Aircraft positioning model

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April 2025

Problem Statement

The aircraft positioning problem involves determining the optimal schedule for a set of jobs within a defined space and time frame.

There are a set of positions, each of them can hold one aircraft at a time.

Each airplane has to undergo a series of operations. We refer to a job as a particular operation for a particular airplane.

The problem consists in deciding the position where each job will be performed and the starting and finishing dates for each job so that a weighted objective function is minimized.

The criteria to minimize are:

- Number of movements.
- Total number of weeks of delay with respect to the latest finishing date of each aircraft.
- Number of customers with delayed aircraft.
- Number of different positions assigned to the aircraft of a given customer.

A movement occurs when two consecutive jobs of a given airplane are assigned to different positions or when one airplane has to be removed and transferred back to a given position for allowing another airplane to access another position (as described below).

All jobs must be scheduled.

Each job has a given and fixed duration.

Each job cannot be interrupted and must be performed entirely in a single position. Therefore, there is a single position assigned to each job.

Each aircraft has an earliest starting date and a latest finishing date. These constraints are soft: aircraft should not start before their earliest starting date and should finish by their latest finishing date. Delays are allowed, but penalized in the objective function.

Some positions block other positions. If position p blocks position p', it implies that if a job starts at position p while there is a job being done in position p', the airplane in position p' will have to be removed for accessing position p, after which, the airplane will be moved back to position p'.

Some jobs are non-interruptible. If a non-interruptible job is assigned to a position that blocks any other position, moving airplanes from or to the corresponding blocked positions will not be possible while the job is performed.

Each airplane corresponds to a given customer. It is desirable that all airplanes of a given customer are assigned to the same position, and that all jobs of a given airplane are also assigned to the same position.

Delays might occur and it is desirable to minimize the number of customers with delays. It is more desirable to have a single customer with a long delay than multiple customers with small delays.

1 Sets

$\mathcal{S}$	:	Slots
$\mathcal{J}$	:	Jobs
$\mathcal{R}$	:	Airplanes
$\mathcal{J}_r$	:	Jobs corresponding to plane $r \in \mathcal{R}$
$\mathcal{P}$	:	Positions
$\mathcal{C}$	:	Clients

2 Parameters

H	:	Time horizon
D_j	:	Duration of job $j \in \mathcal{J}$
$B_{pp'}$	:	1 if positions $p \in \mathcal{P}$ and $p' \in \mathcal{P}$ interfere with each other, 0 otherwise
$P_{jj'}$	:	1 if job $j \in \mathcal{J}$ immediately precedes $j' \in \mathcal{J}$, 0 otherwise
R_j	:	Airplane of job $j \in \mathcal{J}$
C_{cr}	:	Airplane $r \in R$ of client $c \in C$
L_{jr}	:	1 if job $j \in \mathcal{J}$ is the last job of the plane $r \in \mathcal{R}$
T_r	:	Latest time in which all the jobs corresponding to plane $r \in \mathcal{R}$ must be finished

3 Variables

x_{spj}	:	1 if job $j \in \mathcal{J}$ is performed in slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$
y_{spr}	:	1 if airplane $r \in \mathcal{R}$ is in slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$
z_{sp}	:	1 if job in slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$ is different from the job in slot $s+1 \in \mathcal{S}$
w_{rp}	:	1 if some job of plane $r \in \mathcal{R}$ is performed in position $p \in \mathcal{P}$
d^S_{sp}	:	duration of slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$
s_j	:	global start time of job $j \in \mathcal{J}$
f_j	:	global finishing time of job $j \in \mathcal{J}$
s^J_{spj}	:	starting time of slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$ for job $j \in \mathcal{J}$
f^J_{spj}	:	finishing time of slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$ for job $j \in \mathcal{J}$
d^J_{spj}	:	duration of slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$ for job $j \in \mathcal{J}$
s^S_{sp}	:	starting time of slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$
f^S_{sp}	:	finishing time of slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$
π_{cp}	:	position $p \in \mathcal{P}$ occupied by client $c \in \mathcal{C}$
$\alpha_{ss'pp'}$	:	slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$ interferes with slot $s' \in \mathcal{S}$ of position $p' \in \mathcal{P}/B_{pp'} = 1$. This means that either the starting or the finishing times of slot s are in between the starting and the finishing times of slot s'
$\beta^S_{ss'pp'}$	:	1 if starting time of slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$ is earlier than the starting time of $s' \in \mathcal{S}$ of position $p' \in \mathcal{P}/B_{pp'} = 1$, 0 otherwise

$\beta^F_{ss'pp'}$	:	1 if finishing time of slot $s \in \mathcal{S}$ of position $p \in \mathcal{P}$ is later than the starting time of $s' \in \mathcal{S}$ of position $p' \in \mathcal{P}/B_{pp'} = 1$, 0 otherwise
δ_c	:	Accumulative delay of the jobs of client $c \in \mathcal{C}$
γ_r	:	Delay of jobs performed on airplane $r \in \mathcal{R}$

4 Constraints

Each slot of each position can have one job at a time:

$$\sum_{j \in \mathcal{J}} x_{spj} \leq 1 \; \forall s \in \mathcal{S}, \forall p \in \mathcal{P} \tag{1}$$

Calculation of the slot for job duration:

$$d^J_{spj} = f^J_{spj} - s^J_{spj} \; \forall s \in \mathcal{S}, \forall p \in \mathcal{P}, \forall j \in \mathcal{J} \tag{2}$$

Starting and finishing times are 0 if the job is not assigned to a position

$$s^J_{spj} \;\leq\; H \cdot x_{spj} \qquad \forall\, s \in \mathcal{S},\; \forall\, p \in \mathcal{P},\; \forall\, j \in \mathcal{J} \tag{3}$$

$$f^J_{spj} \;\leq\; H \cdot x_{spj} \qquad \forall\, s \in \mathcal{S},\; \forall\, p \in \mathcal{P},\; \forall\, j \in \mathcal{J} \tag{4}$$

Calculating the starting and finishing times for each job.

$$s_j = \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} s^J_{spj} \quad \forall\, s \in S,\; p \in P,\; j \in J \tag{5}$$

$$f_j = \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} f^J_{spj} \quad \forall\, s \in S,\; p \in P,\; j \in J \tag{6}$$

$$s_j \;\leq\; f_j \quad \forall\, j \in J. \tag{7}$$

Calculating the delay of each plane and each client:

$$H \cdot \gamma_r \geq f_j - T_r \quad \forall\, r \in R,\; j \in J \tag{8}$$

$$\delta_c \geq \gamma_r \quad \forall\, c \in C,\; r \in R \tag{9}$$

$$\gamma_r \geq 0 \quad r \in R \;;\; \delta_c \geq 0 \quad c \in C \tag{10}$$

Initial and finishing times for each slot:

$$s^S_{sp} = \sum_{j\in\mathcal{J}} s^J_{spj} \qquad \forall\, s\in\mathcal{S},\ \forall\, p\in\mathcal{P} \tag{11}$$

$$f^S_{sp} = \sum_{j\in\mathcal{J}} f^J_{spj} \qquad \forall\, s\in\mathcal{S},\ \forall\, p\in\mathcal{P} \tag{12}$$

Slot within the same position are sequenced:

$$s^S_{sp} \geq f^S_{s-1\,p},\ \forall s\in\mathcal{S} : s>1,\ \ p\in\mathcal{P} \tag{13}$$

Jobs are sequenced:

$$s_{j'} \ \geq\ f_j \quad \forall\, j,\, j'\in\mathcal{J}\ /\ P_{jj'}=1 \tag{14}$$

A given slot is not used unless all the previous ones have been used

$$\sum_{j\in J} x_{spj} \ = \sum_{j\in J} x_{s-1\,p\,j} \qquad \forall\, s=2,\ldots,|S|,\ \forall\, p\in P \tag{15}$$

Each job has to be assigned to a single slot of a position:

$$\sum_{p\in P}\sum_{s\in S} x_{spj} = 1 \quad \forall j\in\mathcal{J} \tag{16}$$

If a job is not assigned to a slot of a position, the duration of that job in that slot of that position is zero:

$$d^J_{spj} = D_j x_{spj},\ \forall s\in\mathcal{S},\ p\in\mathcal{P},\ j\in\mathcal{J} \tag{17}$$

The duration of a slot is that of the slot assigned to that job

$$d^S_{sp} = \sum_{j\in\mathcal{J}} d^J_{spj},\ \forall s\in\mathcal{S},\ \forall p\in\mathcal{P} \tag{18}$$

Airplane-job consistency assignment

$$y_{spr} = \sum_{j\in\mathcal{J}_r} x_{spj}\ \forall s\in\mathcal{S},\ \forall p\in\mathcal{P},\ \forall r\in\mathcal{R} \tag{19}$$

Airplane with some job in a position.

$$w_{rp} \geq y_{spr} \; \forall s \in \mathcal{S}, \, \forall p \in \mathcal{P}, \, \forall r \in \mathcal{R} \tag{20}$$

Client c with some airplane in position p:

$$\pi_{cp} \geq w_{rp} \cdot C_{cr} \quad \forall p, c \; / C_{cr} = 1 \tag{21}$$

Computing if the starting time of slot s in position p is earlier than the starting time of slot s' in position p':

$$H \times \beta^{S}_{s,s',p,p'} + s^{S}_{sp} \geq s^{S}_{s'p'}, \; \forall s, s' \in \mathcal{S}, \, \forall p, p' \in \mathcal{P}/B_{pp'} = 1 \tag{22}$$

Computing if the finishing time of slot s in position p is later than the starting time of slot s' in position p'

$$H \times \beta^{F}_{s,s',p,p'} + s^{S}_{s'p'} \geq f^{S}_{sp}, \; \forall s, s' \in \mathcal{S}, \, \forall p, p' \in \mathcal{P}/B_{pp'} = 1 \tag{23}$$

The starting time of slot s in position p is earlier than the starting time of slot s' in position p' and finishing time of slot s in position p is later than the starting time of slot s' in position p'.

$$1 + \alpha_{ss'pp'} \geq \beta^{S}_{ss'pp'} + \beta^{F}_{ss'pp'}, \, \forall s \in \mathcal{S}, \, s' \in \mathcal{S}, \, \forall p \in \mathcal{P}, \, \forall p' \in \mathcal{P}/B_{pp'} = 1 \tag{24}$$

Subsequent tasks in a position correspond to different planes:

$$1 + z_{sp} \geq y_{spr} + y_{s+1pr'}, \, \forall s \in \mathcal{S}, \, \forall p \in \mathcal{P}, \, \forall r, r' \in \mathcal{R}/r \neq r' \tag{25}$$

5 Objective function

$$\text{min.} \sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{P}} \sum_{j \in \mathcal{J}} x_{spj} \tag{26}$$

$$+ \sum_{s \in \mathcal{S}} \sum_{s' \in \mathcal{S}} \sum_{p \in \mathcal{P}} \sum_{p' \in \mathcal{P}} \alpha_{ss'pp'} \tag{27}$$

$$+ \sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{P}} \sum_{r \in \mathcal{R}} y_{spr} \tag{28}$$