Data Representation
Number System, 2's complement

Course: CPSC 1150
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Lecture 5

Learning Outcomes

- Convert any positive integer decimal numbers in other bases
- Convert any positive integer number in any bases to decimal
- Represent negative numbers in 2's complement
- Read a negative number
- Represent any real decimal number to binary system
- Convert a real binary number to decimal number
- Represent characters (text) in binary system

Binary System

– Decimal is base 10 and has 10 digit symbols:

0,1,2,3,4,5,6,7,8,9

– Binary is base 2 and has 2 digit symbols:

0,1

– Using **binary system** instead of the decimal system simplifies the design of computers and related technologies

– All data stored inside a **computer** is stored in binary (also called machine language) and interpreted to display on the screen in human language

Converting Decimal to Binary

What is 13 (in base 10) in base 2?

$13 \div 2$ quotient = 6 remainder 1
$6 \div 2$ quotient = 3 remainder 0
$3 \div 2$ quotient = 1 remainder 1
$1 \div 2$ quotient = 0 remainder 1

order for reading the remainder digits

Stop, because the quotient is now zero

Answer: 1 1 0 1

Converting Decimal to Other Bases

– Algorithm for converting a number in base 10 to other bases:

1. Repeat while (the quotient is not zero)

   1.1 Divide the decimal number by the new base

   1.2 Make the remainder the next digit to the left in the answer

   1.3 Replace the original decimal number with the quotient

Bases Higher Than 10

– Use alphabet characters as symbols needed for the base higher than 10.

Example

Base 16 (Hexadecimal) needs 16 digits, we can use the following symbols to represent all the digits:
0,1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F
A, B, C, D, E and F are used for 10, 11, 12, 13, 14 and 15 respectively.

Example

$1247_{(10)} = 4DF_{(16)}$

Positional Notation

Positional notation of (642) in Base 10:

$$
\begin{aligned}
& 6 \times 10^2 = 6 \times 100 = 600 \\
+\ & 4 \times 10^1 = 4 \times 10 = 40 \\
+\ & 2 \times 10^0 = 2 \times 1 = 2
\end{aligned}
$$

642 in base 10

The power indicates the position of the digit

This number is in base 10

– Use positional notation to convert numbers in other systems to its decimal value

Converting Other systems to Decimal

– *What if 642 has the base of 13?*

$642_{(13)} = 6 \times 13^2$ $(6 \times 169 = 1014)$

$+ 4 \times 13^1$ $(4 \times 13 = 52)$

$+ 2 \times 13^0$ $(2 \times 1 = 2)$

$= 1068_{(10)}$

– What is $11001010_{(2)}$ in decimal system?

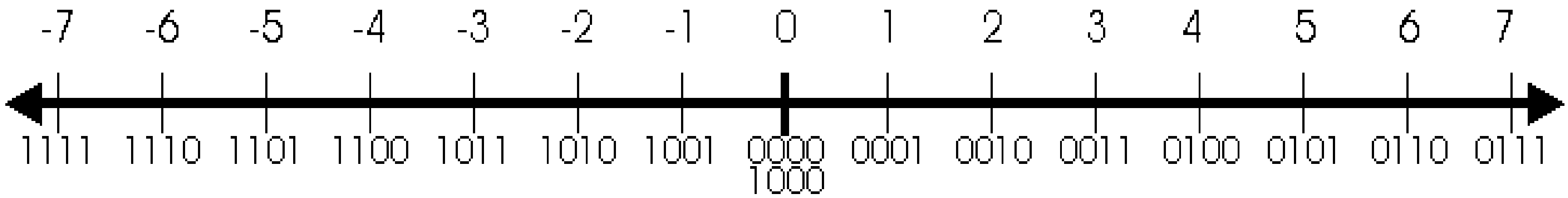
$11001010_{(2)} = 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3$

$+ 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 128 + 64 + 0 + 0 + 8 + 0 + 2 + 0$

$= 202_{(10)}$

Negative numbers : Signed Magnitude



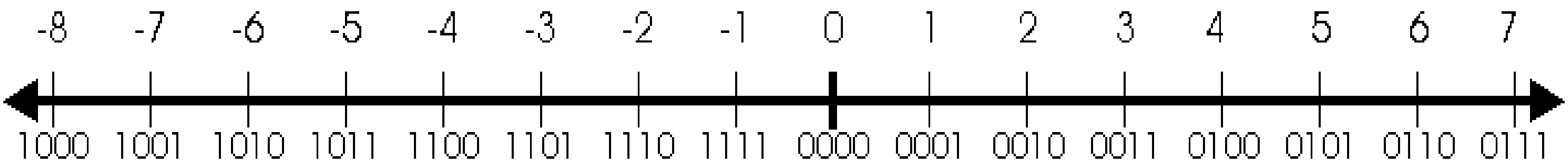
- With signed magnitude, we are able to represent **15 numbers (given n = 4):** -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, and 7 (**2 zero representations!**)

- **Missing one bit pattern:**

  How many possible bit patterns can be created with 4 bits? Easy, we know that's 2^4, or 16 (but, there are only 15!)

Negative numbers : 2's Complement

– Use patterns starts with 1 for negative numbers and patterns starts with 0 for positive numbers
  - given n = 4, the range of numbers: -8 to 7



– Quick identification of negative numbers
  - All negative numbers have the leftmost bit set to '1'

– Remember the range of integers (4 bytes) is $[-2^{31}, 2^{31} -1]$

How to Calculate 2's Complement

– Take the positive representation, flip the bits and add 1.

Negative	Positive	Flip	Add 1
-7	0111	1000	1001
-3	0011	1100	1101

– What about **subtraction**? Does it work properly?

4 -7 would be: 4 + (-7) = -3

How do I know this is -3?

Again, flip and add 1

0100 (+4)
1001 (-7)

1101 (-3)

Number Overflow

- With **4 bits** we could represent **- 8 to +7** (slide 15)
- Using **8 bits** (represents numbers in range **-128** to **+127**)
- Lets sum **127 and 3**

```
 1111111        carry
 01111111       +127
 00000011       +3
 --------------------
 10000010
```

Sign bit

It is -126

number overflow

What number is it?

Practice

- Assuming an **8-bit 2’s complement** representation, what is 11101011 in decimal?
- What is the 2’s complement of 89 in one byte?
- What is the 2’s complement of 89 in two bytes?

Representing real numbers

- **Real numbers** are numbers with an **integer part** and a **fractional part** (either of which may be zero)

104.32

0.999999

357.0

3.14159

In decimal, positions to the **right** of the decimal point are the tenths, hundredths, thousandths, etc.:

10^{-1}, 10^{-2} , 10^{-3} … **104.32=?**

Converting real binary to decimal

- Same rules apply in binary as in decimal
- “**Radix point**” is general term for “**decimal point**” in binary system
- Positions to the right of the radix point in binary:

  2^{-1} (halves position),
  2^{-2} (quarters position),
  2^{-3} (eighths position)
  …
- Convert this binary number (101.11) to decimal!

Converting real decimal to binary

- First, convert **integer value** from base 10 to binary
- Then convert the fractional part to binary as follows:
  - multiply the fraction by 2 rather than dividing,
  - Take the integer part of new value as part of binary number
  - Take the fractional part of new value and keep going until the fractional part becomes 0

Example

$7.25_{(10)} = ?_{(2)}$

$7.25_{(10)} = 111.01_{(2)}$

$0.25 * 2 = 0.5,$

$0.5 * 2 = 1.00$

Representing text

Definition

Text is a collection of characters

- What must be done to represent text (non numerical data)?
  - The number of characters to represent is limited, so list them all and assign each a binary code
  - This is really what a character set does

Definition

Character set is a list of characters and the corresponding codes used to represent each one. For example **ASCII** (1 byte) and **Unicode** (2 bytes) are two character sets

ASCII Character Set Mapping

	ASCII									
Left Digit(s) / Right Digit	0	1	2	3	4	5	6	7	8	9
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT
1	LF	VT	FF	CR	SO	SI	DLE	DC1	DC2	DC3
2	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS
3	RS	US	□	!	"	#	$	%	&	'
4	(	)	*	+	,	–	.	/	0	1
5	2	3	4	5	6	7	8	9	:	;
6	<	=	>	?	@	A	B	C	D	E
7	F	G	H	I	J	K	L	M	N	O
8	P	Q	R	S	T	U	V	W	X	Y
9	Z	[	\	]	^	_	`	a	b	c
10	d	e	f	g	h	i	j	k	l	m
11	n	o	p	q	r	s	t	u	v	w
12	x	y	z	{	|	}	~	DEL		

CR (Carriage Return) is a control character or mechanism used to reset a device's position to the beginning of a line of text

More Practice

- Convert 180 to binary.
  - Represent 180 in binary using an int variable
  - Represent 180 in binary using a short variable
  - Represent 180 in binary using a byte variable.
  - In which of the above representation, overflow happens?
- Convert 8.33 to binary.
  - Can you store 8.33 in memory with 100% precision?
  - Is there any chance of having roundoff error? Explain it.
- Show the binary value that Java uses to store character 'G' in memory?