Microeconomics 1 Lecture Notes

Enrico Mattia Salonia

October 8, 2025

Contents

Preamble

These notes are a first draft for the opening part of the PhD microeconomics sequence. They cover choice under uncertainty and general equilibrium. The write-up is not yet fully polished, and I will keep updating it. If you spot mistakes or typos, please let me know. I have aimed for a conversational style rather than the more formal tone of, say, Mas-Colell et al. (1995). I assumed no knowledge of the topics, but, as usual, mathematical sophistication helps (I hope you develope it during the class!). Each lecture outlines what we discuss in class, followed by exercises and suggestions for further reading.

Before we start, you may enjoy some non-technical background that frames the topics we'll study: Kreps (1988, ch. 1), Debreu (1959, pp. ix–xi), Myerson (1997, pp. 1–7), and Gilboa (2009, chs. 1–2).

You will occasionally see smaller text like this. These remarks are not essential for following the main exposition, but they add context or point to related ideas. Feel free to skip them on a first pass.

These notes draw on several sources. Below is a short reading list. If you would like more pointers on any topic, just ask—I am happy to discuss questions and related material, just drop me an email.

Have fun!

Choice under uncertainty.

- Kreps (1988).
- Fishburn (1970).
- Mas-Colell et al. (1995), ch. 6.
- Savage (1972).
- Gilboa (2009).
- Kreps (2013), ch. 7.

General equilibrium theory.

- Mas-Colell et al. (1995), chs. 15–17.
- Kreps (2013), chs. 10–11.

- Debreu (1959).
- McKenzie (2005).
- Fleurbaey & Maniquet (2011) (for the part related to no-envy).

References

Debreu, G. (1959). *Theory of value: An axiomatic analysis of economic equilibrium* (Vol. 17). Yale University Press. 1, 2

Fishburn, P. C. (1970). *Utility theory for decision making*. New York: Wiley. 1

Fleurbaey, M., & Maniquet, F. (2011). *A theory of fairness and social welfare* (Vol. 48). Cambridge University Press. 2

Gilboa, I. (2009). *Theory of decision under uncertainty* (Vol. 45). Cambridge university press. 1

Kreps, D. M. (1988). *Notes on the theory of choice*. Westview Press. 1

Kreps, D. M. (2013). *Microeconomic foundations* (Vol. 1). Princeton university press. 1

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic theory*. Oxford university press New York. 1

McKenzie, L. W. (2005). Classical general equilibrium theory. *MIT Press Books*, *1*. 2

Myerson, R. B. (1997). *Game theory: Analysis of conflict*. Cambridge, Massachusetts, USA: Harvard University Press. 1

Savage, L. J. (1972). *The foundations of statistics* (2nd rev. ed.). New York: Dover Publications. 1

Lecture 1

Introduction to uncertainty

1.1 How to model uncertainty

The outcomes of our decisions are often uncertain, so we want a choice theory that takes uncertainty into account. Let's start by thinking about how to represent uncertainty. Suppose you make a bet with a friend: if a fair coin toss results in heads, you get 10 euros; otherwise you pay 10 euros to your friend. There are two outcomes, 10 and -10, and since the coin is fair, each occurs with probability $1/2$. What are the main ingredients of this example?

First, we started from a set of outcomes, in this case the monetary transfers 10 and -10. Second, we specified the probability of each outcome occurring, $1/2$ for both. We call such an object—a set of outcomes, each associated with a probability—a **lottery**. Denote the set of outcomes by X. Generic elements of X will be denoted x, y, z or sometimes $x_1, x_2, \ldots$. For simplicity, assume that X is finite. Outcomes alone are not enough to describe a lottery: we need probability distributions over outcomes, as in the $1/2$–$1/2$ distribution of the fair coin above. The set of lotteries over X is denoted by $\Delta(X)$.[1] Each element of $\Delta(X)$ is a function $p : X \to [0, 1]$ such that $\sum_{x \in X} p(x) = 1$; it maps each outcome x to a number $p(x) \in [0, 1]$, representing the probability that x occurs.[2] We can write a lottery as a vector, for example $p = (p(x), p(y), p(z))$ if $X = \{x, y, z\}$.

Example 1.1. In the example above, the set of outcomes is $\{10, -10\}$ and the lottery $p \in \Delta(\{10, -10\})$ induced by the fair coin toss satisfies $p(10) = p(-10) = 1/2$.

We can depict lotteries using a tree diagram, as in Figure 1.1.

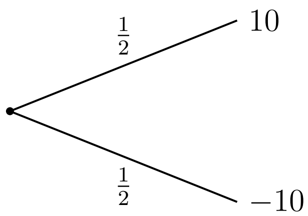


Figure 1.1: Lottery from Example 1.1.

Remark 1.1. *Notice that, in this setup, we are missing something: whether the coin lands on heads or tails is irrelevant; only the probabilities of outcomes matter, not the*

[1] Why the notation Δ? You will realise soon.

[2] Why do we write a sum $\sum_{x \in X} p(x) = 1$ and not an integral?

events that induced them. This is a limitation of this model, which we will address when we introduce a state-space representation of uncertainty.

The set of lotteries $\Delta(X)$ has *structure*: we can combine its elements in a meaningful way. For example, consider a lottery r that yields the lottery p with probability α and q with probability $1-\alpha$, where $\alpha \in [0,1]$. This is a *compound lottery*. It is still an element of $\Delta(X)$, and we write $r = \alpha p + (1-\alpha)q$. For instance, if $p(10) = 1/2$ and $q(10) = 1/4$, the associated compound lottery is represented on the left of Figure 1.2. We can compute the probability that outcome 10 occurs in this compound lottery: $\alpha \times 1/2 + (1-\alpha) \times 1/4 = \frac{1+\alpha}{4}$. By computing the probability of each outcome in a compound lottery, we *reduce* it to a simple lottery, as represented on the right of Figure 1.2.

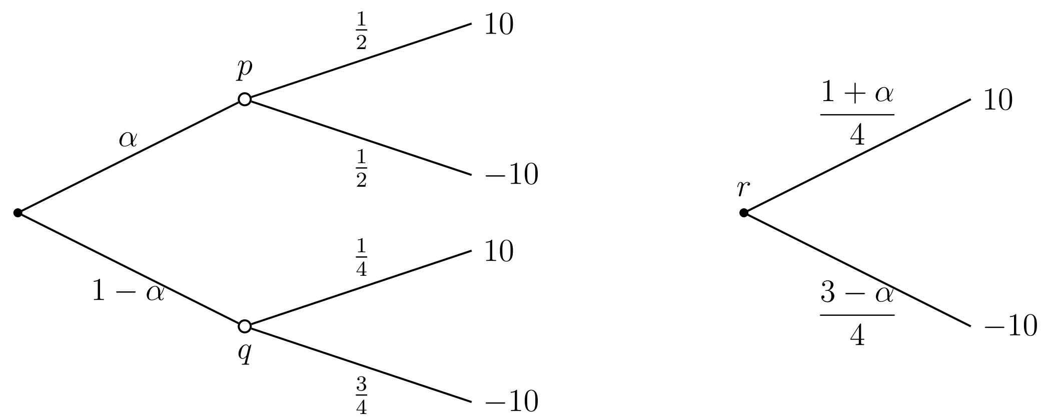


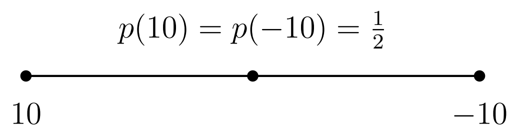
Figure 1.2: Compound lottery (left) and its reduced form (right).

We assume *reduction of compound lotteries*: individuals are indifferent between any compound lottery and its reduced form, i.e., any two lotteries that induce the same probabilities over outcomes are treated as the same.

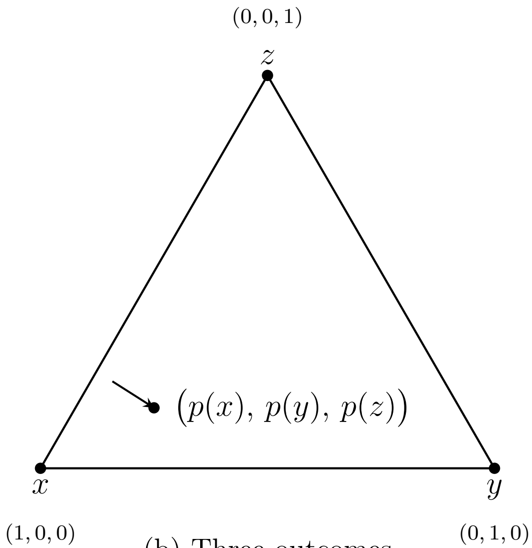
Can you think of reasons why someone might *not* be indifferent between a compound lottery and its reduced form? Violations of reduction generate interesting phenomena studied in behavioural economics. See, e.g., Segal (1990) and Dillenberger & Raymond (2020).

This lottery *mixing* operation is not possible with an unstructured set of outcomes. As an illustration, suppose the set of outcomes comprises fruits. We can have an apple or a banana, but there is no fruit that is a mixture of an apple and a banana. Imposing structure on the set of elements to be ranked is a key move of microeconomic theory. In fact, we will later assume that the set of outcomes is $\mathbb{R}$, the set of real numbers representing monetary outcomes, which lets us say more than with a generic set of outcomes.

There is another way to represent lotteries graphically. Consider once again the coin toss: obtaining 10 euros with probability $1/2$ and -10 euros with probability $1/2$. We can represent this lottery as the midpoint of the line segment whose endpoints correspond to the degenerate lotteries yielding 10 and -10 with probability 1; see panel (a) of Figure 1.3. More generally, with n outcomes we can represent a lottery as a point in an $(n-1)$-dimensional simplex. For example, with three outcomes we can represent lotteries as points in an equilateral triangle, as in panel (b).[3] The vertices of the triangle correspond to degenerate lotteries that yield one outcome with probability 1. Any other point in the triangle corresponds to a lottery that yields each of the three outcomes with some probability. Roughly, the farther a point is from a vertex, the lower the probability of the corresponding outcome. For example, the lottery p in panel (b) yields outcome x with relatively high probability and outcomes y and z with relatively low probabilities.



(a) Two outcomes



(b) Three outcomes

Figure 1.3: Lotteries as points in simplexes: (a) a two–outcome lottery lies on a line segment; (b) with three outcomes, lotteries lie in an equilateral triangle.

For a finite outcome set X, the probability simplex over X is

$$\Delta(X) = \left\{ p : X \to [0,1] \mid \sum_{x \in X} p(x) = 1 \right\}$$

or equivalently

$$\left\{ (p(x_1), \ldots, p(x_n)) \in \mathbb{R}^n \mid p(x_i) \geq 0, \sum_i p(x_i) = 1 \right\}.$$

[3] That's why the Δ notation!

This is an $n-1$-dimensional simplex whose vertices are the degenerate lotteries (unit vectors), e.g. $(1, 0, \dots, 0), \dots, (0, \dots, 0, 1)$.

1.2 Preferences over lotteries

Our aim is to understand how individuals choose between lotteries, whether they like or dislike risk, and how to compare different individuals' attitudes towards risk. We therefore need to make statements such as "an individual weakly prefers lottery p to lottery q". To do so, introduce a binary relation $\succsim$ over $\Delta(X)$, so that $p \succsim q$ reads "the individual weakly prefers lottery p to lottery q".[4] Compared to choice under certainty, we are now comparing lotteries—i.e., probability distributions over outcomes—rather than outcomes themselves.

Technically, $\succsim$ is a subset of $\Delta(X) \times \Delta(X)$, i.e., a set of ordered pairs of lotteries. For example, if $p, q, r \in \Delta(X)$, the statements p *is (weakly) preferred to* q imply $(p, q) \in \succsim$.

Recall that we can define strict preference and indifference in terms of weak preference: $p \succ q$, which reads "p is strictly preferred to q" if and only if $p \succsim q$ but not $q \succsim p$; and $p \sim q$, which reads "p is indifferent to q" if and only if both $p \succsim q$ and $q \succsim p$.

In what follows, we consider what assumptions preferences over lotteries might satisfy, and what these assumptions imply for behaviour.

1.3 Things to read

See Kreps (1988, pp. 31–33) for a brief intuitive introduction to the lottery model in this chapter. If you want to read a similar treatment from a textbook read Mas-Colell et al. (1995, pp. 168–170).

1.4 Exercises

Exercise 1.1. Can we still represent the set of lotteries and compound lotteries on the simplex if there is no indifference between a compound lottery and its reduced form? Why or why not?

Exercise 1.2. Assume there are three outcomes x, y, z. Draw in the simplex the set of lotteries that yield each outcome with the same probability and the lottery that yields

[4] Are you curious why we use the symbol $\succsim$ for preferences and not $\geq$? An historian of economic theory, Ivan Boldyrev, told me it is from Herstein & Milnor (1953), who used the symbol in their famous paper where they provided an axiomatic characterisation of expected utility, which we will do soon.

x with certainty. Now draw the set of all mixtures of these two lotteries. Assume that the individual is indifferent between the lottery yielding each outcome with the same probability, the lottery yielding x with certainty, and any mixture of the two. Which part of the simplex does this indifference "curve" correspond to? Is it really a curve?

Exercise 1.3. Assuming three outcomes x, y, z, draw in the simplex the set of lotteries that yield outcome x with probability at least $1/2$.

Exercise 1.4. In the main text we assumed that individuals are indifferent between a compound lottery and its reduced form. State this indifference formally as a condition on the preference relation $\succsim$, using the notation introduced.

References

Dillenberger, D., & Raymond, C. (2020). *Additive-belief-based preferences* (PIER Working Paper No. 20-020). Philadelphia, PA, USA: Penn Institute for Economic Research (PIER). 4

Herstein, I. N., & Milnor, J. (1953). An axiomatic approach to measurable utility. *Econometrica, Journal of the Econometric Society*, 291–297. 6

Kreps, D. M. (1988). *Notes on the theory of choice.* Westview Press. 6

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic theory.* Oxford university press New York. 6

Segal, U. (1990). Two-stage lotteries without the reduction axiom. *Econometrica: Journal of the Econometric Society*, 349–377. 4

Lecture 2

Expected utility theory

2.1 Assumptions on preferences

We now impose properties on preferences over lotteries. But first, a brief methodological aside on what we are doing. Before discussing properties of $\succsim$, we should explicit what the interpretation of $\succsim$ is. Different methodological stances are possible. Is $\succsim$ tracking what an individual has in mind? What he would say if asked? How he chose in the past?

Under *revealed preference theory*, we interpret $\succsim$ as a description of how an individual chooses. Therefore, there is no psychological content to $\succsim$. Revealed preference theory has been the standard methodological stance in economics for a long time. But why? Wouldn't it be better to develop a theory that exploits psychological insights? Revealed preference theory is exclusively a methodological stance, not a psychological (or, for what matters, a moral) one. The assumption is not that choices are not driven by psychological motives, but that we abstract from these motives and attempt to find patterns in choices directly. There is a strong advantage in doing so: psychological motives are hard to observe, while choices can be observed easily. The implication is that a choice theory based on revealed preferences is more easily testable: if we observe choices that violate the assumptions of the theory, we can reject the it. Therefore, revealed preference theory is **not** a stance on how individuals make choices or what matters for choices, it is just silent about these issues. This is often misunderstood: there is a plethora of papers claiming that economics views individuals as cold robots.[1]

Such critics mostly come from behavioural economics, which is a field that attempts to incorporate psychological insights into economic models. Is therefore impossible to do behavioural economics under the revealed preference approach? Not at all. Good behavioural theories do what the name of the field suggests: characterise the behavioural content of the theory, so that, as economist, we know how different individuals behave. Two theories with different psychological content that are observationally equivalent, i.e., they make the same predictions about choices, are not equally useful for economists.[2]

An interesting case study is Masatlioglu & Raymond (2016), where the authors show that the famous model by Köszegi & Rabin (2007) is behaviourally equivalent to the intersection of rank-dependent utility and quadratic utility, two older models.

[1] If you are interested, you can read Thoma (2021) for a discussion of the current status of revealed preference theory.

[2] There is a huge debate on this topic. Among many, I suggest you to read Gul & Pesendorfer (2011) and the response by Camerer (2008). A more recent piece is Spiegler (2019).

In what follows, you can have in mind the interpretation of $\succsim$ that you prefer, but remember that it is important to be clear about it.

Before discussing the properties of preferences over lotteries, let's consider a reasonable functional form for preferences. A natural candidate is the following: the utility of a lottery p is given by

$$\sum_{x \in X} p(x)u(x) \tag{2.1}$$

for some function $u\colon X \to \mathbb{R}$, assigning to each outcome x a number representing its utility $u(x)$. The idea is simple, the outcome x realises with probability p, and when x realises, the individual gets utility $u(x)$. The functional form in Equation 2.1 is called **expected utility**. This is because it is the expectation, computed with the probability p, of the utility the individual gets. Before turning to the properties of preferences that will lead us to this functional form, let's make some observations.

Having expected utility preferences over lotteries implies that indifference curves on the simplex are straight lines. That is, say that $p \sim q$. Then, for any $\alpha \in (0,1)$ it holds that $\alpha p + (1-\alpha)q \sim p$, as illustrate in Figure 2.1.

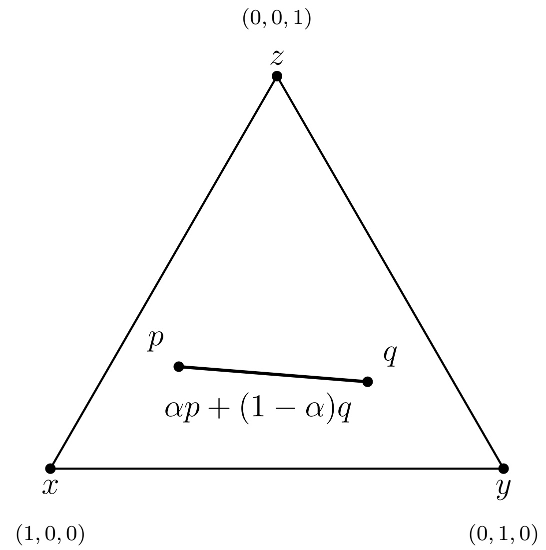


Figure 2.1: If $p \sim q$, then any mixture of p and q is also indifferent to p and q.

Let's show this formally. Assume that $p \sim q$. Then, by definition of expected utility, we have

$$\sum_{x \in X} p(x)u(x) = \sum_{x \in X} q(x)u(x).$$

By applying expected utility again, for any $\alpha \in (0,1)$, the utility of the lottery $\alpha p + (1-\alpha)q$ is given by

$$
\begin{aligned}
\sum_{x\in X}\big(\alpha p(x)+(1-\alpha)q(x)\big)u(x) &= \sum_{x\in X}\alpha p(x)u(x)+\sum_{x\in X}(1-\alpha)q(x)u(x)\\
&= \alpha\sum_{x\in X}p(x)u(x)+(1-\alpha)\sum_{x\in X}q(x)u(x)\\
&= \alpha\sum_{x\in X}q(x)u(x)+(1-\alpha)\sum_{x\in X}q(x)u(x)\\
&= \sum_{x\in X}q(x)u(x).
\end{aligned}
$$

Indifference curves are also parallel, you are asked to show this in Exercise 2.2.

Let's now turn to the properties of $\succsim$ we will consider. First, we assume that preferences are a **weak order**.

Axiom 2.1. *(**Weak order**) Preferences $\succsim$ are complete and transitive.*

Recall that preferences are **complete** if for any two lotteries p, q, either $p \succsim q$ or $q \succsim p$, or both. They are transitive if for any three lotteries p, q, r, if $p \succsim q$ and $q \succsim r$, then $p \succsim r$.

Axiom 2.2. *(**Continuity**) For any three lotteries p, q, r, if $p \succ q \succ r$ then there exist $\alpha, \beta \in (0, 1)$ such that $\alpha p + (1-\alpha)r \succ q \succ \beta p + (1-\beta)r$.*

Axiom 2.3. *(**Independence**) For any three lotteries p, q, r and for any $\alpha \in (0, 1)$, we have $p \succsim q$ if and only if $\alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r$.*

2.2 Expected utility representation

Theorem 2.1. *Preferences over lotteries $\succsim$ satisfy Weak order, Continuity, Independence if and only if then there exists a utility function $u\colon X \to \mathbb{R}$ such that:*

$$p \succsim q \text{ if and only if } \sum_{x\in X}p(x)u(x) \geq \sum_{x\in X}q(x)u(x). \tag{2.2}$$

*We say that u **represents** $\succsim$.*

Proof. Come va. □

Corollary 2.1. *If u represents $\succsim$, then a function $u'\colon X \to \mathbb{R}$ represents $\succsim$ if and only if there exist real numbers $a > 0$ and b such that $u' = au + b$.*

2.3 Things to read

There is already quite a lot to read that I mentioned in the main text. If you want a textbook source of the content of this lecture, check Mas-Colell et al. (1995, pp. 170–178).

2.4 Exercises

Exercise 2.1. Prove the direction of Theorem 2.1 that we did not prove. If u represents $\succsim$, then show that $\succsim$ satisfies Weak order, Continuity, and Independence.

Exercise 2.2. Show that, if preferences are represented by an expected utility function, then indifference curves in the triangle are parallel lines.

Solution to Exercise 2.2. Let $\{x, y, z\}$ be the outcomes and let

$$U(p) = \sum_{w \in \{x,y,z\}} u(w)\, p(w)$$

be the expected utility of lottery p. Since $p(z) = 1 - p(x) - p(y)$, we can write

$$U(p) \;=\; \big(u(x) - u(z)\big)\, p(x) \;+\; \big(u(y) - u(z)\big)\, p(y) \;+\; u(z).$$

Fix a utility level c. The indifference set $\{p : U(p) = c\}$ satisfies

$$\big(u(x) - u(z)\big)\, p(x) \;+\; \big(u(y) - u(z)\big)\, p(y) \;=\; c - u(z).$$

Solving for $p(y)$ as a function of $p(x)$,

$$p(y) \;=\; \frac{c - u(z)}{u(y) - u(z)} \;-\; \frac{u(x) - u(z)}{u(y) - u(z)}\, p(x).$$

The coefficient of $p(x)$ is

$$-\frac{u(x) - u(z)}{u(y) - u(z)},$$

which does not depend on c. Changing c only changes the intercept $\frac{c-u(z)}{u(y)-u(z)}$. Therefore, all indifference lines, the portions of these lines that lie inside the simplex, have the same slope and are parallel. □

References

Camerer, C. (2008). The case for mindful economics. *Foundations of positive and normative economics*, 43–69. 8

Gul, F., & Pesendorfer, W. (2011). The case for mindless economics. In *The Foundations of Positive and Normative Economics: A Hand Book.* Oxford University Press. 8

Köszegi, B., & Rabin, M. (2007). Reference-dependent risk attitudes. *American Economic Review*, *97*(4), 1047–1073. 8

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic theory.* Oxford university press New York. 10

Masatlioglu, Y., & Raymond, C. (2016). A behavioral analysis of stochastic reference dependence. *American Economic Review*, *106*(9), 2760–2782. 8

Spiegler, R. (2019). Behavioral economics and the atheoretical style. *American Economic Journal: Microeconomics*, *11*(2), 173–194. 8

Thoma, J. (2021). In defence of revealed preference theory. *Economics & Philosophy*, *37*(2), 163–187. 8

Lecture 3

Money lotteries and risk aversion

3.1 Hello

ciao

Aczél (1966)

References

Aczél, J. (1966). *Lectures on functional equations and their applications* (Vol. 19). New York: Academic Press. 13