Identifying Belief-dependent Preferences

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January 30, 2025
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Motivation

Individuals **avoid** or **distort** new information:

- medical patients avoid testing (Golman et al., 2017);
- charity donors ignore the impact of their donation (Niehaus, 2014);
- financial investors overreact to news (Daniel & Hirshleifer, 2015).

One possible explanation is that individuals value beliefs *intrinsically*.

Belief-dependent preferences

Assuming that individuals' well-being depends directly on beliefs has implications:

- information is processed to increase well-being;
- belief revision depends on preferences over beliefs;
- hard to empirically identify beliefs, preferences and belief revision rules.

How to intervene if we do not know:

- how beliefs affect well-being;
- what beliefs individuals hold;
- how individuals revise beliefs?

This paper

Dynamic model of belief-dependent preferences:

- The individual well-being depends on beliefs;
- she **distorts beliefs** away from Bayesian updating to increase her well-being;
- she is then **tempted** to act according to her distorted beliefs.

Main result: axiomatic characterisation of the model.

Corollary: identification of beliefs, preferences and revision rules from observables.

Policy Implication: Commitment devices are welfare enhancing.

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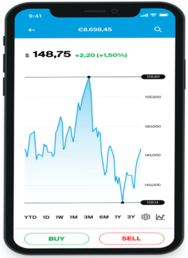
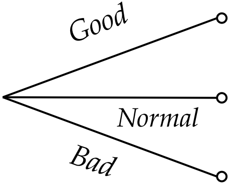
An investor chooses whether to check her portfolio or not.

If she does, she observes prices and can invest or do nothing.

Action

States

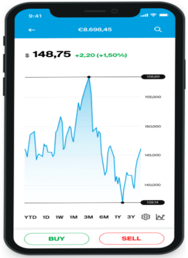
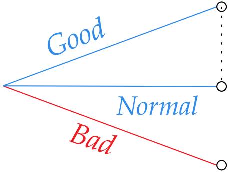




Illustrative Example

An investor chooses whether to check her portfolio or not.

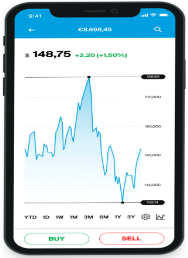
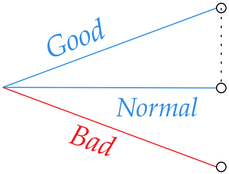
If she does, she observes prices and can invest or do nothing.

Action	*States*	*Information*
		*high price* *low price*

Illustrative Example

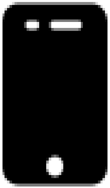
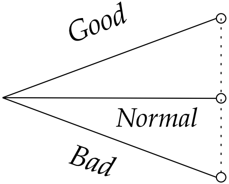
An investor chooses whether to check her portfolio or not.

If she does, she observes prices and can invest or do nothing.

Action	*States*	*Information*	*Menus of actions*
		*high price*	$\{invest,\ nothing\}$
		low price	$\{invest,\ nothing\}$

An investor chooses whether to check her portfolio or not.

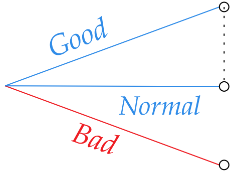
If she does not check, she observe no information and can only do nothing.

Action	*States*	*Information*	*Menus of actions*
		*no information*	$\{nothing\}$

Illustrative Example

An investor chooses whether to check her portfolio or not.

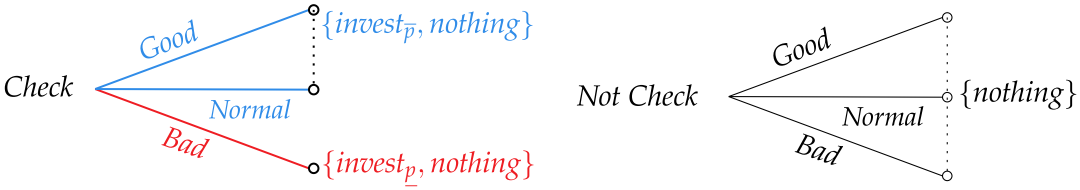
If she does, she observes prices and can invest or do nothing.

Action	*States*	*Menus of actions*
Check		$\{invest_{\overline{p}}, nothing\}$ $\{invest_{\underline{p}}, nothing\}$

Investing is financially optimal only in the *Good* state.

An investor chooses whether to check her portfolio or not.

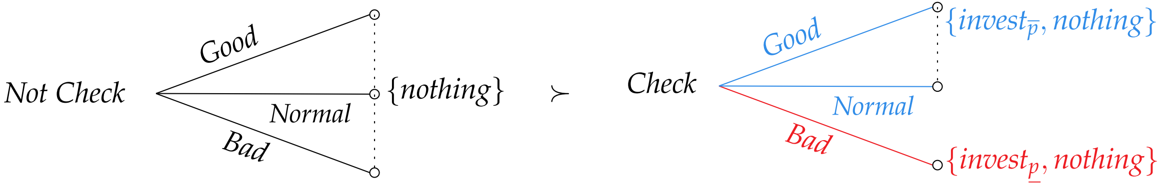
If she does, she observes prices and can invest or do nothing.



Prices both constitute a **signal** and induce a **menu** of feasible actions.

The investor likes believing the state is good.

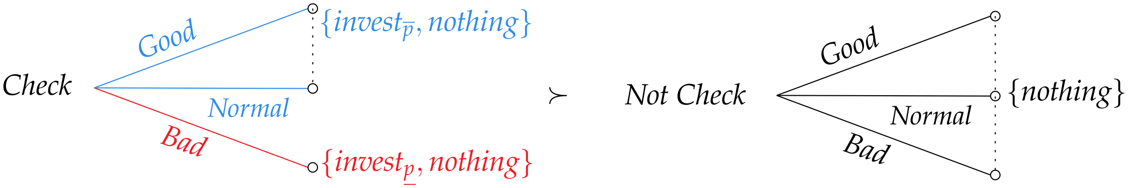
She might prefer not to check because she is afraid of the bad signal.



Illustrative Example

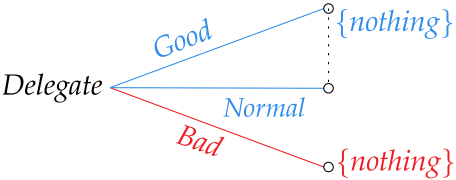
Under $\overline{p}$ she distorts the signal to be more positive . . . and overinvests.

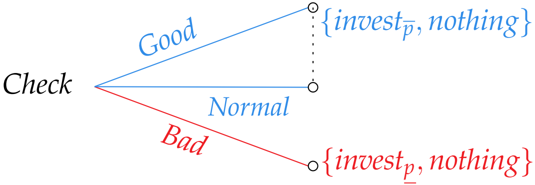
Implication: receiving pleasant information vs acting under a distorted belief.



Both **excessive trading** and **information avoidance** constitute empirical puzzles in finance (Daniel & Hirshleifer, 2015; Golman et al., 2017).

Variant: commit not to invest, e.g. by delegating to a financial advisor or algorithm.

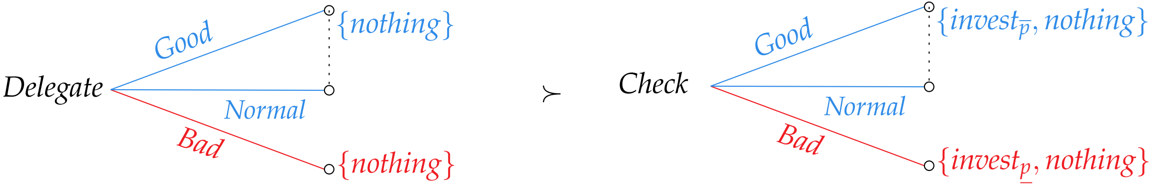




Receiving pleasant information but ~~acting under a distorted belief~~.

Illustrative Example

Variant: commit not to invest, e.g. by delegating to a financial advisor or algorithm.



Receiving pleasant information but ~~acting under a distorted belief~~.

Commitment might be welfare enhancing under belief-dependent preferences.

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Literature

- *Belief-Dependent Preferences.* Brunnermeier & Parker (2005), Eliaz & Spiegler (2006), Bénabou & Tirole (2016), Golman et al. (2017), Battigalli & Dufwenberg (2022).

  Contribution: **Generality, derivation of belief revision rule, identification**.

- *Decision Theory.* Liang (2017), Dillenberger & Raymond (2020) Rommeswinkel et al. (2023).

  Contribution: **Belief revision rule, identification**.

- *Menu Choice.* Gul & Pesendorfer (2001), Ozdenoren (2002), Epstein (2006), Epstein & Kopylov (2007).

  Contribution: **Novel primitive object of choice**.

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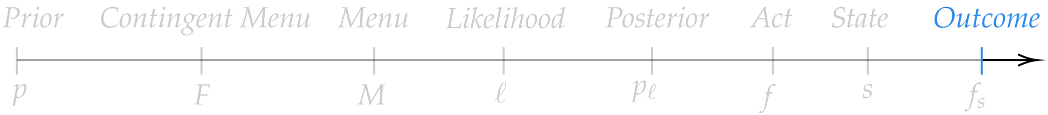
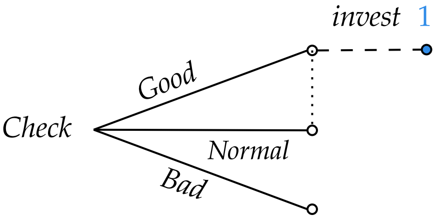
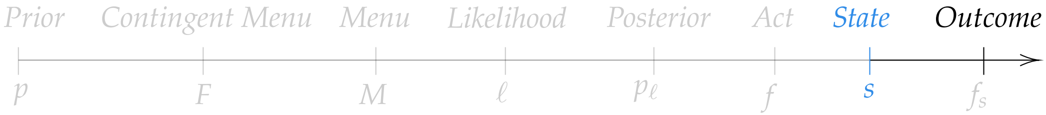
MODEL: ACTS

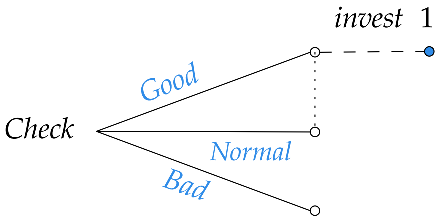




- outcome set (net financial return);

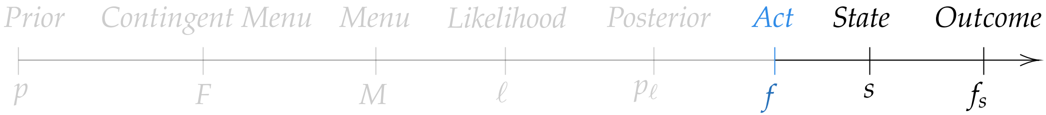
Model: acts

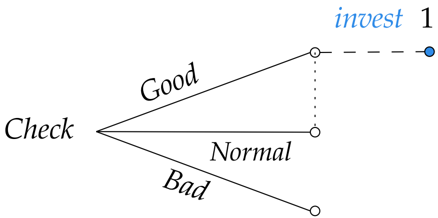




- outcome set;
- state set;

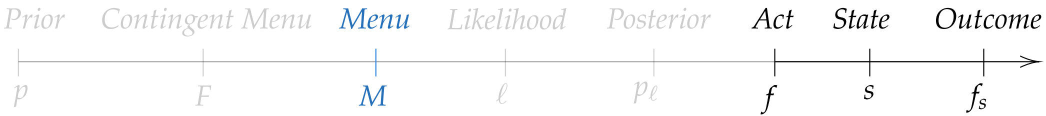
MODEL: ACTS

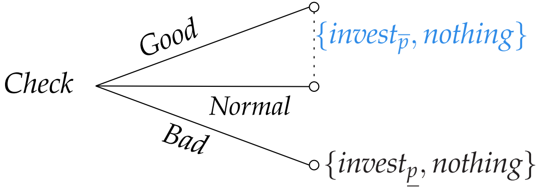




- outcome set;
- state set;
- acts (actions) f;

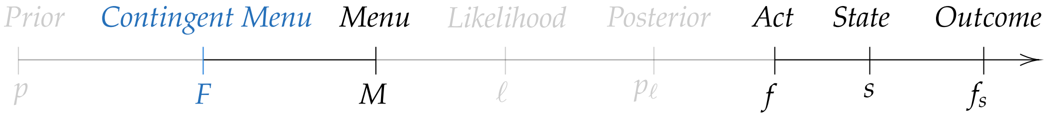
Model: menus and contingent menus

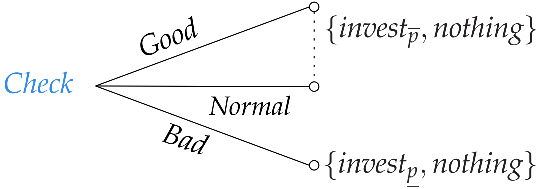




- outcome set;
- state set;
- acts (actions) f;
- a set of acts is a menu M;

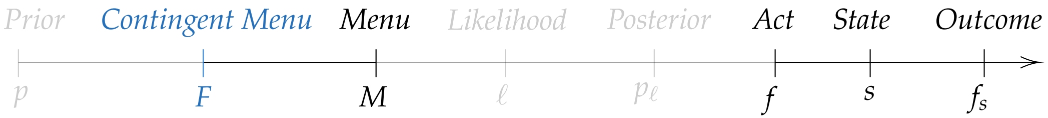
Model: menus and contingent menus

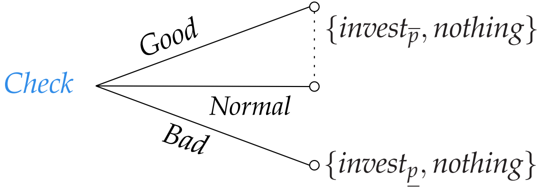




- outcome set;
- state set;
- acts (actions) f;
- a set of acts is a menu M;
- a contingent menu is $F : States \rightarrow Menus$.

Model: menus and contingent menus

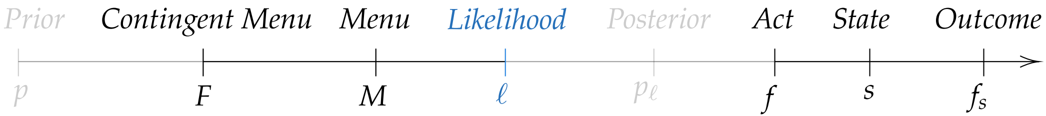


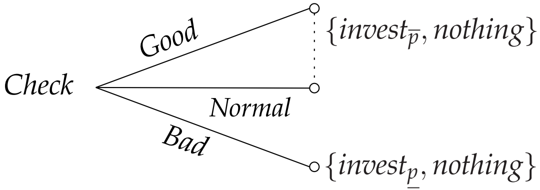


- outcome set;
- state set;
- acts (actions) f;
- a set of acts is a menu M;
- a contingent menu is

$$\begin{aligned} F:\ States &\longrightarrow Menus \\ Good &\mapsto \{invest_{\overline{p}}, nothing\}. \end{aligned}$$

Model: Information



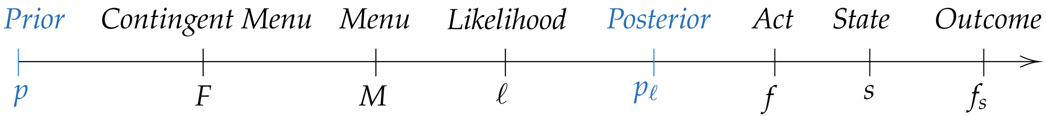


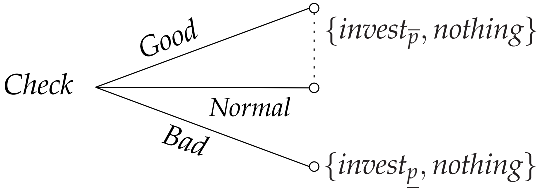
- a contingent menu is $F : States \to Menus$;

The likelihood of state *Good* after realisation of menu $\{invest_{\overline{p}}, nothing\}$ is

$$\ell\,(Good) = \frac{1}{1+1} = \frac{1}{2}.$$

Model: Information



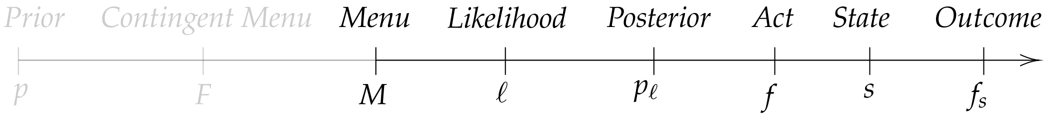


- a contingent menu is $F : States \to Menus$;
- prior belief p;
- Bayesian posterior p_ℓ.

The likelihood of state *Good* after realisation of menu $\{invest_{\overline{p}}, nothing\}$ is

$$\ell\,(Good) = \frac{1}{1+1} = \frac{1}{2}.$$

UTILITY: ACTS



Expected utility of act f at likelihood ℓ

$$\sum_s p_\ell(s)\, u(f_s; \ell)\,.$$

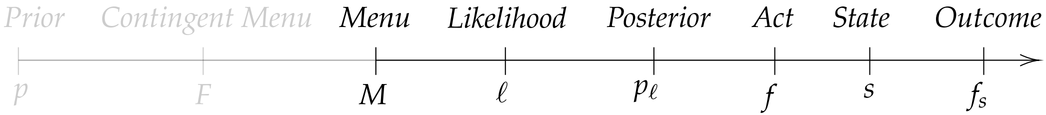
Individual distorts the likelihood to ℓ^*

$$\sum_s p_{\ell^*}(s)\, u(f_s; \ell^*)\,.$$

The main theorem identifies conditions under which the individual solves

$$\max_{f \in M} \left\{ \underbrace{\sum_s p_\ell(s) u(f_s; \ell)}_{EU} + \alpha_\ell \underbrace{\sum_s p_{\ell^*}(s)\, u(f_s; \ell^*)}_{\text{distorted } EU} \right\}.$$

Utility: acts



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UTILITY: CONTINGENT MENUS

Prior	*Contingent Menu*	*Menu*	*Likelihood*	*Posterior*	*Act*	*State*	*Outcome*
p	F	M	ℓ	p_ℓ	f	s	f_s

The individual also suffers a cost of temptation

$$\mathcal{U}\left(M;\ell\right) = \max_{f \in M}\left\{\sum_s p_\ell\left(s\right) u\left(f_s;\ell\right) + \alpha_\ell \sum_s p_{\ell^*}\left(s\right) u\left(f_s;\ell^*\right)\right\} - \max_{f' \in M} \alpha_\ell \sum_s p_{\ell^*}\left(s\right) u\left(f'_s;\ell^*\right).$$

The positive number α_ℓ captures the *strength of motivated reasoning*.

Alpha? Cost?

Utility: contingent menus

$$\mathcal{U}\left(M;\ell\right)=\max_{f\in M}\left\{\sum_{s}p_{\ell}\left(s\right)u\left(f_{s};\ell\right)+\ \alpha_{\ell}\sum_{s}p_{\ell^{*}}\left(s\right)u\left(f_{s};\ell^{*}\right)\right\}$$
$$-\max_{f'\in M}\alpha_{\ell}\sum_{s}p_{\ell^{*}}\left(s\right)u\left(f'_{s};\ell^{*}\right).$$

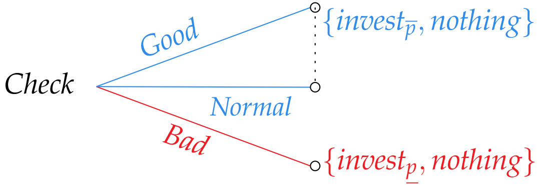
Expected utility of contingent menu F is

$$\mathscr{U}\left(F\right)=\sum_{s}p\left(s\right)\mathcal{U}\left(F_{s};\ell_{F_{s}}\right).$$

Choices over contingent menus are sufficient for identification of $u, p, \ell^{*}, \alpha_{\ell}$.

Distorted likelihood

Each likelihood ℓ is consistent with one set of states.

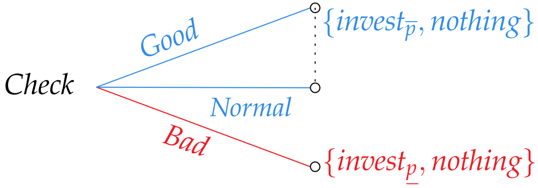


The distorted likelihood ℓ^*_S at states in S is the best one according to $u(\cdot, \ell)$:

$$\ell^*_S \in \arg\max_{\ell \in \Delta(S)} u(x; \ell) \quad \ldots \quad \text{not well defined!}$$

Distorted likelihood

Each likelihood ℓ is consistent with one set of states.



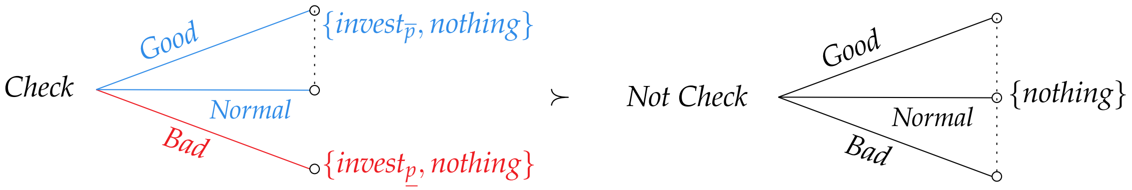
The distorted likelihood ℓ_S^* at states in S is the best one *under the best outcome*:

$$\ell_S^* \in \arg\max_{\ell \in \Delta(S)} \max_x \; u\left(x; \ell\right).$$

Asymmetric updating: preferred information is not distorted (Eil & Rao, 2011).

From choice?

Back to the example



Preferences over financial gains (x) and posterior beliefs (p_ℓ) are:

$$u(x; \ell) = v(x) + p_\ell(\text{Good}).$$

The investor expects to distort ℓ so that $p_{\ell^*}(Good) = 1$.

Optimistic beliefs lead her to invest more than what prescribed by Bayes rule.

BDP imply non-Bayesian updating

Say the true likelihood ℓ coincides with the distorted ℓ^*:

$$\mathcal{U}(M;\ell) = \max_{f\in M}\left\{\sum_s p_\ell(s)\,u(f_s;\ell) + \alpha_\ell \sum_s p_\ell(s)\,u(f_s;\ell)\right\} - \max_{f'\in M} \alpha_\ell \sum_s p_\ell(s)\,u(f'_s;\ell).$$

The second and third terms cancel out, only EU under Bayesian updating remains.

If u does not depend on ℓ, preferences over likelihoods are flat.

A novelty of the model is that BDP **imply** non-Bayesian updating.

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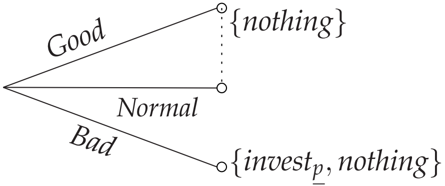
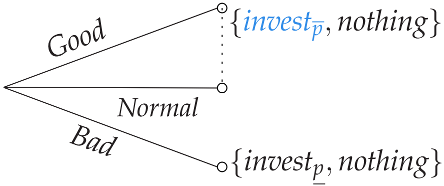
Axiom

*(**Informal**) The individual only satisfies independence for mixtures of contingent menus inducing the same inference for each of their menu realisations.* Full

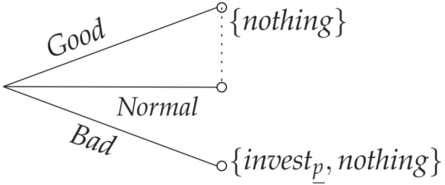


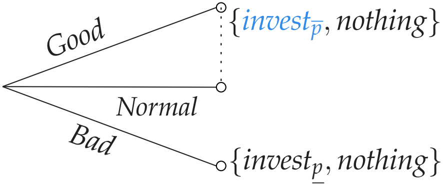


Relaxing independence leads to dynamic inconsistency.

Axiom

*(**Informal**) There is no temptation when, all else equal, the best choices based on the Bayesian update and the favourite posterior in the available menu are the same.* Full





AXIOM

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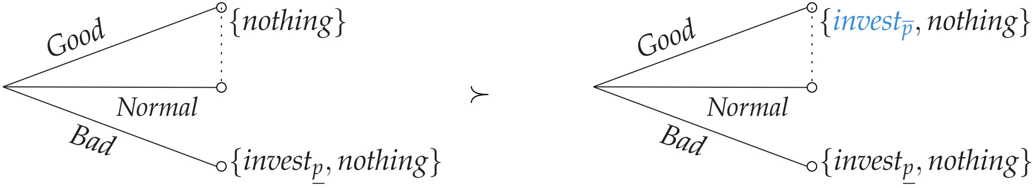

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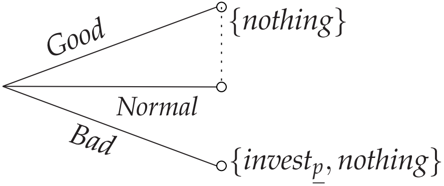
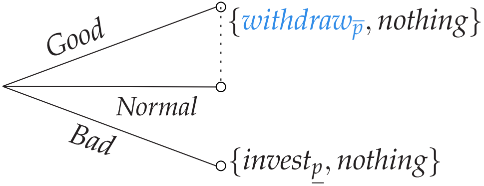
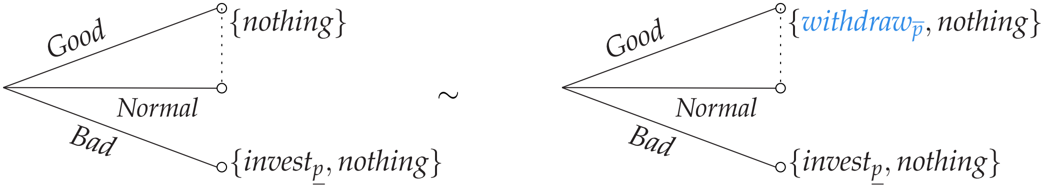




Main Axiom: Strategic Rationality for Best Likelihood

Axiom

(***Informal***) *There is no temptation when, all else equal, the best choices based on the Bayesian update and the favourite posterior in the available menu are the same.* Full



Under no BDP, all posteriors are "favourite" and the axiom implies no temptation.

No temptation implies the model reduces to Expected utility and Bayesian updating.

Theorem

*(**Informal**) Preferences over contingent menus are represented by (1) and (2) if and only if they satisfy **Strategic Rationality for Best Likelihood** and other "standard" axioms.*

$$\mathscr{U}(F) = \sum_s p(s)\, \mathcal{U}(F_s; \ell_{F_s}). \tag{1}$$

$$\begin{aligned} \mathcal{U}(M;\ell) = \max_{f \in M} &\left\{ \sum_s p_\ell(s)\, u(f_s;\ell) + \alpha_\ell \sum_s p_{\ell^*}(s)\, u(f_s;\ell^*) \right\} \\ &- \max_{f' \in M} \alpha_\ell \sum_s p_{\ell^*}(s)\, u(f'_s;\ell^*). \end{aligned} \tag{2}$$

Prior belief p, utilities u, distorted likelihoods ℓ^* and weights α_ℓ are unique. Axioms?

Why this model

1. Generality: how do individuals choose between information sources?

2. Refutability: predictions of previous theories overlap, also with non BDP.

3. Identification: how to intervene if preferences and beliefs are confused?

4. Dual-self: which self matters for welfare analysis?

Conclusion

Theory of belief-dependent preferences tested via choices of contingent menus:

- individual anticipates she distorts beliefs;
- asymmetric updating and no distortion of zero probability events.

Main Implication: Commitment might be welfare enhancing.

Other applications in the paper:

- **Political economy**: politicians induce belief polarisation;
- **Pro-social preferences**: donors avoid information about their impact.

References

Battigalli, P., & Dufwenberg, M. (2022). Belief-dependent motivations and psychological game theory. *Journal of Economic Literature*, *60*(3), 833–882.

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Cost of self-control

Identification of α_ℓ allows elaborating on its behavioural meaning

$$\alpha_\ell = \frac{\mathcal{U}\left(\{f, x\}\,; \ell\right) - \mathcal{U}\left(\{f, x'\}\,; \ell\right)}{u\left(x; \ell\right) - u\left(x'; \ell\right)}.$$

It is the marginal cost of self-control at likelihood ℓ. Back

Alternative representation with cost

The representation can be written as either

$$\mathcal{U}(M;\ell) = \max_{f\in M}\left\{\sum_s p_\ell(s)\, u(f_s;\ell) + \alpha_\ell \sum_s p_{\ell^*_{S_\ell}}(s)\, u\left(f_s;\ell^*_{S_\ell}\right)\right\} - \max_{f'\in M} \alpha_\ell \sum_s p_{\ell^*_{S_\ell}}(s)\, u\left(f'_s;\ell^*_{S_\ell}\right)$$

or

$$\mathcal{U}(M;\ell) = \max_{f\in M}\left\{\underbrace{\sum_s p_\ell(s) u(f_s;p_\ell)}_{EU} - \alpha_\ell \underbrace{c(f,u,S_\ell)}_{\text{Cost of temptation}}\right\}.$$

Back

Axioms: Basics (Informal)

Axiom

*(**Order**). Preferences over contingent menus are a continuous weak order.* Full

Axiom

*(**Nondegeneracy**). There exist at least one outcome better than another.* Full

Axiom

*(**State Independence**). Preferences over outcomes do not depend on the state.*

Axiom

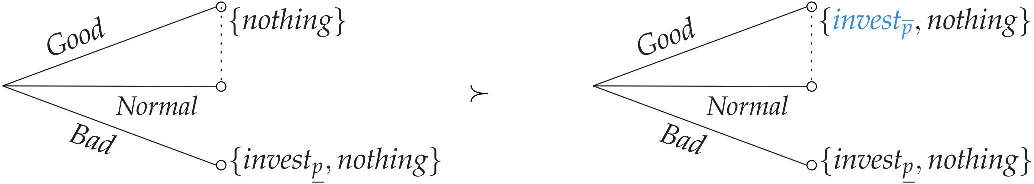
*(**Full support**). The individual assigns ex-ante positive probability to all states.*

Back

Axioms: Set-Betweenness (Informal)

Axiom

The individual is weakly worse if a menu is enhanced with ex-ante dominated options, because these induce temptation.



Full

Axioms: Basics I

Axiom

*(**Order**). The ranking $\succsim$ is complete and transitive.*

Axiom

*(**Continuity**). For all contingent menus F the sets*

$$\{F' \mid F' \succsim F\} \text{ and } \{F' \mid F' \precsim F\}$$

are closed.

Axioms: Basics II

Substitute from F any occurrence of M with M' to get $F_{M \to M'}$.

Axiom

*(**Nondegeneracy**). There exist outcomes y, y' such that $y \succ y'$.*

Axiom

*(**State Independence**). For all contingent menus F, menus L, L', M and states s, s',*

$$F \succsim F_{LsM \to L'sM} \Rightarrow F \succsim F_{Ls'M \to L's'M}.$$

Axiom

*(**Full Support**). For each state s, there exist contingent menus F and F' such that for all menus M it holds that $F_{s'}(M) = F'_{s'}(M)$ for every $s' \neq s$ and $F \nsim F'$.*

Axioms: Identical Inference Independence

The support of F is

$$\mathcal{M}_F := \{M \mid F_s(M) > 0 \text{ for some } s \in S\}.$$

Definition

(**Identical Inference (II)**) Two contingent menus F and F' satisfy **identical inference** if, for each menu $M \in \mathcal{M}_F \cap \mathcal{M}_{F'}$ their likelihood is the same $\ell_{M,F} = \ell_{M,F'}$.

Axiom

*(**II Independence**). For all $0 < \lambda \leq 1$ and contingent menus F, F', F'' such that F and F'' satisfy II and F' and F'' satisfy II, $F \succsim F'$ if and only if $\lambda F + (1-\lambda) F'' \succsim \lambda F' + (1-\lambda) F''$.*

Back

Axioms: Set-Betweenness

Substitute from F any occurrence of M with M' to get $F_{M \to M'}$.

Axiom

*(**Set-Betweenness**). For all contingent menus F and menus M, M',*

$$F \succsim F_{M \to M'} \Rightarrow F \succsim F_{M \to M \cup M'} \succsim F_{M \to M'}.$$

Back

Axioms: Strategic Rationality for Best Likelihood

Substitute from F any occurrence of M with M' to get $F_{M \to M'}$.
For each menu M and likelihood ℓ define the set

$$\mathcal{F}_{M,\ell} := \left\{ f \in M \,\middle|\, F \succsim F_{\{f\} \to \{f'\}} \text{ for all } f' \in M \text{ and some } F \text{ such that } \ell_{\{f\},F} = \ell \right\}.$$

Axiom

*(**Strategic Rationality for Best Likelihood** (**SRBL**)). For each:*

- *couple of menus M, M';*
- *contingent menu F such that $\ell_{M,F} = \ell$;*

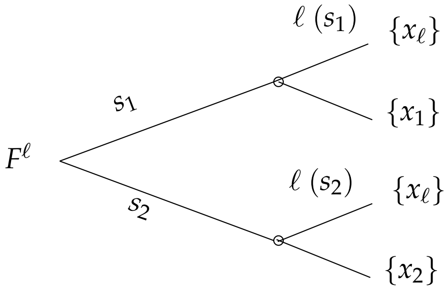
if $\mathcal{F}_{M \cup M', \ell} \cap \mathcal{F}_{M \cup M', \ell^*_{S_\ell}} \neq \emptyset$ for at least one $\ell^*_{S_\ell}$, then

$$F \succsim F_{M \to M'} \Rightarrow F \sim F_{M \to M \cup M'}.$$

Back

Distorted likelihoods from choice

For each ℓ define contingent menus F^{ℓ}.



For each S define distorted likelihoods:

$$\ell_S^* \in \left\{ \ell \in \Delta(S) \;\middle|\; F^{\ell} \succsim F^{\ell'} \text{ for all } \ell' \in \Delta(S) \right\}.$$

Back