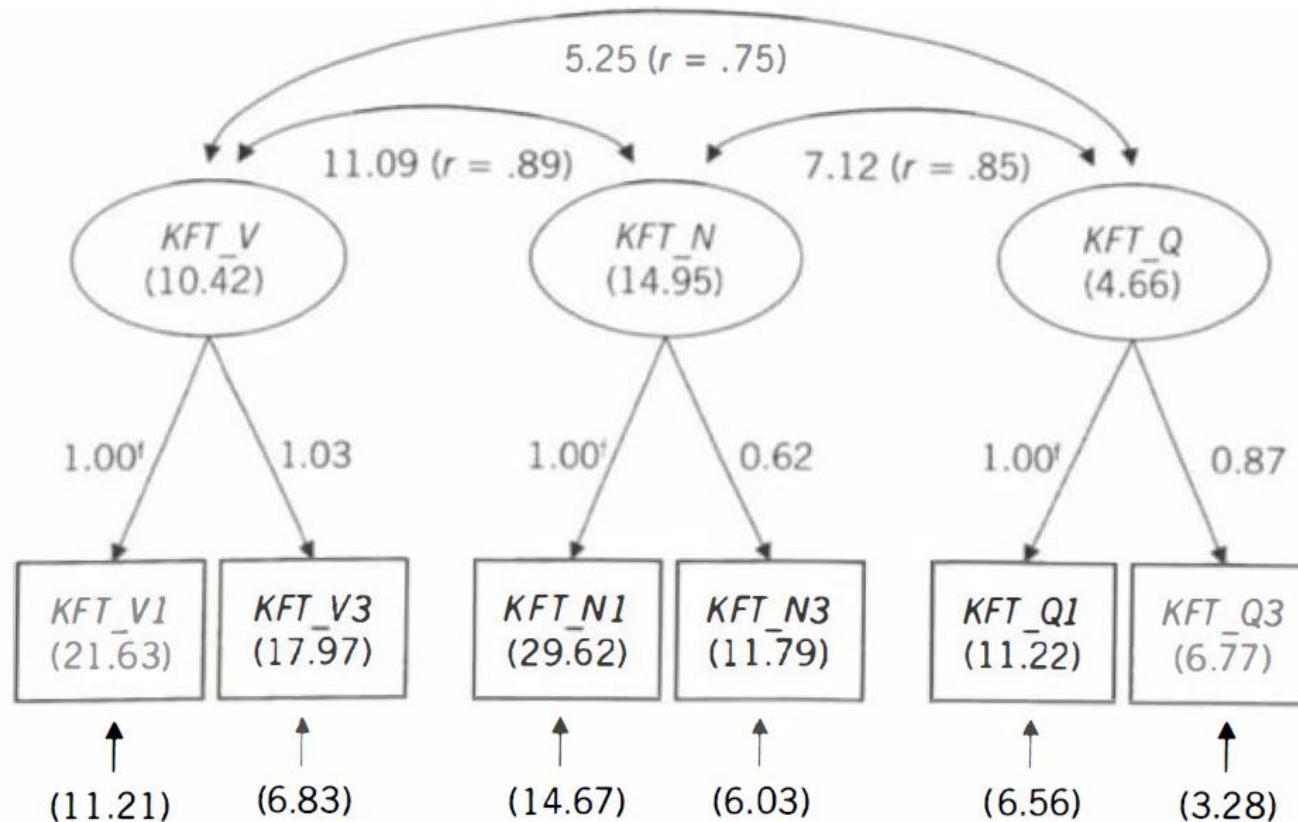
Specific Models  
(types of SEM)

Information regarding  
commands and  
syntaxes

Link:  
[https://www.statmodel.com/download  
usersguide/MplusUserGuideVer\\_8.pdf](https://www.statmodel.com/download/usersguide/MplusUserGuideVer_8.pdf)

# Mplus CFA example

(Geiser, 2013, p. 53)





# Mplus CFA example

(Geiser, 2013, p. 53)

```
title: CFA for the six KFT variables  
       3-Factor model (see Figure 3.8)
```

```
data: file = KFT.dat;  
       listwise = on;
```

```
variable: names = kft_v1 kft_v3 kft_q1 kft_q3 kft_n1 kft_n3;  
       missing = all(-99);
```

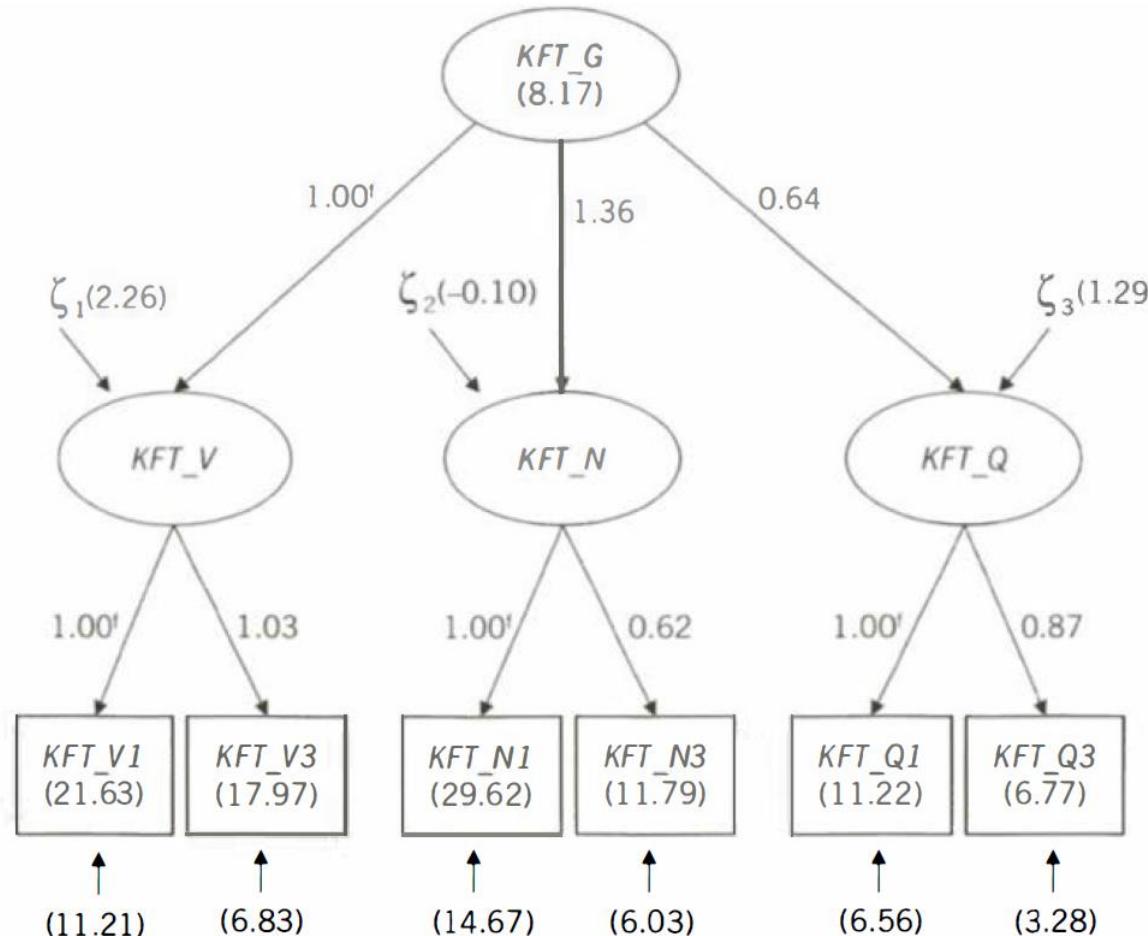
```
analysis: type = general;
```

```
model: KFT_V by kft_v1 kft_v3; ! verbal ability factor  
       KFT_Q by kft_q1 kft_q3; ! numeric ability factor  
       KFT_N by kft_n1 kft_n3; ! figural (non-verbal) ability factor
```

```
output: sampstat stdyx tech4;
```

# Mplus Second-Order CFA example

(Geiser, 2013, p. 59)





# Mplus Second-Order CFA example

(Geiser, 2013, p. 59)

```
title: CFA for the six KFT variables
       Second-order factor model (see Figure 3.11)

data: file = KFT.dat;
       listwise = on;

variable: names = kft_v1 kft_v3 kft_q1 kft_q3 kft_n1 kft_n3;
       missing = all(-99);

analysis: type = general;

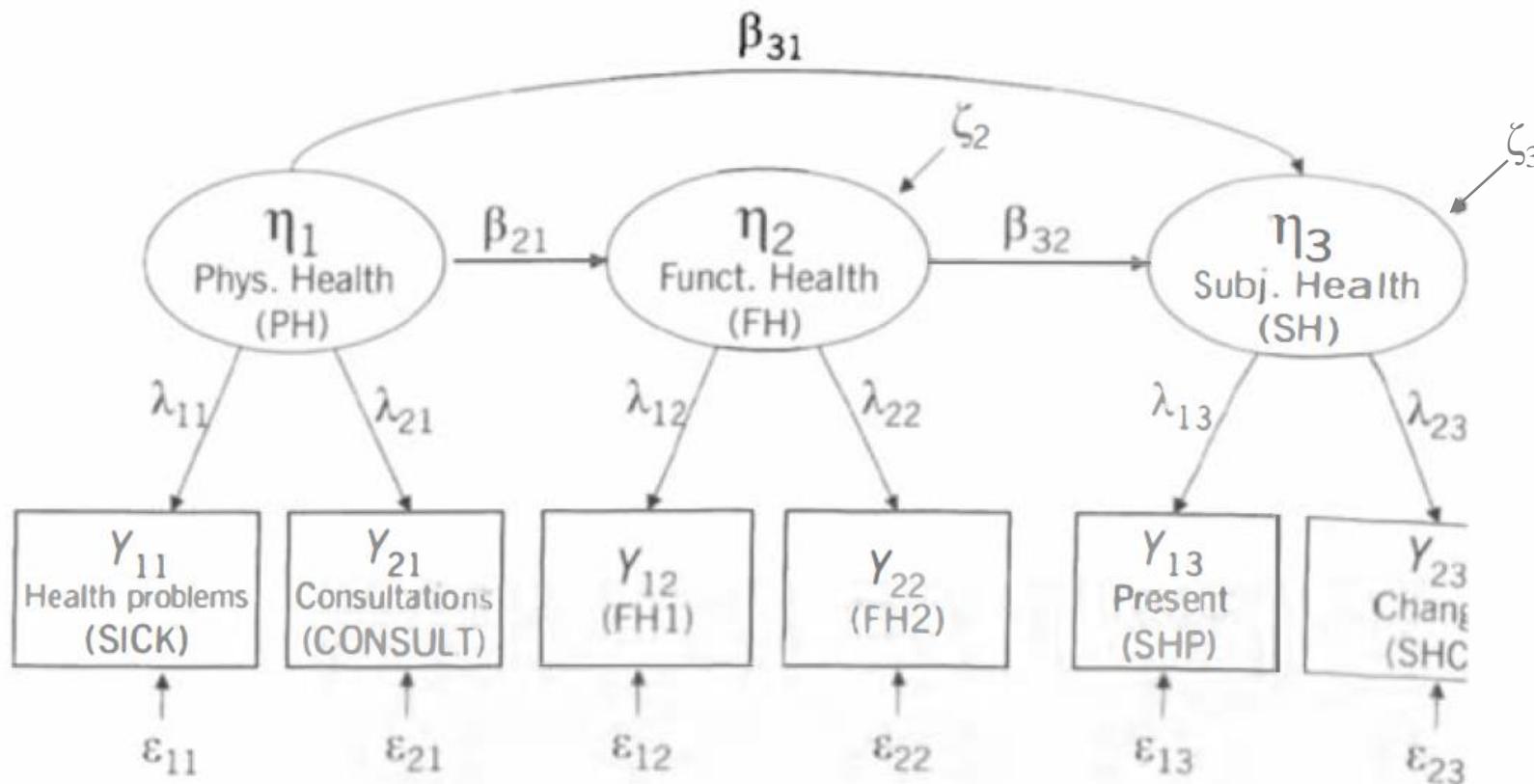
model: ! First-order factors
       KFT_V by kft_v1 kft_v3; ! verbal ability factor
       KFT_Q by kft_q1 kft_q3; ! numeric ability factor
       KFT_N by kft_n1 kft_n3; ! figural (non-verbal) ability factor

       ! Second-order factor
       KFT_G by KFT_V KFT_Q KFT_N;

output: sampstat stdyx;
```

# Mplus SEM example

(Geiser, 2013, p. 74)



# Mplus SEM example

(Geiser, 2013, p. 74)

```
title: Latent path model shown in Figure 3.15
      Relationship between physical health, functional health, and subjective health

data: file = health.dat;

variable: names = SHP ! Subjektive health in the present
           SHC ! Changes in subjective health across the past 6 years
           SICK ! Number of health issues in the past 12 months
           CONSULT ! Number of physician consultations in the past 12 months
           FH ! Total score SF-36 scale on functional health
           FH1 FH2; ! SF-36 scale split into 2 test halves/item parcels

usevar = SHP SHC SICK CONSULT FH1 FH2;

analysis: type = general;

model: SH by SHP SHC; ! Factor/Measurement model subjective health
       PH by SICK CONSULT; ! Factor/Measurement model physical health
       FH by FH1 FH2; ! Factor/Measurement model functional health

! Path analysis at the level of the latent variables
FH on PH;
SH on PH FH;

model indirect: SH ind PH; ! Request output of direct, indirect, and total effects

output: sampstat stdyx;
```



# Mplus SEM example (bootstrap)

(Geiser, 2013, p. 74)

**title:** Latent path model shown in Figure 3.15  
Relationship between physical health, functional health, and subjective health  
With bias-corrected bootstrap confidence intervals

```
data: file = health.dat;

variable: names = SHP ! Subjektive health in the present
           SHC ! Changes in subjective health across the past 6 years
           SICK ! Number of health issues in the past 12 months
           CONSULT ! Number of physician consultations in the past 12 months
           FH ! Total score SF-36 scale on functional health
           FH1 FH2; ! SF-36 scale split into 2 test halves/item parcels

usevar = SHP SHC SICK CONSULT FH1 FH2;

analysis: type = general;
           bootstrap = 10000; ! Request 10000 bootstrap samples

model: SH by SHP SHC; ! Factor/Measurement model subjective health
       PH by SICK CONSULT; ! Factor/Measurement model physical health
       FH by FH1 FH2; ! Factor/Measurement model functional health

! Path analysis at the level of the latent variables
FH on PH;
SH on PH FH;

model indirect: SH ind PH; ! Request output of direct, indirect, and total effects

output: sampstat stdyx
        cinterval(bcbootstrap); ! Request confidence intervals based on bias-corrected bootstrap
```



# Suggested Reading

## Suggested 'Starters' for SEM

- Kline, R. B. (2016). *Principles and practice of structural equation modeling* (4th ed.). The Guilford Press.
- Hoyle, R. K. (2023). *Handbook of Structural Equation Modeling* (2nd ed.). The Guilford Press.
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## Old but gold - Still the best book on SEM mathematical bases

- Bollen, K. A. (1989). *Structural equations with latent variables*. Wiley.

## Longitudinal SEM

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- Wang, J., & Wang, X. (2020). *Structural Equation Modeling: Applications Using Mplus* (2nd ed.). John Wiley & Sons. **[Section 4.8, pp. 241-252]**



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- Enders, C. K. (2022). *Applied missing data analysis* (Chapter 2). The Guilford Press.
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- Heck, R. H., & Thomas, S. L. (2015). *An introduction to multilevel modeling techniques: MLM and SEM approaches using Mplus* (3rd ed.). Routledge.
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## **lavaan (R package)**

- <https://lavaan.ugent.be/tutorial/>
- <https://lavaan.ugent.be/tutorial/tutorial.pdf>
- <https://stats.oarc.ucla.edu/r/seminars/rsem/>
- [https://users.ugent.be/~yrosseel/lavaan/gent2020/lavaan\\_twodays\\_gent2020.pdf](https://users.ugent.be/~yrosseel/lavaan/gent2020/lavaan_twodays_gent2020.pdf)
- Geiser, C. (2023). Structural equation modeling with the *Mplus* and lavaan programs. In R. Hoyle (Ed.), *Handbook of structural equation modeling* (2nd ed., pp. 241-258). Guilford Press.

## **SEM in Python with `semopy` package**

- <https://pypi.org/project/semopy/> and <https://semopy.com/>
- Igolkina, A. A., & Meshcheryakov, G. (2020). Semopy: A Python package for structural equation modeling. *Structural Equation Modeling: A Multidisciplinary Journal*, 27(6), 952-963.

## **Bayesian SEM (BSEM)**

### **Books and Chapters**

- Depaoli, S. (2021). *Bayesian Structural Equation Modeling*. The Guilford Press.
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- Kaplan, D., & Depaoli, S. (2012). Bayesian Structural Equation Modeling. In R. H. Hoyle (Ed.), *Handbook of Structural Equation Modeling* (pp. 650-673). The Guilford Press.
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## Bayesian SEM (BSEM)

### Articles:

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## Machine Learning and Latent Variables/Psychometrics (Part 1)

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- Brandmaier, A. M., & Jacobucci, R. C. (2023). Machine Learning approaches to structural Equation Modeling. In R. H. Hoyle (Ed.), *Handbook of structural equation modeling* (2nd ed., pp. 722-739). The Guilford Press.
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## Machine Learning and Latent Variables/Psychometrics (Part 2)

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And those “latent variables,” are they in the room with us right now?



Thanks for  
your attention

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