Belief-dependent preferences and updating

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Investors overreact to good news, donors are uninformed about their impact, and medical patients avoid testing (Daniel & Hirshleifer, 2015; Golman et al., 2017).

Theories of belief-dependent preferences (BDP) rationalise these behaviours.

Three drawbacks:

- object of choice unobservable (Bénabou & Tirole, 2016; Köszegi, 2010);
- lack of preferences and beliefs identification;
- BDP and non-Bayesian updating are disjoint assumptions.

This paper: testable theory of BDP and non-Bayesian updating.

This Paper

Individuals tastes over outcomes depend on their posterior beliefs.

The individual distorts beliefs away from Bayesian updating to increase her welfare.

She then acts according to her distorted beliefs.

Ex-ante, the individual anticipates the belief distortion and the related choice.

Main result: axiomatic characterisation of BDP preferences and updating rules.

An investor decides whether to check the balance in her portfolio.

If she does, she can invest (i) or withdraw a feasible amount, high ($\overline{w}$) or low (w).

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

Upon observing a high balance, she infers the state of the market is not bad.

When she sees a low balance, she knows the state of the market is bad.

She cannot make any inferences or do anything if she does not check.

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

Not Check

State	Actions
Good	
Normal	0
Bad	

Before checking, she anticipates to overweight evidence and invest too much.

However, she might want to check anyway to obtain pleasant information.

Check			**Not Check**	
State	Actions		State	Actions
Good	$i, [0, \overline{w}]$	$\succ$	Good	0
Normal	$i, [0, \overline{w}]$		Normal	0
Bad	$i, [0, w]$		Bad	0

Trade-off: not receiving information vs acting under a distorted belief.

Illustrative Example

She could delegate to a financial advisor, allowing her to commit not to invest.

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

Check

State	Actions
Good	i, $[0, \overline{w}]$
Normal	
Bad	i, $[0, w]$

Not Check

State	Actions
Good	
Normal	0
Bad	

Commitment allows her to obtain information without being tempted to overinvest.

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

$\succ$

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

$\succ$

Not Check

State	Actions
Good	
Normal	0
Bad	

Table: Commitment under positive prior belief to avoid excessive investment.

She might also prefer not to check because she expects unpleasant information.

Not Check

State	Actions
Good	
Normal	0
Bad	

$\succ$

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

$\succ$

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

Table: Information avoidance under negative prior beliefs, "ostrich effect".

Both excessive trading and the ostrich effect constitute empirical puzzles in finance (Daniel & Hirshleifer, 2015; Golman et al., 2017).

Literature

- *Decision Theory.* Liang (2017), Dillenberger & Raymond (2020) Rommeswinkel et al. (2023).

  Contribution: **Belief revision rule**.

- *Menu Choice.* Gul & Pesendorfer (2001), Ozdenoren (2002), Epstein (2006), Epstein & Kopylov (2007).

  Contribution: **Novel primitive object of choice**.

- *Belief-Dependent Motivations.* Brunnermeier & Parker (2005), Eliaz & Spiegler (2006), Bénabou & Tirole (2016), Golman et al. (2017), Battigalli & Dufwenberg (2022).

  Contribution: **Identification, interaction of tastes and belief revision**.

	Good	Normal	Bad
i	5	0	- 5
n	4	3	-4
$\overline{w}$	2	2	
w			1

Table: Actions payoffs.

- outcome set;

Model: actions

	Good	Normal	Bad
i	5	0	- 5
n	4	3	-4
$\overline{w}$	2	2	
w			1

Table: Actions payoffs.

- outcome set;
- state set;

Model: actions

	Good	Normal	Bad
i	5	0	- 5
n	4	3	-4
$\overline{w}$	2	2	
w			1

Table: Actions payoffs.

- outcome set;
- state set ;
- acts $f\ :\ States\ \rightarrow\ Outcomes;$

Check

State	Actions
Good Normal	$i, [0, \overline{w}]$
Bad	$i, [0, w]$

Table: Menus and Contingent Menus.

- outcome set;
- state set;
- acts f : *States* $\rightarrow$ *Outcomes*;
- a set of acts is a menu M;

Model: menus and contingent menus

Check	
State	Actions
Good Normal	$i, [0, \overline{w}]$
Bad	$i, [0, w]$

Table: Menus and Contingent Menus.

- outcome set;
- state set;
- acts f : *States* $\to$ *Outcomes*;
- a set of acts is a menu M;
- a contingent menu is F : *States* $\to$ *Menus*.

Check

State	Actions
Good	$1 \cdot \{i, [0, \overline{w}]\}$
Normal	
Bad	$1 \cdot \{i, [0, w]\}$

Table: Information.

- a set of acts is a menu M;
- a contingent menu is $F : \mathit{States} \rightarrow \mathit{Menus}$;
- probability menu M realises in state s is $F_s(M)$;

The likelihood of state s after realisation of menu M from the contingent menu F is

$$\ell_{M,F}(\mathit{Good}) = \frac{1}{1+1} = \frac{1}{2}.$$

Check

State	Actions
Good	$1 \cdot \{i, [0, \overline{w}]\}$
Normal	
Bad	$1 \cdot \{i, [0, w]\}$

Table: Information.

- a set of acts is a menu M;
- a contingent menu is $F : States \rightarrow Menus$;
- probability menu M realises in state s is $F_s(M)$;

The likelihood of state s after realisation of menu M from the contingent menu F is

$$\ell_{M,F}(Good) = \frac{1}{1+1} = \frac{1}{2} \qquad \ell_{M,F}(s) = \frac{F_s(M)}{\sum_{s' \in S} F_{s'}(M)}.$$

Prior belief p, posterior belief $p_{\ell_{M,F}}$.

f_s	Good	Normal	Bad
i	5	0	- 5
n	4	3	-4
$\overline{w}$	2	2	
w			1

Table: Actions payoffs.

Expected utility of act f at likelihood $\ell_{M,F}$ is

$$\sum_s p_{\ell_{M,F}}(s)\, u\left(f_s; \ell_{M,F}\right).$$

Under distorted likelihood $\ell^*_{M,F}$

$$\sum_s p_{\ell^*_{M,F}}(s)\, u\left(f_s; \ell^*_{M,F}\right).$$

The individual chooses according to

$$\max_{f \in M} \left[\underbrace{\sum_s p_{\ell_{M,F}}(s) u\left(f_s; \ell_{M,F}\right)}_{EU} + \alpha_{\ell_{M,F}} \underbrace{\sum_s p_{\ell^*_{M,F}}(s)\, u\left(f_s; \ell^*_{M,F}\right)}_{\text{distorted } EU} \right].$$

Utility over menus

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

Expected utility of contingent menu F is

$$\mathscr{U}(F) = \sum_{M} \sum_{s} p(s)\, F_s(M)\, \mathcal{U}\left(M; \ell_{M,F}\right).$$

The individual anticipates her choices and cost of temptation

$$\begin{aligned}\mathcal{U}\left(M; \ell_{M,F}\right) = &\max_{f \in M} \left\{ \sum_{s} p_{\ell_{M,F}}(s)\, u\left(f_s; \ell_{M,F}\right) + \alpha_{\ell_{M,F}} \sum_{s} p_{\ell^*_{M,F}}(s)\, u\left(f_s; \ell^*_{M,F}\right) \right\} \\ &- \max_{f' \in M} \alpha_{\ell_{M,F}} \sum_{s} p_{\ell^*_{M,F}}(s)\, u\left(f'_s; \ell^*_{M,F}\right)\end{aligned}$$

Alpha?

Distorted likelihood

The distorted likelihood ℓ_E^* at event E is the individual's preferred one when she can choose any outcome:

$$\ell_E^* \in \arg\max_{\ell_E} \max_x u(x; \ell_E).$$

If a state has probability 0, its likelihood cannot be distorted.

Asymmetric updating: preferred likelihoods are not distorted.

Axiom

There is no temptation when the available menu comprises the best choices from both the Bayesian update and the favourite posterior.

Delegate

State	Actions
Good Normal	$[0, \overline{w}]$
Bad	$[0, w]$

State	Actions
Good Normal	$i, [0, \overline{w}]$
Bad	$[0, w]$

Full

AXIOM

There is no temptation when the available menu comprises the best choices from both the Bayesian update and the favourite posterior.

Delegate

State	Actions
Good Normal	$[0, \overline{w}]$
Bad	$[0, w]$

$\succ$

State	Actions
Good Normal	$i, [0, \overline{w}]$
Bad	$[0, w]$

Full

Axiom

There is no temptation when the available menu comprises the best choices from both the Bayesian update and the favourite posterior.

Delegate

State	Actions
Good Normal	$[0, \overline{w}]$
Bad	$[0, w]$

State	Actions
Good Normal	$[0, \frac{1}{2}\overline{w}]$
Bad	$[0, w]$

Full

AXIOM

There is no temptation when the available menu comprises the best choices from both the Bayesian update and the favourite posterior.

Delegate

State	Actions
Good, Normal	$[0, \overline{w}]$
Bad	$[0, w]$

$\sim$

State	Actions
Good, Normal	$[0, \frac{1}{2}\overline{w}]$
Bad	$[0, w]$

Under no BDP, all posteriors are "favourite" and the axiom implies no temptation.

No temptation implies the model reduces to Expected utility and Bayesian updating.

Theorem

Individual's preferences over contingent menus represented are by equations 1 and 2 if and only if they satisfy Strategic Rationality for Best Likelihood and other regularity axioms.

$$\mathscr{U}(F) = \sum_M \sum_s p(s) F_s(M) \mathcal{U}\left(M; \ell_{M,F}\right). \tag{1}$$

$$\begin{aligned}\mathcal{U}\left(M; \ell_{M,F}\right) = &\max_{f \in M} \left\{ \sum_s p_{\ell_{M,F}}(s) u\left(f_s; \ell_{M,F}\right) + \alpha_{\ell_{M,F}} \sum_s p_{\ell^*_{M,F}}(s) u\left(f_s; \ell^*_{M,F}\right) \right\} \\ &- \max_{f' \in M} \alpha_{\ell_{M,F}} \sum_s p_{\ell^*_{M,F}}(s) u\left(f'_s; \ell^*_{M,F}\right)\end{aligned} \tag{2}$$

Prior belief p, utilities u, distorted likelihoods ℓ^* and weights α are "unique".

Why this model: observability and identification

The primitive objects of choice, contingent menus, are observable, contrary to

- Choice of beliefs (Brunnermeier & Parker, 2005; Köszegi, 2010);
- Choice of probability to forget (Bénabou & Tirole, 2016).

Choices of information sources do not allow identification (Eliaz & Spiegler, 2006).

These papers only provide "if" results, hard to distinguish from other theories.

Why this model: multiple selves and updating

Individual as the unit of choice, multiple selves render welfare analysis hard.

Under multiple selves, choices are dynamically inconsistent.

BDP and non-Bayesian updating are not disjoint, the first implies the second.

Conclusion

Theory of BDP and belief updating tested via choices of contingent menus.

Dynamically consistent individual anticipates she distorts beliefs to satisfy her BDP.

Asymmetric updating and no distortion of zero probability events.

Identification of BDP, non-Bayesian updating and strength of motivated reasoning.

Other applications in the paper:

- donors avoid and distort information about their impact;
- politicians send poor information to induce polarisation.

References

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Axioms: Basics

Axiom

*(**Order**). Preferences over contingent menus are a continuous weak order.* Full

Axiom

*(**Nondegeneracy**). There exist at least one outcome better than another.* Full

Axiom

*(**State Independence**). Preferences over outcomes do not depend on the state.*

Axiom

*(**Full support**). The individual assigns ex-ante positive probability to all states.*

Axiom

The individual is only indifferent between mixtures of contingent menus inducing the same inference.

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

Not Check

State	Actions
Good	
Normal	0
Bad	

Full

Axioms: Set-Betweenness

Axiom

The individual is weakly worse if a menu is enhanced with ex-ante dominated options, because these induce temptation.

Delegate

State	Actions
Good	$[0, \overline{w}]$
Normal	
Bad	$[0, w]$

$\succ$

Check

State	Actions
Good	$i, [0, \overline{w}]$
Normal	
Bad	$i, [0, w]$

Full

Cost of self-control

Identification of α_ℓ allows elaborating on its behavioural meaning

$$\alpha_\ell = \frac{\mathcal{U}\left(\{f, x\}, \ell\right) - \mathcal{U}\left(\{f, x'\}, \ell\right)}{u\left(x, \ell\right) - u\left(x', \ell\right)}.$$

It is the marginal cost of self-control at likelihood ℓ. Back

Axioms: Basics I

Axiom

*(**Order**). The ranking $\succsim$ is complete and transitive.*

Axiom

*(**Continuity**). For all contingent menus F the sets*

$$\{F' \mid F' \succsim F\} \text{ and } \{F' \mid F' \precsim F\}$$

are closed.

Back

Axioms: Basics II

Axiom

*(**Nondegeneracy**). There exist outcomes y, y' in X for which $y \succ y'$.*

Axiom

*(**State Independence**). For all contingent menus F, menus L, L', M and states s, s',*

$$F \succsim F_{LsM \to L'sM} \;\Rightarrow\; F \succsim F_{Ls'M \to L's'M}.$$

Axiom

*(**Full Support**). For each state s, there exist contingent menus F and F' such that for all menus M it holds that $F_{s'}(M) = F'_{s'}(M)$ for every $s' \neq s$ and $F \nsim F'$.*

Back

Axioms: Identical Inference Independence

The support of F is

$$\mathcal{M}_F := \{M \in \mathcal{M} \mid F_s(M) > 0 \text{ for some } s \in S\}.$$

Definition

(**Identical Inference (II)**) Two contingent menus F and F' satisfy **identical inference** if, for each menu $M \in \mathcal{M}_F \cap \mathcal{M}_{F'}$ their likelihood is the same $\ell_{M,F} = \ell_{M,F'}$.

Axiom

(***II Independence***). *For all $0 < \lambda \leq 1$ and contingent menus F, F', F'' such that F and F'' satisfy II and F' and F'' satisfy II, $F \succsim F'$ if and only if $\lambda F + (1-\lambda) F'' \succsim \lambda F' + (1-\lambda) F''$.*

Back

Axioms: Set-Betweenness

Substitute from F any occurrence of M with M' to get $F_{M \to M'}$.

Axiom

*(**Set-Betweenness**). For all contingent menus F and menus M, M',*

$$F \succsim F_{M \to M'} \Rightarrow F \succsim F_{M \to M \cup M'} \succsim F_{M \to M'}.$$

Back

Axioms: Strategic Rationality for Best Likelihood

Define the preferred likelihood at event E as

$$\ell_E^* \in \arg\max_{\ell_E \in \mathcal{L}_E} \max_{x \in \Delta^0(X)} u\left(x; \ell_E\right).$$

For each menu M and likelihood ℓ define the set

$$M_\ell := \left\{ f \in M \mid F \succsim F_{\{f\} \to \{f'\}} \text{ for all } f' \in M \text{ and } F \text{ such that } \ell_{\{f\},F} = \ell \right\}.$$

Axiom

*(**Strategic Rationality for Best Likelihood**). For all contingent menus F, menus M, M', events E and likelihoods ℓ_E, if $(M \cup M')_{\ell_E} \cap (M \cup M')_{\ell_E^*} \neq \emptyset$ for some ℓ_E^*, then*

$$F \succsim F_{M \to M'} \Rightarrow F \sim F_{M \to M \cup M'}.$$

Back