TD 10

1 Review

h. Considering the estimation of the Cagan model, hyperinflation in the Weimar republic was driven only by an excess in money supply.

▼ *Answer*

False: The Cagan model is all about the interplay between money supply and the expectation of inflation. He shows that also expectations mattered in that episode.

i. An efficient way to end hyperinflations is to change the legal currency used in the country.

▼ *Answer*

Maybe: It can be efficient, although not necessary sufficient. It worked in the historical cases of the Weimar Republic (when the Rentenmark replaced the Papiermark), Zimbabwe (in which the Zimbabwean dollar ceased to exist and was replaced by the US dollar, the Euro and the South-African rand) and in Brazil when the real was introduced. However, people also need to trust that this new currency will be stable and the government needs to have a balanced budget which will not necessitate balancing through by issuing more money.

j. In the Cagan model, prices rise whenever real money demand falls.

▼ *Answer*

Maybe: There are different factors which affect money demand, as shown at page 12 of your notes. Everything else fixed, the statement is true.

k. All else being equal, hyperinflation is more likely to feed on itself (in other words, hyperinflation is more likely to be driven by momentum) if expected future inflation depends a lot on the past realisation of inflation.

▼ *Answer*

True: As we will see in the following exercise, for a low level of λ this is indeed true, check $\pi_t^e = \lambda \pi_{t-1}^e + (1-\lambda)(p_t - p_{t-1})$.

l. All else being equal, momentum-driven hyperinflation is more likely when real money demand does not react much to expected future inflation.

▼ *Answer*

False: We will also see this in the exercise. You can see from $m_t^d - p_t = -\alpha \pi_t^e$ that for higher values of α hyperinflation is more likely

5 Cagan's model

In 1956, P. Cagan published a study of hyper inflation episodes, in which expectations seem to be crucial. This problem is inspired from Cagan 's model, but modifies the way expectations are modelled. We assume that real variables (output, interest rate) are constant, and we study the interactions between the general price level and the money supply, using a money demand equation.

a. The Cagan's money demand equation is specified as follows

$$log\left(\frac{M_t}{P_t}\right) = \alpha_0 + \alpha_1 logY_t + \alpha_2 i_t + u_t$$

where M_t is the monetary aggregate, O_t is the price level, i_t is the nominal interest rate and u_t is the error term reflecting a random shock that affects $log\left(\frac{M_t}{P_t}\right)$, with mean zero. COmmenti this equation. What are a priori the signs of the coefficients?

The relation between money, output and interest rate might remind you the very stylized IS-LM model. If output Y is higher, this means that people need to more money to buy and sell the product produced. We expect α_1 to be positive. On the contrary, every time someone holds money he is renouncing the possibility of investing it to get a return from a fruitful asset. The higher this return, the lower the money demanded. According to this logic, α_2 should be negative.

b. Given the Fisher equation $i_t = r_t + \pi_t^e$ where r is the real interest rate and $\pi_t^e = p_{t+1}^e - p_t$ is expected level of inflation, and under the assumption (in the very short run) that $y_t = y$ and $r_t = r$, show that the money demand can be written as

$$m_t - p_t = \gamma - \alpha\pi_t^e + u_t \tag{1}$$

where m, p and y are the natural logarithms of M, P and Y and γ, α some constants to define.

The only thing we can do is to start from the expression we have. By substituting the logarithms we should reach the solution (remember that $log\left(\frac{x}{y}\right) = log(x) - log(y)$):

$$\begin{aligned} log\left(\frac{M_t}{P_t}\right) &= \alpha_0 + \alpha_1 logY_t + \alpha_2 i_t + u_t \\ m_t - p_t &= \alpha_0 + \alpha_1 y_t + \alpha_2 i_t + u_t \\ m_t - p_t &= \alpha_0 + \alpha_1 y_t + \alpha_2(r_t + \pi_t^e) + u_t \end{aligned}$$

The left hand-side is fine. The error term u_t is also there, together with π_t^e. We just have to understand which is the coefficient of π_t^e and collect everything else as γ:

$$\begin{aligned} m_t - p_t &= \alpha_0 + \alpha_1 y_t + \alpha_2(r_t + \pi_t^e) + u_t \\ m_t - p_t &= \underbrace{\alpha_0 + \alpha_1 y_t + \alpha_2 r_t}_{\gamma} + \underbrace{\alpha_2}_{-\alpha}\pi_t^e + u_t \\ m_t - p_t &= \gamma - \alpha\pi_t^e + u_t \end{aligned}$$

which is the expression we wanted. Notice that this new equation tells us that a higher level of inflation has a negative impact on $log\left(\frac{M_t}{P_t}\right)$. This is because $\alpha_2 < 0 \Rightarrow -\alpha_2 > 0 \Rightarrow \alpha > 0 \Rightarrow -\alpha < 0$. Hence, when π_t^e is high, $log\left(\frac{M_t}{P_t}\right)$ is low.

c. The following table shows Cagan's observations concerning hyperinflation episodes. The last column presents the minimum level of real balances $\frac{M}{P}$ over the period, as a percentage of the initial level of real balances. It reflects how low money demand dipped during the episode. Comment on this table. Are these observations compatible with the money demand equation?

Country	Period	Average inflation rate (% per month)	Real balances (minimum/initial)
Austria	Oct 1921–Aug 1922	47.1	0.35
Germany	Aug 1922–Nov 1923	322	0.03
Greece	Nov 1943–Nov 1944	365	0.007
Hungary	March 1923–Feb 1924	46	0.39
Hungary	Aug 1945–June 1946	19800	0.0003
Poland	Jan 1923–Jan 1924	81.1	0.34
Russia	Dec 1921–Jan 1924	57	0.27

The table confirms the relation we computed above. Consider the second observation for Hungary, as an example. Inflation is very high and, in fact, the minimum value for real balances is close to 0. The following graph plots the correlation coefficient based on the data of this table. You see that the line has clearly a downward slope.

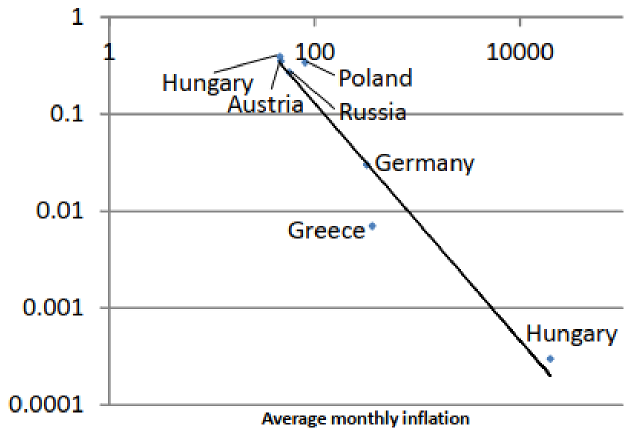

Figure 2: Real balances/initial real balances and inflation during hyperinflation episodes.

d. Why can't we estimate directly equation (1) using available data?

Simply because is very hard to have data about π_t^e, expected inflation. We should ask many people to tell us how much inflation they think there will be in the future (actually, the mere fact of them revealing their expectations might change other people expectations...). Hence, we need to draw some inferences about π_t^e by constructing a theory of how it is determined.

e. Working under the supervision of Friedman, Cagan assumed that expectations were adaptative:

$$\pi_t^e - \pi_{t-1}^e = (1-\lambda)(\pi_{t-1} - \pi_{t-1}^e)$$

Comment this expectation equation.

To interpret it easily we could rewrite it as follows:

$$\pi_t^e = \pi_{t-1} - \lambda\pi_{t-1} - \pi_{t-1}^e + \lambda\pi_{t-1}^e + \pi_{t-1}^e$$
$$\pi_t^e = \lambda\pi_{t-1}^e + (1-\lambda)\pi_{t-1}$$

So expectations about inflation at time t are a combination of realized inflation at time $t-1$ and expectations about it. The variable λ represents the relative weight the agents gives to expectations rather than realised inflation. Notice that if $\pi_{t-1} = \pi_{t-1}^e$ then:

$$\pi_t^e = \pi_{t-1}$$

If the prediction was correct in the past, the agents will not change it.

f. Show that expected inflation π_t^e can be written as a weighted sum of past inflation rates π_{t-i}, assuming $0 < \lambda < 1$. What is the meaning of the λ parameter?

As in one of our previous exercises, we could use a brute force technique here to express present expectations as a function of past expectations. Consider expectations at time t:

$$\begin{aligned}
\pi_t^e &= (1-\lambda)\pi_{t-1} + \lambda\pi_{t-1}^e \\
&= (1-\lambda)\pi_{t-1} + \lambda((1-\lambda)\pi_{t-2} + \lambda\pi_{t-2}^e) \\
&= (1-\lambda)\pi_{t-1} + \lambda(1-\lambda)\pi_{t-2} + \lambda^2\pi_{t-2}^e \\
&= (1-\lambda)\pi_{t-1} + \lambda(1-\lambda)\pi_{t-2} + \lambda^2((1-\lambda)\pi_{t-3} + \lambda\pi_{t-3}^e) \\
&= (1-\lambda)\pi_{t-1} + \lambda(1-\lambda)\pi_{t-2} + \lambda^2(1-\lambda)\pi_{t-3} + \lambda^3\pi_{t-3}^e \\
&\vdots
\end{aligned}$$

You see that any π_{t-i} is multiplied by $(1-\lambda)$ and λ^{i-1}. We may recollect the terms as:

$$\pi_t^e = \sum_{i=1}^{\infty}(1-\lambda)\lambda^{i-1}\pi_{t-i}$$

The last step is to express this as a weighted sum. Of course the weights should be the λ^i , hence it is enough to back out $\frac{1-\lambda}{\lambda}$.

$$\pi_t^e = \frac{1-\lambda}{\lambda}\sum_{i=1}^{\infty}\lambda^i\pi_{t-i}$$

The furthest is π_{t-i} from π_t (high i) the less the agent care about past realisations, as since $\lambda < 1$ we have that $\lambda^i > \lambda^{i+1}$.

g. Using the expectation formula given in e), solve the model to get $m_t - p_t$ as a function of observable variables π_t, $m_t - p_t$, u_t and u_{t-1}.

Now that we have a theory of how expectations are formed, we can use the observable variables that we have to estimate the regression at the beginning of the exercise. Recall that we had:

$$m_t - p_t = \gamma - \alpha\pi_t^e + u_t$$

By plugging in $\pi_t^e = \lambda\pi_{t-1}^e + (1-\lambda)\pi_{t-1}$:

$$\begin{aligned}
m_t - p_t &= \gamma - \alpha\left(\lambda\pi_{t-1}^e + (1-\lambda)\pi_{t-1}\right) + u_t \\
&= \gamma - \alpha\lambda\pi_{t-1}^e - \alpha(1-\lambda)\pi_{t-1} + u_t
\end{aligned}$$

We still have the expectation π_{t-1}^e in the expression, but we can back it out from the previous period regression:

$$m_{t-1}p_{t-1} = \gamma - \alpha\pi_{t-1}^e + u_{t-1} \Rightarrow \pi_{t-1}^e = -\frac{1}{\alpha}\left(m_{t-1} - p_{t-1} - \gamma - u_{t-1}\right)$$

By substituting again:

$$
\begin{aligned}
m_t - p_t &= \gamma - \alpha\lambda\pi^e_{t-1} - \alpha(1-\lambda)\pi_{t-1} + u_t \\
&= \gamma - \alpha\lambda\left[-\frac{1}{\alpha}\left(m_{t-1} - p_{t-1} - \gamma - u_{t-1}\right)\right] - \alpha(1-\lambda)\pi_{t-1} + u_t \\
&= \gamma + \lambda\left[\left(m_{t-1} - p_{t-1} - \gamma - u_{t-1}\right)\right] - \alpha(1-\lambda)\pi_{t-1} + u_t \\
&= (1-\lambda)\gamma + \lambda\left(m_{t-1} - p_{t-1}\right) - \alpha(1-\lambda)\pi_{t-1} + u_t - \lambda u_{t-1}
\end{aligned}
$$

Which gives us a regression on past observable variables.

h. Assume that money supply is exogenous. Rewrite the model's solution as $p_t = f(p_{t-1}, m_t, m_{t-1}, u_t, u_{t-1})$. We further assume that m_t me_t u_t = 0 (what?). Under which condition is the model stable (i.e. converging to a finite limit).

The equation above is exactly a function of $(p_{t-1}, m_t, m_{t-1}, u_t, u_{t-1})$, so it is enough to explicit p_t:

$$
\begin{aligned}
m_t - p_t &= (1-\lambda)\gamma + \lambda\left(m_{t-1} - p_{t-1}\right) - \alpha(1-\lambda)\pi_{t-1} + \\
m_t - p_t &= (1-\lambda)\gamma + \lambda\left(m_{t-1} - p_{t-1}\right) - \alpha(1-\lambda)(p_t - p_{\cdot} \\
-(1-\lambda)\gamma - \lambda m_{t-1} + m_t + (\lambda - \alpha(1-\lambda))\, p_{t-1} - u_t + \lambda u_{t-1} &= (1-\alpha(1-\lambda))\, p_t \\
\frac{-(1-\lambda)\gamma - \lambda m_{t-1} + m_t + (\lambda - \alpha(1-\lambda))\, p_{t-1} - u_t + \lambda u_{t-1}}{(1-\alpha(1-\lambda))} &= p_t
\end{aligned}
$$

The series does not explode if the price does not increase continuosly. This happens when the coefficient of past price on current price is less than 1, which in our model translates into $\frac{\lambda-\alpha(1-\lambda)}{1-\alpha(1-\lambda)} < 1$.

i. Cagan estimates the following equation:

$$
p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2 m_t - \beta_3 m_{t-1} + \varepsilon_t
$$

where β_0 is a constant term and ε_t is a residual. The estimates lead to the following results

Country	Period	Coefficients		
		$\hat\beta_1$ $\frac{\hat\lambda-\hat\alpha(1-\hat\lambda)}{1-\hat\alpha(1-\hat\lambda)}$	$\hat\beta_2$ $\frac{1}{1-\hat\alpha(1-\hat\lambda)}$	$\hat\beta_3$ $\frac{\hat\lambda}{1-\hat\alpha(1-\hat\lambda)}$
Austria	Oct 1921–Aug 1922	0.928	0.556	0.516
Germany	Aug 1922–Nov 1923	3.17	-0.092	-0.292
Greece	Nov 1943–Nov 1944	0.611	0.386	0.236
Hungary	March 1923–Feb 1924	0.23	0.13	0.03
Hungary	Aug 1945–June 1946	0.67	0.455	0.305
Poland	Jan 1923–Jan 1924	0.032	0.31	0.01
Russia	Dec 1921–Jan 1924	5.92	-0.07	-0.421

Are there countries in which hyper inflation can occur without any explosion of the money supply? why?

This table provides the estimates of the coefficient of p_{t-1} on p_t that we computed above. We just realised that if that coefficient is greater than 1 than prices explodes in a finite amount of time. This condition is respected both by Germany and by Russia.

j. The following table shows Cagan 's results from estimating α and λ. Note that there is not a perfect correspondence with $\hat\beta_1$, $\hat\beta_2$, and $\hat\beta_3$ since the model is overidentified.

Country	Period	Param. $\hat{\alpha}$	$\hat{\lambda}$
Austria	Oct 1921–Aug 1922	8.55	0.95
Germany	Aug 1922–Nov 1923	5.46	0.8
Greece	Nov 1943–Nov 1944	4.09	0.85
Hungary	March 1923–Feb 1924	8.7	0.9
Hungary	Aug 1945–June 1946	3.63	0.85
Poland	Jan 1923–Jan 1924	2.3	0.7
Russia	Dec 1921–Jan 1924	3.06	0.65

What do these results implied about inflation expectations? What do they imply about the reaction of money demand to inflation expectations?

Remember that we had $m_t - p_t = \gamma - \alpha\pi_t^e + u_t$. So, in the table we can see that α is quite high. This means that people reaction to inflation is significant, which could in turn lead to a price explosion and thus hyperinflation. However, this effect is countered by the high (sometimes close to 1) λ. Recall that $\pi_t^e = \lambda\pi_{t-1}^e + (1-\lambda)\pi_{t-1}$, so people are strongly anchored to their expectations and will slowly alter their beliefs.