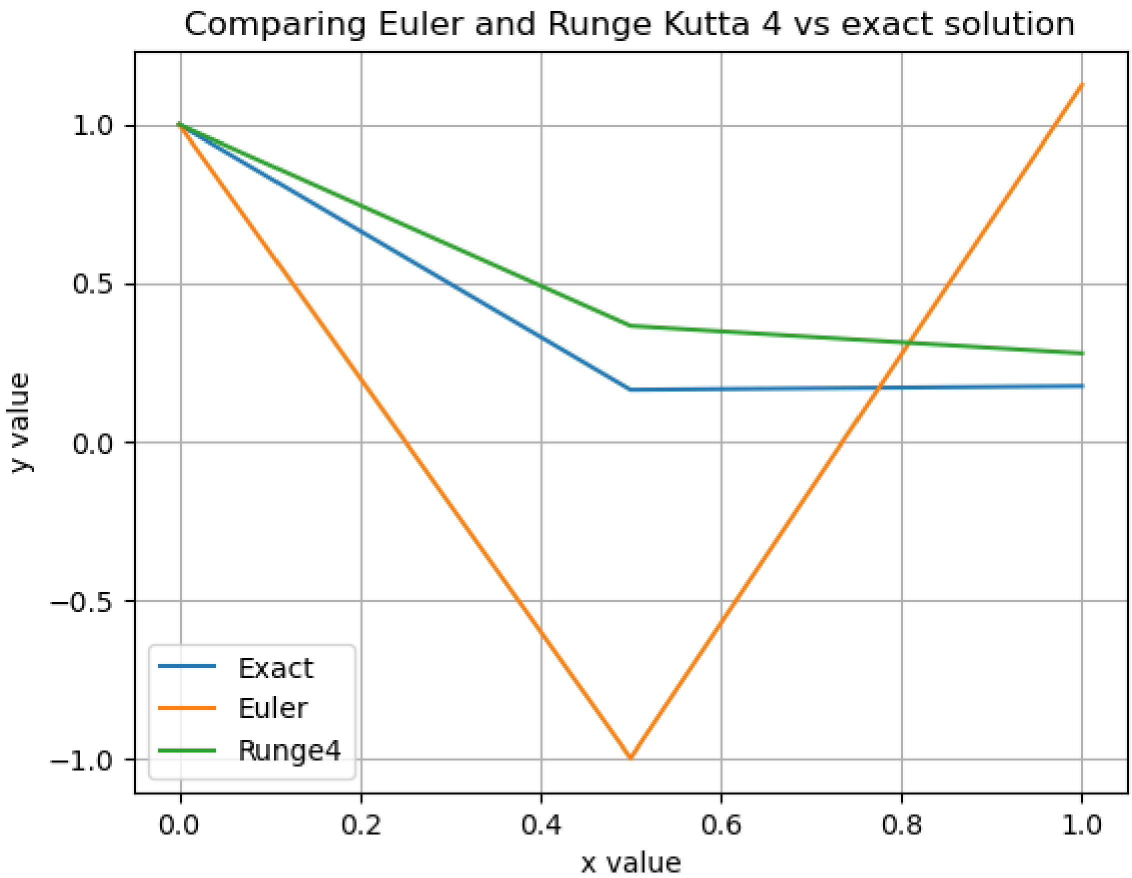
In [47]:
```python
import matplotlib.pyplot as plt
import numpy as np
import math


# Here we will again do example 7.1 page 250, Chapter 7 in the book
# of Kiusalaas J. Numerical methods in Engineering with Pyhton 3. But now the
# 4th order Runge-Kutta method is implemented in addition to the Euler method.
# So we will be able to compare the methods and the exact solution.


# The first order differential equation is dy/dx+4y=x^2, y(0)= 1.
# dy/dx = f(x,y) where f(x,y)=x^2-4y
# Let tend be 1.0 seconds. Start with h = 0.5, then h = 0.2, 0.1, 0.05, 0.01 and
# compare how Runge Kutta4 and Euler method converges
# toward the exact solution

h = 0.5
xstart = 0
xend =1


X=[]
Yexact=[]
Yeuler=[]
Yrunge4=[]

# Save the initial condition y(0)=1 in the arrays

x = xstart
yexact = 1
yeuler = 1
yrunge = 1

X.append(x)
Yexact.append(yexact)
Yeuler.append(yeuler)
Yrunge4.append(yrunge)

while (x<xend):

    h = min(h,xend-x) #  In case we have to reduce the timestep in the end
                      # to fit the simulation time interval.


    #  Here comes the implementation of the Euler method

    yeuler =yeuler+h*(x**2-4*yeuler)

    # Here comes the implementation of the 4th order Runge Kutta method.
    # Recipe see equation 7.10 page 255 in Jan Kiusalaas book.


    K0 = h*(x**2-4*yrunge)
    K1 = h*((x+0.5*h)**2-4*(yrunge+0.5*K0))
    K2 = h*((x+0.5*h)**2-4*(yrunge+0.5*K1))
    K3 = h*((x+h)**2-4*(yrunge+K2))

    yrunge = yrunge+1/6*(K0+2*K1+2*K2+K3)  # This is the same as writing
                                           # ynew = yold + something
```

```
    # Update x and also calculate the exact solution

    x = x+h

    yexact= 31/32*math.exp(-4*x)+1/4*x**2-1/8*x+1/32




    X.append(x)
    Yexact.append(yexact)
    Yeuler.append(yeuler)
    Yrunge4.append(yrunge)


plt.plot(X,Yexact,X,Yeuler,X,Yrunge4)
plt.title('Comparing Euler and Runge Kutta 4 vs exact solution')
plt.grid(True)
plt.xlabel('x value')
plt.ylabel('y value')
plt.legend(['Exact', 'Euler','Runge4'])
plt.show()


print(f'Change h from h = 0.5 to 0.2, 0.1, 0.05!'
      ' Which method converges fastest towards the exact solution?')
```




```
Change h from h = 0.5 to 0.2, 0.1, 0.05! Which method converges fastest towards th
e exact solution?
```

In [ ]: