

# Fundamentals of Machine learning for and with engineering applications

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## Statistics

### Definition

Statistics is the science of acquiring and utilizing data

- It comprises tools for data collection, summarization, and interpretation.
- The aim is identifying the underlying structure, trends, and relationships inherent in the data.
- Is it all statistics then? Yes.
- **Numbers to data, data to information**

## Data properties

Before we talk about machine learning, we need to refresh some terminology.

### Population

The universe of all possible outcomes and events.

### Sample

A finite subset extracted from the population.

### Exhaustivity

The samples covered the population spectra.

### Representativity

The population is properly described by the samples.

## Big data

We speak of big data when dataset are very large: i.e. many instances and features Models have thus a large set of parameters (and often no one has a clue anymore of what is going on).

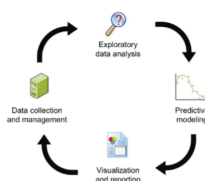
- Volume of data
- Variety different types of data sources with different length and scale.
- Frequency of data generation

## Sampling

Samples shall have no bias (to be randomly selected). If not, the bias has to be corrected for.

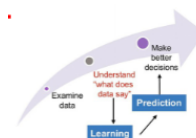
### Cycle of data

- 1 Data is collected
- 2 Checked upon
- 3 Some modelling
- 4 Analysis and visualization



## Data quality

- 1 Data has to be acquired and integrated
- 2 Data are passed to a quality analysis and control
- 3 Data cleaning, consistency check. Most of time goes here



## Preliminary Modeling

### Main tasks:

- 1 Hunt for redundancy
  - 2 Reduce dimensionality
  - 3 AnOmAlles removal
- Descriptive modeling (unsupervised learning)
  - Predictive modeling (supervised learning)
  - The model can be used to guide data acquisition (risky!)

## Visualization and reporting

- The data has to be condensed into a visualization to provide input for decisions.
- Depending on the goal, very very different visualizations are possible.
- Use a model to indicate what is undersampled or oversampled.

### Summarizing and visualizing data as a starting point for more analysis later on.

- Computing summary statistics (e.g. means and variance)
- Determining conditional probabilities of cause+effect relationships
- Calculating correlation and rank correlation coefficient between two variables
- Visualizing univariate, bivariate and multivariate data
- ...

## Exploratory data analysis

### Summarizing and visualizing data as a starting point for more analysis later on.

- ...
- Estimating probability coverage levels for different distributions
- Analyzing behaviour of normal distributions
- Calculating confidence interval and sampling distribution for the mean
- Testing for significance of difference in means
- Comparing two different distributions for statistical equivalence
- Developing a nonparametric regression model from given data
- Reducing data dimensionality
- Grouping data

## Random Variables

- A random variable is a real valued function that assigns a value to each outcome in the sample space
- A random variable (RV) can be either discrete or continuous
  - Discrete RV
  - Continuous RV
- The probability mass function (PMF),  $P$ , of a discrete RV,  $X$ , denotes the probability that the RV is equal to a specified value,  $a$ .

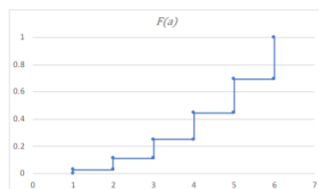
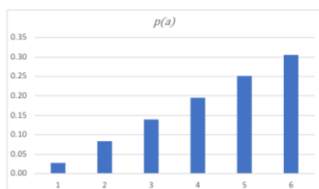
$$p(a) = p(X = a)$$

- The cumulative distribution function (CDF),  $F$ , denotes the sum

$$F(a) = P(X \leq a) = \sum_0^a f(x)dx$$

## Random Variables

$a$	1	2	3	4	5	6
$p(a)$	1/36	3/36	5/36	7/36	9/36	11/36
$F(a)$	1/36	4/36	9/36	16/36	25/36	1



## Sampling

- What are the effective sampling strategies? (Wind turbine example)
- Solar Panels to determine the efficiency of the source (Usage patterns, energy production forecast)
- Drilling (penetration rate)
- Corrosion extension
- Concrete Rigidity
- etc

## Wind turbine example example

## Sampling approaches

### Experimental design

Grid, parallel, series.

### Sampling without replacement

SPR (single point representation).

### Sampling with replacement

The number of the members of the population does not change.

Turbine	Height	X	Y	Wind Speed	Air Density	Temperature	Power Output	Rotor Diameter	Hub Height	Air Pressure	Turbulence Intensity
WT-1	80	752.1	3945	7.5	1.225	15	1500	82	80	1013	0.1
WT-1	80	752.2	3945	8	1.223	15	1600	82	80	1012	0.12
WT-1	80	752.3	3945	7.8	1.224	16	1550	82	80	1013	0.11
WT-2	90	753.5	3946	6.5	1.226	14	1400	85	90	1012	0.15
WT-2	90	753.6	3946	7	1.225	14	1500	85	90	1011	0.13
WT-2	90	753.7	3946	7.2	1.227	14	1520	85	90	1012	0.14

## Univariate statistics

- Easy to displaying data:
  - histogram
  - frequency plots
  - cumulative
- Measures of Location
  - Mean, median, mode
  - Quartiles, Percentiles, Quantiles
- Measure of Dispersion (Spread)
  - Standard deviation (sd)
  - Sariance (Var) or coefficient of variation
- Measures of shape
  - Skewness, modality

## Histograms

- Task 1: make a histogram from a 2d random distribution
- Task 2: make a 2d heat map from a 2d random distribution

## Frequency plots and Histograms

## Class widths

Given a set of data

- 1 Look for min and max values
- 2 Divide the range of values into a number of sensible class intervals (bins)
- 3 Count
- 4 Make a frequency table (or percentage)
- 5 Plot (see jupyter notebook)

Does this histogram represent uncertainty?

No. It shows variability, but it can be used to quantify uncertainty.

- Class widths (bin sizes) are usually CONSTANT
  - the height of each bar is proportional to the number of values in it
- If class width are VARIABLE
  - the AREA of each bar is proportional to the number of values in it
- For small samples, the shape of the histogram can be very sensitive to the number and definition of the class intervals

### Exercise

Plot a histogram from different random number distributions and bin sizes.

## Cumulative Histogram

- Cumulative frequency
- Each data point can be plotted individually
- It helps to read quantiles and compare distributions
- Practice with your jupyter notebook

## Measure of Location: Central Tendency, MEAN

$$m_x = \langle x \rangle = \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

Each point weighted equally by  $\frac{1}{n}$  (assumption)

- Every element in the data set contributes to the values of the mean
- An average provides a common measure for comparing one set of data to another
- The mean is influenced by the extreme values in the data set
- The mean may not be an actual element if the dataset
- The sum of all deviation from the mean is zero, and the sum of squared deviation is minimized when those deviations are measured from the mean

## Means

- Arithmetic
  - Mean of raw data

$$\frac{1}{n} \sum_{i=1}^n x_i$$

- Geometric
  - $n^{th}$  root of product

$$\left( \prod_{i=1}^n x_i \right)^{\frac{1}{n}}$$

- Geometric

\* Mean of logarithms

$$\exp \left( \frac{1}{n} \sum_{i=1}^n \ln(x_i) \right)$$

- Harmonic

- Mean of inverses  $\left( \frac{1}{n} \sum_{i=1}^n \frac{1}{x_i} \right)^{-1}$

## Median

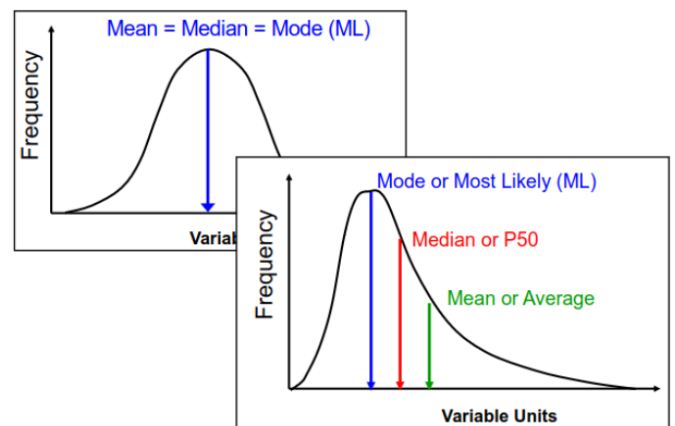
```
if n is odd:
    median = x[(n+1)/2]
else:
    median = x[n/2] + x[(n/2)+1]
```

- On a cumulative density plot, the value of the x-axis that corresponds to 50 % of the y-axis
- Not influenced by extreme values
- May not be contained in the dataset (if n is even)
- For a perfectly symmetrical dataset, means = median

## Mode

- The mode is the most frequently occurring data element or the most likely or most probable value (for a pmf)
- A data set may have more than one mode and it thus called multimodal
- A mode is always a data element in the set
- For a perfectly symmetrical dataset, means = median = mode

## Distribution Descriptors



## Quantiles

### Quartiles

The data split into quarters.

### Deciles

The data are split into tenths. The fifth decile is also the median.

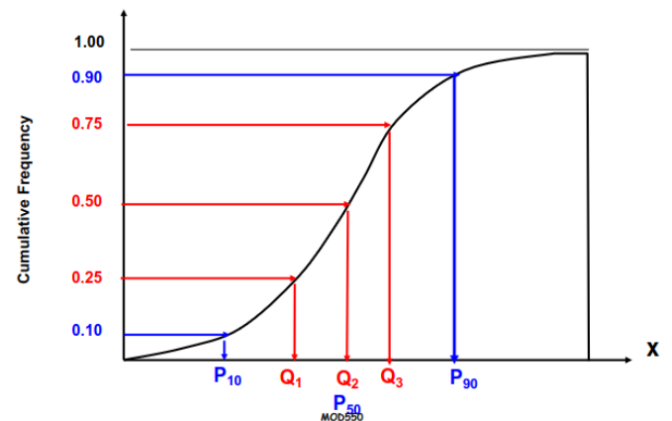
### Percentiles

The data are split into hundredths. P10, P25, P50, P75 and P90 are the most commonly used.

### Quantiles

A generalization of splitting data into any fraction

## Distribution Descriptors



## Dispersion (Spread)

### Range

$$R = \text{maximum} - \text{minimum}$$

### Inter-quantile Range

$$IQR = Q3 - Q1$$

### Mean Deviation from the Mean

$$MD = \sum_{i=1}^n (x_i - \bar{x}) / n$$

### Mean Absolute Deviation

$$MAD = \sum_{i=1}^n |x_i - \bar{x}| / n$$

## Variance

The variance is the average of squared differences between the sample data points and their mean

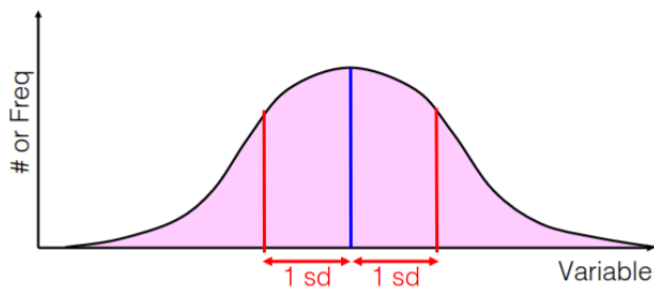
### Variance

$$s_x^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

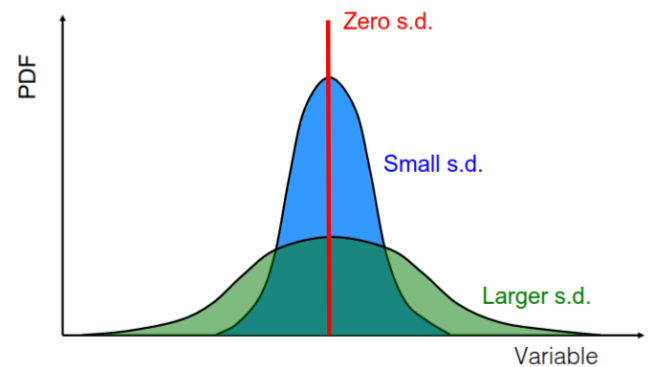
### Standard Deviation (SD)

$$s_x = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

## Standard Deviation



## Standard Deviation



## Measures of dispersion

### Standard Deviation (SD)

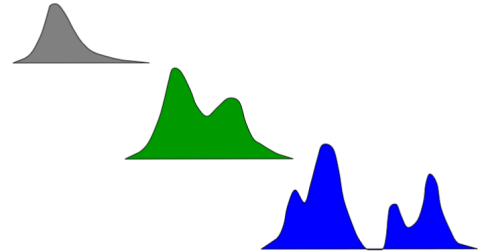
$$SE_x = \frac{s_x}{\sqrt{n}}$$

### Coefficient of Variability

$$CV = \frac{s_x}{\bar{x}}$$

## Modality

- Unimodal
- Bimodal
- Polymodal

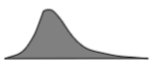


## Skewness

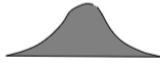
It measures the symmetry in a distribution

$$Sk = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{s^3}$$

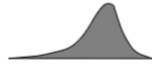
Positive - Values clustered toward the lower end



Zero - Symmetric distribution



Negative - Values clustered toward the higher end



A bit out of fashion with ML

## Distribution Models

### Distribution

means of expressing uncertainty or variability

### Models

- Uniform: useful when only upper and lower bounds are known
- Triangular: useful when estimates of min, max, mode [P10, P50, P90] are available
- Normal: symmetric model of random errors or unbiased uncertainties with mean and standard deviation specified
  - very common for observed data
  - additive processes tend to be normal as a result of the Central Limit Theorem
- log normal comes from multiplicative uncertainties with mean and standard deviation specified

## Uniform Distribution

- The uniform distribution is useful as a rough model for representing low states of knowledge when only the upper and lower bounds are known.
- All possible values within the specified maximum and minimum values are equally likely ( $b=\text{max}$ ,  $a=\text{min}$ ):
- It can express maximum uncertainty

PDF:  $f(x) =$

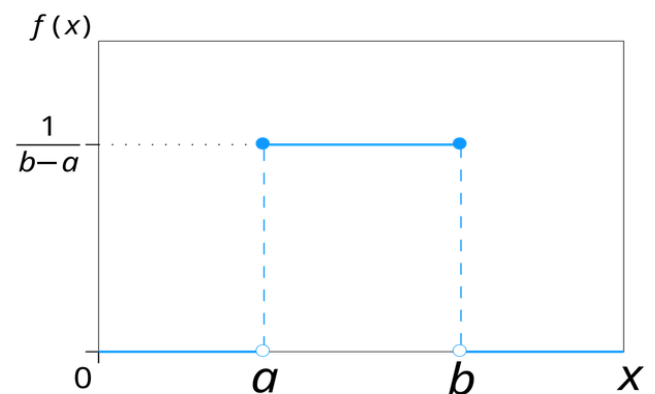
$$\frac{1}{b-a}, a \leq x \leq b$$

CDF:  $F(x) =$

$$\frac{x-a}{b-a}$$

Notation:  $X \sim U(a, b)$

## Uniform Distribution



## Triangular distribution

- The triangular distribution can be used for modeling situations, where non extremal (central) values are more likely than the upper and lower bounds.
- Take min, mode and max as inputs. Typically on the basis of subjective judgement:

PDF:  $f(x) =$

$$\frac{2(x-a)}{(b-a)(c-a)}; \text{ if } a \leq x \leq c$$

$$\frac{2(b-x)}{(b-a)(c-a)}; \text{ if } c \leq x \leq b$$

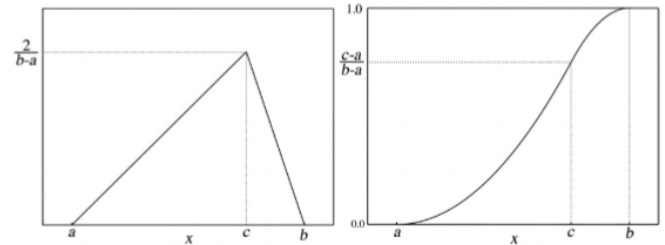
CDF:  $F(x) =$

$$\frac{(x-a)^2}{(b-a)(c-a)}; \text{ if } a \leq x \leq c$$

$$1 - \frac{(b-x)^2}{(b-a)(c-a)}; \text{ if } c \leq x \leq b$$

## Triangular Distribution

Notation:  $X \sim T(a, b, c)$



It can be symmetric or asymmetric

## Normal Distribution

- The normal distribution ('bell curve' or Gaussian) for modeling unbiased uncertainties and random errors of the additive kind of symmetrical distributions of many material processes and phenomena.
- A commonly cited rationale for assuming normal distribution is the central limit theorem, which states that the sum of independent observations asymptotically approaches a normal distribution regardless of the shape of the underlying distributions(s=

PDF:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2 \right\}; \quad -\infty \leq x \leq \infty$$

CDF:  $F(x) =$

has no closed form solution but is often presented using the complementary error function solution

## Normal Distribution

Notation:  $X \sim G(\mu, \sigma)$

It is a Symmetric distribution around the mean

$\mu$  is the mean,  $\sigma$  is the standard deviation

$\mu \pm \sigma$  : 68.3% probability

$\mu \pm 2\sigma$  : 95.4% probability

$\mu \pm 3\sigma$  : 99.7% probability

## Normal Distribution

