

# APPROXIMATE COMMON BELIEFS IN RATIONALITY IN DYNAMIC GAMES

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Discussion  
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## EPISTEMIC GAME THEORY IN 30 SECONDS

Game theory = individual preferences (tastes and beliefs) + equilibrium analysis.

Equilibrium = rationality (expected utility) + common belief in rationality.

Behavioural economics introduces “behavioural heuristics” into games.

How these heuristics relate to classical preferences + equilibrium is often unclear.

Epistemic game theory takes these sums very seriously.

## MOTIVATION OF THIS PAPER

Dynamic refinements (e.g. subgame perfection) use the chronological structure of games.

These refinements often mispredict behaviour in classic experiments.

Question: how robust are these dynamic concepts to small doubts about rationality?

## FRAMEWORK AND CONCEPTS

Finite multistage games with observable actions.

Players hold conjectures over others' strategies.

Dynamic rationalizability: weak (initial), backward (induction), strong (forward induction).

Introduce  $p$ -belief: an event is believed with probability at least  $p < 1$ . (Monderer & Samet, 1989)

Define weak, backward, strong  $p$ -rationalizability.

## MAIN RESULTS: RATIONALIZABILITY

For  $p = 1$ :  $p$ -versions coincide with classical notions.

For any  $p < 1$ : strong and backward  $p$ -rationalizability collapse to weak rationalizability.

Interpretation: even tiny doubts about higher-order rationality destroy the refinement power of dynamic reasoning.

## BEHAVIOURAL IMPLICATIONS

With  $p < 1$ , dynamic refinements lose bite; predictions become essentially static.

Centipede game: cooperation in all but the last stage is  $p$ -rationalisable.

Finitely repeated Prisoner's Dilemma: all threshold strategies except "cooperate in the final round" are  $p$ -rationalisable.

Small doubts about rationality can rationalise non-inductive play in these games.

**But I add:** for any value of  $p$  you could probably rationalise many things!

## COMMENTS

Results induce a conceptual dilemma:

- Are there better dynamic refinements we did not develop yet?
- Is the notion of  $p$ -belief in this paper inadequate?
- (**Pessimistic**): there is no hope.

How much is off-path behaviour responsible?

Structural rationality (Siniscalchi, 2022) may help.

## REFERENCES

- Monderer, D., & Samet, D. (1989). Approximating common knowledge with common beliefs. *Games and Economic behavior*, 1(2), 170–190.
- Siniscalchi, M. (2022). Structural Rationality in Dynamic Games. *Econometrica*, 90(5), 2437–2469.