

IDENTIFYING BELIEF-DEPENDENT PREFERENCES

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MOTIVATION

Individuals **distort** or **avoid** new information:

- investors overreact to good news (Daniel & Hirshleifer, 2015);
- charity donors ignore the impact of their donation (Niehaus, 2014);
- medical patients avoid testing (Golman et al., 2017).

A possible rationalisation is that individuals care about beliefs *per se*:

- well-being depends on beliefs;
- belief revision depends on preferences;
- hard to identify beliefs, preferences and updating rule.

THIS PAPER

This paper:

1. Dynamic model of belief-dependent preferences (BDP);
2. Axiomatic characterisation of such a model;
3. Derivation of individuals' belief updating rule, which depends on preferences.

Main contribution:

- testable predictions;
- joint identification of beliefs, preferences and updating rule.

MODEL IN A NUTSHELL

Key features:

- the individual cares about beliefs per se;
- she **distorts beliefs** away from Bayesian updating to increase her welfare;
- she is then **tempted** to act according to her distorted beliefs;
- ex ante, she **anticipates** such belief and choice distortion. (Cobb-Clark et al., 2022)

Main result: axiomatic characterisation of BDP preferences and updating rules.

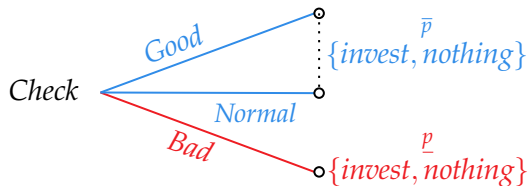
OUTLINE

1. Illustration of the model through an example
2. Comparison with the literature
3. Full model
4. Main axioms and result

ILLUSTRATIVE EXAMPLE

An investor chooses whether to check her portfolio or not.

If she does, she observes prices and can invest or do nothing.

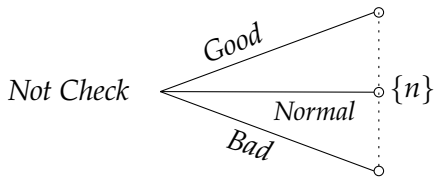
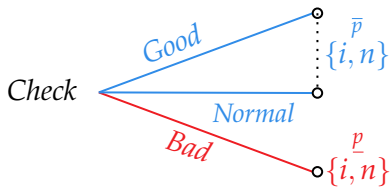


Investing is financially optimal only in the *Good* state.

ILLUSTRATIVE EXAMPLE

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If she does, she observes prices and can invest or do nothing.



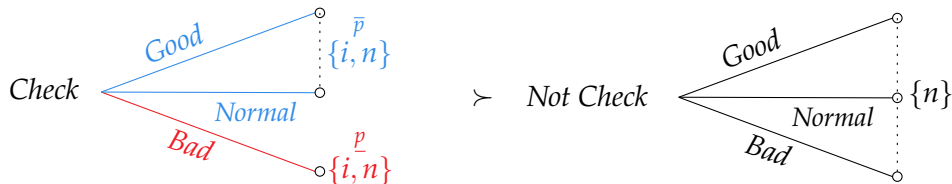
Prices both constitute a **signal** and induce a **menu** of feasible outcomes.

ILLUSTRATIVE EXAMPLE

The investor likes believing the state is good.

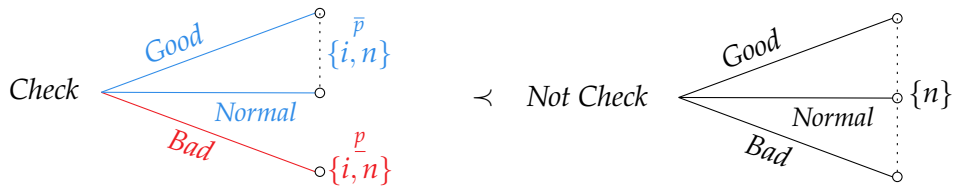
Under \bar{p} she distorts the signal to be more positive and overinvests.

Trade-off: receiving pleasant information but acting under a distorted belief.



ILLUSTRATIVE EXAMPLE

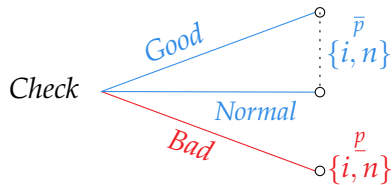
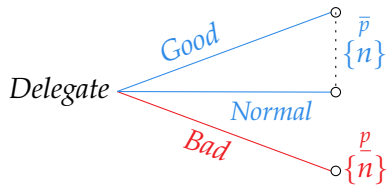
She might also prefer not to check because she is afraid of the bad signal.



Both excessive trading and the ostrich effect constitute empirical puzzles in finance (Daniel & Hirshleifer, 2015; Golman et al., 2017).

ILLUSTRATIVE EXAMPLE

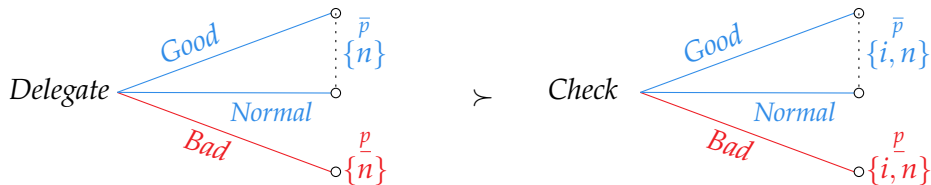
Variante: commit not to invest, e.g. by delegating to a financial advisor or algorithm.



Trade-off: receiving pleasant information but acting under a distorted belief.

ILLUSTRATIVE EXAMPLE

Solution: commit not to invest, e.g. by delegating to a financial advisor or algorithm.



Trade-off: receiving pleasant information but acting under a distorted belief.

Commitment might be welfare enhancing under belief-dependent preferences.

LITERATURE

- *Decision Theory*. Liang (2017), Dillenberger & Raymond (2020) Rommeswinkel et al. (2023).

Contribution: **Belief revision rule, identification.**

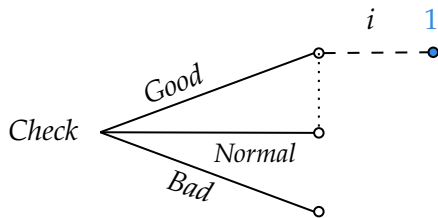
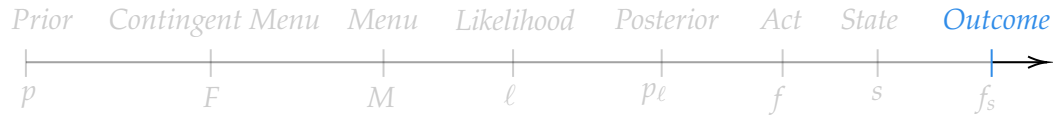
- *Menu Choice*. Gul & Pesendorfer (2001), Ozdenoren (2002), Epstein (2006), Epstein & Kopylov (2007).

Contribution: **Novel primitive object of choice.**

- *Belief-Dependent Preferences*. Brunnermeier & Parker (2005), Eliaz & Spiegel (2006), Bénabou & Tirole (2016), Golman et al. (2017), Battigalli & Dufwenberg (2022).

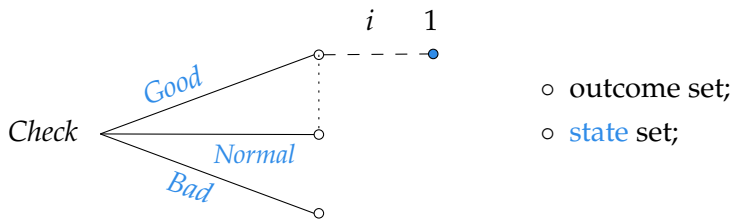
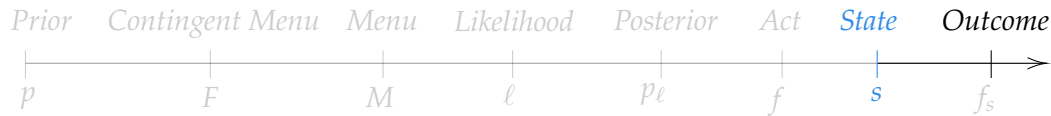
Contribution: **Generality, derivation of belief revision rule, identification.**

MODEL: ACTS

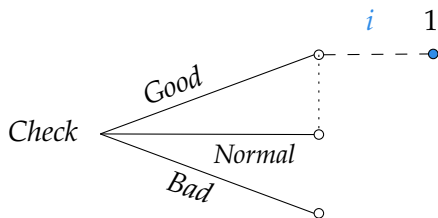
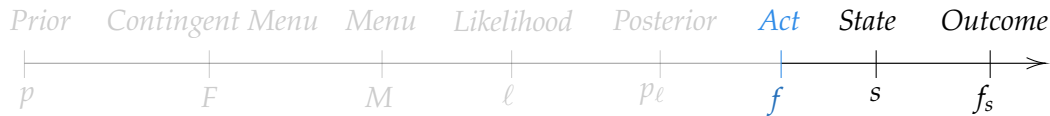


- outcome set (net financial return);

MODEL: ACTS

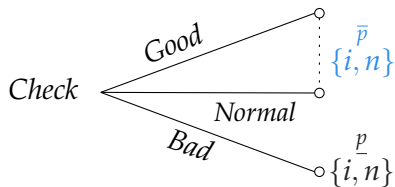


MODEL: ACTS



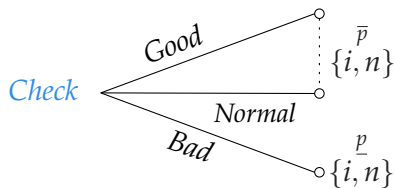
- outcome set;
- state set;
- **acts** $f : \text{States} \rightarrow \text{Outcomes}$;

MODEL: MENUS AND CONTINGENT MENUS



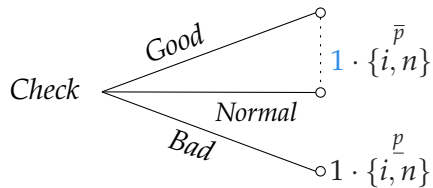
- outcome set;
- state set;
- acts $f : \text{States} \rightarrow \text{Outcomes}$;
- a set of acts is a **menu** M ;

MODEL: MENUS AND CONTINGENT MENUS



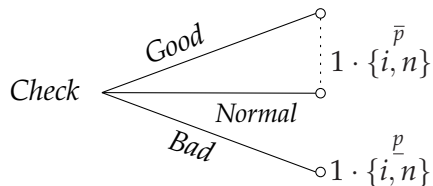
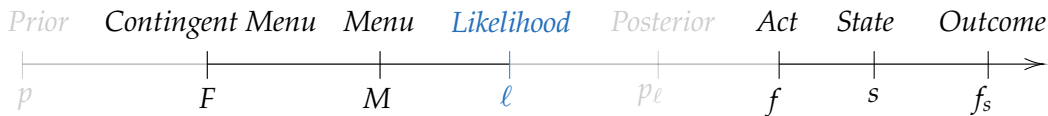
- outcome set;
- state set;
- acts $f : \text{States} \rightarrow \text{Outcomes}$;
- a set of acts is a menu M ;
- a *contingent menu* is $F : \text{States} \rightarrow \text{Menus}$.

MODEL: INFORMATION



- a contingent menu is $F : States \rightarrow Menus$;
- probability M realises in state s is $F_s(M)$;

MODEL: INFORMATION

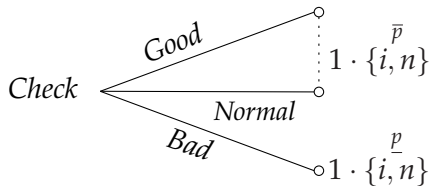
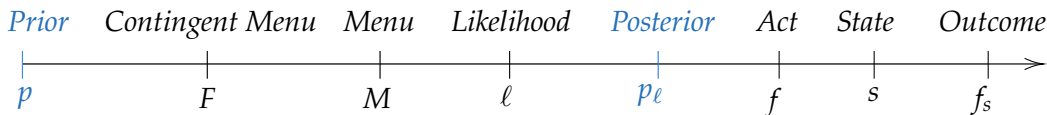


- a contingent menu is $F : States \rightarrow Menus$;
- probability M realises in state s is $F_s (M)$;

The normalised likelihood of state *Good* after realisation of menu $\{i, \bar{n}\}$ is

$$\ell (Good) = \frac{1}{1 + 1} = \frac{1}{2}.$$

MODEL: INFORMATION

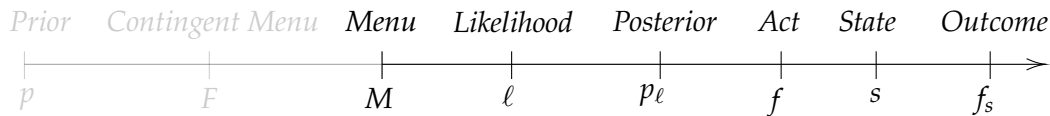


- a contingent menu is $F : States \rightarrow Menus$;
- probability M realises in state s is $F_s(M)$;
- prior p and likelihood ℓ induce posterior p_ℓ .

The likelihood of state s after realisation of menu M from the contingent menu F is

$$\ell_{M,F}(s) = \frac{F_s(M)}{\sum_{s' \in S} F_{s'}(M)}.$$

UTILITY OVER ACTS



Expected utility of act f at likelihood ℓ

$$\sum_s p_\ell(s) u(f_s; \ell).$$

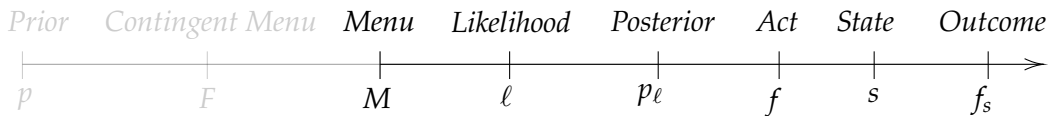
Individual distorts the likelihood to ℓ^*

$$\sum_s p_{\ell^*}(s) u(f_s; \ell^*).$$

The main theorem identifies conditions under which the individual solves

$$\max_{f \in M} \left\{ \underbrace{\sum_s p_\ell(s) u(f_s; \ell)}_{EU} + \alpha_\ell \underbrace{\sum_s p_{\ell^*}(s) u(f_s; \ell^*)}_{\text{distorted EU}} \right\}.$$

UTILITY OVER ACTS



Expected utility of act f at likelihood ℓ

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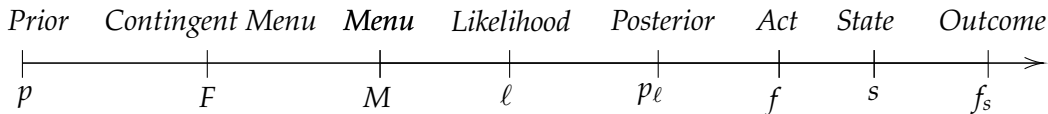
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UTILITY OVER CONTINGENT MENUS



The individual anticipates her choices and cost of temptation

$$\mathcal{U}(M; \ell) = \max_{f \in M} \left\{ \sum_s p_\ell(s) u(f_s; \ell) + \alpha_\ell \sum_s p_{\ell^*}(s) u(\textcolor{blue}{f}_s; \ell^*) \right\} \\ - \max_{f' \in M} \alpha_\ell \sum_s p_{\ell^*}(s) u(\textcolor{red}{f}'_s; \ell^*).$$

The positive number α_ℓ captures the *strength of motivated reasoning*.

Alpha?

Cost?

UTILITY OVER CONTINGENT MENUS

$$\mathcal{U}(M; \ell) = \max_{f \in M} \left\{ \sum_s p_\ell(s) u(f_s; \ell) + \alpha_\ell \sum_s p_{\ell^*}(s) u(f_s; \ell^*) \right\} \\ - \max_{f' \in M} \alpha_\ell \sum_s p_{\ell^*}(s) u(f'_s; \ell^*) .$$

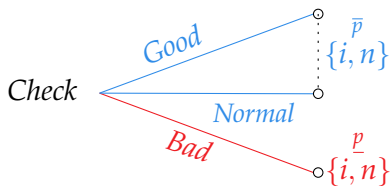
Expected utility of contingent menu F is

$$\mathcal{U}(F) = \sum_M \sum_s p(s) F_s(M) \mathcal{U}(M; \ell_{M,F}) .$$

Choices over **contingent menus** are sufficient for identification of $u, p, \ell^*, \alpha_\ell$.

DISTORTED LIKELIHOOD

Each likelihood ℓ is consistent with one even S_ℓ .

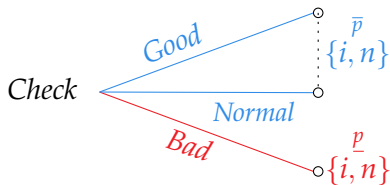


The distorted likelihood ℓ_S^* at event S is the best one according to u :

$$\ell_S^* \in \arg \max_{\ell \in \Delta(S)} u(x; \ell) \quad \dots \quad \text{not well defined!}$$

DISTORTED LIKELIHOOD

Each likelihood ℓ is consistent with one event S_ℓ .



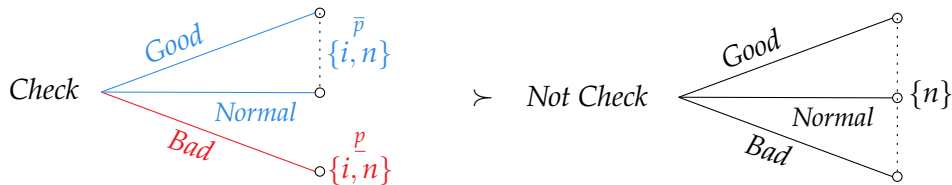
The distorted likelihood ℓ_S^* at event S is the best one *under the best outcome*:

$$\ell_S^* \in \arg \max_{\ell \in \Delta(S)} \max_x u(x; \ell).$$

Asymmetric updating: preferred likelihoods are not distorted.

From choice?

BACK TO THE EXAMPLE



Preferences over financial gains and beliefs are:

$$u(x; \ell) = v(x) + p_{\ell}(\text{Good}).$$

The investor expects to distort ℓ so that $p_{\ell^*}(\text{Good}) = 1$.

Optimistic beliefs lead her to invest more than what prescribed by Bayes rule.

BDP IMPLY NON-BAYESIAN UPDATING

Say the true likelihood ℓ coincides with the distorted ℓ^* :

$$\mathcal{U}(M; \ell) = \max_{f \in M} \left\{ \sum_s p_\ell(s) u(f_s; \ell) + \alpha_\ell \sum_s p_\ell(s) u(f_s; \ell) \right\} \\ - \max_{f' \in M} \alpha_\ell \sum_s p_\ell(s) u(f'_s; \ell) .$$

The second and third terms cancel out, only EU under Bayesian updating remains.

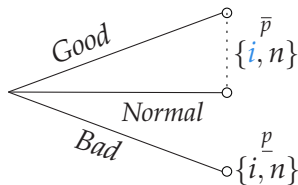
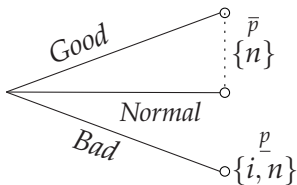
If u does not depend on ℓ , preferences over likelihoods are flat.

A novelty of the model is that BDP **imply** non-Bayesian updating.

AXIOM: IDENTICAL INFERENCE INDEPENDENCE

AXIOM

(Informal) The individual only satisfies independence for mixtures of contingent menus inducing the same inference for each of their menu realisations. Full



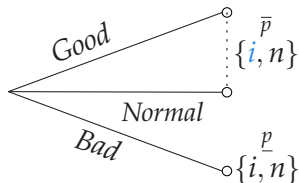
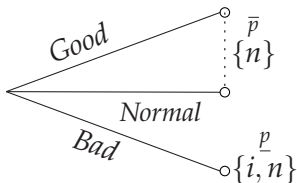
Relaxing independence leads to dynamic inconsistency.

MAIN AXIOM: STRATEGIC RATIONALITY FOR BEST LIKELIHOOD

AXIOM

(Informal) There is no temptation when, all else equal, the available menu comprises the best choices based on both the Bayesian update and the favourite posterior.

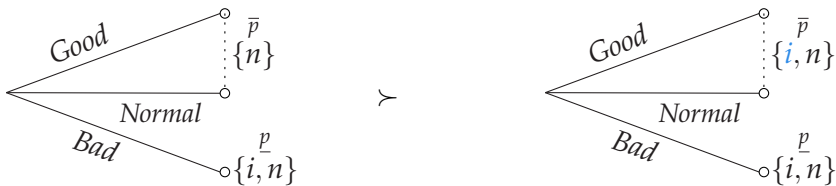
Full



MAIN AXIOM: STRATEGIC RATIONALITY FOR BEST LIKELIHOOD

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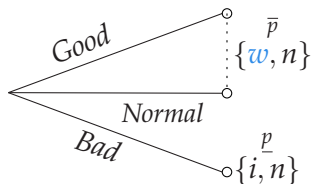
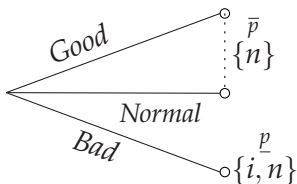


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MAIN AXIOM: STRATEGIC RATIONALITY FOR BEST LIKELIHOOD

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Under no BDP, all posteriors are "favourite" and the axiom implies no temptation.

No temptation implies the model reduces to Expected utility and Bayesian updating.

MAIN RESULT

Theorem

*Preferences over contingent menus are represented by Equations (1) and (2) if and only if they satisfy **Strategic Rationality for Best Likelihood** and other "standard" axioms.*

$$\mathcal{U}(F) = \sum_M \sum_s p(s) F_s(M) \mathcal{U}(M; \ell_{M,F}). \quad (1)$$

$$\mathcal{U}(M; \ell) = \max_{f \in M} \left\{ \sum_s p_\ell(s) u(f_s; \ell) + \alpha_\ell \sum_s p_{\ell_{S_\ell}^*}(s) u(f_s; \ell_{S_\ell}^*) \right\} \\ - \max_{f' \in M} \alpha_\ell \sum_s p_{\ell_{S_\ell}^*}(s) u(f'_s; \ell_{S_\ell}^*). \quad (2)$$

Prior belief p , utilities u , distorted likelihoods ℓ^* and weights α are unique. Axioms?

WHY THIS MODEL

1. Generality: how do individuals choose between information sources?
2. Refutability: predictions of previous theories overlap, also with non BDP.
3. Identification: how to intervene if preferences and beliefs are confused?
4. Dual-self: which self matters for welfare analysis?

CONCLUSION

Theory of BDP and belief updating tested via choices of contingent menus:

- dynamically consistent individual anticipates she distorts beliefs;
- asymmetric updating and no distortion of zero probability events;
- identification of BDP, non-Bayesian updating and strength of motivated reasoning.

Other applications in the paper:

- donors avoid and distort information about their impact;
- politicians send poor information to induce polarisation.

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COST OF SELF-CONTROL

Identification of α_ℓ allows elaborating on its behavioural meaning

$$\alpha_\ell = \frac{\mathcal{U}(\{f; x\}, \ell) - \mathcal{U}(\{f; x'\}, \ell)}{u(x; \ell) - u(x'; \ell)}.$$

It is the marginal cost of self-control at likelihood ℓ .

[Back](#)

ALTERNATIVE REPRESENTATION WITH COST

The representation can be written as either

$$\mathcal{U}(M; \ell) = \max_{f \in M} \left\{ \sum_s p_\ell(s) u(f_s; \ell) + \alpha_\ell \sum_s p_{\ell_{S_\ell}^*}(s) u(f_s; \ell_{S_\ell}^*) \right\} \\ - \max_{f' \in M} \alpha_\ell \sum_s p_{\ell_{S_\ell}^*}(s) u(f'_s; \ell_{S_\ell}^*)$$

or

$$\mathcal{U}(M; \ell) = \max_{f \in M} \left\{ \sum_s p_\ell(s) u(f_s; \ell) - \alpha_\ell c(f, u, S_\ell) \right\}.$$

AXIOMS: BASICS (INFORMAL)

AXIOM

(Order). Preferences over contingent menus are a continuous weak order. [Full](#)

AXIOM

(Nondegeneracy). There exist at least one outcome better than another. [Full](#)

AXIOM

(State Independence). Preferences over outcomes do not depend on the state.

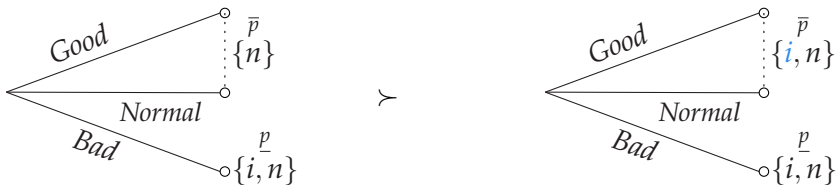
AXIOM

(Full support). The individual assigns ex-ante positive probability to all states.

AXIOMS: SET-BETWEENNESS (INFORMAL)

AXIOM

The individual is weakly worse if a menu is enhanced with ex-ante dominated options, because these induce temptation.



AXIOMS: BASICS I

AXIOM

(Order). The ranking \succsim is complete and transitive.

AXIOM

(Continuity). For all contingent menus F the sets

$$\{F' \mid F' \succsim F\} \text{ and } \{F' \mid F' \precsim F\}$$

are closed.

AXIOMS: BASICS II

Substitute from F any occurrence of M with M' to get $F_{M \rightarrow M'}$.

AXIOM

(Nondegeneracy). There exist outcomes y, y' such that $y \succ y'$.

AXIOM

(State Independence). For all contingent menus F , menus L, L', M and states s, s' ,

$$F \succsim F_{LsM \rightarrow L'sM} \Rightarrow F \succsim F_{Ls'M \rightarrow L's'M}.$$

AXIOM

(Full Support). For each state s , there exist contingent menus F and F' such that for all menus M it holds that $F_{s'}(M) = F'_{s'}(M)$ for every $s' \neq s$ and $F \sim F'$.

AXIOMS: IDENTICAL INFERENCE INDEPENDENCE

The support of F is

$$\mathcal{M}_F := \{M \in \mathcal{M} \mid F_s(M) > 0 \text{ for some } s \in S\}.$$

Definition

(Identical Inference (II)) Two contingent menus F and F' satisfy **identical inference** if, for each menu $M \in \mathcal{M}_F \cap \mathcal{M}_{F'}$ their likelihood is the same $\ell_{M,F} = \ell_{M,F'}$.

AXIOM

(II Independence). For all $0 < \lambda \leq 1$ and contingent menus F, F', F'' such that F and F'' satisfy II and F' and F'' satisfy II, $F \succsim F'$ if and only if $\lambda F + (1 - \lambda) F'' \succsim \lambda F' + (1 - \lambda) F''$.

AXIOMS: SET-BETWEENNESS

Substitute from F any occurrence of M with M' to get $F_{M \rightarrow M'}$.

AXIOM

(Set-Betweenness). For all contingent menus F and menus M, M' ,

$$F \succsim F_{M \rightarrow M'} \Rightarrow F \succsim F_{M \rightarrow M \cup M'} \succsim F_{M \rightarrow M'}.$$

AXIOMS: STRATEGIC RATIONALITY FOR BEST LIKELIHOOD

Substitute from F any occurrence of M with M' to get $F_{M \rightarrow M'}$.

For each menu M and likelihood ℓ define the set

$$\mathcal{F}_{M,\ell} := \{f \in M \mid F \succsim F_{\{f\} \rightarrow \{f'\}} \text{ for all } f' \in M \text{ and some } F \text{ such that } \ell_{\{f\},F} = \ell\}.$$

AXIOM

(Strategic Rationality for Best Likelihood (SRBL)). For each:

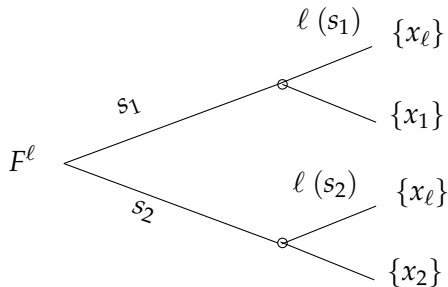
- couple of menus M, M' ;*
- contingent menu F such that $\ell_{M,F} = \ell$;*

if $\mathcal{F}_{M \cup M', \ell} \cap \mathcal{F}_{M \cup M', \ell_{S_\ell}^} \neq \emptyset$ for at least one $\ell_{S_\ell}^*$, then*

$$F \succsim F_{M \rightarrow M'} \Rightarrow F \sim F_{M \rightarrow M \cup M'}.$$

DISTORTED LIKELIHOODS FROM CHOICE

For each ℓ define contingent menus F^ℓ .



For each S define distorted likelihoods:

$$\ell_S^* \in \left\{ \ell \in \Delta(S) \mid F^\ell \succsim F^{\ell'} \text{ for all } \ell' \in \Delta(S) \right\}.$$