

APPROXIMATE COMMON BELIEFS IN RATIONALITY IN DYNAMIC GAMES

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Discussion
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PISTEMIC GAME THEORY IN 30 SECONDS

Game theory = individual preferences (tastes and beliefs) + equilibrium analysis.

Equilibrium = rationality (expected utility) + common belief in rationality.

Behavioural economics introduces “behavioural heuristics” into games.

How these heuristics relate to classical preferences + equilibrium is often unclear.

Epistemic game theory takes these sums very seriously.

MOTIVATION OF THIS PAPER

Dynamic refinements (e.g. subgame perfection) use the chronological structure of games.

These refinements often mispredict behaviour in classic experiments.

Question: how robust are these dynamic concepts to small doubts about rationality?

FRAMEWORK AND CONCEPTS

Finite multistage games with observable actions.

Players hold conjectures over others' strategies.

Dynamic rationalizability: weak (initial), backward (induction), strong (forward induction).

Introduce p -belief: an event is believed with probability at least $p < 1$. (Monderer & Samet, 1989)

Define weak, backward, strong p -rationalizability.

MAIN RESULTS: RATIONALIZABILITY

For $p = 1$: p -versions coincide with classical notions.

For any $p < 1$: strong and backward p -rationalizability collapse to weak rationalizability.

Interpretation: even tiny doubts about higher-order rationality destroy the refinement power of dynamic reasoning.

BEHAVIOURAL IMPLICATIONS

With $p < 1$, dynamic refinements lose bite; predictions become essentially static.

Centipede game: cooperation in all but the last stage is p -rationalisable.

Finitely repeated Prisoner's Dilemma: all threshold strategies except "cooperate in the final round" are p -rationalisable.

Small doubts about rationality can rationalise non-inductive play in these games.

But I add: for any value of p you could probably rationalise many things!

COMMENTS

Results induce a conceptual dilemma:

- Are there better dynamic refinements we did not develop yet?
- Is the notion of p -belief in this paper inadequate?
- (**Pessimistic**): there is no hope.

How much is off-path behaviour responsible?

Structural rationality (Siniscalchi, 2022) may help.

REFERENCES

- Monderer, D., & Samet, D. (1989). Approximating common knowledge with common beliefs. *Games and Economic behavior*, 1(2), 170–190.
- Siniscalchi, M. (2022). Structural Rationality in Dynamic Games. *Econometrica*, 90(5), 2437–2469.