Belief-dependent motivations and belief updating

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Motivation

Investors are overconfident, consumers avoid learning about firms' unethical practices, and patients at health risk do not learn about their condition.

Theories of belief-dependent motivations (BDM) explain these phenomena.

Three drawbacks:

- unknown relation between BDM and belief revision;
- lack of preferences and beliefs identification;
- impossibility to distinguish "desired" from "undesired" beliefs.

This Paper

Axiomatic analysis of BDM in a dynamic setting.

An individual selects which menu to choose from in different states of the world.

Observation of the menu allows to make inferences about the state.

Main result: functional representation of preferences over contingent menus:

- individual anticipates to update beliefs in the direction of her BDM;
- she is tempted to act according to such beliefs rather than the Bayesian update;
- she thus chooses the contingent menu accordingly.

Literature

- *Decision Theory.* Liang (2017), Dillenberger & Raymond (2020) Rommeswinkel et al. (2023);
- *Menu Choice.* Gul & Pesendorfer (2001), Ozdenoren (2002), Epstein (2006), Epstein & Kopylov (2007);
- *Belief-Dependent Motivations.* Eliaz & Spiegler (2006), Bénabou & Tirole (2016), Golman et al. (2017), Battigalli & Dufwenberg (2022).

Example: Moral Wiggle Room (Dana et al., 2007)

A dictator in a laboratory experiment is endowed with 10 euros.

She decides how much to transfer to a recipient she is coupled with.

The transfer is subject to an unknown multiplier, which could be high, medium, or low.

The experimenter allows the dictator to choose the transfer from various menus conditional on the multiplier's value.

Example

State	Actions
High	$h\,\{5\} \;+ (1-h)\,[5,10]$
Medium	$m\,\{5\} \;+ (1-m)\,[0,5]$
Low	$[0,3]$

$\succsim$

State	Actions
High	$h\,\{3\} \;+ (1-h)\,[5,10]$
Medium	$m\,\{3\} \;+ (1-m)\,[0,5]$
Low	$[0,3]$

$\Downarrow$

State	Actions
High	$h\,\{5\} \;+ (1-h)\,[5,10]$
Medium	$m\,\{5\} \;+ (1-m)\,[0,5]$
Low	$[0,3]$

$\succsim$

State	Actions
High	$h\,\{3,5\} \;+ (1-h)\,[5,10]$
Medium	$m\,\{3,5\} \;+ (1-m)\,[0,5]$
Low	$[0,3]$

Table: Set-Betweenness

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succ$

State	Actions
High	$h\{7\} + (1-h)[5,10]$
Medium	$m\{7\} + (1-m)[0,5]$
Low	$[0,3]$

$$\Downarrow$$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\sim$

State	Actions
High	$h\{5,7\} + (1-h)[5,10]$
Medium	$m\{5,7\} + (1-m)[0,5]$
Low	$[0,3]$

Table: Strategic Rationality for Best Likelihood

Model

- compact metric outcome set X, the set of lotteries $\Delta(X)$ is compact metric under the weak convergence topology;
- finite state spaces S, a generic event is $E \in 2^S$;
- the set of Anscombe-Aumann (AA) acts over S is $\Delta(X)^S$, a generic act is $f : S \longrightarrow \Delta(X)$;
- a closed subset M of $\Delta(X)^S$ is a menu of acts over S;
- the set of menus is $\mathcal{M}$, it is compact metric under the Hausdorff metric, the set of lotteries with finite support over it is $\Delta(\mathcal{M})$;
- a contingent menu is $F : S \rightarrow \Delta(\mathcal{M})$, the probability that menu M realises if s is the true state is $F(s)(M)$;
- the set of all contingent menus is $\mathcal{C} = \Delta(\mathcal{M})^S$;
- preference $\succsim$ is defined on $\mathcal{C}$.

If menu M realises, the individual knows the true state is in the event

$$M_F = \{s \in S \mid F(s)(M) > 0 \text{ for some } s \in S\}.$$

Likelihood of state s after realisation of menu M from the contingent menu F is

$$\ell^s(M_F) := \frac{F(s)(M)}{\sum_{s' \in S} F(s')(M)}.$$

Given any contingent menu F and menu M, the likelihood is $\ell(M_F) := \left(\ell^s(M_F)\right)_{s \in S}$.

In the temptation and self-control model, behavior is represented by the following

$$\mathcal{U}(M) = \max_{f \in M} \left\{ U(f(s)) + V(f(s)) - \max_{f' \in M} V\left(f'(s)\right) \right\}$$

where U represents the **commitment** ranking and V is the **temptation** ranking.

Individual's behavior in this paper is represented by the following model

$$\mathscr{U}(F) = \sum_{M \in supp(F)} \left(\sum_{s \in S} F(s)(M) \right) \mathcal{U}(M; \ell(M_F)) \tag{1}$$

$$\begin{aligned} \mathcal{U}(M; \ell(M_F)) = & \max_{f \in M} \left\{ \int_{M_F} u(f(s), \ell(M_F))\, dp(\cdot \mid \ell(M_F)) \right. \\ & \left. + \alpha_{\ell_{M_F}} \int_{M_F} u\left(f(s), \ell'_{M_F}\right) dp\left(\cdot \mid \ell'_{M_F}\right) \right\} \\ & - \max_{f' \in M} \alpha_{\ell_{M_F}} \int_{M_F} u\left(f'(s), \ell'_{M_F}\right) dp\left(\cdot \mid \ell'_{M_F}\right). \end{aligned} \tag{2}$$

When choosing act f from menu M after realisation of the likelihood $\ell(M_F)$, the utility cost of temptation is

$$\alpha_{\ell_{M_F}}\left[\max_{f'\in M}\int_{M_F} u\left(f'(s),\ell'_{M_F}\right)dp\left(\cdot\mid\ell'_{M_F}\right)-\int_{M_F} u\left(f(s),\ell'_{M_F}\right)dp\left(\cdot\mid\ell'_{M_F}\right)\right].$$

Choice at period 2 is described by the following

$$\max_{f\in M}\left[\int_{M_F} u\left(f(s),\ell(M_F)\right)dp\left(\cdot\mid\ell(M_F)\right)+\alpha_{\ell_{M_F}}\int_{M_F} u\left(f(s),\ell'_{M_F}\right)dp\left(\cdot\mid\ell'_{M_F}\right)\right].$$

Axiom

(**Order**). The ranking $\succsim$ is complete and transitive.

(**Continuity**). For all contingent menus F' the sets $\{F \mid F \succsim F'\}$ and $\{F \mid F \precsim F'\}$ are closed.

(**Nondegeneracy**). There exist y, y' in X for which $y \succ y'$.

(**Full Support**). For each state s, there exist contingent menus F and F' such that $F \nsim F'$, where for all menus M it holds that $F(s')(M) = F'(s')(M)$ for every $s' \neq s$.

DEFINITION

(***Identical Inference (II)***) *Two contingent menus F and F' satisfy **identical inference** if, for each $M \in supp(F) \cap supp(F')$ their likelihood is the same $\ell(M_F) = \ell(M_{F'})$.*

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

State	Actions
High	$h\{3,5\} + (1-h)[5,10]$
Medium	$m\{3,5\} + (1-m)[0,5]$
Low	$[0,3]$

AXIOM

(***II Independence***). *For all $0 < \lambda \leq 1$ and contingent menus F, F', F'' such that F and F'' satisfy II and F' and F'' satisfy II, $F \succsim F'$ if and only if $\lambda F + (1-\lambda) F'' \succsim \lambda F' + (1-\lambda) F''$.*

Any contingent menu F, for any menu M, can be denoted with (F_{-M}, M).

The two contingent menus (F_{-M}, M) and $(F_{-M'}, M')$ are equivalent except the realisation M of the first is substituted with M' in the second.

Axiom

*(**Set-Betweenness**). For all contingent menus (F_{-M}, M) and $(F_{-M'}, M')$,*

$$(F_{-M}, M) \succsim \left(F_{-M'}, M'\right) \Rightarrow (F_{-M}, M) \succsim \left(F_{-M \cup M'}, M \cup M'\right) \succsim \left(F_{-M'}, M'\right).$$

$$(F_{-M}, M) \succsim (F_{-M'}, M') \Rightarrow (F_{-M}, M) \succsim (F_{-M \cup M'}, M \cup M') \succsim (F_{-M'}, M')$$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succsim$

State	Actions
High	$h\{3\} + (1-h)[5,10]$
Medium	$m\{3\} + (1-m)[0,5]$
Low	$[0,3]$

$\Downarrow$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succsim$

State	Actions
High	$h\{3,5\} + (1-h)[5,10]$
Medium	$m\{3,5\} + (1-m)[0,5]$
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Preferred Likelihood

The set of likelihoods giving positive weights only to states in event E is $\mathcal{L}_E$.

The notation (F_{-M}, M_ℓ) means M induces likelihood ℓ. For each event E

$$\mathcal{L}_E^* := \left\{\ell \in \mathcal{L}_E \;\middle|\; \left(F_{-\Delta(X)}, \Delta(X)_\ell\right) \succsim \left(F_{-\Delta(X)}, \Delta(X)_{\ell'}\right) \text{ for all } \ell' \in \mathcal{L}_E\right\}.$$

For each menu M and likelihood ℓ_E

$$M_{\ell_E}^* := \left\{f \in M \;\middle|\; \left(F_{-\{f\}}, \{f\}_{\ell_E}\right) \succsim \left(F_{-\{f'\}}, \{f'\}_{\ell_E}\right) \text{ for all } f' \in M\right\}.$$

Axiom

*(**Strategic Rationality for Best Likelihood**). For all not empty menus M, M' and likelihoods $\ell_E \in \mathcal{L}_E$, if $(M \cup M')^*_{\ell_E} \cap (M \cup M')^*_{\ell^*_E} \neq \emptyset$ for some $\ell^*_E \in \mathcal{L}^*_E$, then*

$$\left(F_{-M}, M_{\ell_E}\right) \succsim \left(F_{-M'}, M'_{\ell_E}\right) \Rightarrow \left(F_{-M}, M_{\ell_E}\right) \sim \left(F_{-M \cup M'}, M \cup M'_{\ell_E}\right).$$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succ$

State	Actions
High	$h\{7\} + (1-h)[5,10]$
Medium	$m\{7\} + (1-m)[0,5]$
Low	$[0,3]$

$\Downarrow$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\sim$

State	Actions
High	$h\{5,7\} + (1-h)[5,10]$
Medium	$m\{5,7\} + (1-m)[0,5]$
Low	$[0,3]$

Menus of objective lotteries are denoted with $L \subseteq \Delta(X)$.

The notation fsf' indicates an act being $f(s)$ in state s and $f'(s')$ in all states $s' \neq s$.

For any state s and menus M, M', define the menu $MsM' := \{fsf' \mid f \in M, f' \in M'\}$.

AXIOM

*(**State Independence**). For all states s, s' and menus L, L', M,*

$$\left(F_{-LsM}, LsM\right) \succsim \left(F_{-L'sM}, L'sM\right) \Rightarrow \left(F_{-Ls'M}, Ls'M\right) \succsim \left(F_{-L's'M}, L's'M\right).$$

THEOREM

The ranking $\succsim$ satisfies **Order**, **Continuity**, **II Independence**, **Non Degeneracy**, **Set Betweenness**, **Strategic Rationality for Best Likelihood**, **State Independence** and **Full Support** if and only if it can be represented by Equations 1 and 2.

$$\mathscr{U}(F) = \sum_{M \in supp(F)} \left(\sum_{s \in S} F(s)(M) \right) \mathcal{U}(M; \ell(M_F)) \tag{1}$$

$$\begin{aligned} \mathcal{U}(M; \ell(M_F)) = & \max_{f \in M} \left\{ \int_{M_F} u(f(s), \ell(M_F))\, dp(\cdot \mid \ell(M_F)) \right. \\ & \left. + \alpha_{\ell_{M_F}} \int_{M_F} u\left(f(s), \ell'_{M_F}\right) dp\left(\cdot \mid \ell'_{M_F}\right) \right\} \\ & - \max_{f' \in M} \alpha_{\ell_{M_F}} \int_{M_F} u\left(f'(s), \ell'_{M_F}\right) dp\left(\cdot \mid \ell'_{M_F}\right). \end{aligned} \tag{2}$$

Corollary

Let (u, p, α) represent $\succsim$, then (u', p', α') also represent $\succsim$ if and only if there exists $(a, b) \in \mathbb{R}_{++} \times \mathbb{R}$ such that for all likelihoods ℓ

$$u'(\cdot, \ell) = au(\cdot, \ell) + b \quad and \quad p' = p$$

and either

$$\alpha'(\ell)\left[p\left(\cdot \mid \ell'\right) - p(\cdot \mid \ell)\right] = 0 = \alpha(\ell)\left[p\left(\cdot \mid \ell'\right) - p(\cdot \mid \ell)\right]$$

$$or \quad \alpha'(\ell) = \alpha(\ell).$$

Conclusion

Theory of motivated belief updating based on choices of contingent menus.

A second interpretation is a change of prior rather than a distorsion of the likelihood.

The model predicts similar beliefs for similar preferences.

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Cost of self-control

Identification of α_ℓ allows elaboratig on its behavioral meaning.

$$\alpha_{\ell_E} = \frac{\mathcal{U}\left(\{f, x\}, \ell_E\right) - \mathcal{U}\left(\{f, x'\}, \ell_E\right)}{u\left(x, \ell_E\right) - u\left(x', \ell_E\right)}.$$

It is the marginal cost of self-control at likelihood ℓ.

Example

An investor decides whether to check the status of her portfolio.

Once the investor checks the portfolio, she decides whether to invest more (i) or withdraw any feasible amount of money, which could be high ($\overline{w}$) or low (w).

State	Actions
Outstanding Good	i, $[0, \overline{w}]$
Bad	i, $[0, w]$

When she checks her portfolio, upon observing a high amount in it she infers the status of the market is not bad.

When she sees a low amount, she knows the status of the market is bad.

Example

She can't make any inferences or do anything if she does not check.

<table>
<tr><th>State</th><th>Actions</th></tr>
<tr><td>Outstanding</td><td rowspan="2">i, $[0, \overline{w}]$</td></tr>
<tr><td>Good</td></tr>
<tr><td>Bad</td><td>i, $[0, w]$</td></tr>
</table>

<table>
<tr><th>State</th><th>Actions</th></tr>
<tr><td>Outstanding</td><td rowspan="3">0</td></tr>
<tr><td>Good</td></tr>
<tr><td>Bad</td></tr>
</table>

Example

She could also check and committ not to invest, by delegating a financial advisor.

State	Actions
Outstanding Good	$[0, \overline{w}]$
Bad	$[0, w]$

State	Actions
Outstanding Good	$i, [0, \overline{w}]$
Bad	$i, [0, w]$

State	Actions
Outstanding Good Bad	0

Example

She anticipates to overweight evidence and prefers to committ, but also preferes to obtain information.

State	Actions
Outstanding	$[0, \overline{w}]$
Good	
Bad	$[0, w]$

$\succ$

State	Actions
Outstanding	$i, [0, \overline{w}]$
Good	
Bad	$i, [0, w]$

$\succ$

State	Actions
Outstanding	
Good	0
Bad	

Table: Commitment under positive prior belief to avoid excessive investment.

"Cognitive" non-Bayesian updating (Epstein, 2006) cannot rationalise this behaviour.

Example

If the investor expects the status of the market to be bad, she prefers not to check the portfolio at all to avoid receiving unpleasant information.

State	Actions
Outstanding	
Good	0
Bad	

$\succ$

State	Actions
Outstanding	$[0, \overline{w}]$
Good	
Bad	$[0, w]$

$\succ$

State	Actions
Outstanding	$i, [0, \overline{w}]$
Good	
Bad	$i, [0, w]$

Table: Information avoidance under negative prior beliefs, "ostrich effect".