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In [4]: import matplotlib.pyplot as plt
import numpy as np
import math

# Here will will do example 7.1 page 250, Chapter 7 in the book
# of Kiusalaas J. Numerical methods in Engineering with Pyhton 3

# The first order differential equation is  $dy/dx+4y=x^2$ ,  $y(0)=1$ .
#  $dy/dx = f(x,y)$  where  $f(x,y)=x^2-4y$ 
# We will let the spacing  $h = 0.01$  and we will step from  $x = 0$  to  $0.03$ 
# using the Euler methods and study
# the truncation error and the accumulated error when comparing with the exact
# solution.

h = 0.01
xstart = 0
xend = 0.03

X=[]
Yexact=[]
Yeuler=[]

# Save the initial condition  $y(0)=1$  in the arrays

x = xstart
yexact = 1
yeuler = 1

X.append(x)
Yexact.append(yexact)
Yeuler.append(yeuler)

while (x<xend):

    h = min(h,xend-x) # In case we have to reduce the timestep in the end to
                        # fit the simulation time interval.

    yeuler = yeuler+h*(x**2-4*yeuler) # Note this is an efficient way
                                     # to write  $y_{new} = y_{old} + h*f(x_{old},y_{old})$ 

    # update x and also find exact solution.
    x = x+h
    yexact= 31/32*math.exp(-4*x)+1/4*x**2-1/8*x+1/32

    erroraccumulated = yeuler-yexact

    print(f'x verdi: {x}')
    print(f'Euler: {yeuler}')
    print(f'Exact: {yexact}')
    print(f'Accumulated error: {erroraccumulated}')

    X.append(x)
    Yexact.append(yexact)
    Yeuler.append(yeuler)

print('-----')
print('We observe that the accummulated error increases with approx 0.0007'
      ' for each step when we simulated from 0 to 0.03 s with steps of 0.01 s')

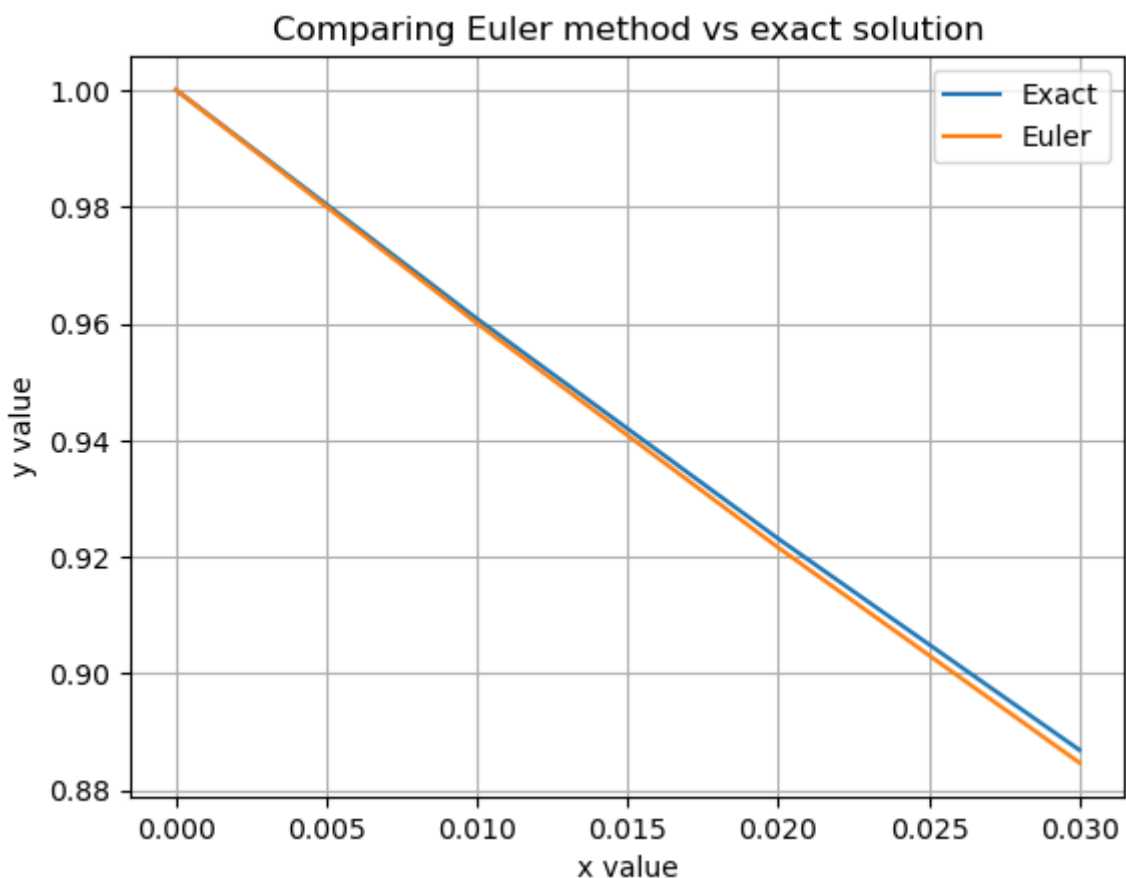
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plt.plot(X,Yexact,X,Yeuler)
plt.title('Comparing Euler method vs exact solution')
plt.grid(True)
plt.xlabel('x value')
plt.ylabel('y value')
plt.legend(['Exact', 'Euler'])
plt.show()

print(f'Change xend to 0.5. Try first with h = 0.01. Then h = 0.1. What is \n'
      'the impact of increasing h ?')
```

```
x verdi: 0.01
Euler: 0.96
Exact: 0.9607897691788132
Accumulated error: -0.0007897691788132377
x verdi: 0.02
Euler: 0.921601
Exact: 0.9231189605620534
Accumulated error: -0.0015179605620534442
x verdi: 0.03
Euler: 0.88474096
Exact: 0.8869291730697463
Accumulated error: -0.0021882130697462587
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```

We observe that the accumulated error increases with approx 0.0007 for each step when we simulated from 0 to 0.03 s with steps of 0.01 s



Change xend to 0.5. Try first with h = 0.01. Then h = 0.1. What is the impact of increasing h ?

In [ ]: