UNIVERSIDAD DE GRANADA

Taller: Identificación del daño estructural

Módulo: MÓDULO FUNDAMENTAL: CALIDAD Y DAÑO
Materia: **Análisis Modal y Detección de Defectos**

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Departamento de Mecánica de Estructuras e Ingeniería Hidráulica

Desarrollo del curso

		FECHA		HORA	PROFESOR	TEMA	
Clase 1	**Lunes**	**1**	**febrero**	**9:30-11:30**	**EGM**	**1**	**Introducción: Análisis modal dentro del marco del mantenimiento de la salud estructural.**
Clase 2	Lunes	8	febrero	9:30-11:30	EGM	2	Fuentes de deterioro, patologías estructurales, y tecnologías de monitorización.
Clase 3	Lunes	15	febrero	9:30-11:30	EGM	3	Taller: procesamiento de señales.
Clase 4	Lunes	22	febrero	9:30-11:30	EGM	4	Análisis modal experimental.
Clase 5	Lunes	15	marzo	9:30-11:30	EGM	5	Análisis modal operacional.
Clase 6	Lunes	12	abril	9:30-11:30	EGM	6	Análisis modal operacional automatizado. Práctica de labotatorio I.
Clase 7	Lunes	19	abril	9:30-11:30	EGM	7	Taller: Identificación del daño estructural.
Clase 8	Lunes	26	abril	9:30-11:30	RCT	8	Técnicas de identificación dinámica basadas en análisis modal operacional.
Clase 9	Lunes	26	abril	12:00-14:00	RCT	9	Práctica de laboratorio II: Test de vibración ambiental.
Clase 10	Martes	27	abril	9:30-11:30	RCT	10	Casos de estudio.
Clase 11	Martes	27	abril	12:00-14:00	RCT		Presentación de trabajos.

ENTREGA DE TRABAJOS Y EVALUACIÓN Del 3 al 28 de mayo

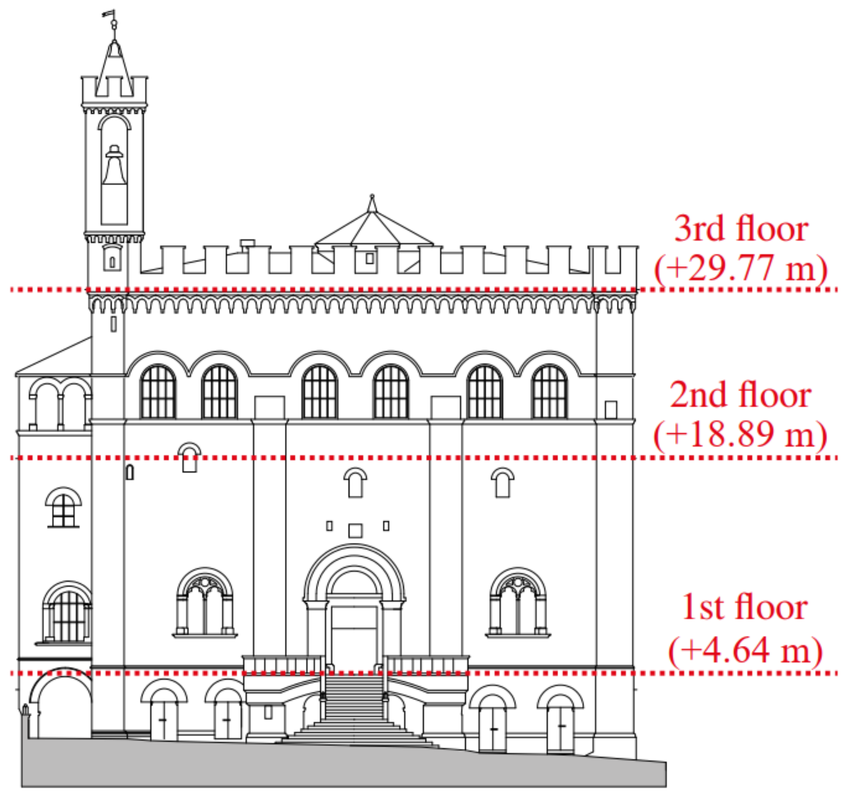
ÍNDICE

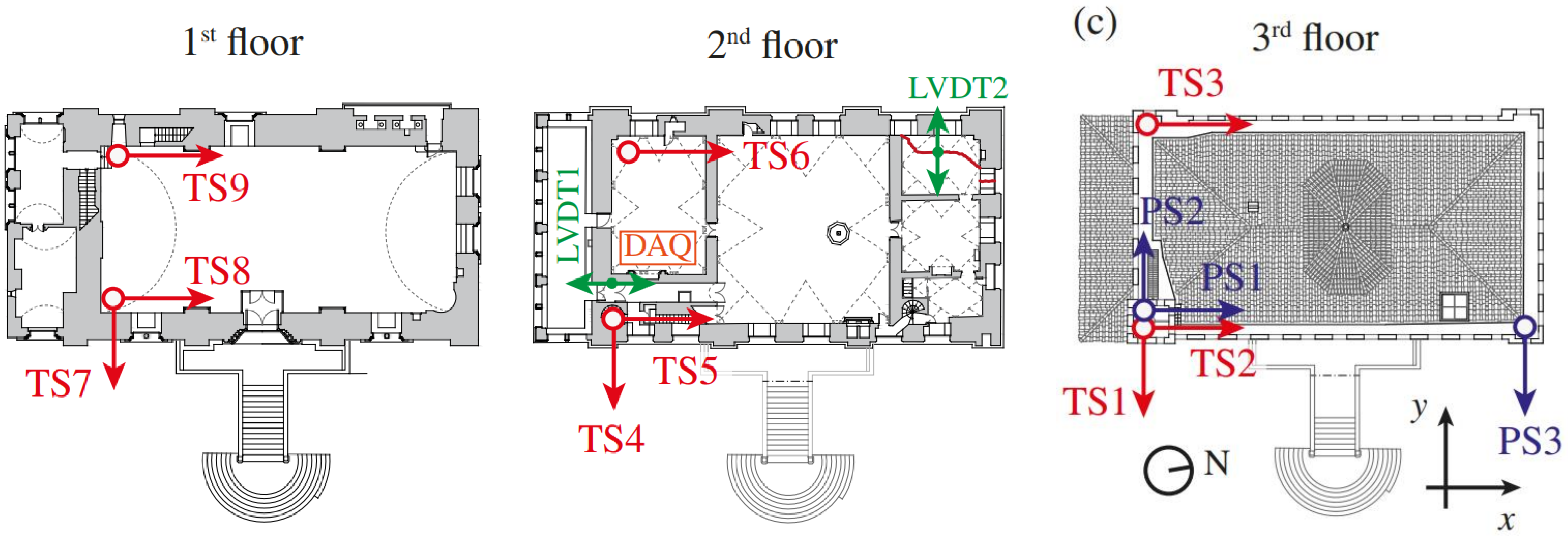
Motivación.

Motivación

Motivation – Consoli Palace in Gubbio (Italy)

Permanent SHM system





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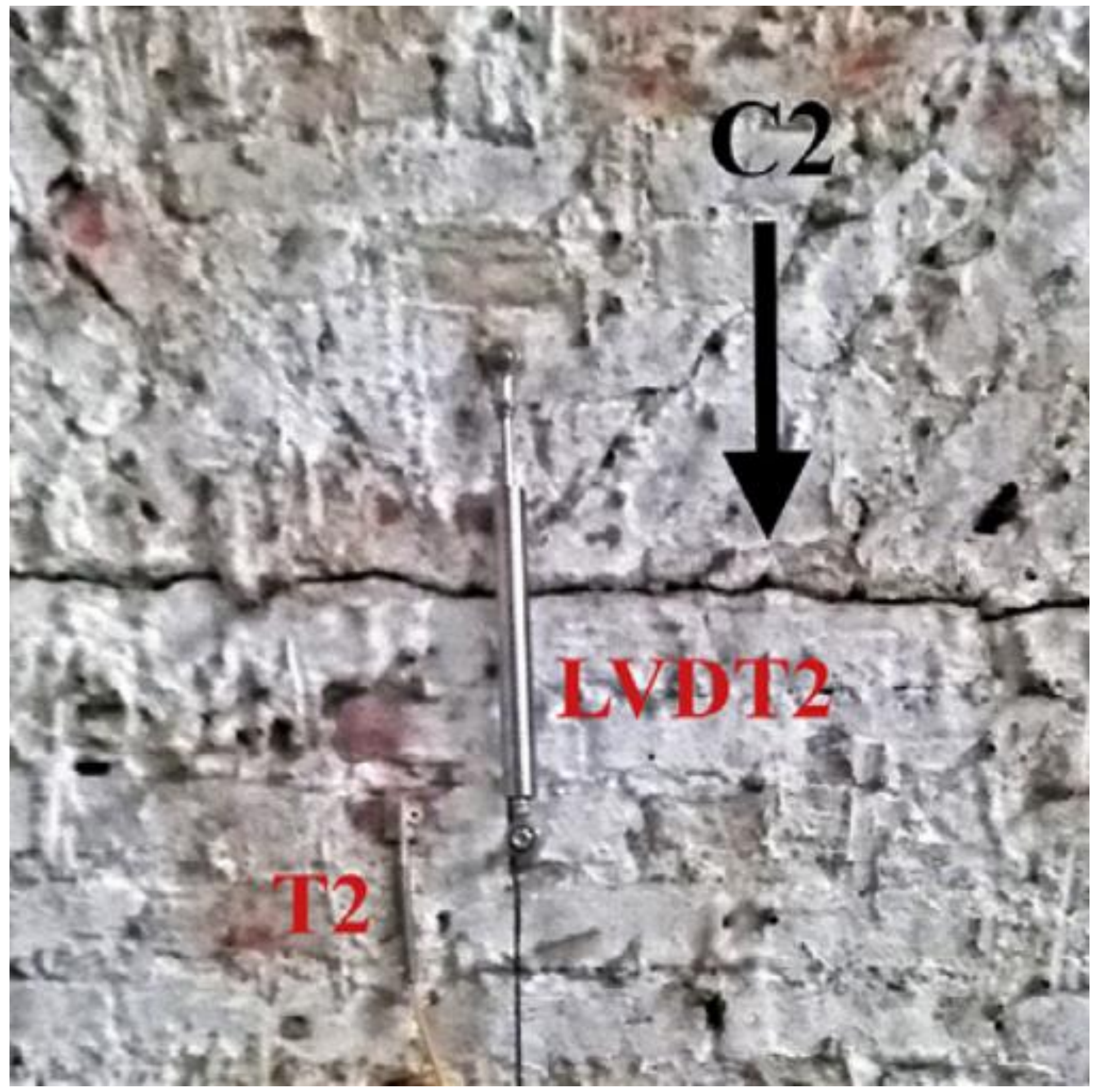
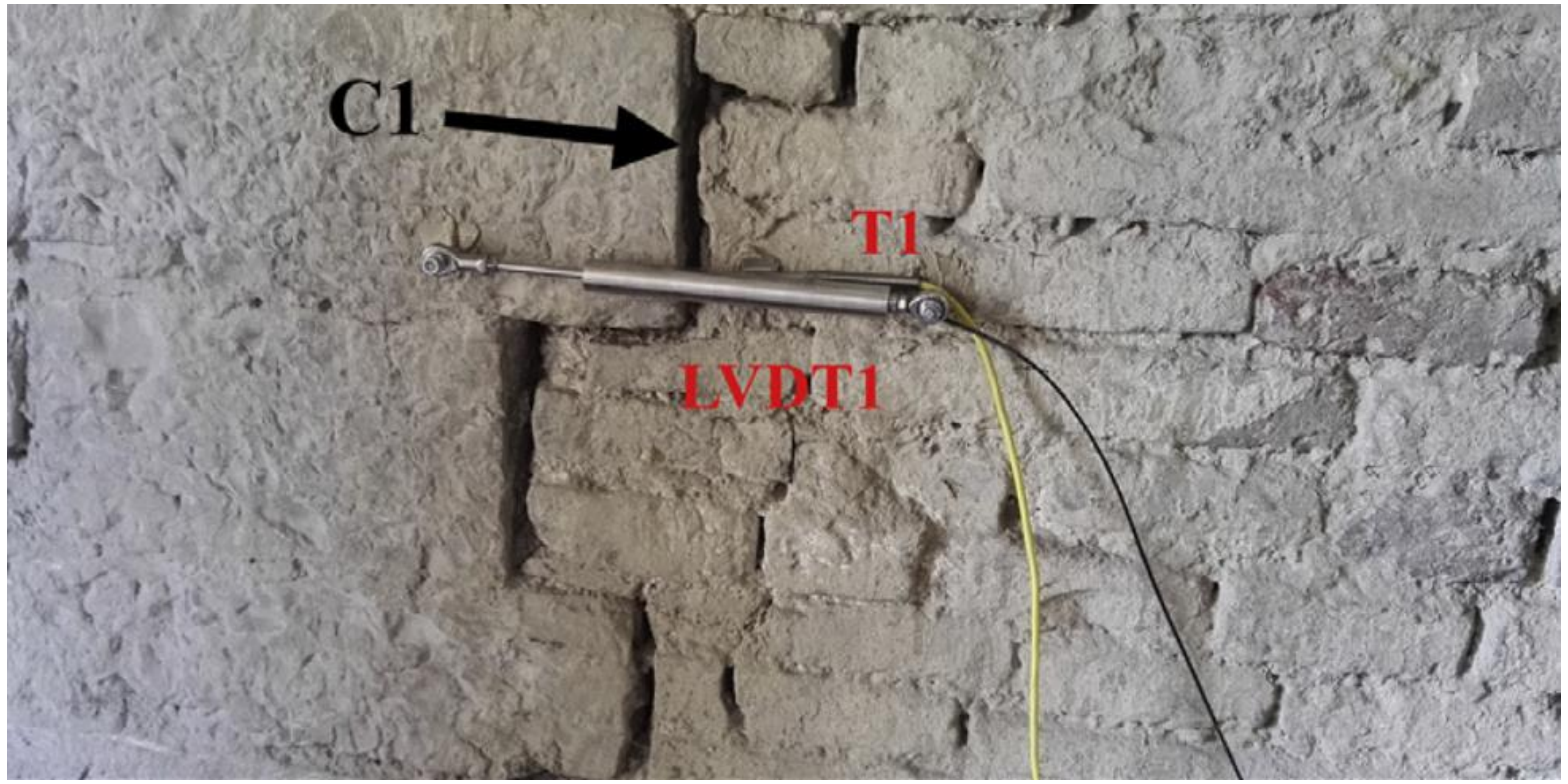
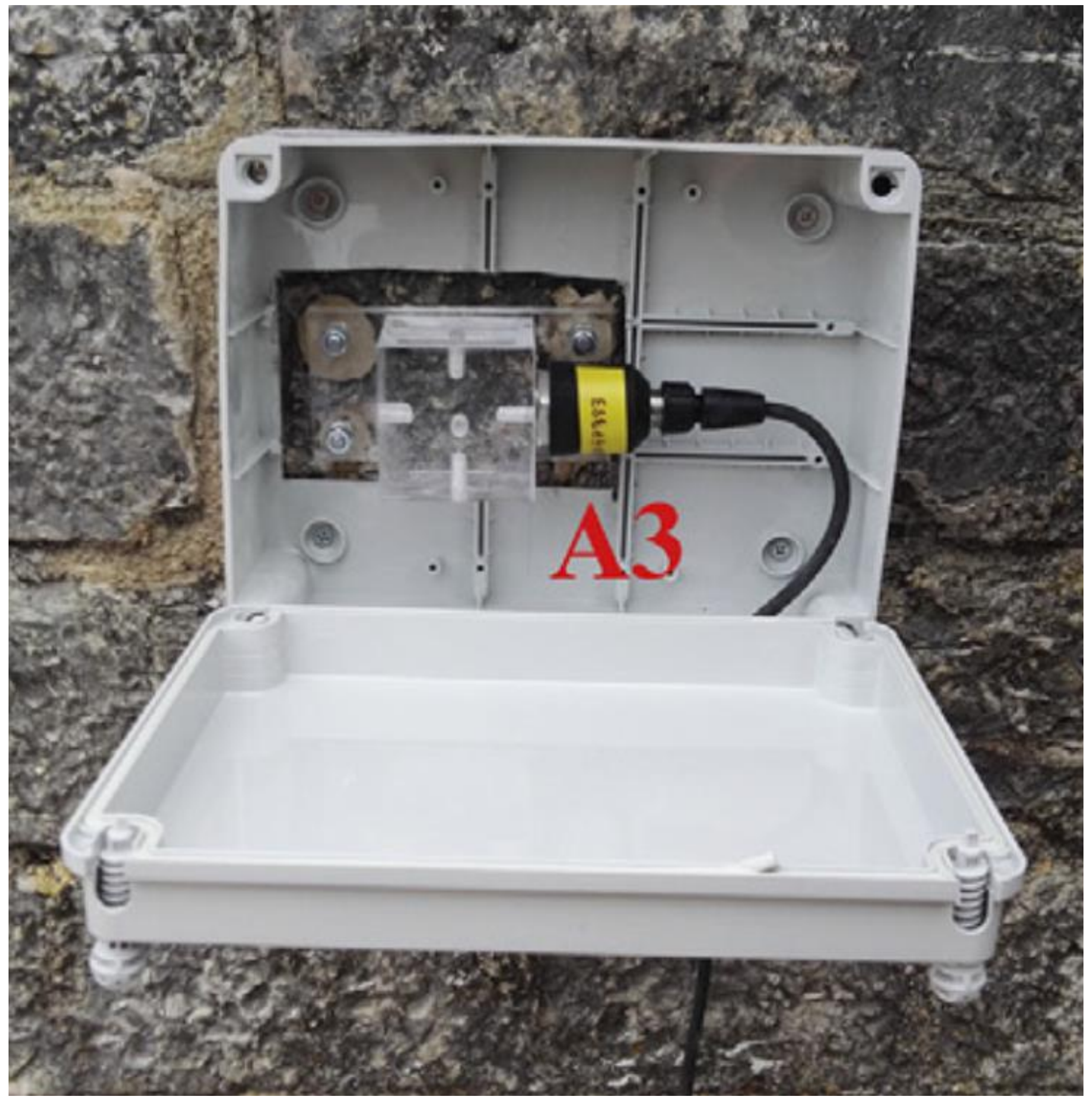
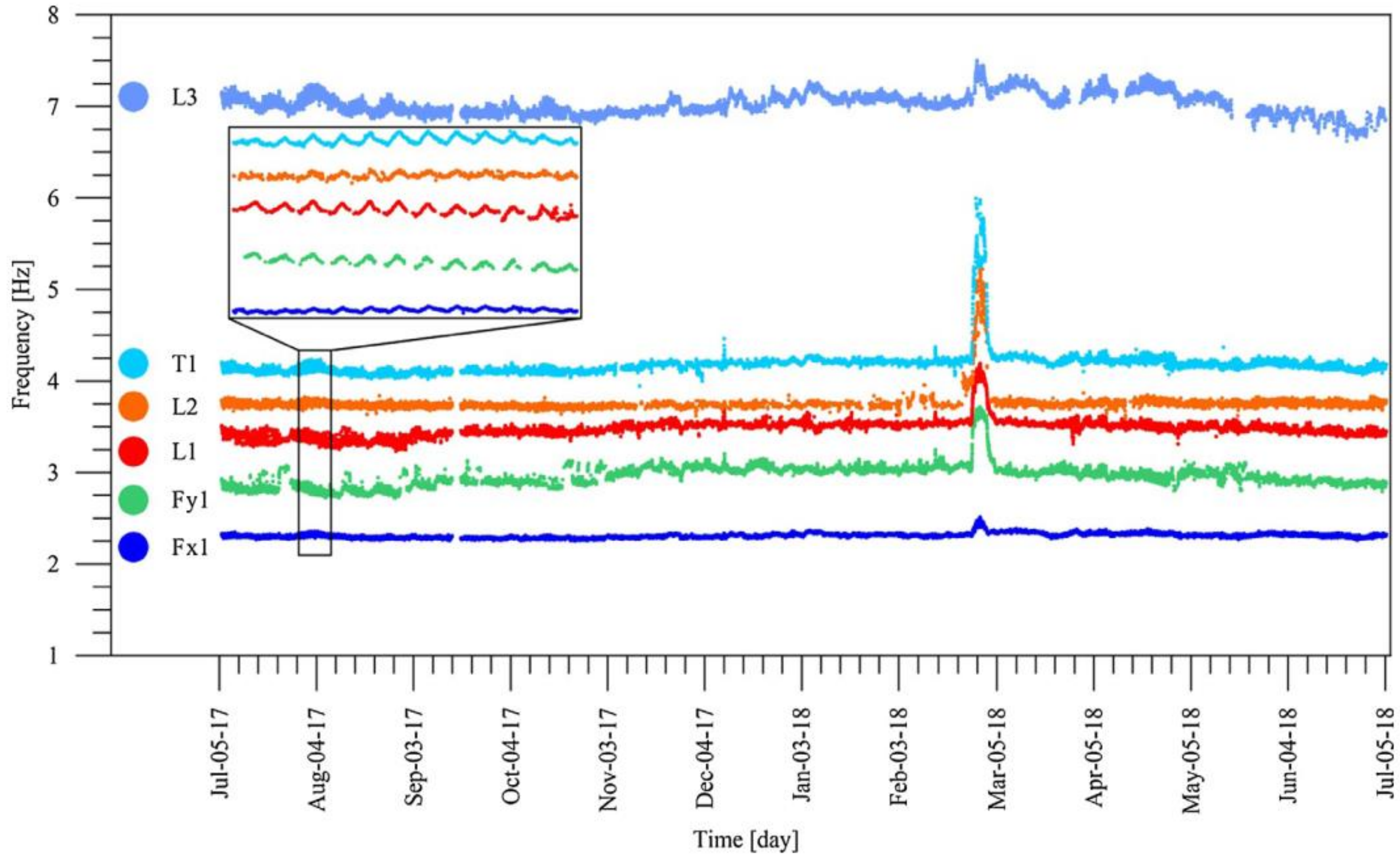




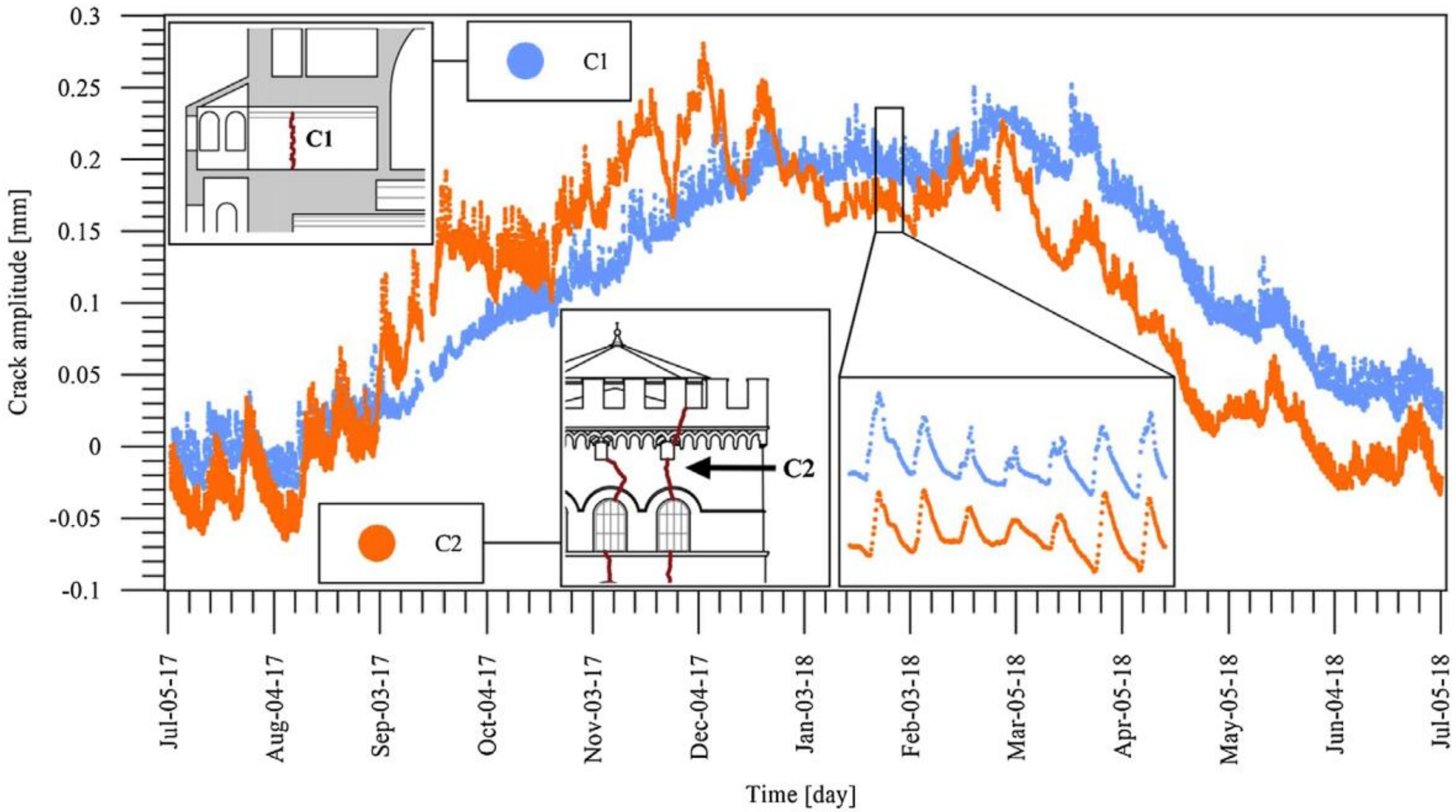


Motivación

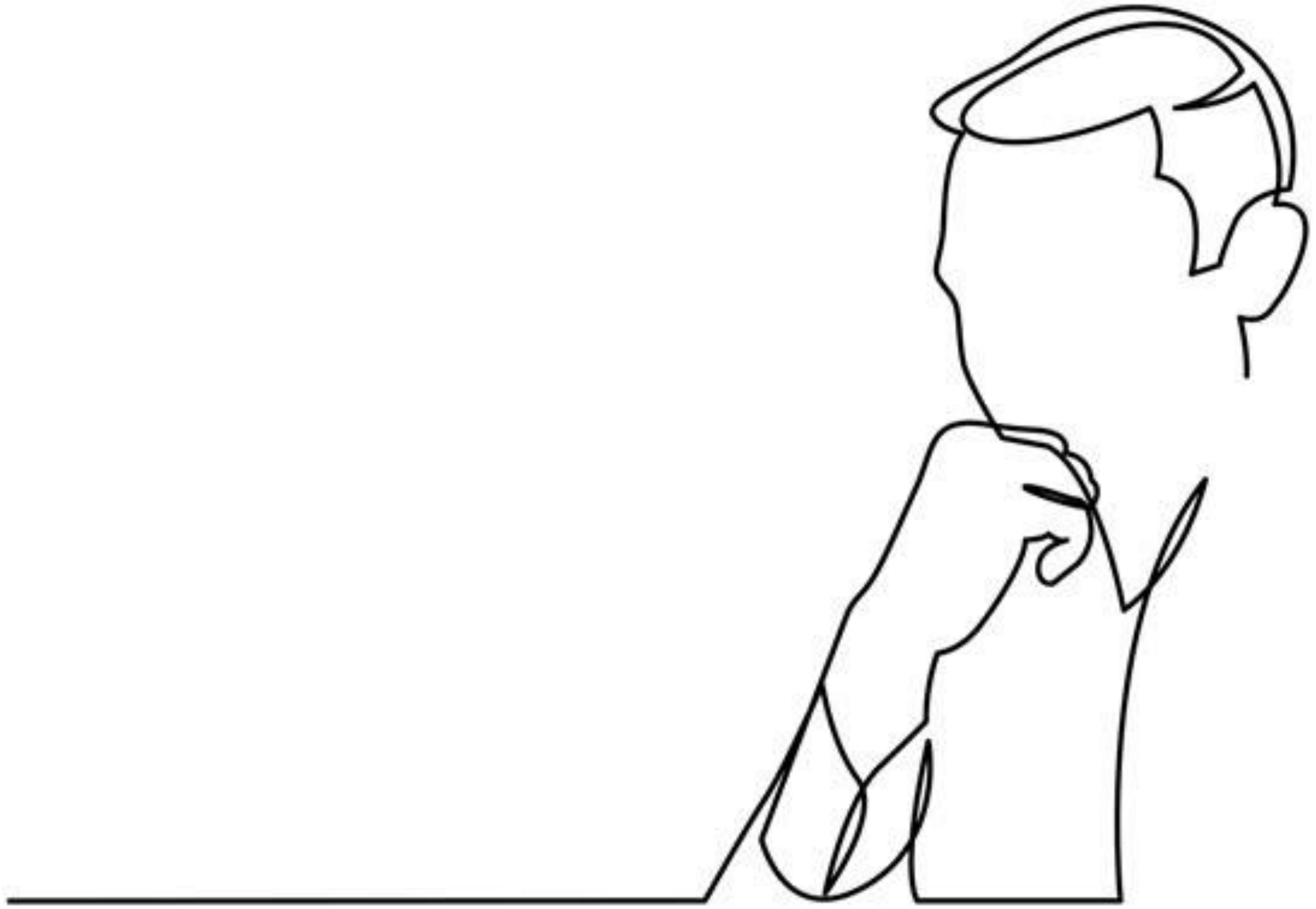
Resonant frequencies



Crack amplitudes



Is it safe?



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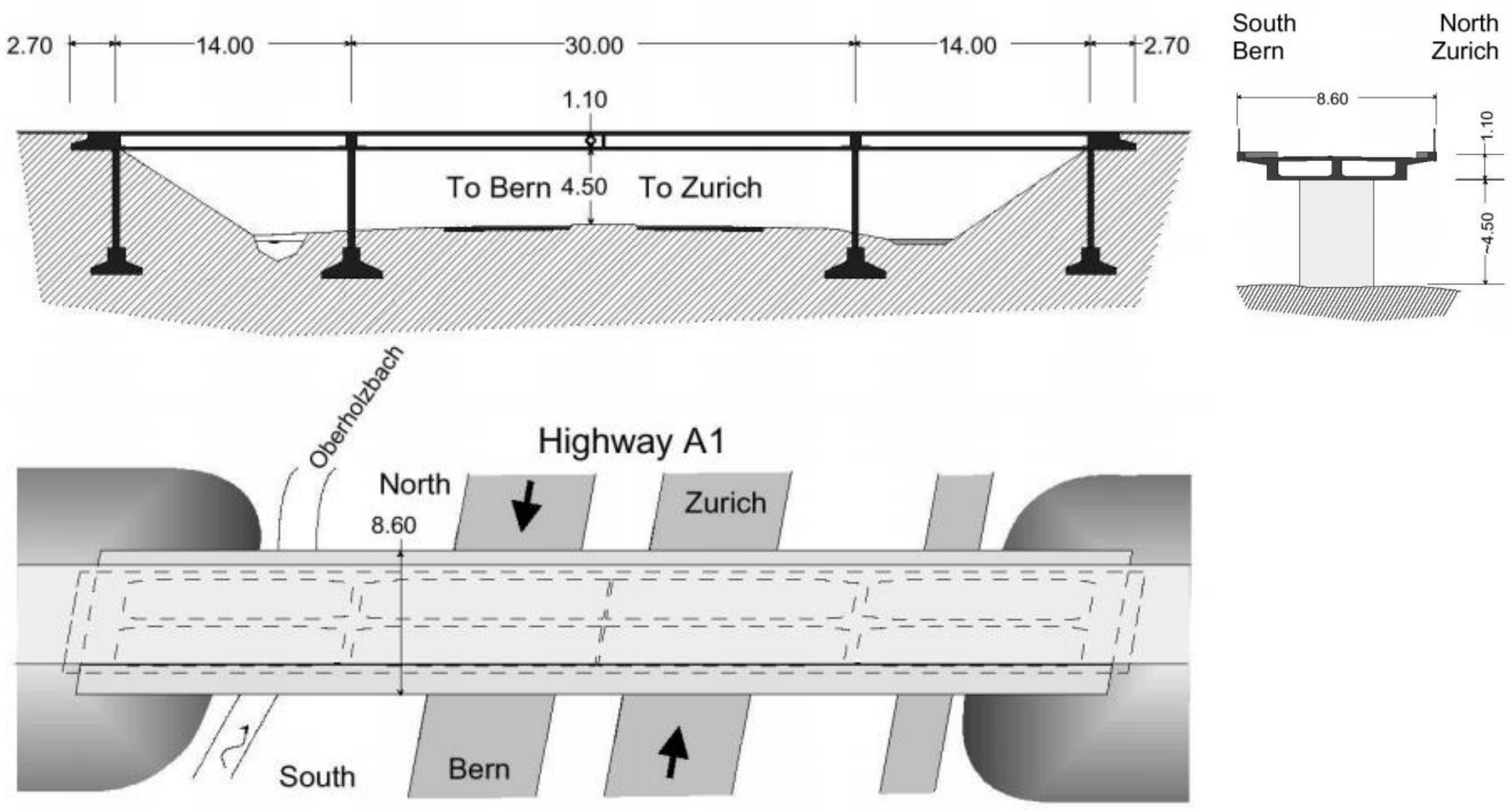
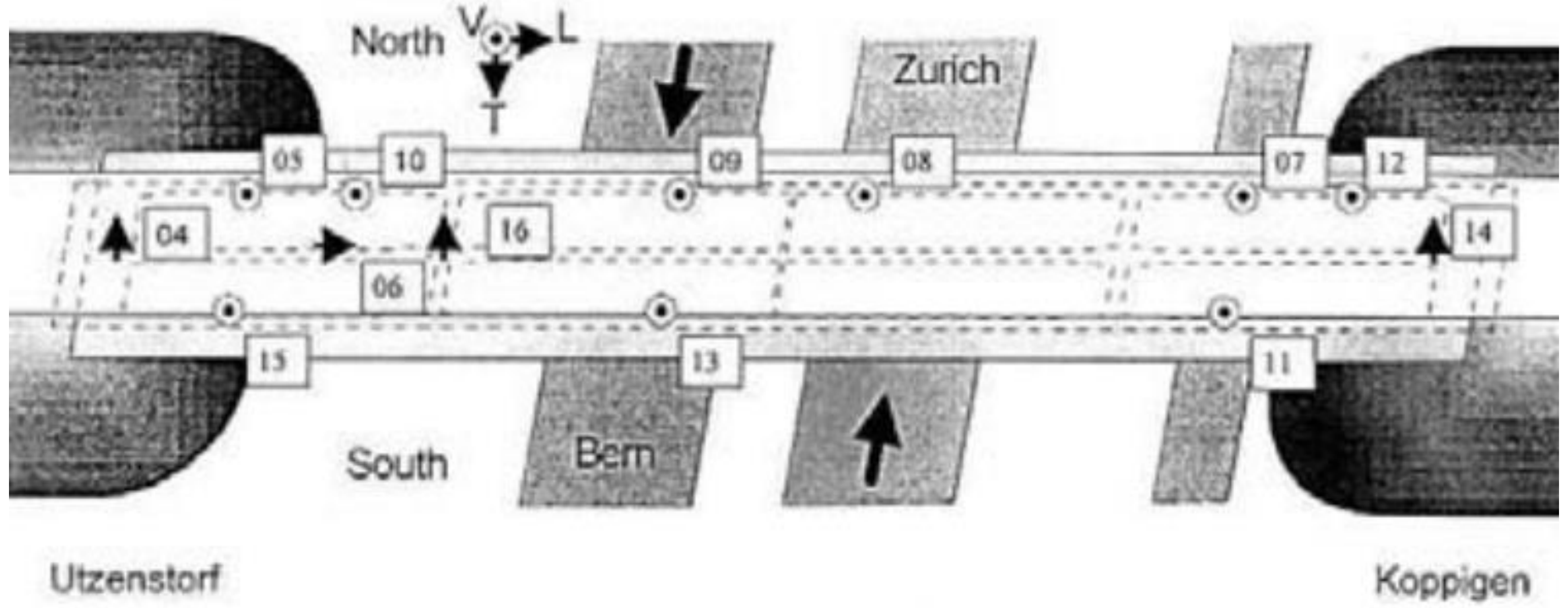
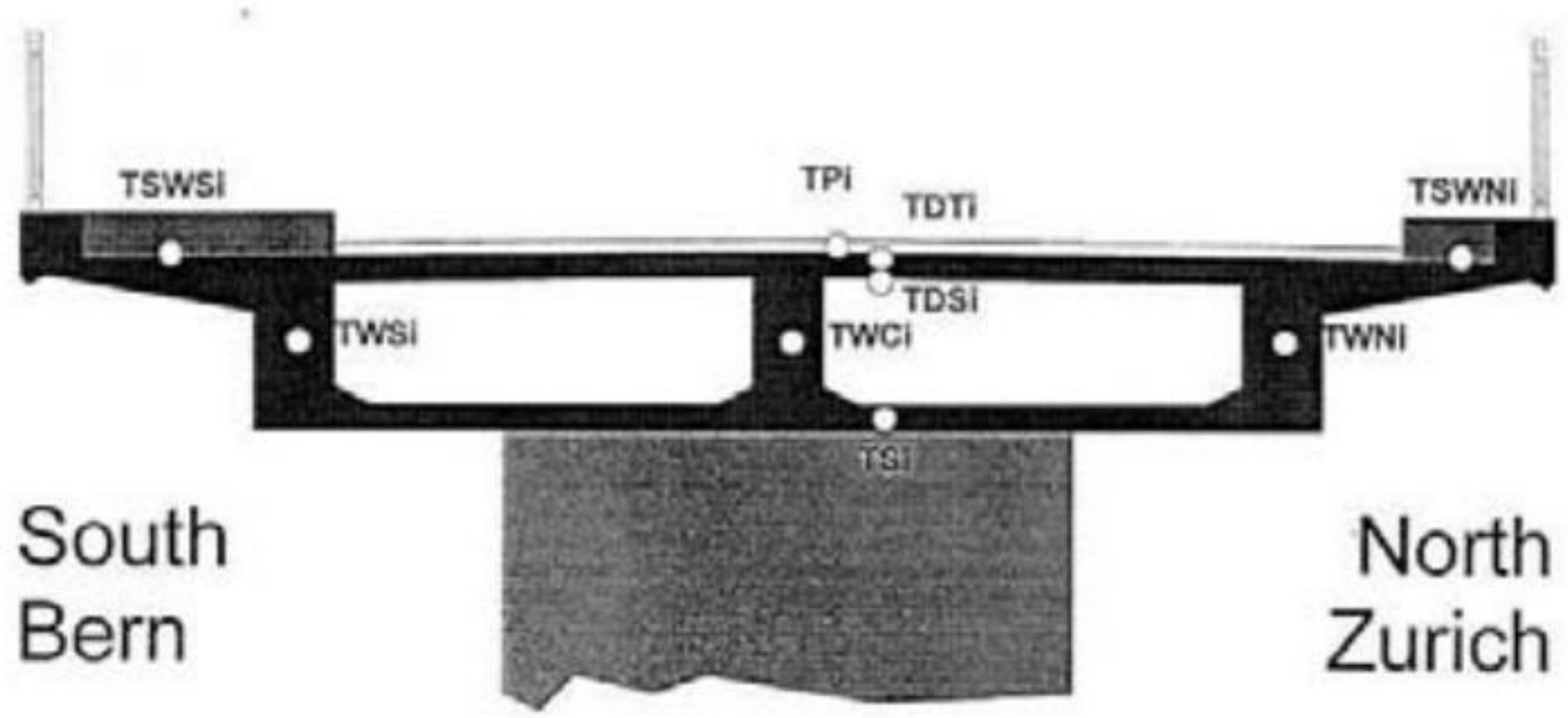


Motivation – Z-24 bridge (Switzerland)

Motivación

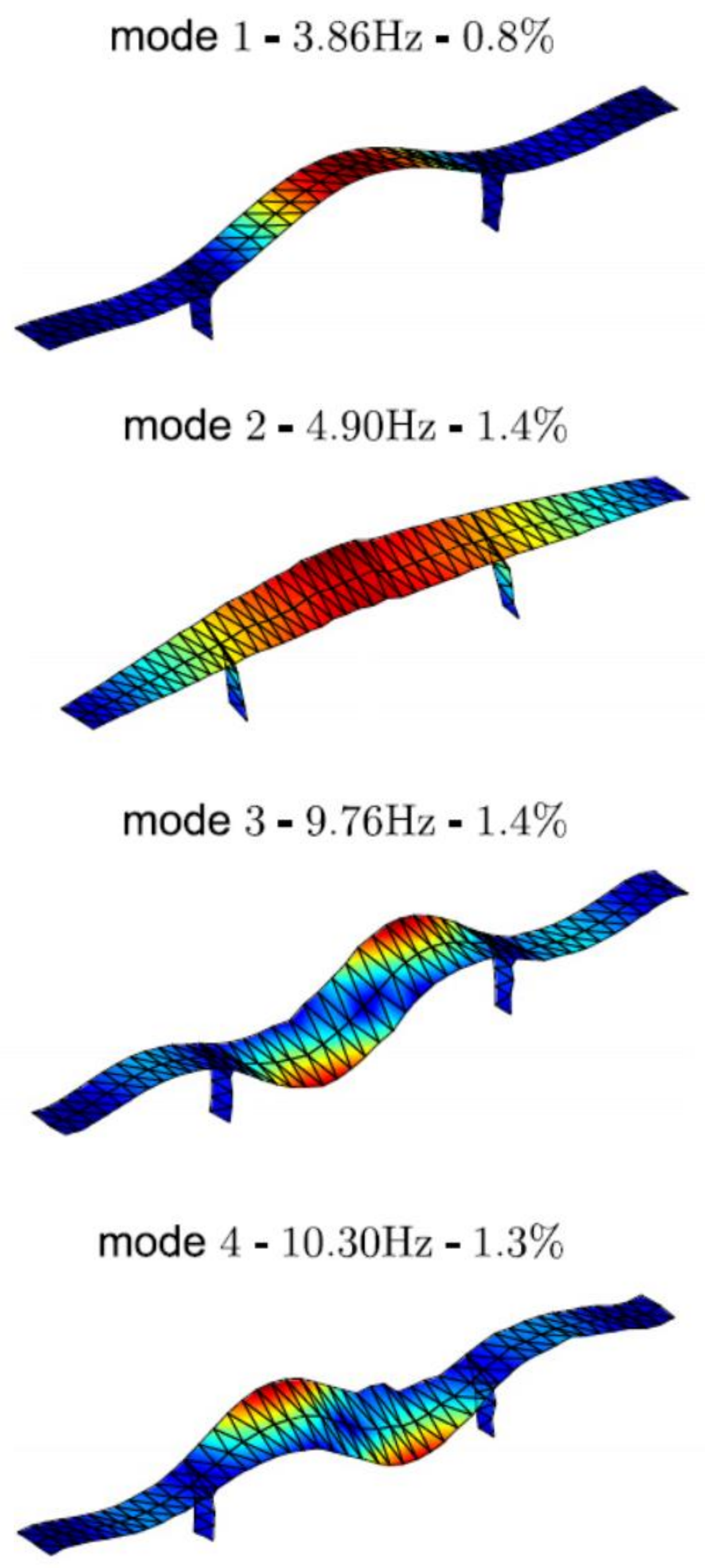


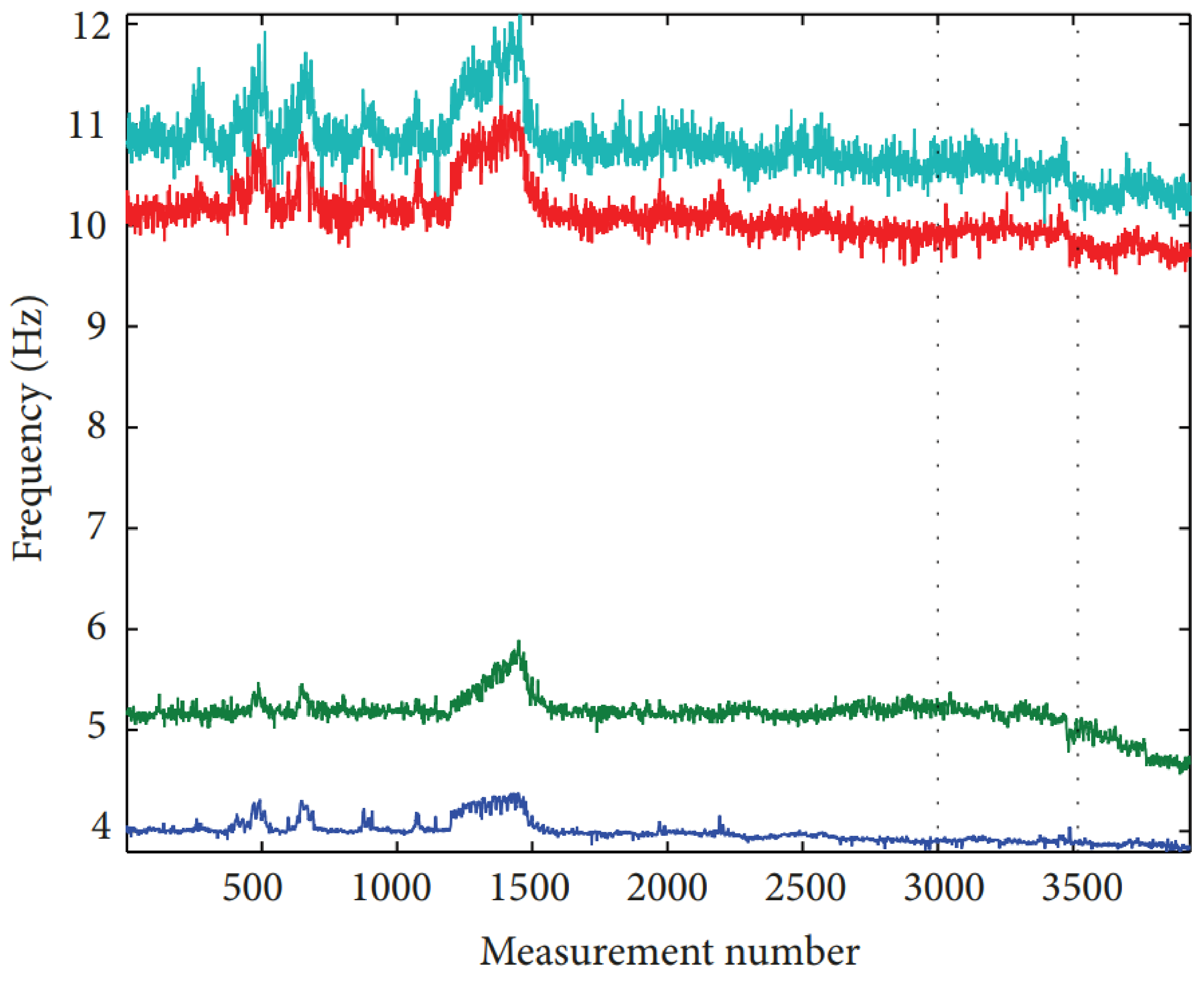
(a) position and direction of accelerometers

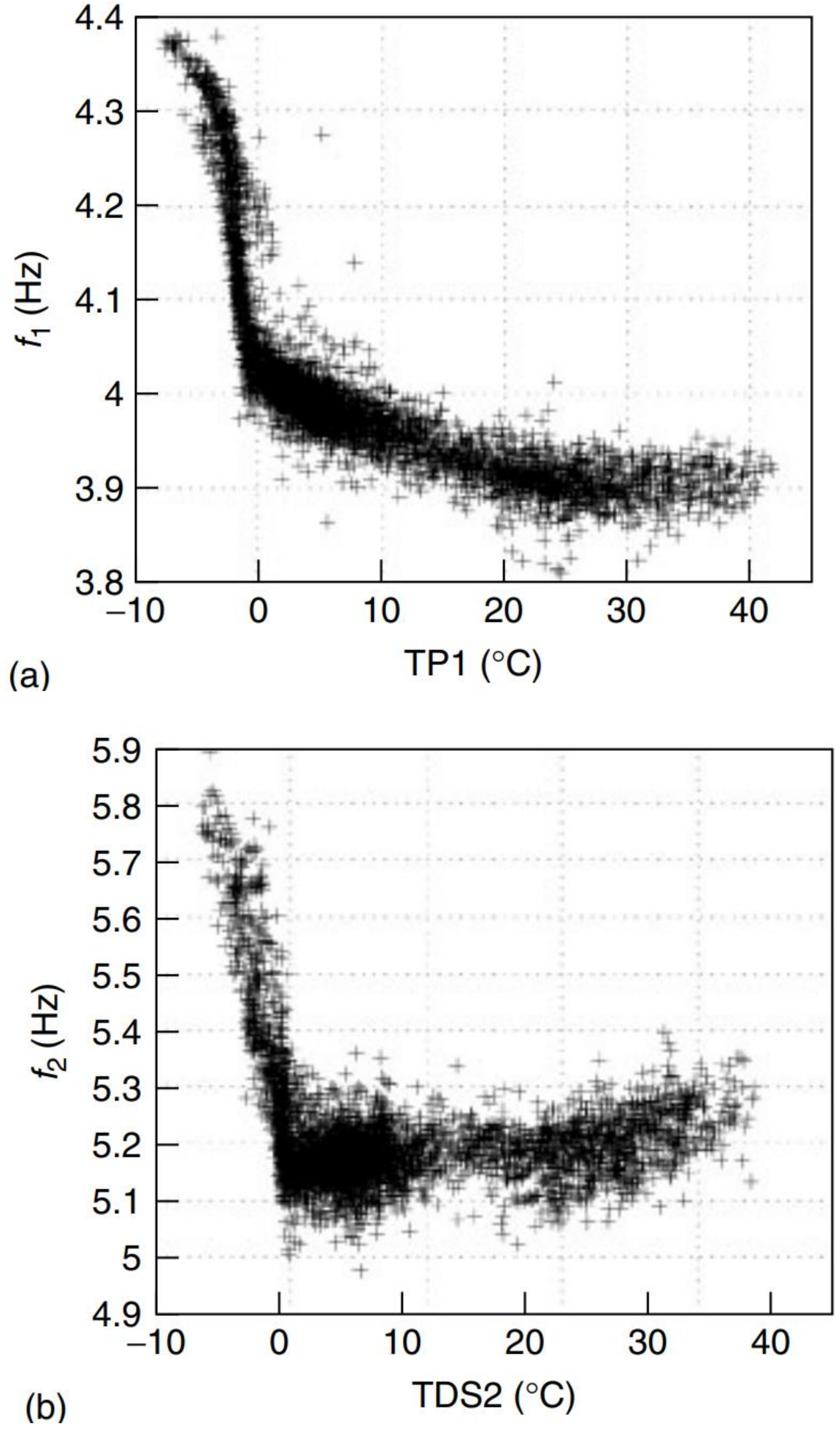


(b) position of temperature sensors

Motivación







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Reynders, E., & Roeck, G. D. (2009). Continuous Vibration Monitoring and Progressive Damage Testing on the Z 24 Bridge. *Encyclopedia of structural health monitoring*.

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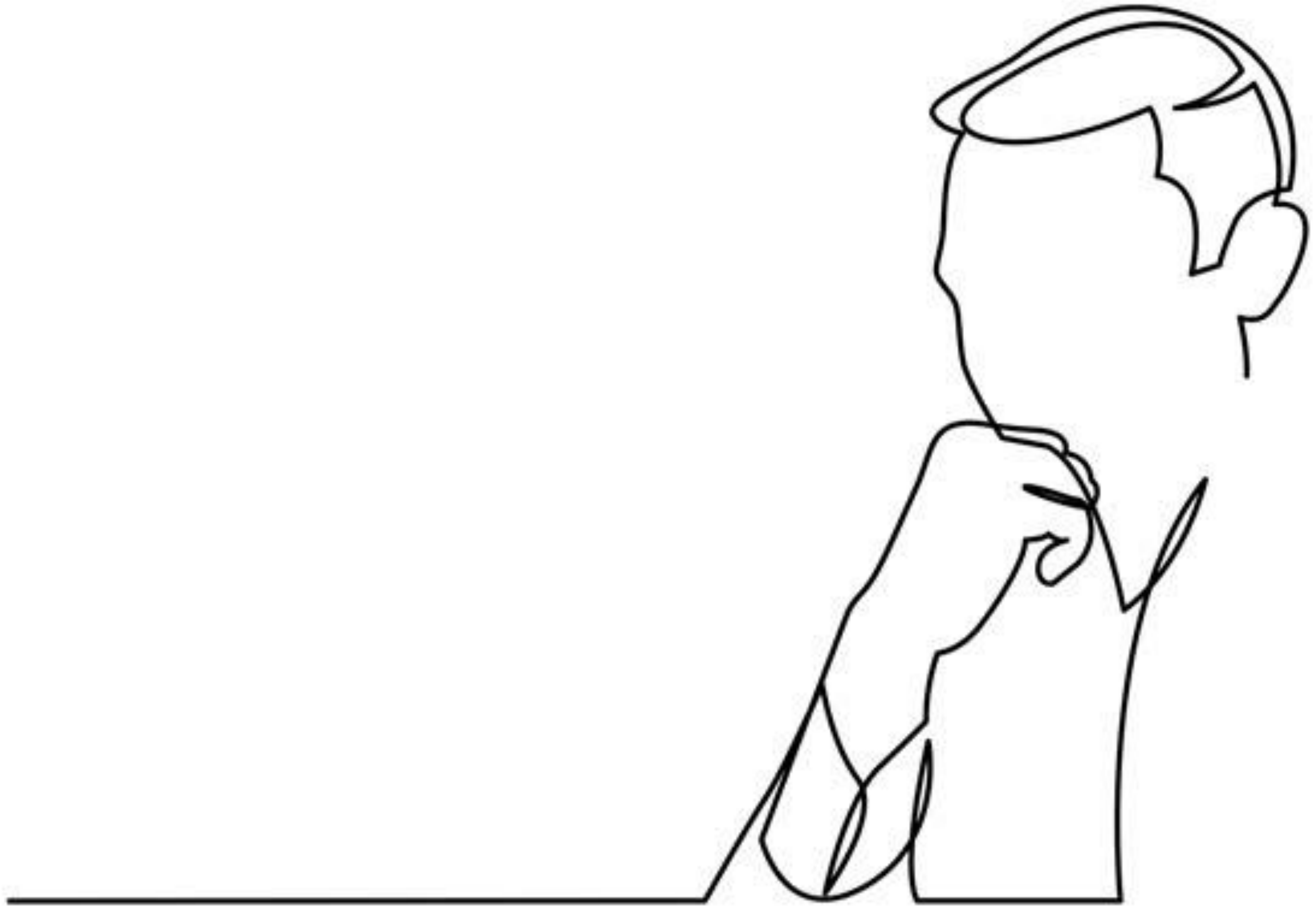
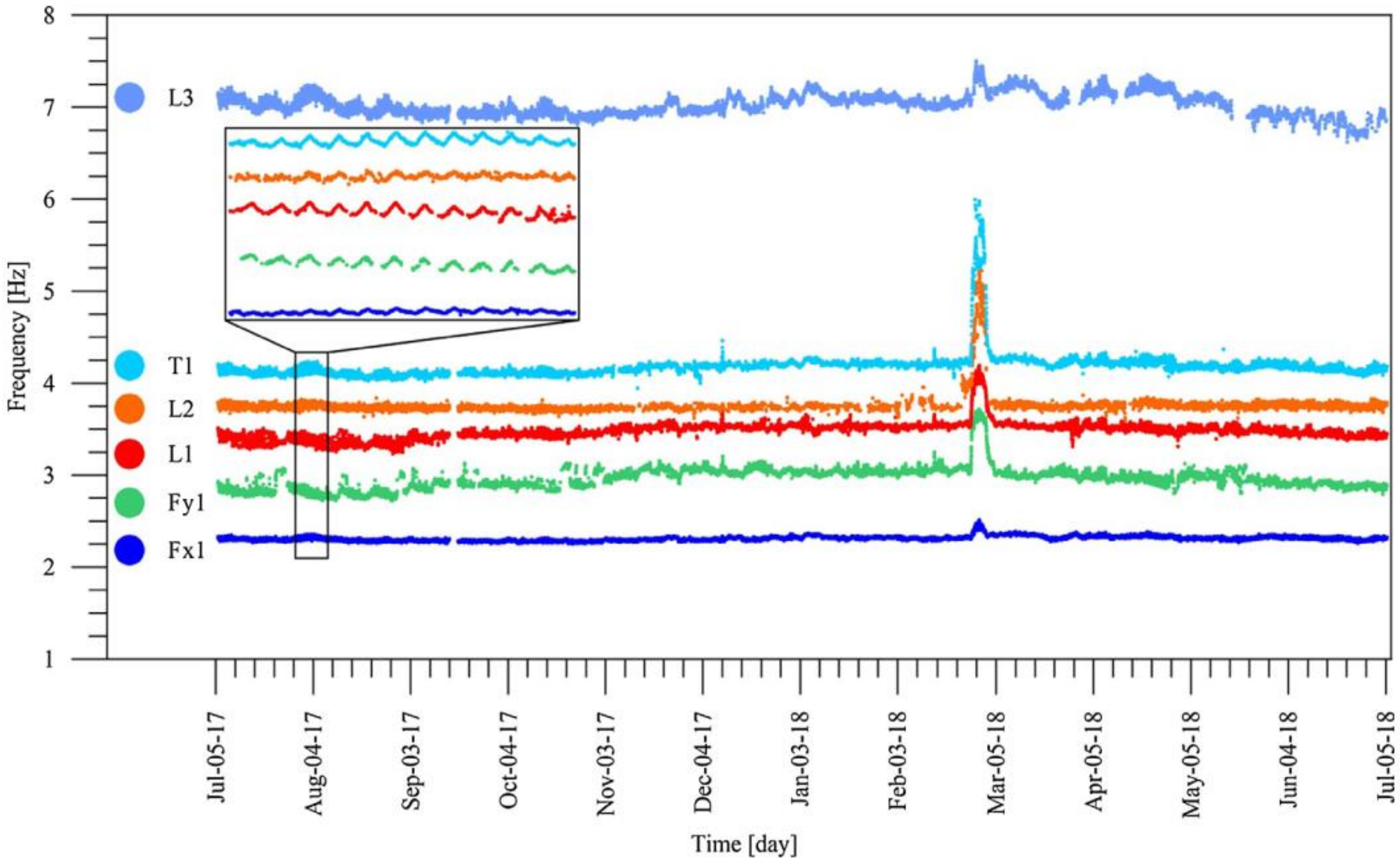


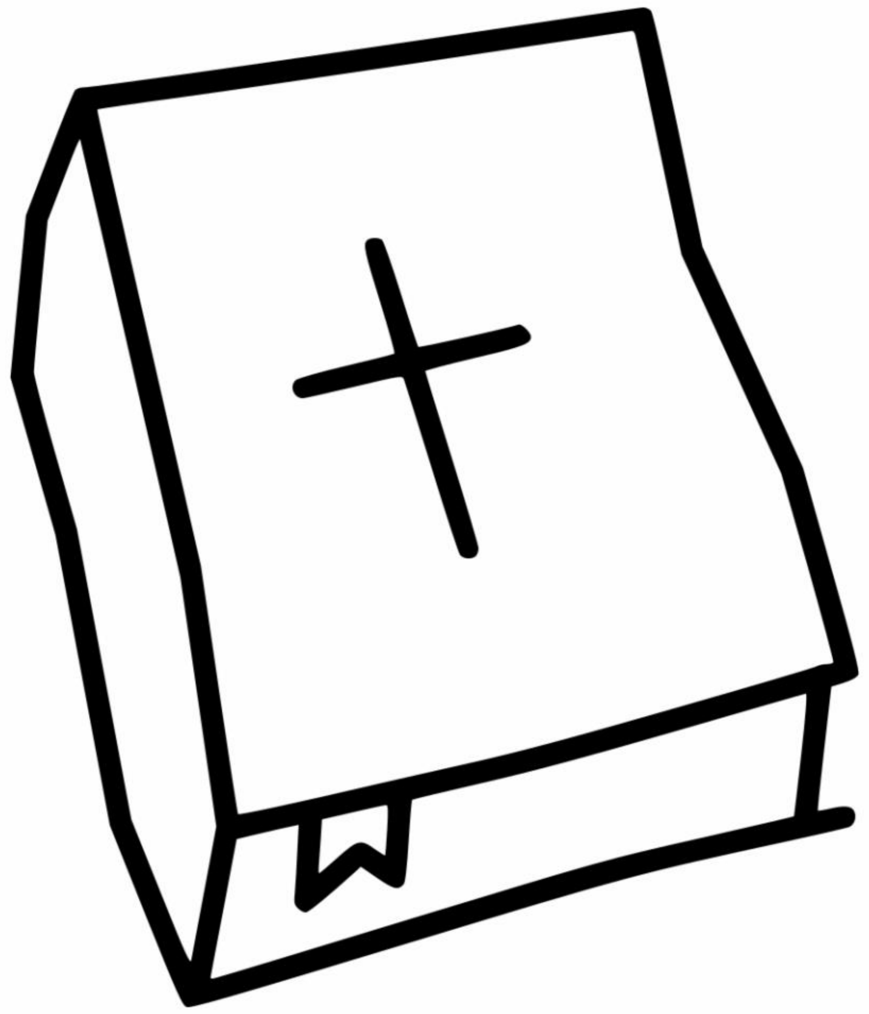
Masking effect of environmental factors



Environmental factors (e.g. temperature, humidity, traffic, wind, …,etc) mask the effects of early-stage damage. To have a proper damage sensitive features, we typically need to filter out environmental effects, that is, perform **pattern recognition**.

SHM como problema de statistical pattern recognition.

Axioms of SHM



Axiomas del SHM

- All materials have inherent flaws and defects
- The assessment of damage requires a comparison between two system states
- Identifying the existence and location of damage can be done in an unsupervised learning mode, but identifying the type of damage present and the damage severity can generally only be done in a supervised learning mode.
- Sensors cannot measure damage. Feature extraction through signal processing and statistical classification is necessary to convert sensor data into damage information.
- Without intelligent feature extraction, the more sensitive a measurement is to damage, the more sensitive it is to changing operational and environmental conditions.
- The length- and time-scales associated with damage initiation and evolution dictate the required properties of the SHM sensing system.
- There is a trade-off between the sensitivity to damage of an algorithm and its noise rejection capability.
- The size of damage that can be detected from changes in system dynamics is inversely proportional to the frequency range of the excitation.

Worden, K., Farrar, C. R., Manson, G., & Park, G. (2007). The fundamental axioms of structural health monitoring. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 463(2082), 1639-1664.

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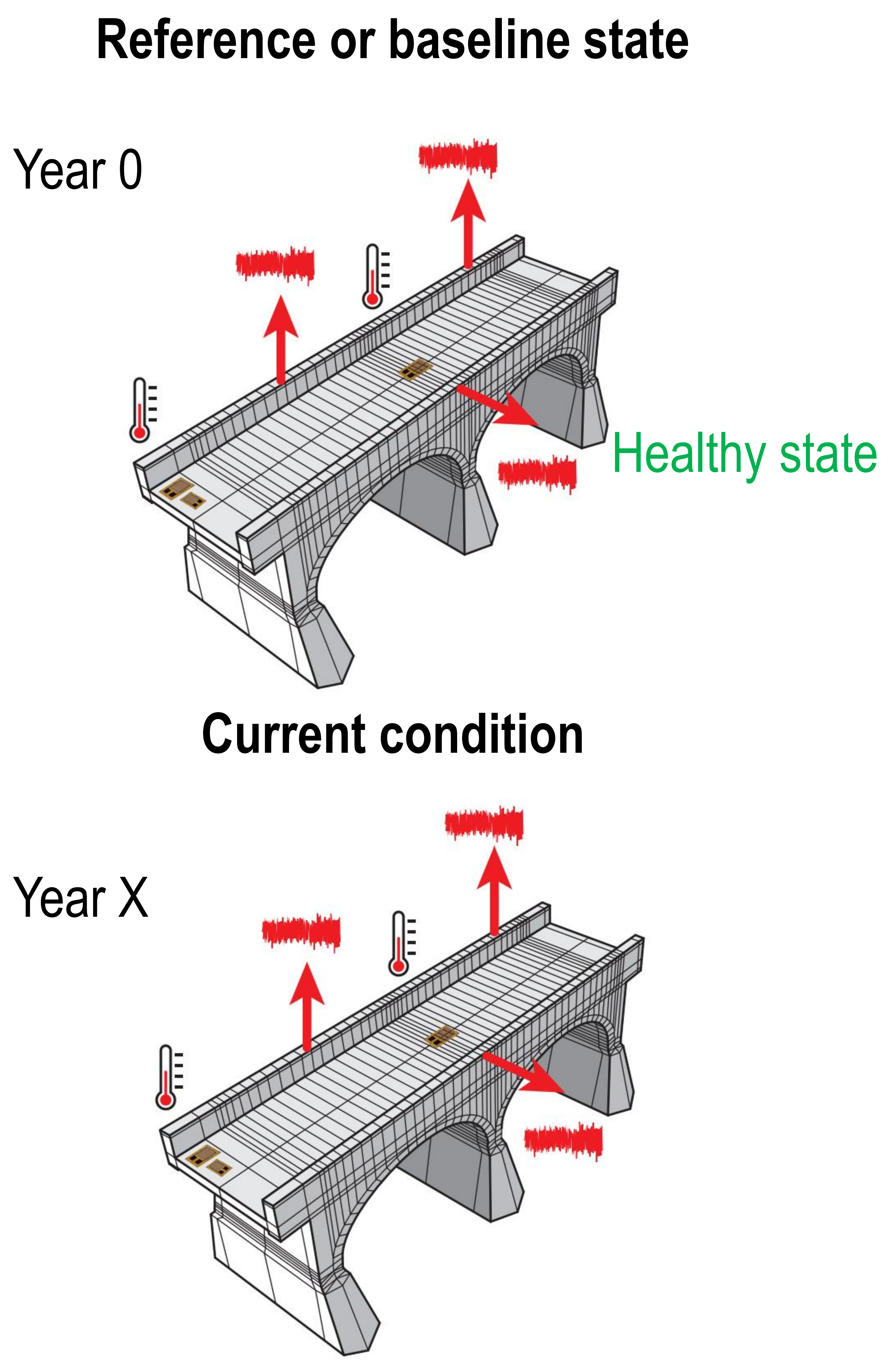

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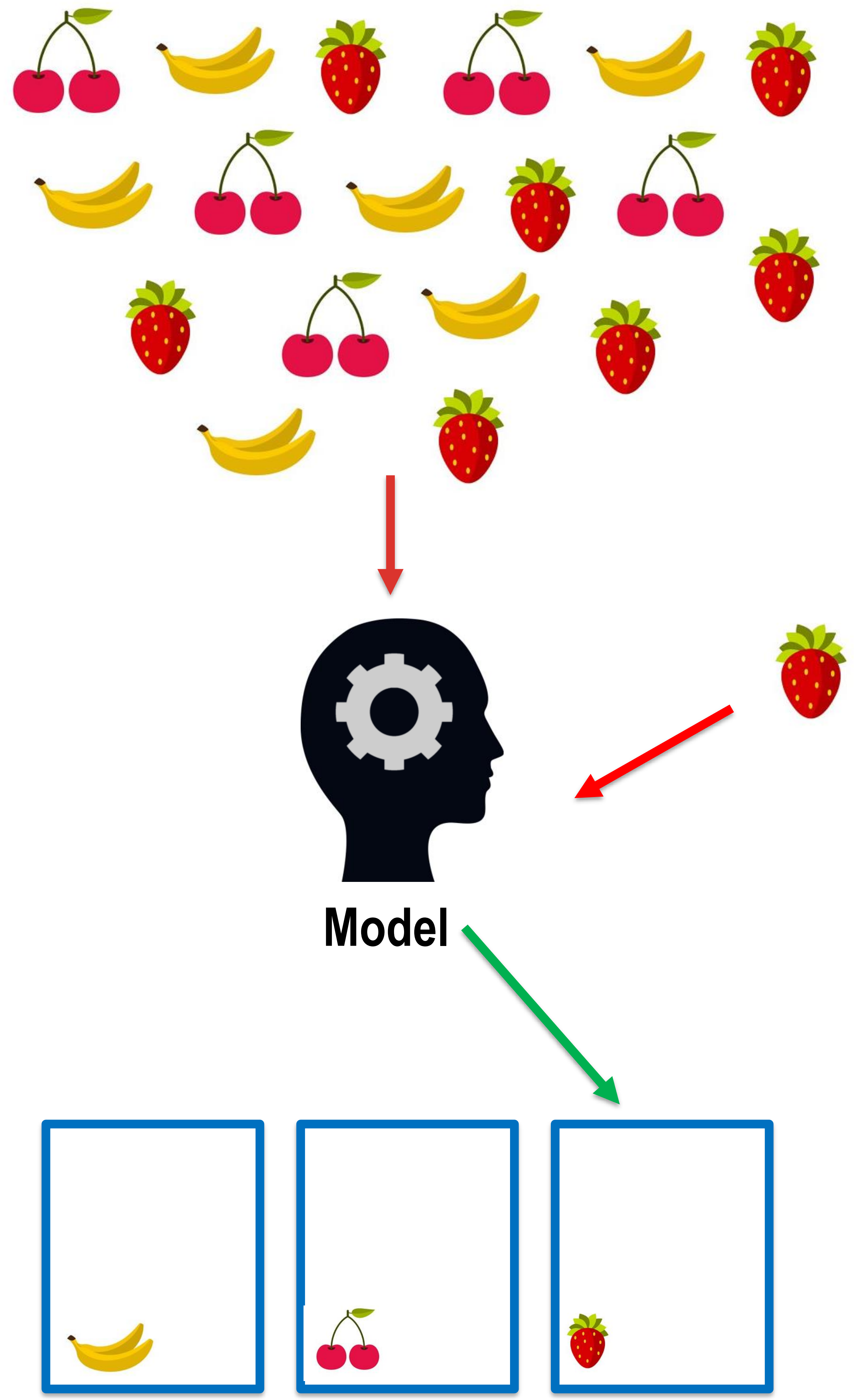

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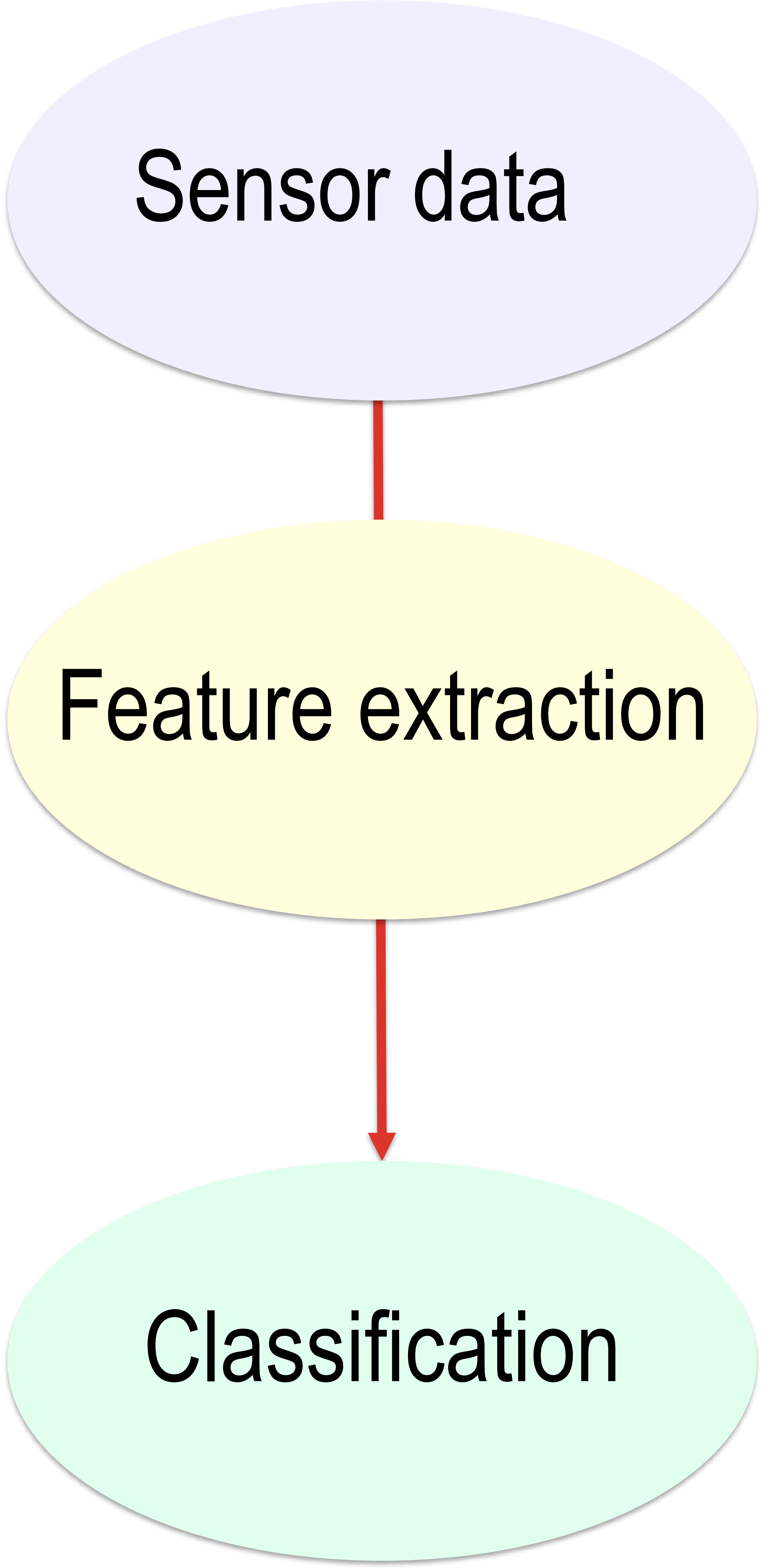

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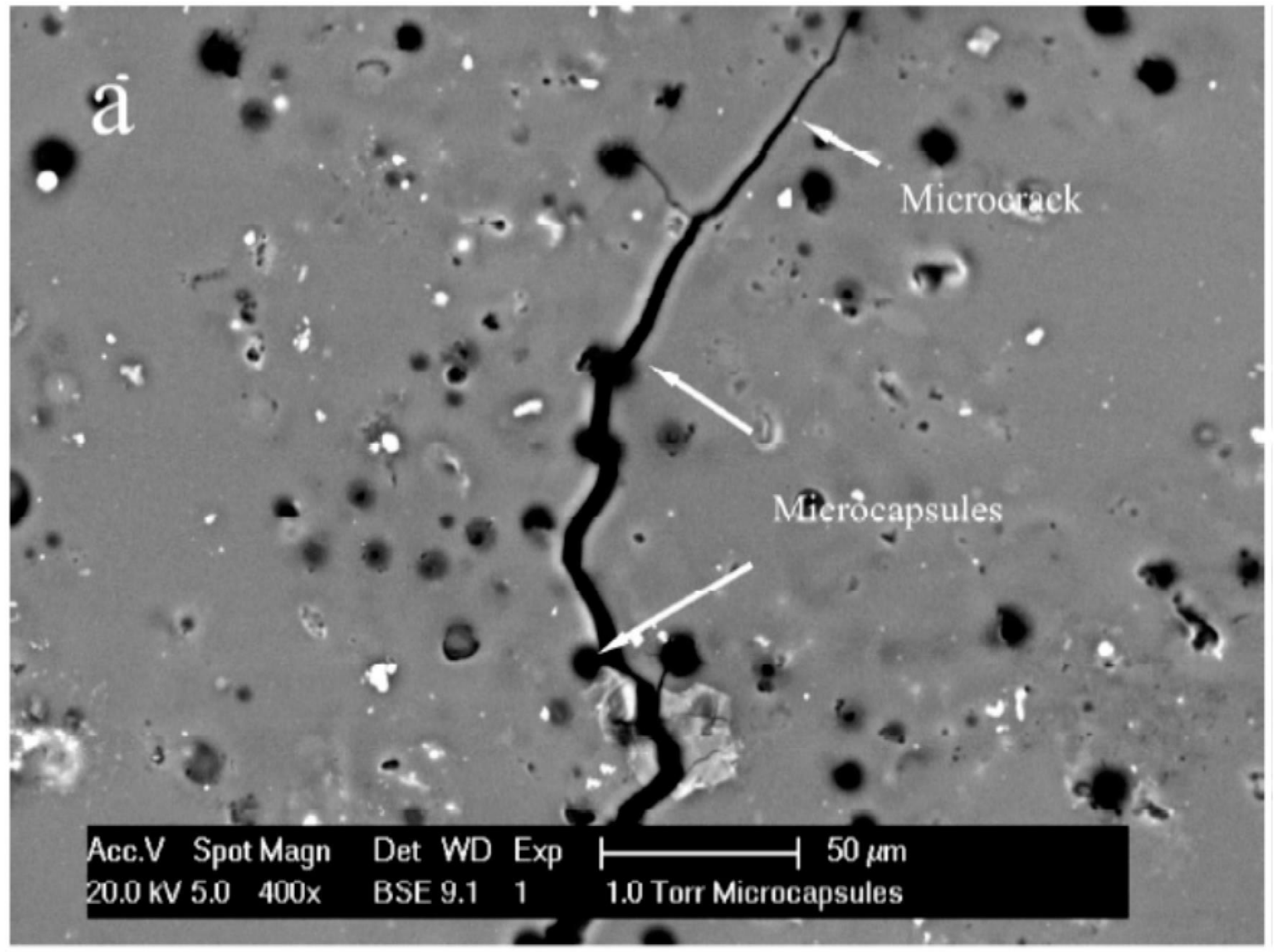

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Signal Processing (SP) – A fundamental challenge in SHM…

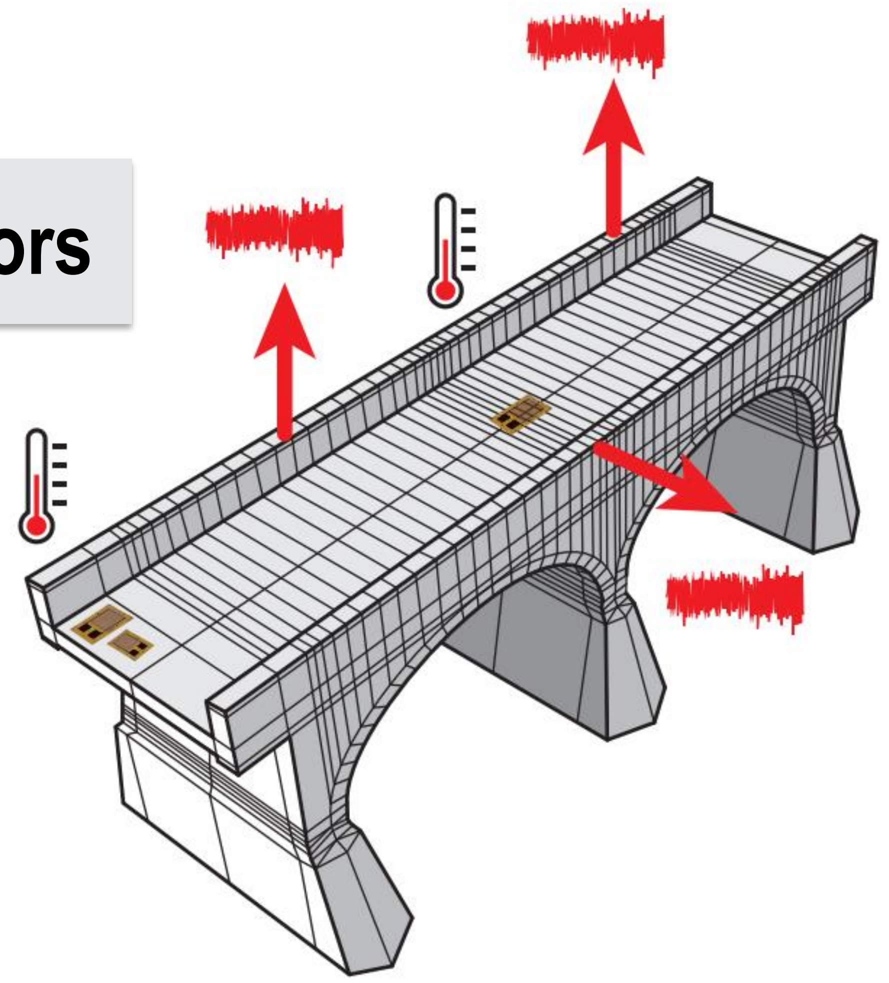
- **Model-Based SP**
  Compara las respuestas físicas con las respuestas del modelo (requiere un modelo o representación precisos)

- **Data-Based SP**
  Compara datos temporal y espacialmente, a menudo utilizando algoritmos de aprendizaje automático (redes neuronales, máquinas de vectores de soporte, etc.)

Introducción al paradigma del mantenimiento de la salud estructural

Monitored physical phenomenon

Sensors

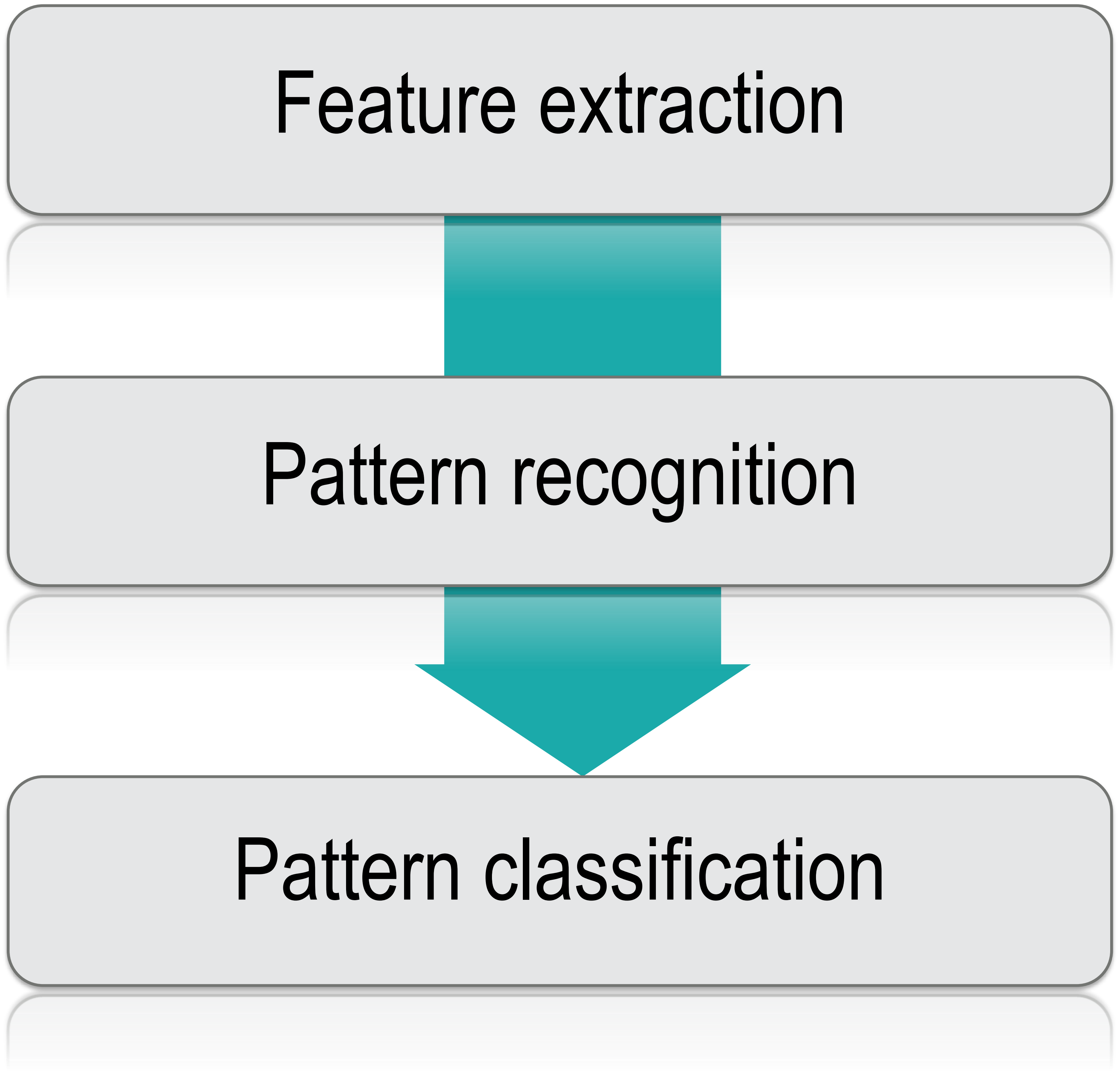


Data transmission (*wired/wireless*) → **DAQ** →

- Signal Pre-processing
- Feature extraction
- Pattern recognition — Damage detection
- Pattern classification — Damage identification
- Prognosis
- Decision

Detección de daños a través del paradigma del SHM como un problema de statistical pattern recognition.

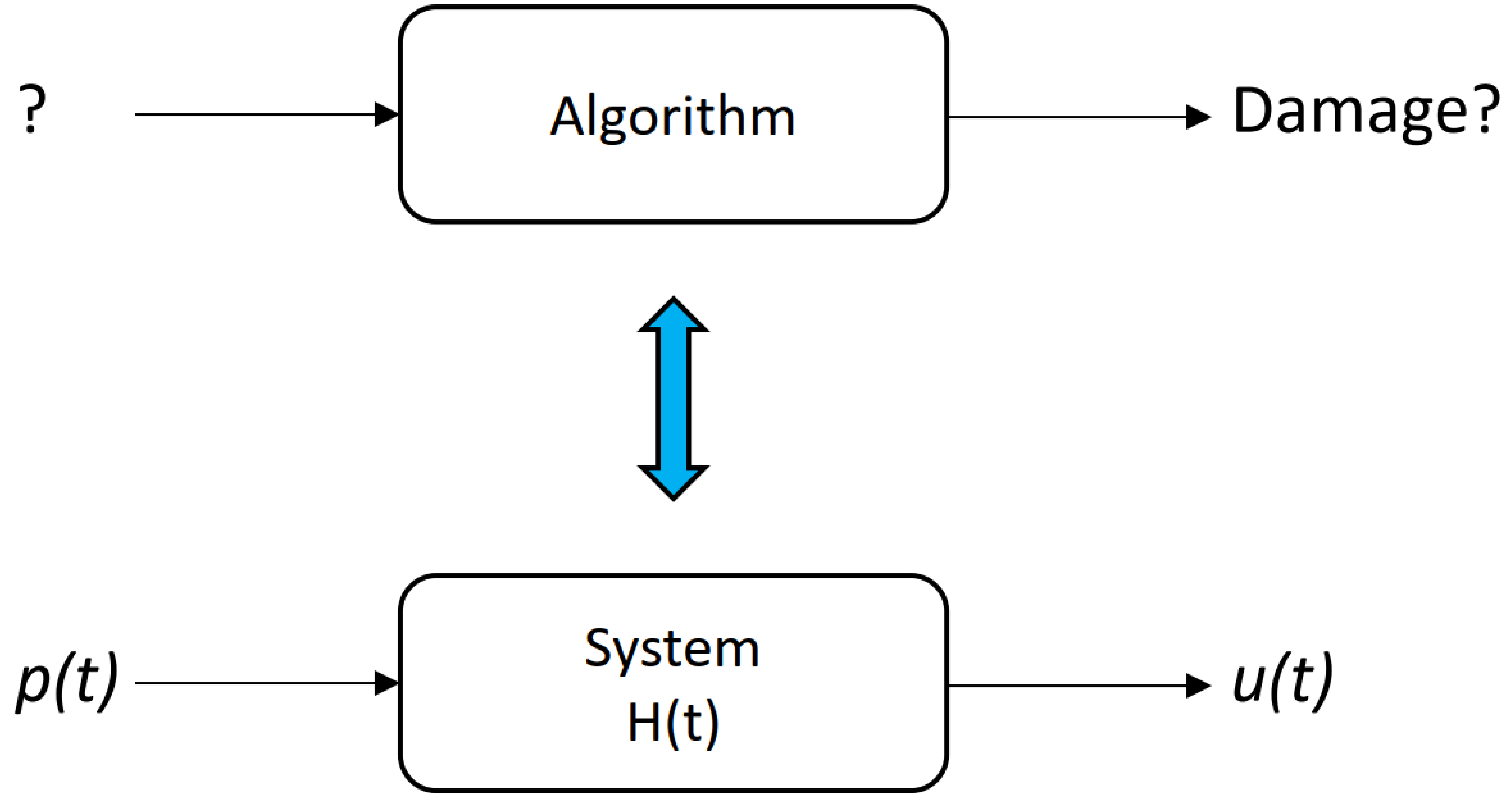
Detección de daños a través del paradigma del SHM como un problema de statistical pattern recognition



Feature extraction

Feature extraction

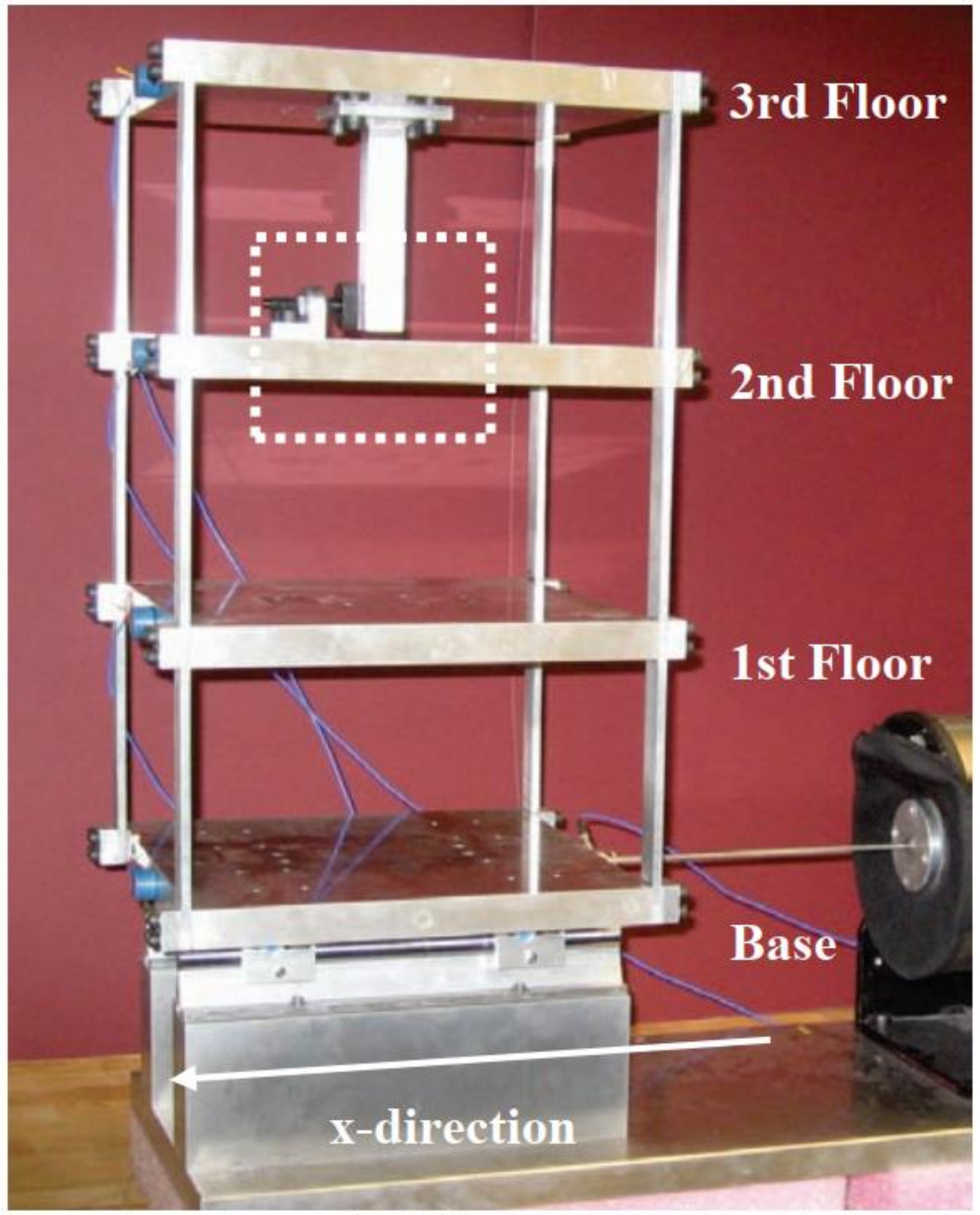
En SHM, una característica de la señal que podemos estudiar para identificar daños (damage-sensitive feature)



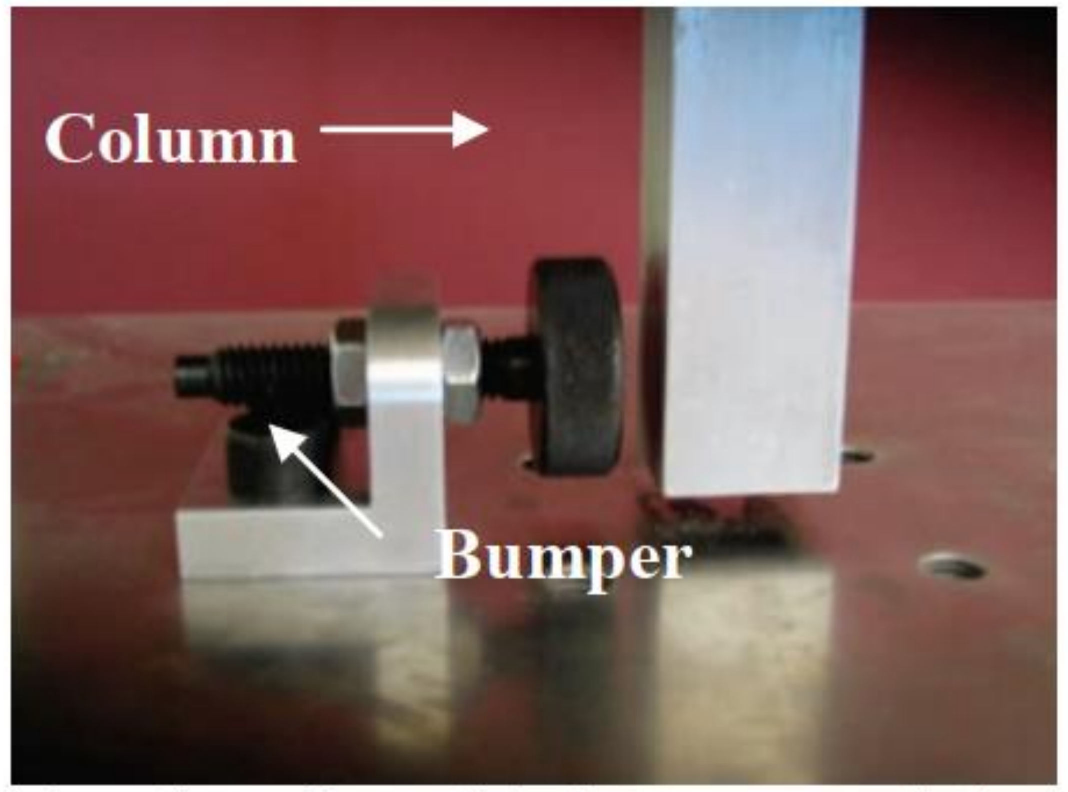
Feature extraction

Las features pueden ser muy variadas:

- Probabilistic-based
- Time domain-based
- Frequency domain-based
- Time-frequency domain-based



(a) Three-story building structure and shaker



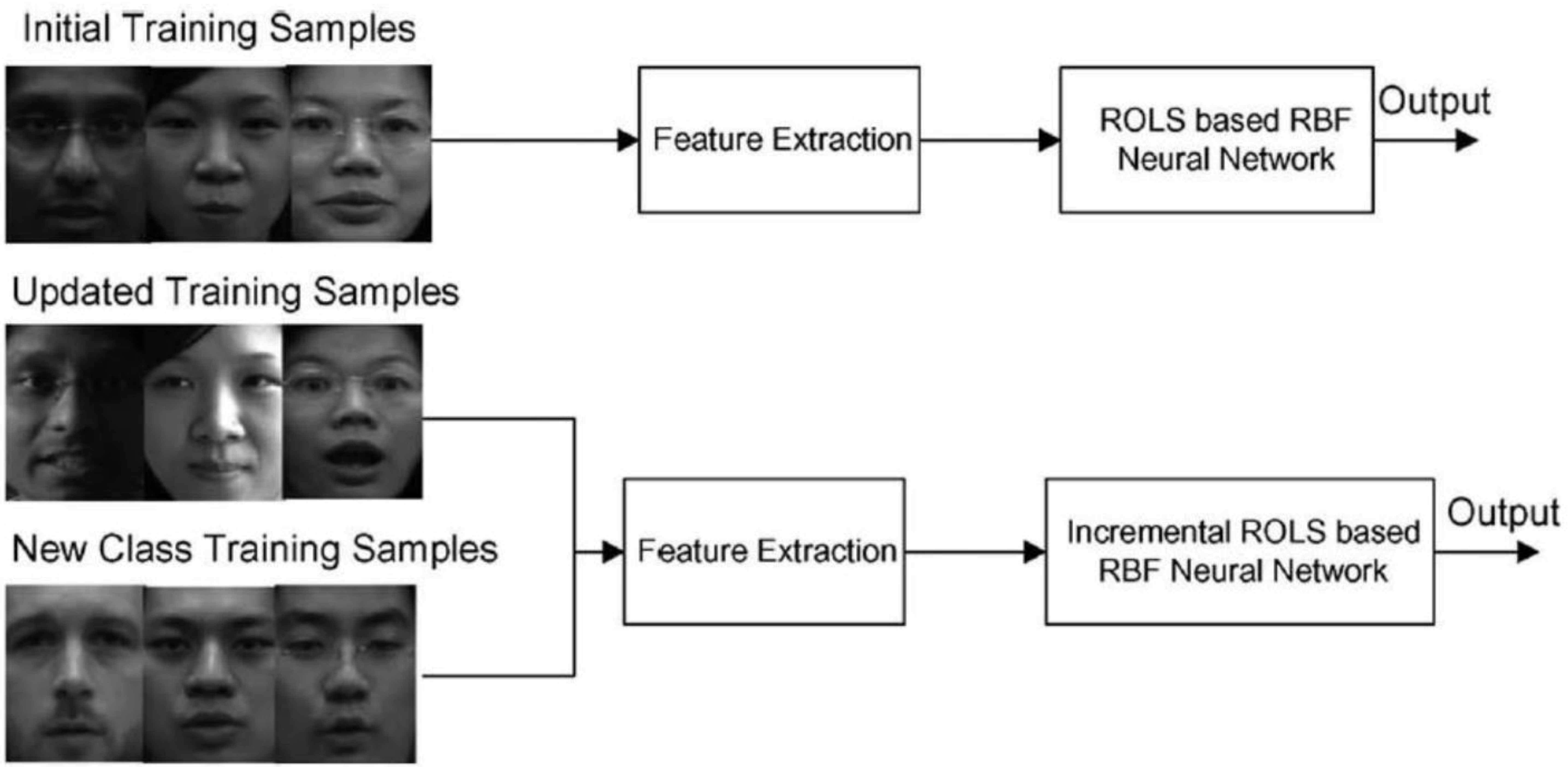
(b) The adjustable bumper and the suspended column

Figueiredo, E., Park, G., Figueiras, J., Farrar, C., & Worden, K. (2009). *Structural health monitoring algorithm comparisons using standard data sets* (No. LA-14393). Los Alamos National Lab.(LANL), Los Alamos, NM (United States).

Pattern Recognition

Pattern recognition

How face recognition works?

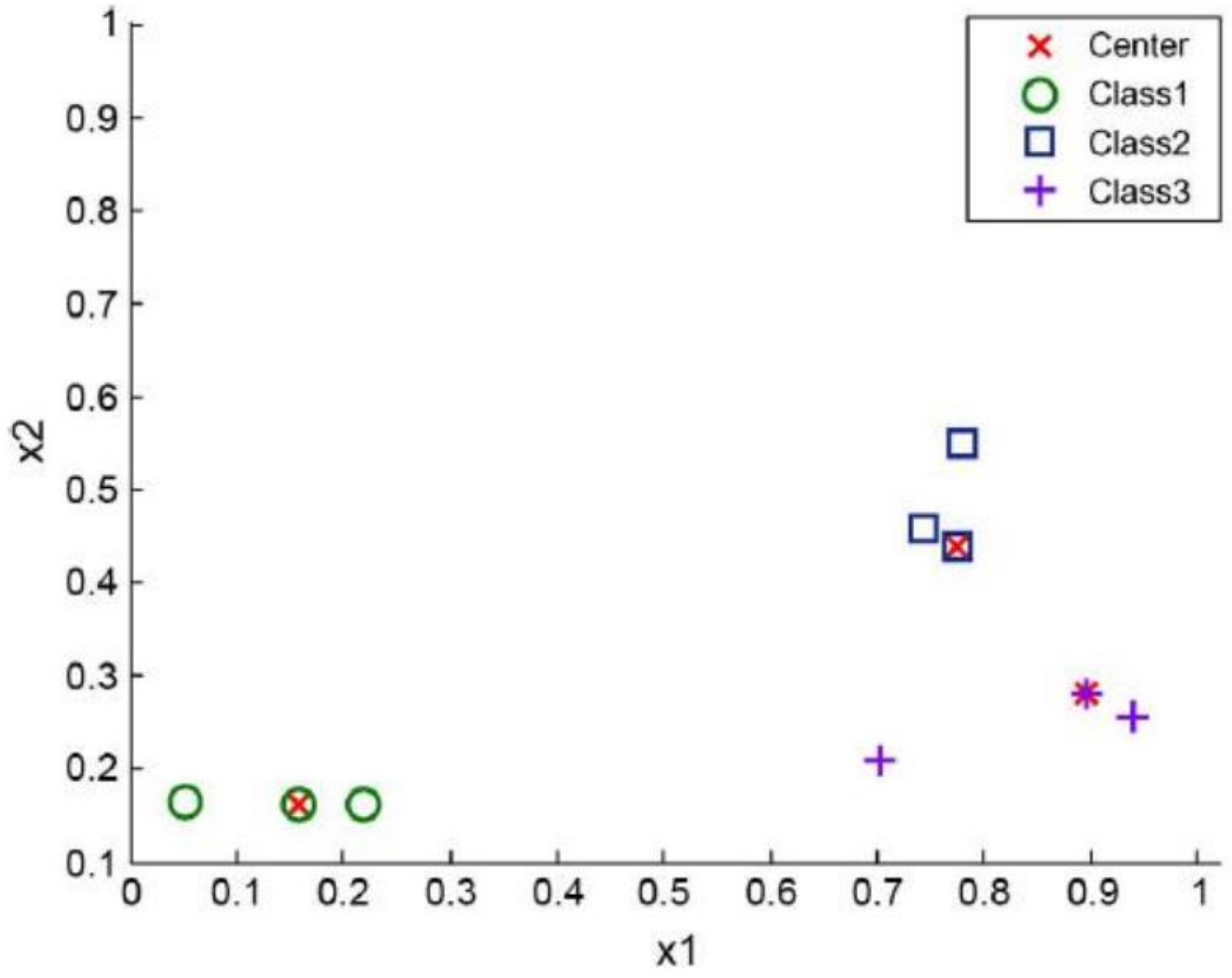


Wong, Y. W., Seng, K. P., & Ang, L. M. (2011). Radial basis function neural network with incremental learning for face recognition. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, *41*(4), 940-949.

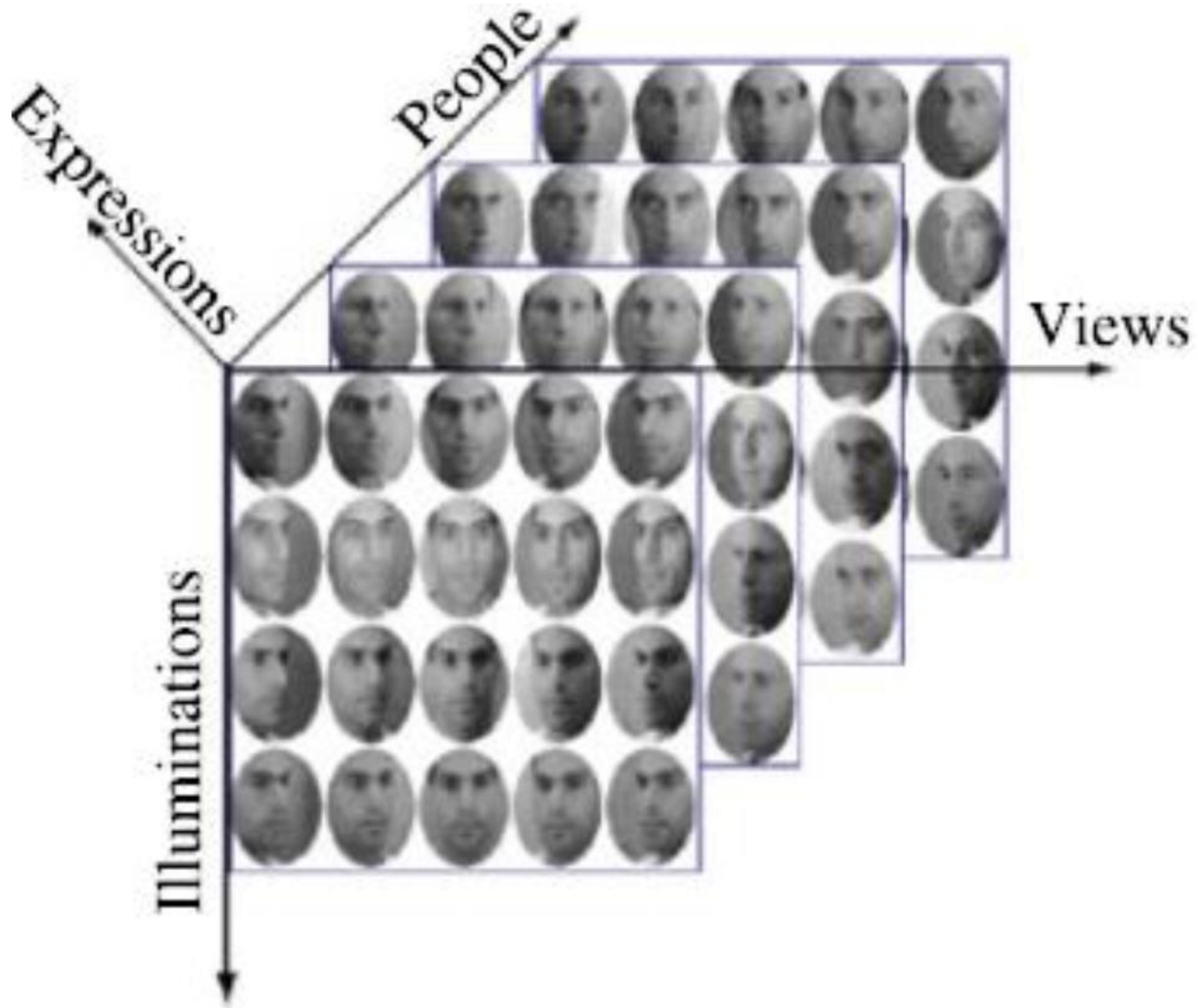
Pattern recognition

Shakhnarovich, G., & Moghaddam, B., *Face Recognition in Subspaces*, Handbook of Face Recognition, Springer London, Chapt. 2, 2011.

Eigenfaces



Tensorfaces



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Pattern recognition

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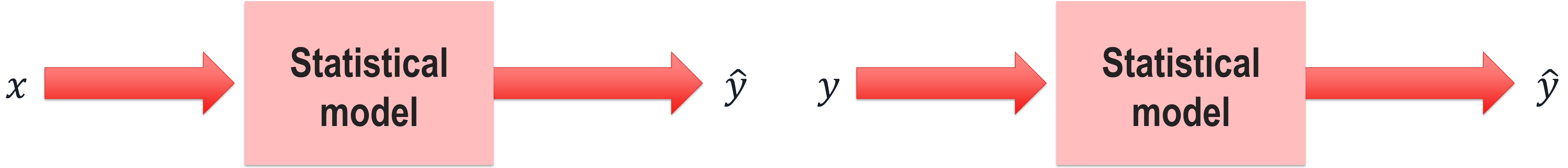
Data normalization: Taller Parte 1

Pattern recognition

Data normalization: Eliminación de efectos ambientales

Input-Output models
- Multiple Linear Regression (MLR)

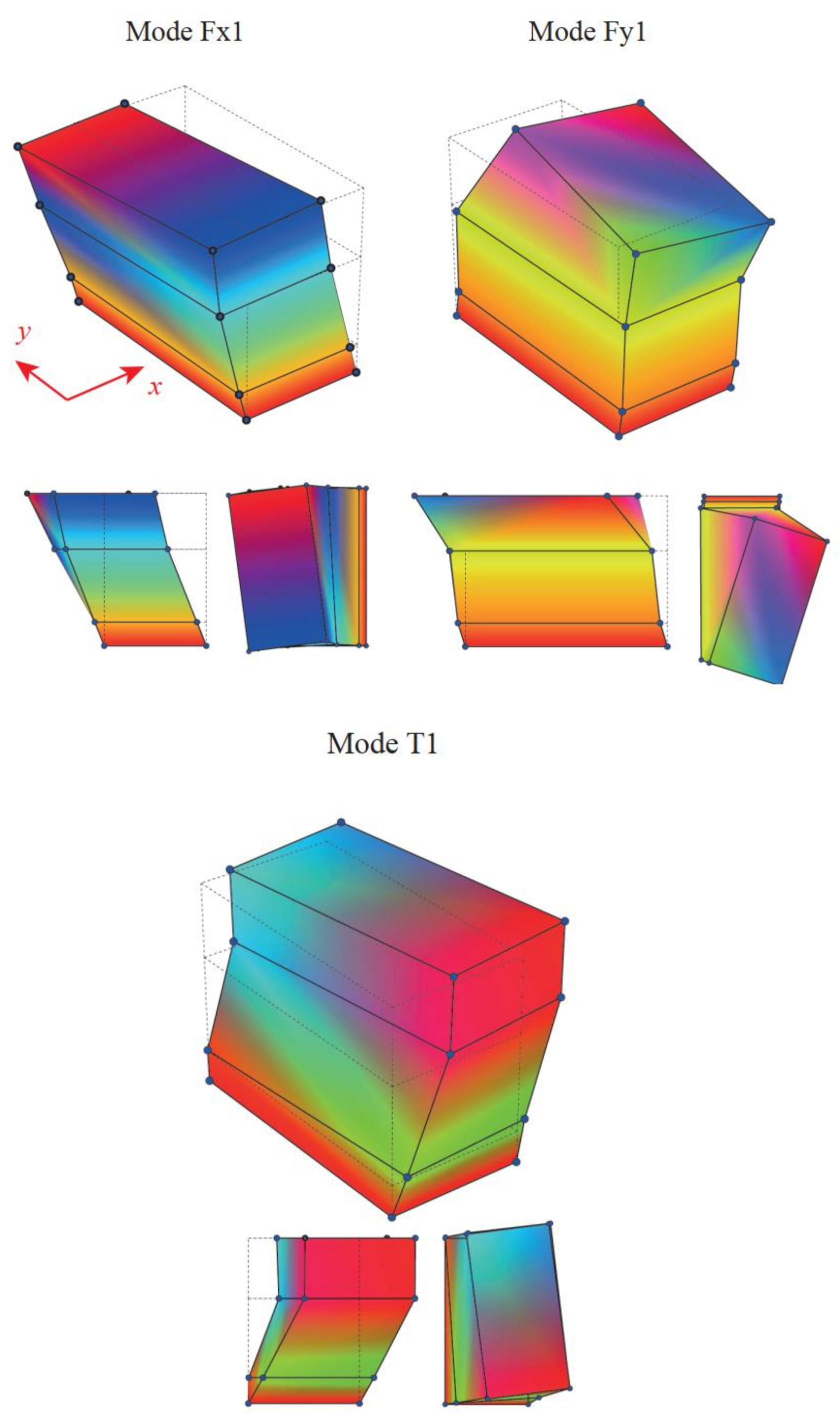
Output only models
- Principal Components Analysis (PCA)

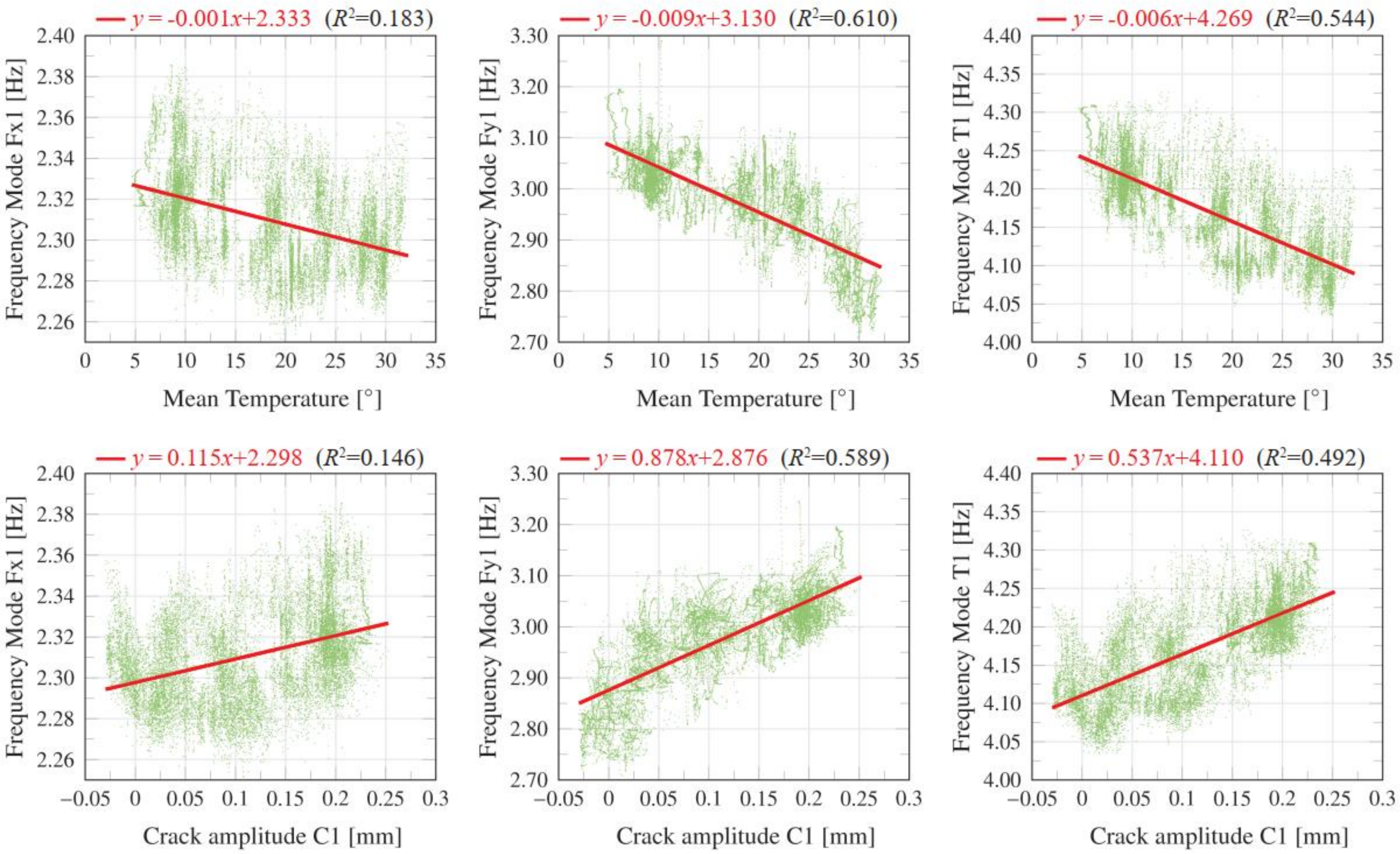


Data normalization: Taller Parte 1

Input-Output --- MLR

Multilinear Regression - Motivation





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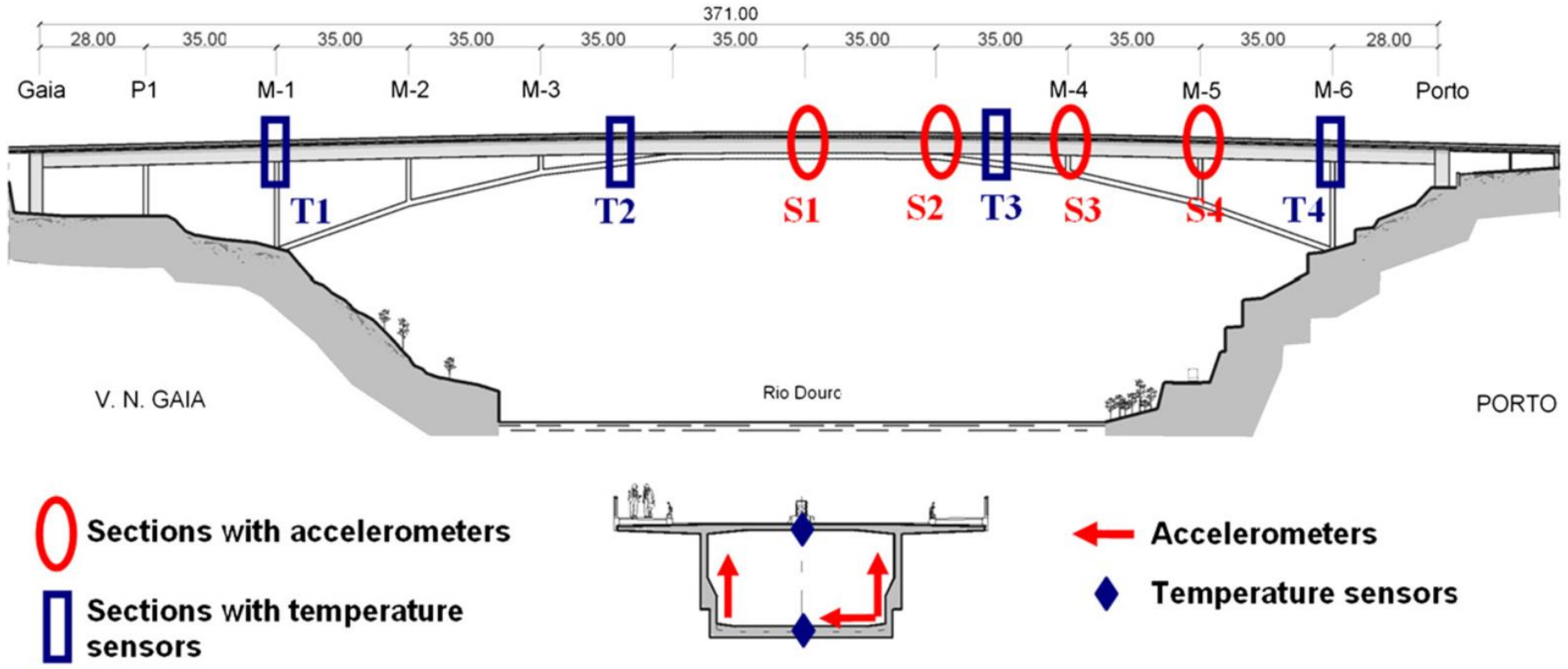
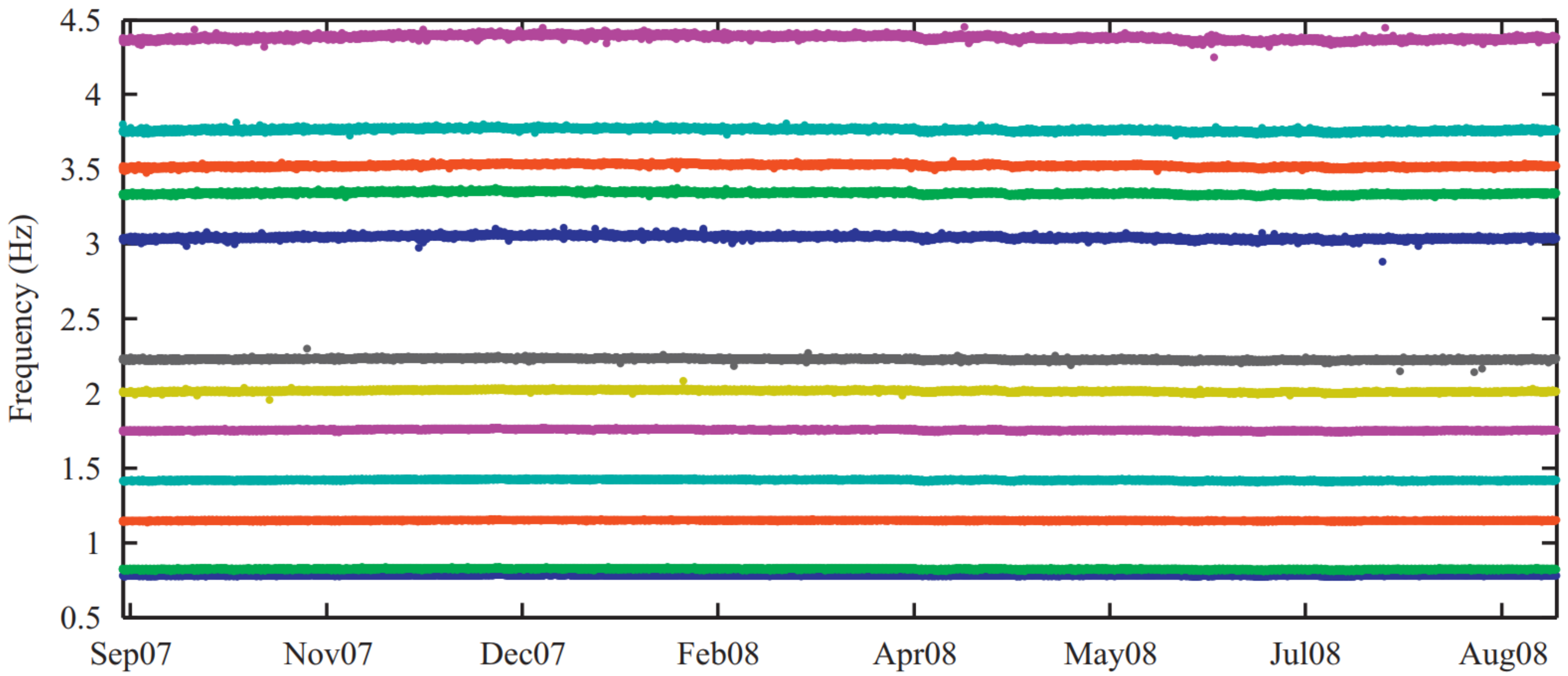
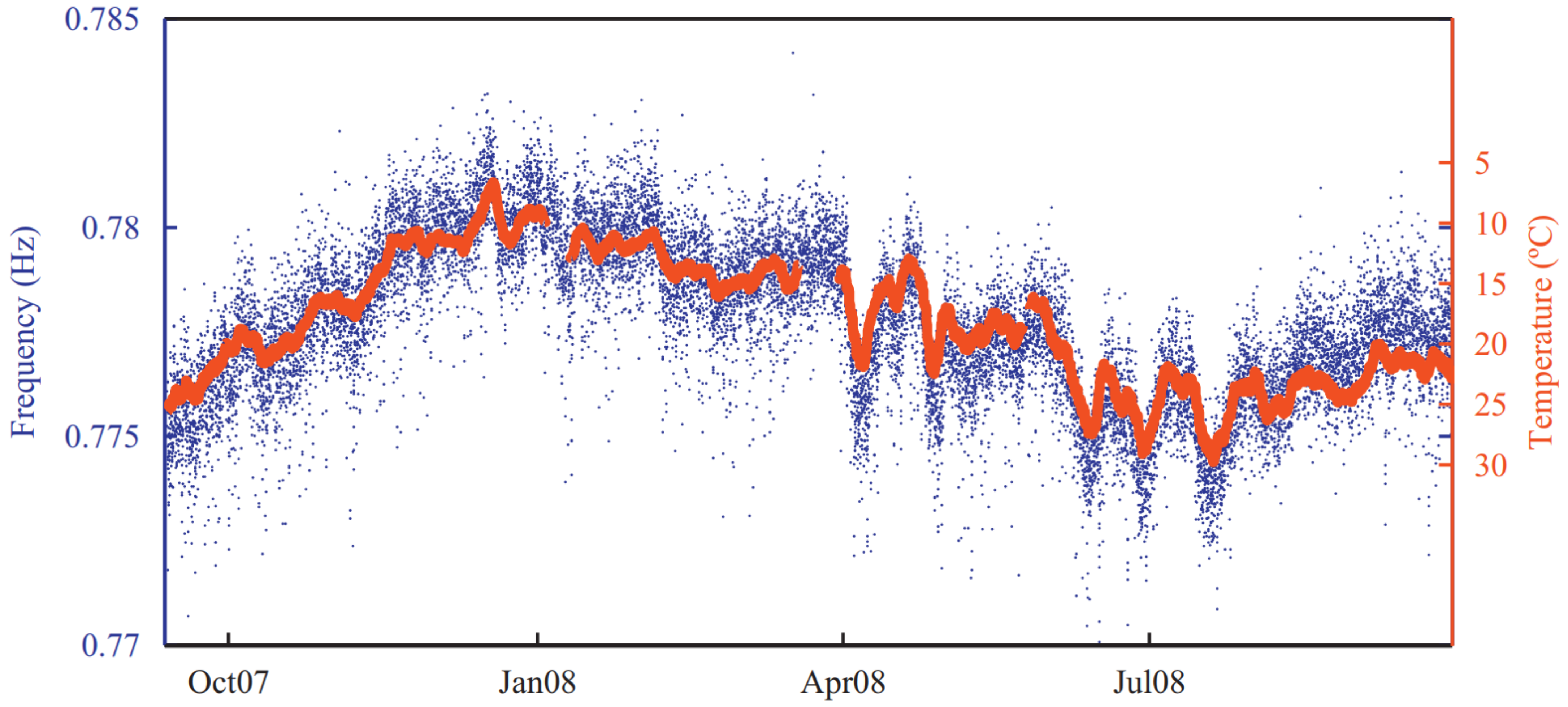
Data normalization: Taller Parte 1

MLR







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Magalhães, F., Cunha, A., & Caetano, E. (2012). Vibration based structural health monitoring of an arch bridge: from automated OMA to damage detection. *Mechanical Systems and Signal Processing*, *28*, 212-228.

Data normalization: Taller Parte 1

We consider the problem of regression when study different variables that depend on more than one explanatory or independent variables. This model generalizes the simple linear regression in two ways. It allows the mean function $E(y)$ to depend on more than one explanatory variable and to have shapes other than straight lines.

The linear model:

Let y denote the dependent variable that is linearly related to k independent variables $x_1, x_2, \dots, x_k$ through parameters $\beta_1, .., \beta_k$

$$y = x_1\beta_1 + \cdots + x_k\beta_k + \varepsilon$$

- $\beta_1, .., \beta_k$ → Regression coefficients
- ε random error component reflecting the difference between the observed and fitted variables (joint effect of those variables not included in the model, random factors which cannot be accounted in the model, etc…)

Data normalization: Taller Parte 1

Note that the j^{th} regression coefficient β_j represents the expected change in y per unit change in x_j. Assuming $E(\varepsilon) = 0$

$$\beta_j = \frac{\partial E(y)}{\partial x_j}$$

In general, we can write the MLR model in matrix notation as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$

Assumptions in the MLR:

I. $E(\boldsymbol{\varepsilon}) = \mathbf{0}$
II. $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \sigma^2\mathbf{I}$
III. $\text{Rank}(\mathbf{X}) = k$
IV. $\mathbf{X}$ is a non-stochastic matrix
V. $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$

Data normalization: Taller Parte 1

MLR

PRINCIPLE OF ORDINARY LEAST SQUARES (OLS)

Let $\mathbf{B}$ be the set of all possible vectors $\boldsymbol{\beta}$. The objective is to find a vector $\boldsymbol{\beta}$ that minimizes the sum of squared deviations of ε_i's

$$S(\boldsymbol{\beta}) = \sum_{i=1}^{n} \varepsilon_i^2 = \boldsymbol{\varepsilon\varepsilon}' = (\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}) = \mathbf{Y}'\mathbf{Y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{Y}$$

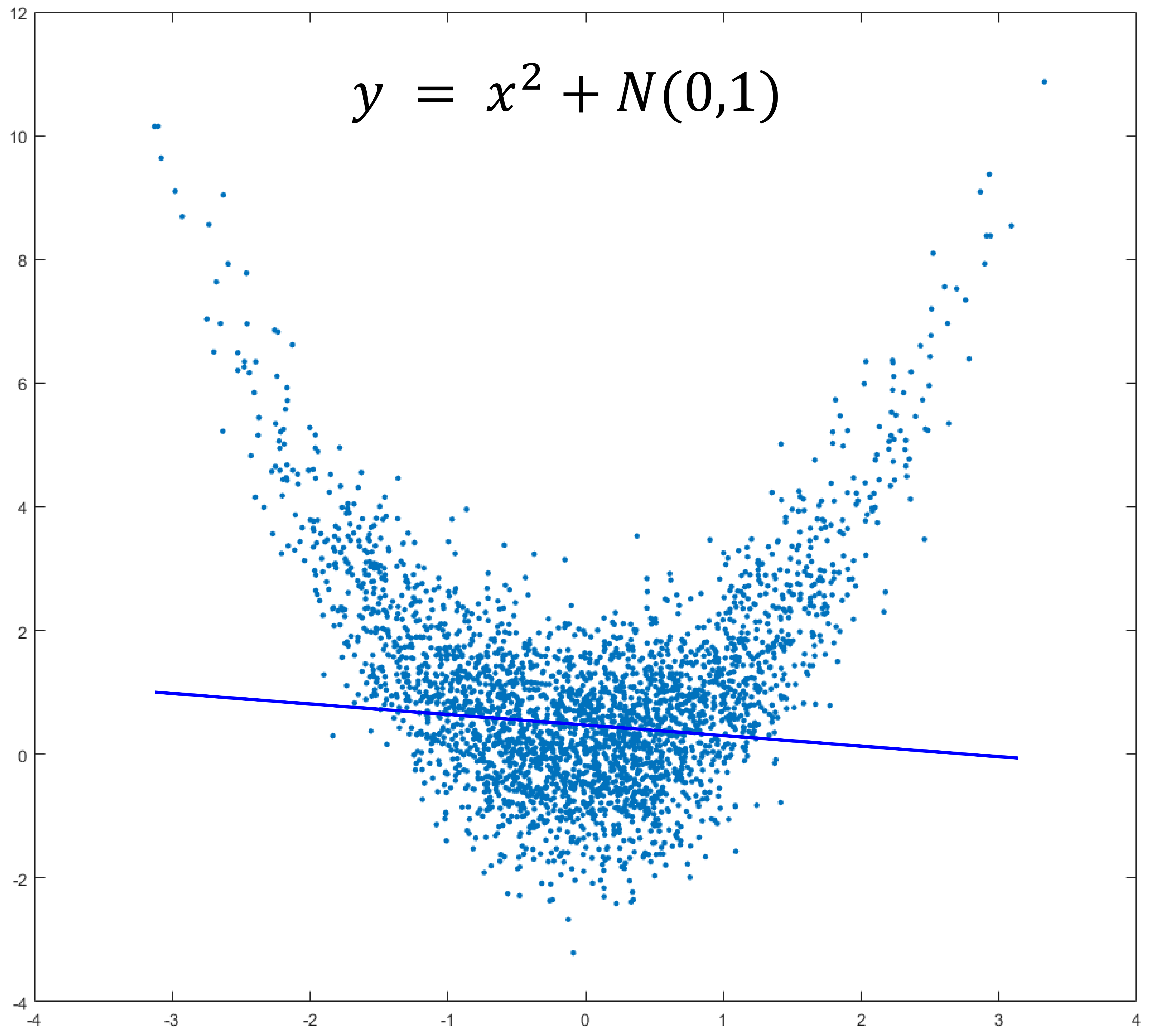
$$\frac{\partial S(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = 2\mathbf{X}'\mathbf{X}\boldsymbol{\beta} - 2\mathbf{X}'\mathbf{Y} = \mathbf{0} \rightarrow \mathbf{X}'\mathbf{X}\boldsymbol{\beta} = \mathbf{X}'\mathbf{Y} \rightarrow (\mathrm{Rank}(\mathbf{X}) = k) \quad \boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$$

FITTED VALUES

$$\hat{\mathbf{Y}} = \mathbf{X}[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}] = \mathbf{HY}$$

Data normalization: Taller Parte 1

MLR



$$\mathbf{Y} = [y_1, \dots, y_\mathrm{N}]^\mathrm{T}$$

$$\mathbf{X} = [x_1, \dots, x_\mathrm{N}]^\mathrm{T}$$

1st order:

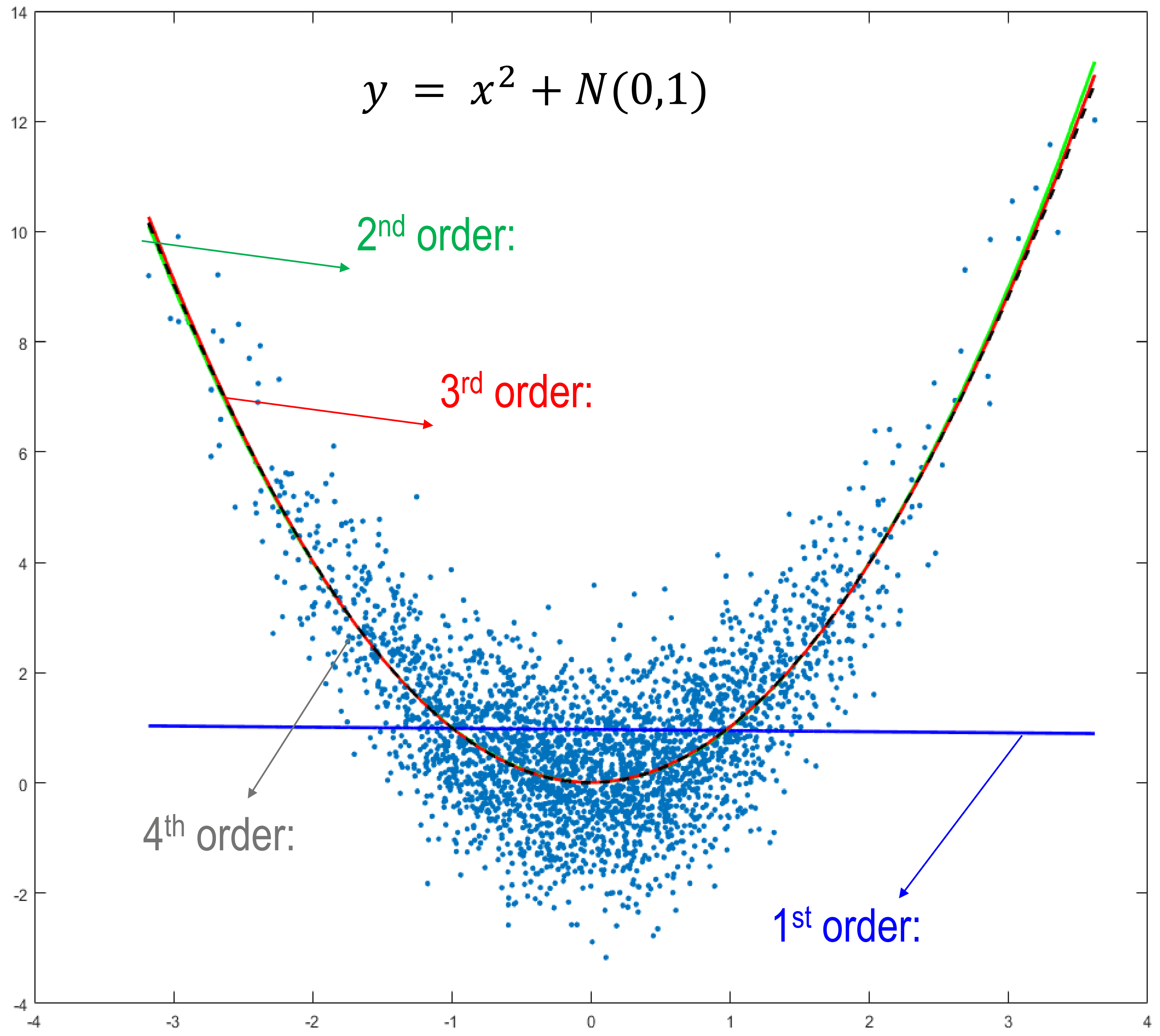
$$\mathbf{Y} = \boldsymbol{\beta}\mathbf{X}$$

Intercept

$$[y_1, \dots, y_\mathrm{N}]^\mathrm{T} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \dots & \dots \\ 1 & x_{\mathrm{N}-1} \\ 1 & x_\mathrm{N} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

$$\boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \longrightarrow \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 1.0272 \\ -0.0097 \end{bmatrix}$$

Data normalization: Taller Parte 1



2st order:

$$[y_1, \dots, y_N]^T = \begin{bmatrix} 1 & x_1 & (x_1)^2 \\ 1 & x_2 & (x_1)^2 \\ \dots & \dots & \dots \\ 1 & x_{N-1} & (x_{N-1})^2 \\ 1 & x_N & (x_N)^2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

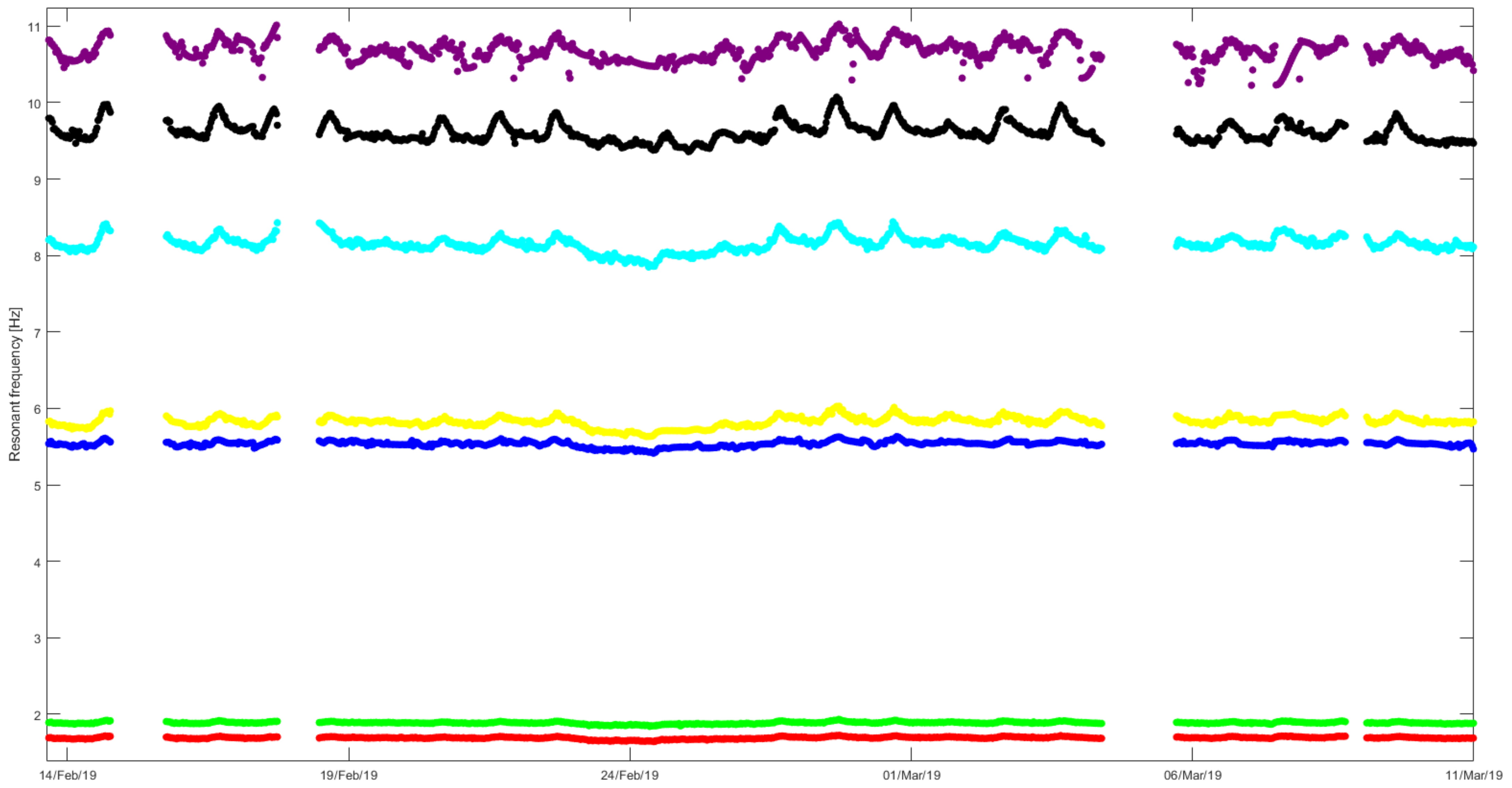
3rd order:

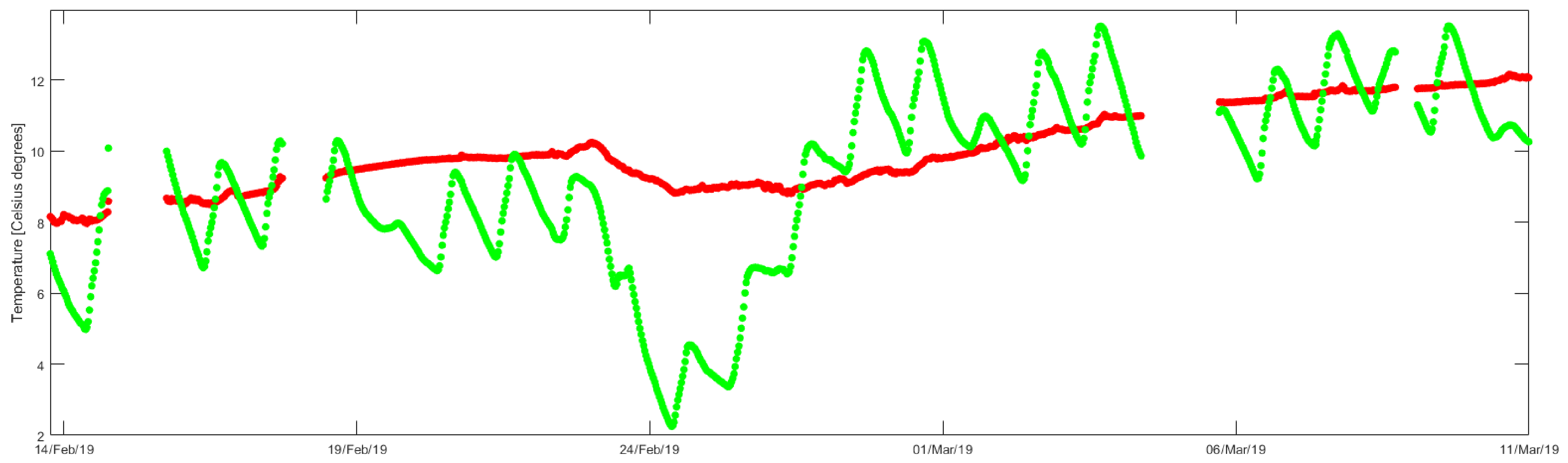
$$[y_1, \dots, y_N]^T = \begin{bmatrix} 1 & x_1 & (x_1)^2 & (x_1)^3 \\ 1 & x_2 & (x_1)^2 & (x_1)^3 \\ \dots & \dots & \dots & \dots \\ 1 & x_{N-1} & (x_{N-1})^2 & (x_{N-1})^3 \\ 1 & x_N & (x_N)^2 & (x_N)^3 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}$$

4th order:

$$[y_1, \dots, y_N]^T = \begin{bmatrix} 1 & x_1 & (x_1)^2 & (x_1)^3 & (x_1)^4 \\ 1 & x_2 & (x_1)^2 & (x_1)^3 & (x_1)^4 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & x_{N-1} & (x_{N-1})^2 & (x_{N-1})^3 & (x_{N-1})^4 \\ 1 & x_N & (x_N)^2 & (x_N)^3 & (x_N)^4 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{bmatrix}$$

Data normalization: Taller Parte 1





Data normalization: Taller Parte 1

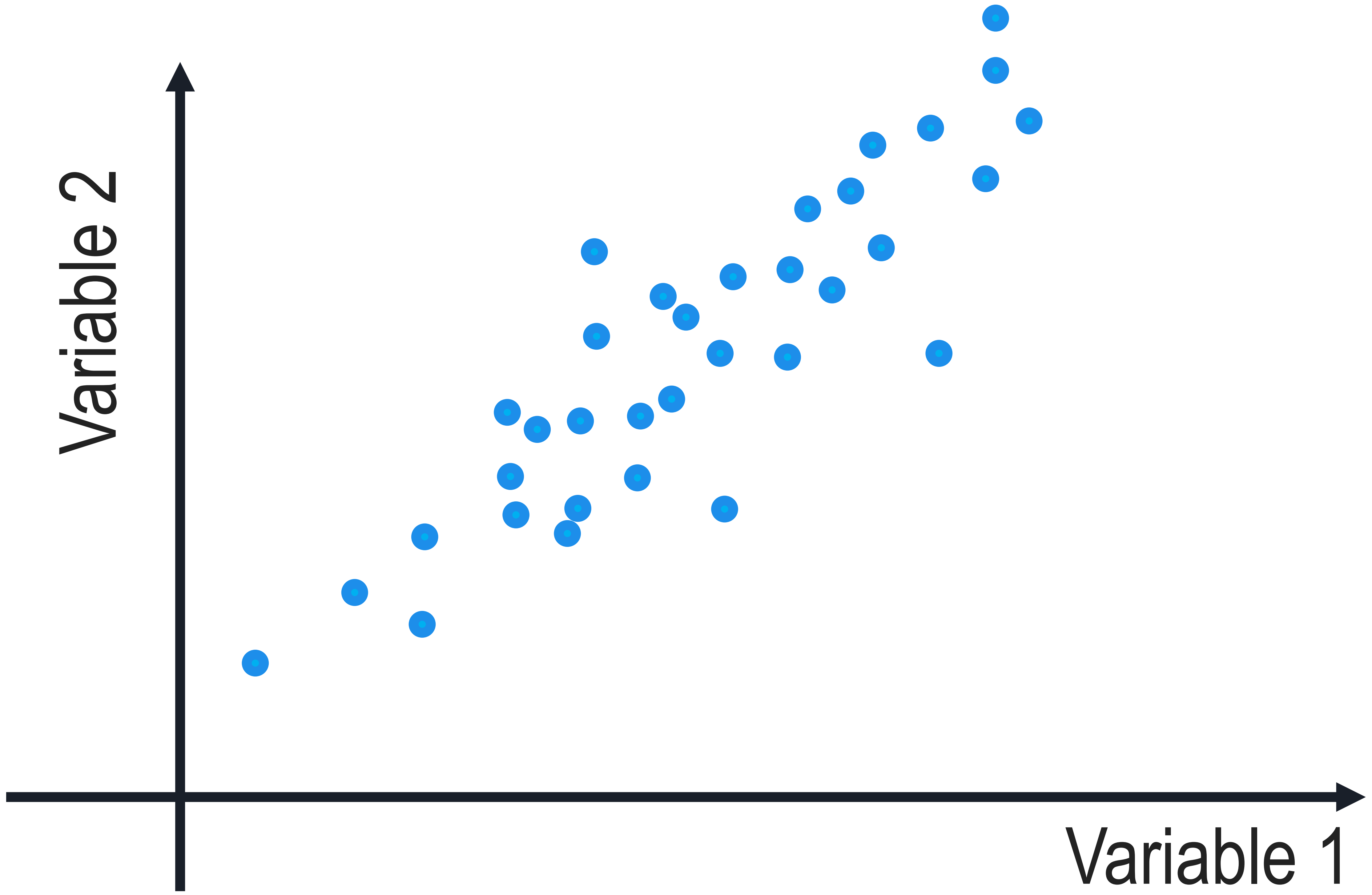
Output only --- PCA

Principal Components Analysis (PCA)

PCA converts a set of observations $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \dots \mathbf{X}_m]$, $\mathbf{X}_\mathrm{i} \in \mathrm{R}^N$ of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components.

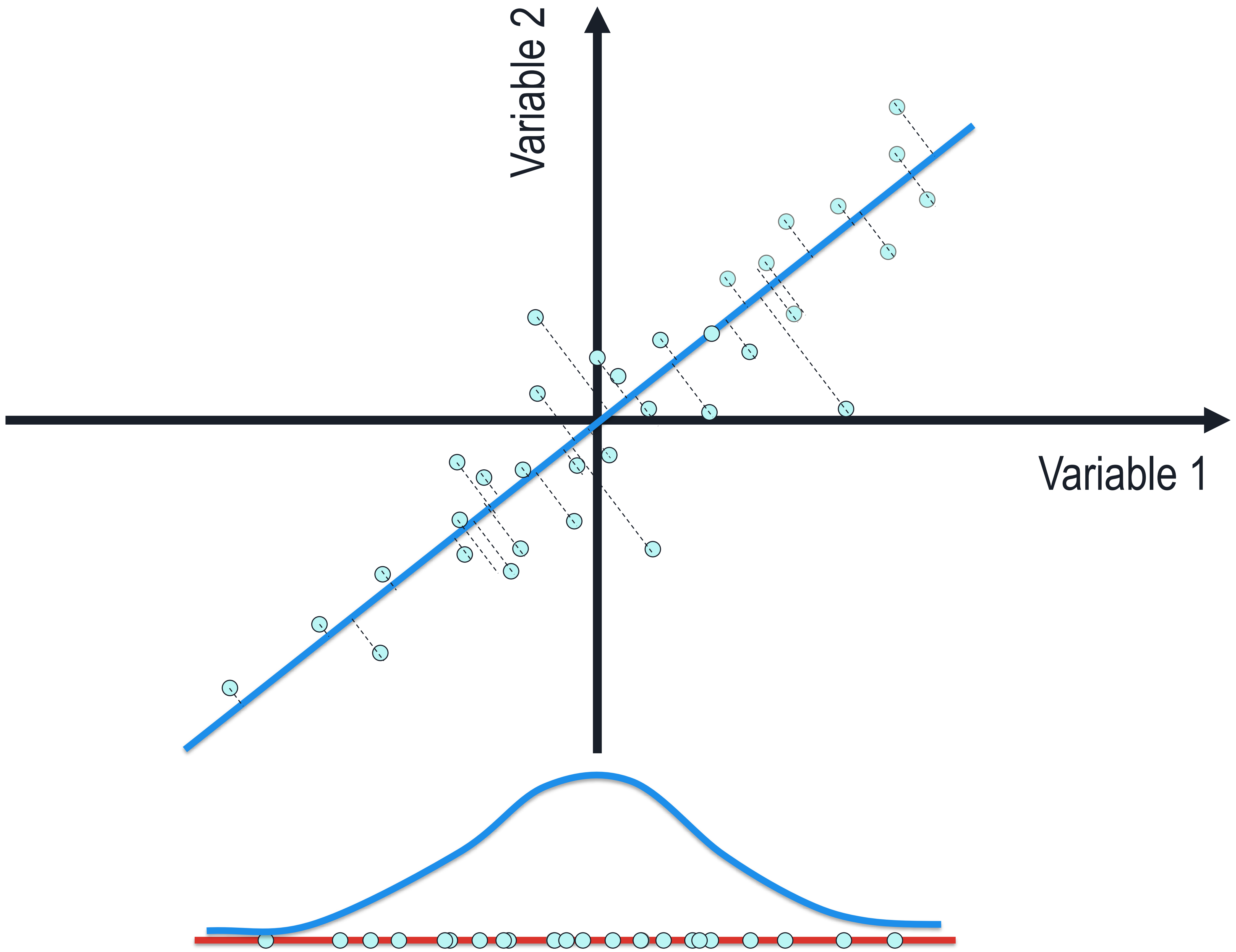
The first step of the PCA algorithm consists of normalizing the variables' mean and variance:

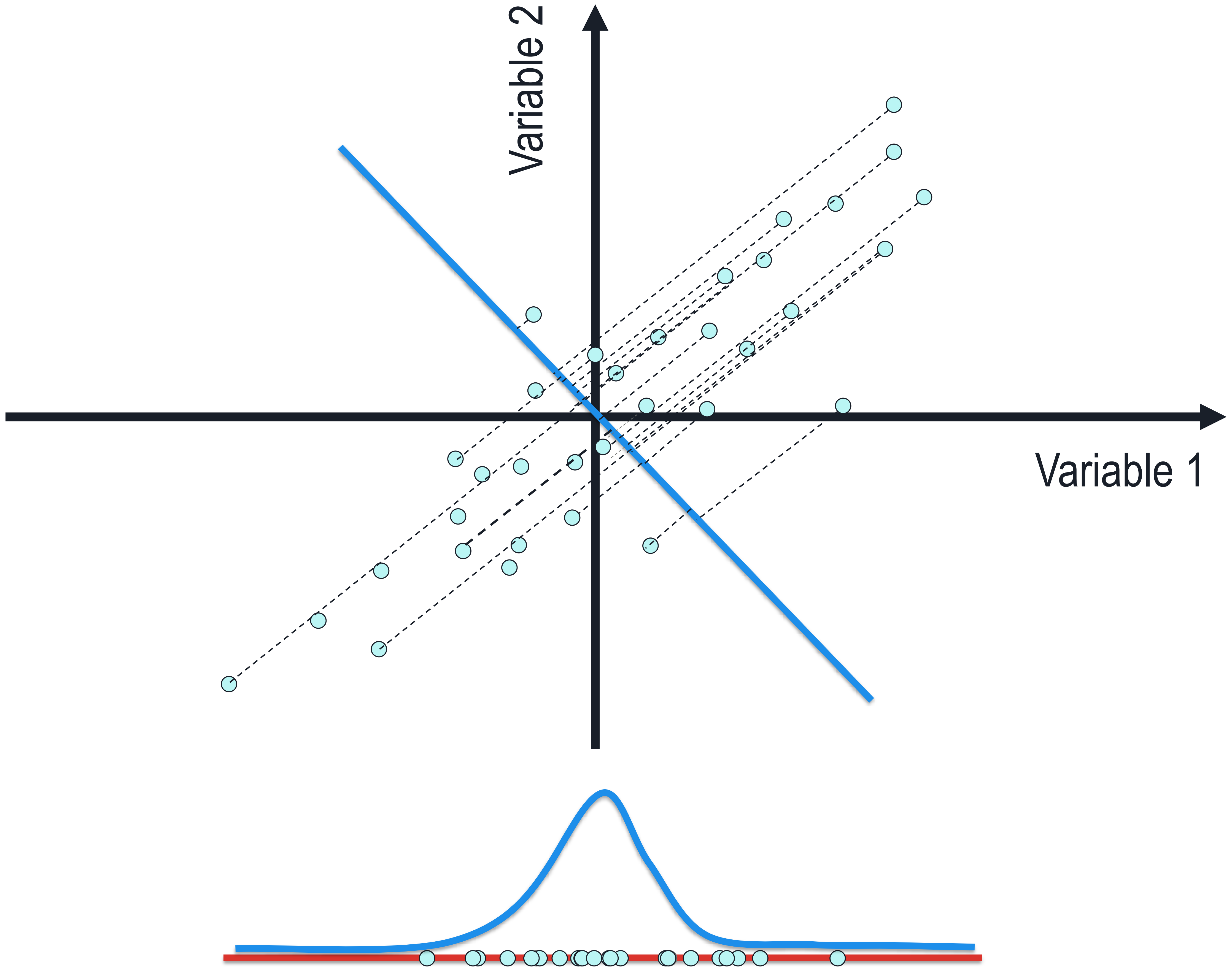
- $\mu_i = \frac{1}{N}\sum_{j=1}^{N} X_{i,j}$
- Replace $\mathbf{X}_i$ by $\mathbf{X}_i - \mu_i$
- $\sigma_i^2 = \frac{1}{N}\sum_{j=1}^{N}\left(X_{i,j}\right)^2$
- Replace $\mathbf{X}_i$ by $\frac{\mathbf{X}_i}{\sigma_i^2}$



Data normalization: Taller Parte 1

Now, we look for the major axis of variation. One may pose this problem as finding the unit vector $\boldsymbol{\Phi}$ so that when the data is projected onto $\boldsymbol{\Phi}$, the variance of the projected data is maximized.





Data normalization: Taller Parte 1

We want to maximize the variance:

$$\boldsymbol{\sigma}_u^2 = \frac{1}{N}\sum_i (\mathbf{X_i u})^2 = \frac{1}{N}(\mathbf{Xu})^T(\mathbf{Xu}) = \frac{1}{N}\mathbf{u}^{\mathrm{T}}\mathbf{X}^{\mathrm{T}}\mathbf{X}\mathbf{u} = \mathbf{u}^{\mathrm{T}}\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\boldsymbol{N}}\mathbf{u} = \mathbf{u}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{u}$$

We can perform the maximization problem using Lagrange multipliers:

$$L(\mathbf{u},\lambda) = \boldsymbol{\sigma}_u^2 - \lambda(\mathbf{u}^{\mathrm{T}}\mathbf{u} - 1)$$

Normalization condition

$$\frac{\partial L}{\partial \lambda} = \mathbf{u}^{\mathrm{T}}\mathbf{u} - 1 = 0$$

$$\frac{\partial L}{\partial \mathbf{u}} = 2\boldsymbol{\Sigma}\mathbf{u} - 2\lambda\mathbf{u} = \mathbf{0} \rightarrow (\boldsymbol{\Sigma} - \lambda\mathbf{I})\mathbf{u} = \mathbf{0}$$

Eigenvalue/Eigenvect
or problem

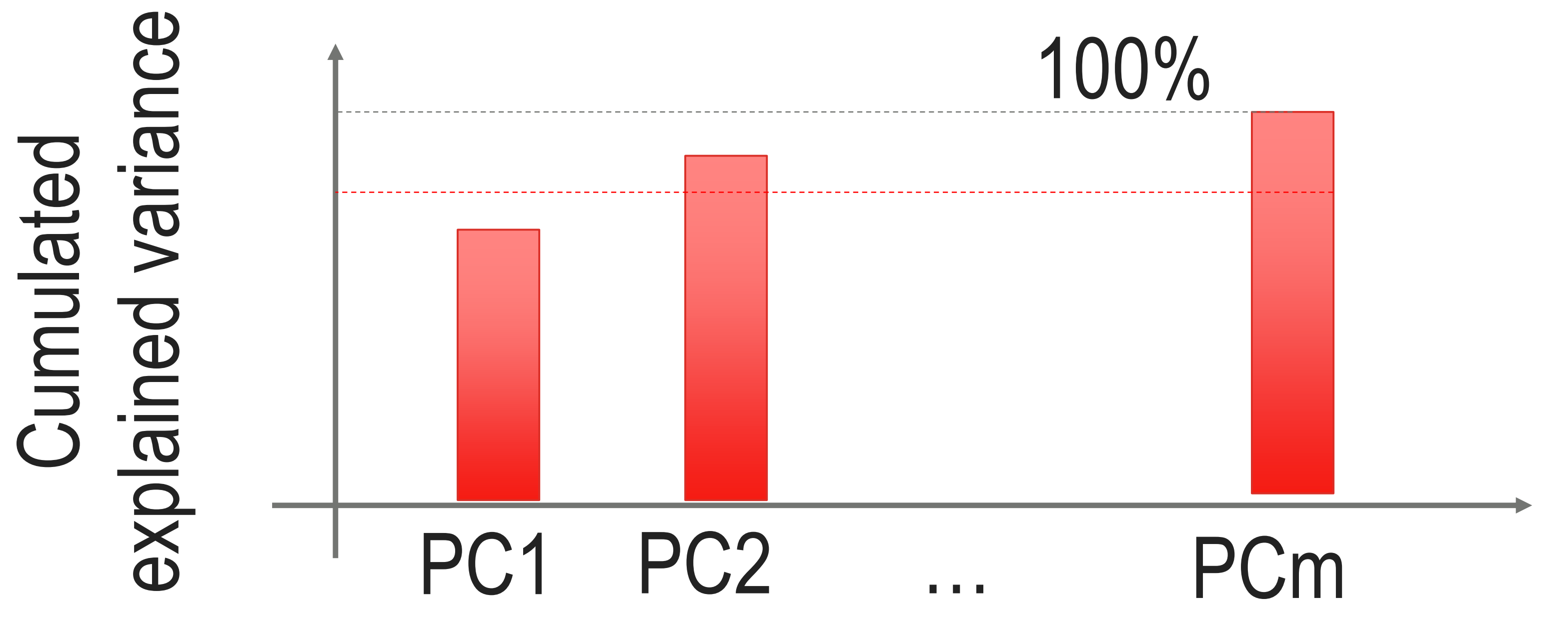
$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & \lambda_m \end{bmatrix}$$

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$$

$$\boldsymbol{U} = [\mathbf{u}_1 \quad \cdots \quad \mathbf{u}_m]$$

PCA

Data normalization: Taller Parte 1



$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & \lambda_m \end{bmatrix}$$

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$$

$$\boldsymbol{U} = [\mathbf{u_1} \quad \cdots \quad \mathbf{u}_m]$$

Total variance: $\sum_i \lambda_i$

Relative variance explained by the PCs: $\frac{100\lambda_i}{\sum_i \lambda_i}$

We retain l PCs – Dimension reduction!

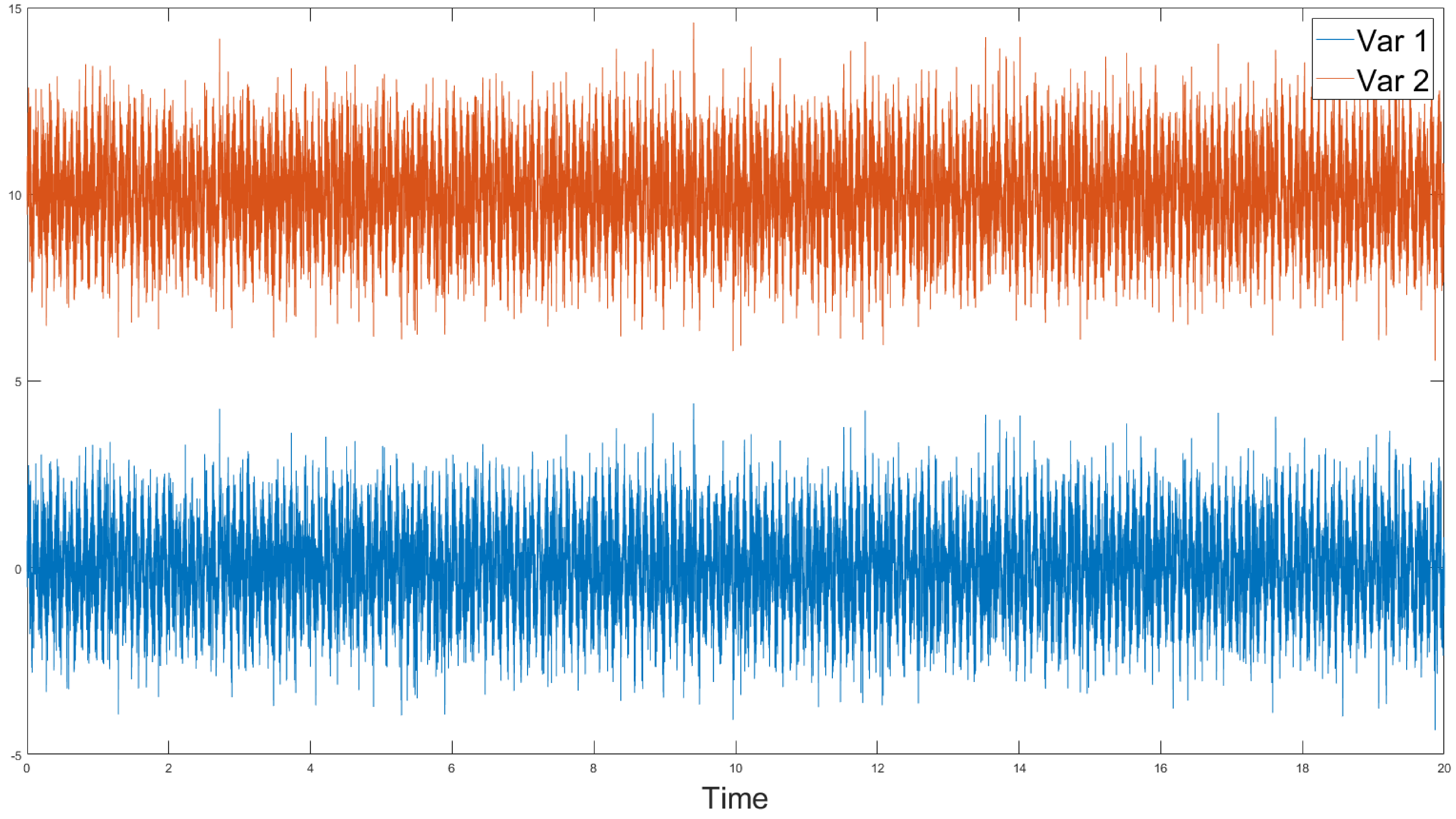
Change of coordinates: $\mathbf{Z} = \mathbf{XU}, \mathbf{Z} \in R^{Nxm}$

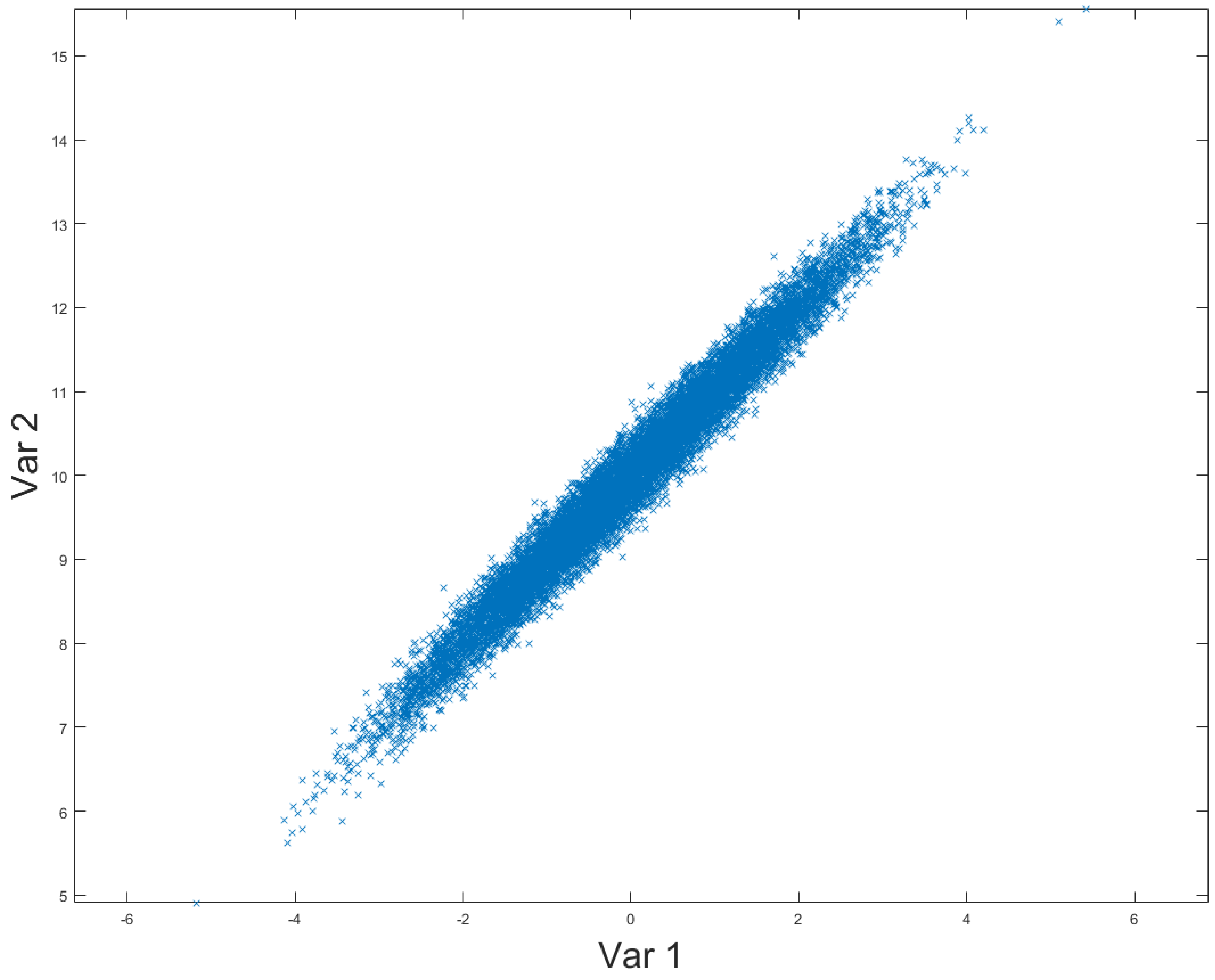
Dimensionality reduction: $\widehat{\boldsymbol{Z}} = \mathbf{Z}(:,1\!:\!\mathrm{l})$

Reconstruction in the original coordinates system:

$\widehat{\boldsymbol{X}} = \widehat{\boldsymbol{Z}}\mathbf{U}(:,1\!:\!\mathrm{l})^{\mathrm{T}}$

Data normalization: Taller Parte 1





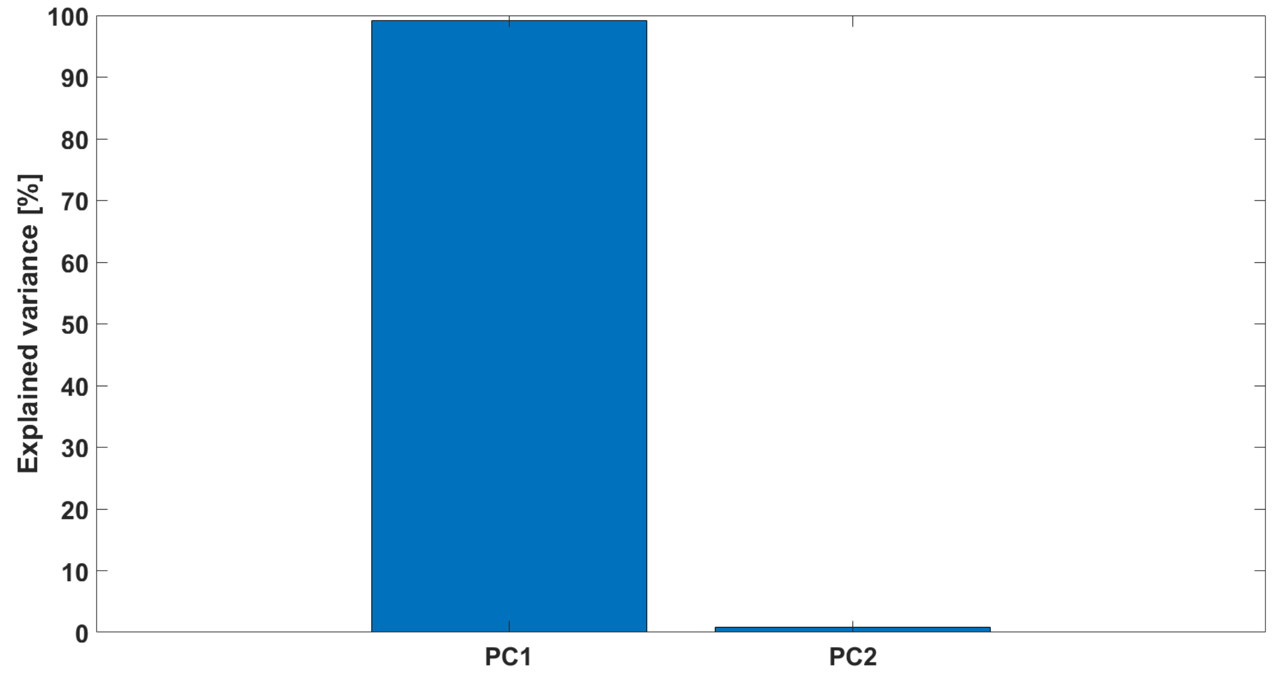
Data normalization: Taller Parte 1

$$\Sigma = \begin{bmatrix} 1.60 & 1.57 \\ 1.57 & 1.60 \end{bmatrix} \cdot 10^4$$

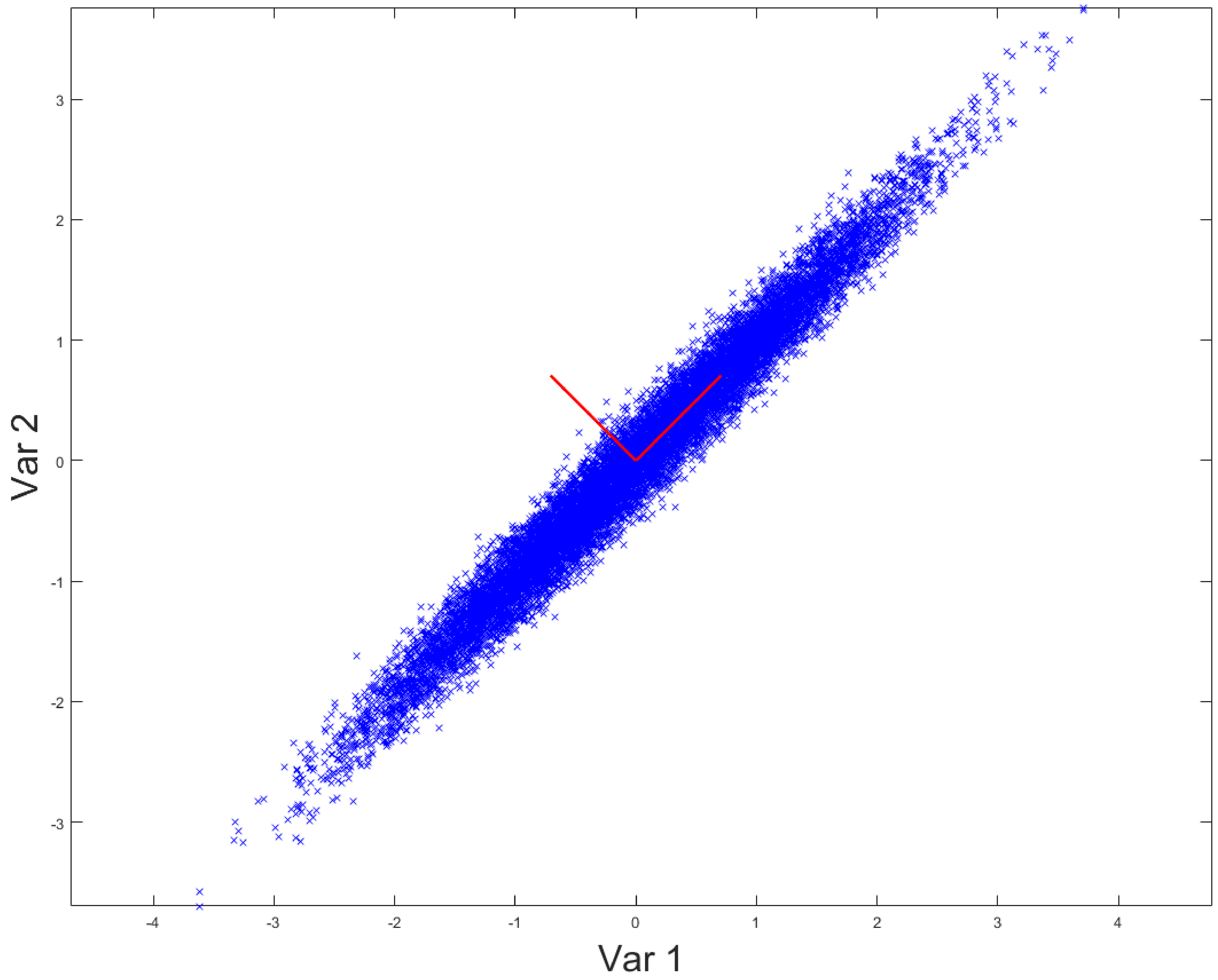
Eigenvalues/vectors

$$\lambda_1 = 3.1739 \cdot 10^4$$
$$\lambda_2 = 0.0261 \cdot 10^4$$

$$\phi_1 = [-0.71 \quad 0.71]$$
$$\phi_2 = [0.71 \quad 0.71]$$

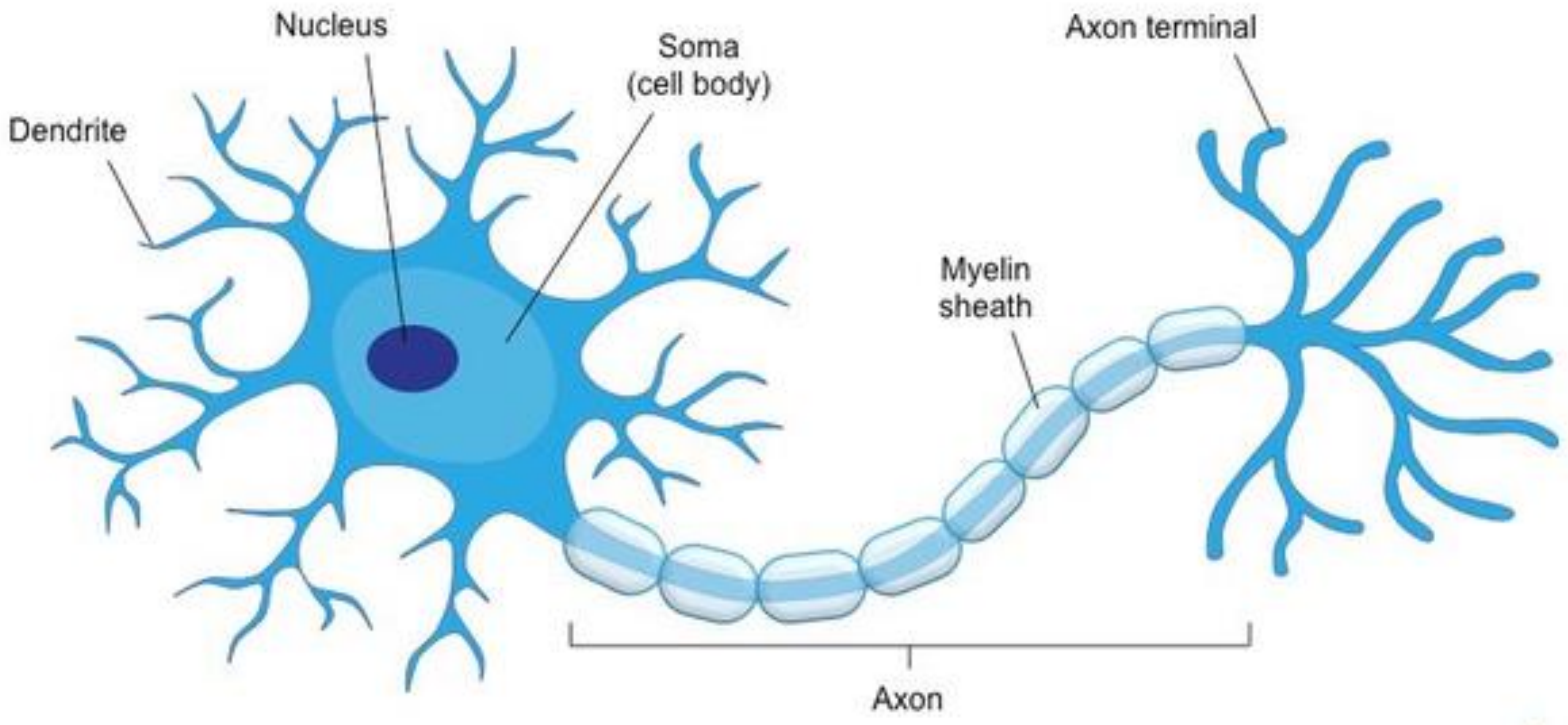


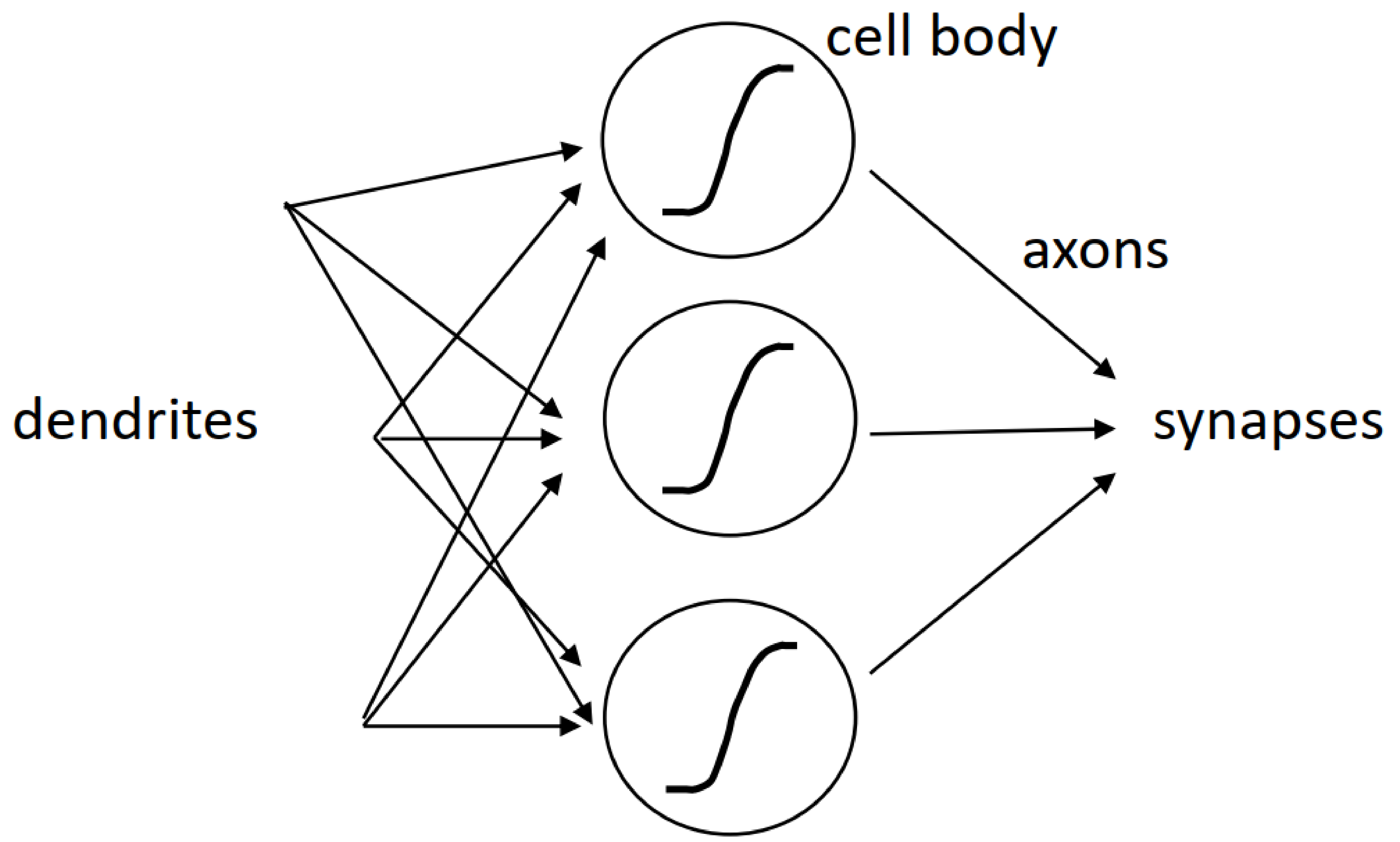
We retain one single PC



Data normalization:

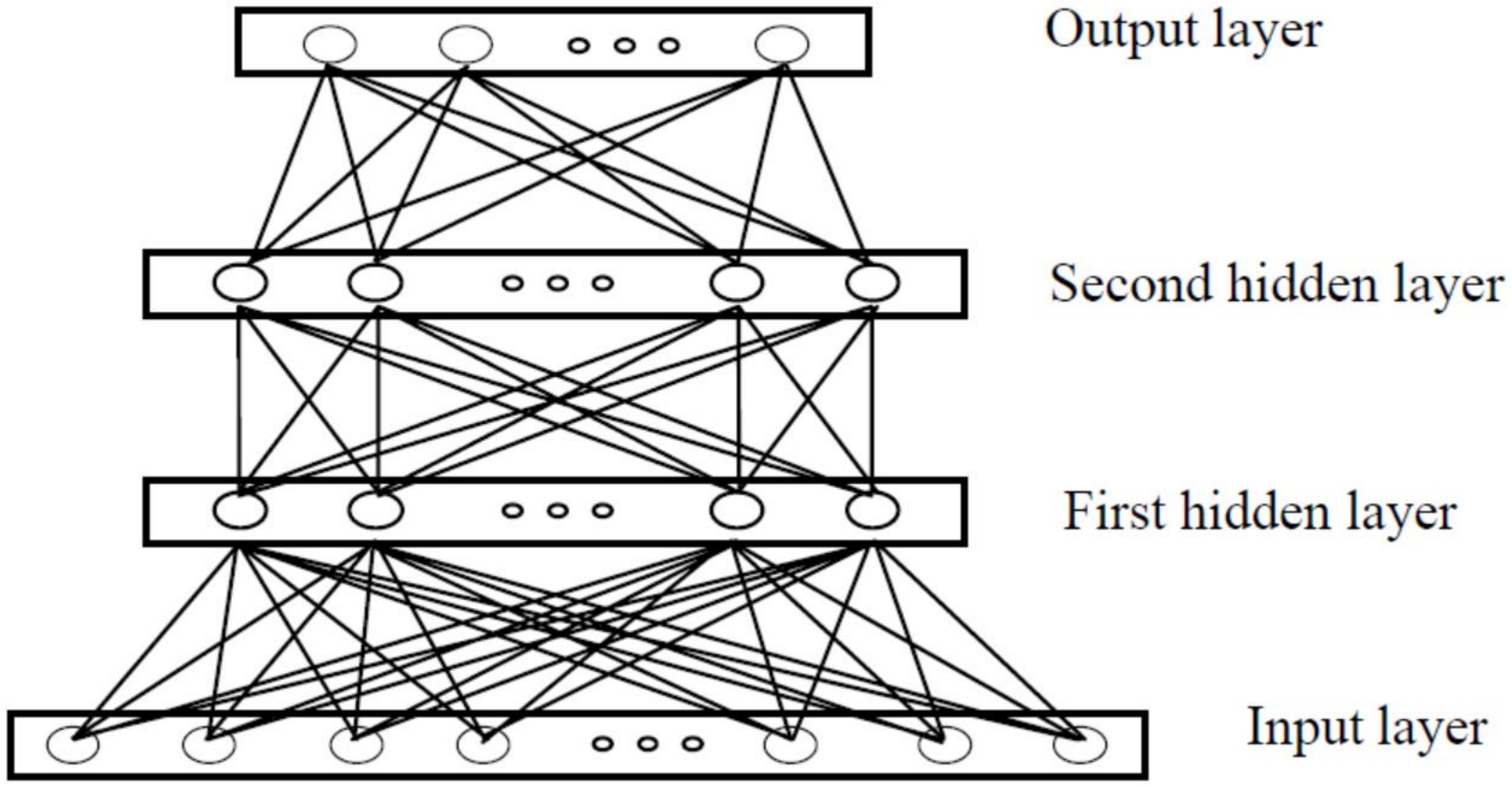
Neural Networks (NN)





Data normalization:

NN



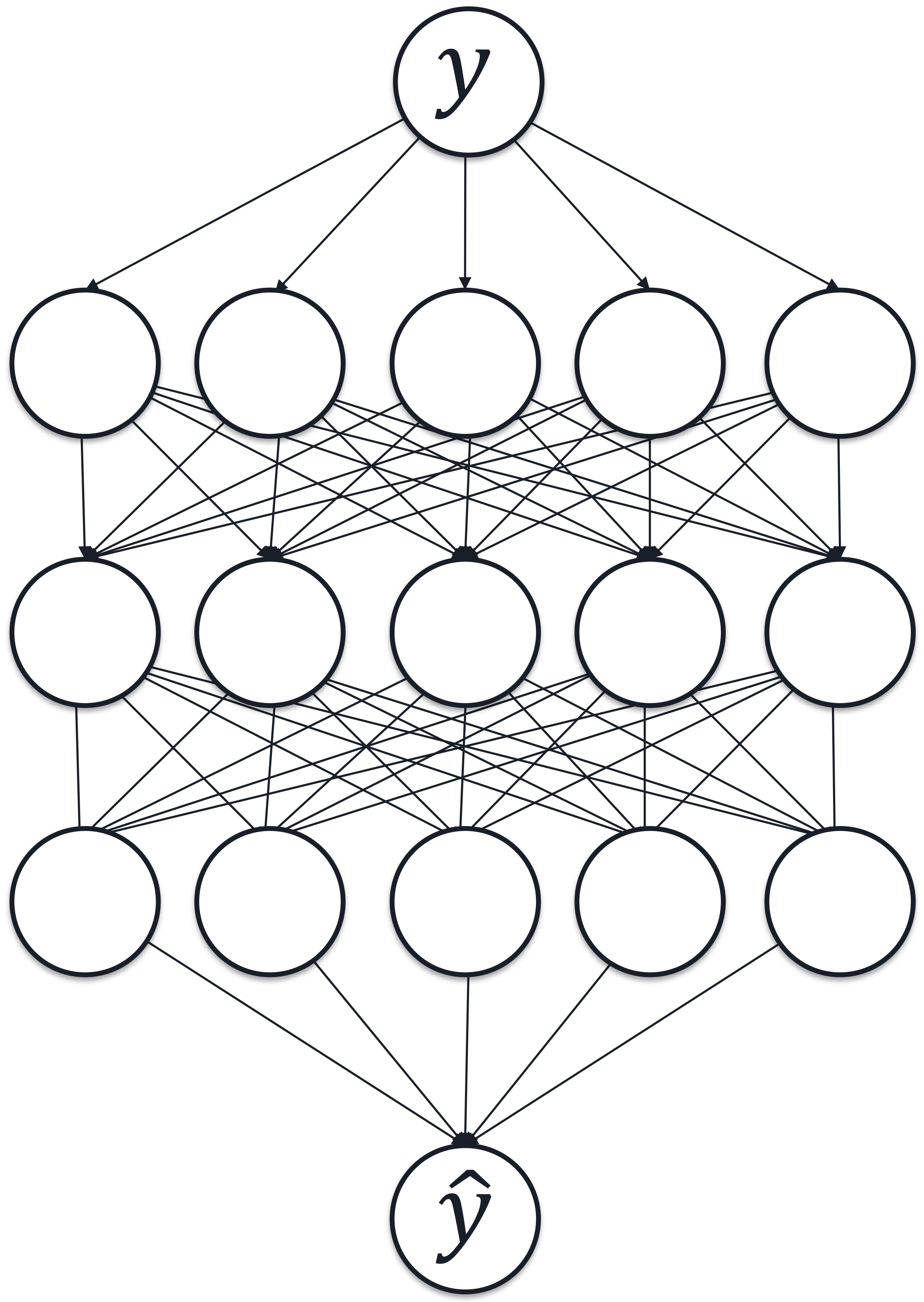
Simply put… a linear summation of nonlinear equations

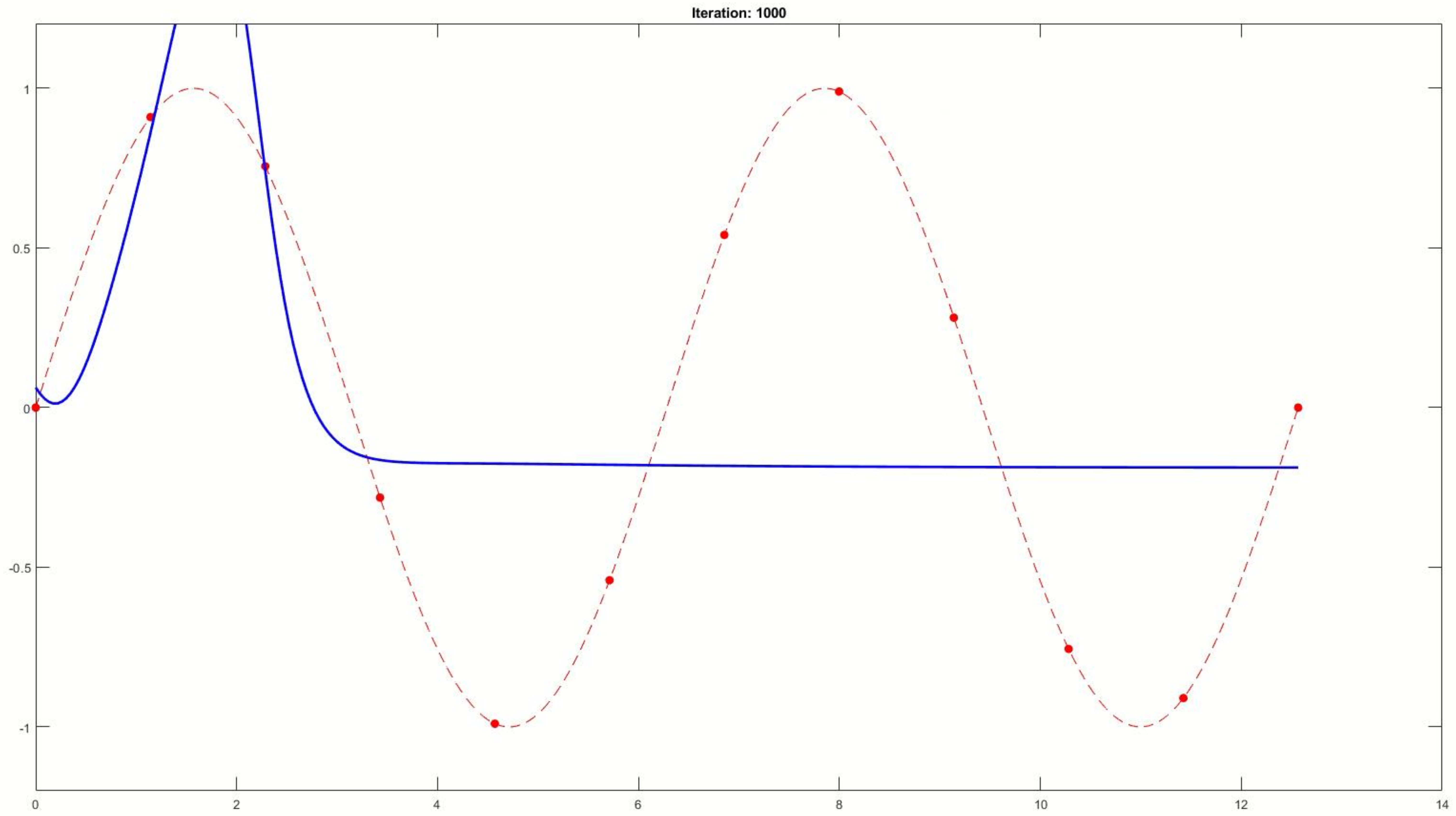
For a single hidden layer:

But how many hidden layers do I need ?

It depends on the NN functions…
- Linear
- Sigmoid
- Gaussian
- Wavelet
- …

Data normalization:





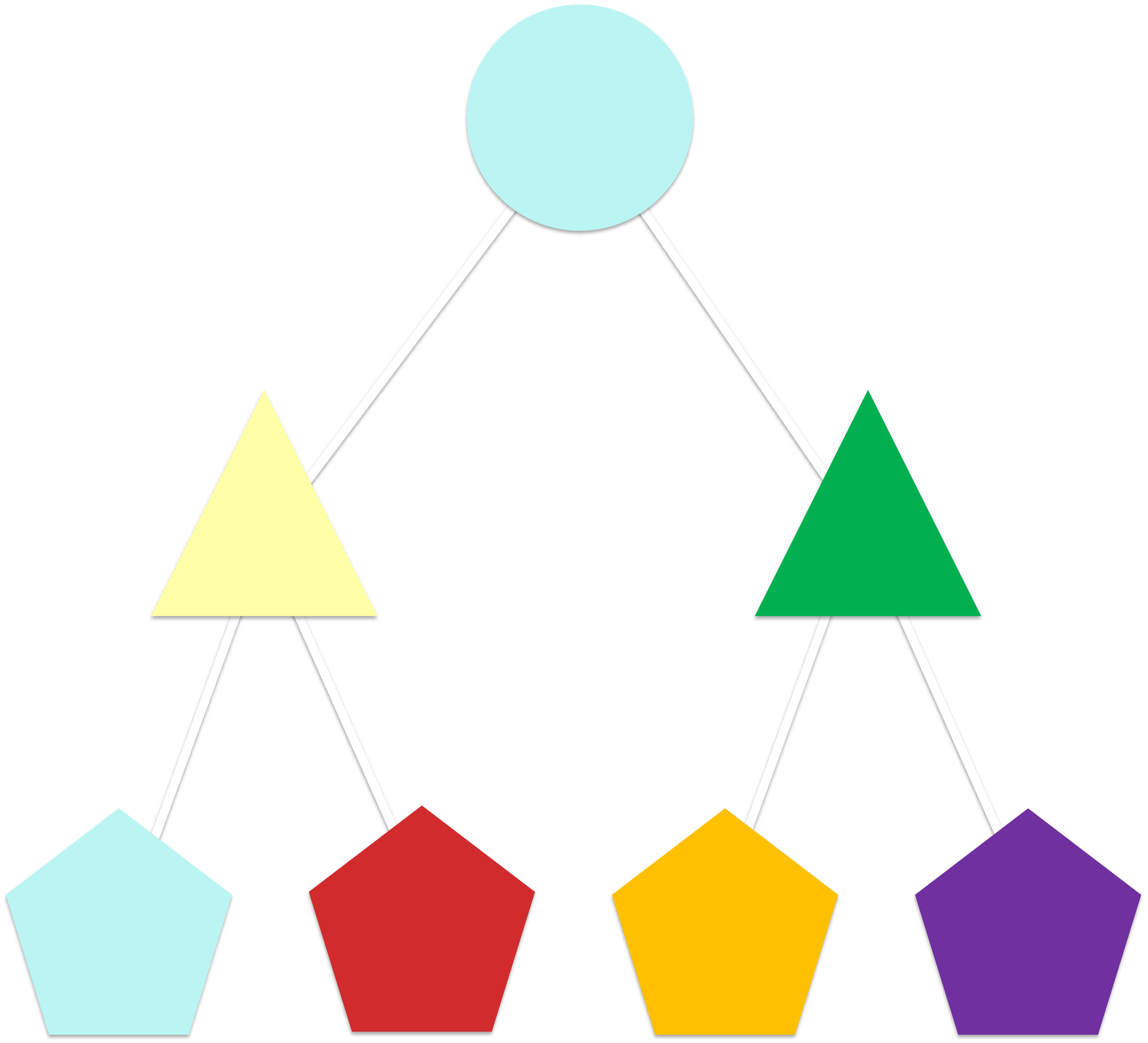
Autoassociative Neural Network (ANN)

Pattern Clasification

Pattern classification:

Pattern classification

- Supervised classification
- Semi-supervised two-class classification
  - ❖Control charts
- Unsupervised classification



Pattern classification:

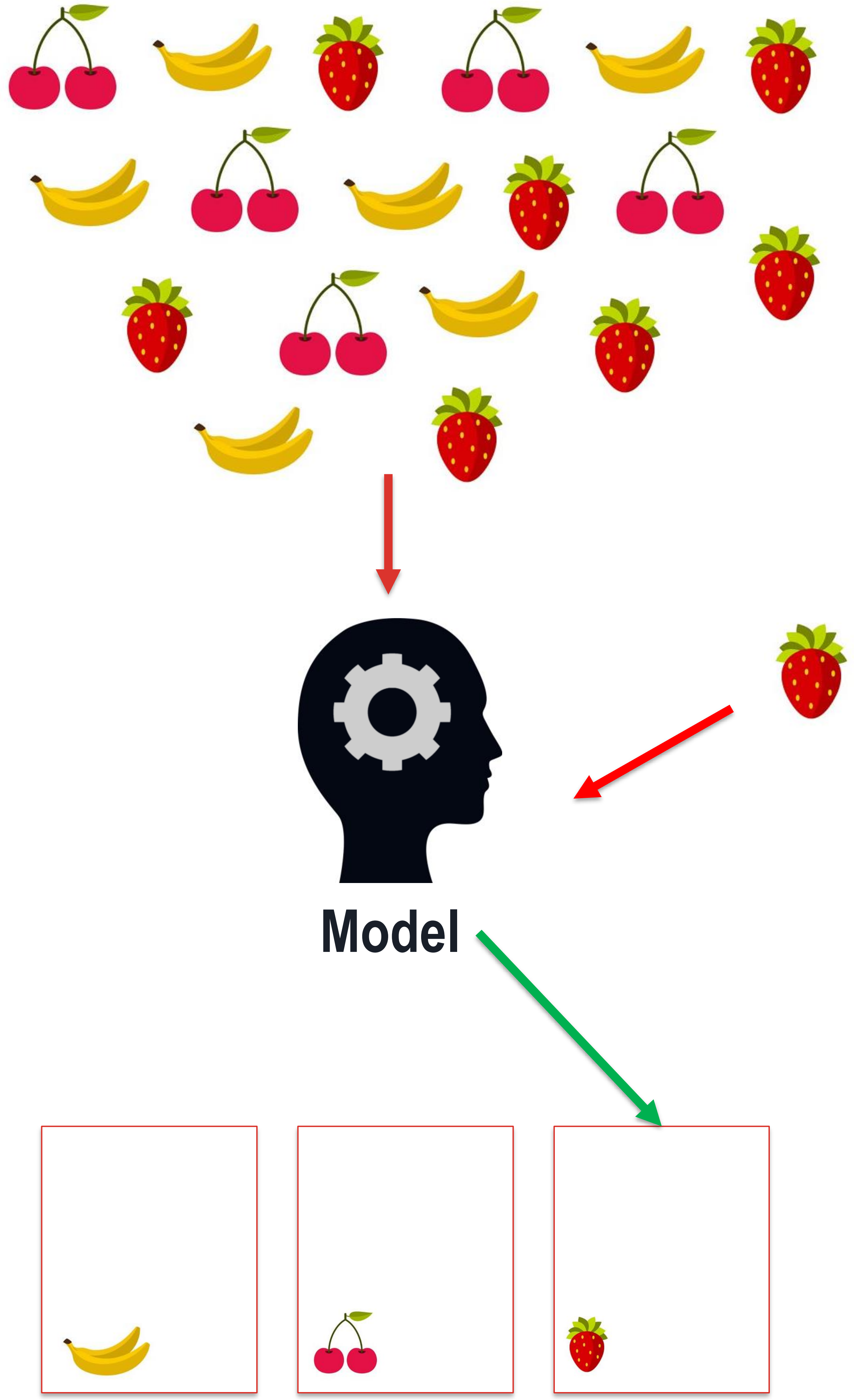
Supervised classification

Supervised learning requires and “external teacher” to provide the network information about the cause-effect relation of damage and change in behavior. This involves characterizing a set of input and output data for an assumed relationship, so that associations might be learnt by a machine learning tool.

- ❑**Classification Problems**: The output variable is typically a category such as Yes or No, or Fraudulent or Not Fraudulent.
- ❑**Regression Problems**: the output variable is a real value, such as Revenues or Volumes

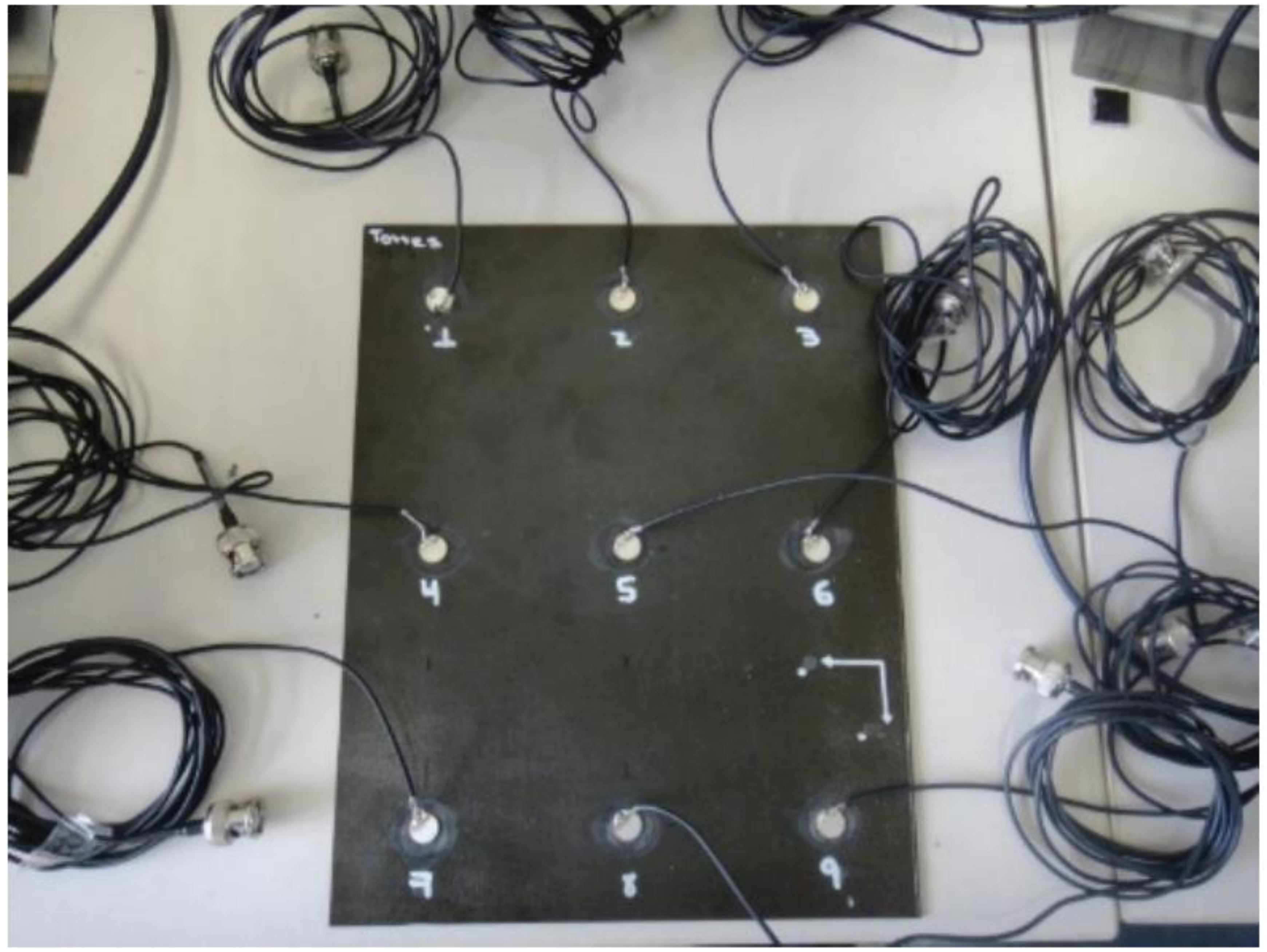
External teacher → Numerical model

External teacher → Experimental data (usually infeasible in CE structures)



Pattern classification: **Classification using experimental data**

Piezoelectric transducers



Carbon fiber-reinforced composite plate

Table 1: Damage description.

Damage Number	Description
1	Delamination: started symmetrically from the right side of the sample at its middle position along the y-axis. Its width along the y-axis is 16 mm and its depth along the x-axis is 10 mm
2	Extended the previous damage to a width of 33 mm and depth of 42 mm
3	A crack of 25 mm length initiated at the middle position along the vertical y-axis and in the parallel direction to the x-axis
4	Extended the previous crack to a length of 30 mm
5	Extended the previous crack to a length of 45 mm
6	Extended the previous crack to a length of 70 mm

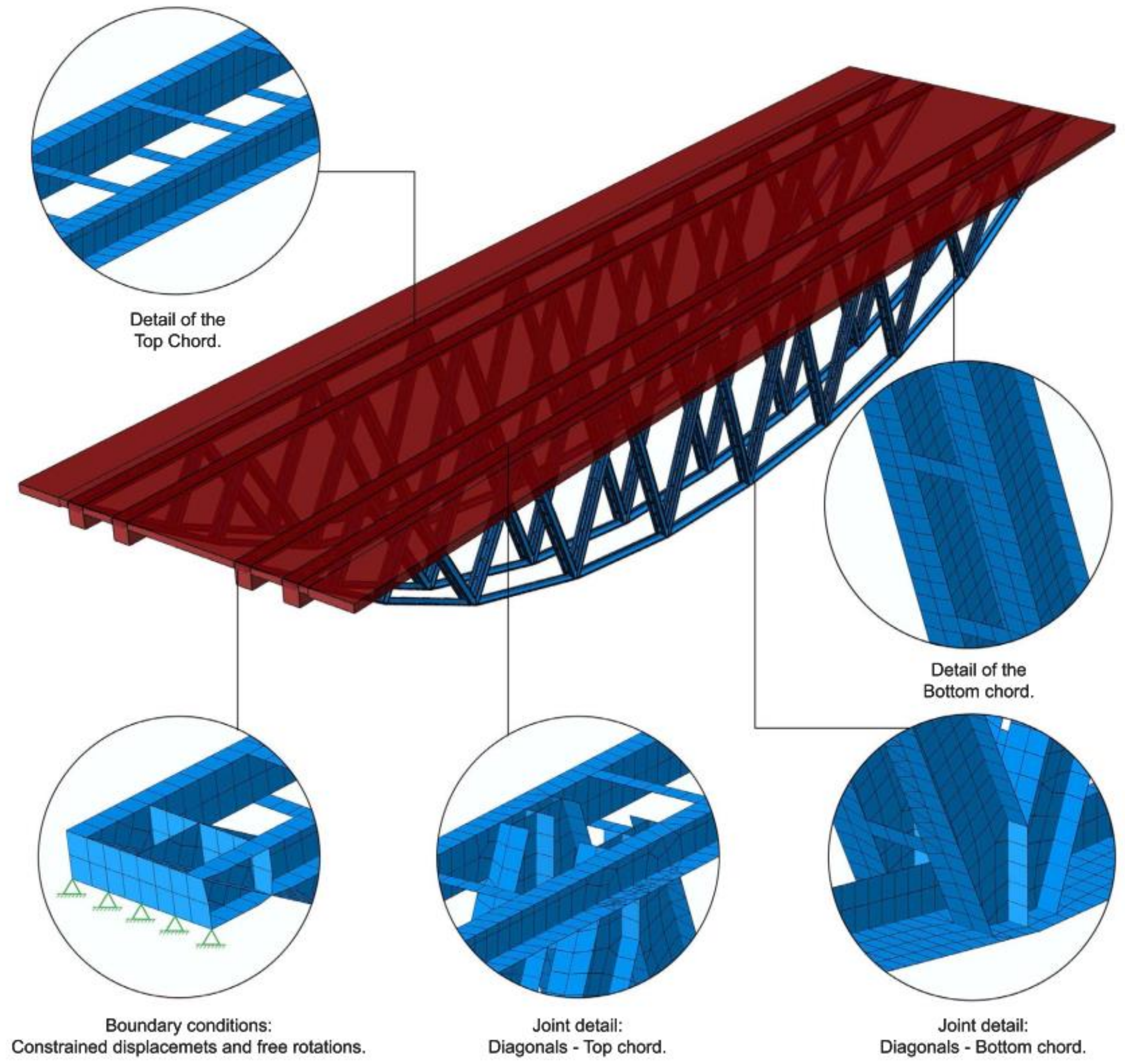
Confusion Matrix - Weighted KNN

True class -> / Predicted class ->	UND	DMG1	DMG2	DMG3	DMG4	DMG5	DMG6	TPR/FNR
UND	96.00% (48)	4.00% (2)	0	0	0	0	0	96.00% (48) / 4.00% (2)
DMG1	0	100.00% (50)	0	0	0	0	0	100.00% (50) / 0.00% (0)
DMG2	0	0	100.00% (50)	0	0	0	0	100.00% (50) / 0.00% (0)
DMG3	0	0	0	100.00% (50)	0	0	0	100.00% (50) / 0.00% (0)
DMG4	0	0	0	0	100.00% (50)	0	0	100.00% (50) / 0.00% (0)
DMG5	0	0	0	0	0	100.00% (50)	0	100.00% (50) / 0.00% (0)
DMG6	0	0	0	0	0	0	100.00% (50)	100.00% (50) / 0.00% (0)

Tibaduiza, D., Torres-Arredondo, M. Á., Vitola, J., Anaya, M., & Pozo, F. (2018). A damage classification approach for structural health monitoring using machine learning. *Complexity*, *2018*.

Pattern classification: **Classification using numerical models**

Numerical Model - FEM



Model parameters: Mechanical properties, boundary conditions, connectivity,…etc

Testing

- Static
- Dynamic
- …

y_e

Output

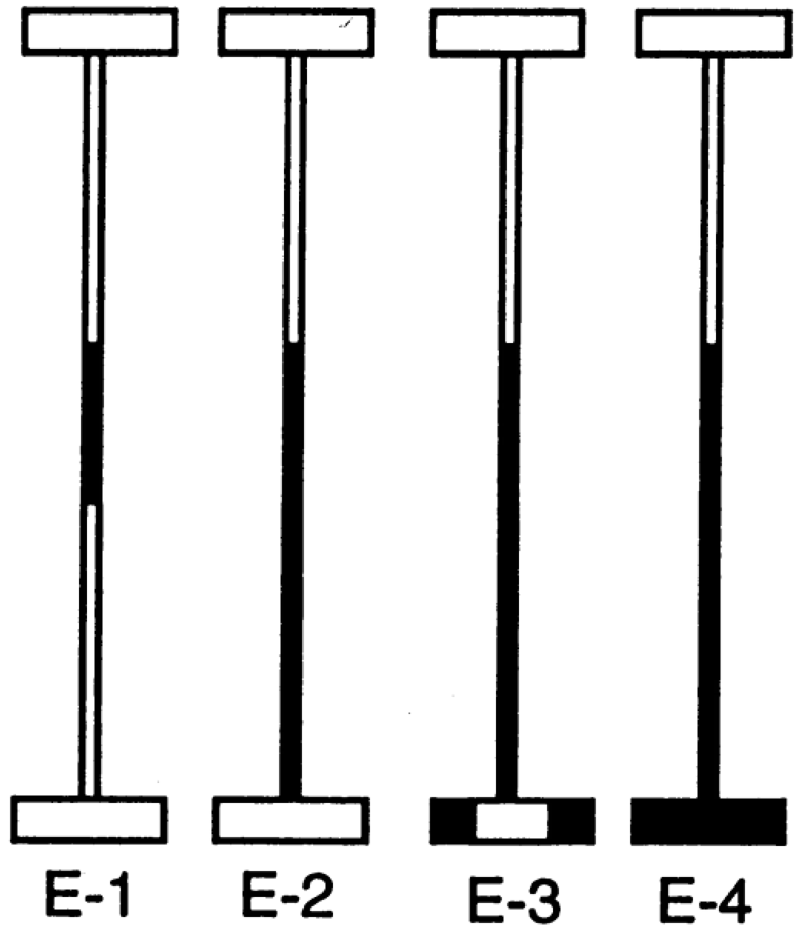
y_t

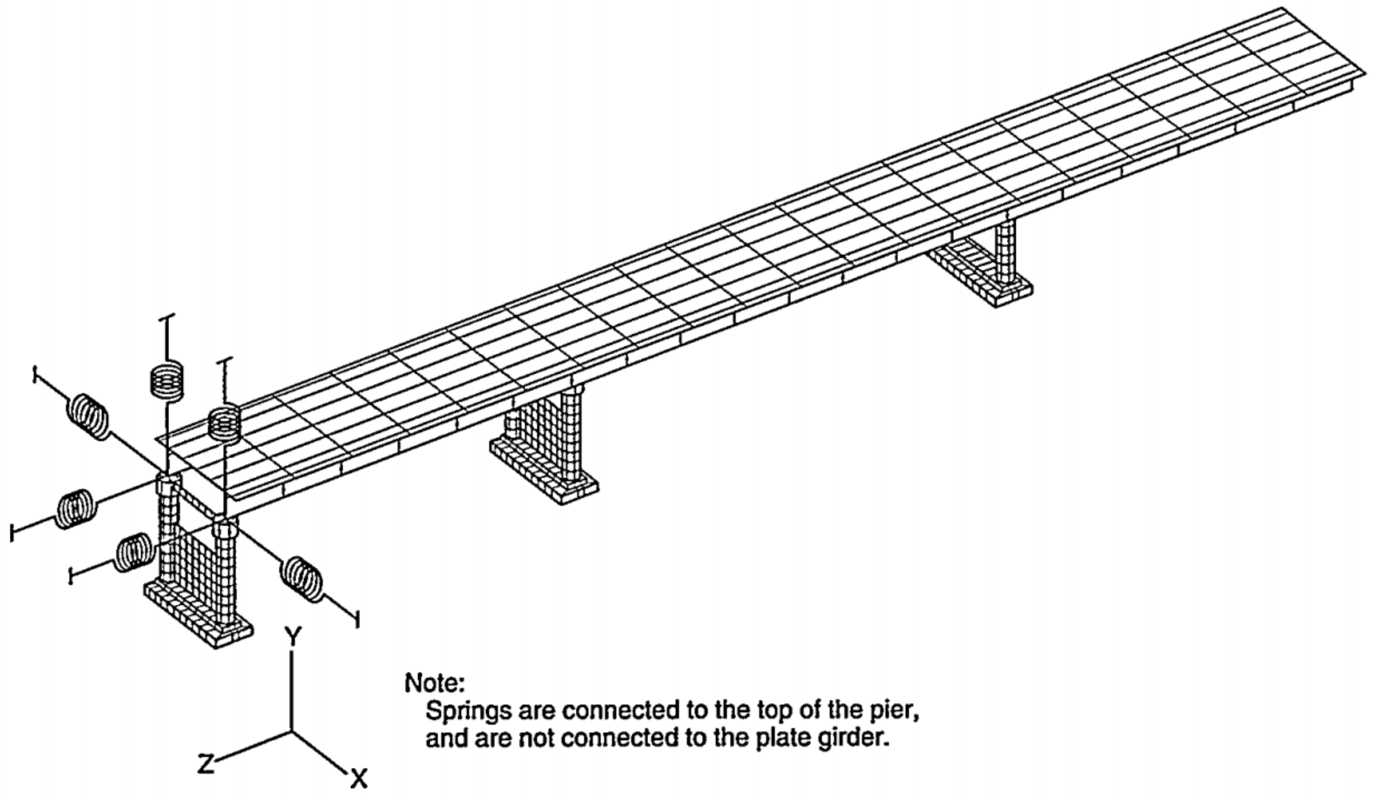
Finite Element Model Updating

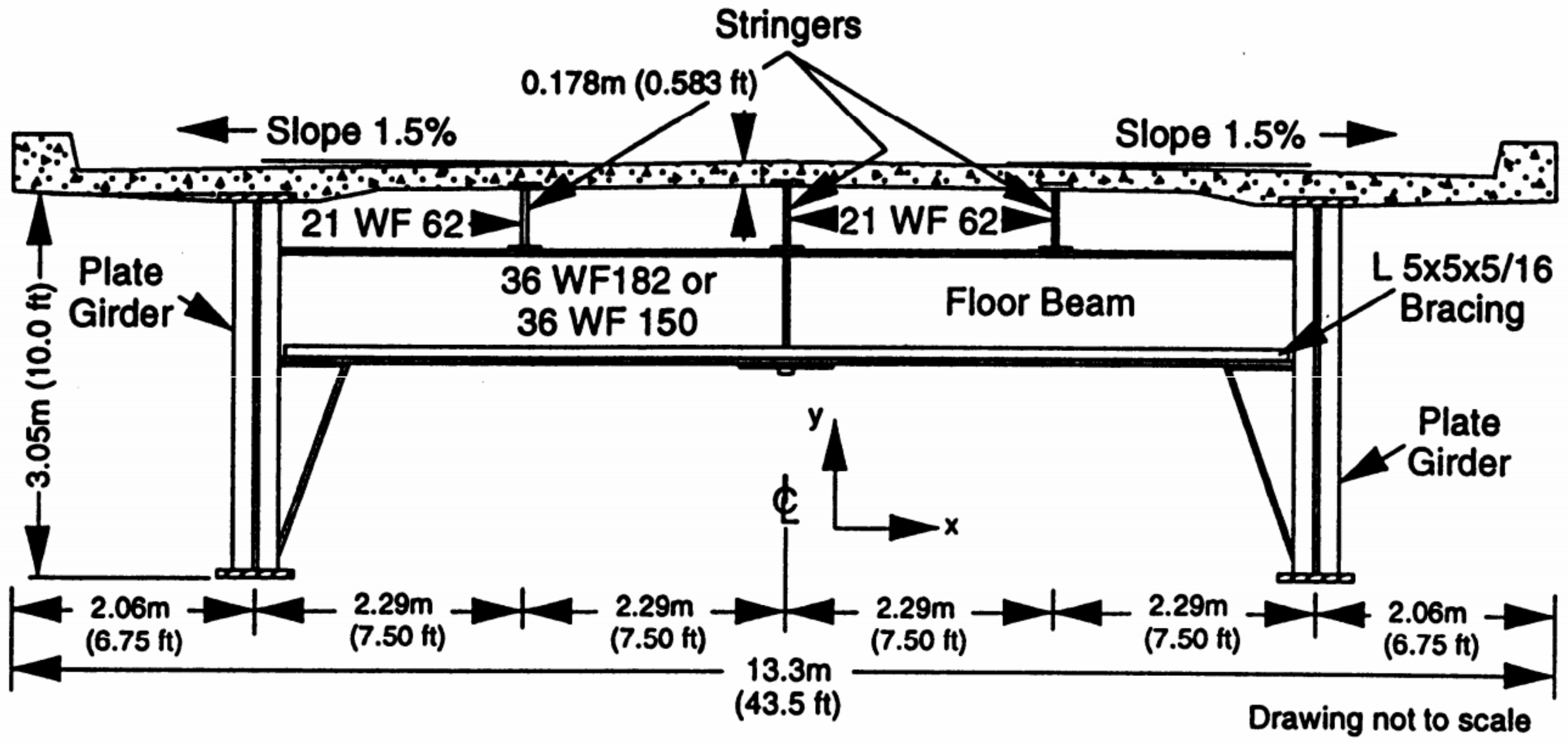
$$\min|y_t - y_e|$$

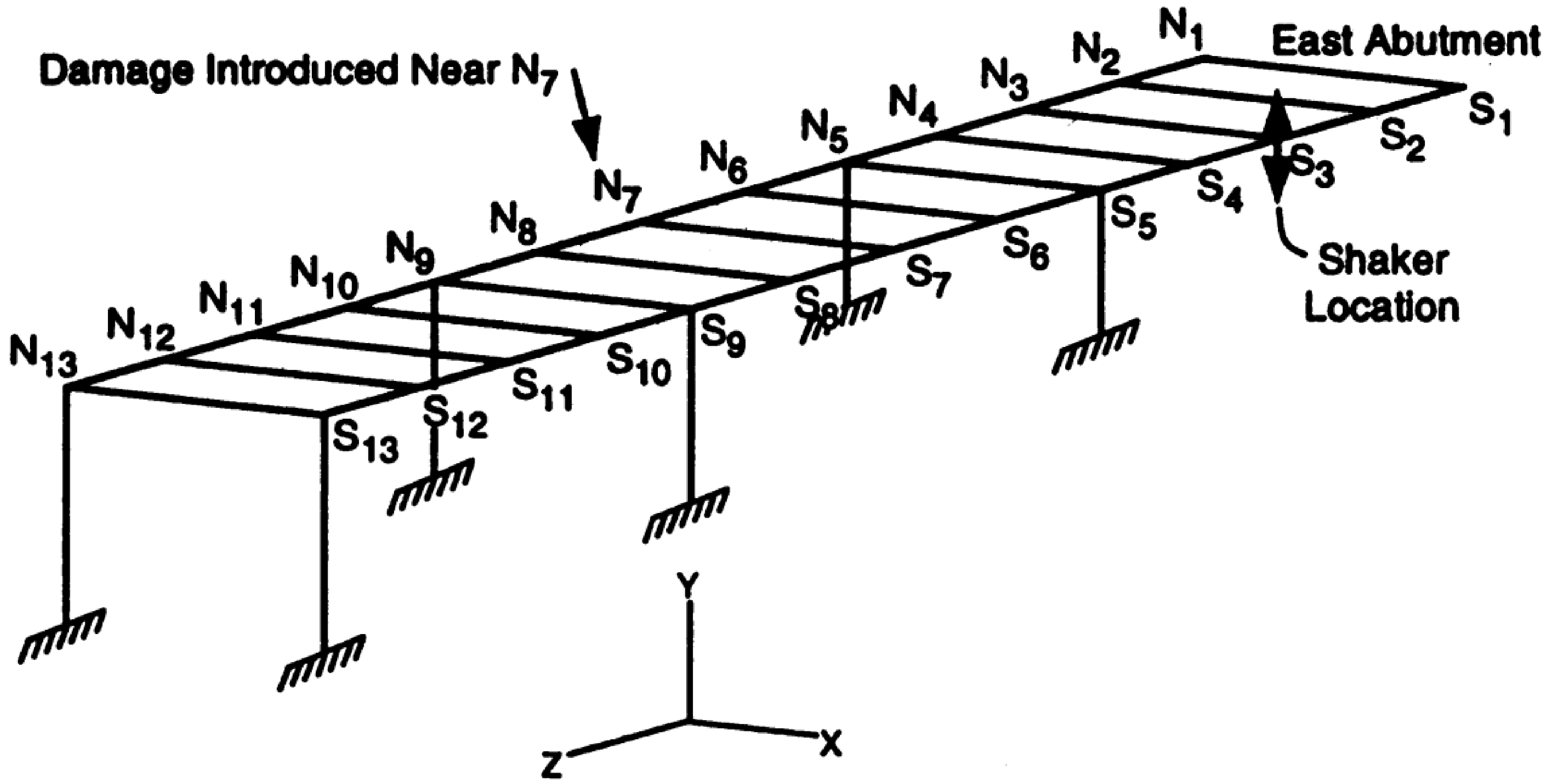
Pattern classification: **Classification using numerical models**

Damage scenarios





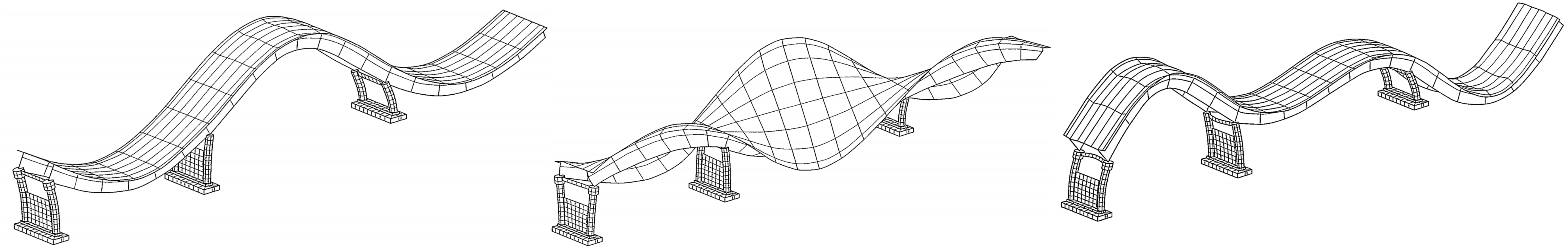




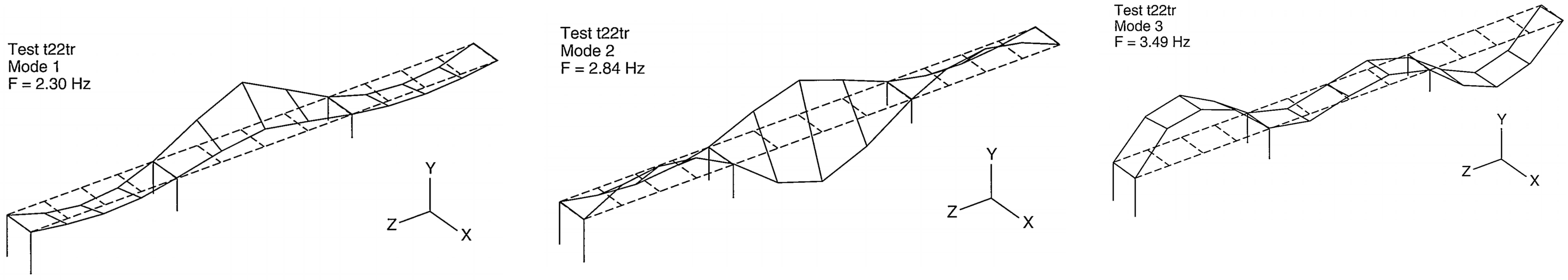
Farrar, C. R., Duffey, T. A., Goldman, P. A., Jauregui, D. V., & Vigil, J. S. (1996). Finite element analysis of the I-40 bridge over the Rio Grande (No. LA-12979-MS). Los Alamos National Lab., NM (United States).

Pattern classification: **Classification using numerical models**

Numerical



Experimental



Farrar, C. R., Duffey, T. A., Goldman, P. A., Jauregui, D. V., & Vigil, J. S. (1996). *Finite element analysis of the I-40 bridge over the Rio Grande* (No. LA-12979-MS). Los Alamos National Lab., NM (United States).

Comparison of Analytical and Experimental Modal Analysis Results

	Resonant Frequency (Hz)						
	Experiment	BR3W	BR3WB	BR3WC	BR3WD	BR3WDSP	BR3WEQ
Mode 1	2.48	2.59	2.59	2.59	2.59	2.63	2.58
Mode 2	2.96	2.78	2.87	2.79	2.88	2.90	2.88
Mode 3	3.50	3.71	3.47	3.71	3.47	3.50	3.44
Mode 4	4.08	4.32	4.11	4.00	4.11	4.23	4.12
Mode 5	4.17	3.96	4.20	4.33	4.21	4.25	4.21
Mode 6	4.63	4.50	4.70	4.56	4.94	4.94	4.77
Ave. % Diff.	--	5.08	1.88	6.39	2.77	3.39	2.24

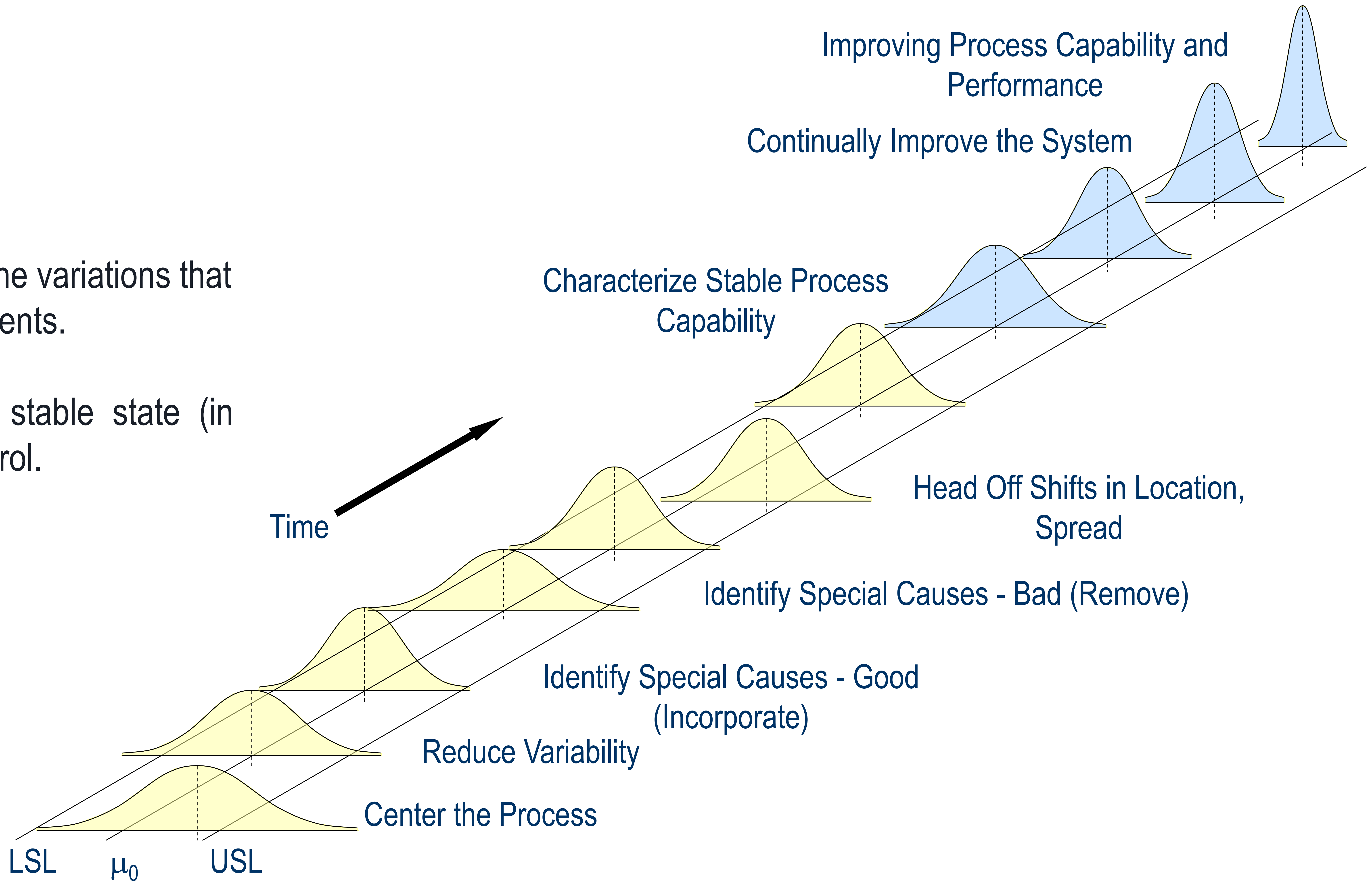
Pattern classification:

Semi-supervised classification

Control charts

A control chart is a means of visualizing the variations that occur in the process data and its components.

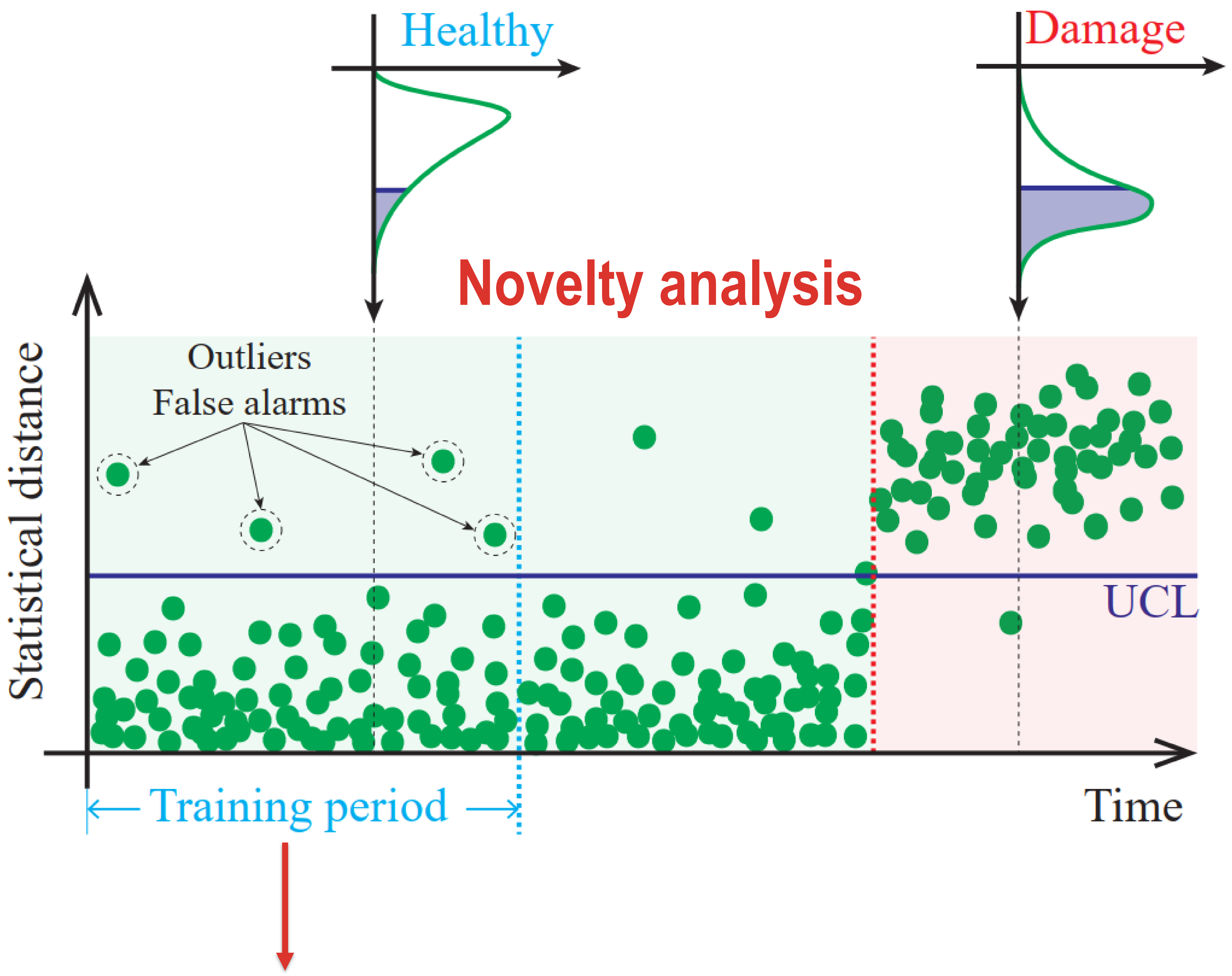
It shows whether the process is in a stable state (in statistical control) or out of statistical control.



Pattern classification:

Given the masking effects of environmental/operational factors, the monitored variables contained in matrix **Y** cannot de directly used as damage-sensitive features. Instead, the residual error matrix **E** can be used instead, that is $\mathbf{E} = \mathbf{Y} - \widehat{\mathbf{Y}}$, with $\widehat{\mathbf{Y}}$ being the predictions computed from a baseline in-control population (training period).

Under the assumption that $\widehat{\mathbf{Y}}$ reproduces the part of the variance of the resonant frequencies corresponding to changes in environmental/operational conditions, **E** only contains the residual variance stemming from identification errors and un-modelled environmental/operational conditions. If certain damage develops, this only affects the data contained in **Y**, while $\widehat{\mathbf{Y}}$ remains unaltered. Therefore, **E** concentrates the damage-induced variance apt for being used for damage detection purposes.



The structures is assumed to be in a healthy state
It must be long enough to capture all the environmental and operational conditions in the structure (typically 1 year)

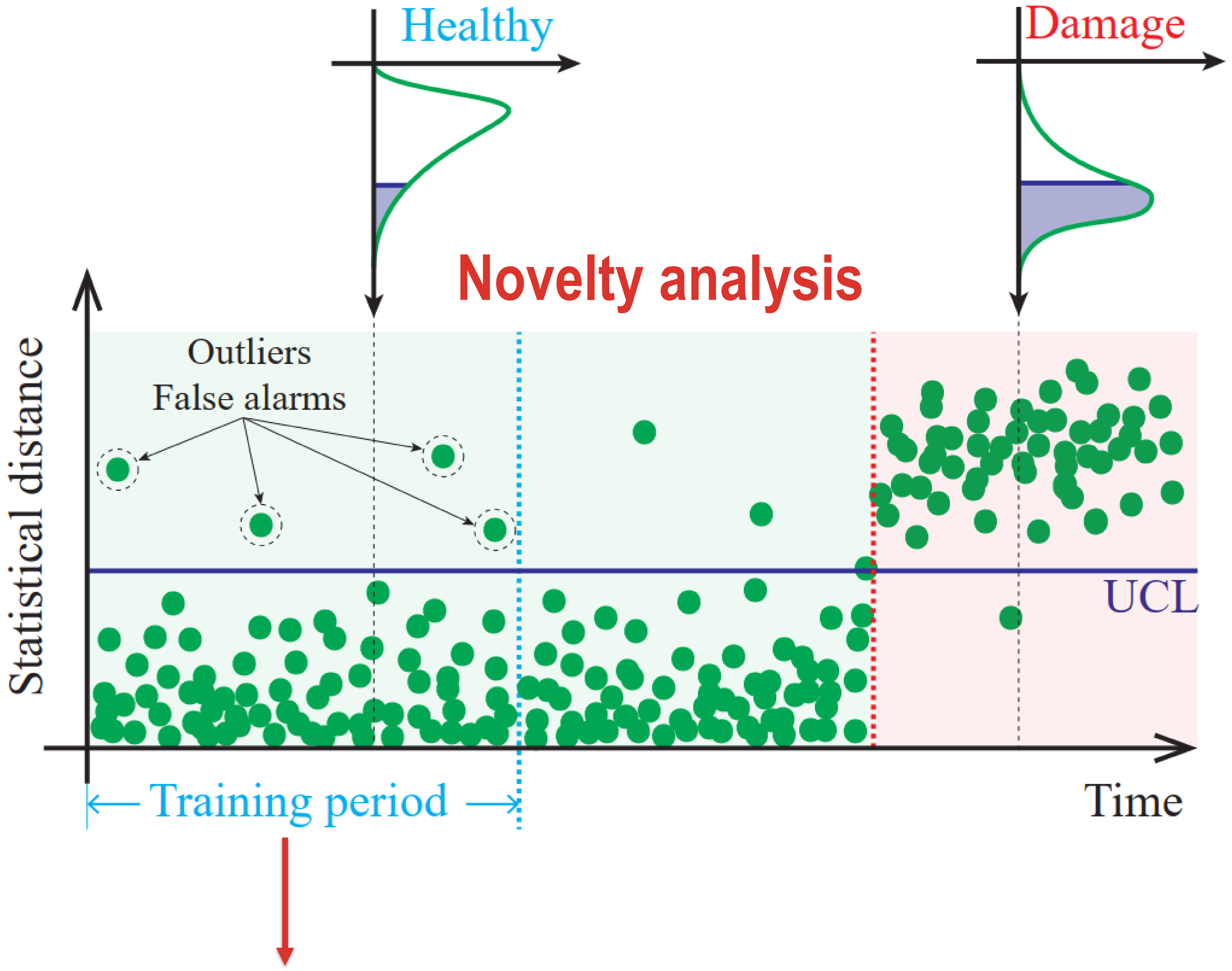
Pattern classification:

Once the residual error matrix **E** is computed, the presence of damage is investigated using control charts. These furnish in time a certain statistical distance accounting for disturbances in the distribution of the residuals contained in **E**. By defining an in-control region, the appearance of out-of-control processes, possibly associated to damage, is detected in the shape of data points violating the in-control region.

<u>**Hotelling's T^2 control chart:**</u>

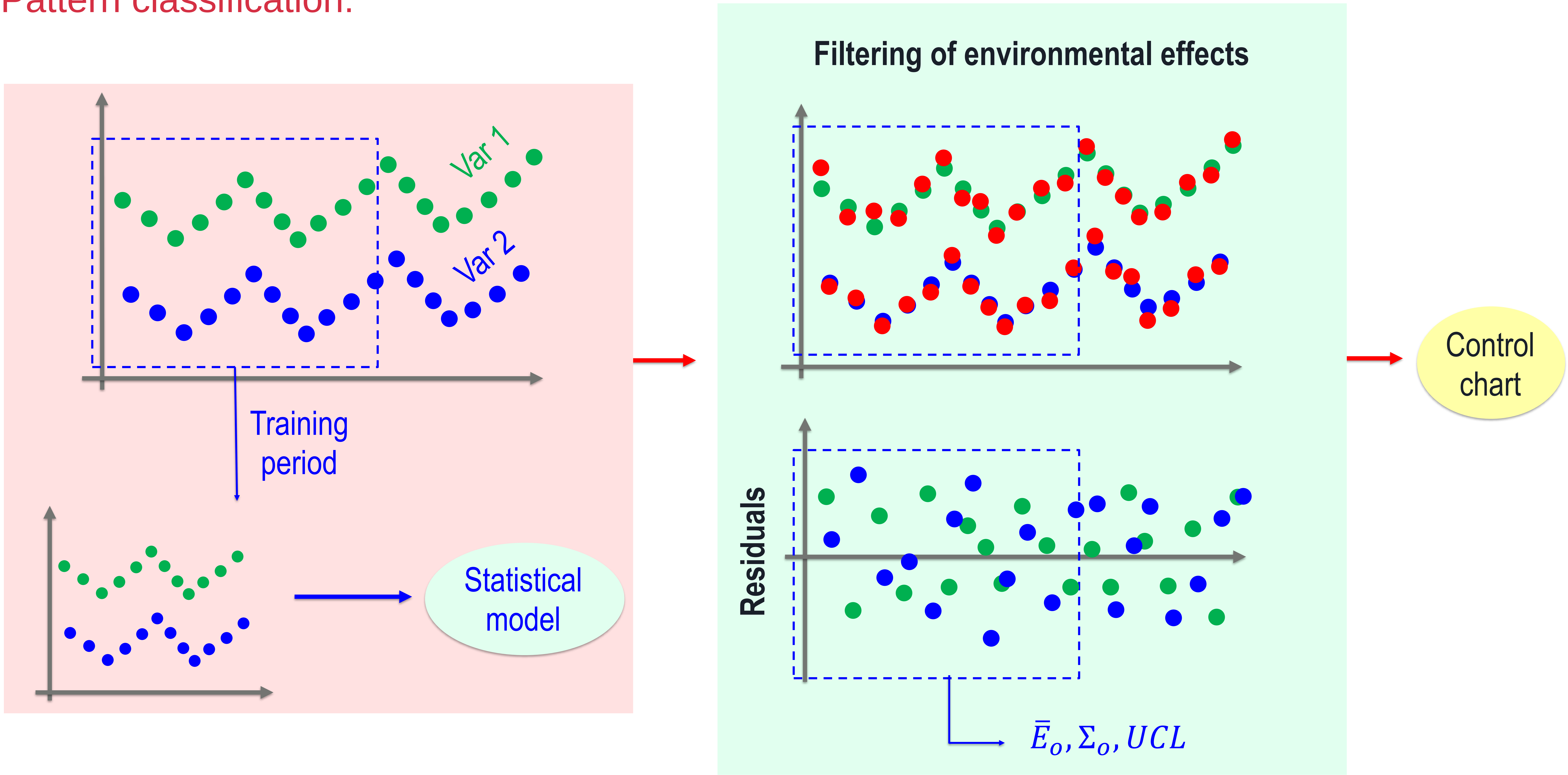
$$T_i^2 = r(\bar{\boldsymbol{E}} - \boldsymbol{E}_o)^T \boldsymbol{\Sigma}_o^{-1}(\bar{\boldsymbol{E}} - \boldsymbol{E}_o)$$

- r – Subgroup size (integer)
- $\bar{\boldsymbol{E}}$ mean of the residuals in the subgroup of the last r observations
- $\boldsymbol{E}_o$ and $\boldsymbol{\Sigma}_o$ are the man values and the covariance matrix of the residuals statistically estimated in the training period.



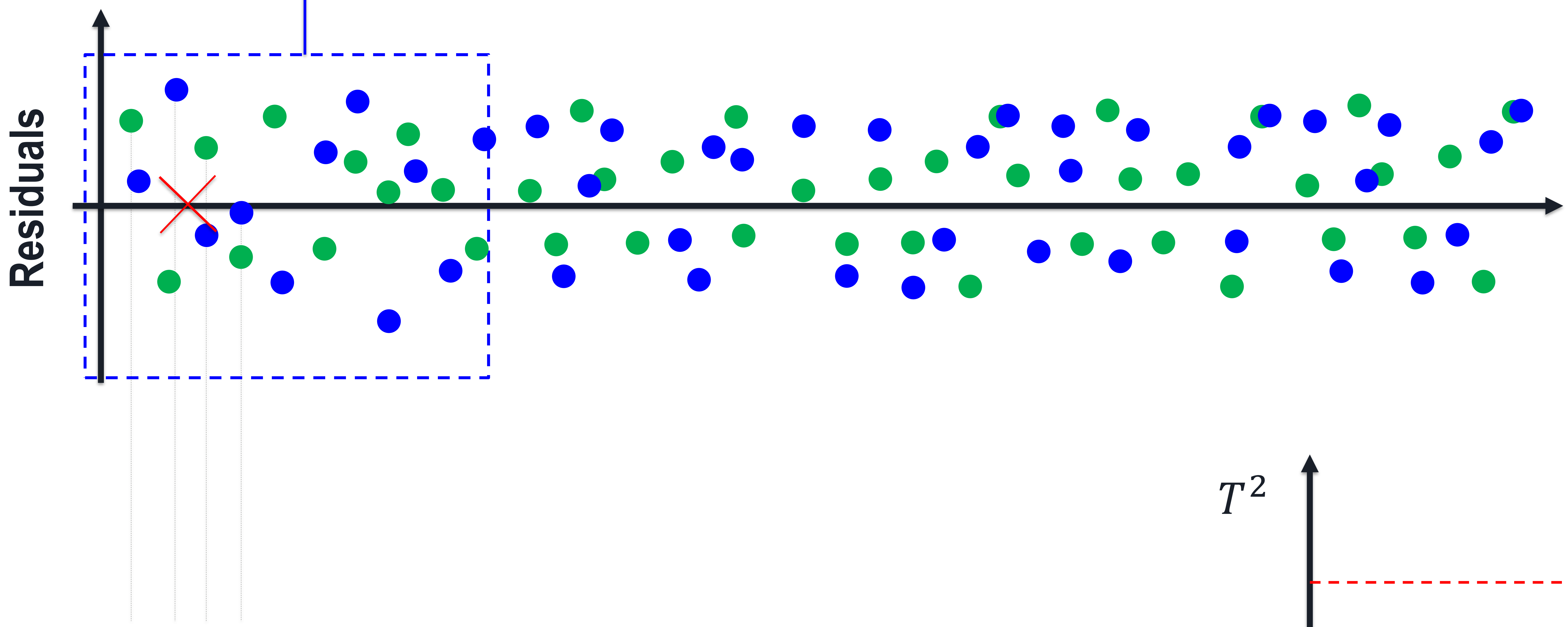
The structures is assumed to be in a healthy state
It must be long enough to capture all the environmental and operational conditions in the structure (typically 1 year)

Pattern classification:



Pattern classification:

$\bar{E}_o, \Sigma_o, UCL$ $(e.g.\ confidence\ level\ of\ 95\%)$



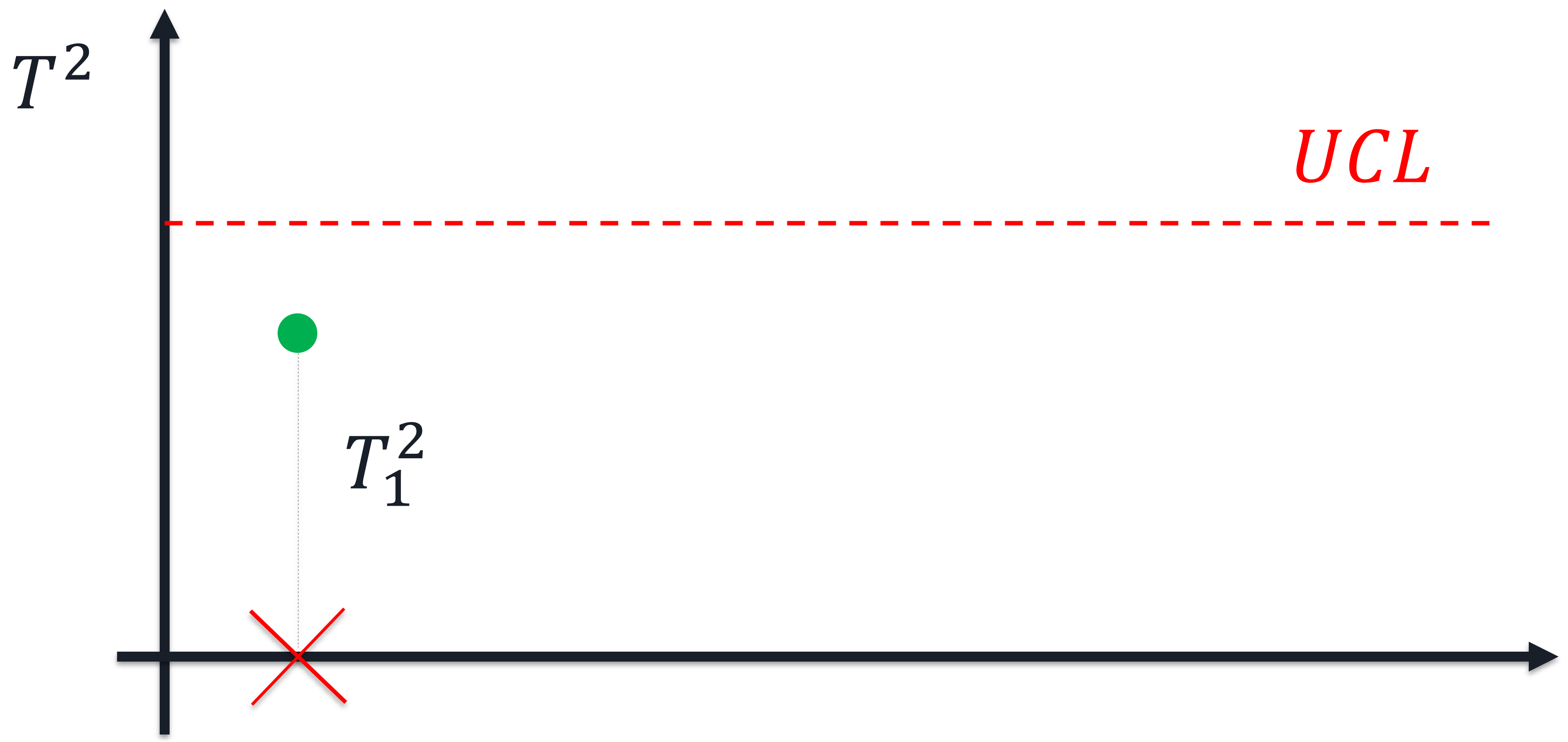
i=1

$\bar{E}_1$

$\bar{E}_1$

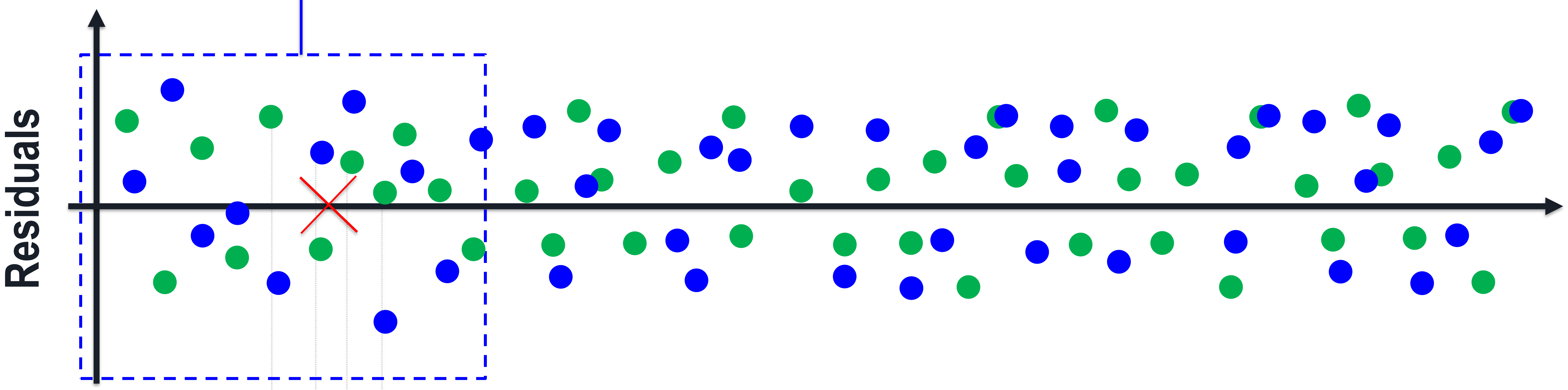
$$\bar{\boldsymbol{E}}_{\mathbf{1}} = [\bar{E}_1, \bar{E}_1]^{\mathrm{T}}$$

$$T_1^2 = r(\bar{\boldsymbol{E}}_{\mathbf{1}} - \boldsymbol{E}_o)^T \boldsymbol{\Sigma}_o^{-1} (\bar{\boldsymbol{E}}_{\mathbf{1}} - \boldsymbol{E}_o)$$



Pattern classification:

$\bar{E}_o, \Sigma_o, UCL$ (e.g. confidence level of 95%)



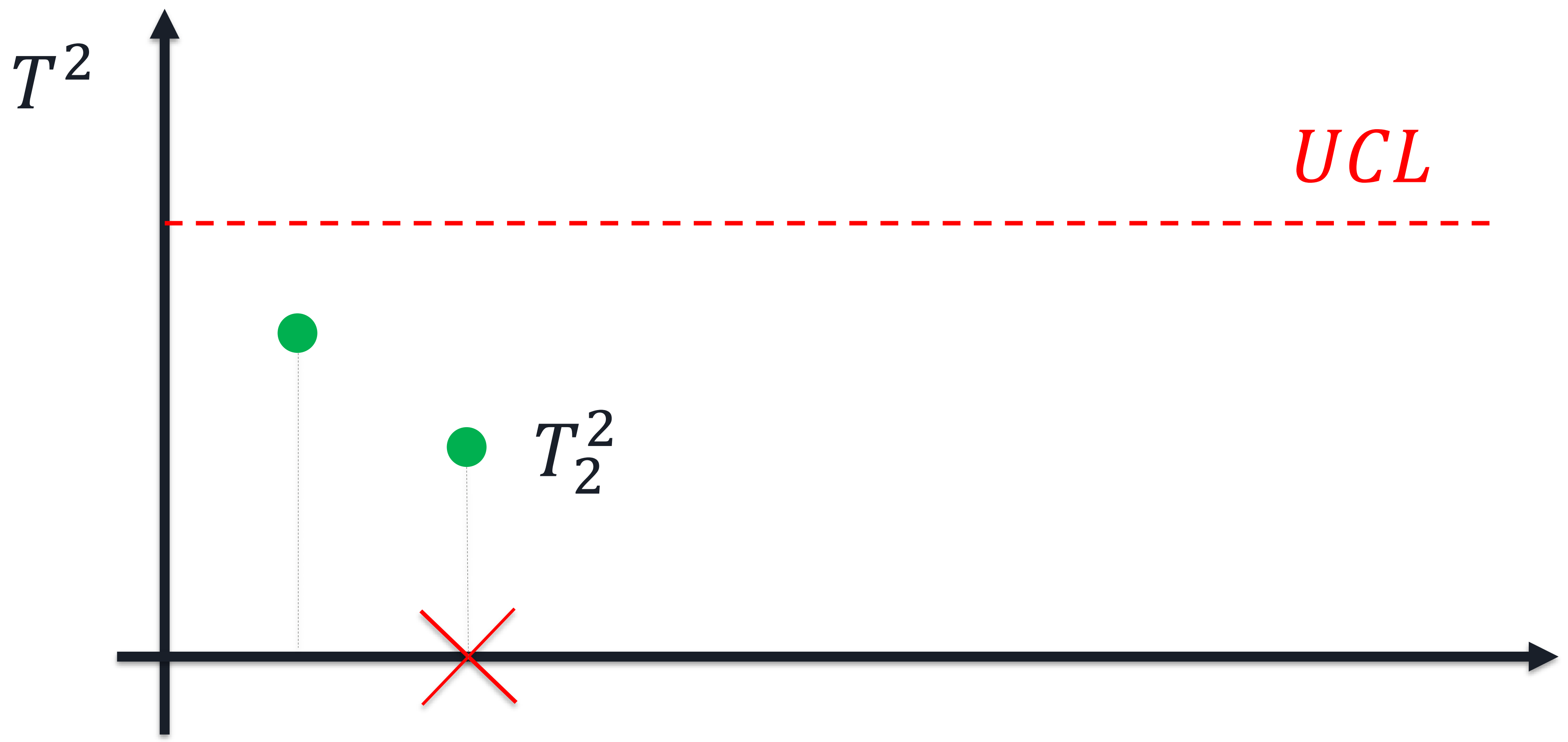
i=2

$\bar{E}_2$

$\bar{E}_2$

$$T_2^2 = r(\bar{\boldsymbol{E}}_2 - \boldsymbol{E}_o)^T \boldsymbol{\Sigma}_o^{-1} (\bar{\boldsymbol{E}}_2 - \boldsymbol{E}_o)$$

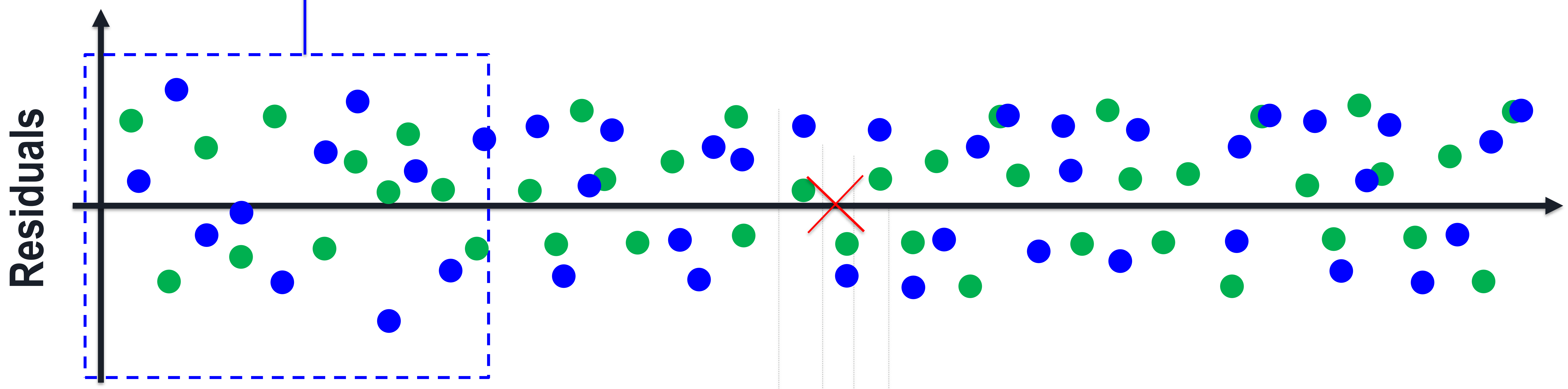
$$\bar{\boldsymbol{E}}_2 = [\bar{E}_2, \bar{E}_2]^{\mathrm{T}}$$



UNIVERSIDAD DE GRANADA

Pattern classification:

$\bar{E}_o, \Sigma_o, UCL\ (e.g.\ confidence\ level\ of\ 95\%)$



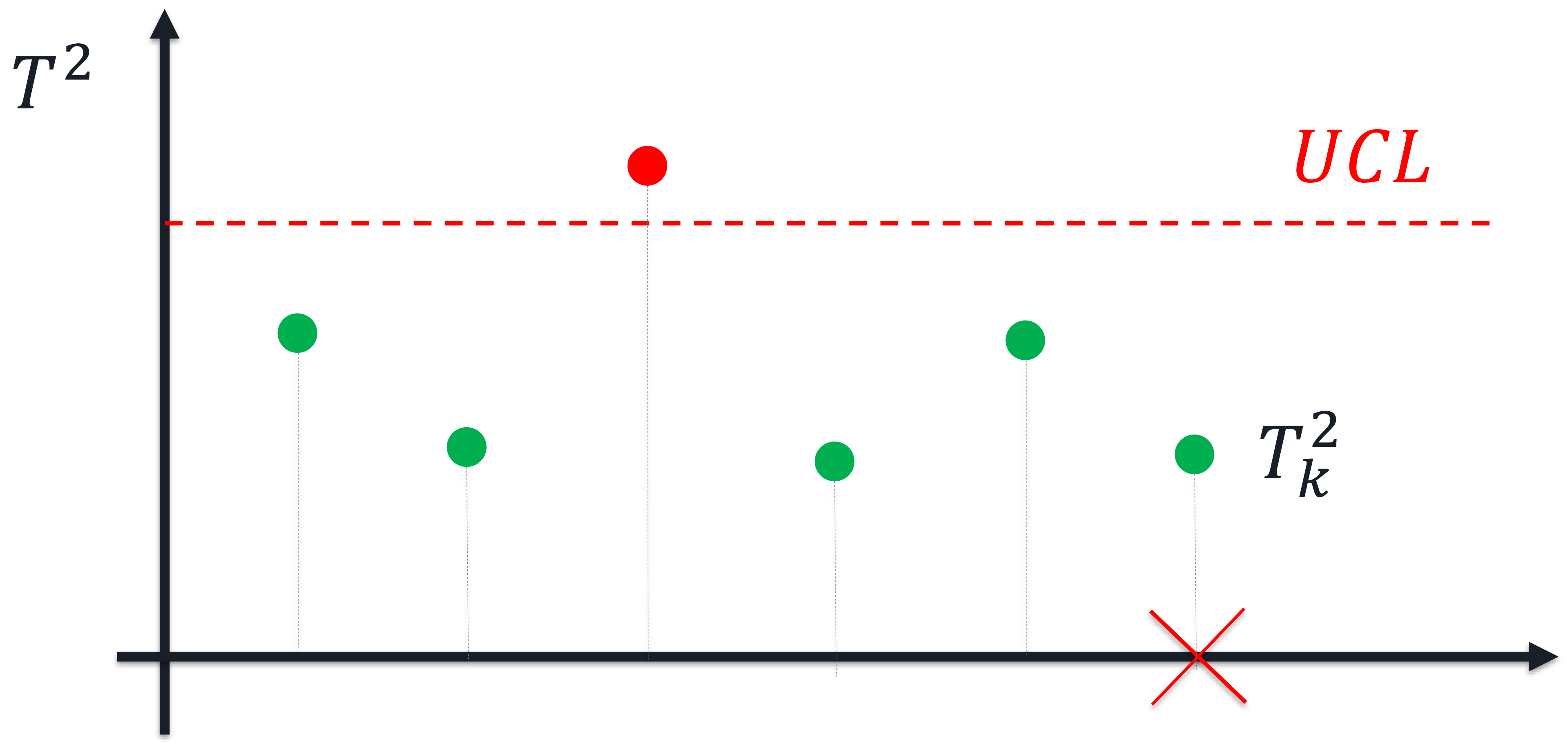
i=k

$\bar{E}_k$

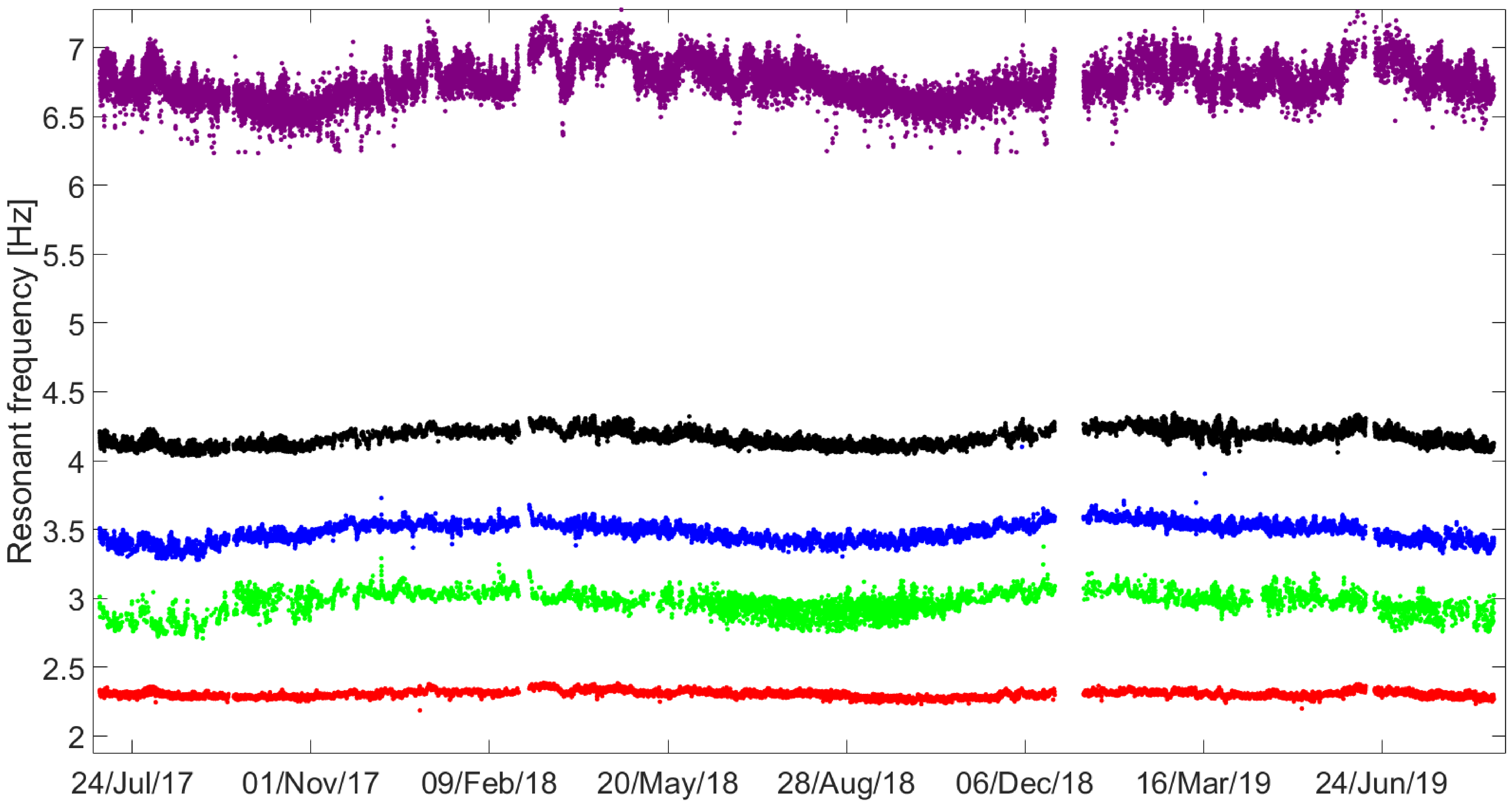
$\bar{E}_k$

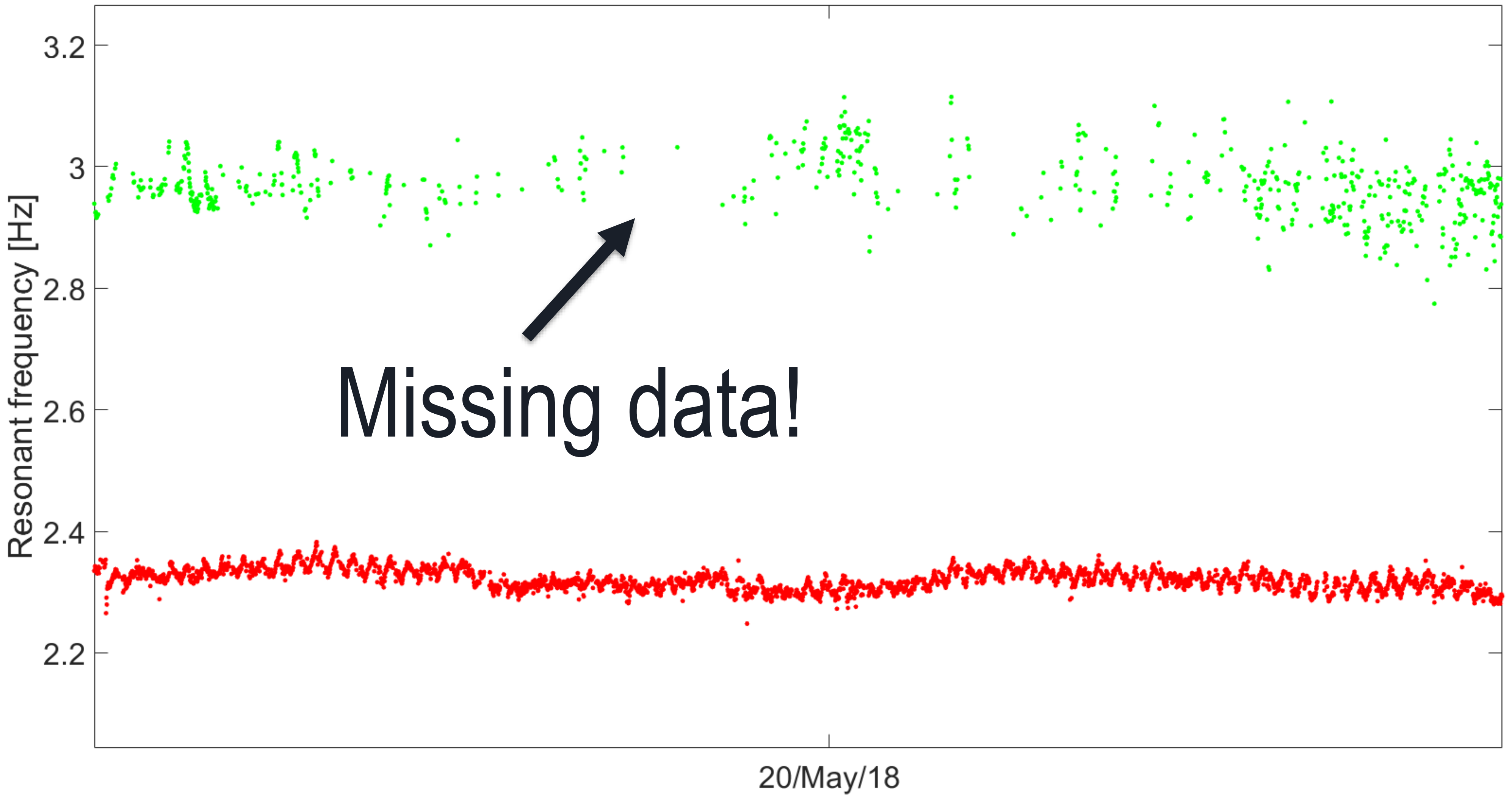
$\bar{\boldsymbol{E}}_{\boldsymbol{k}} = [\bar{E}_k, \bar{E}_k]^{\mathrm{T}}$

$$T_k^2 = r(\bar{\boldsymbol{E}}_{\boldsymbol{k}} - \boldsymbol{E}_o)^T \boldsymbol{\Sigma}_o^{-1} (\bar{\boldsymbol{E}}_{\boldsymbol{k}} - \boldsymbol{E}_o)$$

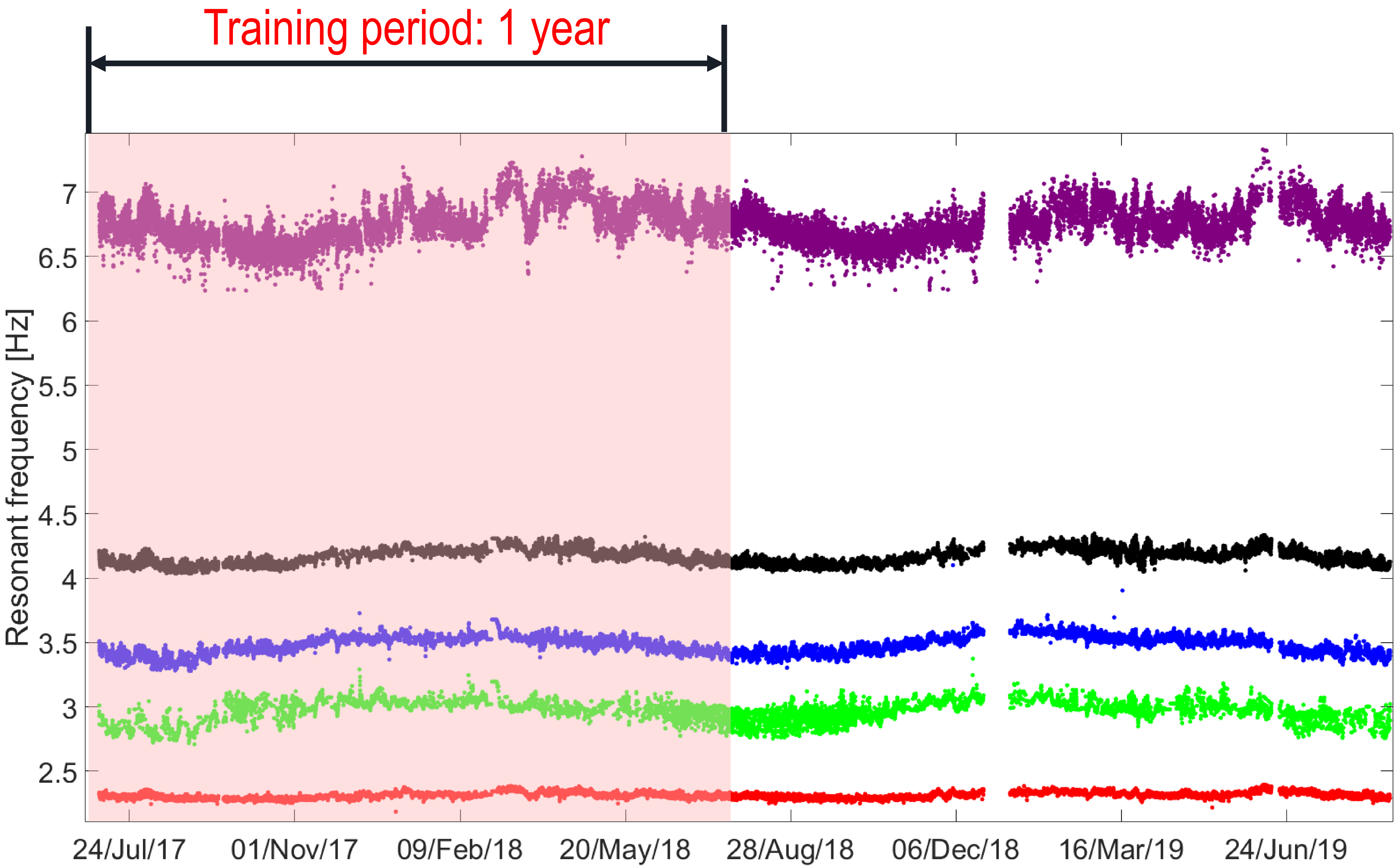


Example: Consoli Palace



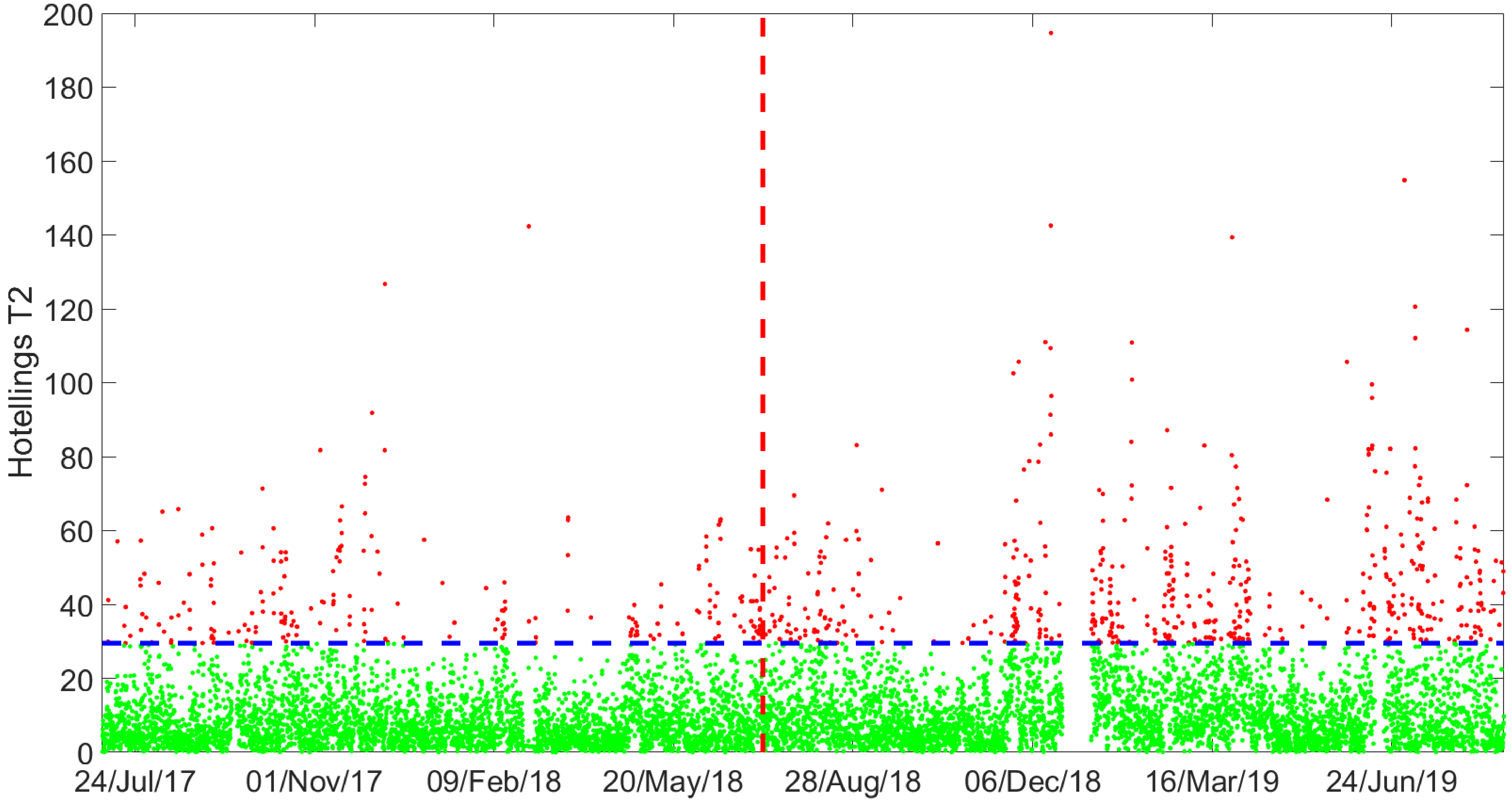


Example: Consoli Palace



Taller Parte 2

Undamaged



Damaged