UNIVERSIDAD DE GRANADA

Máster Universitario en Estructuras
Curso 2020-2021

Tema III: Experimental Modal Analysis

Módulo: MÓDULO FUNDAMENTAL: CALIDAD Y DAÑO
Materia: **Análisis Modal y Detección de Defectos**

Enrique García Macías
enriquegm@ugr.es
Departamento de Mecánica de Estructuras e Ingeniería Hidráulica

Desarrollo del curso

		FECHA		HORA	PROFESOR	TEMA	
Clase 1	**Lunes**	**1**	**febrero**	**9:30-11:30**	**EGM**	**1**	**Introducción: Análisis modal dentro del marco del mantenimiento de la salud estructural.**
Clase 2	Lunes	8	febrero	9:30-11:30	EGM	2	Fuentes de deterioro, patologías estructurales, y tecnologías de monitorización.
Clase 3	Lunes	15	febrero	9:30-11:30	EGM	3	Taller: procesamiento de señales.
Clase 4	Lunes	22	febrero	9:30-11:30	EGM	4	Análisis modal experimental.
Clase 5	Lunes	15	marzo	9:30-11:30	EGM	5	Análisis modal operacional.
Clase 6	Lunes	12	abril	9:30-11:30	EGM	6	Análisis modal operacional automatizado. Práctica de labotatorio I.
Clase 7	Lunes	19	abril	9:30-11:30	EGM	7	Taller: Identificación del daño estructural.
Clase 8	Lunes	26	abril	9:30-11:30	RCT	8	Técnicas de identificación dinámica basadas en análisis modal operacional.
Clase 9	Lunes	26	abril	12:00-14:00	RCT	9	Práctica de laboratorio II: Test de vibración ambiental.
Clase 10	Martes	27	abril	9:30-11:30	RCT	10	Casos de estudio.
Clase 11	Martes	27	abril	12:00-14:00	RCT		Presentación de trabajos.

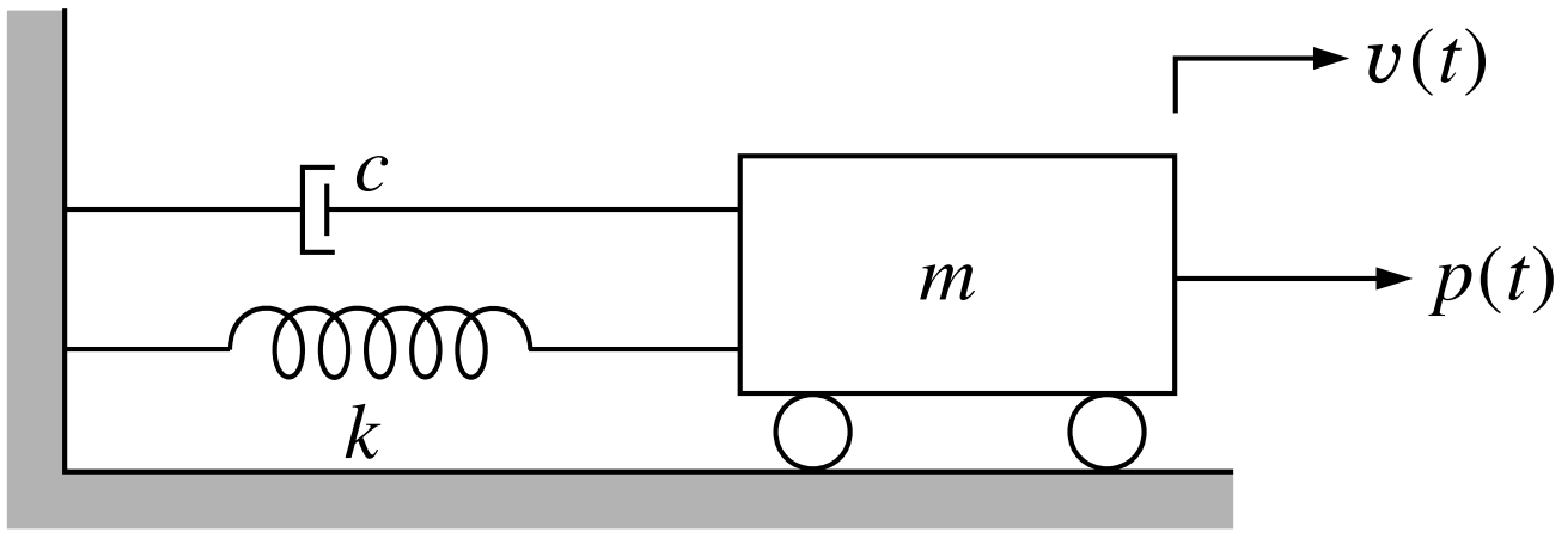
ENTREGA DE TRABAJOS Y EVALUACIÓN Del 3 al 28 de mayo

ÍNDICE

Repaso fundamentos básicos de dinámica de estructuras.

$$\xi = \frac{c}{2m\omega} \qquad \omega = \sqrt{\frac{k}{m}} \qquad \omega_d = \omega\sqrt{1-\xi^2}$$

SDOF



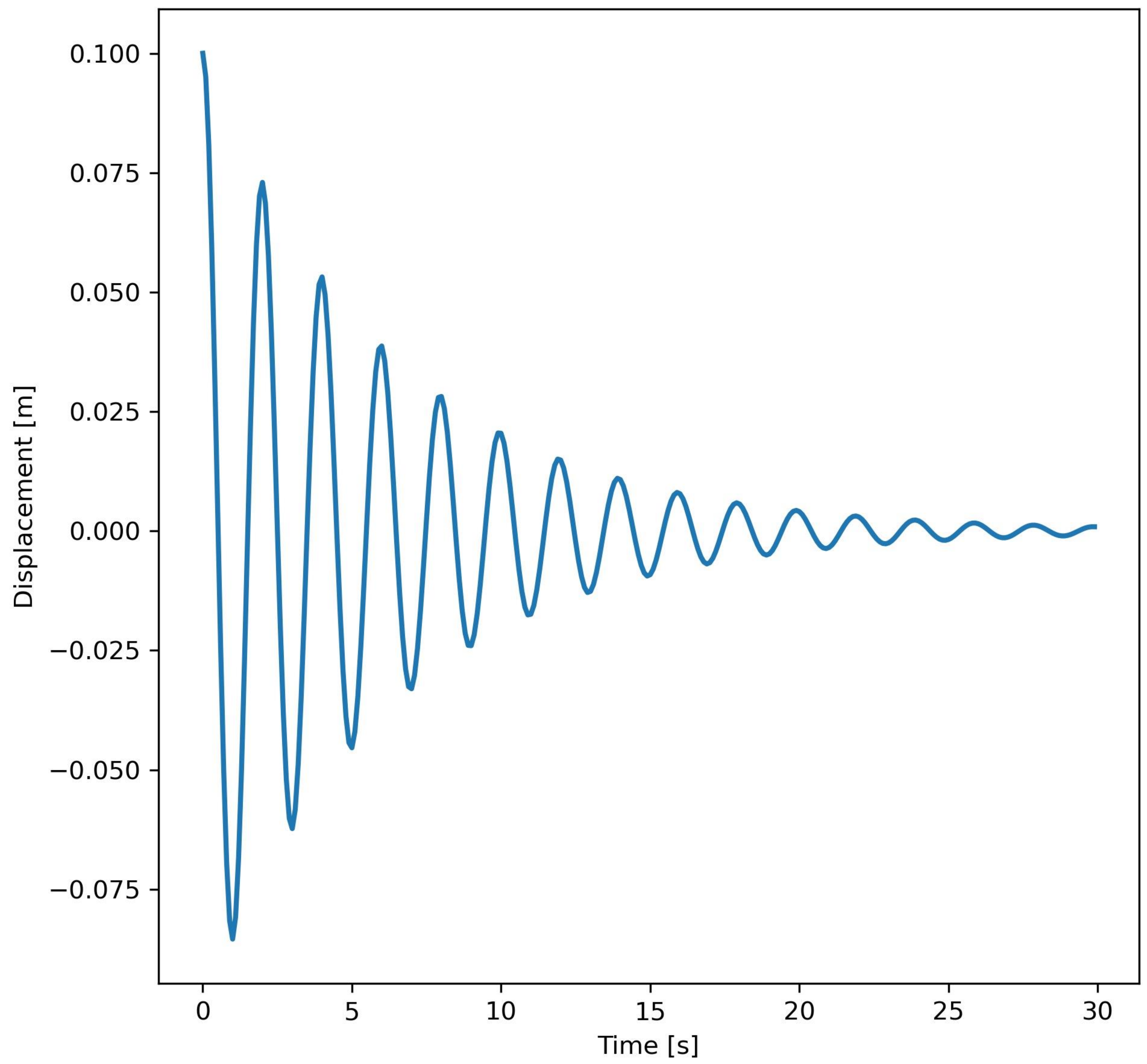
$$m\ddot{x} + c\dot{x} + kx = p(t)$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

Free vibrations

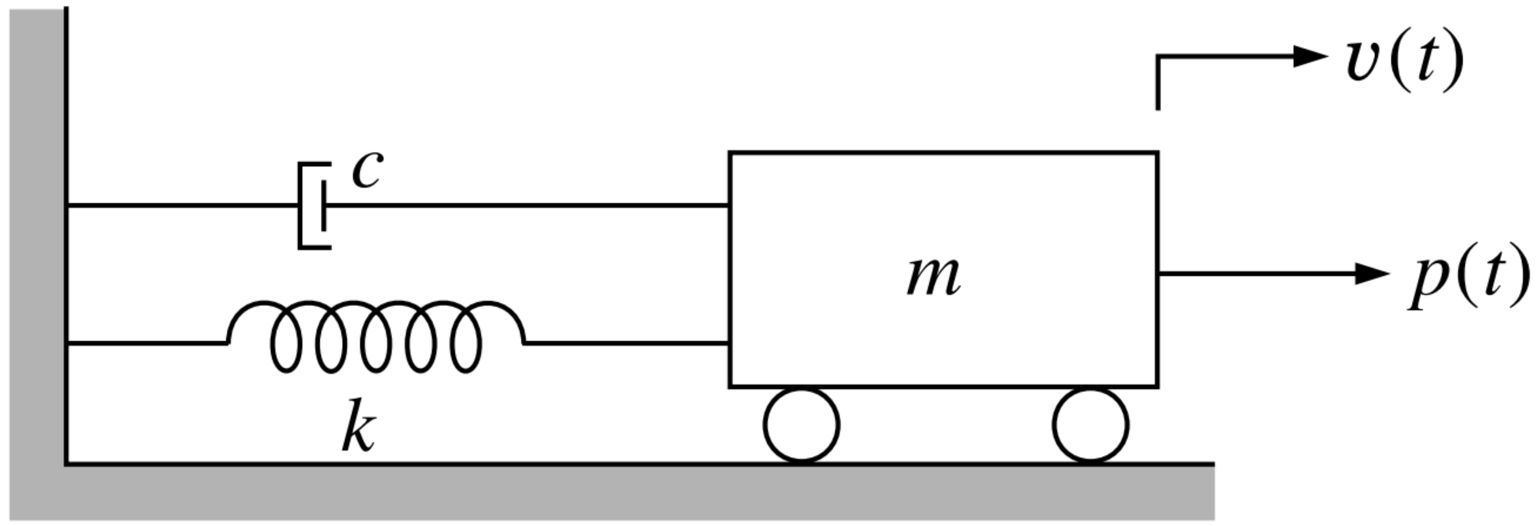
$$x(t) = e^{-\xi\omega t}[A\cos(\omega_d t) + B\sin(\omega_d t)]$$

BC
$$A = u_o$$
$$B = \frac{v_{o+\xi\omega A}}{\omega_d}$$



SDOF_Dynamic_system.py

Harmonic vibrations



$$m\ddot{x} + c\dot{x} + kx = p_o \sin(w_f t)$$

$$x_h(t) = e^{-\xi\omega t}[A\cos(\omega_d t) + B\sin(\omega_d t)]$$

$$x_p(t) = C_1 \sin(w_f t) + C_2 \cos(w_f t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

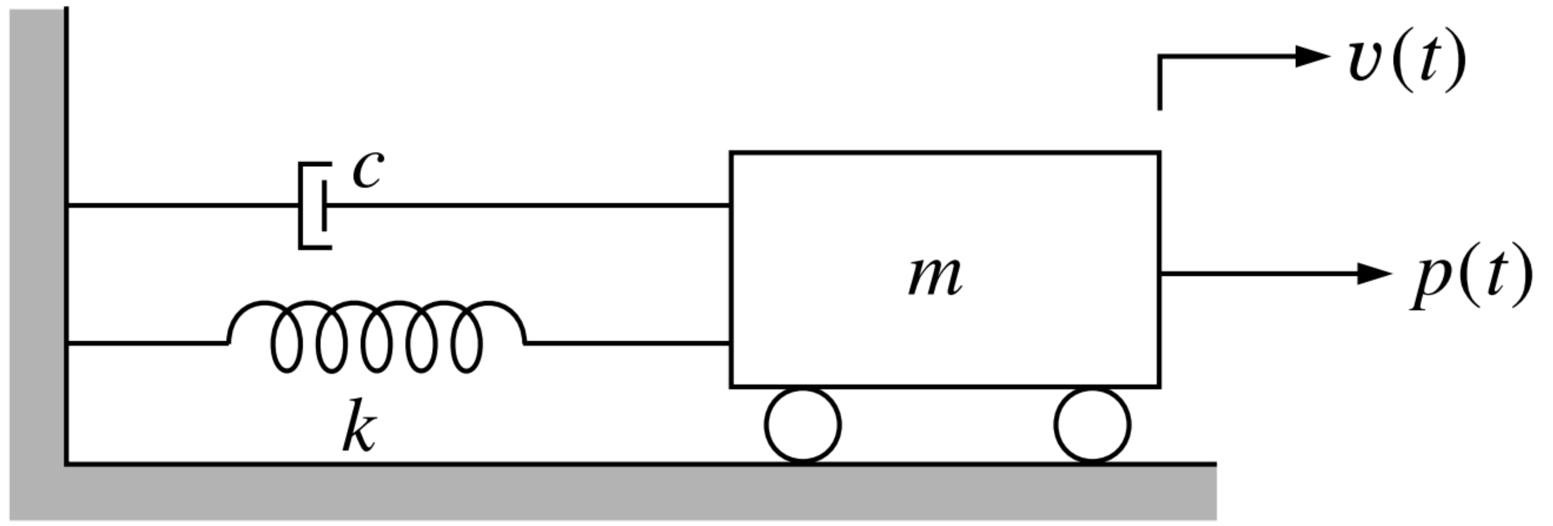
$$\omega_d = \omega\sqrt{1-\xi^2}$$

$$\left[-C_1 w_f^2 + C_2 w_f (2\xi\omega) + C_1\omega^2\right]\cos w_f t + \left[-C_2 w_f^2 - C_1 w_f(2\xi\omega) + C_2\omega^2 - \frac{p_o}{m}\right]\sin w_f t = 0$$

$$\left.\begin{aligned} -C_1 w_f^2 + C_2 w_f(2\xi\omega) + C_1\omega^2 &= 0 \\ -C_2 w_f^2 - C_1 w_f(2\xi\omega) + C_2\omega^2 - \frac{p_o}{m} &= 0 \end{aligned}\right\} \rightarrow \begin{aligned} C_1 &= \frac{p_0}{k}\left[-\frac{2\xi\beta}{(1-\beta^2)+(2\xi\beta)^2}\right] \\ C_2 &= \frac{p_0}{k}\left[-\frac{1-\beta^2}{(1-\beta^2)+(2\xi\beta)^2}\right] \end{aligned}$$

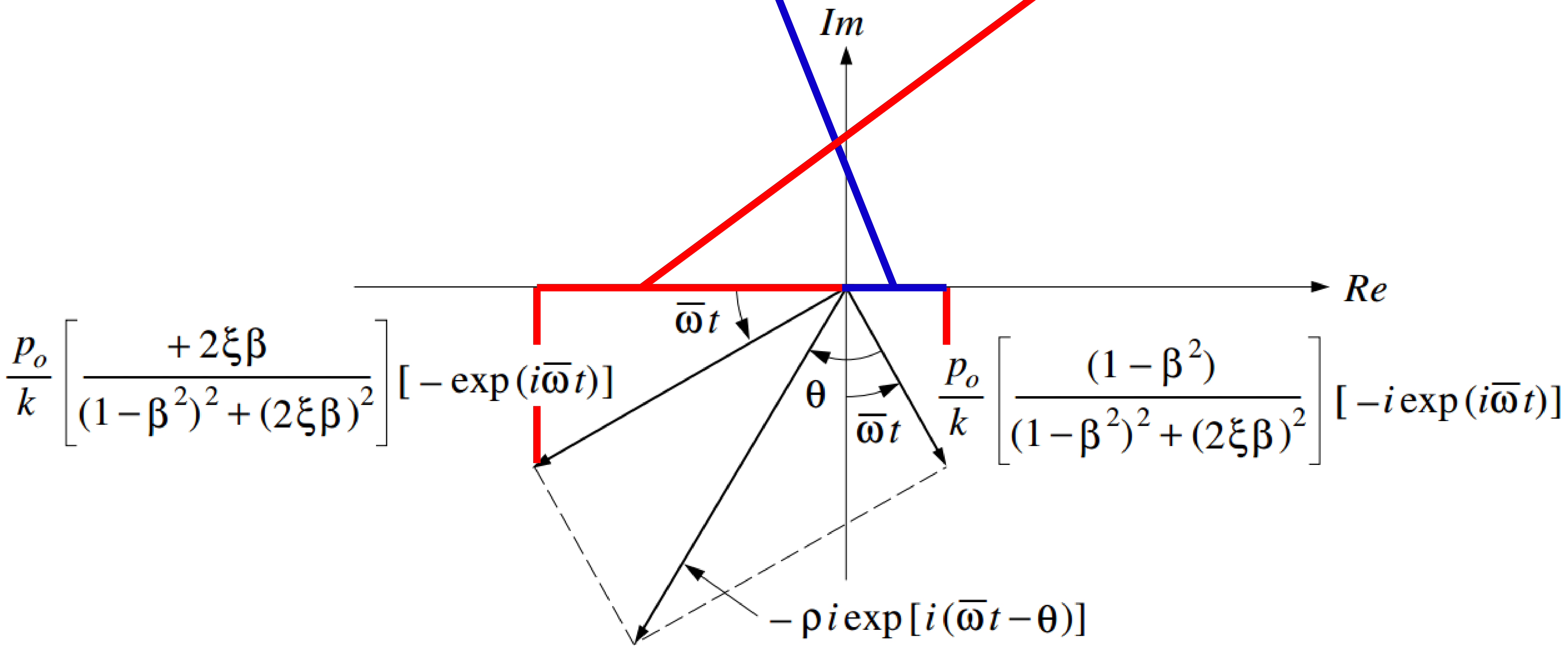
$$\beta = \frac{w_f}{\omega}$$

$$m\ddot{x} + c\dot{x} + kx = p_o \sin(w_f t)$$



$$x(t) = x_h(t) + x_p(t)$$
$$= e^{-\xi\omega t}[A\cos(\omega_d t) + B\sin(\omega_d t)]$$
$$\boxed{+\frac{p_o}{k}\left[\frac{1}{(1-\beta^2)+(2\xi\beta)^2}\right][(1-\beta^2)\sin(w_f t) - 2\xi\beta\cos(w_f t)]}$$

Steady-state harmonic response

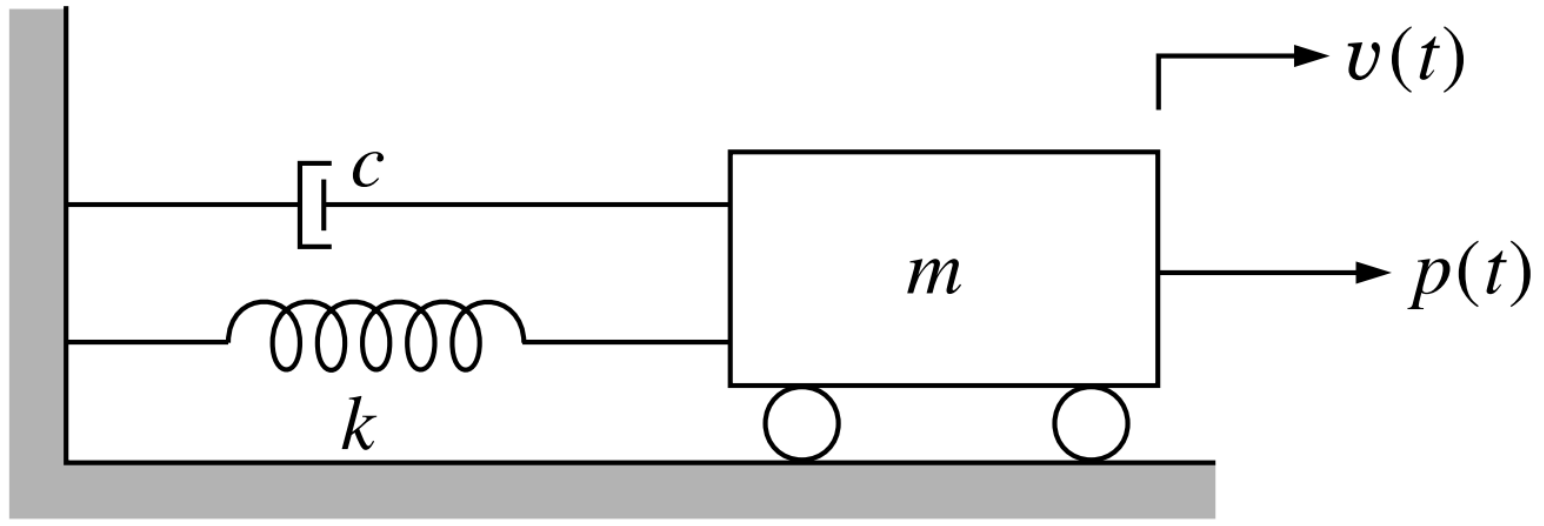


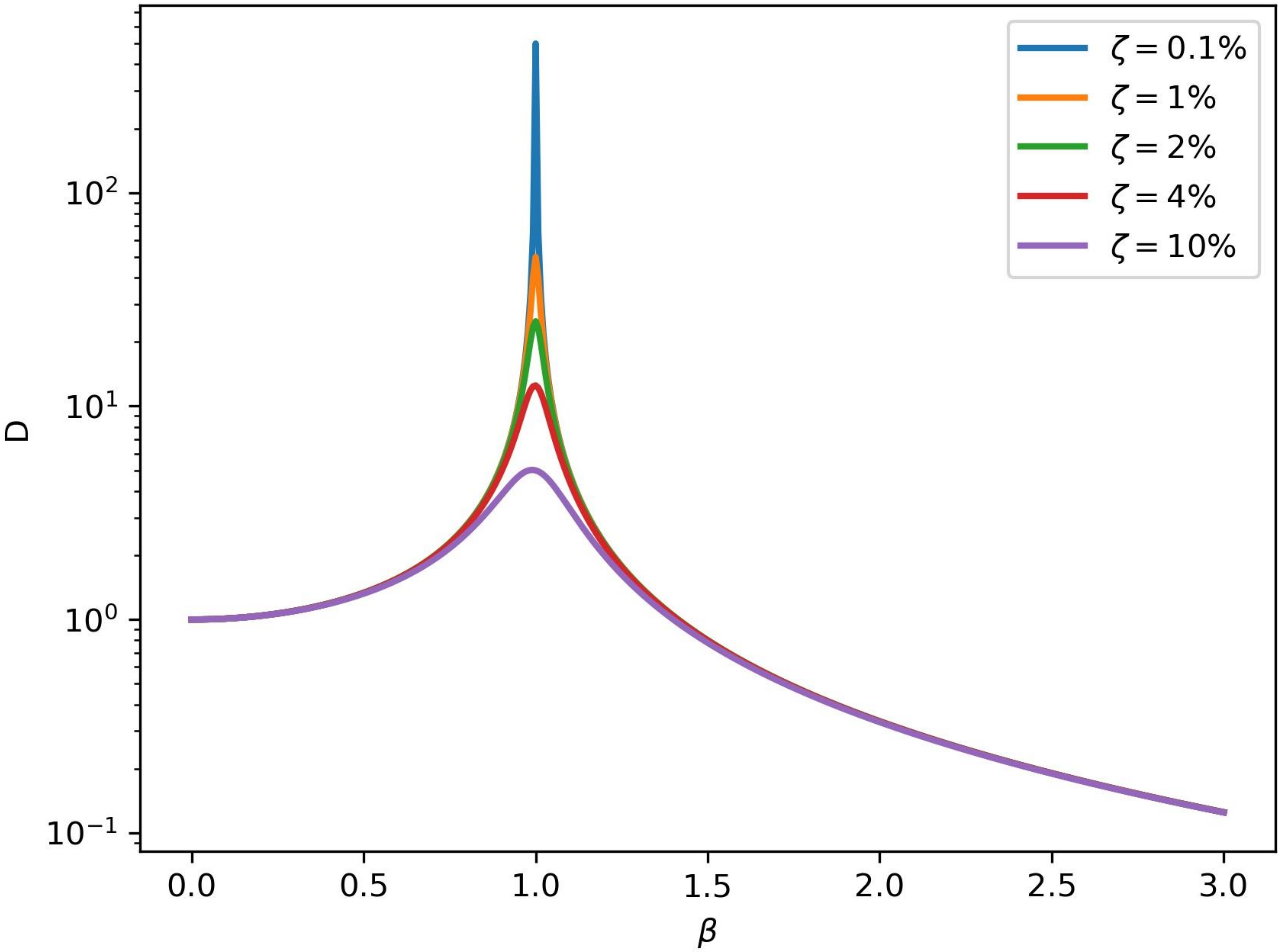
$$x_p(t) = p\sin(w_f t + \theta) \rightarrow p = \boxed{\frac{p_o}{k}[(1-\beta^2)+(2\xi\beta)^2]^{-1/2}}$$

D

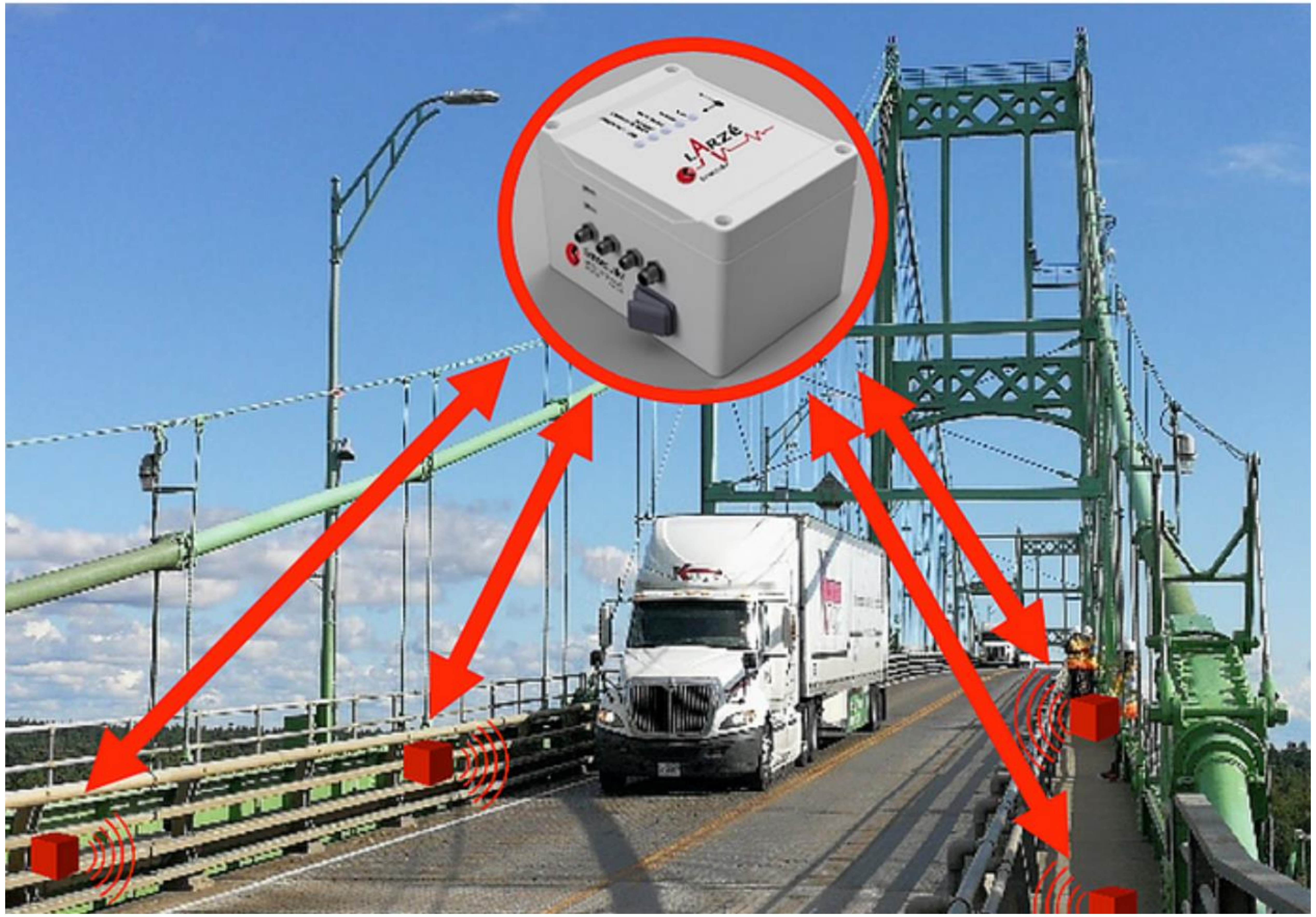
Dynamic amplification factor

$$D = [(1-\beta^2) + (2\xi\beta)^2]^{-1/2}$$

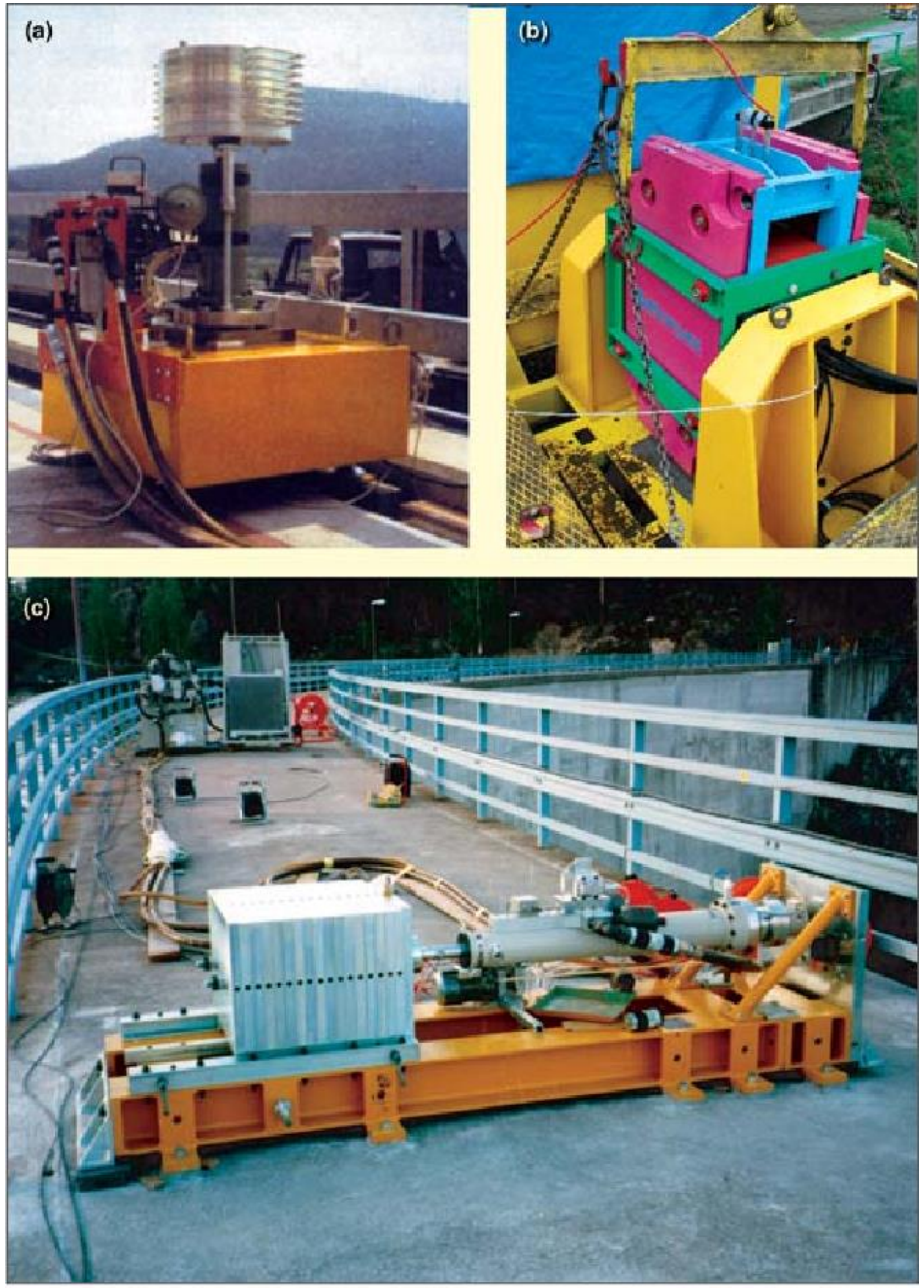




EMA vs OMA.



https://www.sensequake.com/blog-ambient-vibration-test



Cunha, A., & Caetano, E. (2006). Experimental modal analysis of civil engineering structures. Sound & Vibration 40(6)

	Mechanical Engineering	Civil Engineering
Experimental Modal Analysis (EMA)	❑ Artificial excitation ❑ Impact hammer shaker ❑ Controlled blast ❑ Well defined measured input	❑ Artificial excitation ❑ Hydraulic shaker ❑ Drop weights ❑ Pull back tests ❑ Eccentric shakers and exciters ❑ Well-defined measured, or un-measured input ❑ Controlled Blasts
Operational Modal Analysis (OMA)	❑ Artificial excitation ❑ Scratching device ❑ Air flow ❑ Acoustic emissions ❑ Unknown signal ❑ Random in time and space	❑ Natural Excitation ❑ Wind ❑ Waves ❑ Traffic ❑ Unknown signal ❑ Random in time and space with some spatial correlation

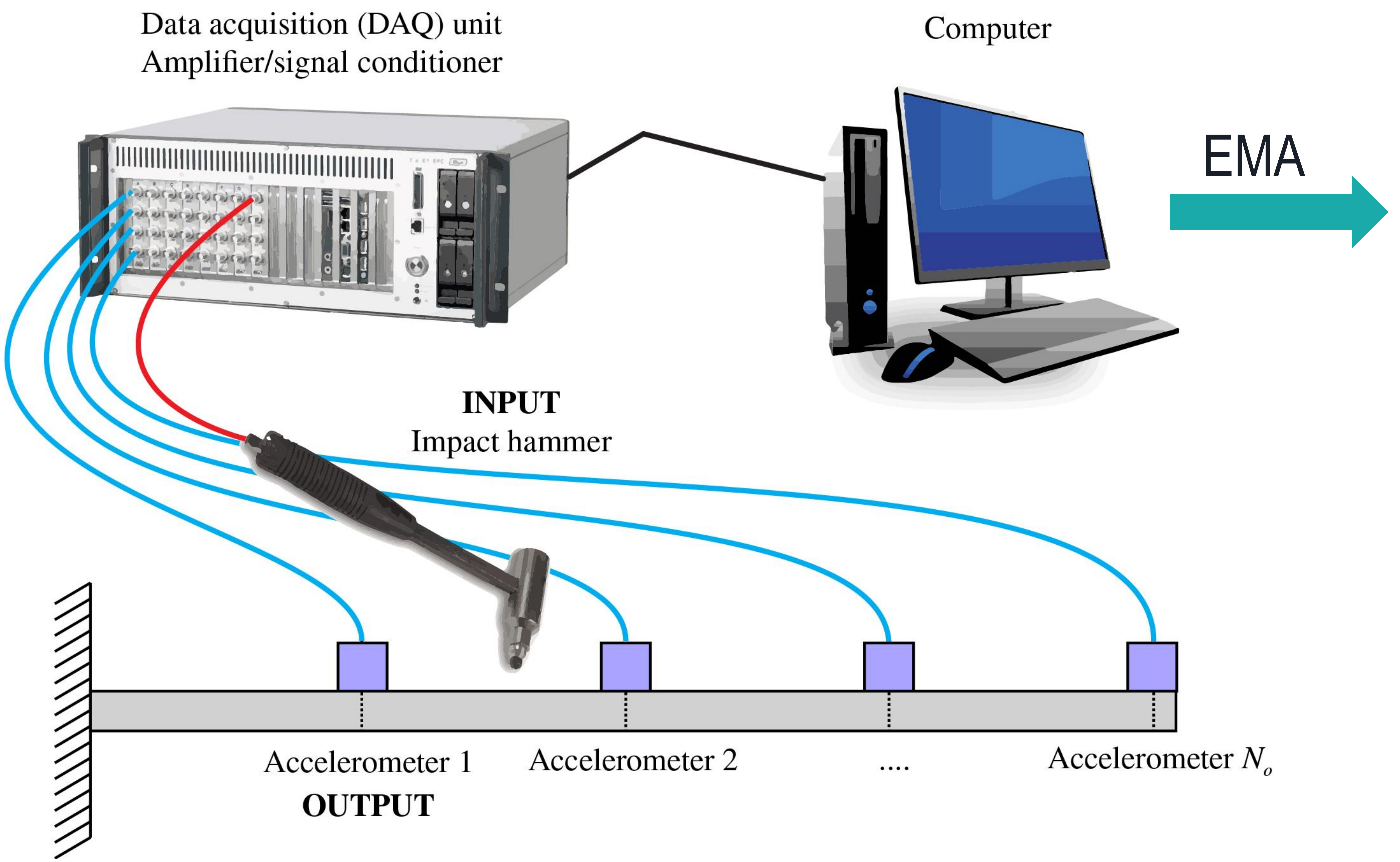
Remarks

- Sensors (accelerometers, or velocity/displacement sensors) cannot be located at nodes.
- The ability of the system for detecting damage depends on its capacity for identifying high-frequency modes.

Advantages/Disadvantages

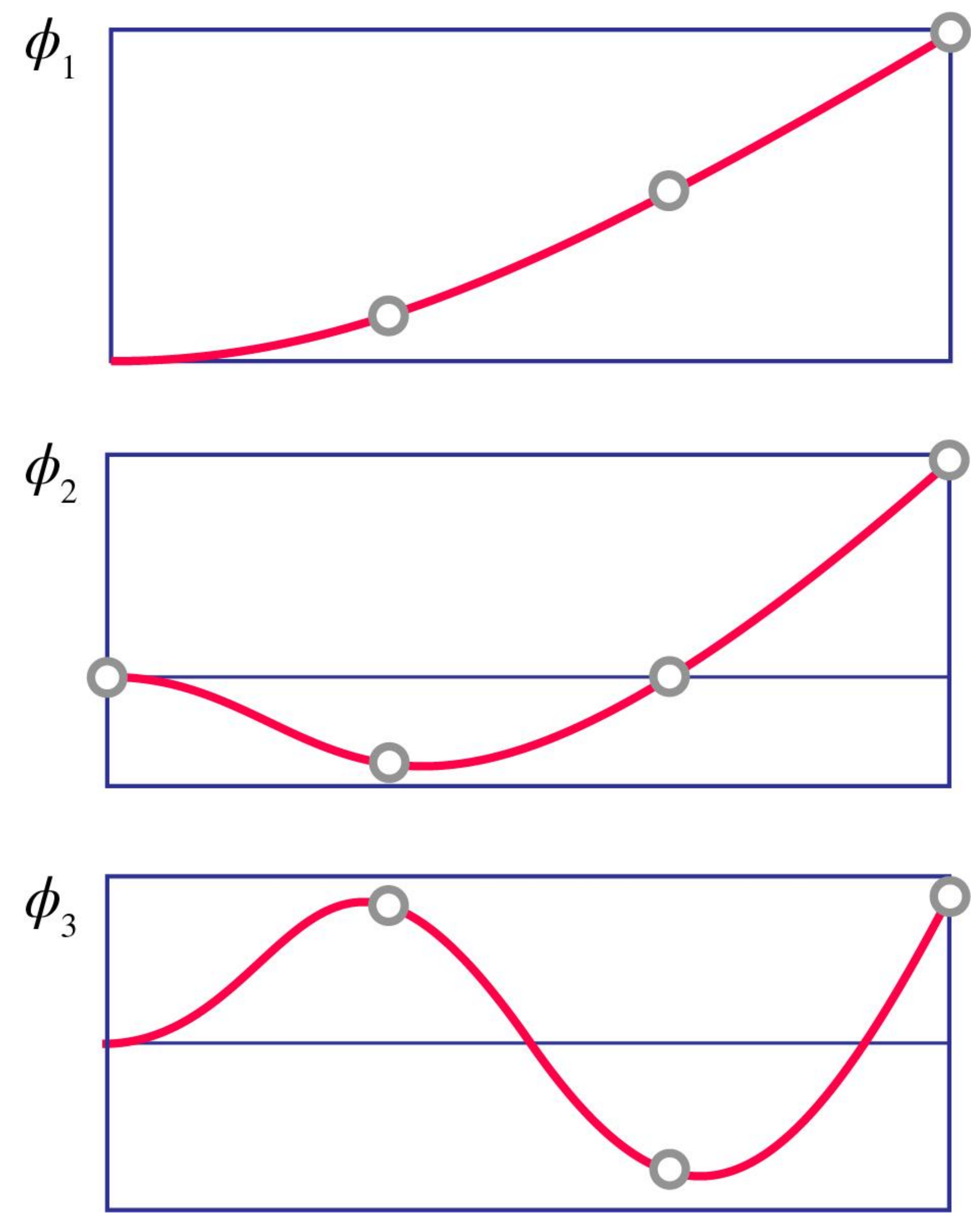
- Global damage identification.
- Local structural pathologies with limited effect on the overall stiffness of structures may go unnoticed.
- Damping (energy dissipation mechanisms) is very sensitive to damage, however its identification is highly dependent on the level of modal excitation and it is usually subject to high levels of uncertainty.
- Non-Destructive.
- It is not necessary to access difficult locations.
- **OMA – The normal operating conditions of the structure remain unaltered – Minimum intrusiveness.**
- **OMA – Readily applicable for continuous monitoring of structures.**

Análisis Modal Experimental

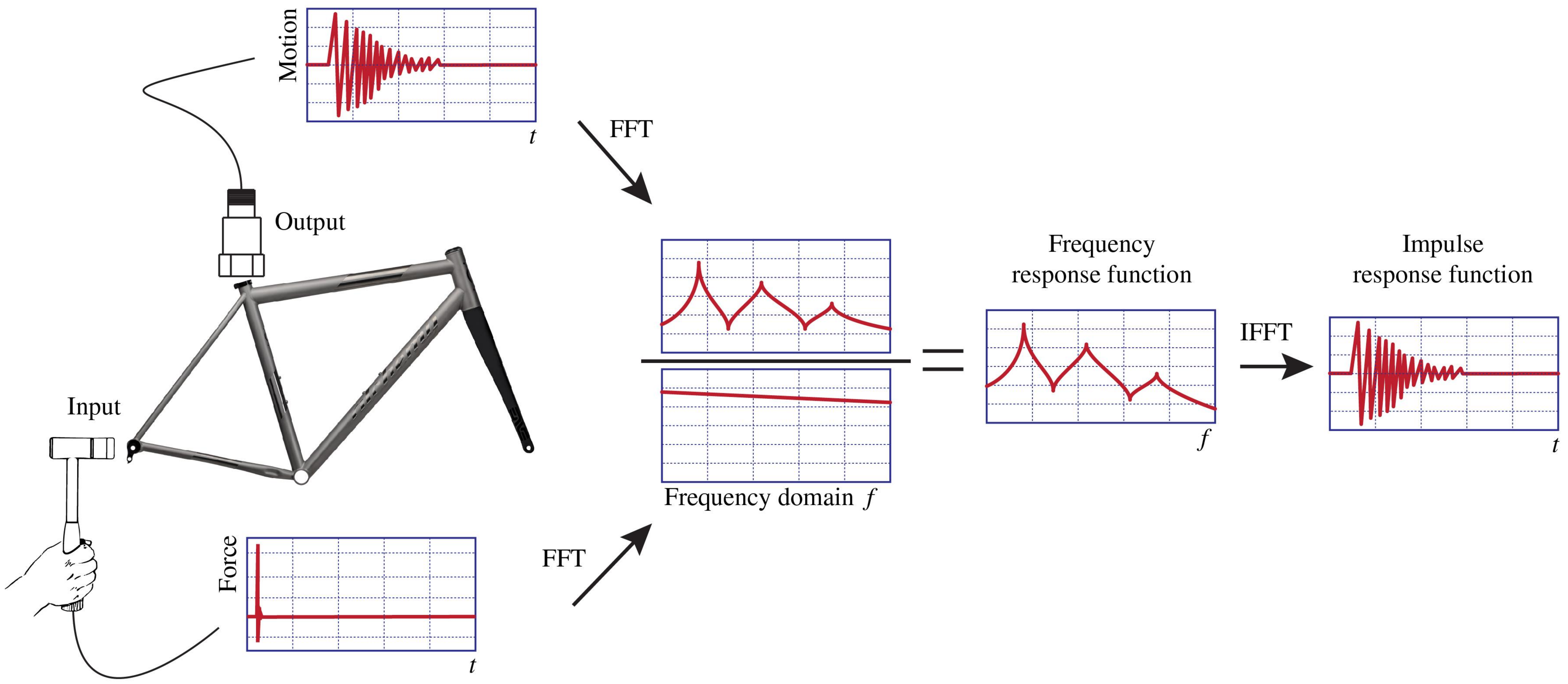


Procedimiento a través del cual se caracteriza el comportamiento dinámico de una estructura en términos de sus propiedades modales

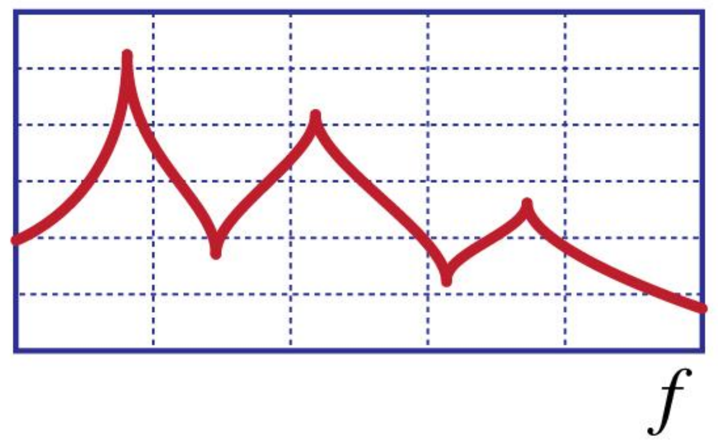
- **Frecuencias de resonancia**
- **Tasas de amortiguamiento**
- **Modos de vibración**



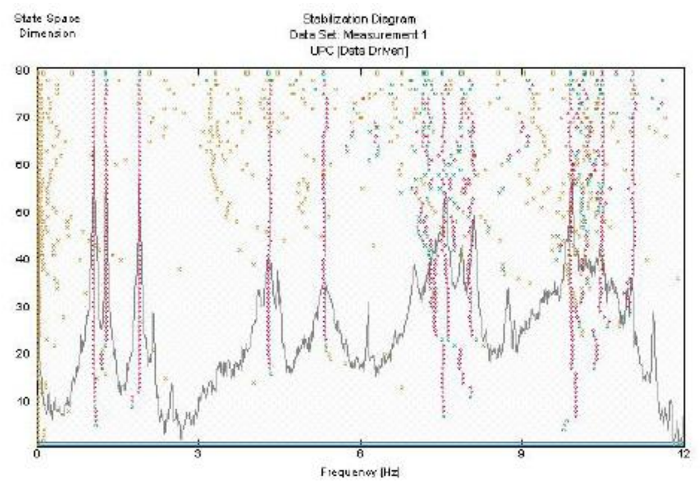
Análisis Modal Experimental



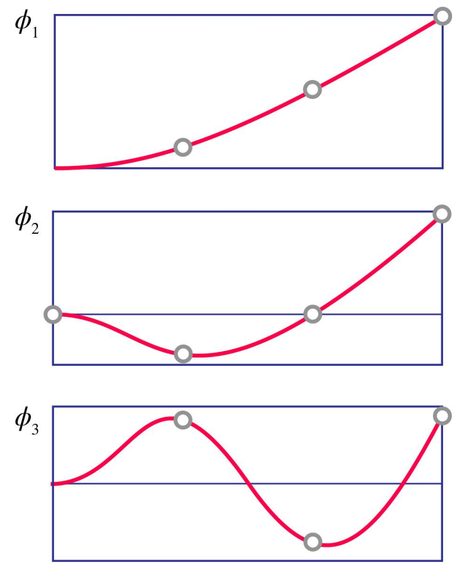
1. Medir FRFs



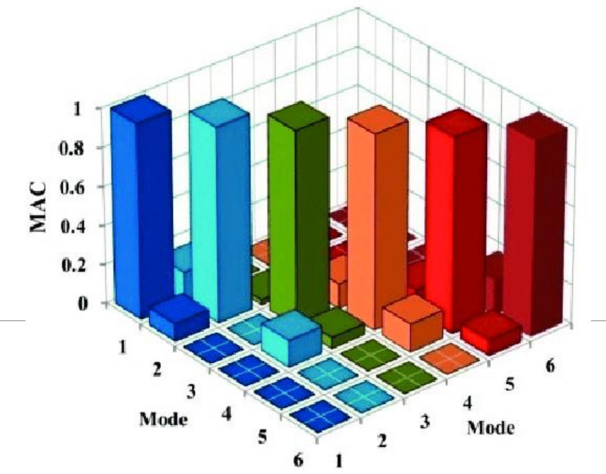
2. Estimar polos del sistema



3. Modos de vibración



4. Validación

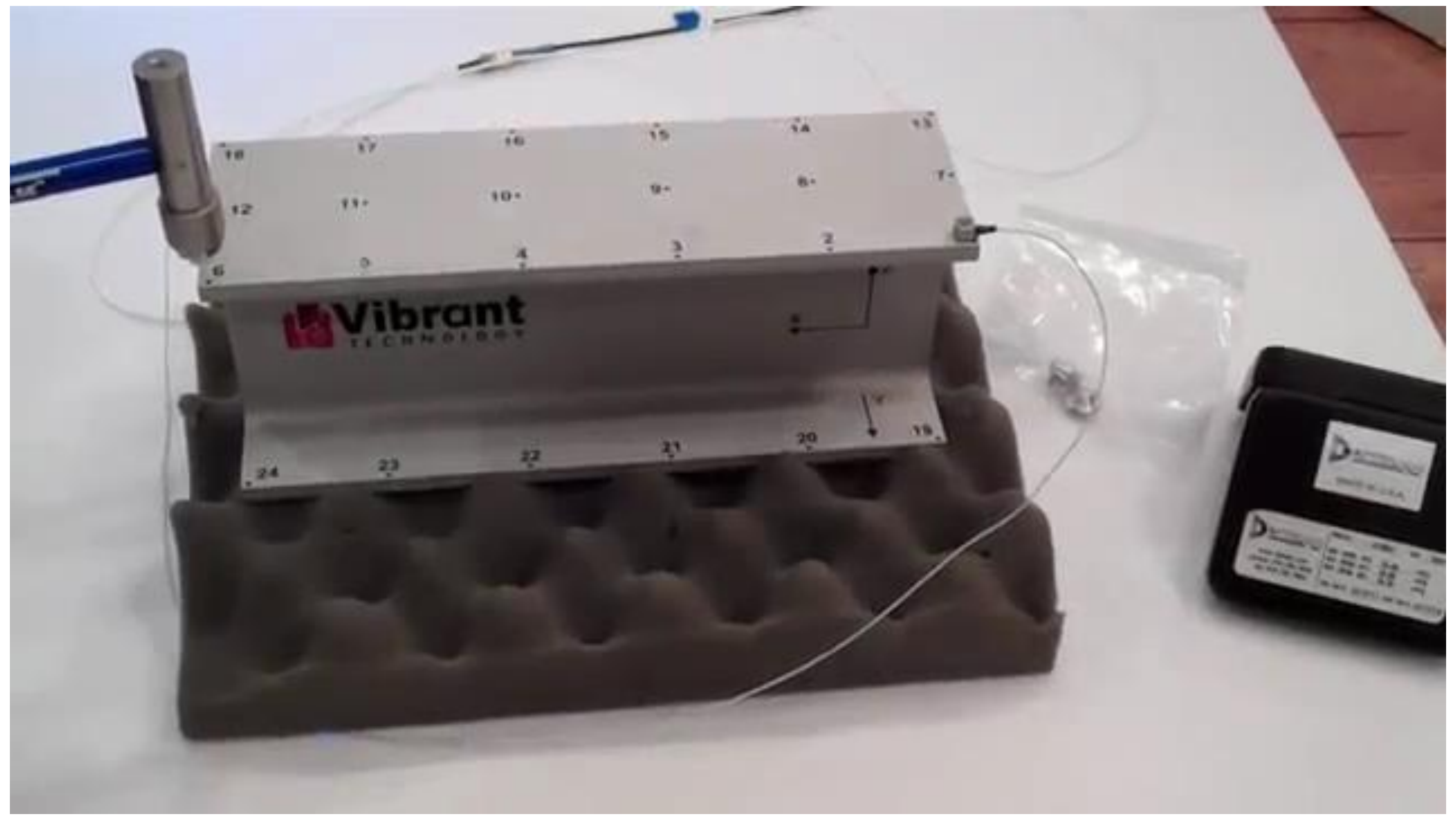


Métodos de excitación en EMA

Impact hammer

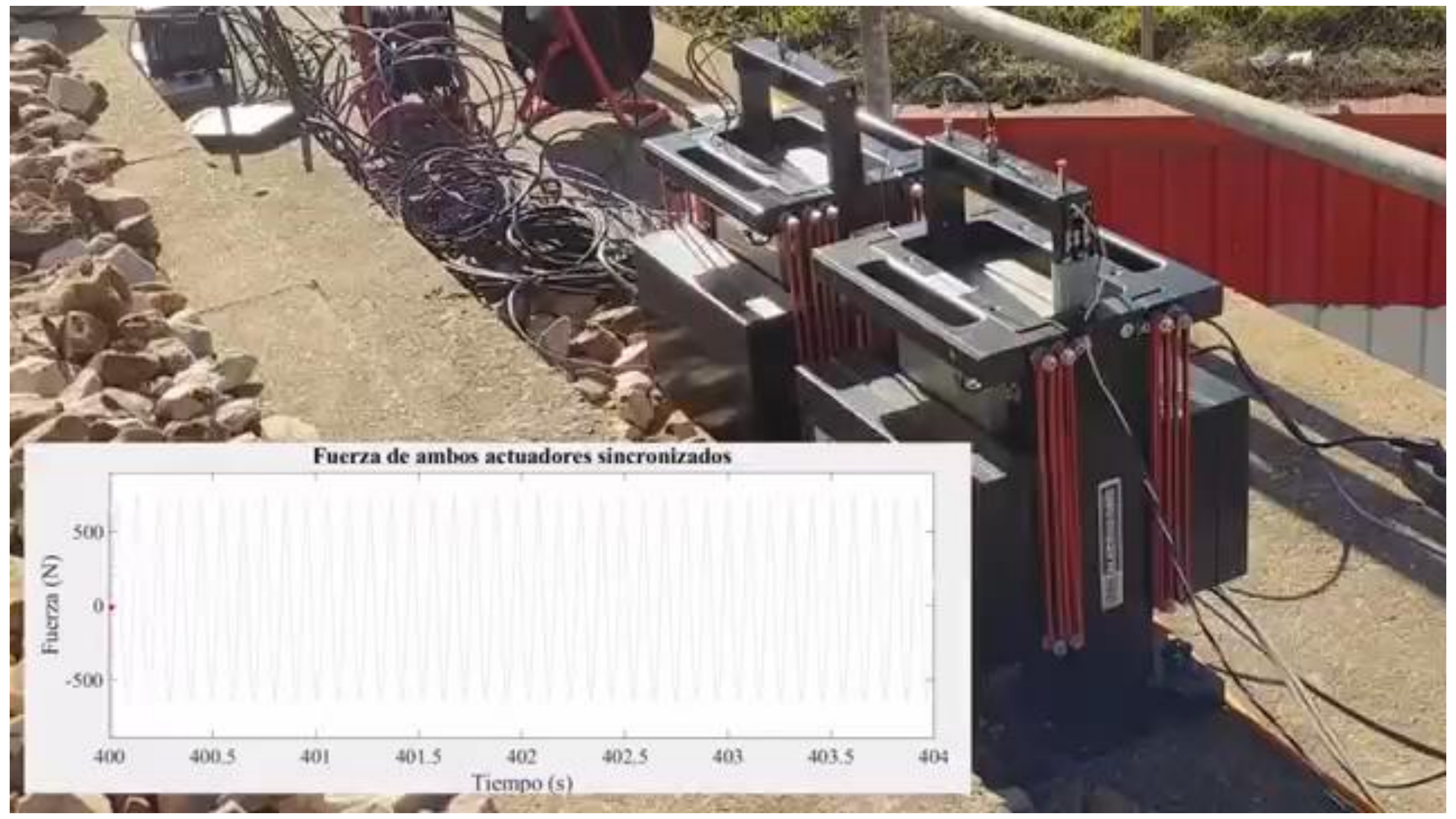
Shaker

Hydraulic actuator



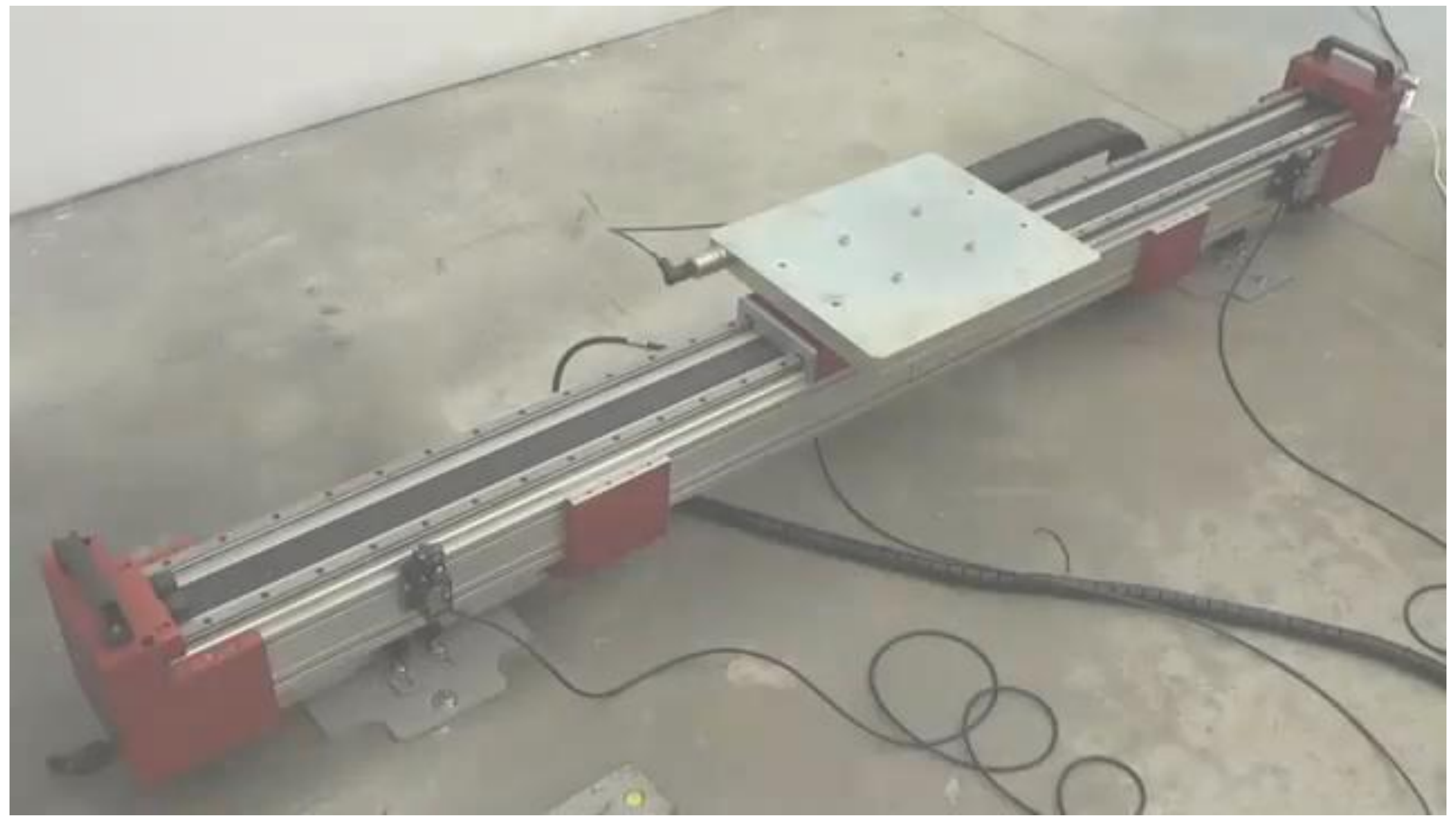
https://www.youtube.com/watch?v=tBRjPN8m6zE

https://www.youtube.com/watch?v=d3U_m-4XOtg



https://www.youtube.com/watch?v=y2nG5uCGoRs&feature=share&fbclid=IwAR12W2yuyYOU7mr_P6XEKFWqVuNoV4elLbLSQ_Gii1QH9m34G83Cb4DftaY

Vibrodyne

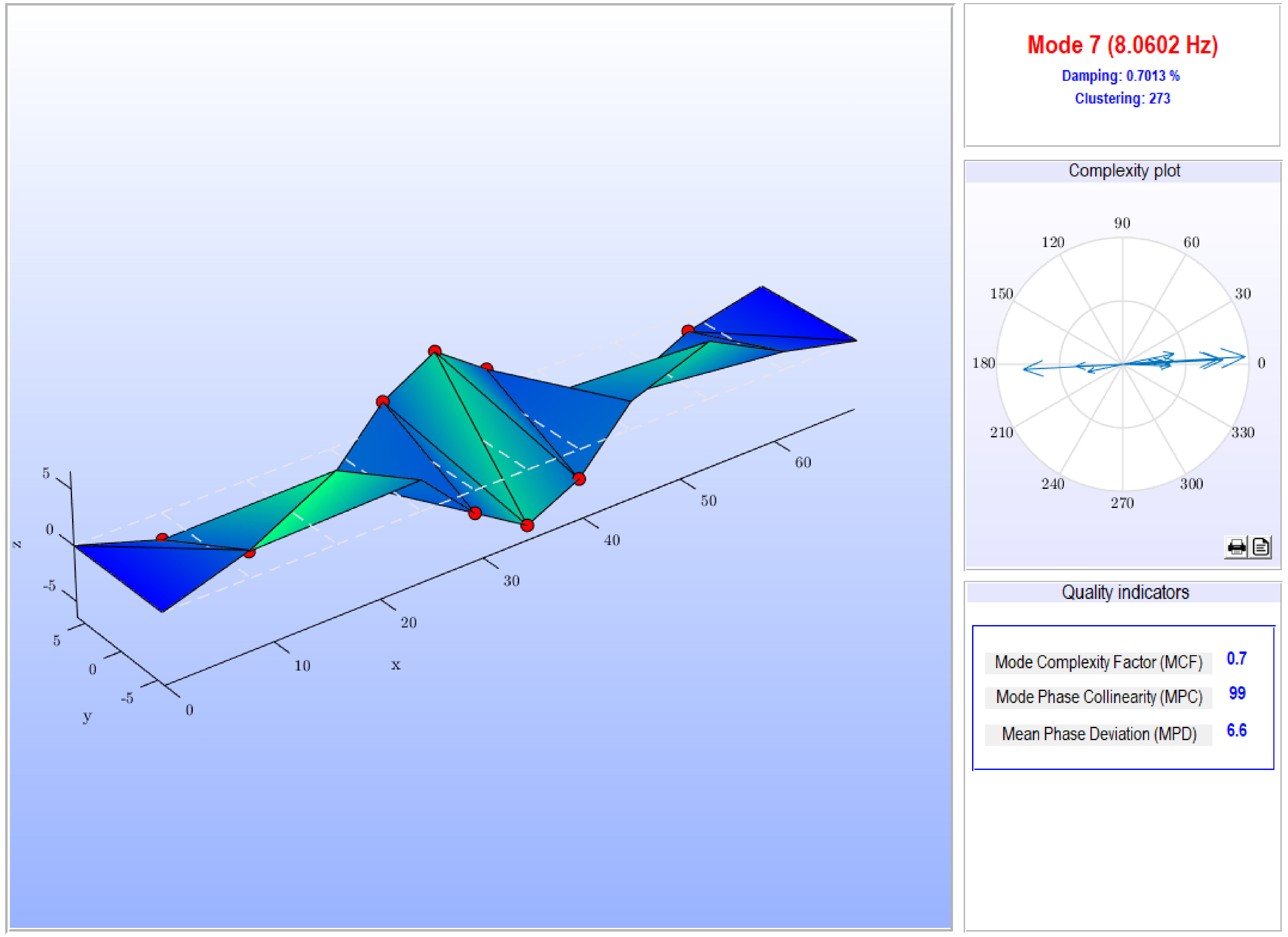


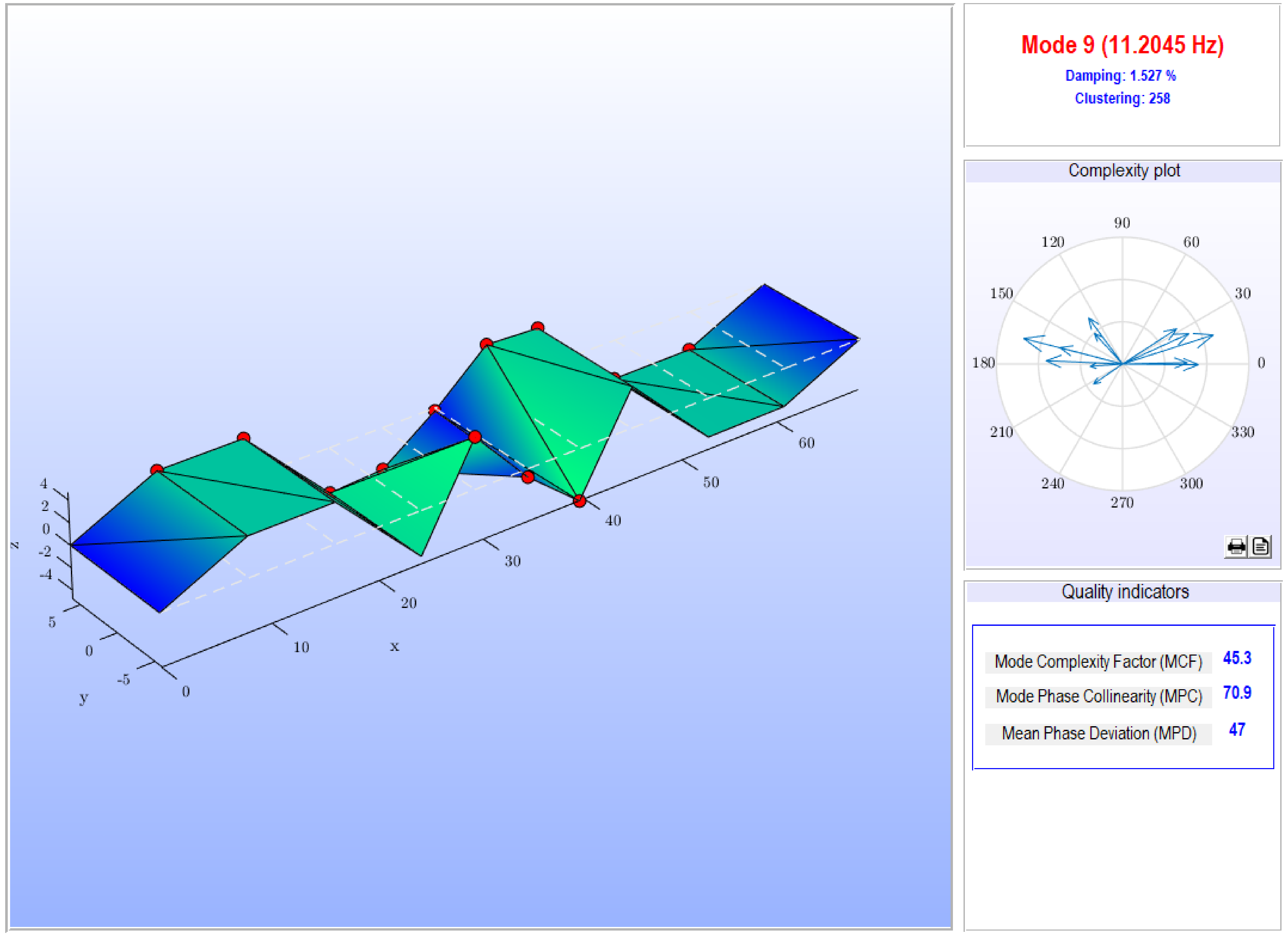
https://www.youtube.com/watch?v=gmWFK-vT6_Q&t=23s

https://www.youtube.com/watch?v=ENO6gqO-Uu0&t=135s

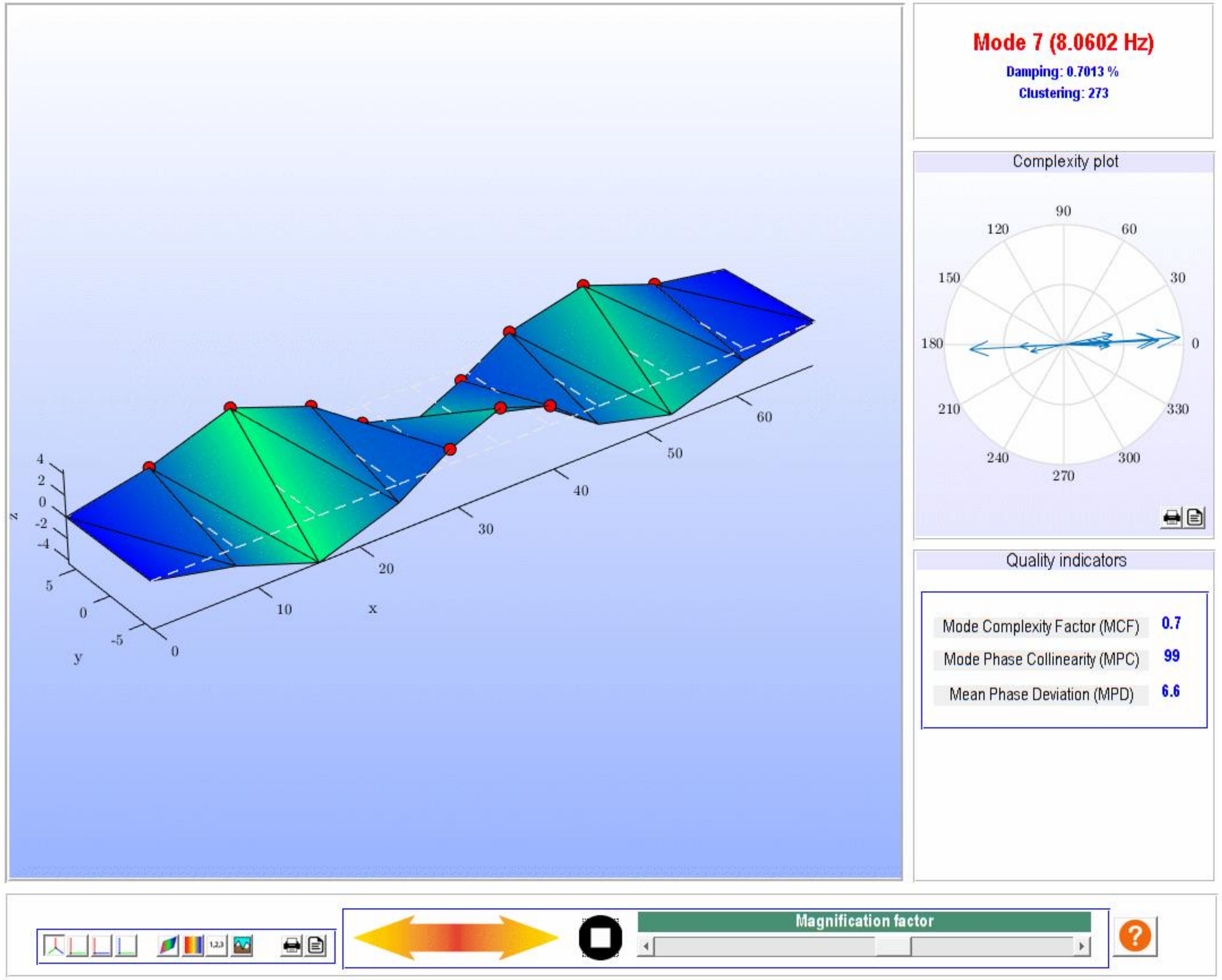
Fundamentos teóricos de EMA (pdf)

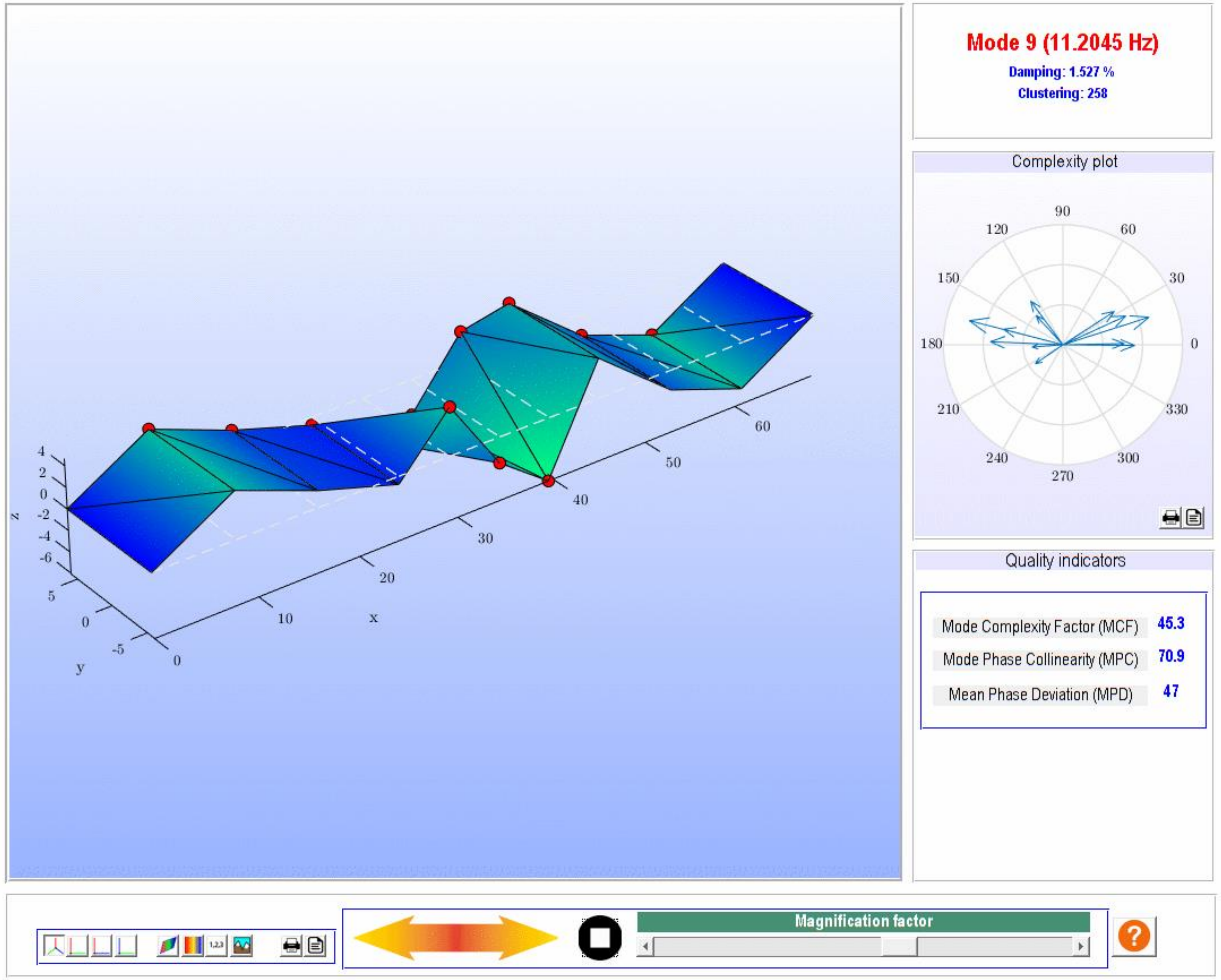
Modos complejos





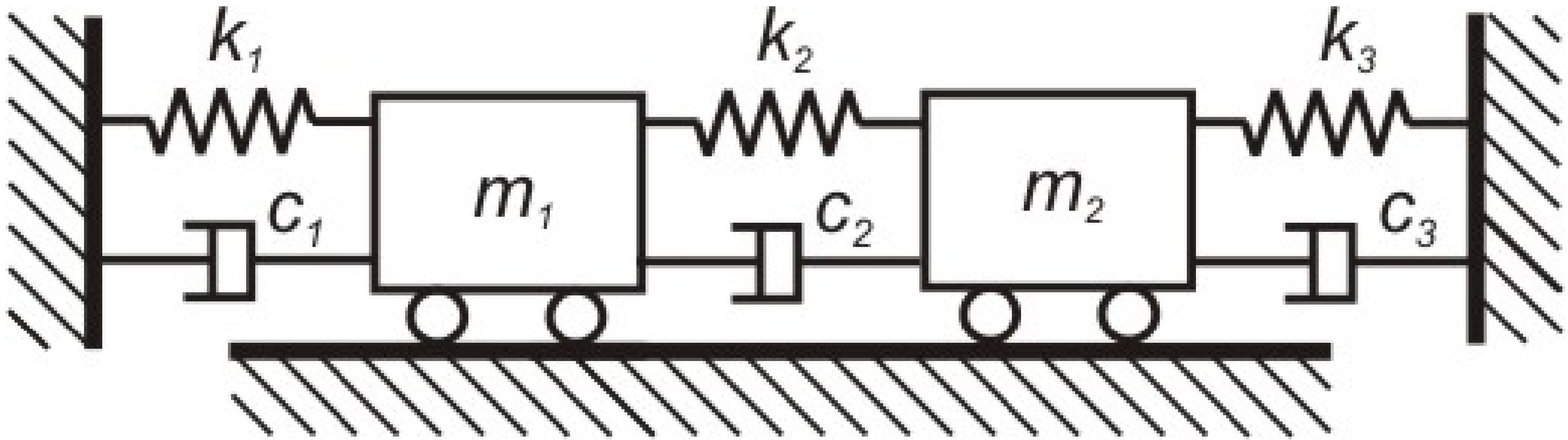
Modos complejos





Fundamentos teóricos de EMA
- Ejercicios

Sistema de 2 GDL con amortiguamiento general



$$m_1 = m_2 = 2 \text{ kg}$$

$$C_1 = 3 \frac{\text{N}}{\text{m/s}}, C_2 = 1 \frac{\text{N}}{\text{m/s}}, C_3 = 4 \frac{\text{N}}{\text{m/s}}$$

$$k_1 = 4000 \frac{\text{N}}{\text{m}}, k_2 = 2000 \frac{\text{N}}{\text{m}}, k_3 = 4000 \frac{\text{N}}{\text{m}}$$

Determinar:

- Polos del sistema
- Modos de vibración
- Matrices de residuos
- Factores de contribución modal

Ecuaciones del movimiento

Dominio del tiempo

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 4 & -1 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \begin{bmatrix} 6000 & -2000 \\ -2000 & 6000 \end{bmatrix} \begin{bmatrix} \mathrm{x}_1\ (t) \\ \mathrm{x}_2\ (t) \end{bmatrix} = F$$

Dominio de Laplace

$$p^2 \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \mathrm{X}(p) + p \begin{bmatrix} 4 & -1 \\ -1 & 5 \end{bmatrix} \mathrm{X}(p) + \begin{bmatrix} 6000 & -2000 \\ -2000 & 6000 \end{bmatrix} \mathrm{X}(p) = \mathrm{F}(p)$$

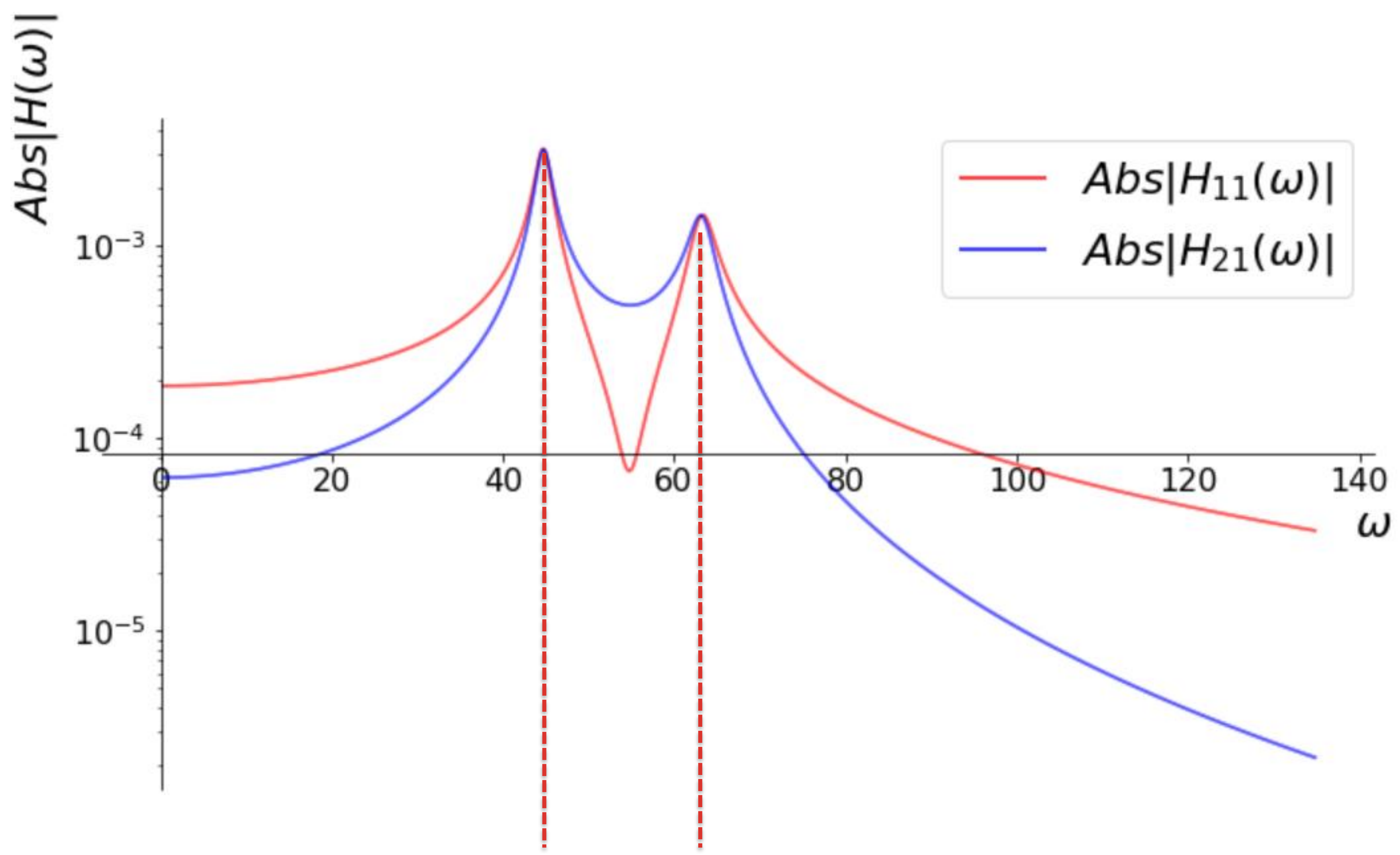
Rigidez dinámica

$$Z = \begin{bmatrix} 2p^2 + 4p + 6000 & -p - 2000 \\ -p - 2000 & 2p^2 + 5p + 6000 \end{bmatrix}$$

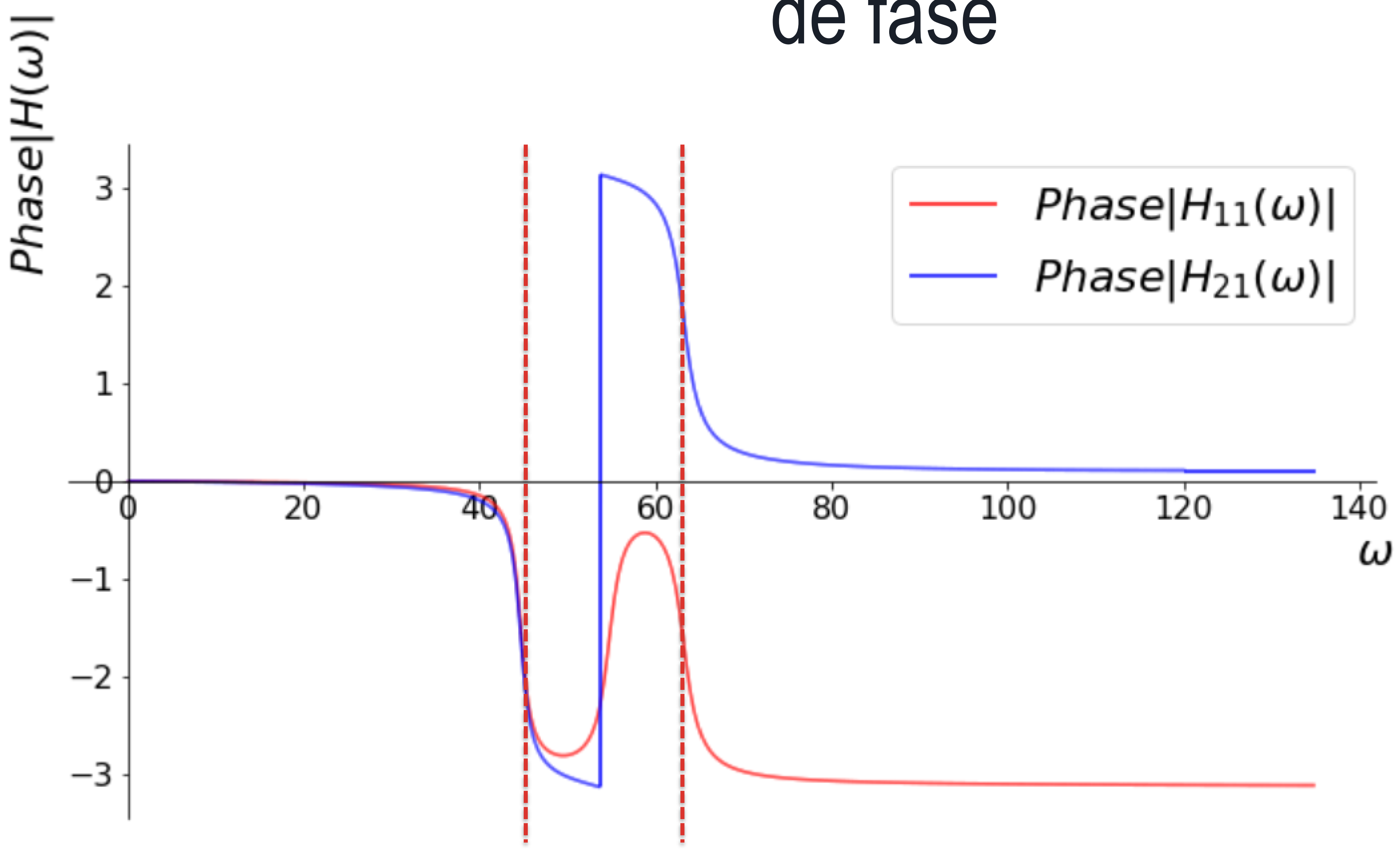
Función de transferencia

$$H(p) = \frac{\mathrm{Adj}[Z(p)]}{\mathrm{Det}[Z(p)]} = \begin{bmatrix} \frac{2p^2+5p+6000}{4p^4+18p^3+24019p^2+50000p+32000000} & \frac{p+2000}{4p^4+18p^3+24019p^2+50000p+32000000} \\ \frac{p+2000}{4p^4+18p^3+24019p^2+50000p+32000000} & \frac{2(p^2+2p+3000)}{4p^4+18p^3+24019p^2+50000p+32000000} \end{bmatrix}$$

Ecuaciones del movimiento



En oposición de fase



En fase

Transformada de Laplace de ecuación diferencial de primer orden

$$\left(p\begin{bmatrix} 0 & \mathbf{M} \\ \mathbf{M} & 0 \end{bmatrix} + \begin{bmatrix} -\mathbf{M} & 0 \\ 0 & \mathbf{K} \end{bmatrix}\right)\mathrm{Y}(p) = \begin{bmatrix} 0 \\ \mathrm{F}(p) \end{bmatrix}$$

$$p \cdot A + B = \begin{bmatrix} -2 & 0 & 2p & 0 \\ 0 & -2 & 0 & 2p \\ 2p & 0 & 4p+6000 & -p-2000 \\ 0 & 2p & -p-2000 & 5p+6000 \end{bmatrix}$$

Autovalores

$$\Lambda = \begin{bmatrix} -1.37499+63.2296i & 0 & 0 & 0 \\ 0 & -1.37499-63.2296i & 0 & 0 \\ 0 & 0 & -0.875012+44.7135i & 0 \\ 0 & 0 & 0 & -0.875012-44.7135i \end{bmatrix}$$

$$\lambda_1 = -1.37499 + 63.2296i \qquad \lambda_1 = 63.2446\angle 91.2457^\circ$$

$$\lambda_2 = -1.37499 - 63.2296i \qquad \lambda_2 = 63.2446\angle -91.2457^\circ$$

$$\lambda_3 = -0.875012 + 44.7135i \qquad \lambda_3 = 44.7221\angle 91.1211^\circ$$

$$\lambda_4 = -0.875012 - 44.7135i \qquad \lambda_4 = 44.7221\angle -91.1211^\circ$$

Transformada de Laplace de ecuación diferencial de primer orden

$$\left(p\begin{bmatrix} 0 & \mathbf{M} \\ \mathbf{M} & 0 \end{bmatrix} + \begin{bmatrix} -\mathbf{M} & 0 \\ 0 & \mathbf{K} \end{bmatrix}\right)\mathrm{Y}(p) = \begin{bmatrix} 0 \\ \mathrm{F}(p) \end{bmatrix}$$

$$p \cdot A + B = \begin{bmatrix} -2 & 0 & 2p & 0 \\ 0 & -2 & 0 & 2p \\ 2p & 0 & 4p+6000 & -p-2000 \\ 0 & 2p & -p-2000 & 5p+6000 \end{bmatrix}$$

Autovalores

$$\begin{aligned}
\omega_1^d &= 63.2296\,\mathrm{rad/s} & \omega_1 &= 63.2446\,\mathrm{rad/s} & \zeta_1 &= 0.0217408 \\
\omega_2^d &= -63.2296\,\mathrm{rad/s} & \omega_2 &= 63.2446\,\mathrm{rad/s} & \zeta_2 &= 0.0217408 \\
\omega_3^d &= 44.7135\,\mathrm{rad/s} & \omega_3 &= 44.7221\,\mathrm{rad/s} & \zeta_3 &= 0.0195655 \\
\omega_4^d &= -44.7135\,\mathrm{rad/s} & \omega_4 &= 44.7221\,\mathrm{rad/s} & \zeta_4 &= 0.0195655
\end{aligned}$$

Transformada de Laplace de ecuación diferencial de primer orden

$$\left(p\begin{bmatrix} 0 & \mathbf{M} \\ \mathbf{M} & 0 \end{bmatrix} + \begin{bmatrix} -\mathbf{M} & 0 \\ 0 & \mathbf{K} \end{bmatrix}\right)\mathrm{Y}(p) = \begin{bmatrix} 0 \\ \mathrm{F}(p) \end{bmatrix}$$

$$p \cdot A + B = \begin{bmatrix} -2 & 0 & 2p & 0 \\ 0 & -2 & 0 & 2p \\ 2p & 0 & 4p+6000 & -p-2000 \\ 0 & 2p & -p-2000 & 5p+6000 \end{bmatrix}$$

Autovectores

$$\begin{bmatrix} -0.706874747861719 + 0.0111794444518016i & -0.706874747861719 - 0.0111794444518016i & 0.706941128535425 & 0.706941128535425 \\ 0.707073663439352 & 0.707073663439352 & 0.706874822414759 - 0.00790516995109802i & 0.706874822414759 + 0.00790516995109802i \\ 0.000419717239670559 + 0.0111703610548922i & 0.000419717239670559 - 0.0111703610548922i & -0.000309281218296151 - 0.015804411179221i & -0.000309281218296151 + 0.015804411179221i \\ -0.000243062098281271 - 0.0111773485135086i & -0.000243062098281271 + 0.0111773485135086i & -0.000485980584153942 - 0.0157994703862086i & -0.000485980584153942 + 0.0157994703862086i \end{bmatrix}$$

Transformada de Laplace de ecuación diferencial de primer orden

$$\left(p\begin{bmatrix} 0 & \mathbf{M} \\ \mathbf{M} & 0 \end{bmatrix} + \begin{bmatrix} -\mathbf{M} & 0 \\ 0 & \mathbf{K} \end{bmatrix}\right)\mathrm{Y}(p) = \begin{bmatrix} 0 \\ \mathrm{F}(p) \end{bmatrix}$$

$$p \cdot A + B = \begin{bmatrix} -2 & 0 & 2p & 0 \\ 0 & -2 & 0 & 2p \\ 2p & 0 & 4p + 6000 & -p - 2000 \\ 0 & 2p & -p - 2000 & 5p + 6000 \end{bmatrix}$$

Autovectores

$$\begin{bmatrix} -0.706874747861719 + 0.0111794444518016i & -0.706874747861719 - 0.0111794444518016i & 0.706941128535425 & 0.706941128535425 \\ 0.707073663439352 & 0.707073663439352 & 0.706874822414759 - 0.00790516995109802i & 0.706874822414759 + 0.00790516995109802i \\ 0.000419717239670559 + 0.0111703610548922i & 0.000419717239670559 - 0.0111703610548922i & -0.000309281218296151 - 0.015804411179221i & -0.000309281218296151 + 0.015804411179221i \\ -0.000243062098281271 - 0.0111773485135086i & -0.000243062098281271 + 0.0111773485135086i & -0.000485980584153942 - 0.0157994703862086i & -0.000485980584153942 + 0.0157994703862086i \end{bmatrix}$$

Matrices de residuos

$$A_1 = (3.12661\cdot 10^{-8} + 9.88245\cdot 10^{-7} i)\begin{bmatrix} -1999.06 - 31.6119i & 1998.63 + 63.2296i \\ 1998.63 + 63.2296i & -1997.69 - 94.8415i \end{bmatrix}$$

$$A_2 = (3.12661\cdot 10^{-8} - 9.88245\cdot 10^{-7} i)\begin{bmatrix} -1999.06 + 31.6119i & 1998.63 - 63.2296i \\ 1998.63 - 63.2296i & -1997.69 + 94.8415i \end{bmatrix}$$

$$A_3 = (-3.12661\cdot 10^{-8} - 1.39783\cdot 10^{-6} i)\begin{bmatrix} 1998.56 + 67.0682i & 1999.13 + 44.7135i \\ 1999.13 + 44.7135i & 1999.44 + 22.3547i \end{bmatrix}$$

$$A_4 = (-3.12661\cdot 10^{-8} + 1.39783\cdot 10^{-6} i)\begin{bmatrix} 1998.56 - 67.0682i & 1999.13 - 44.7135i \\ 1999.13 - 44.7135i & 1999.44 - 22.3547i \end{bmatrix}$$

Factores de contribución modal

$$Q_1 = -0.937557 + 15.7925i$$

$$Q_2 = -0.937557 - 15.7925i$$

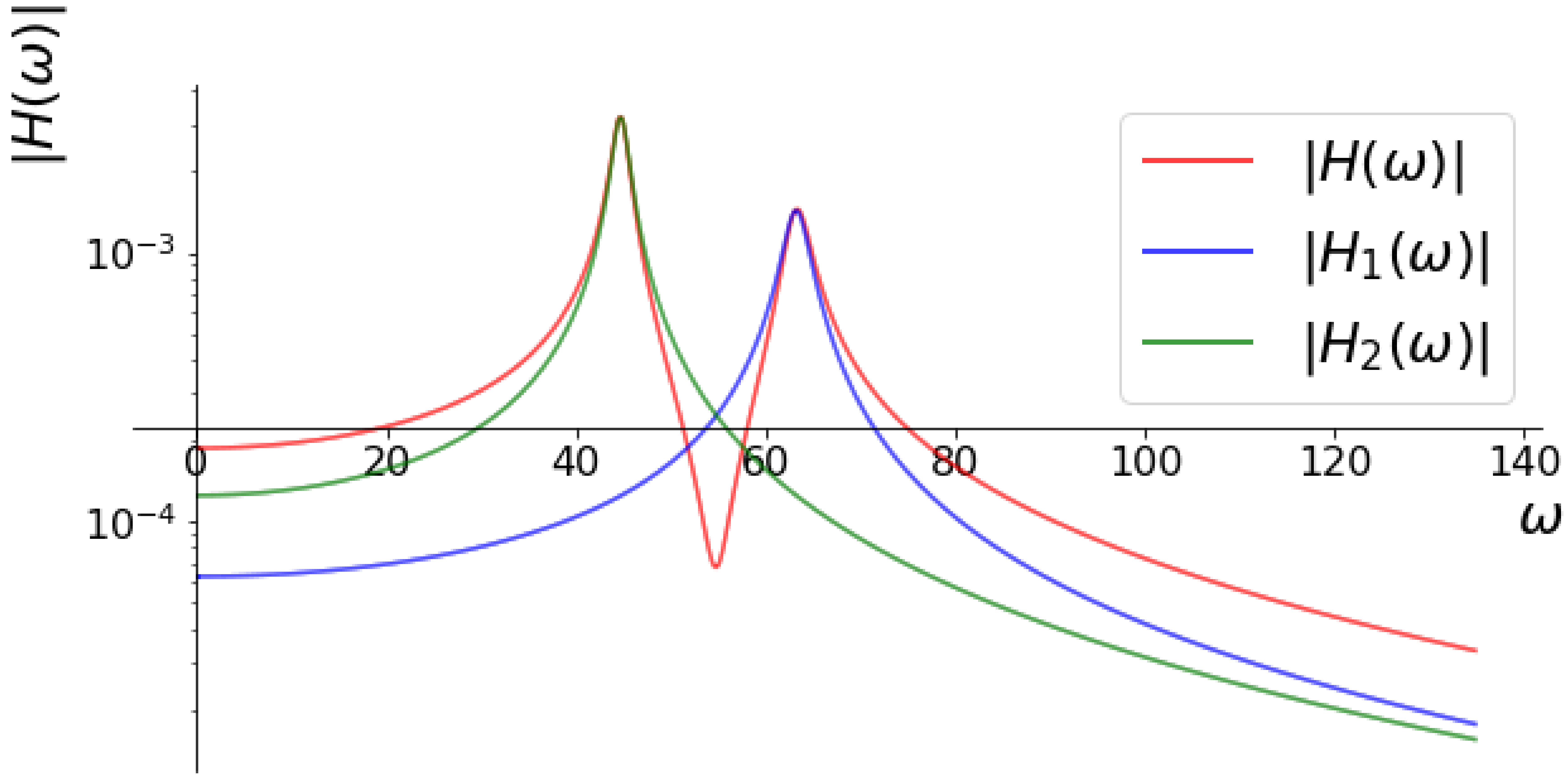
$$Q_3 = -0.562756 + 11.1751i$$

$$Q_4 = -0.562756 - 11.1751i$$

$$Q_2\cdot\psi\cdot\psi^T = (-0.937557 + 15.7925i)\cdot\begin{bmatrix} -0.000124601 + 9.37679\cdot 10^{-6} i & 0.000124753 - 7.40642\cdot 10^{-6} i \\ 0.000124753 - 7.40642\cdot 10^{-6} i & -0.000124874 + 5.43358\cdot 10^{-6} i \end{bmatrix} =$$

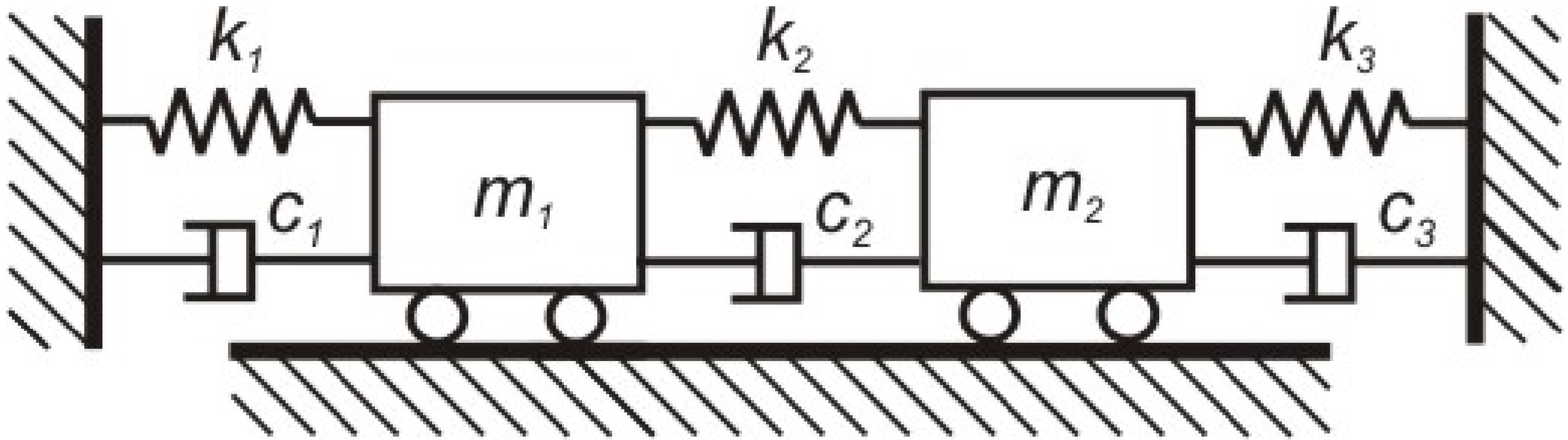
$$= \begin{bmatrix} -3.12627\cdot 10^{-5} - 0.00197655i & 2.92845\cdot 10^{-9} + 0.00197711i \\ 2.92845\cdot 10^{-9} + 0.00197711i & 3.12666\cdot 10^{-5} - 0.00197717i \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -3.12627\cdot 10^{-5} - 0.00197655i & 2.94571\cdot 10^{-9} + 0.00197711i \\ 2.94571\cdot 10^{-9} + 0.00197711i & 3.12666\cdot 10^{-5} - 0.00197717i \end{bmatrix}$$



$|H(\omega)|$
10^{-3}
10^{-4}
0
20
40
60
80
100
120
140
ω
$|H(\omega)|$
$|H_1(\omega)|$
$|H_2(\omega)|$

Sistema de 2 GDL con amortiguamiento proporcional



$$m_1 = m_2 = 2\ \mathrm{kg}$$

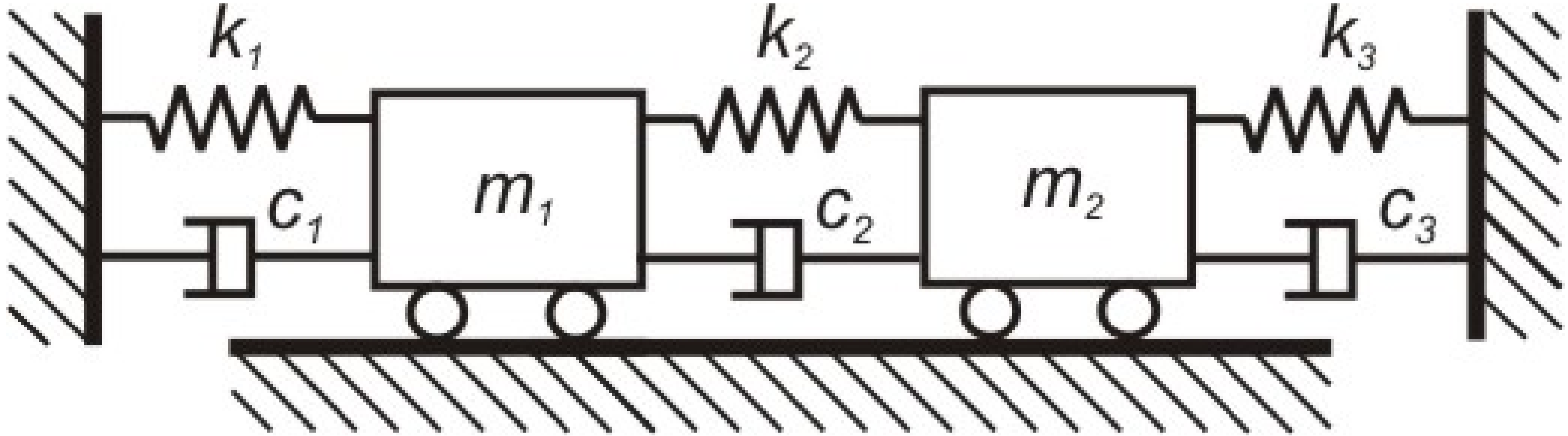
$$C_1 = 3\,\frac{\mathrm{N}}{\mathrm{m/s}}, C_2 = 2\,\frac{\mathrm{N}}{\mathrm{m/s}}, C_3 = 3\,\frac{\mathrm{N}}{\mathrm{m/s}}$$

$$k_1 = 4000\,\frac{\mathrm{N}}{\mathrm{m}}, k_2 = 2000\,\frac{\mathrm{N}}{\mathrm{m}}, k_3 = 4000\,\frac{\mathrm{N}}{\mathrm{m}}$$

Determinar:

- Polos del sistema
- Modos de vibración
- Matrices de residuos
- Factores de contribución modal

Sistema de 2 GDL con amortiguamiento proporcional



$$C = \alpha \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \beta \begin{bmatrix} 6000 & -2000 \\ -2000 & 6000 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 5 \end{bmatrix}$$

$$\alpha = -\frac{1}{2} 1/s$$

$$\beta = \frac{1}{1000} 1/s$$

Determinar:

- Polos del sistema
- Modos de vibración
- Matrices de residuos
- Factores de contribución modal

Sistema de 2 GDL con amortiguamiento proporcional

Autovalores/Autovectores

$$\lambda_1 = -1.75 + 63.2213i = 63.2456\angle 91.5856^\circ$$

$$\lambda_2 = -1.75 - 63.2213i = 63.2456\angle -91.5856^\circ$$

$$\lambda_3 = -0.75 + 44.7151i = 44.7214\angle 90.9609^\circ$$

$$\lambda_4 = -0.75 - 44.7151i = 44.7214\angle -90.9609^\circ$$

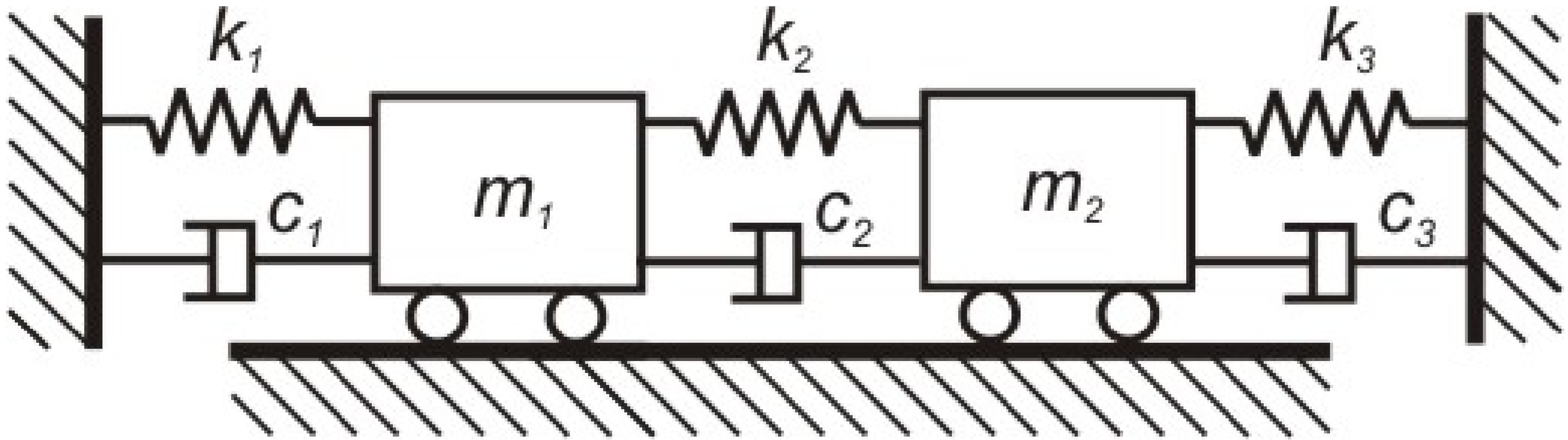
$$\mathrm{Abs}(\lambda_1) = \begin{bmatrix} 0.707018409408262 \\ 0.707018409408263 \\ 0.0111789426069976 \\ 0.0111789426069976 \end{bmatrix}, \ \mathrm{Ang}(\lambda_1) \begin{bmatrix} 180.0 \\ 0.0 \\ 88.4144274491374 \\ -91.5855725508626 \end{bmatrix} [^\circ]$$

$$\mathrm{Abs}(\lambda_2) = \begin{bmatrix} 0.707018409408262 \\ 0.707018409408263 \\ 0.0111789426069976 \\ 0.0111789426069976 \end{bmatrix}, \ \mathrm{Ang}(\lambda_2) \begin{bmatrix} -180.0 \\ 0.0 \\ -88.4144274491374 \\ 91.5855725508626 \end{bmatrix} [^\circ]$$

$$\mathrm{Abs}(\lambda_3) = \begin{bmatrix} 0.706930070754903 \\ 0.706930070754902 \\ 0.015807436935467 \\ 0.015807436935467 \end{bmatrix}, \ \mathrm{Ang}(\lambda_3) \begin{bmatrix} 0.0 \\ 3.97368448908243 \cdot 10^{-14} \\ -90.9609244805382 \\ -90.9609244805381 \end{bmatrix} [^\circ]$$

$$\mathrm{Abs}(\lambda_4) = \begin{bmatrix} 0.706930070754903 \\ 0.706930070754902 \\ 0.015807436935467 \\ 0.015807436935467 \end{bmatrix}, \ \mathrm{Ang}(\lambda_4) \begin{bmatrix} 0.0 \\ -3.97368448908243 \cdot 10^{-14} \\ 90.9609244805382 \\ 90.9609244805381 \end{bmatrix} [^\circ]$$

Sistema de 2 GDL sin amortiguamiento



$$m_1 = m_2 = 2\ \mathrm{kg}$$

$$C_1 = 0\,\frac{\mathrm{N}}{\mathrm{m/s}}, C_2 = 0\,\frac{\mathrm{N}}{\mathrm{m/s}}, C_3 = 0\,\frac{\mathrm{N}}{\mathrm{m/s}}$$

$$k_1 = 4000\,\frac{\mathrm{N}}{\mathrm{m}}, k_2 = 2000\,\frac{\mathrm{N}}{\mathrm{m}}, k_3 = 4000\,\frac{\mathrm{N}}{\mathrm{m}}$$

Determinar:

- Polos del sistema
- Modos de vibración
- Matrices de residuos
- Factores de contribución modal

Sistema de 2 GDL con amortiguamiento proporcional

Autovalores/Autovectores

$$\lambda_1 = 63.2456i = 63.2456\angle 90.0^\circ$$

$$\lambda_2 = -63.2456i = 63.2456\angle -90.0^\circ$$

$$\lambda_3 = 4.71845 \cdot 10^{-15} + 44.7214i = 44.7214\angle 90.0^\circ$$

$$\lambda_4 = 4.71845 \cdot 10^{-15} - 44.7214i = 44.7214\angle -90.0^\circ$$

$$\mathrm{Abs}(\lambda_1) = \begin{bmatrix} 0.707018409408262 \\ 0.707018409408262 \\ 0.0111789426069976 \\ 0.0111789426069976 \end{bmatrix}, \ \mathrm{Ang}(\lambda_1) \begin{bmatrix} 0.0 \\ -180.0 \\ -89.9999999999999 \\ 90.0 \end{bmatrix} [^\circ]$$

$$\mathrm{Abs}(\lambda_2) = \begin{bmatrix} 0.707018409408262 \\ 0.707018409408262 \\ 0.0111789426069976 \\ 0.0111789426069976 \end{bmatrix}, \ \mathrm{Ang}(\lambda_2) \begin{bmatrix} 0.0 \\ 180.0 \\ 89.9999999999999 \\ -90.0 \end{bmatrix} [^\circ]$$

$$\mathrm{Abs}(\lambda_3) = \begin{bmatrix} 0.706930070754903 \\ 0.706930070754902 \\ 0.015807436935467 \\ 0.015807436935467 \end{bmatrix}, \ \mathrm{Ang}(\lambda_3) \begin{bmatrix} 0.0 \\ 9.70873569144084 \cdot 10^{-15} \\ -90.0 \\ -90.0 \end{bmatrix} [^\circ]$$

$$\mathrm{Abs}(\lambda_4) = \begin{bmatrix} 0.706930070754903 \\ 0.706930070754902 \\ 0.015807436935467 \\ 0.015807436935467 \end{bmatrix}, \ \mathrm{Ang}(\lambda_4) \begin{bmatrix} 0.0 \\ -9.70873569144084 \cdot 10^{-15} \\ 90.0 \\ 90.0 \end{bmatrix} [^\circ]$$