Identifying Belief-dependent Preferences

Enrico Mattia Salonia

Toulouse School of Economics

December 4, 2024

Motivation

Individuals **avoid** and **distort** information:

- investors overreact to good news (Daniel & Hirshleifer, 2015);
- donors are uninformed about their impact (Niehaus, 2014);
- medical patients avoid testing (Golman et al., 2017).

A possible rationalisation is that individuals care about beliefs *per se*:

- no separation between beliefs and preferences.
- belief revision depends on preferences;
- hard to identify beliefs, preferences and updating rule.

This paper

This paper:

1. Dynamic model of belief-dependent preferences (BDP);
2. Axiomatic characterisation of such a model;
3. Derivation of individuals' belief updating rule.

Main contribution:

- testable predictions to reject the theory;
- joint identification of beliefs, preferences and updating rule.

Model in a nutshell

Key features:

- the individual cares about beliefs per se;
- she **distorts beliefs** away from Bayesian updating to increase her welfare;
- she is then **tempted** to act according to her distorted beliefs;
- ex ante, she **anticipates** such belief and choice distortion. (Cobb-Clark et al., 2022)

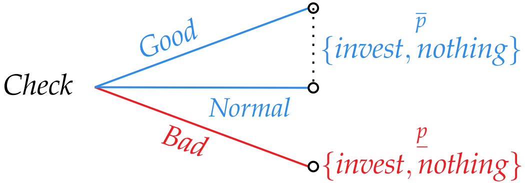
Main result: axiomatic characterisation of BDP preferences and updating rules.

Outline

1. Illustration of the model through an example
2. Comparison with the literature
3. Full Model
4. Main axioms and result

An investor chooses whether to check her portfolio or not.

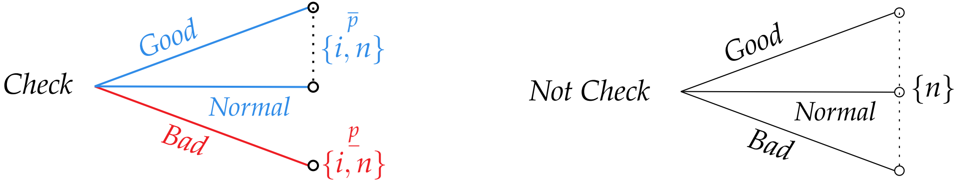
If she does, she observes prices and can invest or do nothing.



Investing is optimal only in the *Good* state.

An investor chooses whether to check her portfolio or not.

If she does, she observes prices and can invest or do nothing.

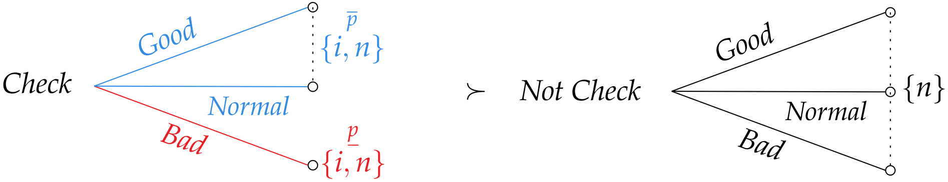


Prices both constitute a **signal** and induce a **menu** of feasible outcomes.

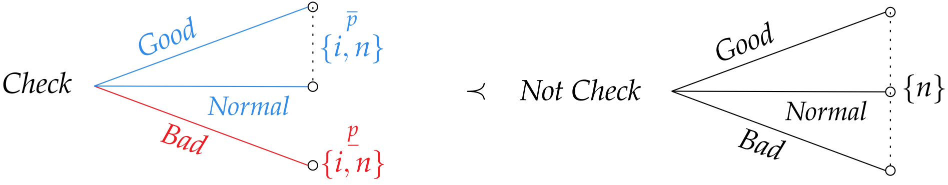
She likes believing the state is good:

- under $\overline{p}$: overweights the positive signal and invests;
- under $\underline{p}$: suffers from the bad news.

Trade-off: receiving pleasant information but acting under a distorted belief.

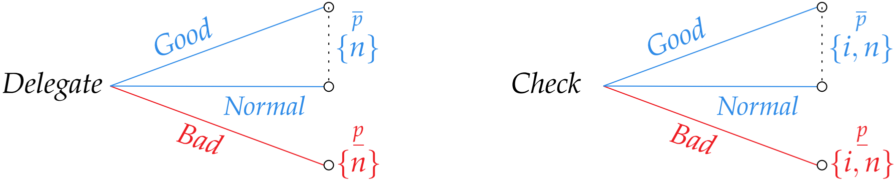


She might also prefer not to check because she expects the bad signal.



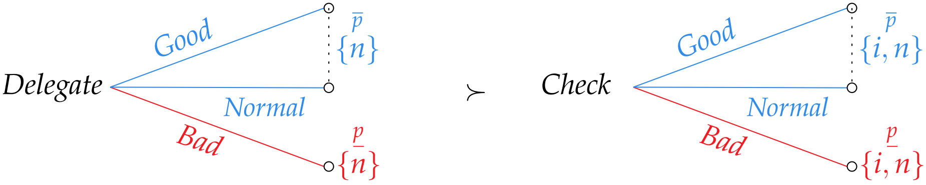
Both excessive trading and the ostrich effect constitute empirical puzzles in finance (Daniel & Hirshleifer, 2015; Golman et al., 2017).

Solution: commit not to invest, e.g. by delegating to a financial advisor or algorithm.



Trade-off: receiving pleasant information but ~~acting under a distorted belief~~.

Solution: commit not to invest, e.g. by delegating to a financial advisor or algorithm.



Trade-off: receiving pleasant information but ~~acting under a distorted belief~~.

Commitment might be welfare enhancing under belief-dependent preferences.

Literature

- *Decision Theory.* Liang (2017), Dillenberger & Raymond (2020) Rommeswinkel et al. (2023).

  Contribution: **Belief revision rule, identification**.

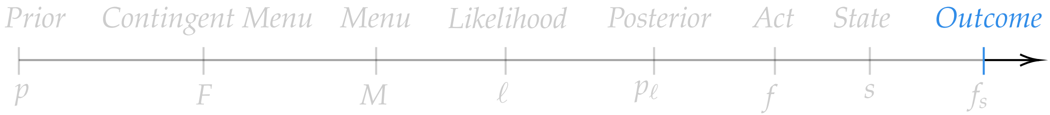
- *Menu Choice.* Gul & Pesendorfer (2001), Ozdenoren (2002), Epstein (2006), Epstein & Kopylov (2007).

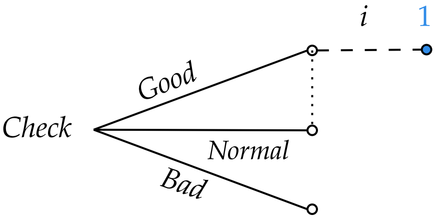
  Contribution: **Novel primitive object of choice**.

- *Belief-Dependent Preferences.* Brunnermeier & Parker (2005), Eliaz & Spiegler (2006), Bénabou & Tirole (2016), Golman et al. (2017), Battigalli & Dufwenberg (2022).

  Contribution: **Generality, derivation of belief revision rule, identification**.

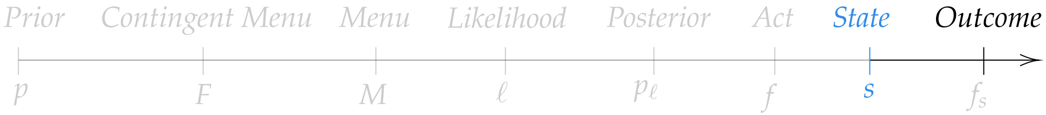
Model: acts

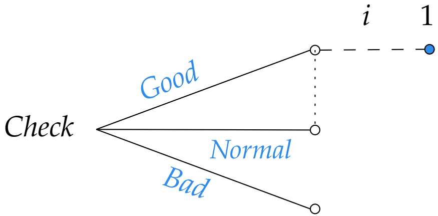




- outcome set (net financial return);

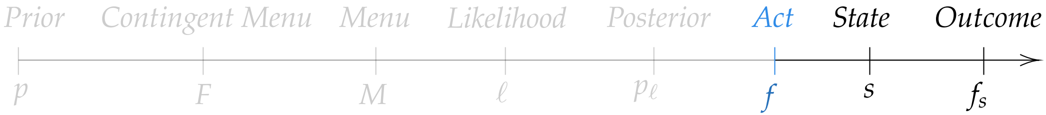
Model: acts

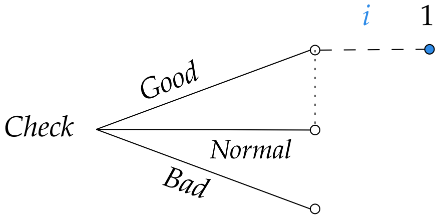




- outcome set;
- state set;

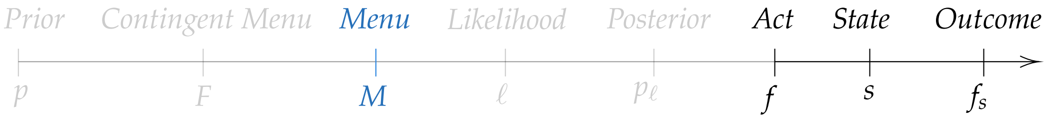
Model: acts

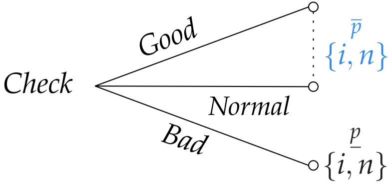




- outcome set;
- state set;
- acts $f : States \rightarrow Outcomes$;

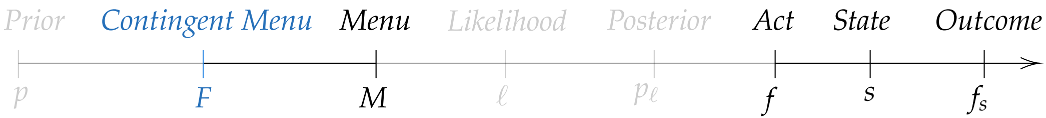
Model: menus and contingent menus

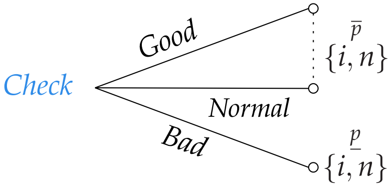




- outcome set;
- state set;
- acts $f : States \to Outcomes$;
- a set of acts is a menu M;

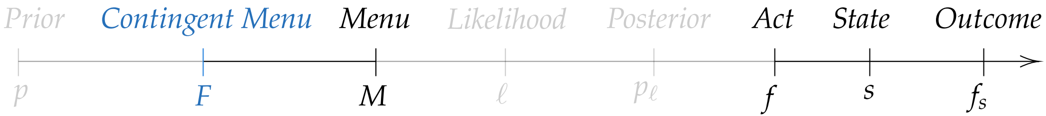
Model: menus and contingent menus

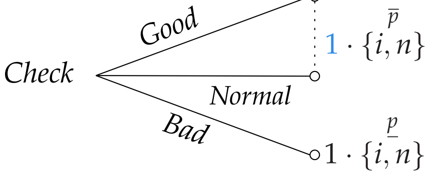




- outcome set;
- state set;
- acts $f : \mathit{States} \to \mathit{Outcomes}$;
- a set of acts is a menu M;
- a contingent menu is $F : \mathit{States} \to \mathit{Menus}$.

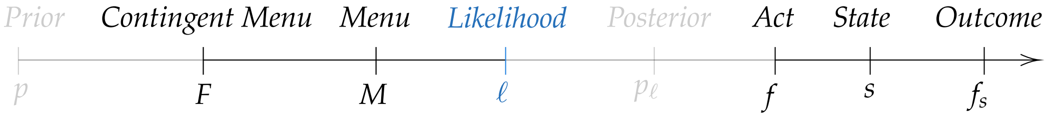
Model: Information

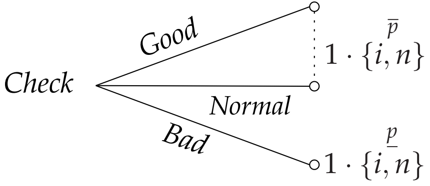




- a contingent menu is $F : States \to Menus$;
- probability M realises in state s is $F_s(M)$;

Model: Information



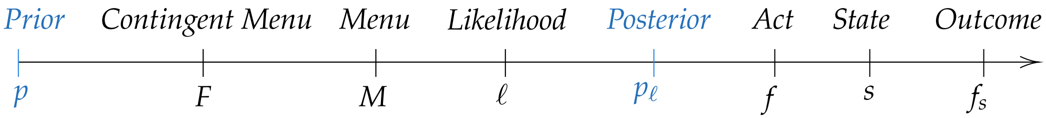


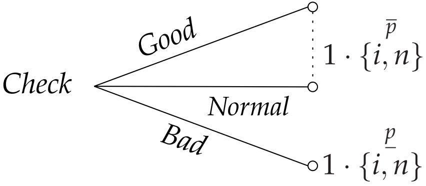
- a contingent menu is $F : \mathit{States} \to \mathit{Menus}$;
- probability M realises in state s is $F_s(M)$;

The normalised likelihood of state *Good* after realisation of menu $\{\overset{\overline{p}}{i}, n\}$ is

$$\ell\,(\mathit{Good}) = \frac{1}{1+1} = \frac{1}{2}.$$

Model: Information





- a contingent menu is $F : States \rightarrow Menus$;
- probability M realises in state s is $F_s(M)$;
- prior p and likelihood ℓ induce posterior p_ℓ.

The likelihood of state s after realisation of menu M from the contingent menu F is

$$\ell_{M,F}(s) = \frac{F_s(M)}{\sum_{s' \in S} F_{s'}(M)}.$$

Utility over acts

Prior	Contingent Menu	Menu	Likelihood	Posterior	Act	State	Outcome
p	F	M	ℓ	p_ℓ	f	s	f_s

Expected utility of act f at likelihood ℓ

$$\sum_s p_\ell (s)\, u\,(f_s; \ell)\,.$$

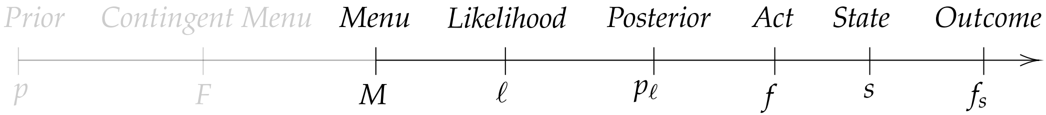
Individual distorts the likelihood to ℓ^*

$$\sum_s p_{\ell^*} (s)\, u\,(f_s; \ell^*)\,.$$

The main theorem identifies conditions under which the individual solves

$$\max_{f \in M} \left\{ \underbrace{\sum_s p_\ell(s) u\,(f_s; \ell)}_{EU} + \alpha_\ell \underbrace{\sum_s p_{\ell^*}(s)\, u\,(f_s; \ell^*)}_{\textit{distorted EU}} \right\}.$$

Utility over acts



Expected utility of act f at likelihood ℓ

$$\sum_s p_\ell(s)\, u\left(f_s; p_\ell\right).$$

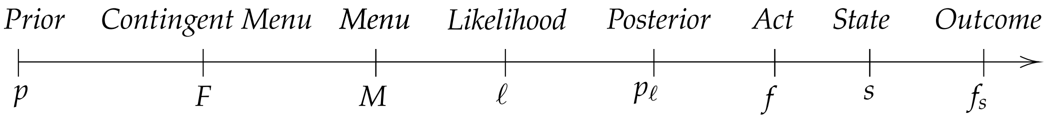
Individual distorts the likelihood to ℓ^*

$$\sum_s p_{\ell^*}(s)\, u\left(f_s; p_{\ell^*}\right).$$

The main theorem identifies conditions under which the individual solves

$$\max_{f \in M} \left\{ \underbrace{\sum_s p_\ell(s) u\left(f_s; p_\ell\right)}_{EU} + \alpha_\ell \underbrace{\sum_s p_{\ell^*}(s)\, u\left(f_s; p_{\ell^*}\right)}_{\text{distorted } EU} \right\}.$$

Utility over contingent menus



The individual anticipates her choices and cost of temptation

$$\mathcal{U}\left(M;\ell\right) = \max_{f \in M} \left\{ \sum_s p_\ell\left(s\right) u\left(f_s;\ell\right) + \alpha_\ell \sum_s p_{\ell^*}\left(s\right) u\left(f_s;\ell^*\right) \right\} - \max_{f' \in M} \alpha_\ell \sum_s p_{\ell^*}\left(s\right) u\left(f'_s;\ell^*\right).$$

The positive number α_ℓ captures the *strength of motivated reasoning*.

Utility over contingent menus

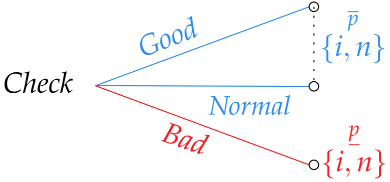
$$\mathcal{U}\left(M;\ell\right)=\max_{f\in M}\left\{\sum_{s}p_{\ell}\left(s\right)u\left(f_{s};\ell\right)+\ \alpha_{\ell}\sum_{s}p_{\ell^{*}}\left(s\right)u\left(f_{s};\ell^{*}\right)\right\}$$
$$-\max_{f'\in M}\alpha_{\ell}\sum_{s}p_{\ell^{*}}\left(s\right)u\left(f_{s}';\ell^{*}\right).$$

Expected utility of contingent menu F is

$$\mathscr{U}\left(F\right)=\sum_{M}\sum_{s}p\left(s\right)F_{s}\left(M\right)\mathcal{U}\left(M;\ell_{M,F}\right).$$

Choices over contingent menus are sufficient for identification of $u, p, \ell^{*}, \alpha_{\ell}$.

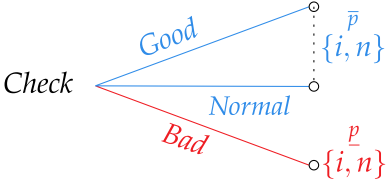
Each likelihood ℓ is consistent with one even S_ℓ.



The distorted likelihood ℓ^*_S at event S is the best one according to u:

$$\ell^*_S \in \arg\max_{\ell \in \Delta(S)} u(x; \ell) \quad \ldots \quad \text{not well defined!}$$

Each likelihood ℓ is consistent with one even S_ℓ.

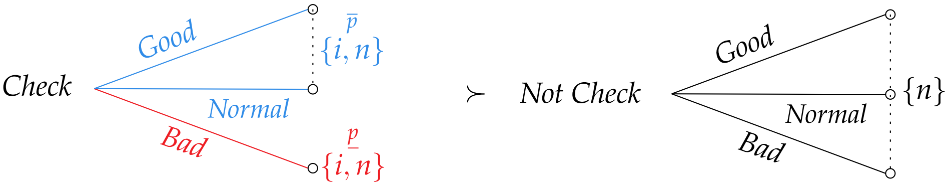


The distorted likelihood ℓ_S^* at event S is the best one *under the best outcome*:

$$\ell_S^* \in \arg\max_{\ell \in \Delta(S)} \max_x \; u\left(x; \ell\right).$$

Asymmetric updating: preferred likelihoods are not distorted. From choice?

Back to the example



Preferences over financial gains and beliefs are:

$$u(x; \ell) = v(x) + p_\ell(\text{Good}).$$

The investor expects to distort ℓ so that $p_{\ell^*}(Good) = 1$.

Optimistic beliefs lead her to invest more than what prescribed by Bayes rule.

BDP imply non-Bayesian updating

Say the true likelihood ℓ coincides with the distorted ℓ^*:

$$\mathcal{U}(M;\ell) = \max_{f\in M}\left\{\sum_s p_\ell(s)\,u(f_s;\ell) + \alpha_\ell \sum_s p_\ell(s)\,u(f_s;\ell)\right\} - \max_{f'\in M} \alpha_\ell \sum_s p_\ell(s)\,u(f'_s;\ell).$$

The second and third them cancel out, only EU under Bayesian updating remains.

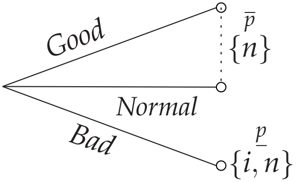
If u does not depend on ℓ, preferences over likelihoods are flat.

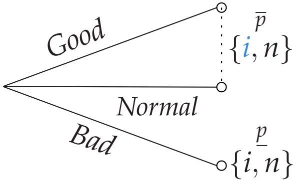
A novelty of the model is that BDP **imply** non-Bayesian updating.

Axiom: Identical Inference Independence

Axiom

*(**Informal**) The individual only satisfies independence for mixtures of contingent menus inducing the same inference for each of their menu realisations.* Full

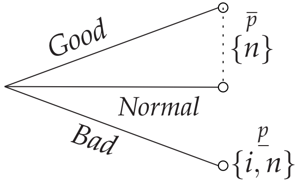


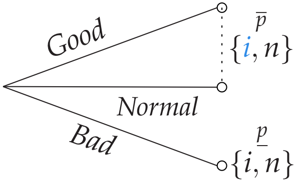


Relaxing independence leads to dynamic inconsistency.

Axiom

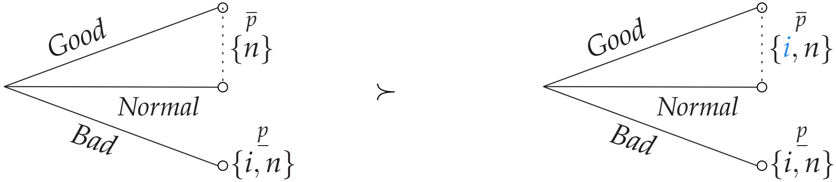
*(**Informal**) There is no temptation when, all else equal, the available menu comprises the best choices based on both the Bayesian update and the favourite posterior.* Full





Axiom

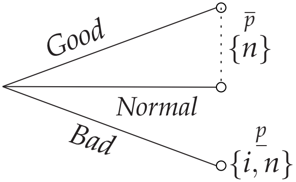
*(**Informal**) There is no temptation when, all else equal, the available menu comprises the best choices based on both the Bayesian update and the favourite posterior.* Full

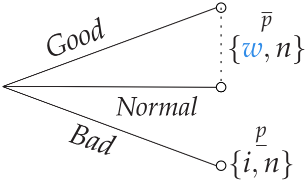


Main Axiom: Strategic Rationality for Best Likelihood

Axiom

*(**Informal**) There is no temptation when, all else equal, the available menu comprises the best choices based on both the Bayesian update and the favourite posterior.* *Full*





Main Axiom: Strategic Rationality for Best Likelihood

Axiom

*(**Informal**) There is no temptation when, all else equal, the available menu comprises the best choices based on both the Bayesian update and the favourite posterior.* Full

Under no BDP, all posteriors are "favourite" and the axiom implies no temptation.

No temptation implies the model reduces to Expected utility and Bayesian updating.

Theorem

*Preferences over contingent menus are represented by Equations (1) and (2) if and only if they satisfy **Strategic Rationality for Best Likelihood** and other "standard" axioms.*

$$\mathscr{U}(F) = \sum_{M} \sum_{s} p(s) F_s(M) \mathcal{U}\left(M; \ell_{M,F}\right). \tag{1}$$

$$\begin{aligned} \mathcal{U}(M;\ell) = \max_{f \in M} \left\{ \sum_{s} p_{\ell}(s) u(f_s;\ell) + \alpha_{\ell} \sum_{s} p_{\ell^*_{S_\ell}}(s) u\left(f_s; \ell^*_{S_\ell}\right) \right\} \\ - \max_{f' \in M} \alpha_{\ell} \sum_{s} p_{\ell^*_{S_\ell}}(s) u\left(f'_s; \ell^*_{S_\ell}\right). \end{aligned} \tag{2}$$

Prior belief p, utilities u, distorted likelihoods ℓ^* and weights α are unique. Axioms?

Why this model

1. Generality: how do individuals choose between information sources?
2. Refutability: predictions of previous theories overlap, also with non BDP.
3. Identification: how to intervene if preferences and beliefs are confused?
4. Dual-self: which self matters for welfare analysis?

Conclusion

Theory of BDP and belief updating tested via choices of contingent menus:

- dynamically consistent individual anticipates she distorts beliefs;
- asymmetric updating and no distortion of zero probability events;
- identification of BDP, non-Bayesian updating and strength of motivated reasoning.

Other applications in the paper:

- donors avoid and distort information about their impact;
- politicians send poor information to induce polarisation.

References

Battigalli, P., & Dufwenberg, M. (2022). Belief-dependent motivations and psychological game theory. *Journal of Economic Literature*, *60*(3), 833–882.

Bénabou, R., & Tirole, J. (2016). Mindful economics: The production, consumption, and value of beliefs. *Journal of Economic Perspectives*, *30*(3), 141–64.

Brunnermeier, M. K., & Parker, J. A. (2005). Optimal expectations. *American Economic Review*, *95*(4), 1092–1118.

Cobb-Clark, D. A., Dahmann, S. C., Kamhöfer, D. A., & Schildberg-Hörisch, H. (2022). The predictive power of self-control for life outcomes. *Journal of Economic Behavior & Organization*, *197*, 725–744.

Daniel, K., & Hirshleifer, D. (2015). Overconfident investors, predictable returns, and excessive trading. *Journal of Economic Perspectives*, *29*(4), 61–88.

Dillenberger, D., & Raymond, C. (2020). Additive-belief-based preferences. *PIER Working Paper*.

Eliaz, K., & Spiegler, R. (2006). Can anticipatory feelings explain anomalous choices of information sources? *Games and Economic Behavior*, *56*(1), 87–104.

Epstein, L. G. (2006). An axiomatic model of non-Bayesian updating. *The Review of Economic Studies*, *73*(2), 413–436.

Epstein, L. G., & Kopylov, I. (2007). Cold feet. *Theoretical Economics*, *2*, 231–259.

Golman, R., Hagmann, D., & Loewenstein, G. (2017). Information avoidance. *Journal of economic literature*, *55*(1), 96–135.

Gul, F., & Pesendorfer, W. (2001). Temptation and Self-Control. *Econometrica*, *69*(6), 1403–1435.

Liang, Y. (2017). Information-dependent expected utility. *Available at SSRN 2842714*.

Niehaus, P. (2014). A theory of good intentions. *San Diego, CA: University of California and Cambridge, MA: NBER*, *111*.

Ozdenoren, E. (2002). Completing the state space with subjective states. *Journal of Economic Theory*, *105*(2), 531–539.

Rommeswinkel, H., Chang, H.-C., & Hsu, W.-T. (2023). Preference for Knowledge. *Journal of Economic Theory*, *214*, 105737.

Cost of self-control

Identification of α_ℓ allows elaborating on its behavioural meaning

$$\alpha_\ell = \frac{\mathcal{U}\left(\{f, x\}, \ell\right) - \mathcal{U}\left(\{f, x'\}, \ell\right)}{u\left(x, \ell\right) - u\left(x', \ell\right)}.$$

It is the marginal cost of self-control at likelihood ℓ. Back

Alternative representation with cost

The representation can be written as either

$$\mathcal{U}(M;\ell) = \max_{f \in M} \left\{ \sum_s p_\ell(s)\, u(f_s;\ell) + \alpha_\ell \sum_s p_{\ell^*_{S_\ell}}(s)\, u\left(f_s;\ell^*_{S_\ell}\right) \right\} - \max_{f' \in M} \alpha_\ell \sum_s p_{\ell^*_{S_\ell}}(s)\, u\left(f'_s;\ell^*_{S_\ell}\right).$$

or

$$\mathcal{U}(M;\ell) = \max_{f \in M} \left\{ \sum_s p_\ell(s)\, u(f_s;\ell) - \alpha_\ell\, c(f,u,S_\ell) \right\}.$$

Back

Axioms: Basics

Axiom

*(**Order**). Preferences over contingent menus are a continuous weak order.* Full

Axiom

*(**Nondegeneracy**). There exist at least one outcome better than another.* Full

Axiom

*(**State Independence**). Preferences over outcomes do not depend on the state.*

Axiom

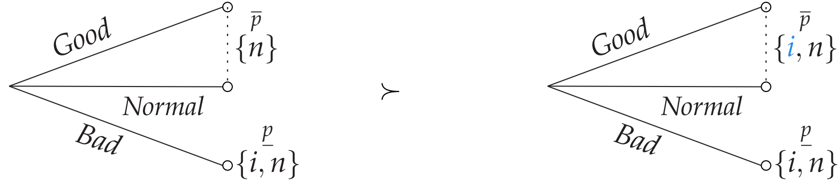
*(**Full support**). The individual assigns ex-ante positive probability to all states.*

Back

Axioms: Set-Betweenness

Axiom

The individual is weakly worse if a menu is enhanced with ex-ante dominated options, because these induce temptation.



Full

Axioms: Basics I

Axiom

*(**Order**). The ranking $\succsim$ is complete and transitive.*

Axiom

*(**Continuity**). For all contingent menus F the sets*

$$\{F' \mid F' \succsim F\} \text{ and } \{F' \mid F' \precsim F\}$$

are closed.

Back

Axioms: Basics II

Substitute from F any occurrence of M with M' to get $F_{M \to M'}$.

Axiom

*(**Nondegeneracy**). There exist outcomes y, y' such that $y \succ y'$.*

Axiom

*(**State Independence**). For all contingent menus F, menus L, L', M and states s, s',*

$$F \succsim F_{LsM \to L'sM} \;\Rightarrow\; F \succsim F_{Ls'M \to L's'M}.$$

Axiom

*(**Full Support**). For each state s, there exist contingent menus F and F' such that for all menus M it holds that $F_{s'}(M) = F'_{s'}(M)$ for every $s' \neq s$ and $F \nsim F'$.*

Back

Axioms: Identical Inference Independence

The support of F is

$$\mathcal{M}_F := \{M \in \mathcal{M} \mid F_s(M) > 0 \text{ for some } s \in S\}.$$

Definition

(**Identical Inference (II)**) Two contingent menus F and F' satisfy **identical inference** if, for each menu $M \in \mathcal{M}_F \cap \mathcal{M}_{F'}$ their likelihood is the same $\ell_{M,F} = \ell_{M,F'}$.

Axiom

*(**II Independence**). For all $0 < \lambda \leq 1$ and contingent menus F, F', F'' such that F and F'' satisfy II and F' and F'' satisfy II, $F \succsim F'$ if and only if $\lambda F + (1-\lambda) F'' \succsim \lambda F' + (1-\lambda) F''$.*

Back

Axioms: Set-Betweenness

Substitute from F any occurrence of M with M' to get $F_{M \to M'}$.

Axiom

*(**Set-Betweenness**). For all contingent menus F and menus M, M',*

$$F \succsim F_{M \to M'} \Rightarrow F \succsim F_{M \to M \cup M'} \succsim F_{M \to M'}.$$

Back

Axioms: Strategic Rationality for Best Likelihood

Substitute from F any occurrence of M with M' to get $F_{M \to M'}$.
For each menu M and likelihood ℓ define the set

$$\mathcal{F}_{M,\ell} := \left\{ f \in M \,\middle|\, F \succsim F_{\{f\} \to \{f'\}} \text{ for all } f' \in M \text{ and some } F \text{ such that } \ell_{\{f\},F} = \ell \right\}.$$

Axiom

*(**Strategic Rationality for Best Likelihood** (**SRBL**)). For each:*

- *couple of menus M, M';*
- *contingent menu F such that $\ell_{M,F} = \ell$;*

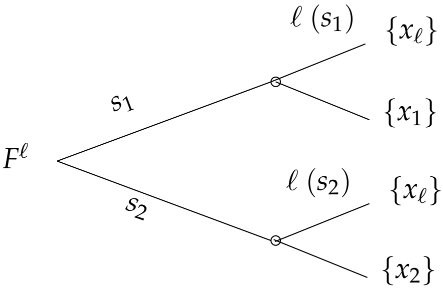
if $\mathcal{F}_{M \cup M',\ell} \cap \mathcal{F}_{M \cup M',\ell^*_{S_\ell}} \neq n$ for at least one $\ell^*_{S_\ell}$, then

$$F \succsim F_{M \to M'} \Rightarrow F \sim F_{M \to M \cup M'}.$$

Back

Distorted likelihoods from choice

For each ℓ define contingent menus F^{ℓ}.



For each S define distorted likelihoods:

$$\ell_S^* \in \left\{ \ell \in \Delta(S) \;\middle|\; F^{\ell} \succsim F^{\ell'} \text{ for all } \ell' \in \Delta(S) \right\}.$$

Back