Belief-dependent motivations and updating

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Motivation

Investors overreact to information, consumers refrain from learning about firms' unethical practices, and patients at health risk do not learn about their condition.

These observations are at odds with expected utility theory.

Theories of belief-dependent motivations (BDM) explain these phenomena.

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These observations are at odds with expected utility theory.

Theories of belief-dependent motivations (BDM) explain these phenomena.

It is unclear whether Bayesian updating and BDM are consistent assumptions.

Question: how do BDM affect belief updating?

This Paper

I develop a theory of BDM in a dynamic setting encompassing previous models.

An individual receives information and takes action under uncertainty.

She distorts her Bayesian update to satisfy her belief-dependent preferences.

Maximises a combination of true and distorted belief-dependent expected utility.

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I develop a theory of BDM in a dynamic setting encompassing previous models.

An individual receives information and takes action under uncertainty.

She distorts her Bayesian update to satisfy her belief-dependent preferences.

Maximises a combination of true and distorted belief-dependent expected utility.

Key observation: Choices over menus allow identyfing the model components.

Main result: axiomatic representation of BDM preferences and updating rules.

Example: overinvestment and information avoidance

A financial investor likes believing the market is in good state.

She might receive a neutral or bad message about the market state.

The neutral message does not rule out the good state, the bad is conclusive evidence.

After the neutral message, she overweights the evidence and overinvests.

After the bad message, she suffers from the news.

When choosing whether to receive information, she weighs these two outcomes.

Literature

- *Belief-Dependent Motivations.* Eliaz & Spiegler (2006), Bénabou & Tirole (2016), Golman et al. (2017), Battigalli & Dufwenberg (2022).

  Contribution: **Interaction between preferences and belief revision**.

- *Decision Theory.* Liang (2017), Dillenberger & Raymond (2020) Rommeswinkel et al. (2023).

  Contribution: **Belief revision rule**.

- *Menu Choice.* Gul & Pesendorfer (2001), Ozdenoren (2002), Epstein (2006), Epstein & Kopylov (2007).

  Contribution: **Novel primitive object of choice**.

Model

Primitives:

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- compact metric outcome set X;
- finite state spaces S with prior $p \in \Delta(S)$;
- acts map states to outcomes $f : S \longrightarrow X$;
- experiments map states to finite distribution of messages $F : S \to \Delta(M)$;
- Bayesian posterior after observing message m from experiment F is $p_{F,m}$.

Utility

The individual:

1. observes message m from experiment F and computes Bayesian posterior $p_{F,m}$;
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Comments

$$U\left(f, p_{F,m}\right) = \sum_{s} u\left(f_s, p_{F,m}\right) p_{F,m}\left(s\right) + \alpha_{p_{F,m}} \sum_{s} u\left(f_s, p^*_{F,m}\right) p^*_{F,m}\left(s\right).$$

The distorted posterior p^* is the Bayesian update under a different experiment.

It depends on preferences, it is the preferred posterior according to the utility u.

The probability of states wich are ruled out by the message remains zero.

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If the message induces the preferred p^*, then the model collapses to:

$$U\left(f, p^*_{F,m}\right) = \sum_s u\left(f_s, p^*_{F,m}\right) p^*_{F,m}(s).$$

If u does not depend on beliefs, it collapses to expected utility.

Interpretation

The individual's might:

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- exhibit motivated inattention (Caplin & Dean, 2015);
- be optimistic (Sharot, 2011).

All these interpretations are consistent with the model.

Distorsion is with respect to the individual's **subjective beliefs**.

Example: overinvestment and information avoidance

The market might be in a high or low state.
Experiment delivers a neutral n or bad b message.
Preferences are increasing in the posterior on the good state.

$$u\left(\cdot, p^*_{F,m}\right) = v\left(\cdot, p^*_{F,m}\right) - d\left(p^*_{F,m}, p_{F,m}\right)$$

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$$U\left(f, p_{F,n}\right) = \sum_s v\left(f_s, p_{F,n}\right) p_{F,n}\left(s\right) + \alpha_{F,n}\left[\sum_s v\left(f_s, p^*_{F,m_n}\right) p^*_{F,n}\left(s\right) - d(p^*_{F,n}, p_{F,n})\right].$$

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$$U\left(f, p_{F,b}\right) = \sum_s v\left(f_s, p_{F,b}\right) p_{F,b}(s).$$

Example: Contingent Menus

Say the investor can choose wheter to check the status of her portfolio.

By checking, she gets a signal on the state of the market and can invest or withdraw.

If she does not check, she gets no information and can do nothing.

Checking delivers information and induces a **state contingent menu of choices**.

Example: Temptation

When choosing to check, the investor anticipates she will overweight the evidence.

She knows she will be tempted to act following her distorted beliefs and overinvest.

She wants to receive information but prefers to commit not to face temptation.

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She knows she will be tempted to act following her distorted beliefs and overinvest.

She wants to receive information but prefers to commit not to face temptation.

If the Bayesian posterior coincides with the preferred one, there is no distortion.

If beliefs are not distorted, there is no temptation.

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- contingent menus map states to distribution of menus $F : S \to \Delta(M)$;
- time 0 preference on contingent menus.

Main Result

If the individual does not exhibit temptation at her preferred posterior, then her behaviour is represented by equation 1 and 2:

$$\mathscr{U}(F) = \sum_{m} \left(\sum_{s} F_s(m) \right) \mathcal{U}\left(m; p_{F,m}\right) \ ; \tag{1}$$

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Utility u, beliefs p, distorted beliefs p^* and cost of self-control α are identified.

Choice at period 2 is described by the following

$$\max_{f \in m} \left[\sum_s u\left(f_s; p_{F,m}\right) p_{F,m}(s) \right.$$

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When choosing act f from menu m at posterior $p_{F,m}$, the cost of temptation is

$$\alpha_{p_{F,m}}\left[\max_{f'\in m}\sum_{s} u\left(f'_s;p^{*}_{F,m}\right)p^{*}_{F,m}\left(s\right)-\sum_{s} u\left(f_s;p^{*}_{F,m}\right)p^{*}_{F,m}\left(s\right)\right].$$

Conclusion and future directions

Theory of BDM and belief updating tested via choices of contingent menus.

BDM and non Bayesian updating should be considered together.

The model predicts same beliefs for same preferences, thus assortativity of beliefs.

Value of information for individuals with BDM preferences.

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Cost of self-control

Identification of α_ℓ allows elaboratig on its behavioral meaning

$$\alpha_\ell = \frac{\mathcal{U}\left(\{f, x\}, \ell\right) - \mathcal{U}\left(\{f, x'\}, \ell\right)}{u\left(x, \ell\right) - u\left(x', \ell\right)}.$$

It is the marginal cost of self-control at likelihood ℓ.

Example: Moral Wiggle Room (Dana et al., 2007)

A dictator in a laboratory experiment is endowed with 10 euros.

She decides how much to transfer to a recipient she is coupled with.

The transfer is subject to an unknown multiplier, which could be high, medium, or low.

The experimenter allows the dictator to choose the transfer from various menus conditional on the multiplier's value.

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

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State	Actions
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Medium	$m\{3\} + (1-m)[0,5]$
Low	$[0,3]$

$$\Downarrow$$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succsim$

State	Actions
High	$h\{3,5\} + (1-h)[5,10]$
Medium	$m\{3,5\} + (1-m)[0,5]$
Low	$[0,3]$

Table: Set-Betweenness

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succ$

State	Actions
High	$h\{7\} + (1-h)[5,10]$
Medium	$m\{7\} + (1-m)[0,5]$
Low	$[0,3]$

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$$\succ$$

State	Actions
High	$h\{7\} + (1-h)[5,10]$
Medium	$m\{7\} + (1-m)[0,5]$
Low	$[0,3]$

$$\Downarrow$$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$$\sim$$

State	Actions
High	$h\{5,7\} + (1-h)[5,10]$
Medium	$m\{5,7\} + (1-m)[0,5]$
Low	$[0,3]$

Table: Strategic Rationality for Best Likelihood

Illustrative Example

An investor decides whether to check the state of her portfolio.

After checking, she decides whether to invest more (i) or withdraw any feasible amount of money, which could be high ($\overline{w}$) or low (w).

Check

State	Actions
Good Normal	$i, [0, \overline{w}]$
Bad	$i, [0, w]$

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Upon observing a high amount in it she infers the state of the market is not bad.

When she sees a low amount, she knows the state of the market is bad.

Illustrative Example

She can't make any inferences or do anything if she does not check.

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

Not Check

State	Actions
Good	
Normal	0
Bad	

Illustrative Example

She could also check and committ not to invest, by delegating to a financial advisor.

Delegate

State	Actions
Good Normal	$[0, \overline{w}]$
Bad	$[0, w]$

Check

State	Actions
Good Normal	$i, [0, \overline{w}]$
Bad	$i, [0, w]$

Not Check

State	Actions
Good Normal Bad	0

ILLUSTRATIVE EXAMPLE

She anticipates to overweight evidence and invest too much.

Therefore, she prefers to commit, but also wants to obtain information.

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

$\succ$

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

$\succ$

Not Check

State	Actions
Good	
Normal	0
Bad	

Table: Commitment under positive prior belief to avoid excessive investment.

"Cognitive" non-Bayesian updating (Epstein, 2006) cannot rationalise this behaviour.

Illustrative Example

If the investor expects the state of the market to be bad, she prefers not to check the portfolio at all to avoid receiving unpleasant information.

Not Check

State	Actions
Good	
Normal	0
Bad	

$\succ$

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

$\succ$

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

Table: Information avoidance under negative prior beliefs, "ostrich effect".

Both excessive trading and the ostrich effect constitutes empirical puzzles in finance (Daniel & Hirshleifer, 2015; Golman et al., 2017).