Belief-dependent preferences and updating

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September 30, 2024

Motivation

Individuals derive pleasure or pain from having specific beliefs.

Consequently, they avoid or distort information:

- investors overreact to good news (Daniel & Hirshleifer, 2015);
- donors are uninformed about their impact (Niehaus, 2014);
- medical patients avoid testing (Golman et al., 2017).

Economists need models aligned with these phenomena for prediction and policy.

Belief-dependent preferences

Proposal: Belief-dependent preferences (Bénabou & Tirole, 2016; Köszegi, 2010).

Three drawbacks:

- object of choice unobservable, such as beliefs, probability to forget;
- no identification, many pairs of beliefs and preferences are choice equivalent;
- BDP and non-Bayesian updating are disjoint assumptions.

This paper:

- propose novel observable choice data allowing identification (if and only if);
- BDP imply a specific form of non-Bayesian updating.

This Paper

Preferences depend on the individual's posterior beliefs.

The individual distorts beliefs away from Bayesian updating to increase her welfare.

She then acts according to her distorted beliefs.

Ex-ante, she anticipates such belief and choice distortion (Cobb-Clark et al., 2022).

Main result: axiomatic characterisation of BDP preferences and updating rules.

An investor decides whether to check the balance in her portfolio.

If she does, she observes price p and can invest (i_p) or withdraw (w_p).

Check

State	Menus
Good	i_p, w_p
Normal	
Bad	i_q, w_q

Upon observing price p, she infers the state of the market is not bad.

When she sees price q, she knows the state of the market is bad.

Illustrative Example

Before checking, she anticipates to overweight evidence and invest too much.

However, she might want to check anyway to obtain pleasant information.

Check

State	Menus
Good	i_p, w_p
Normal	
Bad	i_q, w_q

$\succ$

Not Check

State	Menus
Good	
Normal	0
Bad	

Table: Excessive investment.

Trade-off: not receiving information vs acting under a distorted belief.

She might also prefer not to check because she expects unpleasant information.

Check

State	Menus
Good	i_p, w_p
Normal	
Bad	i_q, w_q

$\prec$

Not Check

State	Menus
Good	
Normal	0
Bad	

Table: Information avoidance under negative prior beliefs, "ostrich effect".

Both excessive trading and the ostrich effect constitute empirical puzzles in finance (Daniel & Hirshleifer, 2015; Golman et al., 2017).

Solution: delegate to a financial advisor, allowing her to commit not to invest.

Delegate

State	Menus
Good	w_p
Normal	
Bad	w_q

Check

State	Menus
Good	i_p, w_p
Normal	
Bad	i_q, w_q

Not Check

State	Menus
Good	
Normal	0
Bad	

Commitment allows her to obtain information without being tempted to overinvest.

Delegate

State	Menus
Good	w_p
Normal	
Bad	w_q

$\succ$

Check

State	Menus
Good	i_p, w_p
Normal	
Bad	i_q, w_q

$\succ$

Not Check

State	Menus
Good	
Normal	0
Bad	

Table: Commitment under positive prior belief to avoid excessive investment.

Commitment might be welfare enhancing under belief-dependent preferences.

Literature

- *Decision Theory.* Liang (2017), Dillenberger & Raymond (2020) Rommeswinkel et al. (2023).

  Contribution: **Belief revision rule**.

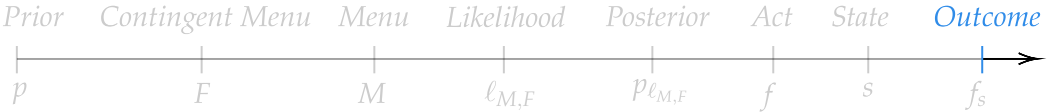
- *Menu Choice.* Gul & Pesendorfer (2001), Ozdenoren (2002), Epstein (2006), Epstein & Kopylov (2007).

  Contribution: **Novel primitive object of choice**.

- *Belief-Dependent Motivations.* Brunnermeier & Parker (2005), Eliaz & Spiegler (2006), Bénabou & Tirole (2016), Golman et al. (2017), Battigalli & Dufwenberg (2022).

  Contribution: **Identification, interaction of tastes and belief revision**.

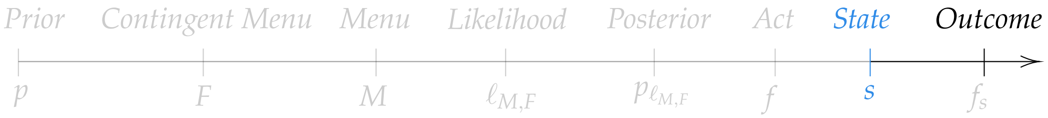
Model: acts



State	Menus
Good	i_p, w_p
Normal	
Bad	i_q, w_q

- outcome set (net financial gains);

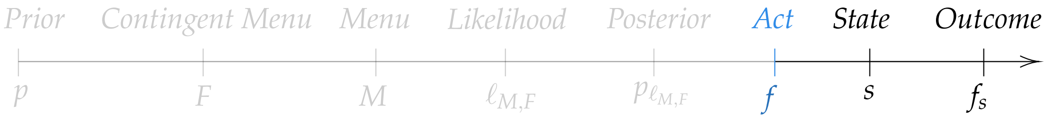
Model: acts



State	Menu
Good	i_p, w_p
Normal	
Bad	i_q, w_q

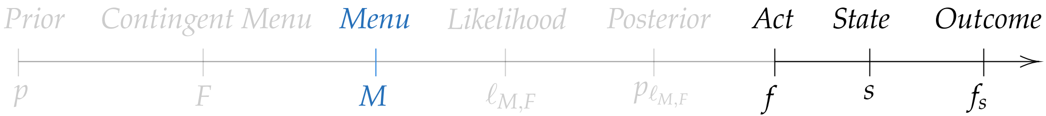
- outcome set;
- state set;

Model: acts



State	Menu
Good	i_p, w_p
Normal	
Bad	i_q, w_q

- outcome set;
- state set ;
- acts $f\ :\ \mathit{States} \rightarrow \mathit{Outcomes}$;



State	Menu
Good	i_p, w_p
Normal	i_p, w_p
Bad	i_q, w_q

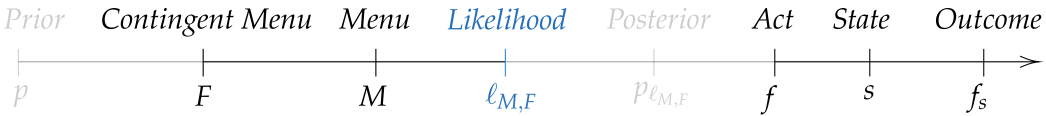
- outcome set;
- state set;
- acts f : *States* $\rightarrow$ *Outcomes*;
- a set of acts is a menu M;

Model: menus and contingent menus

Prior	*Contingent Menu*	*Menu*	*Likelihood*	*Posterior*	*Act*	*State*	*Outcome*
p	F	M	$\ell_{M,F}$	$p_{\ell_{M,F}}$	f	s	f_s

State	Menu
Good	i_p, w_p
Normal	
Bad	i_q, w_q

- outcome set;
- state set;
- acts $f : States \rightarrow Outcomes$;
- a set of acts is a menu M;
- a contingent menu is $F : States \rightarrow Menus$.



State	Menu
Good	$1 \cdot \{i_p, w_p\}$
Normal	
Bad	$1 \cdot \{i_q, w_q\}$

- a contingent menu is $F : \mathit{States} \rightarrow \mathit{Menus}$;
- probability menu M realises in state s is $F_s(M)$;

The likelihood of state s after realisation of menu M from the contingent menu F is

$$\ell_{M,F}(\mathit{Good}) = \frac{1}{1+1} = \frac{1}{2}.$$

Information

Prior	*Contingent Menu*	*Menu*	*Likelihood*	*Posterior*	*Act*	*State*	*Outcome*
p	F	M	$\ell_{M,F}$	$p_{\ell_{M,F}}$	f	s	f_s

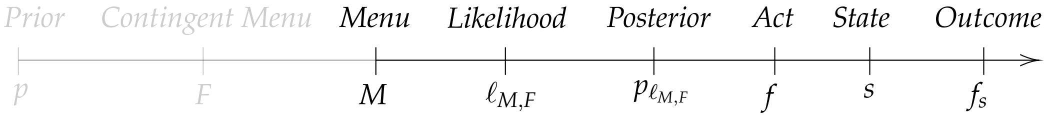
State	Menu
Good	$1 \cdot \{i_p, w_p\}$
Normal	
Bad	$1 \cdot \{i_q, w_q\}$

- a contingent menu is $F : \mathit{States} \to \mathit{Menus}$;
- probability menu M realises in state s is $F_s(M)$;
- prior p and likelihood $\ell_{M,F}$ induce posterior $p_{\ell_{M,F}}$.

The likelihood of state s after realisation of menu M from the contingent menu F is

$$\ell_{M,F}(\mathit{Good}) = \frac{1}{1+1} = \frac{1}{2} \qquad \ell_{M,F}(s) = \frac{F_s(M)}{\sum_{s' \in S} F_{s'}(M)}.$$

Utility over acts



Expected utility of act f at likelihood $\ell_{M,F}$

$$\sum_s p_{\ell_{M,F}}(s)\, u\left(f_s; \ell_{M,F}\right).$$

Under distorted likelihood $\ell^*_{M,F}$

$$\sum_s p_{\ell^*_{M,F}}(s)\, u\left(f_s; \ell^*_{M,F}\right).$$

The main theorem identifies conditions under which the individual solves

$$\max_{f \in M} \left[\underbrace{\sum_s p_{\ell_{M,F}}(s) u\left(f_s; \ell_{M,F}\right)}_{EU} + \alpha_{\ell_{M,F}} \underbrace{\sum_s p_{\ell^*_{M,F}}(s)\, u\left(f_s; \ell^*_{M,F}\right)}_{\text{distorted } EU} \right].$$

Utility over contingent menus

Prior	*Contingent Menu*	*Menu*	*Likelihood*	*Posterior*	*Act*	*State*	*Outcome*
p	F	M	$\ell_{M,F}$	$p_{\ell_{M,F}}$	f	s	f_s

Expected utility of contingent menu F is

$$\mathscr{U}(F) = \sum_M \sum_s p(s) F_s(M) \mathcal{U}\left(M; \ell_{M,F}\right).$$

The individual anticipates her choices and cost of temptation

$$\mathcal{U}\left(M; \ell_{M,F}\right) = \max_{f \in M} \left\{ \sum_s p_{\ell_{M,F}}(s)\, u\left(f_s; \ell_{M,F}\right) + \alpha_{\ell_{M,F}} \sum_s p_{\ell^*_{M,F}}(s)\, u\left(f_s; \ell^*_{M,F}\right) \right\} - \max_{f' \in M} \alpha_{\ell_{M,F}} \sum_s p_{\ell^*_{M,F}}(s)\, u\left(f'_s; \ell^*_{M,F}\right).$$

Alpha? Cost?

Expected utility of contingent menu F is

$$\mathscr{U}(F) = \sum_{M} \sum_{s} p(s) F_s(M) \mathcal{U}\left(M; \ell_{M,F}\right).$$

Only choices over contingent menus are needed for identification.

$$\mathcal{U}\left(M; \ell_{M,F}\right) = \max_{f \in M} \left\{ \sum_{s} p_{\ell_{M,F}}(s)\, u\left(f_s; \ell_{M,F}\right) + \alpha_{\ell_{M,F}} \sum_{s} p_{\ell^*_{M,F}}(s)\, u\left(f_s; \ell^*_{M,F}\right) \right\}$$
$$- \max_{f' \in M} \alpha_{\ell_{M,F}} \sum_{s} p_{\ell^*_{M,F}}(s)\, u\left(f'_s; \ell^*_{M,F}\right).$$

$$\mathscr{U}(F) = \sum_M \sum_s p(s) F_s(M) \mathcal{U}(M; \ell_{M,F}).$$

Say the true likelihood ℓ coincides with the distorted ℓ^*:

$$\mathcal{U}(M; \ell^*) = \max_{f \in M} \left\{ \sum_s p_{\ell^*}(s) u(f_s; \ell^*) + \alpha_{\ell^*} \sum_s p_{\ell^*}(s) u(f_s; \ell^*) \right\}$$
$$- \max_{f' \in M} \alpha_{\ell^*} \sum_s p_{\ell^*}(s) u(f'_s; \ell^*).$$

The second and third them cancel out, only EU under Bayesian updating remains.

BDP **imply** non-Bayesian updating.

Distorted likelihood

The distorted likelihood $\ell^*_{S'}$ at event S' is the best one when any outcome is available:

$$\ell^*_{S'} \in \arg\max_{\ell_{S'}} \max_x u\left(x; \ell_{S'}\right).$$

If a state has probability 0, its likelihood cannot be distorted.

Asymmetric updating: preferred likelihoods are not distorted.

Axiom

There is no temptation when, all else equal, the available menu comprises the best choices from both the Bayesian update and the favourite posterior.

Delegate

State	Menu
Good	w_p
Normal	
Bad	w_q

State	Menu
Good	i_p, w_p
Normal	
Bad	w_q

Full

AXIOM

There is no temptation when, all else equal, the available menu comprises the best choices from both the Bayesian update and the favourite posterior.

Delegate

State	Menu
Good	w_p
Normal	
Bad	w_q

$\succ$

State	Menu
Good	i_p, w_p
Normal	
Bad	w_q

Full

Axiom

There is no temptation when, all else equal, the available menu comprises the best choices from both the Bayesian update and the favourite posterior.

Delegate

State	Menu
Good	w_p
Normal	w_p
Bad	w_q

State	Menu
Good	0
Normal	0
Bad	w_q

Full

Main Axiom: Strategic Rationality for Best Likelihood

Axiom

There is no temptation when, all else equal, the available menu comprises the best choices from both the Bayesian update and the favourite posterior.

Delegate

State	Menu
Good	w_p
Normal	w_p
Bad	w_q

$\sim$

State	Menu
Good	0
Normal	0
Bad	w_q

Under no BDP, all posteriors are "favourite" and the axiom implies no temptation.

No temptation implies the model reduces to Expected utility and Bayesian updating.

Theorem

Preferences over contingent menus are represented by equations 1 and 2 if and only if they satisfy ***Strategic Rationality for Best Likelihood*** *and other regularity axioms.*

$$\mathscr{U}(F) = \sum_M \sum_s p(s)\, F_s(M)\, \mathcal{U}\left(M; \ell_{M,F}\right). \tag{1}$$

$$\begin{aligned} \mathcal{U}\left(M; \ell_{M,F}\right) = & \max_{f \in M} \left\{ \sum_s p_{\ell_{M,F}}(s)\, u\left(f_s; \ell_{M,F}\right) + \alpha_{\ell_{M,F}} \sum_s p_{\ell^*_{M,F}}(s)\, u\left(f_s; \ell^*_{M,F}\right) \right\} \\ & - \max_{f' \in M} \alpha_{\ell_{M,F}} \sum_s p_{\ell^*_{M,F}}(s)\, u\left(f'_s; \ell^*_{M,F}\right) \end{aligned} \tag{2}$$

Prior belief p, utilities u, distorted likelihoods ℓ^* and weights α are "unique". Axioms?

Why this model: observability and identification

The primitive objects of choice, contingent menus, are observable, contrary to

- Choice of beliefs (Brunnermeier & Parker, 2005; Köszegi, 2010);
- Choice of probability to forget (Bénabou & Tirole, 2016).

Choices of information sources do not allow identification (Eliaz & Spiegler, 2006).

These papers only provide "if" results, hard to distinguish from other theories.

Why this model: multiple selves and updating

Individual as the unit of choice, multiple selves render welfare analysis hard.

Under multiple selves, choices are dynamically inconsistent.

BDP and non-Bayesian updating are not disjoint, the first implies the second.

Conclusion

Theory of BDP and belief updating tested via choices of contingent menus.

Dynamically consistent individual anticipates she distorts beliefs to satisfy her BDP.

Asymmetric updating and no distortion of zero probability events.

Identification of BDP, non-Bayesian updating and strength of motivated reasoning.

Other applications in the paper:

- donors avoid and distort information about their impact;
- politicians send poor information to induce polarisation.

References

Battigalli, P., & Dufwenberg, M. (2022). Belief-dependent motivations and psychological game theory. *Journal of Economic Literature*, *60*(3), 833–882.

Bénabou, R., & Tirole, J. (2016). Mindful economics: The production, consumption, and value of beliefs. *Journal of Economic Perspectives*, *30*(3), 141–64.

Brunnermeier, M. K., & Parker, J. A. (2005). Optimal expectations. *American Economic Review*, *95*(4), 1092–1118.

Cobb-Clark, D. A., Dahmann, S. C., Kamhöfer, D. A., & Schildberg-Hörisch, H. (2022). The predictive power of self-control for life outcomes. *Journal of Economic Behavior & Organization*, *197*, 725–744.

Daniel, K., & Hirshleifer, D. (2015). Overconfident investors, predictable returns, and excessive trading. *Journal of Economic Perspectives*, *29*(4), 61–88.

Dillenberger, D., & Raymond, C. (2020). Additive-belief-based preferences. *PIER Working Paper*.

Eliaz, K., & Spiegler, R. (2006). Can anticipatory feelings explain anomalous choices of information sources? *Games and Economic Behavior*, *56*(1), 87–104.

Epstein, L. G. (2006). An axiomatic model of non-Bayesian updating. *The Review of Economic Studies*, *73*(2), 413–436.

Epstein, L. G., & Kopylov, I. (2007). Cold feet. *Theoretical Economics*, *2*, 231–259.

Golman, R., Hagmann, D., & Loewenstein, G. (2017). Information avoidance. *Journal of economic literature*, *55*(1), 96–135.

Gul, F., & Pesendorfer, W. (2001). Temptation and Self-Control. *Econometrica*, *69*(6), 1403–1435.

Köszegi, B. (2010). Utility from anticipation and personal equilibrium. *Economic Theory*, *44*, 415–444.

Liang, Y. (2017). Information-dependent expected utility. *Available at SSRN 2842714*.

Niehaus, P. (2014). A theory of good intentions. *San Diego, CA: University of California and Cambridge, MA: NBER*, *111*.

Ozdenoren, E. (2002). Completing the state space with subjective states. *Journal of Economic Theory*, *105*(2), 531–539.

Rommeswinkel, H., Chang, H.-C., & Hsu, W.-T. (2023). Preference for Knowledge. *Journal of Economic Theory*, *214*, 105737.

Cost of self-control

Identification of α_ℓ allows elaborating on its behavioural meaning

$$\alpha_\ell = \frac{\mathcal{U}\left(\{f, x\}, \ell\right) - \mathcal{U}\left(\{f, x'\}, \ell\right)}{u\left(x, \ell\right) - u\left(x', \ell\right)}.$$

It is the marginal cost of self-control at likelihood ℓ. Back

Alternative representation with cost

The representation can be written as either

$$\mathcal{U}\left(M;\ell_{M,F}\right)=\max_{f\in M}\left\{\sum_{s}p_{\ell_{M,F}}(s)\,u\left(f_s;\ell_{M,F}\right)+\alpha_{\ell_{M,F}}\sum_{s}p_{\ell^{*}_{S_{M,F}}}(s)\,u\left(f_s;\ell^{*}_{S_{M,F}}\right)\right\}$$
$$-\max_{f'\in M}\alpha_{\ell_{M,F}}\sum_{s}p_{\ell^{*}_{S_{M,F}}}(s)\,u\left(f'_s;\ell^{*}_{S_{M,F}}\right),$$

or

$$\mathcal{U}\left(M;\ell_{M,F}\right)=\max_{f\in M}\left\{\sum_{s}p_{\ell_{M,F}}(s)\,u\left(f_s;\ell_{M,F}\right)-\alpha_{\ell_{M,F}}c\left(f,u,S_{M,F}\right)\right\}.$$

Axioms: Basics

Axiom

*(**Order**). Preferences over contingent menus are a continuous weak order.* Full

Axiom

*(**Nondegeneracy**). There exist at least one outcome better than another.* Full

Axiom

*(**State Independence**). Preferences over outcomes do not depend on the state.*

Axiom

*(**Full support**). The individual assigns ex-ante positive probability to all states.*

Back

AXIOM

The individual is only indifferent between mixtures of contingent menus inducing the same inference.

Delegate

State	Menu
Good	i_p, n
Normal	
Bad	n

State	Menu
Good	n, w
Normal	
Bad	n

Full

Axiom

The individual is weakly worse if a menu is enhanced with ex-ante dominated options, because these induce temptation.

Delegate

State	Menu
Good	n
Normal	
Bad	n

$\succ$

State	Menu
Good	i_p, n
Normal	
Bad	n

Full

Axioms: Basics I

Axiom

(***Order***). *The ranking* $\succsim$ *is complete and transitive.*

Axiom

(***Continuity***). *For all contingent menus F the sets*

$$\{F' \mid F' \succsim F\} \text{ and } \{F' \mid F' \precsim F\}$$

are closed.

Axioms: Basics II

Axiom

*(**Nondegeneracy**). There exist outcomes y, y' in X for which $y \succ y'$.*

Axiom

*(**State Independence**). For all contingent menus F, menus L, L', M and states s, s',*

$$F \succsim F_{LsM \to L'sM} \;\Rightarrow\; F \succsim F_{Ls'M \to L's'M}.$$

Axiom

*(**Full Support**). For each state s, there exist contingent menus F and F' such that for all menus M it holds that $F_{s'}(M) = F'_{s'}(M)$ for every $s' \neq s$ and $F \nsim F'$.*

Back

Axioms: Identical Inference Independence

The support of F is

$$\mathcal{M}_F := \{M \in \mathcal{M} \mid F_s(M) > 0 \text{ for some } s \in S\}.$$

Definition

(**Identical Inference (II)**) Two contingent menus F and F' satisfy **identical inference** if, for each menu $M \in \mathcal{M}_F \cap \mathcal{M}_{F'}$ their likelihood is the same $\ell_{M,F} = \ell_{M,F'}$.

Axiom

(***II Independence***). *For all $0 < \lambda \leq 1$ and contingent menus F, F', F'' such that F and F'' satisfy II and F' and F'' satisfy II, $F \succsim F'$ if and only if $\lambda F + (1-\lambda) F'' \succsim \lambda F' + (1-\lambda) F''$.*

Back

Axioms: Set-Betweenness

Substitute from F any occurrence of M with M' to get $F_{M\to M'}$.

Axiom

(***Set-Betweenness***). *For all contingent menus F and menus M, M',*

$$F \succsim F_{M\to M'} \Rightarrow F \succsim F_{M\to M\cup M'} \succsim F_{M\to M'}.$$

Back

Axioms: Strategic Rationality for Best Likelihood

Substitute from F any occurrence of M with M' to get $F_{M \to M'}$.

For each menu M and likelihood ℓ define the set

$$M_\ell := \left\{ f \in M \;\middle|\; F \succsim F_{\{f\} \to \{f'\}} \text{ for all } f' \in M \text{ and } F \text{ such that } \ell_{\{f\},F} = \ell \right\}.$$

Axiom

*(**Strategic Rationality for Best Likelihood**). For all contingent menus F, menus M, M', events E and likelihoods ℓ_E, if $(M \cup M')_{\ell_E} \cap (M \cup M')_{\ell_E^*} \neq \emptyset$ for some ℓ_E^*, then*

$$F \succsim F_{M \to M'} \Rightarrow F \sim F_{M \to M \cup M'}.$$

Back