Belief-dependent preferences and updating

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Do Beliefs exist?

"Some scholars we met seemed skeptical to the idea of belief-dependent motivation and in the sequel to the idea of eliciting beliefs . . .

These would be people who revere "revealed preference," who argue that beliefs are not real, or at least not observable. Beliefs are merely a theory feature, something that should be viewed only as part of a preference representation . . .

In our view, this position has little merit." (Battigalli & Dufwenberg, 2022)

Reconcile the two views: agents behave **as if** they were motivated by beliefs.

Literature

In the literature of beliefs-dependent motivations (*BDM*), **objective beliefs** are present in various forms.

- *Psychological games*: system of conditional beliefs (Battigalli & Dufwenberg, 2022).
- *Psychological expected utility*: lotteries over future prizes (Caplin & Leahy, 2001).

These models make revealed preference preachers angry.

This Presentation

I sketch the contours of a theory of *BDM* in a Subjective Expected Utility (*SEU*) framework (Savage, 1972).

A model of *BDM* should feature the followings:

- Beliefs are produced by the agent;
- Trade-off between accuracy and wishful thinking;
- Beliefs depend on material reward. (Problematic!)

I showcase these features in examples and attempt to account for them.

Example: guilt aversion

I am a cab driver, and I have expectations regarding the tip I receive from a client.

The client suffers guilt if he does not match my expectations.

It would be very convenient for me to have high expectations.

If the client has 10$, I believe he will tip 10$.

But if he has 50$, I believe he will tip 50$.

Example: a bet

	p	$1-p$
	H	L
a_H	1	0
a_L	0	$1+b$

A *BDM* agent's preferences increase in p.

(Production)

Without *BDM*, a_H is preferred to a_L when $p \geq \frac{1+b}{2+b}$.

When $p \geq \frac{1+b}{2+b}$, there is no "cost" of increasing p, it is "free" to set $p = 1$. (Trade-off)

For different b, the agent will have different p. (Dependence on material outcome)

What does it mean to "elicit beliefs"?

Savage Model

A **finite** decision problem comprises:

- Set of **uncertain states** S;
- Set of **outcomes** X;
- **Acts** mapping uncertain states to outcomes $a : S \to X$;
- **Preferences** $\succsim$ on the set of acts A, represented by

$$U(a) = \sum_{s \in S} u(a(s)) \cdot p(s) \,. \tag{SEU}$$

Probability dependent outcome

Define the **outcome valuation at** p as $t(\cdot, p) : X \to X$ for each belief p.

Preferences are now represented by

$$U(a) = \sum_{s \in S} u\left(t\left(a\left(s\right), p\right)\right) \cdot p(s) \tag{PDOSEU}$$

$$U(a) = \sum_{s \in S} u(a(s)) \cdot p(s)\ . \tag{SEU}$$

This model is inspired by Karni (1992), set in an objective probability framework.

Previous Example

	p	$1-p$
	H	L
a_H	1	0
a_L	0	$1+b$

Set $t(x,p) = x + \mathbf{1}\left\{p \geq \frac{1+b}{2+b}\right\}$.

If $p \geq \frac{1+b}{2+b}$, the outcome evaluation is "kicked" without a change in beliefs.

The quantity $x + \mathbf{1}\left\{p \geq \frac{1+b}{2+b}\right\}$ represents a "compensation" for not distorting beliefs.

Beliefs are not affected by a change in b.

Conclusion

Current modeling of belief-dependent motivations relies on objective beliefs.

Attempt to capture belief-dependent motivations in subjective expected utility.

Hard to disentangle probabilities affecting conjectures on uncertain states and probabilities involving outcome evaluations.

References

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