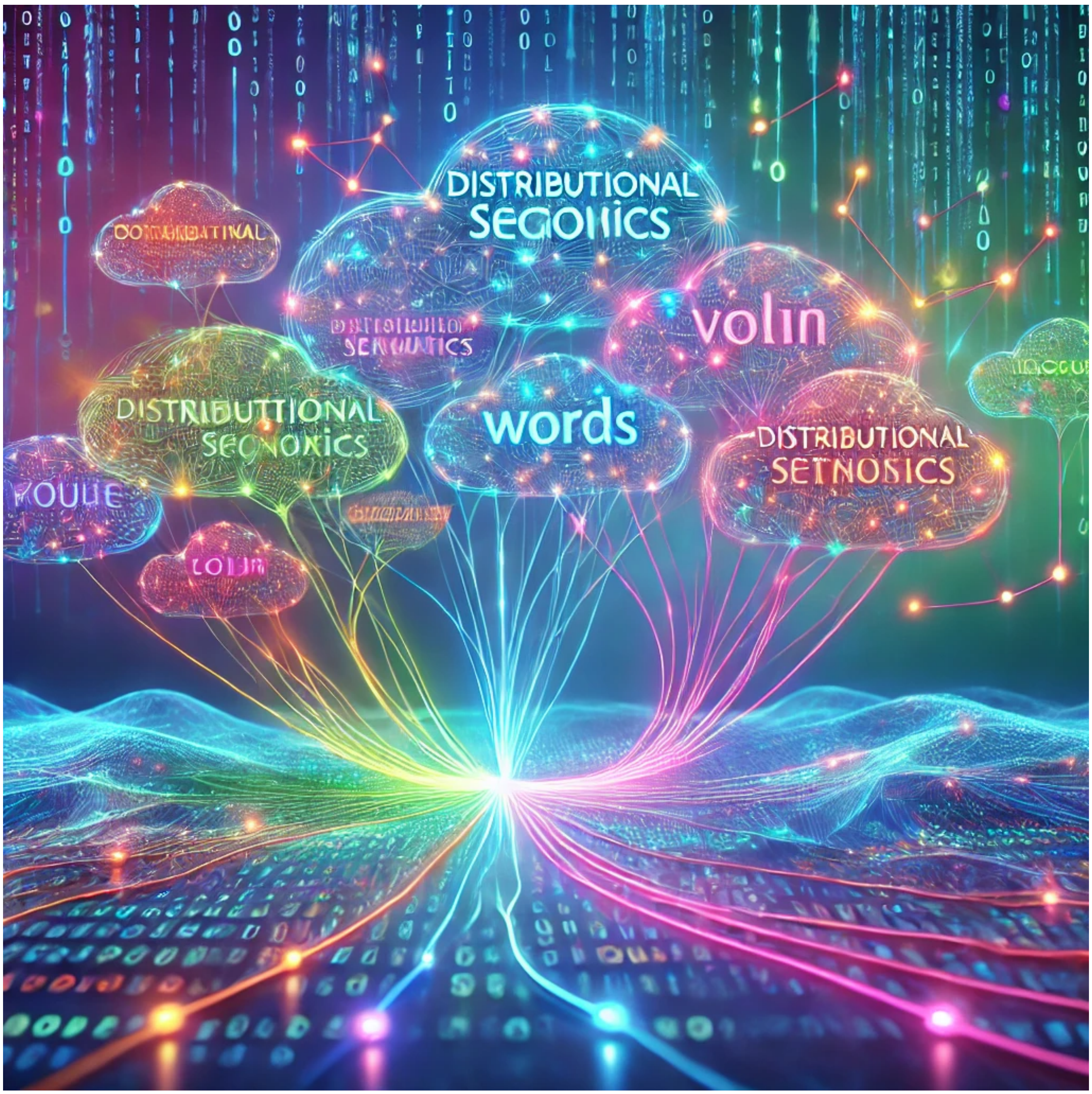
Introduction to Natural Language Processing, Assignment 2

Enrique Mesonero Ronco   Sergio Sánchez García   Ismael Cross Moreno

December 10, 2024



Contents

1 Distributional Semantics

1.1 Raw Co-occurrence Vectors

Given the following raw co-occurrence counts of words with contexts jealous (c_1) and gossip (c_2):

	c_1(jealous)	c_2(gossip)
w_1	2	5
w_2	3	0
w_3	4	0
w_4	0	4

1. **Compute the TF-IDF Weighted Co-occurrence Matrix**
   Use the following formulas:

$$\text{tf}(w, c) = \log\left(\frac{\text{freq}(w, c)}{\max_{w'} \text{freq}(w', c)} + 1\right)$$

$$\text{idf}(c) = \log\left(\frac{|V|}{|\{w \in V : \text{freq}(w, c) > 0\}|}\right)$$

   Where $|V| = 4$.

   TF:

   - $\text{tf}(w_1, c_1) = \log\left(\frac{2}{4} + 1\right) = 0.176$
   - $\text{tf}(w_2, c_1) = \log\left(\frac{3}{4} + 1\right) = 0.243$
   - $\text{tf}(w_3, c_1) = \log\left(\frac{4}{4} + 1\right) = 0.301$
   - $\text{tf}(w_4, c_1) = \log\left(\frac{0}{4} + 1\right) = 0$
   - $\text{tf}(w_1, c_2) = \log\left(\frac{5}{5} + 1\right) = 0.301$
   - $\text{tf}(w_2 c_2) = \log\left(\frac{0}{5} + 1\right) = 0$
   - $\text{tf}(w_3, c_2) = \log\left(\frac{0}{5} + 1\right) = 0$
   - $\text{tf}(w_4, c_2) = \log\left(\frac{4}{5} + 1\right) = 0.255$

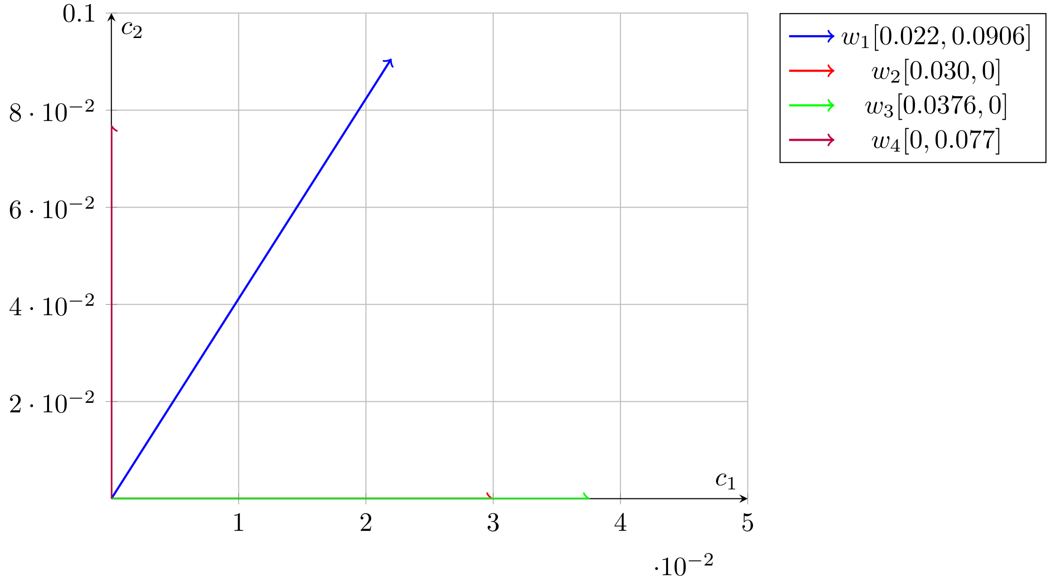
   IDF:

   - $\text{idf}(c_1) = \log(\frac{4}{3}) = 0.125$
   - $\text{idf}(c_2) = \log(\frac{4}{2}) = 0.301$

   TF-IDF

   - $\text{tf-idf}(w_1, c_1) = 0.176 \cdot 0.125 = 0.022$
   - $\text{tf-idf}(w_2, c_1) = 0.243 \cdot 0.125 = 0.030$
   - $\text{tf-idf}(w_3, c_1) = 0.301 \cdot 0.125 = 0.0376$
   - $\text{tf-idf}(w_4, c_1) = 0 \cdot 0.125 = 0$
   - $\text{tf-idf}(w_1, c_2) = 0.301 \cdot 0.301 = 0.0906$
   - $\text{tf-idf}(w_2, c_2) = 0 \cdot 0.301 = 0$
   - $\text{tf-idf}(w_3, c_2) = 0 \cdot 0.301 = 0$
   - $\text{tf-idf}(w_4, c_2) = 0.255 \cdot 0.301 = 0.077$

2. **Represent Each Word as a TF-IDF Vector**



3. **Compute the Euclidean Distance Between:**

$$d(x,y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}$$

   (a) w_1 and $w_2 \to d(w_1, w_2) = \sqrt{(0.022 - 0.030)^2 + (0.0906 - 0)^2} = 0.091$

   (b) w_2 and $w_3 \to d(w_1, w_2) = \sqrt{(0.0307 - 0.0376)^2 + (0 - 0)^2} = 0.0076$

4. **Discussion**
   Based on the Euclidean distances computed, evaluate whether Euclidean distance is an appropriate measure for capturing the relationships between the words.
   Answer: Is not a valid option because the Euclidean Distance is affected by vectors length.

1.2 Prediction Based Word Vectors

- Why does Word2Vec use separate input vectors (u_w) and output vectors (v_w) for each word, and how does this benefit the model's performance?
  Answer: Two random vectors (of dimension $d << |V|$) assigned to each word:
  - $u_w \to$ the "input" vector of the word w, when it is used to predict another word.
  - $v_w \to$ the "output" vector of the word w, when it is the one being predicted.
- What are the primary differences between the Skip-Gram and Continuous Bag-of-Words (CBOW) models in Word2Vec, and in what scenarios might one outperform the other?
  Answer:
  - Skip-Gram predicts each context words from the center word. More efficient with less amounts of data.
  - CBOW predicts the center word from the whole context. Faster, more efficient with big amounts of data.
- How does negative sampling improve the efficiency of training Word2Vec models compared to using the full softmax function?
  Answer: Denominator in softmax sums over words in V_N, instead of the whole $V \to N << |V|$

- How does the choice of window size in Word2Vec affect the type of semantic relationships the model captures?
  Answer:
  - Small windows → more sintactic relationships catched
  - Big windows → more semantic relationships catched
- What strategies canWord2Vec employ to handle out-of-vocabulary (OOV) words, and what are the implications of these strategies?
  Answer: Tokenization: enrich word embeddings with subword information (low frequency "token" and "ization" more frequent). Learn subwords vectors (char n-grams) along with word-level embeddings.

2 Topic Modeling

Consider a simple corpus with the following characteristics:

- **Vocabulary (V)**: {apple, banana, cherry}
- **Number of Topics (K)**: 2
- **Number of Documents (M)**: 2

The initial topic distributions over words (ϕ_k) and document distributions over topics (θ_m) are randomly initialized as follows:

$$\phi_1 = \left[\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}\right], \quad \phi_2 = \left[\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}\right]$$

$$\theta_1 = \left[\tfrac{1}{2}\,\tfrac{1}{2}\right], \quad \theta_2 = \left[\tfrac{1}{2}\,\tfrac{1}{2}\right]$$

Documents

- **Document 1**: apple, banana
- **Document 2**: banana, cherry

Steps to Solve

1. Compute Topic Assignment Probabilities

For each word in each document, compute the probability of assigning it to each topic using the current ϕ and θ values. Specifically, calculate:

$$P(z_{mn} = k) \propto \phi_k[w] \times \theta_m[k]$$

for each word w in document m.

The probability of assigning each word w in document m to topic k is calculated as:

$$P(z_{mn} = k) \propto \phi_k[w] \cdot \theta_m[k]$$

For both topics ($k = 1, 2$), and all words in the documents:

$$\phi_k[w] = \frac{1}{3}, \quad \theta_m[k] = \frac{1}{2}$$

Thus:

$$P(z_{mn} = k) \propto \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

Normalization of $P(Z_{11} = k)$

Given:

$$P(Z_{11} = 1) \propto \frac{1}{6}, \quad P(Z_{11} = 2) \propto \frac{1}{6}$$

The sum of unnormalized probabilities is:

$$\text{Sum} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

Normalized probabilities:

$$P(Z_{11} = 1) = \frac{\frac{1}{6}}{\frac{2}{6}} = \frac{1}{2}$$

$$P(Z_{11} = 2) = \frac{\frac{1}{6}}{\frac{2}{6}} = \frac{1}{2}$$

Thus, both probabilities are:

$$P(Z_{11} = 1) = \frac{1}{2}, \quad P(Z_{11} = 2) = \frac{1}{2}$$

The probabilities for all words are equal, so no preference exists between topics.

2. Assign New Topics

Based on the probabilities computed earlier, assign a new topic to each word in each document. Assume you sample deterministically by choosing the topic with the higher probability.

Since the probabilities are equal, we assign topics deterministically:

- **Document 1:** Assign topic 1 to "apple" and topic 2 to "banana".
- **Document 2:** Assign topic 1 to "banana" and topic 2 to "cherry".

3. Update Distributions

Update the ϕ_k and θ_m distributions based on the new topic assignments. Compute the new probabilities:

$$P(w|k) = \frac{C(w,k)}{\sum_{w'} C(w',k)}$$

$$P(k|d) = \frac{C(k,d)}{\sum_{k'} C(k',d)}$$

where $C(w,k)$ is the count of word w assigned to topic k across all documents, and $C(k,d)$ is the count of topic k in document d. **Updated ϕ_k:**

$$P(w|k) = \frac{C(w,k)}{\sum_{w'} C(w',k)}$$

$$C(w,k) : \begin{array}{lll} C(\text{apple}, 1) = 1, & C(\text{banana}, 1) = 1, & C(\text{cherry}, 1) = 0 \\ C(\text{apple}, 2) = 0, & C(\text{banana}, 2) = 1, & C(\text{cherry}, 2) = 1 \end{array}$$

$$\phi_1 = \left[\tfrac{1}{2}\tfrac{1}{2}0\right], \quad \phi_2 = \left[0\tfrac{1}{2}\tfrac{1}{2}\right]$$

Updated θ_m:

$$P(k|d) = \frac{C(k,d)}{\sum_{k'} C(k',d)}$$

$$C(k,d) : \begin{array}{ll} C(1, \text{Doc 1}) = 1, & C(2, \text{Doc 1}) = 1 \\ C(1, \text{Doc 2}) = 1, & C(2, \text{Doc 2}) = 1 \end{array}$$

$$\theta_1 = \left[\tfrac{1}{2}\tfrac{1}{2}\right], \quad \theta_2 = \left[\tfrac{1}{2}\tfrac{1}{2}\right]$$