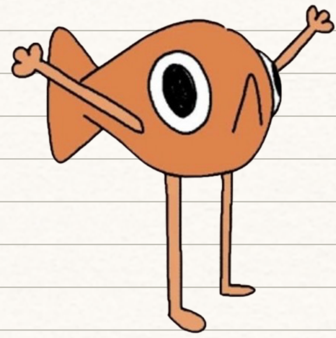
01 - octubre - 2025



# Poisson ( Poisson )

con media $\lambda > 0$

Si $X \sim \text{Poisson}(\lambda)$ entonces

*) $X$ cuenta el número de eventos que ocurren en cierto periodo de tiempo, considerando que los eventos ocurren de forma independiente sin que ocurran dos eventos al mismo tiempo.

*) $$\boxed{\mathbb{P}[X=x] = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!} & \text{si } x = 0, 1, 2, \dots \\ 0 & \text{c.o.c.} \end{cases}}$$

*) $\mathbb{E}[X] = \lambda = \text{Var}[X]$

máxima parte entera de $\lambda$

*) $$\text{moda} = \begin{cases} \lfloor \lambda \rfloor & \text{si } \lambda \text{ no es un número entero} \\ (\lambda) \text{ y } (\lambda - 1) & \text{si } \lambda \text{ es un número entero} \end{cases}$$

*) Relación recursiva:

$$\mathbb{P}[X = n+1] = \mathbb{P}[X=n] \cdot \frac{\lambda}{n+1}$$

The number of brake repair jobs a particular bus needs in a year is modeled by a Poisson distribution. The probability that the bus needs at least one brake repair job this year is 0.10.

Calculate the probability that the bus needs at least two brake repair jobs this year.

**solución:**

Sea $N$ la v.a. que cuenta el número de reparaciones.
$N \sim \text{Poisson}(\lambda = ?)$

$$0.1 = \mathbb{P}(N \geq 1) = 1 - \mathbb{P}(N<1) = 1 - \mathbb{P}(N \leq 0) = 1 - \mathbb{P}(N=0)$$

$$\mathbb{P}(N=0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda} \rightarrow 1 - \mathbb{P}(N=0) = 1 - e^{-\lambda}$$

$$\rightarrow 0.10 = 1 - e^{-\lambda} \rightarrow e^{-\lambda} = 0.90 \rightarrow -\lambda = \ln(0.9) \rightarrow \lambda = -\ln(0.9)$$

$$\mathbb{P}(N \geq 2) = 1 - \mathbb{P}(N<2) = 1 - \mathbb{P}(N \leq 1)$$

$$= 1 - [\mathbb{P}(N=1) + \mathbb{P}(N=0)]$$

$$= 1 - \left[\frac{e^{-\lambda}\lambda^1}{\cancel{1!}} + \frac{e^{-\lambda}\cancel{\lambda^0}}{\cancel{0!}}\right]$$

$$= 1 - \lambda e^{-\lambda} - e^{-\lambda}$$

$$= 1 - (-\ln(0.9))\cancel{e}^{-(-\cancel{\ln}(0.9))} - e^{-(-\ln(0.9))}$$

$$= 1 + \ln(0.9)(0.9) - (0.9)$$

$$\boxed{\mathbb{P}(N \geq 2) = 0.005175536}$$

This year, the number of tooth fillings a policyholder undergoes is Poisson distributed. The probability that the policyholder undergoes no tooth fillings this year is 0.18.

Calculate the mode of the number of tooth fillings the policyholder undergoes this year.

**solución:**

Sea N la v.a. que cuenta el número de caries a tratar.

$N \sim \text{Poisson}(\lambda = ?)$

$$0.18 = \mathbb{P}(N=0) = \frac{e^{-\lambda}\cdot\lambda^0}{0!} = e^{-\lambda} \rightarrow \lambda = -\ln(0.18)$$

Si $\lambda \approx 1.7147$ entonces $\lambda$ no es un número entero $(\lambda \notin \mathbb{Z})$

Cuando $\lambda \in \mathbb{Z} \rightarrow$ bimodal $\rightleftarrows \begin{matrix}\lambda \\ \lambda - 1\end{matrix}$

Cuando $\lambda \notin \mathbb{Z} \rightarrow$ moda: $\lfloor \lambda \rfloor$

$\boxed{\text{Moda de N está en } n = 1.}$

The annual number of claims per policy for a class of insurance policies is modeled by a Poisson distribution with mean 1.20.

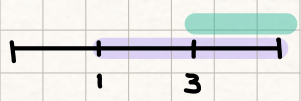
A policy is randomly selected from those policies that had at least one claim in the past year.

Calculate the probability that the selected policy had at least three claims in the past year.

**solución:**

$$N \sim \text{Poisson}\,(\lambda = 1.2) \rightsquigarrow \mathbb{P}(N=n) = \frac{e^{-\lambda}\cdot\lambda^n}{n!} \quad \text{para } n:0,1,\dots$$

$$\mathbb{P}(N \geq 3 \mid N \geq 1) = \frac{\mathbb{P}(N\geq 1 \cap N \geq 3)}{\mathbb{P}(N \geq 1)}$$




$$= \frac{\mathbb{P}(N\geq 3)}{\mathbb{P}(N\geq 1)} = \frac{1-\mathbb{P}(N<3)}{1-\mathbb{P}(N<1)} = \frac{1-\mathbb{P}(N\leq 2)}{1-\mathbb{P}(N\leq 0)}$$

$$= \frac{1-\mathbb{P}(N=0)-\mathbb{P}(N=1)-\mathbb{P}(N=2)}{1-\mathbb{P}(N=0)}$$

$$= \frac{1-0.8794}{1-0.301194} = 0.17258$$

$$\boxed{\mathbb{P}(N\geq 3 \mid N \geq 1) = 0.1725}$$

# Geométrica (geometric)

con parámetro $0 \leq p \leq 1$

Si $X \sim Geom(p)$ entonces

ensayos ind.

*) $X$ cuenta el número de fallos antes del 1er éxito asociado a la proba $p$.

*) $$\mathbb{P}[X=x] = \begin{cases} p(1-p)^x & \text{si } x=0,1,\dots \\ 0 & \text{c.o.c.} \end{cases}$$

*) $F_X(x) = \mathbb{P}(X \leq x) = 1-(1-p)^{x+1}$

*) $\mathbb{E}[X] = \dfrac{1-p}{p}$

*) $Var[X] = \dfrac{1-p}{p^2}$

*) Propiedad de pérdida de memoria (memoryless property):

$\mathbb{P}[X=n+k \mid X \geq n] = \mathbb{P}[X=k]$ ; $\mathbb{P}[X \geq n+k \mid X \geq n] = \mathbb{P}[X \geq k]$

A scientist plans to repeat an experiment until a successful result is achieved. On each trial the probability of a successful result is 0.25. The outcomes of the trials are mutually independent.

Calculate the probability that more than three trials are needed to get a successful result.

**solución:**

Sea $N$ la v.a. que cuenta el núm. de ensayos necesarios para obtener un resultado éxitoso.

$N \sim Geom(p=0.25)$ ; $n=\{1,2,3,\dots\} \rightarrow \mathbb{P}(N=n)=(0.25)(0.75)^{n-1}$

$$\begin{aligned}\mathbb{P}(N>3) &= 1-\mathbb{P}(N\leq 3)\\ &= 1-[\mathbb{P}(N=1)+\mathbb{P}(N=2)+\mathbb{P}(N=3)]\\ &= 1-[(0.25)(0.75)^{1-1}+(0.25)(0.75)^{2-1}+(0.25)(0.75)^{3-1}]\end{aligned}$$

$$\rightarrow \boxed{\mathbb{P}(N>3) = 1-(0.25)[1+(0.75)+(0.75)^2] = 0.421875}$$

Alternativamente si $Y = X+1$

$Y \sim Geom(p)$

*) $Y$ cuenta el número de ensayos necesarios para alcanzar el 1er éxito.

*) $\mathbb{P}[Y=y] = p(1-p)^{y-1}$ si $y = 1, 2, \dots$

*) $F_Y(y) = 1-(1-p)^y$

*) $\mathbb{E}[Y] = \dfrac{1}{p}$

*) $Var[Y] = \dfrac{1-p}{p^2}$

$Var[Y] = Var[X+1] = Var[X]$

*) Propiedad de pérdida de memoria

$\mathbb{P}[Y=n+k \mid Y \geq n] = \mathbb{P}[Y=k]$

$\mathbb{P}[Y>n+k \mid Y>n] = \mathbb{P}[Y>k]$

Patients in a study are tested for sleep apnea, one at a time, until a patient is found to have this disease. Each patient independently has the same probability of having sleep apnea. Let $r$ represent the probability that at least four patients are tested.

Determine the probability that at least twelve patients are tested given that at least four patients are tested.

(A) $r^{\frac{11}{3}}$
(B) $r^{3}$
(C) $r^{\frac{8}{3}}$
(D) $r^{2}$
(E) $r^{\frac{1}{3}}$

**solución:**

$N \sim Geom(p = ?)$ ; $n = \{1, 2, 3, \dots\}$

$$r = \mathbb{P}(N \geq 4) = 1 - \mathbb{P}(N < 4) = 1 - \mathbb{P}(N \leq 3)$$

$$= 1 - F_N(3)$$

$$= \cancel{1} - (\cancel{1} - (1-p)^3) = (1-p)^3$$

$$\mathbb{P}(N \geq 12 \mid N \geq 4) = \frac{\mathbb{P}(N \geq 12)}{\mathbb{P}(N \geq 4)} = \frac{1 - \mathbb{P}(N \leq 11)}{r} = \frac{(1-p)^{11}}{(1-p)^3}$$

$$= (1-p)^{11-3} = (1-p)^{8 \cdot 1} = (((1-p)^3)^{\frac{1}{3}})^8 = r^{\frac{8}{3}}$$

Each person in a certain large population independently has probability 0.0625 of having a certain disease. People in this population are tested for the disease, until somebody is found to have the disease.

Calculate the mode of the number of people tested.

**solución:**

Sea $Y$ la v.a. que cuenta el núm. de personas que son testeadas para hallar una con la enfermedad.

$Y \sim Geom(p = 0.0625)$ con $y \in \{1, 2, 3, \dots\}$

$$\mathbb{P}(Y = y) = p(1-p)^{y-1} = (0.0625)(1 - 0.0625)^{y-1}$$

La función $\mathbb{P}(Y = y)$ se maximiza cuando $(1 - 0.0625)^{y-1}$ lo hace. Pero $x^a < x$ con $a > 0$ y con $x \in (0,1)$

$$\rightarrow (1 - 0.0625)^{y-1} < (1 - 0.0625)^1 < (1 - 0.0625)^0 = 1$$

Entonces, la función alcanza su máximo en $\boxed{y = 1}$

# Binomial negativa (negative binomial)

con parámetros $r > 0$ y $0 \leq p \leq 1$

Si $X \sim BinNeg(r, p)$ entonces

*) $X$ es una generalización de la distribución geométrica que cuenta el número de fracasos que ocurren antes del $r$-ésimo éxito.

*) $$\boxed{\mathbb{P}[X = x] = \begin{cases} \binom{r+x-1}{x} p^r (1-p)^x & \text{si } x = 0, 1, \ldots \\ 0 & \text{c.o.c.} \end{cases}}$$

Por la simétría de Pascal

$$\binom{x+r-1}{x} = \binom{x+r-1}{r-1}$$

*) $$\mathbb{E}[X] = \frac{r(1-p)}{p}$$

*) $$Var[X] = \frac{r(1-p)}{p^2}$$

A representative of a market research firm contacts consumers by phone to conduct surveys. The specific consumer contacted by each phone call is randomly determined. The probability that a phone call produces a completed survey is 0.25.

Calculate the probability that the eighth phone call produces the third completed survey.

**solución:**

$r$ = 3 éxitos
$x$ = 5 fracasos

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| ü | ü | x | x | x | x | x | ü |
| ü | x | ü | x | x | x | x | ü |
| x | x | x | x | x | ü | ü | ü |
| | | | ⋮ | | | | |

$$\Big\} \; (0.25)(0.25)(0.75)(0.75)(0.75)(0.75)(0.75)(0.25) = (0.25)^3 (0.75)^5$$

$X \sim BinNeg(r = 3, p = 0.25)$

$$\mathbb{P}(X=5) = (0.25)^3 (0.75)^5 \cdot \binom{7}{2} = (0.25)^3 (0.75)^5 \cdot \binom{7}{5}$$

$$\boxed{\mathbb{P}(X=5) = 0.0778656}$$

On average, a certain word processing software program has a fatal crash once in every 50 instances of saving a document.

The instances of fatal crashes, while saving, are independent from one another.

Calculate the probability that the second fatal crash, while saving, occurs on the fourth instance of saving a document.

**solución:**

Sea N la v.a. que cuenta el núm. de veces que el programa logra salvar el documento sin crashearse.

$$N \sim \text{Bin Neg}\left(r = 2,\ p = 1/50\right)$$

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1) | F | OK | OK | F |
| 2) | OK | F | OK | F |
| 3) | OK | OK | F | F |

$r = 2$

$\boxed{n = 2}$

$$\boxed{\mathbb{P}(N=2)} = (1/50)^2 (1 - 1/50)^2 \cdot 3$$

$$= \binom{2+2-1}{2} (1/50)^2 (1 - 1/50)^2 = \binom{2+2-1}{1} \dots$$

$$\boxed{\mathbb{P}(N=2) = .001152}$$

A fair die is rolled until three sixes are obtained. Let the random variable $X$ be the total number of rolls required.

Calculate Var $(X)$.

**solución:**

Éxito: obtener un "6" al lanzar el dado

$p_{\text{éxito}} = 1/6$

$Y \sim \text{Bin Neg}(r=3, p = 1/6)$

$r = 3$

| | | | | | | |
|---|---|---|---|---|---|---|
| $x = 3$ | 6 | 6 | 6 | | | $\leftarrow y = 0$ |
| $x = 4$ | 4 | 6 | 6 | 6 | | $\leftarrow y = 1$ |
| $x = 5$ | 6 | 6 | 3 | 2 | 6 | $\leftarrow y = 2$ |
| $\vdots$ | | | | | | $\vdots$ |

$$x = Y + r$$

$$\boxed{\text{Var}(X) = \text{Var}(Y + r) = \text{Var}(Y) = 3 \cdot \frac{(1 - 1/6)}{(1/6)^2} = 90}$$

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# Hipergeométrica ( hypergeometric )

con 3 parámetros enteros $M > 0$, $0 \leq K \leq M$, $1 \leq n \leq M$

Si $X \sim HG(M, K, n)$ entonces

*) En un grupo de M objetos, si K son de tipo I y M-K son de tipo II, X contará el número de elementos de tipo I en una muestra de tamaño n.

*) 
$$\boxed{\mathbb{P}[X=x] = \begin{cases} \dfrac{\binom{K}{x}\cdot\binom{M-K}{n-x}}{\binom{M}{n}} & \text{si } \max\{0, n-(M-K)\} \leq x \text{ y } x \leq \min\{n,K\} \\ 0 & \text{c.o.c.} \end{cases}}$$

*) $$\mathbb{E}[X] = \frac{n \cdot K}{M}$$

*) $$Var[X] = \frac{n \cdot K \cdot (M-K) \cdot (M-n)}{M^2 \cdot (M-1)}$$

Each of ten homeowners independently has the same probability of experiencing at least one loss this year. Eight of these ten homeowners are insured.

Three of the ten homeowners experience at least one loss.

Calculate the probability that at least two of these three homeowners are insured.

**solución:**

M : 10 propietarios → 8 asegurados : K ; 2 no asegurados : M − K

n : 3 → $X \sim HG(M=10, K=8, n=3)$
X cuenta el núm. de prop. asegurados
$x = 1, 2, 3$

$$\mathbb{P}(X \geq 2) = \mathbb{P}(X=2) + \mathbb{P}(X=3) = 1 - \mathbb{P}(X=1)$$
$$= \frac{\binom{8}{2}\binom{2}{1}}{\binom{10}{3}} + \frac{\binom{8}{3}\binom{2}{0}}{\binom{10}{3}} = 14/15$$

$$\boxed{\mathbb{P}(X \geq 2) = 14/15}$$

Se establece el $\min\{n,K\}$ como cota superior para que el valor de X, que equivale al núm. de elementos de K en la muestra, sea a lo más n.

Tomamos el $\max\{0, n-(M-K)\}$ porque anticipamos que el tamaño de la muestra pudiera ser más grande que el número de elementos que no son de tipo K. Por ejemplo, $M=5$, $K=3$, $M-K=2$ si $n=3$ entonces $\max\{0, 3-2\} = 1$. Por lo que siempre habrá por lo menos 1 elemento de tipo K en cada muestra.

A homeowner hires a moving company to transport ten pieces of electronic equipment to a new home. Each piece is equally likely to be damaged during the move, and the event of damage to any one piece is independent of the events of damage to the other pieces.

The homeowner fully insures two of these pieces, partially insures three of them, and leaves the other five pieces uninsured.

During the move, three pieces are damaged.

Calculate the probability that at least one damaged piece is fully insured <u>and</u> at least one damaged piece is partially insured.

**solución:**

M : 10 piezas → 2 totalmente aseg. (T) ; 3 parcialmente aseg. (P) ; 5 no aseg. (N)

n : 3

$$\mathbb{P}(T \geq 1 \cap P \geq 1) = \frac{\begin{matrix} T=1 \\ P=1 \\ N=1 \end{matrix} + \begin{matrix} T=2 \\ P=1 \\ N=0 \end{matrix} + \begin{matrix} T=1 \\ P=2 \\ N=0 \end{matrix}}{\text{"tomar 3 de 10"}}$$

$$= \frac{\cancelto{2}{\binom{2}{1}}\cancelto{3}{\binom{3}{1}}\cancelto{5}{\binom{5}{1}} + \cancelto{1}{\binom{2}{2}}\cancelto{3}{\binom{3}{1}}\cancelto{1}{\binom{5}{0}} + \cancelto{2}{\binom{2}{1}}\cancelto{3}{\binom{3}{2}}\cancelto{1}{\binom{5}{0}}}{\binom{10}{3}}$$

$$= \frac{2 \cdot 3 \cdot 5 + 3 + 2 \cdot 3}{120}$$

$$= \frac{39}{120}$$

$$= 0.325$$

$$\boxed{\mathbb{P}(T \geq 1 \cap P \geq 1) = 0.325}$$

A state is starting a lottery game. To enter this lottery, a player uses a machine that randomly selects six distinct numbers from among the first 30 positive integers. The lottery randomly selects six distinct numbers from the same 30 positive integers. A winning entry must match the same set of six numbers that the lottery selected.

The entry fee is 1, each winning entry receives a prize amount of 500,000, and all other entries receive no prize.

Calculate the probability that the state will lose money, given that 800,000 entries are purchased.

**solución:**

Sea X la v.a. que cuenta el núm. de veces que se otorga el premio de $500,000.

$\mathbb{P}(X \geq 2)$

$X \sim Bin(n = 800{,}000,\ p = ?)$

$$p = \frac{\cancel{\binom{6}{6}}\cancel{\binom{24}{0}}}{\binom{30}{6}}$$

M : 30 números → 6 seleccionados : K ; 24 otros : M−K

n: 6

$\rightarrow p = \binom{30}{6}^{-1} \approx 0.0000001684$

$\rightarrow \mathbb{P}(X \geq 2) = 1 - \mathbb{P}(X < 2) = 1 - \mathbb{P}(X \leq 1)$

$= 1 - \mathbb{P}(X=0) - \mathbb{P}(X=1)$

$= 1 - \binom{800{,}000}{0}\left(\binom{30}{6}^{-1}\right)^{0}\left(1-\binom{30}{6}^{-1}\right)^{800{,}000}$

$- \binom{800{,}000}{1}\binom{30}{6}^{-1}\left(1-\binom{30}{6}^{-1}\right)^{799{,}999}$

$$\boxed{\mathbb{P}(X \geq 2) = 0.389844}$$