GEOL/PHYS 6670
Geophysical Inverse Theory
Fall 2021
Tues 3-4:50, Benson Rm 455

Class Zoom link: https://cuboulder.zoom.us/j/93267047976
(primary mode of instruction is in-person, Zoom link is for occasional use)

Prof. Anne Sheehan
Anne.Sheehan @ colorado.edu

Office: Benson Rm 440A

Office hours: Monday 3-4, Tuesday 11-12

Class web page on Canvas

Today

- Class format/ logistics
- Introductions
- Overview of course
- Introduction – forward and inverse problems

For next time –

Do Homework 1

Fill out questionnaire

Read Menke Chapter 1

Course Goals

- Gain familiarity with inverse methods, which are used to determine model parameters from a set of observations

- Learn to formulate and solve inverse problems

- Understand the nonuniqueness and resolution associated with inversions. Learn to quantify uncertainty associated with models.

- Gain competence in applying inverse methods to geophysical problems and your own research

- Explore a topic of interest in greater depth for your term project

Class page on Canvas

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Dashboard

Courses

Calendar

Inbox

History

Studio

Help

GEOL-PHYS 6670-001 > Syllabus

2021 Fall Term

- Home
- Announcements
- Discussions
- Grades
- People
- **Syllabus**
- Modules
- BigBlueButton (Formerly Conferences)
- Collaborations
- Chat
- My Media
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Recent Announcements

Welcome to GEOL/PHYS 6670 Geophysical Inverse Theory!
Welcome to GEOL/PHYS 6670 Geophysical In...

Posted on:
Aug 17, 2021 at 5:36pm

GEOL/PHYS 6670-001: Geophysical Inverse Theory

GEOL/PHYS 6670 Geophysical Inverse Theory

Fall 2021, 2 credits

Tuesday 3-4:50 p.m., Benson Rm 455

Professor: Anne Sheehan, Anne.Sheehan@colorado.edu

Office: Benson Rm. 440A, Office phone: (303)492-4597

Office hours: Monday 3-3:50, Tuesday 11-11:50. Feel free to email me anytime.

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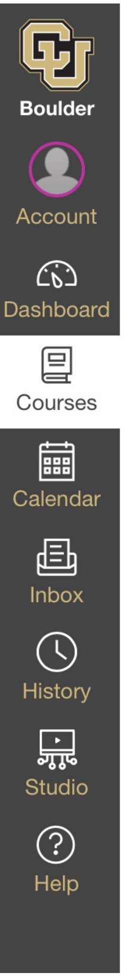
To Do

Welcome to GEOL/PH...
Aug 17 at 5:36pm



Grading:

Homeworks	60%
Term Paper	20%
Term Paper presentation	10%
Peer reviews, in-class presentations	10%

Late Policy: Homework assignments are due on the date specified on the assignment. If you have exceptional circumstances, contact me prior to the due date.

Academic Honesty: Working together is allowed and encouraged, but work turned in for credit must be your own and not copied. List any collaborators and acknowledge help received. Using the internet to find examples and teach yourself material is fine, but copying solutions from anywhere is forbidden.

Tentative Lecture Schedule: GEOL/PHYS 6670, Fall 2021

Check back often for updates!

Week	Textbook Chapter	Topics
1. Aug 24	I, 1	Introduction, linear algebra
2. Aug 31	2	Probability

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Calendar

Inbox

History

Studio

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Collapse All

▾ Week 1 - Aug 24

- **GEOL6670syllabus_2021.pdf**
- **Menke Chapter-1---Describing-Inverse-Problems_2018_Geophysical-Data-Analysis.pdf**

▾ Menke Textbook

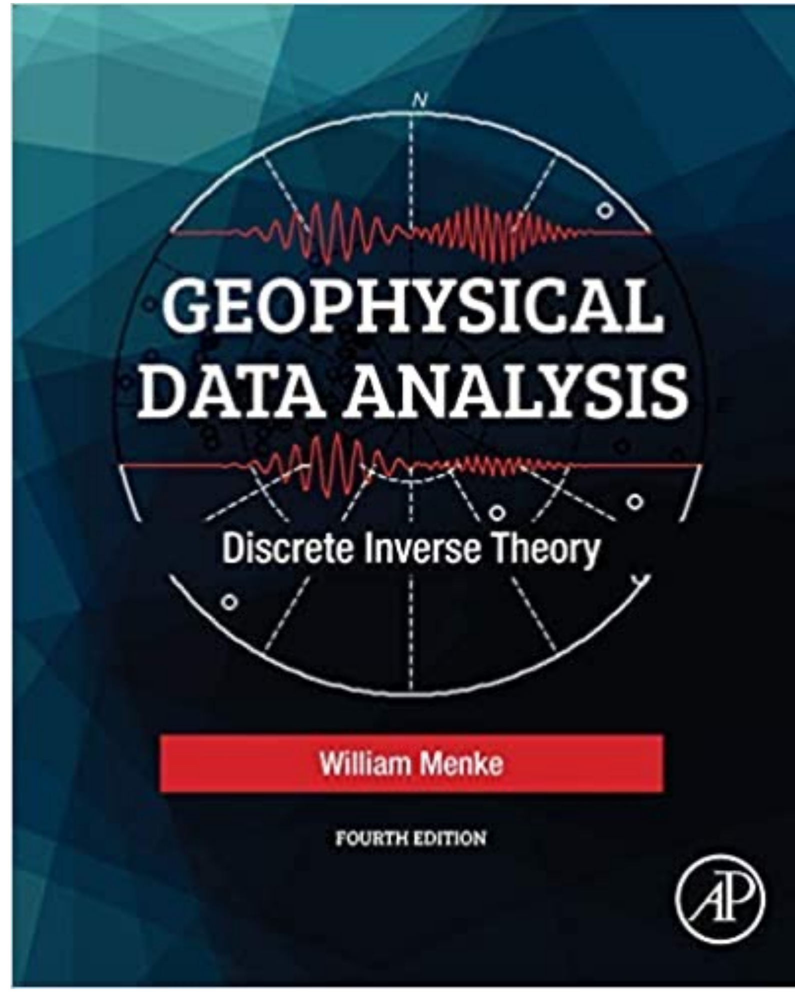
- **Menke-TOC-4thEd.pdf**
- **Index_2018_Geophysical-Data-Analysis.pdf**
- **Introduction_2018_Geophysical-Data-Analysis.pdf**

Textbook

Geophysical Data Analysis: Discrete Inverse Theory, Matlab Edition, 4th Edition, William Menke, Academic Press

Available in CU bookstore, electronic version available through CU library

Assigned reading listed on the syllabus



Textbook

Geophysical Data Analysis: Discrete Inverse Theory, Matlab Edition, 4^{th} Edition, William Menke, Academic Press

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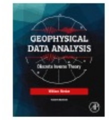
ScienceDirect



Geophysical Data Analysis

Discrete Inverse Theory

Book • Fourth Edition • 2018

Authors:

William Menke

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Table of contents

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Book chapter Full text access

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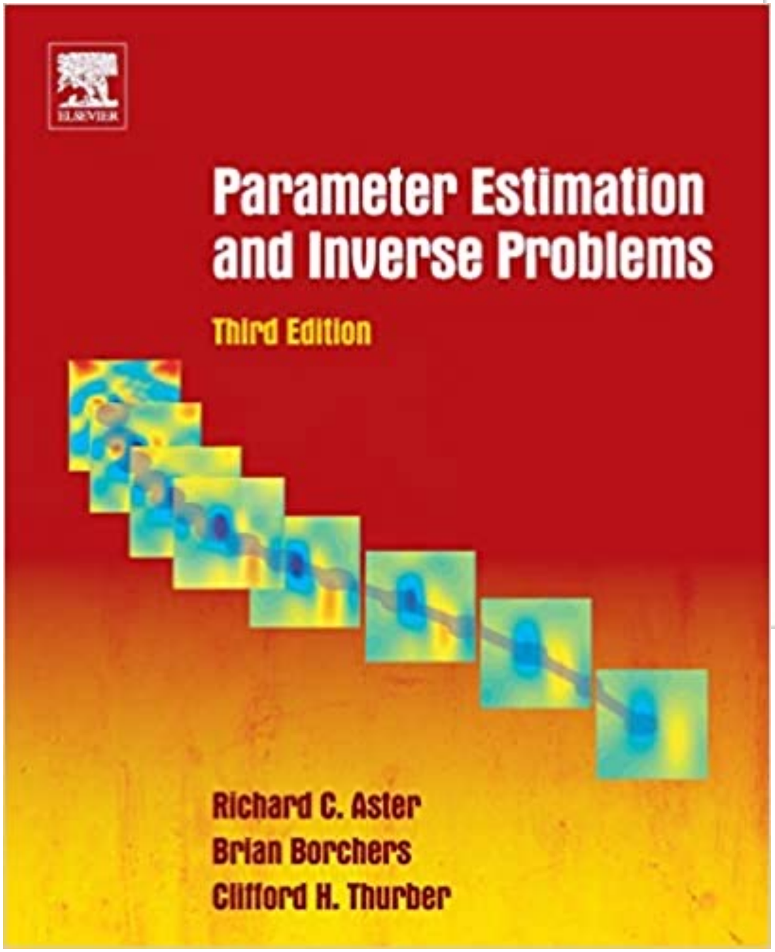

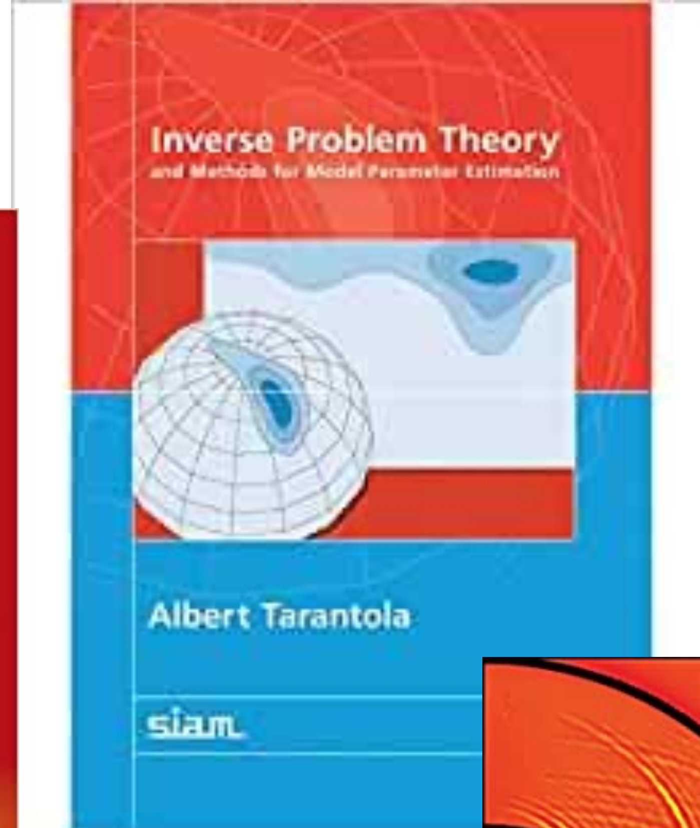

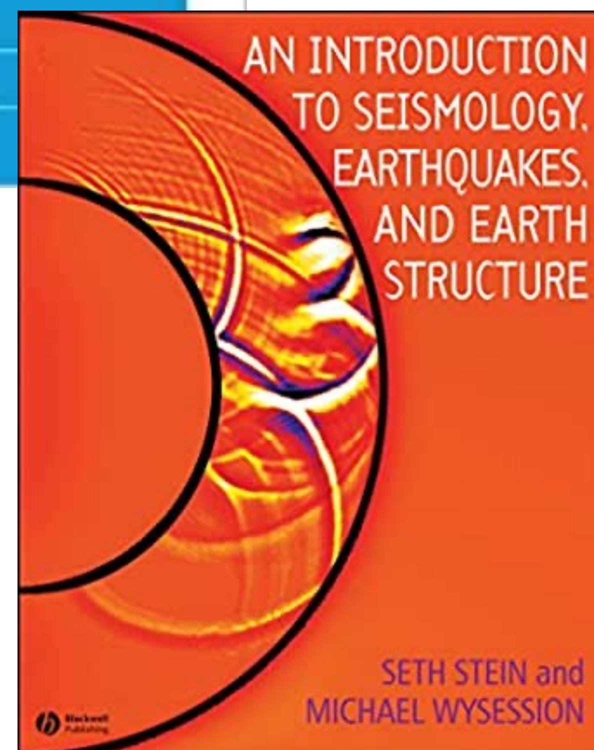

Lecture Schedule

Week	Chapter	Topic
1. Aug 24	I, 1	Introduction, linear algebra
2. Aug 31	2	Probability
3. Sep 7	3	Least Squares
4. Sep 14	4	Generalized Inverse
5. Sep 21	6	Nonuniqueness,generalized averages
6. Sep 28	7	Singular value decomposition
7. Oct 5	handouts	Genetic Algorithms
8. Oct 12	handouts	Data Assimilation
9. Oct 19	9	Nonlinear inverse problems
10. Oct 26	9	Nonlinear inverse problems
11. Nov 2	10	Factor analysis
12. Nov 9	12	Example inverse problems
13. Nov 16		TBA
14. Nov 23 No class		Fall Break
15. Nov 30		Student presentations
16. Dec 7		Student presentations
AGU Dec 14-18		

*Reading from Menke textbook

Grading

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Computing

Computing: Many of the assignments will require programming. The textbook gives examples in Matlab, but you may use Python or other languages if you wish.

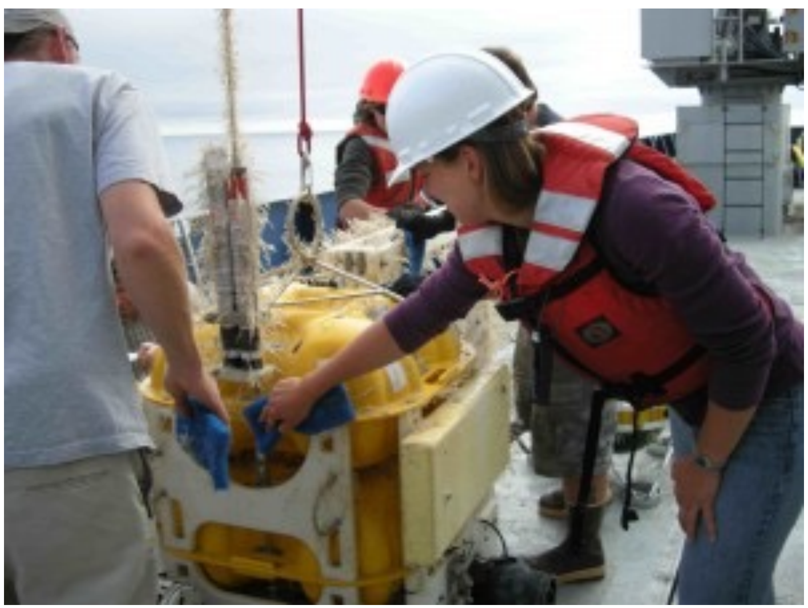
Matlab is available on the computers in the Benson 3rd floor computer lab, and CU has a full license for Matlab for students, faculty, and staff

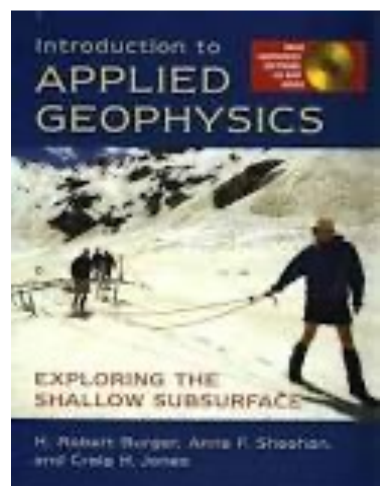
https://oit.colorado.edu/software-hardware/software-catalog/matlab

Matlab tutorials can be found at http://www.mathworks.com/academia/student_center/tutorials/launchpad.html and on LinkedIn Learning.

What I do:
(Prof. Anne Sheehan)

- Use geophysics to study crust and mantle structure
- Structure and tectonics of mountain ranges (seismology, geodesy, electromagnetic)
- Ocean bottom seismometer studies
- Induced seismicity
- Tsunami studies





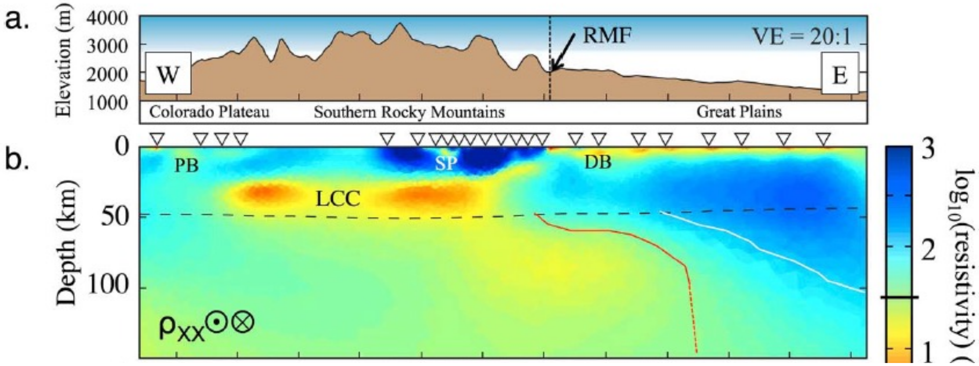
ENVIRONMENT

Injection Well Halted By COGCC As Greeley Quakes Again

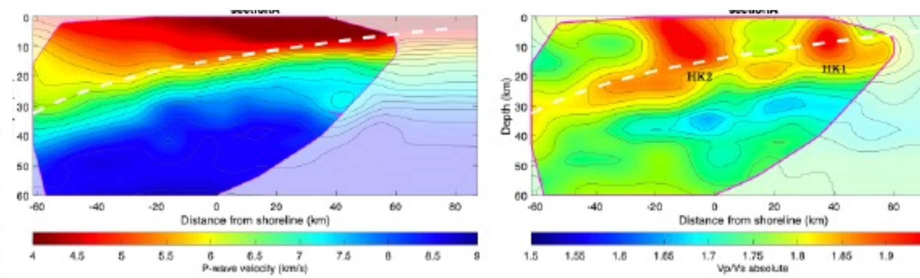
By Stephanie Paige Ogburn

Jenny Nakai, a seismologist and doctoral student and Matthew Weingarten, a hydrogeologist and doctoral candidate from the University of Colorado align, level and bury a seismometer outside of Gill, Colo., June 4, 2014.

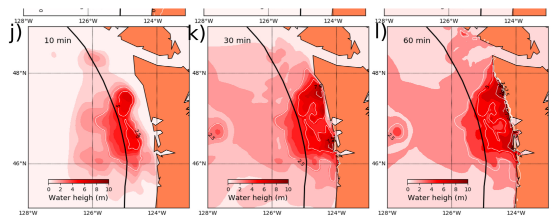
Some inversion examples from my research group



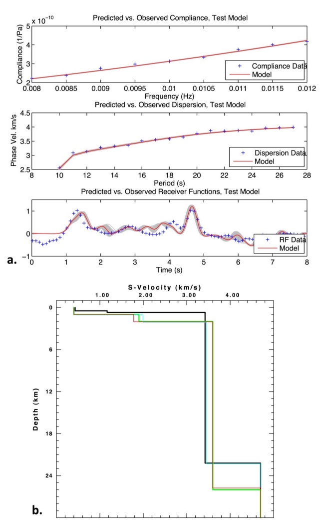
Inversion of magnetic and electric field data for lithospheric resistivity structure. Feucht et al., 2017



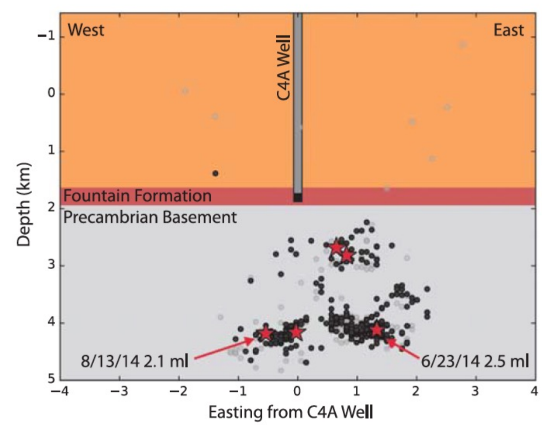
Inversion of earthquake travel time residuals for seismic velocity variations. Yarce et al., 2021

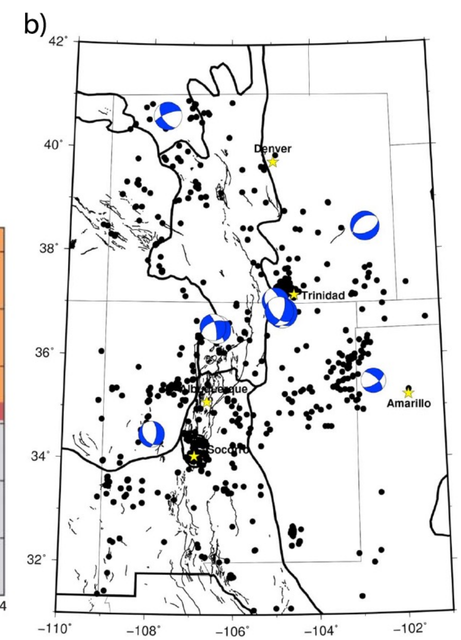


Assimilation of ship GNSS data for tsunami forecasts. Hossen et al., 2021



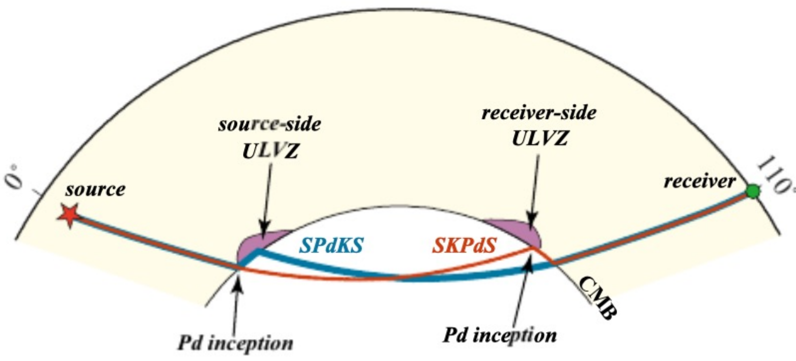
Estimation of seafloor shear velocity structure from surface wave dispersion, seafloor compliance, and receiver functions. Ball et al., 2014



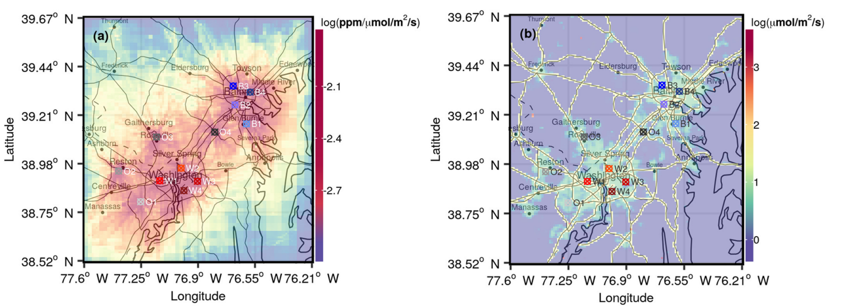


Inversion of earthquake arrivals times for earthquake hypocenters. Yeck et al., 2016; Nakai et al., 2017

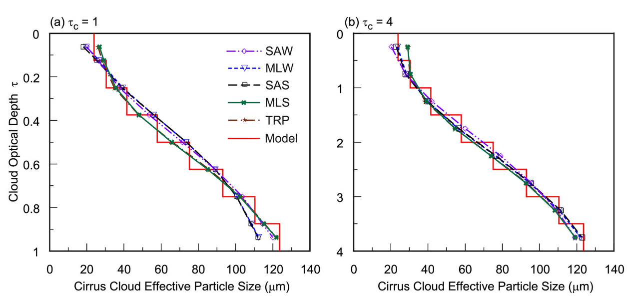
Other inversion examples



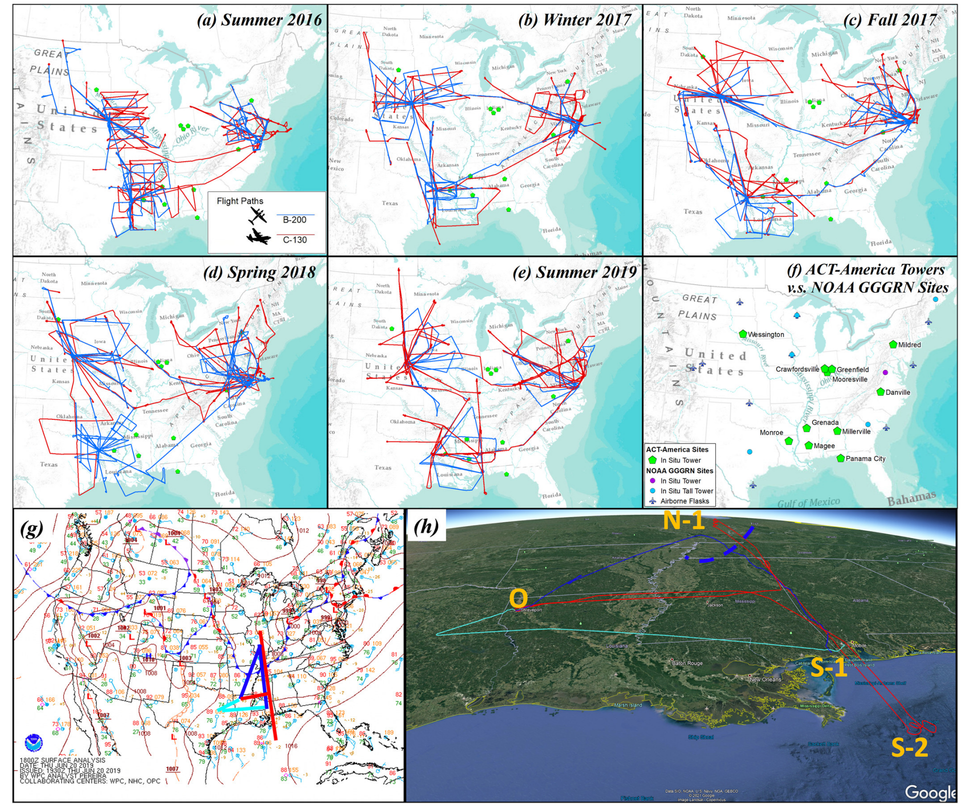
Inversion of seismic travel times for lower mantle low velocity zones. Thorne et al., G-cubed, 2021



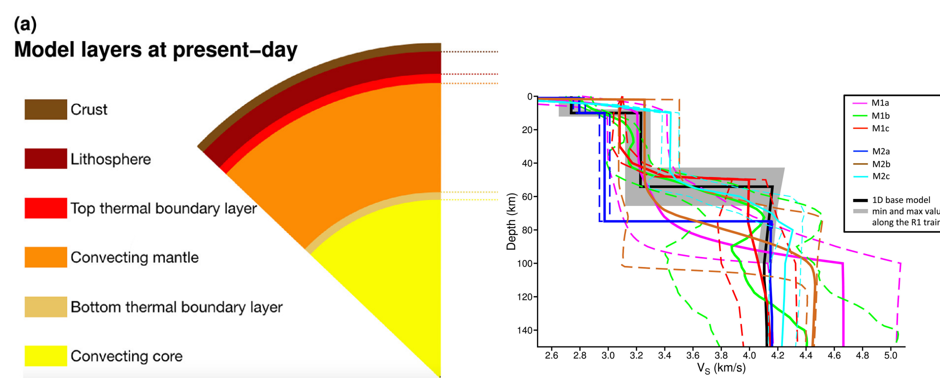
Accounting for errors in atmospheric inversions that estimate GHG emissions in urban regions. Ghosh et al., ESS, 2021



Determine ice crystal in cirrus clouds, Gu, ESS, 2021



Data sets for inverse modeling to infer regional carbon fluxes from atmospheric carbon observations. Wei et al., ESS, 2021



Marsquake inversion. Drilleau, Earth and Space Science, 2020.

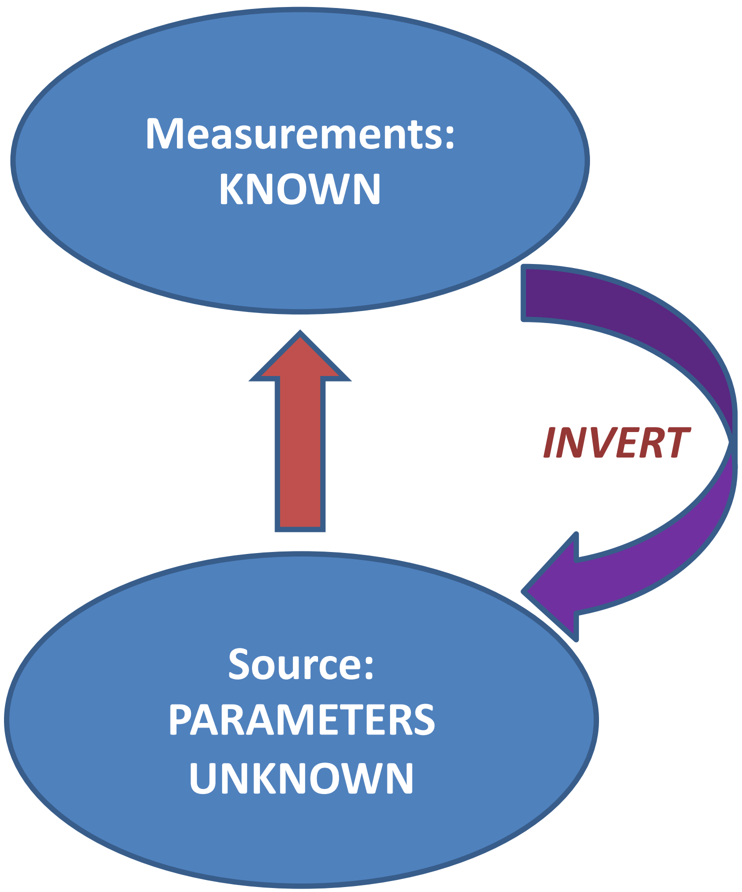
Today’s Lecture

Distinguish **forward** and **inverse** problems

Setting up inverse problems

Some examples

What is an inversion?



- Some physical source (in our case *geophysical*) causes an external change (geodetic height changes, groundwater contamination, material property changes that slow down earthquake waves, etc.).
- We can measure the changes, and then calculate backwards to estimate the parameters for that source – that's *an inversion*.

What are the necessary parts of any inversion?

- Data/measurements, hopefully with realistic error estimates.
- One or more potential source models.
- An appropriate technique for inverting that data. These include both linear and nonlinear inversion methods.
- In addition, a good inversion requires an understanding of the limits of the inversion techniques, the sensitivity of the results to the source parameters and their interplay, and the propagation of the data error into the solution.

data, $\mathbf{d} = [d_1, d_2, \ldots d_N]^{\mathrm{T}}$

T denotes transpose

things that are measured in an experiment or observed in nature…

model parameters, $\mathbf{m} = [m_1, m_2, \ldots m_M]^{\mathrm{T}}$

things you want to know about the world …

quantitative model (or ***theory***)

relationship between data and model parameters data, $\mathbf{d} = [d_1, d_2, \ldots d_N]^T$

gravitational accelerations
travel time of seismic waves

model parameters, $\mathbf{m} = [m_1, m_2, \ldots m_M]^T$

density
seismic velocity

quantitative model (or *theory*)

Newton's law of gravity
seismic wave equation

data, $\mathbf{d} = [d_1, d_2, \ldots d_N]^T$

gravitational accelerations
travel time of seismic waves

Can you think of other examples of geophysical data and corresponding models?

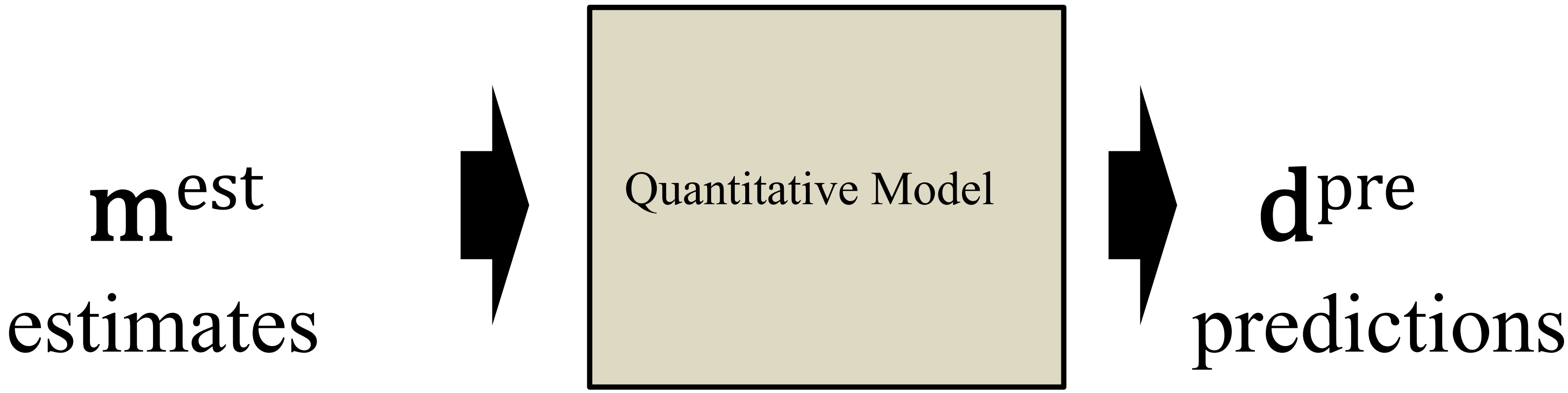
model parameters, $\mathbf{m} = [m_1, m_2, \ldots m_M]^T$

density
seismic velocity

quantitative model (or *theory*)

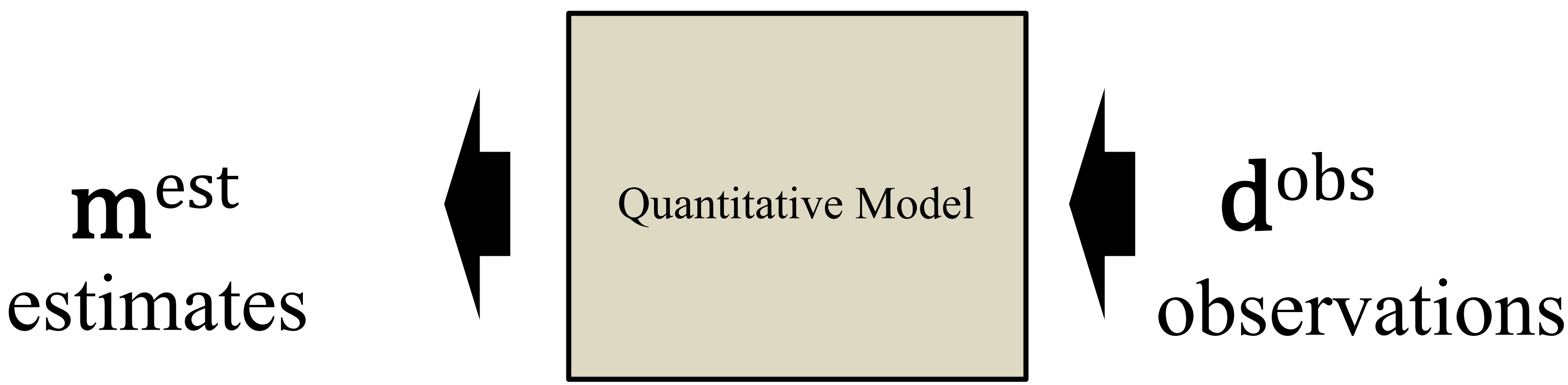
Newton's law of gravity
seismic wave equation

Forward Modeling



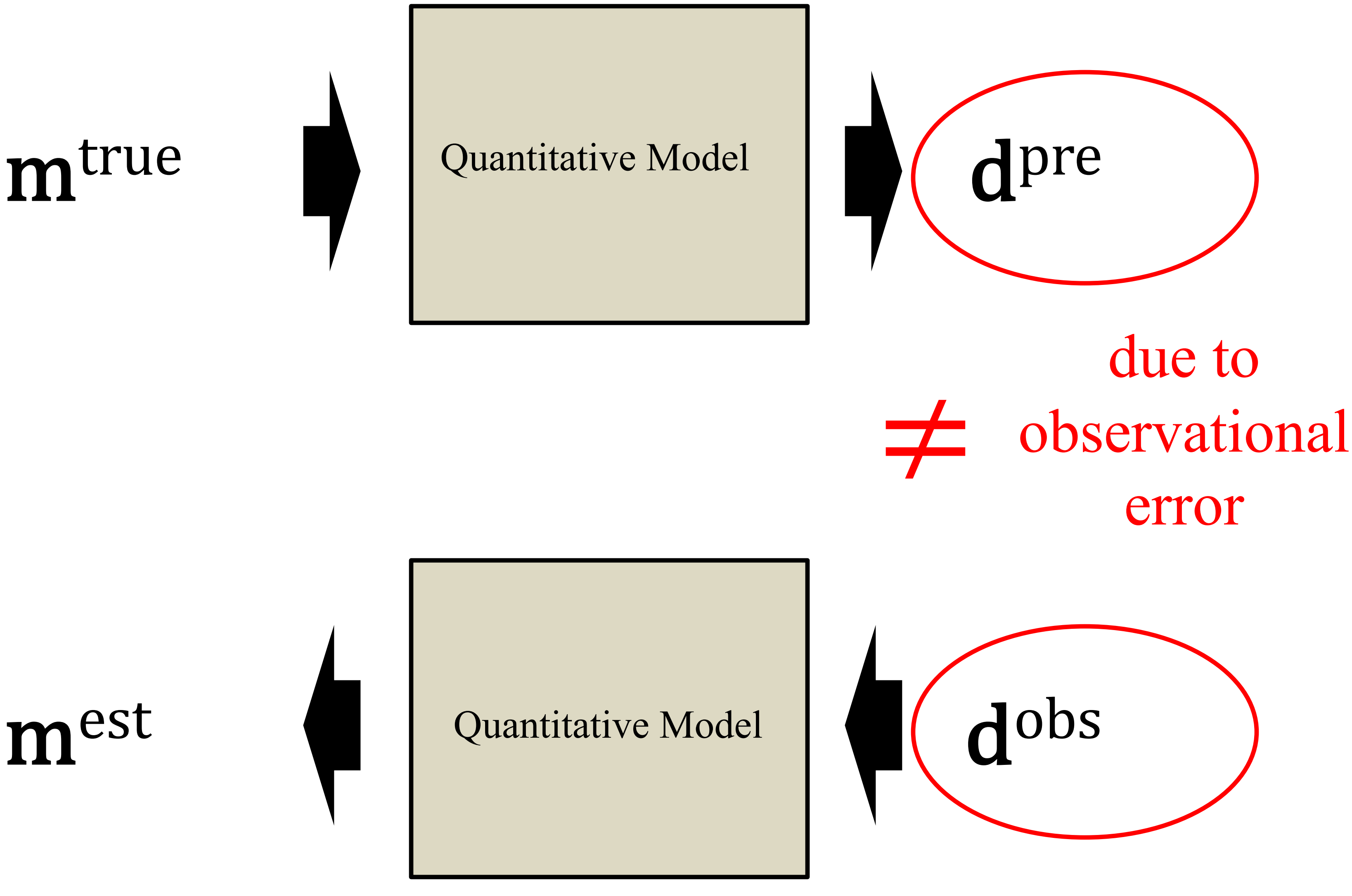
estimates of model parameters ---> quantitative model → predictions of data.

Inverse Modeling

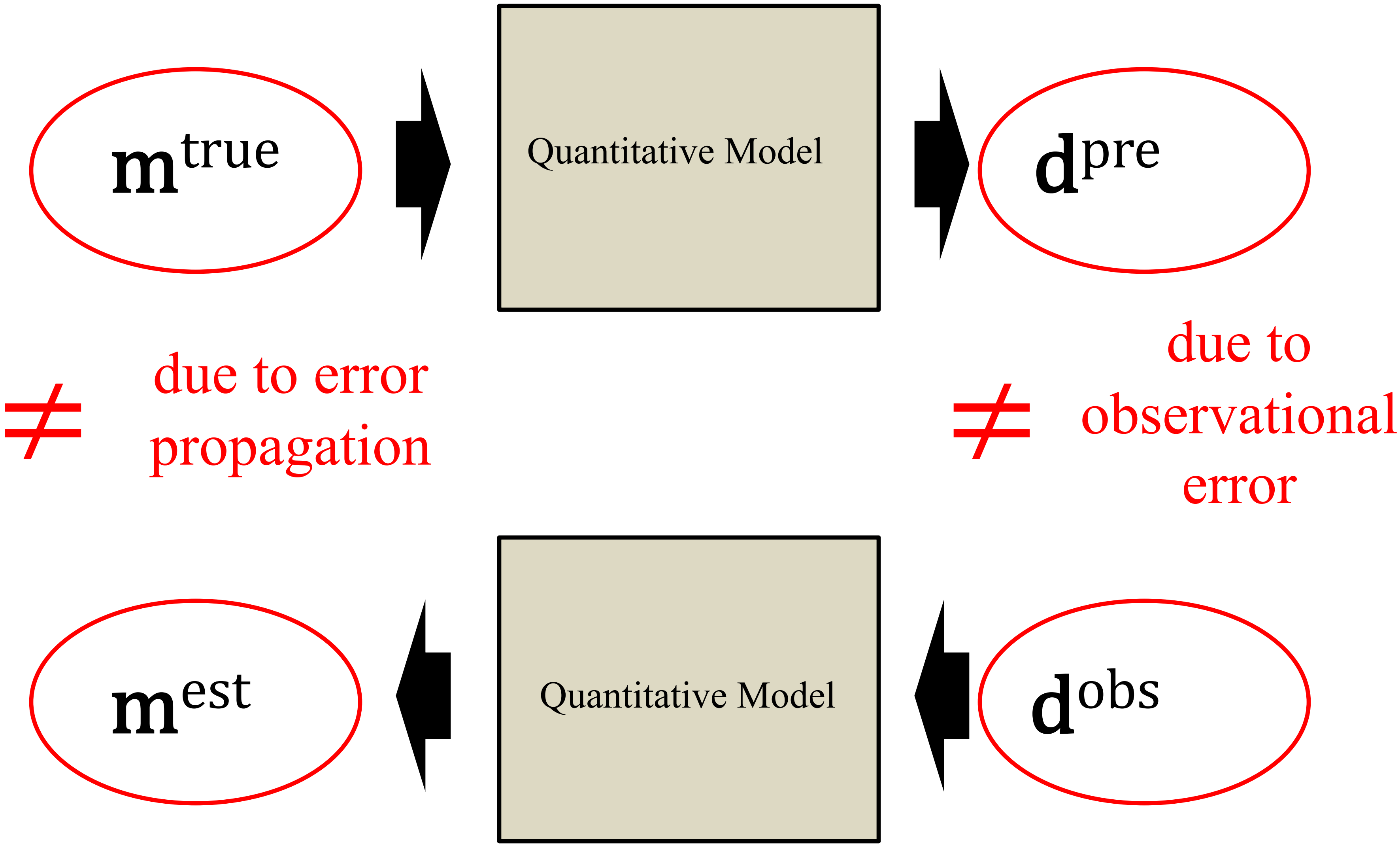


Observations (data) → quantitative model → estimates of the model parameters

Suppose you knew the true values of the model parameters. The data predicted by them is rarely equal to the observed data, due to observational error.



Consequently, the estimates of the model parameters are never equal to their true values. Error has propagated through the solution process.



What are the necessary parts of any inversion?

- Data/measurements, hopefully with realistic error estimates.
- One or more potential quantitative models.
- An appropriate technique for inverting that data. These include both linear and nonlinear inversion methods.
- In addition, a good inversion requires an understanding of the limits of the inversion techniques, the sensitivity of the results to the source parameters and their interplay, and the propagation of the data error into the solution.

Linear Inversion Techniques

- Assume that a function, G, is our geophysical theory – and that it is a good approximation of the true physics.
- m is our model, or the important physical parameters that describe the behavior of G.
- d are the data measurements resulting from the action of G and m. Then,

$$G(m) = d$$

Linear Inversion Techniques

- For most linear inversions, G will be characterized as a set of linear equations, coded as a matrix.
- Remember that almost all data has some level of error associated with it. Therefore,

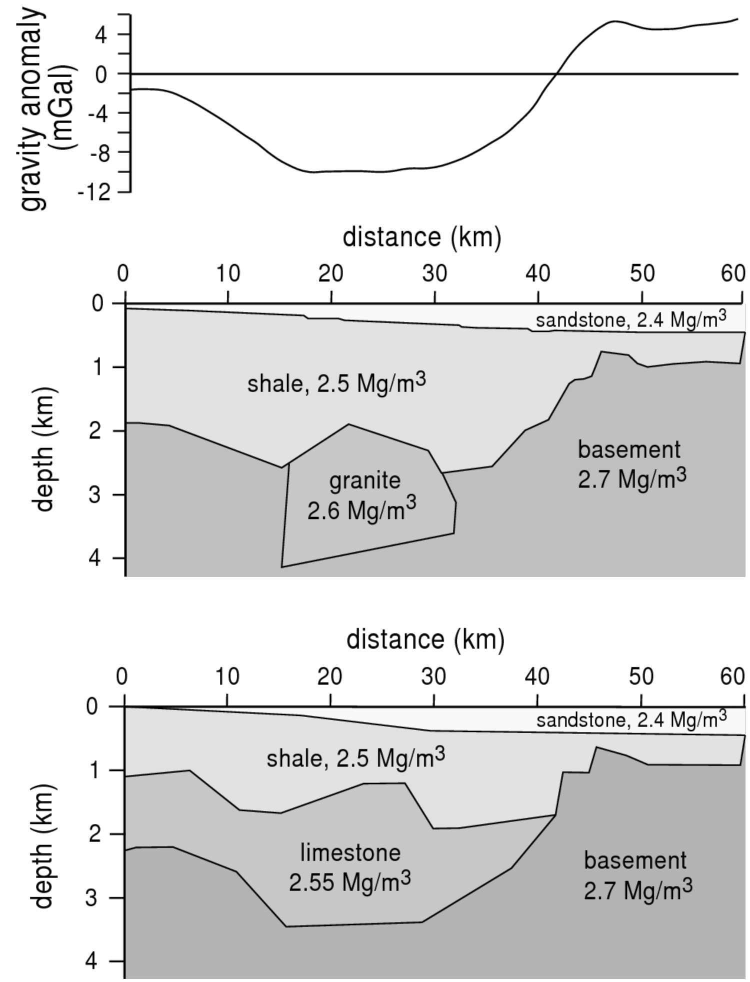
$$d = d_{true} + \eta, \quad \text{where}$$

$$d_{true} = G(m_{true}), \text{ and}$$

η is the measurement noise

Inversions

- What is the estimate of *m*, given *d*?
- Typicaly many models, *m*, can produce *d*, for one *G* – and there are often more than one potential operator, *G*. η only adds to the difficulty of finding m_{true}.



Linear Inverse Problems

the function $\mathbf{g}(\mathbf{m})$ is a matrix $\mathbf{G}$ times $\mathbf{m}$

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

$\mathbf{G}$ has N rows and M columns

(what are the dimensions of d and m?)

Linear Inverse Problems

the function $\mathbf{g}(\mathbf{m})$ is a matrix $\mathbf{G}$ times $\mathbf{m}$

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

$\mathbf{G}$ has N rows and M columns

d is an Nx1 vector and m is an Mx1 vector

Linear Inverse Problems

the function $\mathbf{g}(\mathbf{m})$ is a matrix $\mathbf{G}$ times $\mathbf{m}$

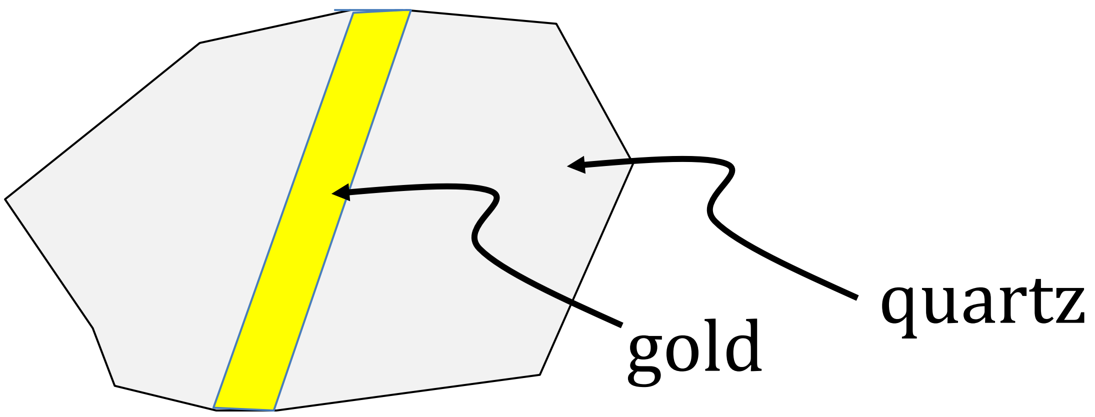
$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

"data kernel"

$\mathbf{G}$ has N rows and M columns

Example

-You want to know how much gold is in the rock.
-You measure mass and volume.
-You want to infer the volume of the gold.



Example

-You want to know how much gold is in the rock.
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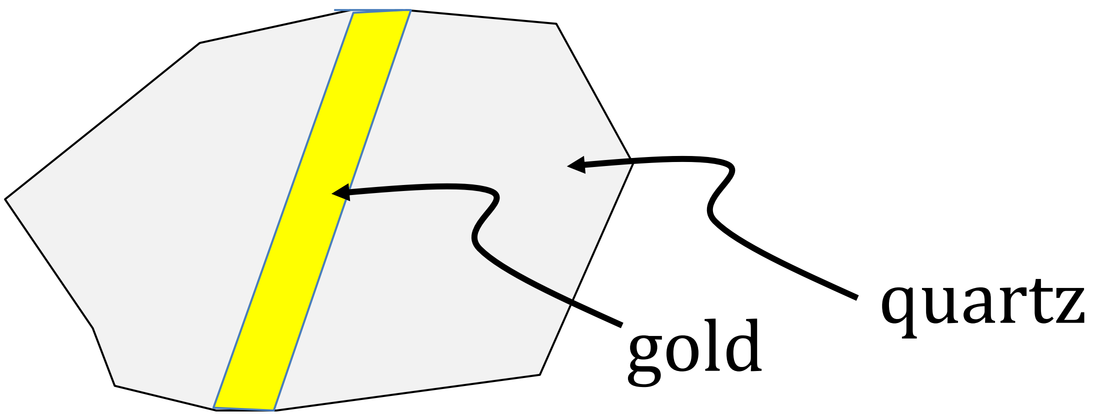


$$\text{total volume} = \text{volume of gold} + \text{volume of quartz}$$

$$V = V_g + V_q$$

$$\text{total mass} = \text{density of gold} \times \text{volume of gold} + \text{density of quartz} \times \text{volume of quartz}$$

$$M = \rho_g \times V_g + \rho_q \times V_q$$

$$V = V_g + V_q$$

$$M = \rho_g \times V_g + \rho_q \times V_q$$

Data (measurements) are

$V = d_1$

$M = d_2$

$\mathbf{d} = [d_1, d_2]^T$ and $N=2$

Model you want to determine

$V_g = m_1$

$V_q = m_2$

$\mathbf{m} = [m_1, m_2]^T$ and $M=2$

assume

$$\left.\begin{matrix}\rho_g \\ \rho_g\end{matrix}\right\}\ \text{known}$$

$$\begin{bmatrix} V \\ M \end{bmatrix} = \begin{pmatrix} 1 & 1 \\ \rho_g & \rho_q \end{pmatrix} \begin{bmatrix} \mathbf{Vg} \\ \mathbf{Vq} \end{bmatrix}$$

$$V = V_g + V_q$$

$$M = \rho_g \times V_g + \rho_q \times V_q$$

Data (measurements) are

$V = d_1$

$M = d_2$

$\mathbf{d}=[d_1, d_2]^T$ and $N=2$

Model you want to determine

$V_g = m_1$

$V_q = m_2$

$\mathbf{m}=[m_1, m_2]^T$ and $M=2$

assume

$$\left.\begin{matrix}\rho_g \\ \rho_g\end{matrix}\right\} \text{known}$$

$$\mathbf{d} = \begin{pmatrix} 1 & 1 \\ \rho_g & \rho_q \end{pmatrix} \mathbf{m}$$

in all these examples **m** is discrete

$$d_i = \sum_{j=1}^{M} G_{ij} m_j$$

discrete inverse theory

one could have a continuous *m(x)* instead

$$d_i = \int G_i(x)\, m(x)\ \mathrm{d}x$$

continuous inverse theory

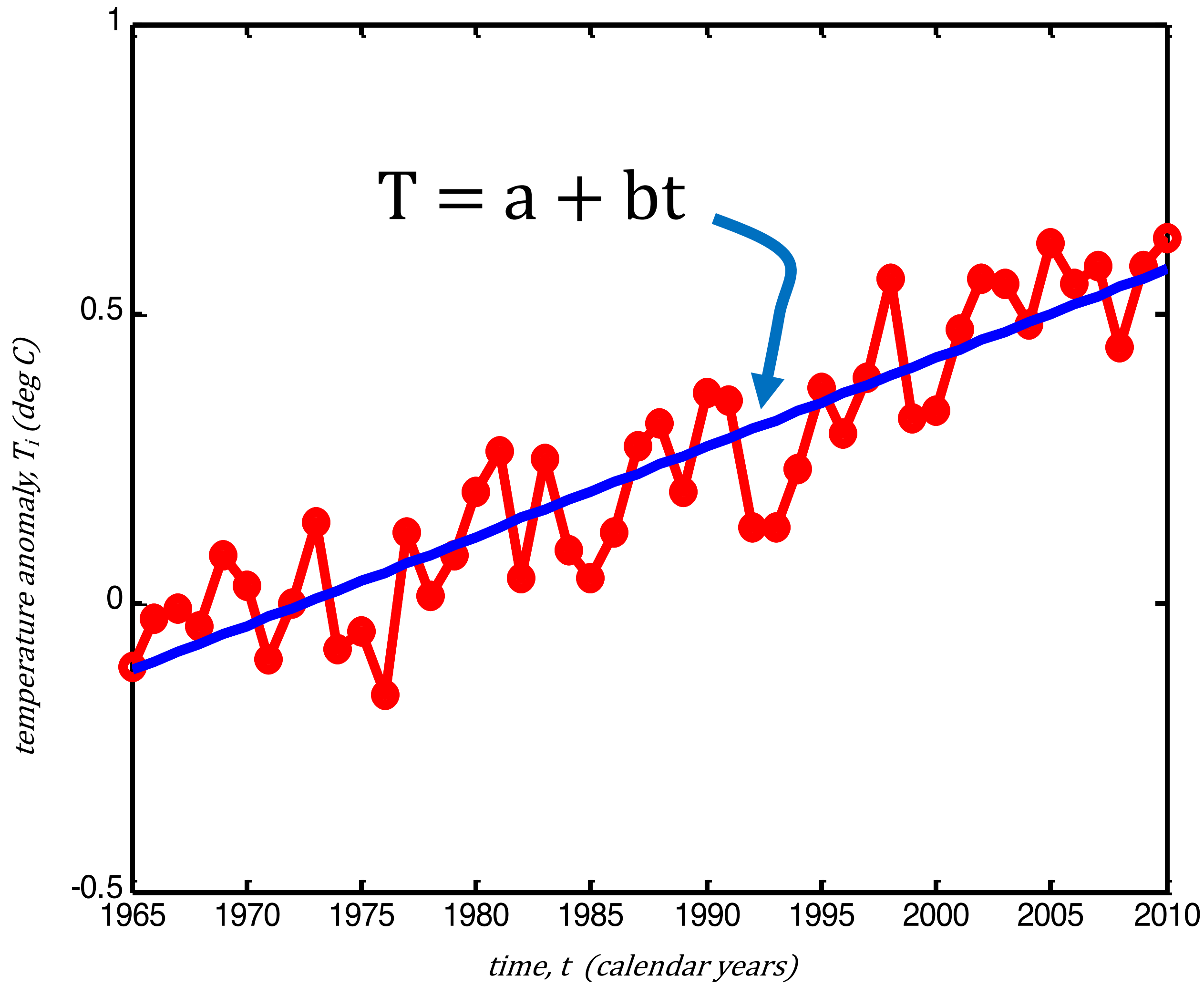
in this course we will usually approximate a continuous $m(x)$

as a discrete vector $\mathbf{m}$

but we will spend some time later in the course dealing with the continuous problem directly

Some Examples

Example 1. Fitting a straight line to data



The straight line has two model parameters, intercept (a) and slope (b).

$$
\begin{aligned}
T_1 &= a + bt_1 \\
T_2 &= a + bt_2 \\
&\vdots \\
T_N &= a + bt_N
\end{aligned}
$$

each data point

is predicted by a
straight line

matrix formulation

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_N \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\mathbf{d} \quad = \quad \mathbf{G} \quad \mathbf{m}$$

Example 2. Fitting a parabola to data

$$T = a + bt + ct^2$$

$$
\begin{aligned}
T_1 &= a + bt_1 + ct_1^2 \\
T_2 &= a + bt_2 + ct_2^2 \\
&\vdots \\
T_N &= a + bt_N + ct_N^2
\end{aligned}
$$

each data point

is predicted by a
quadratic curve

matrix formulation

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ \vdots & \vdots & \vdots \\ 1 & t_N & t_N^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\mathbf{d} = \mathbf{G} \quad \mathbf{m}$$

$M=3$

straight line

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_N \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

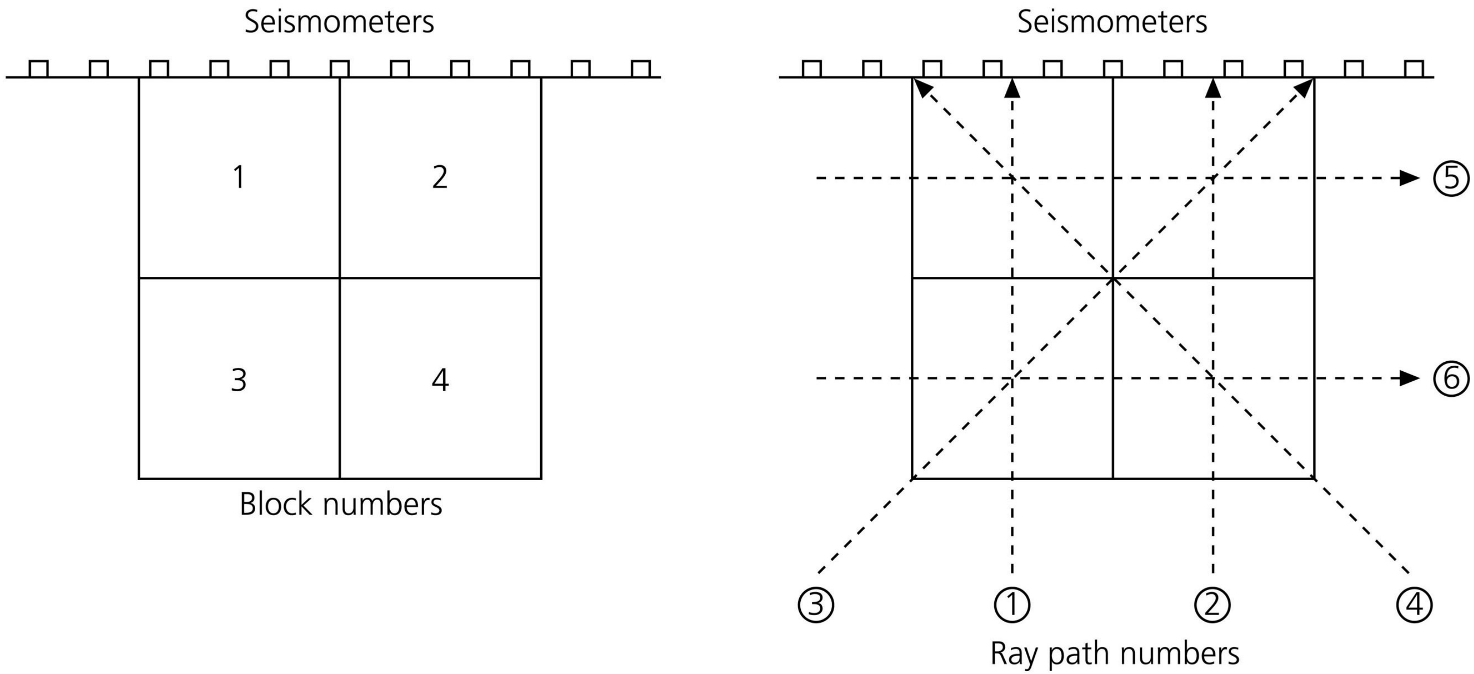
parabola

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ \vdots & \vdots & \vdots \\ 1 & t_N & t_N^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

note similarity

Example 3: Tomography

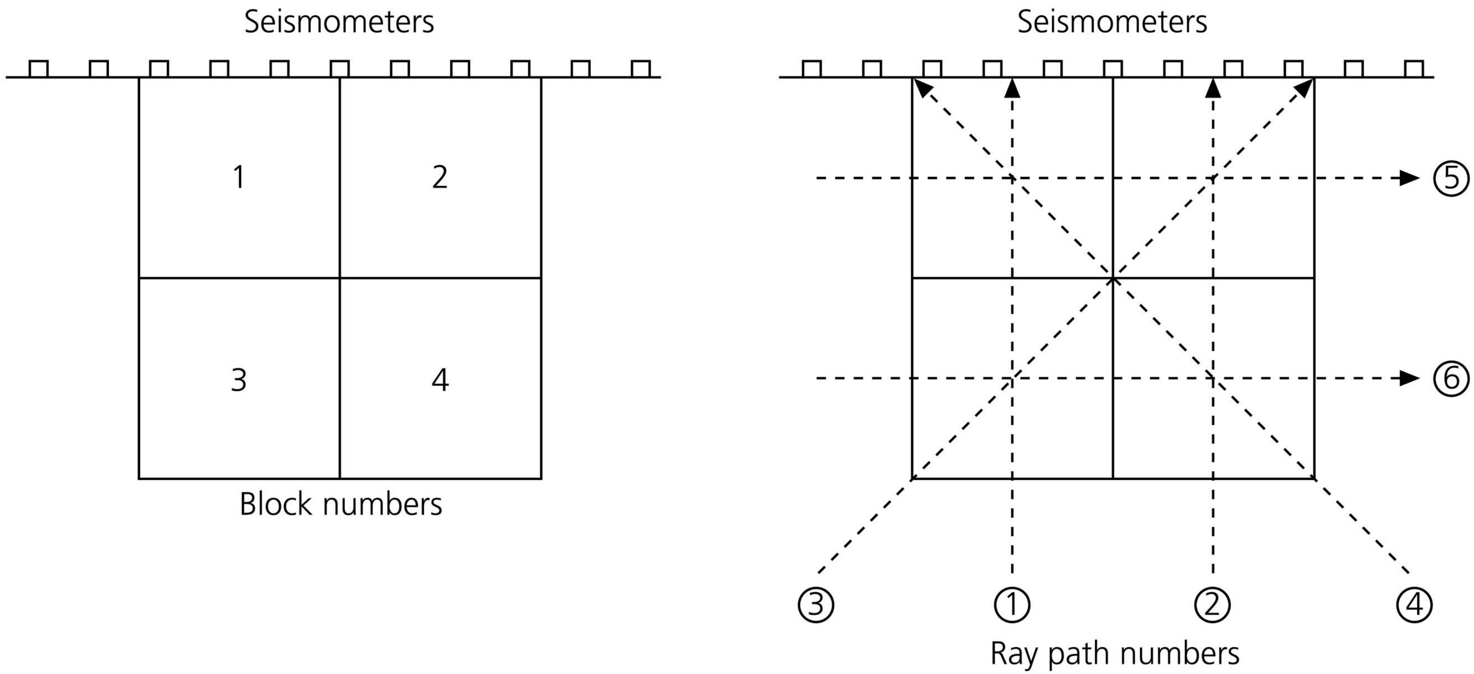
Figure 7.3-2: Ray path and block geometry for an idealized tomographic experiment.



Stein and Wysession, 2003

Intro to Tomography

Figure 7.3-2: Ray path and block geometry for an idealized tomographic experiment.

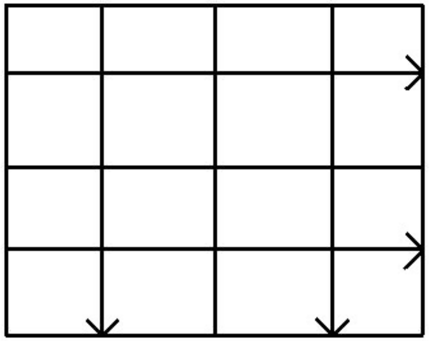


Forward problem: Given velocity structure, calculate travel times

Inverse problem: Given travel times, calculate velocity structure

Stein and Wysession, 2003

Intro to Tomography

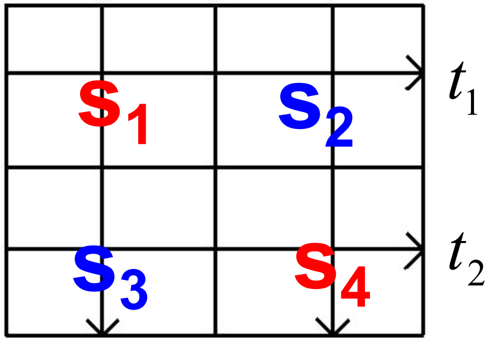


$$t = s \cdot d$$

$$s = slowness \quad = 1/velocity$$

d = distance
t = travel time

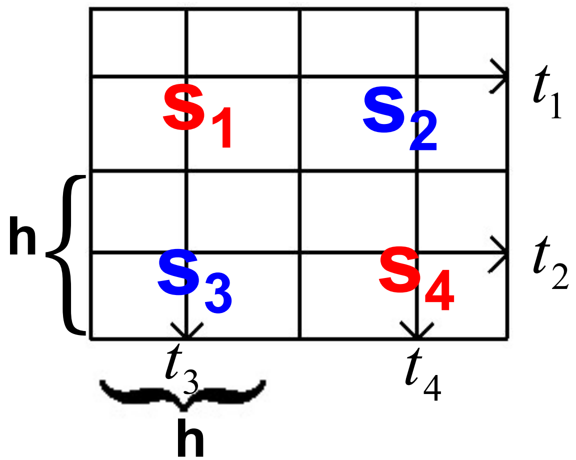
Intro to Tomography



$$t = s \cdot d$$

$$s = slowness$$ = 1/velocity

Intro to Tomography



$$t = s \cdot d$$

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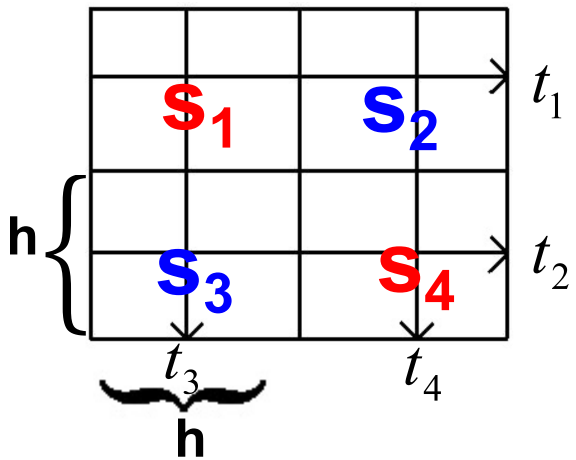
Intro to Tomography



$$t = s \cdot d$$

$$s = slowness$$ $= 1/velocity$

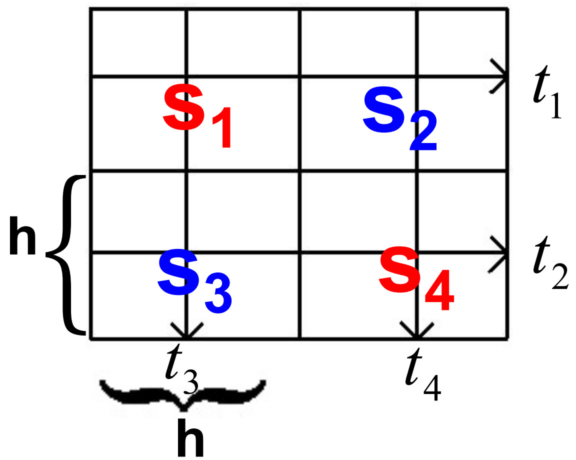
$$t_1 = s_1 h + s_2 h$$

$$t_2 = s_3 h + s_4 h$$

$$t_3 = \quad ?$$

$$t_4 = \quad ?$$

Intro to Tomography



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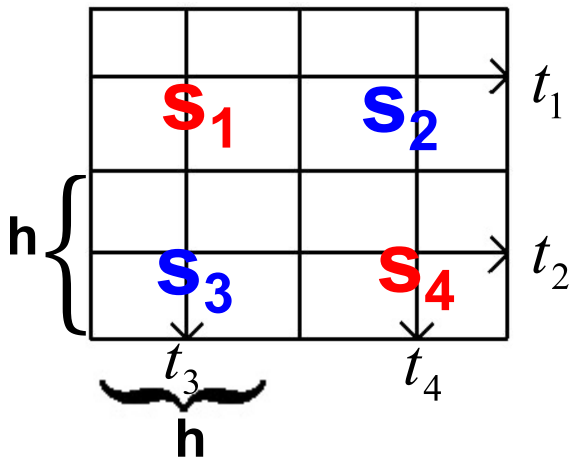
Intro to Tomography



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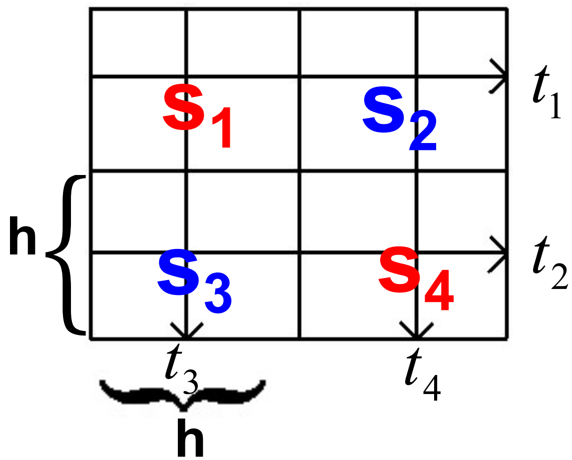
Rewrite equations in matrix form

Intro to Tomography



$$t = s \cdot d$$

$$s = slowness \quad = 1/velocity$$

$$t_1 = s_1 h + s_2 h$$

$$t_2 = s_3 h + s_4 h$$

$$t_3 = s_1 h + s_3 h$$

$$t_4 = s_2 h + s_4 h$$

$$\begin{bmatrix} t_1 \\ \\ \\ t_n \end{bmatrix} = \begin{bmatrix} h & h & 0 & 0 \\ 0 & 0 & h & h \\ h & 0 & h & 0 \\ 0 & h & 0 & h \end{bmatrix} \begin{bmatrix} s_1 \\ \\ \\ s_n \end{bmatrix}$$

$$\mathbf{d} = \mathbf{G} \quad \mathbf{m}$$

data

model

$$d = G\,m \quad \Rightarrow \quad m = G^{-1}\,d$$

$$m = (G^T\,G)^{-1}\,G^{T}\,d$$

Can't always

get G^{-1}

inverse

$$2 \times \frac{1}{2} = 1$$

$$G \times G^{-1} = I$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Flavors of Least Squares

damped least squares

$$\min\,(d - Gm)^T (d - Gm) + m^T m$$

$$m = (G^T G + \varepsilon^2 I)^{-1} G^T d$$

weighted least squares

$$m = (G^T W_m^{-1} G)^{-1} G^T W_m^{-1} d$$

damped weighted least squares

$$m = (G^T W_m^{-1} G + \varepsilon^2 I)^{-1} G^T W_m^{-1} d$$

- **Grid search –> Forward modeling**

  try many models, calculate misfit

  look for minimum difference in misfit

  ok for small number of parameters

- Genetic algorithms - similar to grid search

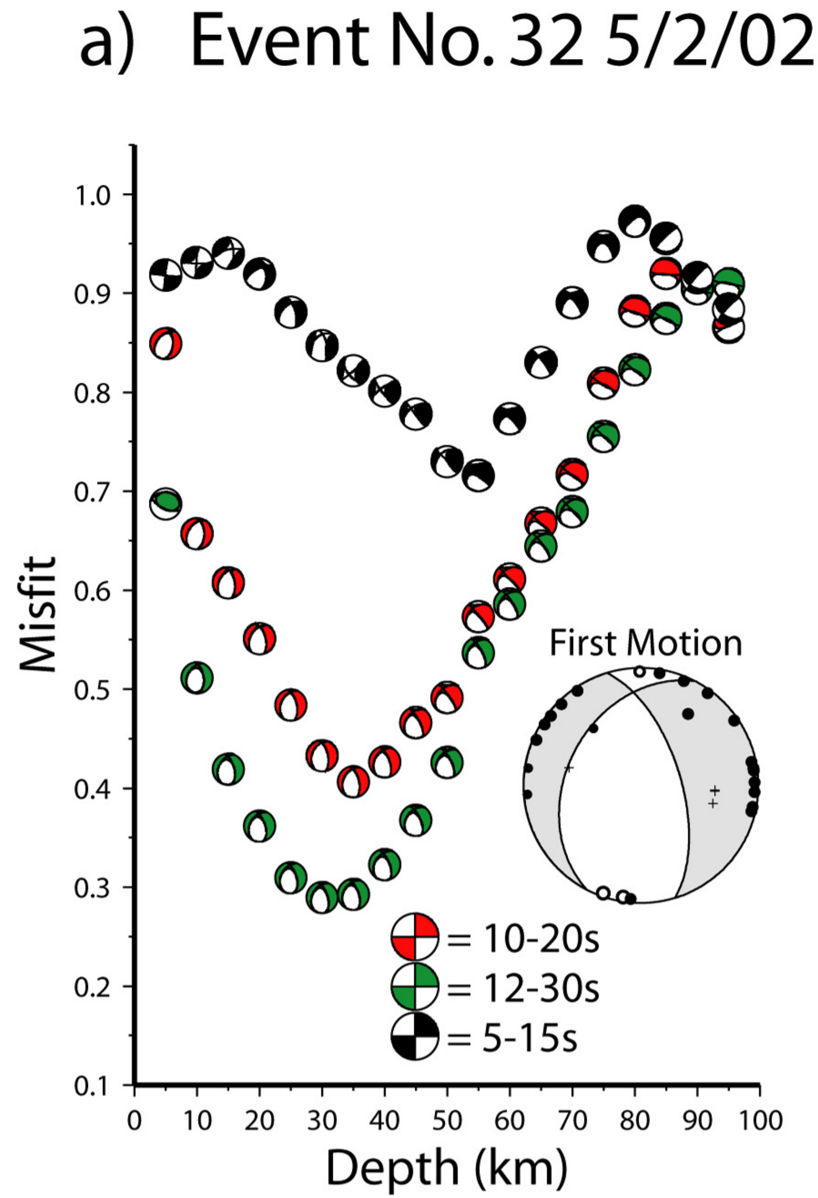
  keep and combine best models

- Monte Carlo

- Neighborhood Algorithm

Grid Search Example:

Find earthquake depth that minimizes
misfit in moment tensor inversion



Setting up inverse problems

1. Find the equations that relate your data to a model
2. Be able to do the forward problem
3. Check whether your system of equations is linear or not
4. Decide how to solve
   ie damping, weighting, parameterization of model, linearize, grid search, etc.
5. Evaluate solution(s)
   Does the solution make sense? What does the data suggest?
   Uniqueness - there is usually not a single model that is the only possibility

For Next Time:

- Read Introduction and Chapter 1 of Menke.
- After that, we will move on to Chapter 2, Probability, before coming back to linear inversion theory.
- Homework 1 (matrix algebra review) due next class (next Tuesday)