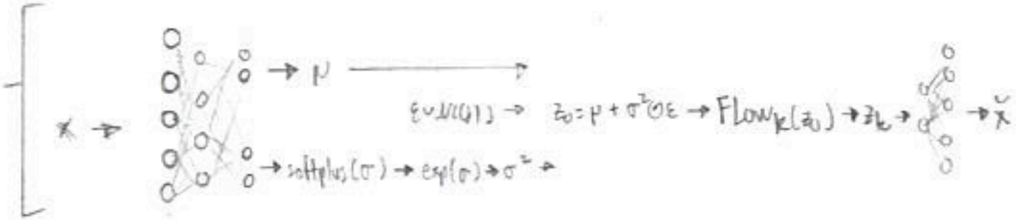
- MLE Normal: $\sum \left(-\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\sigma^2) - \frac{1}{2}\left(\frac{z-\mu}{\sigma^2}\right)^2\right)$
- MLE standard Normal: $\sum \left(-\frac{1}{2}\log(2\pi) + z^2\right)$



- ELBO:

$$
\begin{aligned}
\log p_\theta(x) &= \log \int p_\theta(x|z)\, p(z)\, dz \\
&= \log \int \frac{q_\phi(z|x)}{q_\phi(z|x)}\, p_\theta(x|z)\, p(z)\, dz \\
&\geq -D_{KL}\left(q_\phi(z|x) \,\|\, p(z)\right) + \mathbb{E}_q\left[\log p_\theta(x|z)\right] \\
&= -\mathcal{F}(x)
\end{aligned}
$$

- Normal Flow:

$$
\begin{aligned}
z_k &= f_k \circ \dots \circ f_2 \circ f_1(z_0) \\
\ln q_k(z_k) &= \ln q_0(z_0) - \sum_{k=1}^{K} \ln\left|\det \frac{\partial f_k}{\partial z_{k-1}}\right|
\end{aligned}
$$

$\boxed{\text{Variational Inference with Normalizing Flows}}$

- VAE-Flow loss: (normal prior-latent)

$$
\begin{aligned}
\Rightarrow \mathcal{F}(x) &= \mathbb{E}_{q_\phi(z|x)}\left[\log q_\phi(z|x) - \log p(x, z)\right] \\
&= \mathbb{E}_{q_0(z_0)}\left[\ln q_k(z_k) - \log p(x, z_k)\right] \\
&= \mathbb{E}_{q_0(z_0)}\left[\ln q_k(z_k) - \log p(x|z_k) - \log p(z_k)\right] \\
&= \underbrace{\mathbb{E}_{q_0}(z_0)\left[\ln q_0(z_0)\right]}_{\text{MLE of Normal}} - \underbrace{\mathbb{E}_{q_0(z_0)}\left[\log p(x|z_k)\right]}_{\text{encoder output through sigmoid, then cross-entropy against } x\text{-input}} - \underbrace{\mathbb{E}_{q_0}(z_0)\left[\log p(z_k)\right]}_{\text{MLE of standard Normal}} - \underbrace{\mathbb{E}_{q_0}\left[\ln\left|\det\left(\frac{\partial F(z_k)}{\partial z_0}\right)\right|\right]}_{\text{log-det term of chosen Flow}(\cdot)}
\end{aligned}
$$