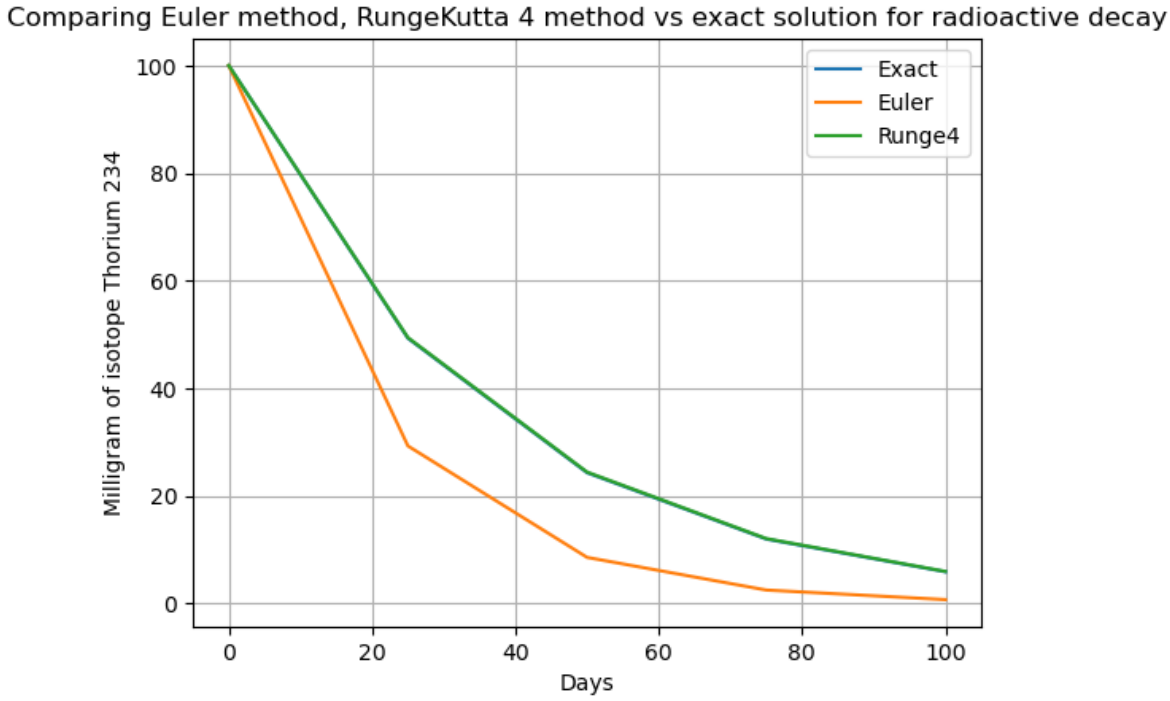
In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
import math

# Repetition of example 2 with the fourth order Runge Kutta method implemented.
# Radioactive decay of Thorium Isotope-234
# Here we will solve the differential equation
# dQ/dt=-rQ,  r = 0.02828 days-1, q(0)=100 mg. t is in days
# Example is taken from the book:  Elementary Differential Equations and
# Boundary Value Problems.by William E. Boyce and Richard D. Prima. page 44.
# The exact solution can be found to be:  Q(t)=100*exp(-rt)
# Here we compare the Euler method and fourth order Runge Kutta method.
# In this case we simulate for 100 days
# to capture more of the radioactive decay


dt = 25 # days
tstart = 0
tend  = 100 # days
r = 0.02828  # days-1

T=[]
Qexact=[]
Qeuler=[]
Qrunge4=[]

# Save the initial condition y(0)=1 in the arrays

t = tstart
qexact = 100 # mg
qeuler = 100 # mg
qrunge = 100 # mg

T.append(t)
Qexact.append(qexact)
Qeuler.append(qeuler)
Qrunge4.append(qrunge)

while (t<tend):
    dt = min(dt,tend-t)

    # Euler method
    qeuler =qeuler+dt*(-r*qeuler)

    # 4th order Runge Kutta method (page 255 in book)

    K0=dt*(-r*qrunge)
    K1=dt*(-r*(qrunge+0.5*K0))
    K2=dt*(-r*(qrunge+0.5*K1))
    K3=dt*(-r*(qrunge+K2))

    qrunge=qrunge+1/6*(K0+2*K1+2*K2+K3)

    # update time and exact solution
    t = t+dt
    qexact= 100*math.exp(-r*t)

    T.append(t)
    Qexact.append(qexact)
    Qeuler.append(qeuler)
    Qrunge4.append(qrunge)
```

```
plt.plot(T,Qexact,T,Qeuler,T,Qrunge4)
plt.title('Comparing Euler method, RungeKutta 4 method vs exact solution for radioa
plt.grid(True)
plt.xlabel('Days')
plt.ylabel('Milligram of isotope Thorium 234')
plt.legend(['Exact', 'Euler','Runge4'])
plt.show()

print('-----------------------------------------------------------------------')


print('Start with dt = 25 days (step size) and then reduce the timestep')
print()
print('What timestep do you recommend for 4th order Runge Kutta method?')
print()
print('What timestep do you recommend for Euler method?')
```




```
-----------------------------------------------------------------------
Start with dt = 25 days (step size) and then reduce the timestep

What timestep do you recommend for 4th order Runge Kutta method?

What timestep do you recommend for Euler method?
```

In [ ]:

In [ ]:

In [ ]: