Enrico Mattia Salonia
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The Ultimatum Game

Ann has 10 euros. She can choose whether to give all of them to Bob, split them between the two, or give nothing to Bob. Bob can later accept Ann's offer or reject it. If Bob accepts, Ann's offer is implemented. If he rejects, they both get nothing. The interaction is represented in Figure 1.

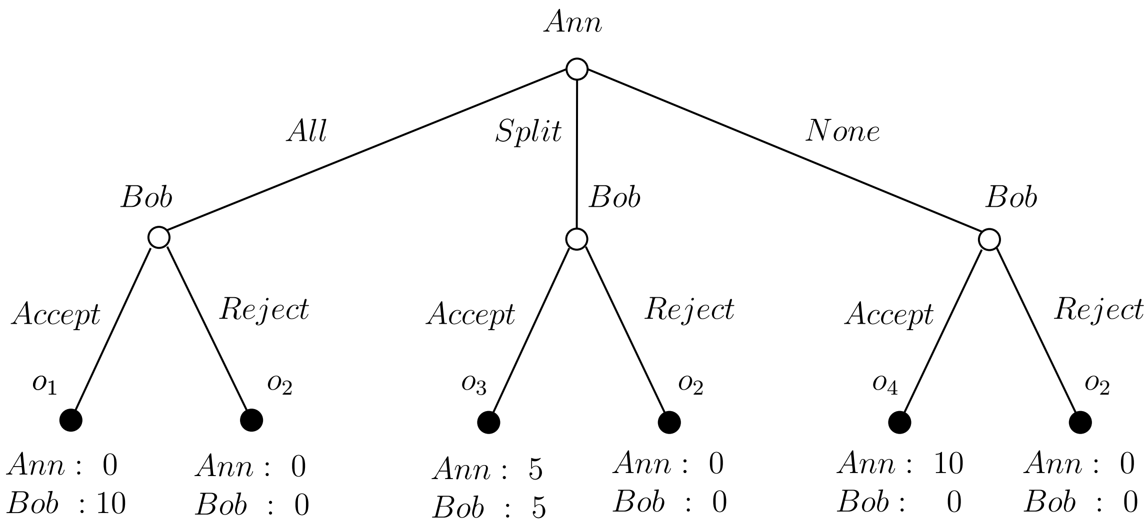

Figure 1: An extensive form for Ann and Bob.

The picture as an extensive form game with perfect information

This interaction is called the **Ultimatum Game**. Ann makes an offer to Bob, who can either accept or reject it; if he rejects, both players receive nothing. Let us study how elements of the definition of extensive form with perfect information map to this example.

The picture is a **rooted directed tree**.

- The *root* is the initial node, where *Ann* can choose what to do.
- Each arrow is a *directed edge* and points from a node to one of its *successors*.
- All the white dots are *decision nodes*. The black dots at the bottom are the *terminal nodes*.

As a **finite extensive form**, the tree comes with the following objects.

- The rooted directed tree defined above.
- The set of *players* is $I = \{Ann, Bob\}$.
- The *player function* assigns to each nonterminal node the player who moves there: *Ann* moves at the root, and *Bob* moves at the three nodes on the second level.

- For each nonterminal node, the set of *actions* available is given by the labels on the outgoing edges from the white dot:
  - at the root, where *Ann* moves: the available actions are the three offers *All*, *Split*, and *None*;
  - at each of *Bob*'s nodes: the available actions are *Accept* and *Reject*.
- The set of outcomes is $O = \{o_1, o_2, o_3, o_4\}$, where

$$o_1 = (0, 10), \qquad o_2 = (0, 0), \qquad o_3 = (5, 5), \qquad o_4 = (10, 0),$$

  with the first component giving Ann's monetary outcome and the second component giving Bob's monetary outcome.
- Each terminal node is assigned an *outcome*, shown next to the corresponding black dot.

This is a finite extensive form *with perfect information* because at each decision node the moving player knows exactly where in the tree they are. To complete the description of the game, we should add preferences. The following table records examples of preference relations.

Player	**Preference ordering**
Ann (selfish)	$o_4 \succ o_3 \succ o_1 \sim o_2$
Ann (fair, benevolent)	$o_3 \succ o_4 \succ o_1 \succ o_2$
Bob (selfish)	$o_1 \succ o_3 \sim o_4 \sim o_2$
Bob (fair, spiteful)	$o_3 \succ o_1 \succ o_2 \succ o_4$

We now have a **finite extensive game with perfect information**.

- The finite extensive form with perfect information we have above.
- A preference $\succsim_i$ over the set of outcomes O, one for each player $i \in I$.

Backward Induction

Say Ann is selfish and Bob is fair and spiteful, we can represent their preferences by the following utility functions.

Utility function	o_1	o_2	o_3	o_4
U_{Ann} (selfish)	2	1	3	4
U_{Bob} (fair, spiteful)	3	2	4	1

With these preferences, the game looks as in Figure 2.

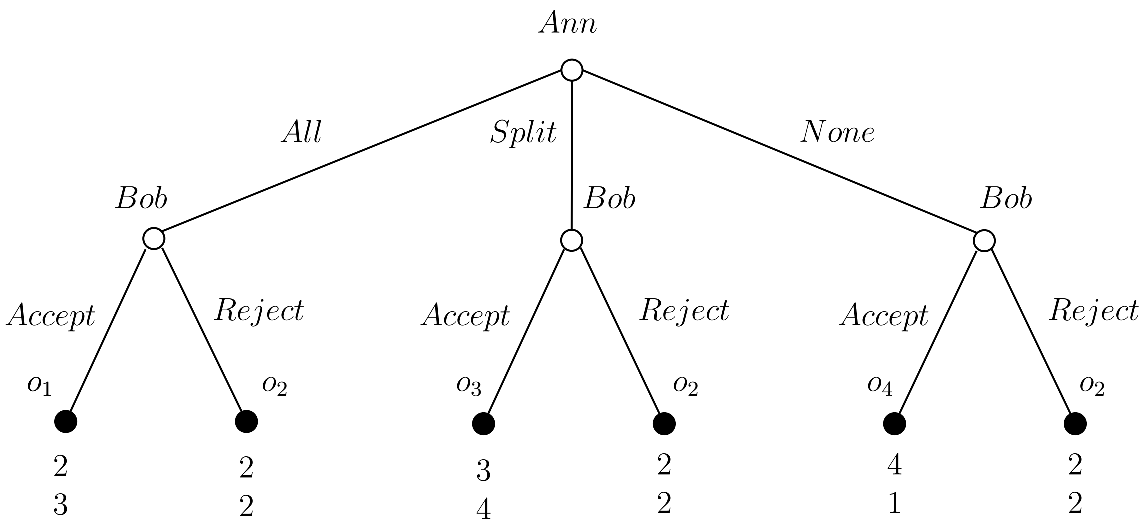

Figure 2: The ultimatum game if Ann is selfish and Bob is fair and spiteful.

Players could reason as follows. Start from each decision node immediately before a terminal node, and consider what the player assigned to that decision node would do. Such reasoning is called **Backward Induction**. Consider Bob's choice after Ann chooses to give *All* to him. He prefers o_1 to o_2, and therefore chooses to *Accept*. After Ann chooses to *Split*, he chooses to *Accept*, because he prefers o_3 to o_2. After Ann chooses *None*, Bob chooses to *Reject*, since he prefers o_2 to o_4. The colored edges in Figure 3 represent Bob's choices.

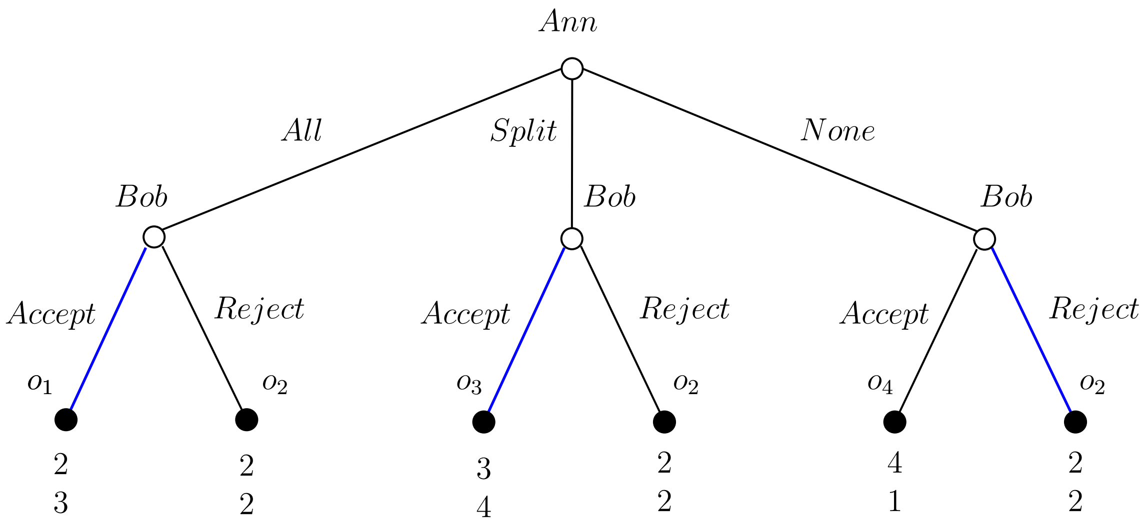

Figure 3: Bob's choice at each of his decision nodes in blue.

Ann can then anticipate Bob's choices. If she chooses *All*, Bob will induce outcome o_1; if she chooses *Split*, Bob will induce outcome o_3; and if she chooses *None*, Bob will induce outcome o_2. Since she prefers o_3 to o_1 and o_2, she chooses *Split*. The entire path we just described is represented in Figure 4.

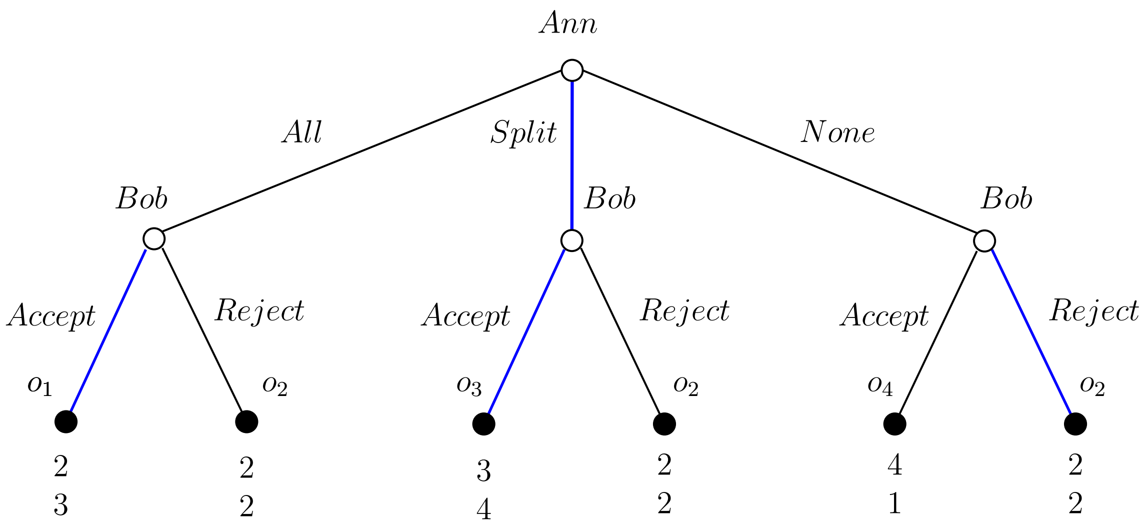

Figure 4: Ann's and Bob's choices at each decision node in blue.

What is the moral of the story? Bob's spitefulness induces Ann to split the money, even if she is selfish.

Exercises

> "Human nature is too weak to acquire an art of which it has no experience."
> (Plato, 1881, p. 111)

Exercise 1. Repeat the Backward Induction analysis under the assumption that both Ann and Bob are selfish. Do you notice any issue? Write the game in strategic form and find the Nash Equilibria.

Exercise 2. In this exercise, you study the **Dictator Game**. It is a variant of the ultimatum game in which the second player (Bob) cannot act; thus, the first player's (Ann) choice determines the outcome.

1. Describe this game as a finite extensive form and draw a diagram of it.

2. Suppose you are a researcher. You do not know Ann's preferences, but you observe her choices in this game. What, if anything, can you infer about her preferences?

3. Elaborate on how, in general, observing players' choices in a game frame or extensive form allows researchers to infer their preferences.

References

Plato. (1881). *The theaetetus of plato* (B. H. Kennedy, Ed. & Trans.) [With translation and notes]. Cambridge University Press.