Belief-dependent motivations and updating

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Motivation

Investors are overconfident, consumers avoid learning about firms' unethical practices, and patients at health risk do not learn about their condition.

Theories of belief-dependent motivations (BDM) explain these phenomena

$$U(p) = \sum_{x \in X} p(x)\, u(x, p)\,.$$

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Three drawbacks:

- lack of preferences and beliefs identification;
- impossibility to distinguish "desired" from "undesired" beliefs;
- unknown relation between BDM and belief revision.

This Paper

Axiomatic analysis of BDM in a dynamic setting.

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Main result: functional representation of preferences over contingent menus:

- individual anticipates to update beliefs in the direction of her BDM;
- she is tempted to act according to such beliefs rather than the Bayesian update;
- she thus chooses the contingent menu accordingly.

Example

An investor decides whether to check the status of her portfolio.

Once the investor checks the portfolio, she decides whether to invest more (i) or withdraw any feasible amount of money, which could be high ($\overline{w}$) or low (w).

Check

State	Actions
Outstanding Good	i, $[0, \overline{w}]$
Bad	i, $[0, w]$

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Bad	$i, [0, w]$

When she checks her portfolio, upon observing a high amount in it she infers the status of the market is not bad.

When she sees a low amount, she knows the status of the market is bad.

Example

She can't make any inferences or do anything if she does not check.

Check

State	Actions
Outstanding Good	i, $[0, \overline{w}]$
Bad	i, $[0, w]$

Not Check

State	Actions
Outstanding Good Bad	0

She could also check and committ not to invest, by delegatin to a financial advisor.

Delegate

State	Actions
Outstanding	$[0, \overline{w}]$
Good	
Bad	$[0, w]$

Check

State	Actions
Outstanding	$i, [0, \overline{w}]$
Good	
Bad	$i, [0, w]$

Not Check

State	Actions
Outstanding	0
Good	
Bad	

Example

She anticipates to overweight evidence and prefers to committ, but also preferes to obtain information.

Delegate			**Check**			**Not Check**	
State	Actions		State	Actions		State	Actions
Outstanding Good	$[0, \overline{w}]$	$\succ$	Outstanding Good	$i, [0, \overline{w}]$	$\succ$	Outstanding Good	0
Bad	$[0, w]$		Bad	$i, [0, w]$		Bad	

Table: Commitment under positive prior belief to avoid excessive investment.

"Cognitive" non-Bayesian updating (Epstein, 2006) cannot rationalise this behaviour.

Example

If the investor expects the status of the market to be bad, she prefers not to check the portfolio at all to avoid receiving unpleasant information.

Not Check			Delegate			Check	
State	Actions		State	Actions		State	Actions
Outstanding Good Bad	0	$\succ$	Outstanding Good	$[0, \overline{w}]$	$\succ$	Outstanding Good	$i, [0, \overline{w}]$
			Bad	$[0, w]$		Bad	$i, [0, w]$

Table: Information avoidance under negative prior beliefs, "ostrich effect".

Both excessive trading and the ostrich effect constitutes empirical puzzles in finance (Daniel & Hirshleifer, 2015; Golman et al., 2017).

Literature

- *Decision Theory.* Liang (2017), Dillenberger & Raymond (2020) Rommeswinkel et al. (2023);
- *Menu Choice.* Gul & Pesendorfer (2001), Ozdenoren (2002), Epstein (2006), Epstein & Kopylov (2007);
- *Belief-Dependent Motivations.* Eliaz & Spiegler (2006), Bénabou & Tirole (2016), Golman et al. (2017), Battigalli & Dufwenberg (2022).

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- a contingent menu is $F : S \to \Delta^0(\mathcal{M})$, the probability that menu M realises if s is the true state is $F_s(M)$;
- the set of all contingent menus is $\mathcal{C} = \Delta^0(\mathcal{M})^S$;
- time 0 preference $\succsim$ is defined on $\mathcal{C}$.

The likelihood of state s after realisation of menu M from the contingent menu F is

$$\ell_{M,F}(s) := \frac{F_s(M)}{\sum_{s' \in S} F_{s'}(M)}.$$

Given any contingent menu F and menu M, the vector of likelihoods is $\ell_{M,F}$.

Gul & Pesendorfer (2001)

In the temptation and self-control model, behavior is represented by the following

$$\mathcal{U}(M) = \max_{f \in M} \left\{ U(f_s) + V(f_s) - \max_{f' \in M} V\left(f'_s\right) \right\}.$$

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Cost of self-control is

$$\max_{f' \in M} V\left(f'_s\right) - V(f_s).$$

Utility

Individual's behavior in this paper is represented by the following model

$$\mathscr{U}(F) = \sum_{M \in \mathcal{M}} \left(\sum_{s \in S} F_s(M) \right) \mathcal{U}\left(M; \ell_{M,F}\right) \ ; \tag{1}$$

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$$\begin{aligned} \mathcal{U}\left(M; \ell_{M,F}\right) = & \max_{f \in M} \left\{ \sum_{s \in S} u\left(f_s; \ell_{M,F}\right) p_{\ell_{M,F}}(s) + \alpha_{\ell_{M,F}} \sum_{s \in S} u\left(f_s; \ell^*_{M,F}\right) p_{\ell^*_{M,F}}(s) \right\} \\ & - \max_{f' \in M} \alpha_{\ell_{M,F}} \sum_{s \in S} u\left(f'_s; \ell^*_{M,F}\right) p_{\ell^*_{M,F}}(s) \; . \end{aligned} \tag{2}$$

When choosing act f from menu M after realisation of the likelihood $\ell_{M,F}$, the utility cost of temptation is

$$\alpha_{\ell_{M,F}} \left[\max_{f' \in M} \sum_{s \in S} u\left(f'_s; \ell^*_{M,F}\right) p_{\ell^*_{M,F}}(s) - \sum_{s \in S} u\left(f_s; \ell^*_{M,F}\right) p_{\ell^*_{M,F}}(s) \right].$$

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$$\alpha_{\ell_{M,F}}\left[\max_{f'\in M}\sum_{s\in S}u\left(f'_s;\ell^*_{M,F}\right)p_{\ell^*_{M,F}}(s)-\sum_{s\in S}u\left(f_s;\ell^*_{M,F}\right)p_{\ell^*_{M,F}}(s)\right].$$

Choice at period 2 is described by the following

$$\max_{f\in M}\left[\sum_{s\in S}u\left(f_s;\ell_{M,F}\right)p_{\ell_{M,F}}(s)+\alpha_{\ell_{M,F}}\sum_{s\in S}u\left(f_s;\ell^*_{M,F}\right)p_{\ell^*_{M,F}}(s)\right].$$

Conclusion

Theory of motivated belief updating based on choices of contingent menus.

The model predicts similar beliefs for similar preferences, assortativity of beliefs.

A second interpretation is a change of prior rather than a distorsion of the likelihood.

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Cost of self-control

Identification of α_ℓ allows elaboratig on its behavioral meaning

$$\alpha_\ell = \frac{\mathcal{U}\left(\{f, x\}, \ell\right) - \mathcal{U}\left(\{f, x'\}, \ell\right)}{u\left(x, \ell\right) - u\left(x', \ell\right)}.$$

It is the marginal cost of self-control at likelihood ℓ.

Example: Moral Wiggle Room (Dana et al., 2007)

A dictator in a laboratory experiment is endowed with 10 euros.

She decides how much to transfer to a recipient she is coupled with.

The transfer is subject to an unknown multiplier, which could be high, medium, or low.

The experimenter allows the dictator to choose the transfer from various menus conditional on the multiplier's value.

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

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State	Actions
High	$h\,\{5\} + (1-h)\,[5,10]$
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$\succsim$

State	Actions
High	$h\{3\} + (1-h)[5,10]$
Medium	$m\{3\} + (1-m)[0,5]$
Low	$[0,3]$

$$\Downarrow$$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succsim$

State	Actions
High	$h\{3,5\} + (1-h)[5,10]$
Medium	$m\{3,5\} + (1-m)[0,5]$
Low	$[0,3]$

Table: Set-Betweenness

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$\succ$

State	Actions
High	$h\{7\} + (1-h)[5,10]$
Medium	$m\{7\} + (1-m)[0,5]$
Low	$[0,3]$

Example

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$$\succ$$

State	Actions
High	$h\{7\} + (1-h)[5,10]$
Medium	$m\{7\} + (1-m)[0,5]$
Low	$[0,3]$

$$\Downarrow$$

State	Actions
High	$h\{5\} + (1-h)[5,10]$
Medium	$m\{5\} + (1-m)[0,5]$
Low	$[0,3]$

$$\sim$$

State	Actions
High	$h\{5,7\} + (1-h)[5,10]$
Medium	$m\{5,7\} + (1-m)[0,5]$
Low	$[0,3]$

Table: Strategic Rationality for Best Likelihood