

# MOD500 Decision Analysis with Artificial Intelligence Support

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1 Metrics and distributions

2 Information Theory

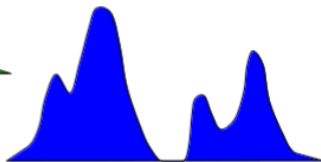
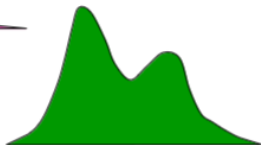
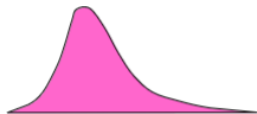
3 Comparing information

# Probability distribution

A metric is needed

Amount of uncertainty

Number of Modes: unimodal, bimodal, polymodal



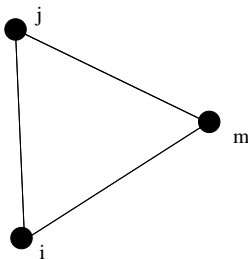
# Metrics

## Triangle inequality

Considering a vectors in an N-dimensional space, to be a **distance** it must satisfy the **triangle inequality**:

$$d_{ij} + d_{im} \geq d_{jm}$$

If also  $d_{jj} = 0$ , if  $i = j$  and  $d_{jj} - d_{ii} = 0$ , then we call it a *metric*.



## Common metrics:

- Counting
- Euclidean

$$d_{ij}^{(E)} = \left[ \sum_{k=1}^N (x_{ik} - x_{jk})^2 \right]^{\frac{1}{2}}$$

Categorical variables do not necessarily satisfy these relations.

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# Task assignment

Code a discrete probability distribution in Python

Calculate the Mean and Standard Deviation

How to get an experiment out of this distribution?

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# How to relate a distribution to information?

First, the origins:

- Daddy: Claude Shannon (1940)
- His initial work has been done on signal transmission.
- It uses Entropy as key measurement of information uncertainty.

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# How Entropy can alter a decisions?

It is an interface between data and decisions.

A question to sum up the idea

Does more data bring value?

It has permitted the advances of several fields:

cryptography, neurobiology, signal processing, linguistics,  
bioinformatics, statistical physics, black holes, quantum computing,  
information retrieval, intelligence gathering, plagiarism detection,  
pattern recognition, anomaly detection, etc

zip files, phones, internet!

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# Entropy of an information source

$$H_X = - \sum_i (p_i) \log(p_i)$$

$H_X$  of a discrete random variable  $X$  is a measure of the amount of uncertainty associated with the value of  $X$  when only its distribution is known

What is  $p_i$ ?

It is a numerical descriptions of how likely an event is to occur

Do not mix the concepts!

Assigned probability and computed probability are different

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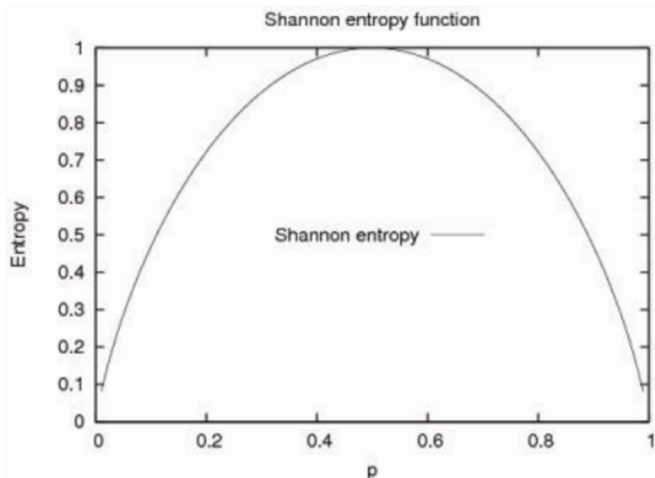
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# Shannon's entropy shape

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Calculate Shannon's Entropy as a function of the number of experiments

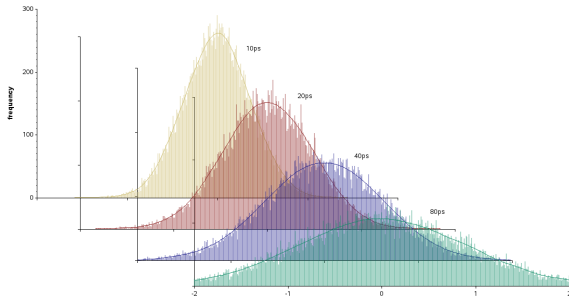
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# Comparing information

A distribution can be the sum of multiple distributions



# Kullback–Leibler divergence (information gain)

How to compare multiple information sources?

$$D_{KL} = \sum_i (p_i) \log\left(\frac{p_i}{q_i}\right)$$

- It is a divergence, as it is asymmetric
- It is NOT a metric



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