Belief-dependent motivations and updating

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Motivation

Investors overreact to information, consumers avoid learning about firms' unethical practices, and patients at health risk do not learn about their condition.

Theories of belief-dependent motivations (BDM) explain these phenomena

$$\sum_{x \in X} p(x)\, u(x, p)\,.$$

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$$\sum_{x \in X} p(x)\, u(x, p).$$

Three drawbacks:

- lack of preferences and beliefs identification;
- impossibility to distinguish "desired" from "undesired" beliefs;
- unknown relation between preferences and belief revision.

Question: can we develop a testable theory of BDM?

This Paper

I develop a theory of BDM in a dynamic setting encompassing previous model.

The individual deviates from Bayesian updating to satisfy her preferences.

Axiomatic analysis identifies preferences, prior beliefs and distorted posteriors.

Main result: representation of BDM preferences and belief updating rules.

Illustrative Example

An investor decides whether to check the status of her portfolio.

After checking, she decides whether to invest more (i) or withdraw any feasible amount of money, which could be high ($\overline{w}$) or low (w).

Check

State	Actions
Good Normal	i, $[0, \overline{w}]$
Bad	i, $[0, w]$

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State	Actions
Good Normal	$i, [0, \overline{w}]$
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Upon observing a high amount in it she infers the status of the market is not bad.

When she sees a low amount, she knows the status of the market is bad.

She can't make any inferences or do anything if she does not check.

Check

State	Actions
Good	i, $[0, \overline{w}]$
Normal	
Bad	i, $[0, w]$

Not Check

State	Actions
Good	
Normal	0
Bad	

She could also check and committ not to invest, by delegating to a financial advisor.

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

Check

State	Actions
Good	i, $[0, \overline{w}]$
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Bad	i, $[0, w]$

Not Check

State	Actions
Good	
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Illustrative Example

She anticipates to overweight evidence and invest too much.

Therefore, she prefers to commit, but also wants to obtain information.

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

$\succ$

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

$\succ$

Not Check

State	Actions
Good	
Normal	0
Bad	

Table: Commitment under positive prior belief to avoid excessive investment.

"Cognitive" non-Bayesian updating (Epstein, 2006) cannot rationalise this behaviour.

Illustrative Example

If the investor expects the status of the market to be bad, she prefers not to check the portfolio at all to avoid receiving unpleasant information.

Not Check

State	Actions
Good	
Normal	0
Bad	

$\succ$

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

$\succ$

Check

State	Actions
Good	$i, [0, \overline{w}]$
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Table: Information avoidance under negative prior beliefs, "ostrich effect".

Both excessive trading and the ostrich effect constitutes empirical puzzles in finance (Daniel & Hirshleifer, 2015; Golman et al., 2017).

- *Decision Theory.* Liang (2017), Dillenberger & Raymond (2020) Rommeswinkel et al. (2023).

  Contribution: **Belief revision rule**.

- *Menu Choice.* Gul & Pesendorfer (2001), Ozdenoren (2002), Epstein (2006), Epstein & Kopylov (2007).

  Contribution: **Novel primitive object of choice**.

- *Belief-Dependent Motivations.* Eliaz & Spiegler (2006), Bénabou & Tirole (2016), Golman et al. (2017), Battigalli & Dufwenberg (2022).

  Contribution: **Interaction between preferences and belief revision**.

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- the set of all contingent menus is $\mathcal{C} = \Delta^0(\mathcal{M})^S$;
- time 0 preference $\succsim$ is defined on $\mathcal{C}$.

The likelihood of state s after realisation of menu M from the contingent menu F is

$$\ell_{M,F}(s) := \frac{F_s(M)}{\sum_{s' \in S} F_{s'}(M)}.$$

Given any contingent menu F and menu M, the vector of likelihoods is $\ell_{M,F}$.

In the temptation and self-control model, behavior is represented by the following

$$\mathcal{U}\left(M\right)=\max_{f\in M}\left\{U\left(f\right)+V\left(f\right)-\max_{f'\in M}V\left(f'\right)\right\}.$$

Gul & Pesendorfer (2001)

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Cost of self-control is

$$\max_{f' \in M} V\left(f'\right) - V\left(f\right).$$

Utility

Individual's behavior in this paper is represented by the following model

$$\mathscr{U}(F) = \sum_{M} \left(\sum_{s} F_s(M) \right) \mathcal{U}\left(M; \ell_{M,F}\right) \ ; \tag{1}$$

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$$\mathcal{U}\left(M; \ell_{M,F}\right) = \max_{f \in M} \left\{ \sum_{s} u\left(f_s; \ell_{M,F}\right) p_{\ell_{M,F}}(s) \right.$$

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$$\mathcal{U}\left(M; \ell_{M,F}\right) = \max_{f \in M} \left\{ \sum_s u\left(f_s; \ell_{M,F}\right) p_{\ell_{M,F}}(s) + \alpha_{\ell_{M,F}} \sum_s u\left(f_s; \ell^*_{M,F}\right) p_{\ell^*_{M,F}}(s) \right\}$$

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Belief-dependent motivations **imply** non-Bayesian updating.

When choosing act f from menu M after realisation of the likelihood $\ell_{M,F}$, the utility cost of temptation is

$$\alpha_{\ell_{M,F}} \left[\max_{f' \in M} \sum_{s} u\left(f'_s; \ell^*_{M,F}\right) p_{\ell^*_{M,F}}(s) - \sum_{s} u\left(f_s; \ell^*_{M,F}\right) p_{\ell^*_{M,F}}(s) \right].$$

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Choice at period 2 is described by the following

$$\max_{f\in M}\left[\sum_{s}u\left(f_s;\ell_{M,F}\right)p_{\ell_{M,F}}(s)+\alpha_{\ell_{M,F}}\sum_{s}u\left(f_s;\ell^*_{M,F}\right)p_{\ell^*_{M,F}}(s)\right].$$

Conclusion

Theory of BDM and motivated updating tested via choices of contingent menus.

Future:

- **Applications.**

  1. **Polarisation.** The model predicts similar beliefs for similar preferences, and thus assortativity of beliefs.
  2. **Moral wiggle room.** Experimental design to test motivated belief updating to rationalise selfishness.

- **Theory.** Value of information for individuals with BDM preferences.

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Cost of self-control

Identification of α_ℓ allows elaboratig on its behavioral meaning

$$\alpha_\ell = \frac{\mathcal{U}\left(\{f, x\}, \ell\right) - \mathcal{U}\left(\{f, x'\}, \ell\right)}{u\left(x, \ell\right) - u\left(x', \ell\right)}.$$

It is the marginal cost of self-control at likelihood ℓ.

Example: Moral Wiggle Room (Dana et al., 2007)

A dictator in a laboratory experiment is endowed with 10 euros.

She decides how much to transfer to a recipient she is coupled with.

The transfer is subject to an unknown multiplier, which could be high, medium, or low.

The experimenter allows the dictator to choose the transfer from various menus conditional on the multiplier's value.

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

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State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$$\succsim$$

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State	Actions
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$\succsim$

State	Actions
High	$h\{3\} + (1-h)[5,10]$
Medium	$m\{3\} + (1-m)[0,5]$
Low	$[0,3]$

$$\Downarrow$$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succsim$

State	Actions
High	$h\{3,5\} + (1-h)[5,10]$
Medium	$m\{3,5\} + (1-m)[0,5]$
Low	$[0,3]$

Table: Set-Betweenness

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succ$

State	Actions
High	$h\{7\} + (1-h)[5,10]$
Medium	$m\{7\} + (1-m)[0,5]$
Low	$[0,3]$

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succ$

State	Actions
High	$h\{7\} + (1-h)[5,10]$
Medium	$m\{7\} + (1-m)[0,5]$
Low	$[0,3]$

$$\Downarrow$$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\sim$

State	Actions
High	$h\{5,7\} + (1-h)[5,10]$
Medium	$m\{5,7\} + (1-m)[0,5]$
Low	$[0,3]$

Table: Strategic Rationality for Best Likelihood