Optimal Asset Allocation in Asset Liability Management*

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Abstract

We study the impact of regulations on the investment decisions of a defined benefits pension plan. We assess the influence of ex ante (preventive) and ex post (punitive) risk constraints on the gains to dynamic, as opposed to myopic, decision making. We find that preventive measures, such as Value-at-Risk constraints, tend to decrease the gains to dynamic investment. In contrast, punitive constraints, such as mandatory additional contributions from the sponsor when the plan becomes underfunded, lead to very large utility gains from solving the dynamic program. We also show that financial reporting rules have real effects on investment behavior. For example, the current requirement to discount liabilities at a rolling average of yields, as opposed to at current yields, induces grossly suboptimal investment decisions.

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1 Introduction

The investment behavior of corporate pension plans is receiving increasing attention. A recent study by the Pension Benefit Guarantee Corporation shows that the number of pension plans that are more than $50 million short of promised benefit levels has risen from 221 in 2000 to 1,108 in 2004 with an aggregate of $786.8 billion in assets to cover $1.14 trillion in liabilities.[1] In response to this crisis, a set of funding rule reforms has been proposed to strengthen the pension system. These proposed reforms include limits on the deviation of the actuarial values of assets and liabilities from their market values and more stringent guidelines for pension plans to make up shortfalls in the value of their assets, relative to that of their liabilities, through additional financial contributions (AFCs) to the plan. We study in this paper the optimal asset allocation decisions of an investment manager of a defined benefit pension plan as a function of the plan's funding ratio (defined as the ratio of its assets to liabilities), interest rates, and the equity risk premium. We compare the optimal investment decisions under several policy alternatives to understand better the real effects of financial reporting and risk management rules.

We focus on two general mechanisms a regulator or principal can apply to keep an agent from taking undesirable actions: prevention and punishment. To illustrate these mechanisms, consider the example of a truck driver. The carrier rewards the truck driver for delivering the goods he transports early, but wants to keep him from taking excessive risk when speeding. One way to achieve the latter is to provide the driver with a truck that mechanically cannot exceed the speed limit (prevention). Alternatively, the carrier can make the driver pay for the damage in case he is involved in an accident, for example in the form of a salary cut (punishment). In the context of investment management, ex ante (preventive) risk constraints, such as Value-at-Risk (VaR) constraints, short-sale constraints and a maximum weight in stocks, restrict the investment manager's set of allowable portfolio weights. The manager is required to adhere to these ex ante constraints but is not held responsible for bad return realizations ex post. In contrast, ex post (punitive) risk constraints, when triggered, lead to a punitive action that decreases the investment manager's utility either through loss of personal compensation or reputation. Such ex post constraints do not restrict investment choices ex ante, but the manager is held accountable for a bad return realizations ex post. The ex post constraint we focus on, is the requirement to draw AFCs from the plan sponsor whenever the plan becomes underfunded.[2]

[1] 2004 Pension Insurance Data Book, Pension Benefit Guarantee Corporation.

[2] Another important example of an ex post constraint is a firing rule for investment managers based on their performance.

At first glance, ex ante and ex post risk constraints may seem similar as both aim to decrease the risk-taking behavior of the manager. However, we show that they have profoundly different implications for the gains to dynamic, as opposed to myopic, decision making. We show that ex ante (preventive) constraints tend to *decrease* the gains to dynamic investment. Ex post (punitive) constraints, in contrast, largely *increase* the utility gains from solving the dynamic program. In other words, under ex ante constraints, the myopic solution provides a good approximation for the optimal solution whereas under ex post constraints it requires dynamic optimization to make the optimal investment decision. As such, ex post constraints induce the manager to behave strategically.

Another important aspect of the asset liability management (ALM) problem is the discount factor used for computing the present value of a pension plan's liabilities. Recently, the discount factor has received a lot of attention for its impact on the reported financial position of the plan. On one hand, discounting by current yields guarantees an accurate description of the fund's financial situation. On the other hand, using a constant yield smoothes out temporary fluctuations in the present value of the liabilities and gives a more long-term description of the fund's financial condition. Under current regulations, the discount factor equals a four-year rolling average of the 30-year government (or corporate) bond yield, which constitutes a compromise between the two options described above. What has received much less attention, however, is the effect that these financial reporting rules have on the optimal decisions of the investment manager of the pension plan. We show that the way liabilities are computed can drive an important wedge between the fund manager's long-term objective of maximizing the funding ratio and his short-term objective (and/or requirement) of satisfying risk constraints and avoiding AFCs from the plan sponsor as described above. The key to this wedge is the fact that these risk constraints are based on the (smoothed) reported liabilities instead of on the actual liabilities that enter into the investment manager's long-term objective.

We thus examine two important issues in a stylized ALM problem. First, we address the role of hedging demands. ALM problems are inherently long-horizon problems with potentially important strategic aspects.[3] They differ from standard portfolio choice problems (Markowitz (1952), Merton (1969,1971), Samuelson (1969) and Fama (1970)), not only because of the short position in the pension liabilities, but also because of the regulatory risk constraints and mandatory AFCs discussed above. We assume that the investment manager dislikes drawing AFCs from the plan sponsor and directly model this dislike as a utility cost. We interpret this utility cost as a reduced form for the loss of compensation or

[3]Recent strategic asset allocation studies include Kim and Omberg (1996), Campbell and Viceira (1999), Brandt (1999,2005), Aït-Sahalia and Brandt (2001), and Sangvinatsos and Wachter (2005).

reputation of the investment manager. In other words, drawing mandatory AFCs serves as an ex post (punitive) risk constraint. The associated utility cost introduces a kink in the value function of the investment manager's dynamic optimization problem that causes the manager to become first-order risk averse whenever the (reported) funding ratio approaches the critical threshold that triggers AFCs. We show that this kink in the value function leads to substantial hedging demands and large certainty equivalent utility gains from dynamic investment. We also find that ex ante risk constraints, such as Value-at-Risk constraints, decrease the gains to dynamic investment. Such constraints do not introduce a kink in the value function, leading to the relatively flat peak of the value function that is common for intertemporal investment problems with power utility. This flat peak implies that even large deviations from the optimal decision lead to only small welfare losses (e.g., Cochrane (1989)). In addition, the ex ante constraints decrease the menu of available portfolio weights, thereby inhibiting the manager to execute his desired hedging demands. As a consequence the gains from dynamic (strategic) investment are small.

Second, we show that under the assumption that the plan manager dislikes AFCs and in the long run wishes to maximize (the utility from) the actual funding ratio, smoothing the reported liabilities induces grossly sub-optimal investment behavior. The associated welfare loss is a direct consequence of the misalignment of long- and short-term incentives. The investment manager is torn between his long-term objective of maximizing the actual funding ratio (assets to *actual* liabilities) and his short-term concerns about risk constraints and drawing AFCs based on the reported funding ratio (assets to *smoothed* liabilities). Interestingly, we find that risk controls based on the smoothed liability measure can inadvertently induce the manager to take *riskier*, instead of less risky, positions.

The investment behavior of corporate pension plans has been studied by Sundaresan and Zapatero (1997) and by Boulier, Trussant and Florens (2005). Sundaresan and Zapatero (1997) model the marginal productivity of the workers of a firm and solve the investment problem of its pension plan assuming a constant investment opportunity set consisting of a risky and a riskless asset. We instead allow for a time-varying investment opportunity set including cash, bonds, and stocks. More importantly, we consider the ALM problem from the perspective of the investment manager as a decision maker and investigate how regulatory rules influence the optimal investment decisions. In order to focus attention on the asset allocation side of the ALM problem, we model the liabilities of the pension plan in reduced form by assuming a constant duration of 15 years.

Boulier, Trussant, and Florens (2005) also assume a constant investment opportunity set with a risky and a riskfree asset. In their problem, the investment manager chooses his portfolio weights to minimize the expected discounted value of the contributions over a fixed time horizon, with the constraint that the value of the assets cannot fall below that of the liabilities at the terminal date. This problem setup implicitly assumes that the pension plan terminates at some known future date and that the investment manager's horizon is equal to this terminal date. By taking the investment manager's preferences and horizon as the primitive, our perspective is different. The manager has a motive to minimize (the disutility from) the sponsor's contributions, captured by the AFCs in our case. However, the manager also has a bequest motive by wanting to maximize the funding ratio at the end of his investment horizon. The end of the manager's investment horizon may be long before the pension plan terminates, which is why we hold the duration of the liabilities fixed.

Our contributions to the literature are the following. First, we attempt to bridge further the gap between the dynamic portfolio choice literature and the ALM literature.[4] We pose the ALM problem as a standard dynamic portfolio choice problem by defining terminal utility over the ratio of assets and liabilities, as opposed to over assets only. This approach allows a parsimonious representation of the ALM problem under a time-varying investment opportunity set. Solving this dynamic program is relatively straightforward compared to the usual, more complicated, stochastic programming techniques. We then assess the interplay between dynamic hedging demands, risk constraints, and first-order risk aversion. We show that the solution to the ALM problem under ex post (punitive) constraints involves economically significant hedging demands, whereas ex ante (preventive) constraints decrease the gains from dynamic investment. Finally, we explicitly model the trade-off between the long-term objective of maximizing terminal utility and the short-term objective of satisfying VaR constraints and avoiding AFCs from the plan sponsor.[5] We show that if these short-term objectives are based on reported liabilities that are different from actual liabilities, they can lead to large utility losses with respect to the long-term objective.

In order to focus on these contributions, there are at least three important aspects of the ALM problem that we do not address explicitly. First, there is a literature, starting with Sharpe (1976), that explores the value of the so-called "pension put" arising from the fact that U.S. defined-benefit pension plans are insured through the Pension Benefit Guarantee Corporation. Sharpe (1976) shows that if insurance premiums are not set correctly, the

[4] Campbell and Viceira (2002) and Brandt (2005) survey the dynamic portfolio choice literature.

[5] We could easily incorporate other short-term objectives, such as beating a benchmark portfolio over the course of the year (see also Basak, Shapiro, and Teplá (2006) and Basak, Pavlova, and Shapiro (2007)). Whenever this short-term objective is defined with respect to reported liabilities that are different from actual liabilities, this leads to a similar misalignment of incentives as the one we explore in this paper. It is interesting to note that in practice pension fund managers are often assessed relative to an assets-only benchmark, which is a benchmark that implicitly assumes constant liabilities (see van Binsbergen, Brandt and Koijen (2006)).

optimal investment policy of the pension plan may be to maximize the difference between the value of the insurance and its cost. This obviously induces perverse incentives. Sharpe also shows that if insurance premiums are set correctly, the pension put does not affect investment behavior. Even if insurance premiums are not set correctly, however, it is not clear how realistic it is to assume that a corporate pension plan gives its investment manager a mandate to exploit the insurance system. Second, we do not incorporate inflation. Besides affecting the allocation to real versus nominal assets (Hoevenaars et al. (2004)), inflation drives another wedge between the long-term objective of maximizing the real funding ratio, computed with liabilities that are usually pegged to real wage levels, and the short-term objective of satisfying risk controls and avoiding AFCs based on nominal valuations. Third, we ignore the taxation issues described by Black (1980) and Tepper (1981).

The paper proceeds as follows. Section 2 describes the return dynamics, the preferences of the investment manager, and the constraints under which the manager operates. We model the dynamics of stock returns, short-term bond yields, and long-term bond yields, as a first-order vector autoregression (VAR). The investment manager has power utility with constant relative risk aversion (CRRA) but incurs a linear utility cost every time the pension plan becomes underfunded and is forced to draw AFCs from its sponsor. Section 3 describes our numerical solution method for the dynamic optimization problem, which is a version of the simulation-based algorithm developed by Brandt, Goyal, Santa-Clara, and Stroud (2005). Section 4 presents our results. Because the impact of yield smoothing can easily be illustrated in a simple one-period model, we first address this issue before addressing the role of hedging demands in the multi-period setting. We show that smoothing yields to compute the value of the liabilities can lead to grossly suboptimal investment decisions. We then assess the gains to dynamic, as opposed to myopic investment for four cases: *i.* a standard CRRA investment problem with a time-varying investment opportunity set (no liabilities, no sponsor contributions, no risk constraint), *ii.* a standard ALM problem (no sponsor contributions and no risk constraint), *iii.* an ALM problem with a VaR constraint (no sponsor contributions), and *iv.* an ALM problem with sponsor contributions. We show that ex ante risk constraints decrease the already small gains from dynamic investment in the absence of AFCs. However, when we introduce the utility cost of AFCs, the gains from dynamic investment become economically very large.

2 ALM problem

The ALM problem requires that we specify the investment opportunity set (or return dynamics), the preferences of the investment manager, and the risk constraints the investment manager faces. The next three sections describe these three items in turn.

2.1 Return dynamics

We consider a pension plan that can invest in three asset classes: stocks, bonds, and the riskfree asset. Stocks are represented by the Standard and Poors (S&P) 500 index, bonds by a 15-year constant maturity Treasury bond, and the riskfree asset by a one-year Treasury bill. We consider an annual rebalancing frequency. Reducing the investment opportunity set to only three asset classes may seem restrictive. However, these asset classes should be interpreted as broader categories where long-term bonds represent assets that are highly correlated with the liabilities; stocks and one-year Treasury bills represent assets that have a low correlation with liabilities and have respectively a high risk/high return and low risk/low return profile. We assume that the one-year and 15-year log yield levels follow a first-order VAR process. We model stock returns with a time-varying risk premium that depends on the level and slope of the yield curve (e.g., Ang and Bekaert (2005)). Formally the return dynamics are:

$$\begin{bmatrix} r_{s,t} \\ \ln(y_{1,t}) \\ \ln(y_{15,t}) \end{bmatrix} = A + B \begin{bmatrix} \ln(y_{1,t-1}) \\ \ln(y_{15,t-1}) \end{bmatrix} + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim \text{MVN}(0, \Sigma), \tag{1}$$

where $r_{s,t}$ is the annual log return on the S&P 500 index (including distributions), $y_{1,t}$ and $y_{15,t}$ are the annualized continuously compounded yields on the one-year Treasury bill and the 15-year constant maturity Treasury bond, A and B are respectively a 3×1 vector and a 3×2 matrix of parameters, ε_t is a 3×1 vector of innovations, and Σ is a 3×3 covariance matrix. We model the dynamics of the log yields in the spirit of Black and Karasinski (1991) to ensure that yields are positive.[6] The VAR(1) representation allows for both a time-varying risk free rate, time-varying expected bond returns, and a time-varying equity risk premium.[7] The estimation results are given in Appendix A.

[6]Note that we assume that asset returns are homoskedastic. Recent evidence by Chacko and Viceira (2005) suggests that the volatility of stock returns is not persistent and variable enough to create sizeable hedging demands.

[7]For recent work on return predictability see Ang and Bekaert (2005), Lewellen (2004), Campbell and Yogo (2005), and Torous, Valkanov, and Yan (2005) for stock returns, as well as Dai and Singleton (2002) and Cochrane and Piazzesi (2005) for bond returns.

Our stylized reduced form model of the term structure is not arbitrage-free. The forecasting power of the VAR model may improve when no-arbitrage conditions are imposed in the estimation of the VAR (e.g., Ang and Piazessi (2002)). However, it is not our purpose to predict future yields, but rather to investigate the optimal behavior of the investment manager facing a reasonable description of a time-varying investment opportunity set.

We assume that the pension plan has liabilities with a fixed duration of 15 years. We measure the value of these liabilities in three ways. First, we compute the actual present value of the liabilities by discounting by the actual 15-year government bond yield:[8]

$$L_t = \exp(-15 y_{15,t}). \tag{2}$$

Our second measure is based on current regulations prescribing that the appropriate discount factor is the four-year *average* long-term bond yield, which implies:

$$\hat{L}_t = \exp(-15 \hat{y}_{15,t}), \tag{3}$$

where

$$\hat{y}_{15,t} = \frac{1}{4}(y_{15,t} + y_{15,t-1} + y_{15,t-2} + y_{15,t-3}). \tag{4}$$

Finally, we compute the value of the liabilities using a constant yield equal to the steady state value of the long-term bond yield $\bar{y}_{15}$ implied by the VAR (see Appendix A):

$$\bar{L}_t = \exp(-15 \bar{y}_{15}) \tag{5}$$

Note that with all three measures the liabilities follow a stationary stochastic process. The model could easily be extended to include a deterministic time trend representing demographic factors. However, to maintain a parsimonious representation, we focus on the detrended series. Our specification also abstracts from inflows (premium payments) and outflows (pension payments) to the fund. We assume that in each year the inflows equal the outflows, which allows us to focus purely on the investment management part of the fund. The only inflows we consider are cash injections by the plan sponsor required to meet the regulator's minimal funding level. Note further that the three measures of liabilities described above are driven by only one risk factor, the 15-year government bond yield. This could suggest that a one-factor model for the term structure would suffice in our model.

[8]To maintain a parsimonious representation we use the 15-year bond yield to determine the discount factor instead of the 30-year bond yield. Since the dynamics of both yields are very similar, this simplification does not influence our results.

However, we assume a two-factor model to allow for a time-varying riskfree rate.

We compute the simple gross returns on the three asset classes as follows:

$$\begin{aligned} R_{f,t} &= \exp(y_{1,t-1}) \qquad (6) \\ R_t &= \begin{bmatrix} R_{s,t} \\ R_{b,t} \end{bmatrix} = \begin{bmatrix} \exp(r_{s,t}) \\ \exp(-14y_{15,t})/\exp(-15y_{15,t-1}) \end{bmatrix}, \end{aligned}$$

where $R_{s,t}$ is the simple gross return on stocks, $R_{f,t}$ is the return on the one-year T-bill (riskfree), and $R_{b,t}$ is the simple gross return on long-term bonds. Our expression for the bond return assumes that the yield curve is flat between 14 and 15 years to maturity.

The funding ratio of the pension plan is defined as the ratio of its assets to liabilities:

$$S_t = \frac{A_t}{L_t}, \qquad (7)$$

where assets evolve from one period to the next according to:

$$A_t = A_{t-1}\left(R_{f,t} + \alpha_{t-1} \cdot (R_t - R_{f,t})\right) + c_t \exp(-15y_{15,t}) \quad \text{for } t \geq 1 \qquad (8)$$

and $\alpha_t \equiv [\alpha_{s,t}, \alpha_{b,t}]'$ denotes the portfolio weights in stocks and bonds. We let c_t denote the contributions of the plan sponsor at time t as a percentage of the liabilities which, under actual discounting, are equal to $\exp(-15y_{15,t})$. Note that defining the contributions as a percentage of the liabilities is equivalent to expressing contributions in future $(t + 15)$ dollars. When liabilities are determined through constant discounting or four-year average discounting, we define the contributions as a percentage of those liability measures and the last term in expression (8) is adapted accordingly. We use $\hat{S}_t$ and $\bar{S}_t$ to denote the funding ratios computed using the liability measures $\hat{L}_t$ and $\bar{L}_t$, respectively.

Finally, we define A_t^* as the assets in period t before the contributions are received, and S_t^* as the ratio of A_t^* and the liabilities:

$$\begin{aligned} A_t^* &= A_{t-1}\left(R_{f,t} + \alpha_{t-1} \cdot (R_t - R_{f,t})\right) \quad \text{for } t \geq 1 \qquad (9) \\ S_t^* &= \frac{A_t^*}{L_t} \qquad (10) \end{aligned}$$

2.2 Preferences

We take the perspective of an investment manager facing a realistic regulatory environment. We assume that the manager's utility is an additively separable function of the funding ratio at the end of the investment horizon (S_T) and the requested extra contributions from its sponsor as a percentage of the liabilities (c_t). We assume that the manager suffers disutility in the form of unmodeled reputation loss or loss in personal compensation for requesting these contributions. The utility function of the manager is given by:

$$\begin{aligned} U\left(S_T, \{c_t\}_{t=1}^{T-1}\right) =& \mathrm{E}_0\left[u\left(S_T\right) - \sum_{t=1}^{T} v\left(c_t, t\right)\right] \\ =& \mathrm{E}_0\left[\beta^T \frac{S_T^{1-\gamma}}{1-\gamma} - \lambda \sum_{t=1}^{T} \beta^t c_t\right] \text{ where } \gamma \geq 0 \text{ and } \lambda \geq 0. \end{aligned} \tag{11}$$

The first term in the utility function is the standard power utility specification with respect to the funding ratio at the end of the investment horizon, S_T. We call this wealth utility. We assume that this wealth utility always depends on the actual funding ratio. That is, we use the actual yields to compute the liabilities in the denominator, regardless of government regulations, as opposed to using a smoothed or constant yield. The motivation for this assumption is that ultimately the manager is interested in maximizing the actual financial position of the fund, which is also the position the pension holders care about.[9]

The term $\sum_{t=1}^{T} \beta^t v\left(c_t\right)$ represents the investment manager's disutility (penalty) for requesting and receiving extra contributions c_t from the plan sponsor. This penalty can be interpreted as loss in reputation or compensation. The linear function just reflects the first-order effect of these penalties and higher order terms could be included in our analysis.[10] Furthermore, when contributions from the sponsor are set equal to the funding ratio shortfall, linearity of the function $v(\cdot)$ implies that the utility penalties are scaled versions of the expected loss, which, next to a VaR constraint, is often used as a risk measure. As noted by Campbell and Viceira (2005), the weakness of a VaR constraint is that it treats all shortfalls greater than the VaR as equivalent, whereas it seems likely that the cost of a shortfall is increasing in the size of the shortfall. They, therefore, propose to incorporate the expected

[9] It is interesting to note that even when both wealth utility and the risk constraints/AFCs are determined through four-year average discounting, there is still a misalignment of incentives for a multi-period investment problem. The risk constraints (which apply in every period) still induce the use of the risk-free asset. This is a consequence of the large reduction of the *conditional* variance that yield smoothing induces: $\mathrm{var}_t(\hat{y}_{15,t+1}) = \mathrm{var}_t[\frac{1}{4}(y_{15,t+1} + y_{15,t} + y_{15,t-1} + y_{15,t-2})] = \frac{1}{16}\mathrm{var}_t[y_{15,t+1}]$. Wealth utility, on the other hand, depends on the funding ratio in year T. The conditional variance of $\hat{y}_{15,T}$ is given by: $\mathrm{var}_t(\hat{y}_{15,T}) = \mathrm{var}_t[\frac{1}{4}(y_{15,T} + y_{15,T-1} + y_{15,T-2} + y_{15,T-3})]$. Note that for a 10-year investment problem, $T = 10$, the yields in year 10, nine, eight and seven (which jointly determine the liabilities in year 10) are all unknown before year seven. Therefore, in periods one through six, long-term bonds are still the preferred instrument to hedge against liability risk when maximizing wealth utility.

[10] Non-linear specifications for the function $v(\cdot)$, e.g. a quadratic form, do not change our qualitative results.

loss directly in the utility function, which in our framework is achieved by the linearity of the function $v(\cdot)$. Finally, the investment manager discounts next period's utility and disutility by the subjective discount factor β.

Another appealing interpretation of our utility specification is the following. In the context of private pension plans, the investment manager acts in the best interest of two stakeholders of the plan, (i) the pension holders who are generally risk averse and (ii) the sponsoring firm which we assume to be risk neutral. The parameter λ then measures the investment manager's tradeoff between these two stakeholders. If one believes that the investment manager merely acts in the best interest of the firm, the value of λ is high. Conversely, if one believes that the investment manager acts mainly in the interest of the beneficiaries, λ is low.

Finally, we can interpret the proposed utility specification in yet two other interesting ways. First we can interpret it as a portfolio choice problem with intermediate consumption and bequest. In the literature on life-time savings and consumption, it is common to assume that utility from consumption is additively separable from bequest utility. The only difference is that, in our case, consumption is strictly negative and not strictly positive. In other words, the investment manager can increase his wealth by suffering negative consumption which leads to a tradeoff between maximizing (the utility from) the funding ratio at the end of the investment horizon and minimizing (the disutility from) the contributions along the way. The second interpretation is that similar utility specifications have been used in the general equilibrium literature with endogenous default, where agents may choose to default on their promises, even if their endowments are sufficient to meet the required payments (e.g., Geanakoplos, Dubey, Shubik (2005)). Agents incur utility penalties which are linearly increasing in the amount of real default. The idea of including default penalties in the utility specification was first introduced by Shubik and Wilson (1977).

The tradeoff between the disutility from contributions and wealth utility is captured by the coefficient λ. When we impose that in each period the sponsor contributions are equal to the funding ratio shortfall, and this shortfall is determined through actual discounting, a value of $\lambda = 0$ implies that the investment manager owns a put option on the funding ratio with exercise level $S^* = 1$. This gives the manager an incentive to take riskier investment positions. When $\lambda \to \infty$, the disutility from contributions is so high that the investment manager will invest conservatively to avoid a funding ratio shortfall when the current funding level is high. Depending on how liabilities are computed, investing conservatively either implies investing fully in the riskfree asset or investing fully in bonds (to immunize the liabilities) or a mixture of the two.[11]

Increasing the funding ratio at time zero affects the expected utility in three ways. First, it increases current wealth and therefore, keeping the investment strategy constant, also increases expected wealth utility. Second, if there is a period-by-period risk constraint, a higher funding ratio will make the risk constraint less binding in the current period and also decreases its expected impact on future decisions. Third, keeping the investment strategy constant, the probability of incurring contribution penalties in future periods decreases.

2.3 Constraints

2.3.1 Short sale constraints

We assume that the investment manager faces short sales constraints on all three assets:

$$\alpha_t \geq 0 \ \text{ and } \ \alpha_t' \iota \leq 1. \tag{12}$$

2.3.2 VaR constraints

Pension funds often operate under Value-at-Risk (VaR) constraints. A VaR constraint is an ex ante (preventive) risk constraint. It is a risk measure based on the probability of loss over a specific time horizon. For pension plans, regulators typically require that over a specific time horizon the probability of underperforming a benchmark is smaller than some specified probability. The most natural candidate for this benchmark is the fund's liabilities. In this case, the VaR constraint requires that in each period the probability of being underfunded in the next period is smaller than probability δ. We set δ equal to 0.025. Depending on prevailing regulations, the relevant benchmark can be the actual liabilities (L_t), constant liabilities ($\bar{L}_t$) or, as under current regulations, $\hat{L}_t$.

We also compute the optimal portfolio weights and certainty equivalents when there are no additional contributions from the sponsor. In that case, there is no external source of funding that guarantees the lower bound of one on the funding ratio. It may therefore be that in some periods the fund is underfunded to begin with. In those cases, the VaR constraint described above can not be applied and requires adaptation. When at the beginning of the

[11] When $\lambda \geq 1$, concavity of the utility function is guaranteed under actual discounting. For $\lambda = 1$, the utility is smooth, but for $\lambda > 1$, it is kinked at $S^* = 1$. The right derivative of the function $\frac{1}{1-\gamma}[\max(S^*, 1)]^{1-\gamma} - \lambda \max(1 - S^*, 0)$ is 1 whereas its left derivative equals λ. The risk neutrality over losses combined with the kinked utility function at $S^* = 1$ resembles elements of prospect theory (Kahneman and Tversky (1979)).

period the fund has less assets than liabilities, we impose that the probability of a decrease in the funding ratio is less than 0.025. In other words, if the fund is underfunded to begin with, the manager faces a VaR constraint as if the funding ratio equals one.

2.3.3 AFCs

Under current regulations, a pension plan is required to receive AFCs from its sponsor whenever it is underfunded. As the manager dislikes drawing AFCs from the plan sponsor, this requirement serves as an ex post (punitive) risk constraint. The government regulation around these mandatory AFCs is not at all trivial. First, the fund is allowed to amortize a realized shortfall over 30 years. Current reform proposals shorten this amortization to 18 years. Furthermore, there is currently a credits system in place which implies that previous excess contributions can be subtracted from current required contributions, regardless of the financial condition of the fund. For example, if a fund is 30 percent underfunded, but in the past the plan sponsor has contributed significantly more than necessary, current regulations exempt the sponsor from having to reduce the shortfall. This credits system can obviously significantly endanger the financial stability of the pension system, which is by now well recognized and major changes have been proposed. In our setup, we set the contributions of the sponsor equal to the funding ratio shortfall in each period. However, the measurement of this shortfall is highly dependent on the way liabilities are computed, which is what we study in this paper. Hence, we do not allow for credits nor do we allow for amortization of the shortfall. Since the latter can easily be mimicked by a bond that amortizes over time, we do not consider this to be a severe restriction in our model.

2.4 Data description and estimation

We use annual data from 1954 through 2004 to estimate the parameters of the return process. For stock returns we take the natural logarithm of the return on the S&P 500 composite index including distributions. For bond yields we use the continuously compounded constant maturity yields as published by the Federal Reserve Bank. Whenever data on 15-year government bonds is missing, we take an average of the 10 and 20-year bond yields. We estimate the model by OLS. We include dummy variables for the period 1978-1983 in our estimation to correct for this exceptional period with high inflation. Excluding the dummies does not change our conclusions. The estimation results are given in Appendix A.

In Figure 1 we plot: *i.* the 15-year bond yield, *ii.* the four-year smoothed 15-year bond yield and, *iii.* the steady state value for the 15-year bond yield that follows from our VAR specification. The graph shows that the unconditional variance of the 15-year bond yield is close to the unconditional variance of the smoothed 15-year bond yield. In other words, the 15-year yield is so persistent that a four-year smoothing period is not long enough to decrease its unconditional variance.[12] To the extent that the purpose of yield smoothing is to create stability in the pension system by decreasing the unconditional variance of the discount factor, we have to conclude that this goal is currently not reached. However, the conditional variance of the smoothed series, given by:

$$\mathrm{var}_t(\hat{y}_{15,t+1}) = \mathrm{var}_t[\frac{1}{4}(y_{15,t+1} + y_{15,t} + y_{15,t-1} + y_{15,t-2})] = \frac{1}{16}\mathrm{var}_t[y_{15,t+1}], \tag{13}$$

is a factor sixteen smaller than the conditional variance of the actual yield series. This conditional variance reduction in combination with the risk constraints induces the perverse incentives that are the scope of this paper.

3 Method

The ALM investment problem, even in stylized form, is a complicated and path-dependent dynamic optimization program. We use the simulation-based method developed by Brandt, Goyal, Santa-Clara, and Stroud (2005) to solve this program. The main idea of their method is to parameterize the conditional expectations used in the backward recursion of the dynamic problem by regressing the stochastic variables of interest across simulated sample paths on a polynomial basis of the state variables.[13] More specifically, we generate $N = 10,000$ paths of length T from the estimated return dynamics. We then solve the dynamic problem recursively backward, starting with the optimization problem at time $T-1$:

$$\max_{\alpha_{T-1}} U\left(S_T\right) = \max_{\alpha_{T-1}} \mathrm{E}_{T-1}\left[\frac{\beta}{1-\gamma}S_T^{1-\gamma} - \lambda\beta c_T\right], \tag{14}$$

subject to equations (1), (2), (6), (7) and (8) as well as the short sale constraints and the definition of the required contributions.

The solution of this problem depends on S_{T-1}. To recover this dependence, we solve a range of problems for S_{T-1} varying between 0.4 and three. For each value of S_{T-1} we optimize over the portfolio weights α_{T-1} by a grid search over the space $[0,1]\times[0,1]$. This grid search over the portfolio weights avoids a number of numerical problems that can occur when taking first order conditions and iterating to a solution. We then evaluate the conditional

[12] As noted before, similar results hold for the 30-year bond yield

[13] This approach is inspired by Longstaff and Schwartz (2001) who first proposed this method to price American-style options by simulation.

expectation $\mathrm{E}_{T-1}\left(S_T^{1-\gamma}\right)$ by regressing for each value of S_{T-1} and each grid point of α_{T-1} the realizations of $S_T^{1-\gamma}$ $(N \times 1)$ on a polynomial basis of the two state variables $y_{15,T-1}$ and $y_{1,T-1}$. Define:

$$z = \left[\begin{array}{cc} z_1 & z_2 \end{array}\right] = \left[\begin{array}{cc} y_{1,T-1} & y_{15,T-1} \end{array}\right], \tag{15}$$

then

$$X = \left[\begin{array}{ccccccc} 1 & z_{1,1} & z_{2,1} & (z_{1,1})^2 & (z_{2,1})^2 & (z_{1,1})(z_{2,1}) & ... \\ 1 & z_{1,2} & z_{2,2} & (z_{1,2})^2 & (z_{2,2})^2 & (z_{1,2})(z_{2,2}) & ... \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & ... \\ 1 & z_{1,N} & z_{2,N} & (z_{1,N})^2 & (z_{2,N})^2 & (z_{1,2})(z_{2,N}) & ... \end{array}\right] \tag{16}$$

and

$$\mathrm{E}_{T-1}\left(S_T^{1-\gamma}\right) = X'\hat{\beta}, \tag{17}$$

where

$$\hat{\beta} = (X'X)^{-1} X' \left(S_T^{1-\gamma}\right). \tag{18}$$

When liabilities are discounted using the rolling four-year average yield, we have to include polynomial expansions of all four lags of both state variables in our solution method. To evaluate the conditional expectation of the contributions in period T, $\mathrm{E}_{T-1}(c_T)$, we first regress $1 - S_T^*$ on X:

$$\hat{\zeta} = (X'X)^{-1} X' (1 - S_T^*). \tag{19}$$

Assuming normality for the error term in the regression and letting $\hat{\sigma}$ denote its standard deviation, we find:

$$\mathrm{E}_{T-1}(c_T) = \mathrm{E}_{T-1}\left[\max(1 - S_T^*, 0)\right] = \Phi\left(\frac{X'\hat{\zeta}}{\hat{\sigma}}\right) X'\hat{\zeta} + \hat{\sigma}\phi\left(\frac{X'\hat{\zeta}}{\hat{\sigma}}\right), \tag{20}$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ denote respectively the cumulative and probability density functions of the standard normal distribution. Note that this conditional expectation also represents the expected loss of the fund over the next period. In the exposition above, contributions are determined by actual discounting. When we use constant or four-year average discounting we simply replace S_T by $\bar{S}_T$ or $\hat{S}_T$ and we replace S_T^* by $\bar{S}_T^*$ or $\hat{S}_T^*$.

Given the solution at time $T-1$, meaning the mapping from S_{T-1} to the optimal α_{T-1}, we iterate backwards through time. The iterative steps are as described above with just a few additions. For ease of exposition we now describe these additions for period $T-2$, but they equally apply for periods $T-3, T-4, \ldots, 1$. At time $T-2$ we determine for each grid point of α_{T-2} the return on the portfolio in path $i \in N$ from $T-2$ to $T-1$. Using this return to compute $S_{T-1,i}$, we can then compute the return in path i from $T-1$ to T by interpolating over the mapping from S_{T-1} to α_{T-1} derived in the previous step. Similarly, we interpolate in each path the expected penalty payments.

We impose in each path and in each period a VaR constraint. For given values of S_t we determine for each α_t the conditional mean and conditional variance of the funding ratio in period $t+1$ through regressions on the polynomial basis of the state variables. By assuming log normality, we then evaluate whether the probability of a funding ratio shortfall (i.e., a funding ratio smaller than one) in period $t+1$ is less than δ. If this requirement is not met, those particular portfolio weights are excluded from the investment manager's choice set. As described above, the VaR can be imposed with respect to S_t^* (discounting at actual yields), $\bar{S}_t^*$ (discounting at constant yields), or $\hat{S}_t^*$ (discounting at the four-year average yield).

4 Results

4.1 The impact of smoothing yields

In this section we investigate the investment manager's optimal portfolio choice when he is faced with risk constraints that are based on the smoothed liability measure. We consider a VaR constraint as the ex ante (preventive) constraint. For the ex post (punitive) constraint, we consider the requirement to draw AFCs whenever the plan becomes underfunded. Note again that both the VaR constraint and the AFCs are short-term considerations based on the *smoothed* liability measure whereas the long-term objective of wealth utility is defined with respect to the *actual* liability measure. We quantify in this section the welfare losses that result from the wedge that yield smoothing drives between these short- and long-term considerations. We first solve a one-period problem to explain the main intuition in a parsimonious setting. We then explain how the results change in a multi-period setup.

Furthermore, it is very interesting to note that both the VaR constraint and AFCs are intended to *decrease* the manager's risky holdings as the funding ratio approaches the critical threshold of one. We show that when these short-term objectives are defined with respect to the smoothed liability measure, they can inadvertently induce the manager to *increase* his risky positions. We therefore conclude that smoothing yields may lead to highly perverse investment behavior and large welfare losses.

4.1.1 Case 1: ALM with a VaR constraint

First we investigate optimal portfolio decisions and corresponding certainty equivalents in a one-period context ($T = 1$) under a VaR constraint. We set the state variables at time zero equal to their long-run averages. We set the VaR probability δ=0.025 and we do not include contributions from the sponsor (i.e., $c_T = 0$). We compare a VaR constraint imposed on S_T (discounting at actual yields) with one imposed on $\bar{S}_T$ (discounting at a constant yield) and one on $\hat{S}_T$ (discounting at the four-year average yield).

Table 1 and 2 present the optimal portfolio weights and scaled certainty equivalents for constant and actual discounting for four different levels of risk aversion. Table 1 addresses the cases where risk aversion equals one and five, and in Table 2 we consider risk aversion levels of eight and 10. Note that the VaR constraint is more binding when the funding ratio at time zero, denoted by S_0, is lower. Therefore, as S_0 decreases, the manager has to substitute away from stocks to satisfy the constraint. The key insight of these results is that under actual discounting the manager substitutes into the long-term bond, whereas under constant discounting he moves into the riskfree asset. Because the utility from wealth depends on the actual funding ratio, which is computed using current yields, investing in the riskless asset leads to large utility losses. The riskless asset does not hedge against liability risk and has a low expected return. In other words, when the VaR is imposed with constant discounting, the manager is torn between the objective of maximizing utility from wealth and satisfying the VaR constraint. When the VaR constraint is based on actual yields these two objectives are more aligned.

The utility loss from constant discounting can be large and up to four percent of wealth. This loss is increasing in the degree of risk aversion. Substituting away from bonds into the riskless asset leaves a larger exposure to liability risk, leading to larger utility losses when the degree of risk aversion is higher. In other words, as risk aversion increases, the manager's preferred position in stocks is lower and he prefers to invest more in bonds to hedge against liability risk. As a consequence, the VaR constraint under actual discounting which requires a substantial weight in bonds does not affect the manager much. The VaR constraint under constant discounting, on the other hand, forces the manager into the riskfree asset leading to large welfare losses.

A VaR constraint based on the smoothed liability measure may lead to a higher expected utility only when the degree of risk aversion is very low. This fact is easiest to understand in the following way. Suppose that the investment manager is risk neutral, meaning that he only cares about the average return on the portfolio. In steady state, the liabilities mainly add uncertainty to the problem which, in this case, the manager does not care about.

Therefore, (ignoring the Jensen term) he maximizes the expected value of the assets only and can assume that the liabilities are constant. As a result, there is no longer a mismatch between the objective of maximizing utility from wealth and satisfying the VaR constraint. In fact, the VaR based on the smoothed liability measure may allow a higher weight in stocks, implying a higher expected return, and a higher certainty equivalent.

The results above indicate that smoothing yields can lead to grossly suboptimal investment decisions. However, under current regulations, liabilities are discounted at the rolling four-year average yield, not at the unconditional average yield. As a consequence, the impact of smoothing is smaller than suggested above. However we show that it is still very large. When moving from period t to $t+1$, the yield at time $t-3$ is dropped from the four-year average and the yield at time $t+1$ is added. This implies that three values in the average stay the same and are known at time t. Only the yield at time $t+1$ causes uncertainty. This has a relatively small impact on the average. In fact, as shown before, the conditional variance of the four-year average is a factor 16 smaller than the variance of the original series. Therefore, the investment manager still employs the riskfree asset to satisfy the VaR constraint. To illustrate this effect in a one-period example, assume that in and before period zero the yields are steady state. In period one, reported liabilities are computed by discounting at $0.75\bar{y}_{15} + 0.25y_{15,t}$. Table 1 and 2 also present the optimal portfolio weights and scaled certainty equivalents for four-year average discounting, for risk aversion levels of one and five (Table 1) and eight and 10 (Table 2). As expected, the resulting portfolio weights are an average of those chosen under constant and actual discounting. The investment manager still wants to invest part of the funds in the riskfree asset. The welfare loss is still large and up to two percent of wealth.

We conclude that imposing a VaR constraint on the smoothed funding ratio can lead to large welfare losses as it induces the use of the riskfree asset. Without the VaR constraint the investment manager would not use the riskfree asset because long-term bonds are a better hedge against long-run liability risk.

4.1.2 Case 2: ALM with AFCs

We now assess the impact of smoothing yields by comparing optimal portfolio decisions and corresponding certainty equivalents when the investment manager has to request AFCs whenever the fund is underfunded. The notion of being underfunded depends strongly on the liability measure used. We show that AFCs based on the smoothed liability measure lead to a similar misalignment of objectives as under the VaR constraint. We set the contributions equal to the realized funding ratio shortfall, which implies $c_t = \max(1-\bar{S}^*_t, 0)$ under constant discounting and $c_t = \max(1 - S_t^*, 0)$ under actual discounting. As before, we consider a one-period setup. We set $\lambda > 1$ to ensure concavity of the utility function and, for ease of exposition, we do not impose the VaR constraint. Finally, we set the state variables equal to their long-run averages at time zero.

Table 3 presents the optimal portfolio weights and certainty equivalents for different values of λ and varying degrees of risk aversion as a function of the funding ratio S_0. The results show that when sponsor contributions and their consequent reputation loss are determined through constant discounting, the investment manager does not substitute into bonds but hedges against the utility penalties through the riskfree asset. This goes against the investment manager's desire to maximize wealth utility, leading to large welfare losses.

There are, however, two main differences compared to the case of a VaR constraint. First, the manager now invests fully in stocks when the fund is highly underfunded, leading to a V-shaped policy function as in Berkelaar and Kouwenberg (2003). This is simply a consequence of our utility specification, which exhibits risk neutrality in the lower tail. Secondly, under constant discounting, as the funding ratio at time zero approaches the critical threshold of one, the first response of the investment manager is to increase his position in stocks. The reason is as follows. Under the VaR constraint, the portfolio weights that do not satisfy the constraint are excluded from the manager's choice set. However, when we consider contributions from the sponsor and reputation loss, no weights are excluded from the choice set and the manager is allowed to take a risky position in stocks. If he does, he just has to bear the consequences if the fund gets underfunded. Investing in the riskless asset leads to a very unfavorable portfolio in terms of wealth utility because it does not hedge the liability risk and has a low expected return. Investing in long-term bonds does not hedge against AFCs because, under constant discounting, long-term bonds are a volatile investment. Therefore, as it turns out, the manager's best response is to increase the weight in stocks. Again we conclude that constant discounting leads to highly perverse incentives. Whereas the AFCs should induce the manager to take less risky positions when the funding ratio approaches the critical threshold of one, under constant discounting they, in fact, induce the manager to take riskier positions. We can therefore conclude once again that smoothing yields can lead to perverse investment behavior accompanied by large welfare losses.

4.1.3 Smoothing yields in a multi-period framework

In the previous section we have assessed the welfare and portfolio choice impact of smoothing yields in a parsimonious one-period framework. We now extend our analysis to a multi-period framework and show that the impact of smoothing yields in a multi-period context

($T = 10$) is very similar to the one in the one-period context. Constant and four-year average discounting can lead to large utility losses in terms of certainty equivalents. There are, however, a few important differences, specifically with respect to the VaR constraint. The case of sponsor contributions (AFCs) is very similar to the one-period case.

Because the VaR constraint only binds for levels of the funding ratio close to (and less than) one and the funding ratio has a positive drift, the VaR becomes on average less binding over time. Therefore, for fully funded pension plans, the impact of the VaR and the difference between constant and actual discounting decreases over time as the plan's funding ratio increases. However, highly underfunded plans can be confronted with the VaR constraint over a very long time span. Therefore, the welfare loss of constant discounting for such underfunded plans, is very large and in the same order of magnitude as in the one-period model, i.e., between two to four percent per year.

We would expect that, as in the one-period model, the impact of four-year average discounting takes an average of the impacts of constant and actual discounting. However, another important disadvantage of smoothing yields now emerges: in around 10-20 percent of the cases, not a single portfolio weight in the choice space satisfies the VaR constraint. The reason is as follows. When we apply actual discounting, the investment manager can achieve an almost perfect hedge of liability risk through the long-term bond. Similarly, under constant discounting, a riskless alternative is available through the riskless asset. However, when we discount at the four-year average yield, it is not possible for the investment manager to construct a riskless asset from the traded securities.

To illustrate this mechanism, consider the following example of a plan with a funding ratio equal to one. Suppose that in the last four periods ($t-3$, $t-2$, $t-1$ and t) the 15-year yield took the path 0.060, 0.055, 0.045 and 0.040, leading to a four-year rolling average of 0.050. Assume further that the short term yield is currently very low at 0.020. The investment manager knows that next year the yield at time $t-3$ (i.e., 0.060) will be dropped from the average and the yield at $t+1$ will be added. The 15-year bond yield is expected to rise, due to mean reversion, but it is unlikely to rise back to 0.060 in one period. Therefore, the-four year average yield will most probably decrease, leading to a deterioration of the fund's position. Investing fully in the riskless asset is not allowed because the expected return is too low to compensate for the deterioration of the fund's position. As a result, investing fully in the riskless asset leads to an almost certain shortfall, whereas the maximum allowed probability of a shortfall under the VaR constraint is only 0.025. Investing in bonds is not too attractive either, as the long-term yield is expected to rise, leading to low returns on long-term bonds. The investment manager needs to hedge against a *drop* in the four-year rolling average yield, whereas the long-term yield is expected to increase. Therefore, the probability of a shortfall will be larger than 0.025 for all available portfolio weighs. Surprisingly, it turns out that in cases like this, the portfolio composition that leads to the lowest probability of a shortfall, is investing 100 percent in stocks, which seems counterintuitive and, more importantly, is highly undesirable. We therefore conclude once again that smoothing yields can lead to highly perverse investment behavior leading to large welfare losses.[14]

4.2 Regulatory constraints and hedging demands

In this section we address the impact of regulatory constraints on hedging demands. Hedging demands are the differences between the dynamic (also called strategic) and myopic portfolio weights. They hedge against future changes in the investment opportunity set. It is well-known that the value function of standard CRRA utility function is relatively flat at the maximum, implying that moderate deviations from the optimal portfolio policy only lead to small utility losses (e.g., Cochrane (1989) and Brandt (2005)). As a consequence, the economic gains to dynamic (strategic) as opposed to myopic (tactical) investment are usually small even when the hedging demands are large in magnitude. Intuitively, ex ante (preventive) risk constraints could enhance these gains as it could be profitable to strategically avoid the constraints in future periods. Specifically when the investment opportunity set is time-varying, the investment manager might want to avoid being constrained in the future when expected returns are high. We show that the exact opposite result holds: ex ante risk constraints further decrease the gains to dynamic investment. However, we also show that under ex post (punitive) constraints such as the requirement to draw AFCs when the plan becomes underfunded, strategically (dynamically) avoiding these contributions can lead to economically large utility gains.

4.2.1 Case 1: Portfolio optimization without liabilities

As a benchmark, we first solve a standard dynamic portfolio optimization problem without liabilities, AFCs, utility penalties, or a VaR constraint. We compare the certainty equivalent achieved under the solution of the dynamic 10-year investment problem with that of a myopic setup. The myopic problem involves solving 10 sequential one-year optimizations. Hence the only difference between the dynamic and the myopic problems is the utility function that the manager maximizes. In the myopic problem, the manager optimizes the one period utility

[14]When long-term yields have been rising consistently, and the short term yield is high, the opposite argument holds, and the investment environment is very favorable to the manager. In this case he is less restricted by the VaR constraint under four-year average discounting than under actual discounting.

function 10 times and in the dynamic problem he optimizes the 10-period utility function. We then use the optimal weights for both problems to compute certainty equivalents with respect to the 10-period utility function. In other words, we use the myopic and dynamic policy functions to compute the certainty equivalent when the investment manager has a 10-year utility function. In this case the myopic policy function is suboptimal. The important question is how suboptimal it is. We define the gains to dynamic investment as the ratio of the dynamic and myopic certainty equivalents.

Table 4 shows for different values of γ the optimal portfolio weights and certainty equivalents for the dynamic and the myopic problem. In the myopic case, the manager spreads his funds between stocks and the riskless asset and hardly invests in long-term bonds. In the dynamic case, however, it is optimal to invest part of the funds in long-term bonds as a hedge against changes in the investment opportunity set for stocks. When there is a drop in the 15-year yield, the return on bonds in the current period are high, which forms a hedge against the lower future risk premium on stocks. Even though the hedging demands can be large, the utility gains, as expressed by the net ratio of the dynamic and myopic certainty equivalents, are relatively small. They vary between zero and 10 basis points per year depending on the degree of risk aversion. These low gains to dynamic investment are caused by the relatively flat peak of the 10-period value function.

It is well known that for $\gamma = 1$ the gains to dynamic investment are zero, as the log utility investor is myopic. However, in this case these gains are also zero for values of γ close to one. This is because both the dynamic and the myopic investor will want to invest all their funds in stocks and both hit the short sale constraints. If we allowed for short positions in long-term bonds and the riskfree asset, there would be a gain to dynamic investment, as the dynamic investor's short position in bonds would be smaller than the myopic investor's. This suggests that a constraint on the choice set (the space of allowed portfolio weights) can significantly reduce the gains to dynamic investment. A similar argument holds for other ex ante (preventive) risk constraint such as the VaR constraints, which we consider later.

4.2.2 Case 2: ALM problem

As a second benchmark, we present the optimal portfolio weights for an ALM problem without sponsor contributions, utility penalties, or VaR constraint. Table 5 shows the optimal portfolio weights and certainty equivalents for the dynamic and the myopic investor for different values of γ. Given the short position in the liabilities, it is no longer optimal to invest in the riskless asset, so the manager spreads his wealth between stocks and bonds. The dynamic investor now substitutes away from long-term bonds into stocks. When investing myopically, the uncertainty caused by the liabilities induces the manager to invest more in long-term bonds, which are a good hedge against liability risk. However, when investing dynamically, liability risk is not as important due to the mean-reverting nature of yields. In other words, future bond returns are negatively correlated with current bond returns. Suppose that the 15-year yield is at its long-run average and is hit by a negative shock. Current liabilities will increase, which leads to a deterioration of the funds financial position. However, yields are consequently expected to increase which will ameliorate the fund's position. In other words, in our setup bonds are a good hedge against changes in the investment opportunities for bonds. Apparently, this effect even dominates the use of bonds as a hedge against changes in the investment opportunities for stocks. The gains to dynamic investment are again small, not exceeding 10 basis points per year.

4.2.3 Case 3: ALM problem with VaR constraint

We now investigate the impact of ex ante (preventive) risk constraints on hedging demands. In particular, we focus on a VaR constraint. We set the VaR probability δ equal to 0.025 and maintain the assumption of no sponsor contributions, so $c_t = 0 \ \forall t$. As noted before, intuitively we would expect the VaR constraint to increase the value of solving the dynamic program. Strategically avoiding the VaR constraint should lead to utility increases.

Our results indicate exactly the opposite: the VaR constraint further reduces the already small gains to dynamic investing. Table 6 and 7 show the optimal portfolio weights and standardized certainty equivalents for $\gamma = 5$ as a function of the funding ratio at time zero for both the dynamic and myopic investor. Similar results hold for other levels of risk aversion. In Table 6, the VaR is imposed with respect to S_t (discounting at current yields) and in Table 7 it is imposed with respect to $\bar{S}_t$ (discounting at a constant yield). Recall that the VaR constraint applies period-by-period for both the dynamic and the myopic investor. For ease of exposition, let us consider low and intermediate levels of the funding ratio at time zero. In this case the VaR constraint binds and it reduces the weight in stocks. That is, the VaR constraint leads the investor away from the preferred portfolio weight in stocks, leading to a utility loss. The upper bound on the weight in stocks as required by the VaR depends on the funding ratio at time zero and is the same for the dynamic and the myopic investor. From the unconstrained ALM problem above we know that to hedge against changes in the investment opportunity set, the dynamic investor wants to invest more in stocks than the myopic one. However, the VaR constraint prevents him from doing so, thereby eliminating the gains to dynamic investment.

Even though the dynamic investor can not invest more in stocks than the myopic one, he could choose to invest less in stocks in the current period, thereby strategically lowering the probability of being constrained by the VaR in the future. However, the current VaR already decreases his weight in stocks, which already lowers the probability of being constrained by the VaR in the future. The current period's portfolio loss of decreasing the weight in stocks even further outweighs the potential future gains. As a result, both investors make the same portfolio choices, leading to almost equal certainty equivalents. More generally speaking, we can conclude that the VaR constraint in the current period is a strong remedy in trying to avoid the VaR constraint in the future. Furthermore, it is interesting to note that the expected returns on stocks are high when the 15-year yield is high. This means that the liabilities are low when stock returns are high. This ameliorates the negative impact of the VaR in future periods and allows the manager to invest in stocks when it is most profitable for him to do so. However, this last argument only holds when the VaR is imposed on the actual funding ratio S_t and may not hold when liabilities are smoothed. This is yet another unattractive feature of yield smoothing.

We conclude that the dynamic investor faces a tradeoff between forming an optimal portfolio in the current period given the current VaR and lowering the probability and impact of hitting the VaR in the future. Our results suggest that the current loss from decreasing the weight in stocks by more than is prescribed by the current VaR outweighs the gain of a lower probability of hitting the VaR in the future. When the 15-year yield at time zero is no longer at its unconditional mean but below it, investing in stocks in the current period becomes less appealing compared to investing in stocks in the future. However it also implies that current liabilities are high and are expected to decrease. This downward trend in liabilities decreases the impact of the VaR constraint in the future under actual discounting but not under constant discounting.

4.2.4 Case 4: ALM problem with AFCs

Finally, we consider ex post (punitive) risk constraints. In particular we focus on the investment manager's requirement to request additional financial contributions (AFCs) from the plan sponsor whenever the plan becomes underfunded. In the previous section we showed that lowering the weight in stocks today to strategically avoid the VaR constraint in the future does not pay off. Now, however, lowering the weight in stocks to lower the probability of being underfunded in the future can lead to very large utility gains. Including sponsor contributions in the utility function leads to a kinked utility function. The induced first order risk aversion enhances the gains to dynamic investment. Furthermore, contributions have a direct utility impact and apply each period as opposed to utility from wealth, which only depends on the funding ratio in time T. We set the subjective discount factor $\beta = 0.90$, and set $\delta = 1$ (no VaR constraint).[15] Recall that λ is the parameter that describes the tradeoff between the sponsor contributions and wealth utility. When λ is set sufficiently high, the gains of lowering the probability of being underfunded in the future will outweigh the portfolio loss of lowering the weight in stocks today. In this case the gains to dynamic investment are very large. Table 8 shows the portfolio weights and certainty equivalents for $\lambda = 2$ when in each period the contributions are set equal to the realized funding ratio shortfall under actual discounting ($c_t = \max(1 - S_t^*, 0)$). The certainty equivalents are in terms of wealth, which assumes that the utility penalties for additional contributions can be converted into monetary amounts according to the tradeoff in the utility function. Both tables show that the gains from dynamic investment are very large. By lowering the weight in stocks today, the investment manager can avoid costly contributions from the sponsor in the future, thereby realizing large utility gains.

5 Conclusion

We addressed in this paper the investment problem of the investment manager of a defined benefits pension plan. We showed that financial reporting and risk control rules have real effects on investment behavior. The current requirement to discount liabilities at a four year rolling average yield can induce grossly suboptimal investment decisions, both myopically and dynamically. More importantly, we compared the influence of preventive and punitive constraints on the gains to dynamic decision making. We concluded that ex ante (preventive) constraints such as VaR constraints, short sale constraints and an upper bound on the share of stocks in the portfolio, decrease the size of the choice set (i.e. the space of admissible portfolio weights) and thereby substantially decrease the gains to dynamic investment. However, ex post (punitive) constraints, such as mandatory additional financial contributions (AFCs) from the plan sponsor, make the investment manager first-order risk averse at the critical threshold that triggers the constraint, leading to large utility gains to dynamic investment. In other words, if the investment manager is concerned about being underfunded and dislikes the resulting AFCs, a dynamic investment strategy leads to large expected utility gains by strategically avoiding to be underfunded in the future.

[15] Setting the subjective discount factor β to 0.95 or 0.99 does not influence our results.

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A Return model parameter estimates

We estimate the Vector Auto Regression (VAR) of order one that describes the return dynamics by OLS, equation by equation. The estimates are given below with their respective standard errors between brackets.

$$A = \begin{bmatrix} 0.1077 & (0.1703) \\ -0.5308 & (0.4409) \\ -0.3789 & (0.1658) \end{bmatrix} \tag{A.1}$$

$$B = \begin{bmatrix} -0.1346 & (0.0771) & 0.1459 & (0.1136) \\ 0.5647 & (0.1996) & 0.2885 & (0.2943) \\ 0.0162 & (0.0750) & 0.8491 & (0.1106) \end{bmatrix} \tag{A.2}$$

$$\Sigma = \begin{bmatrix} 0.0176 & 0.0048 & -0.0038 \\ 0.0048 & 0.1178 & 0.0356 \\ -0.0038 & 0.0356 & 0.0167 \end{bmatrix} \tag{A.3}$$

We can then determine the unconditional expectation of the log 1 year and 15 year yield by:

$$\begin{bmatrix} E\left(\log\left(y_{1,t}\right)\right) \\ E\left(\log\left(y_{15,t}\right)\right) \end{bmatrix} = \begin{bmatrix} 1-0.5647 & -0.2885 \\ -0.0162 & 1-0.8491 \end{bmatrix}^{-1} \begin{bmatrix} -0.5308 \\ -0.3789 \end{bmatrix} = \begin{bmatrix} -3.1040 \\ -2.8438 \end{bmatrix} \tag{A.4}$$

which corresponds to steady state yield levels of:

$$\begin{bmatrix} \bar{y}_1 \\ \bar{y}_{15} \end{bmatrix} = \begin{bmatrix} 0.045 \\ 0.058 \end{bmatrix} \tag{A.5}$$

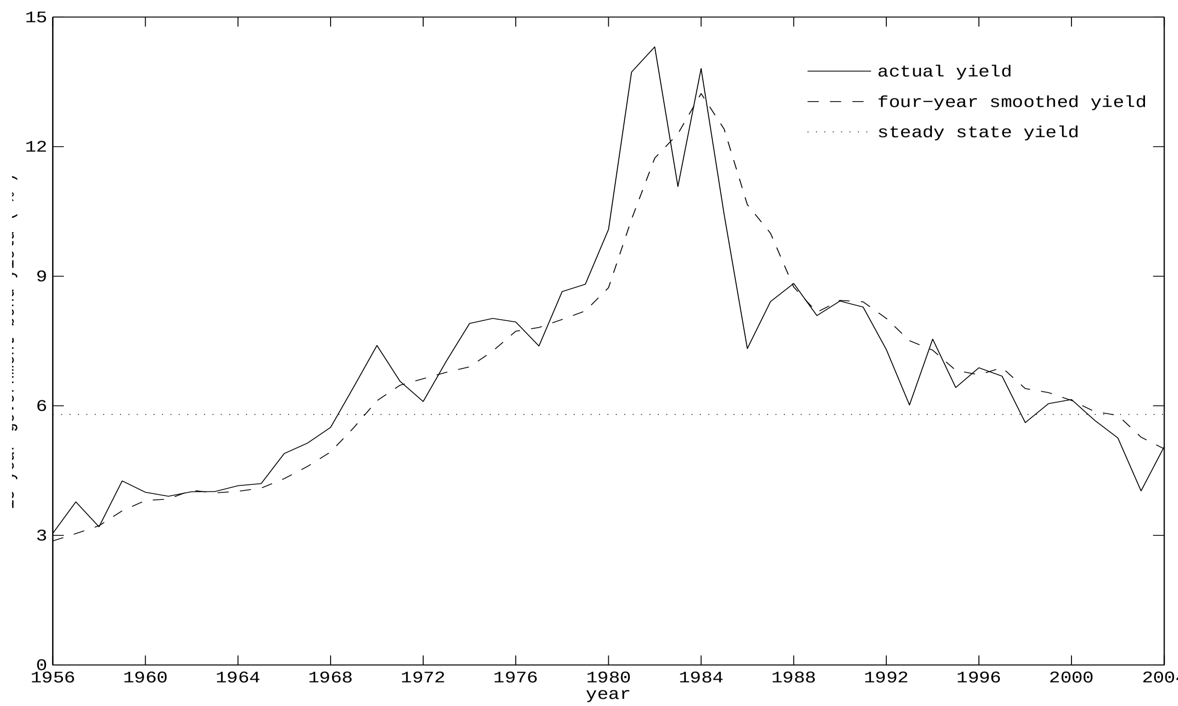

Figure 1: The impact of smoothing the 15-year government bond yield. We plot the 15-year constant maturity government bond yield (actual yield), the four-year smoothed 15-year constant maturity government bond yield (four-year smoothed yield) and the steady state value that follows from our VAR(1) specification (steady state yield) between 1956 and 2004.

Table 1: Portfolio weights and scaled certainty equivalents for a one-period CRRA ALM problem under a Value-at-Risk constraint (δ=0.025). The Value-at-Risk constraint is determined under constant, actual and four-year average discounting respectively. We solve the problem for relative degrees of risk aversion equal to one and five. The certainty equivalents are scaled by the funding ratio at time zero, denoted by S_0.

$\gamma = 1$												
	constant discounting				actual discounting				4y discounting			
S_0	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled
$\leq$ 0.90	0.24	0.74	0.02	1.0734	0.24	0.00	0.76	1.0787	0.28	0.48	0.24	1.0787
0.99	0.24	0.74	0.02	1.0734	0.24	0.00	0.76	1.0787	0.28	0.48	0.24	1.0787
1.00	0.24	0.74	0.02	1.0734	0.24	0.00	0.76	1.0787	0.28	0.48	0.24	1.0787
1.01	0.30	0.70	0.00	1.0778	0.28	0.00	0.72	1.0815	0.32	0.40	0.28	1.0821
1.02	0.36	0.64	0.00	1.0824	0.34	0.00	0.66	1.0858	0.38	0.36	0.26	1.0864
1.05	0.50	0.46	0.04	1.0931	0.46	0.00	0.54	1.0934	0.54	0.22	0.24	1.0978
1.08	0.66	0.34	0.00	1.1040	0.58	0.00	0.42	1.1018	0.68	0.08	0.24	1.1075
1.10	0.74	0.22	0.04	1.1099	0.66	0.00	0.34	1.1069	0.78	0.00	0.22	1.1141
1.12	0.84	0.16	0.00	1.1162	0.74	0.00	0.26	1.1117	0.86	0.00	0.14	1.1188
1.15	0.96	0.02	0.02	1.1242	0.86	0.00	0.14	1.1188	0.98	0.00	0.02	1.1255
$\geq$ 1.20	1.00	0.00	0.00	1.1266	1.00	0.00	0.00	1.1266	1.00	0.00	0.00	1.1266

$\gamma = 5$												
	constant discounting				actual discounting				4y discounting			
S_0	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled
$\leq$ 0.90	0.20	0.70	0.10	1.0507	0.24	0.00	0.76	1.0748	0.22	0.40	0.38	1.0641
0.99	0.20	0.70	0.10	1.0507	0.24	0.00	0.76	1.0748	0.22	0.40	0.38	1.0641
1.00	0.20	0.70	0.10	1.0507	0.24	0.00	0.76	1.0748	0.22	0.40	0.38	1.0641
1.01	0.26	0.64	0.10	1.0550	0.28	0.00	0.72	1.0765	0.26	0.34	0.40	1.0674
1.02	0.28	0.56	0.16	1.0591	0.34	0.00	0.66	1.0787	0.34	0.30	0.36	1.0708
1.05	0.42	0.38	0.02	1.0689	0.46	0.00	0.54	1.0818	0.44	0.10	0.46	1.0789
1.08	0.50	0.20	0.30	1.0765	0.58	0.00	0.42	1.0833	0.62	0.00	0.38	1.0834
1.10	0.58	0.08	0.34	1.0809	0.62	0.00	0.38	1.0834	0.62	0.00	0.38	1.0834
1.12	0.62	0.00	0.38	1.0834	0.62	0.00	0.38	1.0834	0.62	0.00	0.38	1.0834
1.15	0.62	0.00	0.38	1.0834	0.62	0.00	0.38	1.0834	0.62	0.00	0.38	1.0834
$\geq$ 1.20	0.62	0.00	0.38	1.0834	0.62	0.00	0.38	1.0834	0.62	0.00	0.38	1.0834

Table 2: Portfolio weights and scaled certainty equivalents for a one-period CRRA ALM problem under a Value-at-Risk constraint (δ=0.025). The Value-at-Risk constraint is determined under constant, actual and four-year average discounting respectively. We solve the problem for relative degrees of risk aversion equal to eight and 10. The certainty equivalents are scaled by the funding ratio at time zero, denoted by S_0.

$\gamma = 8$												
	constant discounting				actual discounting				4y discounting			
S_0	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled
$\leq$ 0.90	0.16	0.70	0.14	1.0370	0.24	0.00	0.76	1.0721	0.22	0.40	0.38	1.0564
0.99	0.16	0.70	0.14	1.0370	0.24	0.00	0.76	1.0721	0.22	0.40	0.38	1.0564
1.00	0.16	0.70	0.14	1.0370	0.24	0.00	0.76	1.0721	0.22	0.40	0.38	1.0564
1.01	0.18	0.64	0.18	1.0413	0.28	0.00	0.72	1.0729	0.26	0.34	0.40	1.0597
1.02	0.22	0.56	0.22	1.0467	0.34	0.00	0.66	1.0730	0.30	0.28	0.42	1.0627
1.05	0.28	0.38	0.34	1.0575	0.38	0.00	0.62	1.0739	0.36	0.08	0.56	1.0712
1.08	0.36	0.20	0.44	1.0662	0.38	0.00	0.62	1.0739	0.38	0.00	0.62	1.0739
1.10	0.44	0.08	0.48	1.0705	0.38	0.00	0.62	1.0739	0.38	0.00	0.62	1.0739
1.12	0.38	0.00	0.62	1.0739	0.38	0.00	0.62	1.0739	0.38	0.00	0.62	1.0739
1.15	0.38	0.00	0.62	1.0739	0.38	0.00	0.62	1.0739	0.38	0.00	0.62	1.0739
$\geq$ 1.20	0.38	0.00	0.62	1.0739	0.38	0.00	0.62	1.0739	0.38	0.00	0.62	1.0739

$\gamma = 10$												
	constant discounting				actual discounting				4y discounting			
S_0	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled
$\leq$ 0.90	0.12	0.70	0.18	1.0295	0.24	0.00	0.76	1.0703	0.18	0.40	0.42	1.0517
0.99	0.12	0.70	0.18	1.0295	0.24	0.00	0.76	1.0703	0.18	0.40	0.42	1.0517
1.00	0.12	0.70	0.18	1.0295	0.24	0.00	0.76	1.0703	0.18	0.40	0.42	1.0517
1.01	0.14	0.64	0.22	1.0345	0.28	0.00	0.72	1.0707	0.20	0.34	0.46	1.0553
1.02	0.22	0.56	0.22	1.0398	0.30	0.00	0.70	1.0707	0.22	0.28	0.50	1.0587
1.05	0.28	0.38	0.34	1.0519	0.30	0.00	0.70	1.0707	0.28	0.10	0.62	1.0671
1.08	0.30	0.22	0.48	1.0612	0.30	0.00	0.70	1.0707	0.30	0.00	0.70	1.0707
1.10	0.32	0.12	0.56	1.0659	0.30	0.00	0.70	1.0707	0.30	0.00	0.70	1.0707
1.12	0.38	0.00	0.62	1.0700	0.30	0.00	0.70	1.0707	0.30	0.00	0.70	1.0707
1.15	0.30	0.00	0.70	1.0707	0.30	0.00	0.70	1.0707	0.30	0.00	0.70	1.0707
$\geq$ 1.20	0.30	0.00	0.70	1.0707	0.30	0.00	0.70	1.0707	0.30	0.00	0.70	1.0707

Table 3: Portfolio weights and standardized certainty equivalents for a one-period CRRA ($\gamma = 5$) ALM problem with sponsor contributions and reputation loss for λ equal to two and five. The sponsor contributions are set equal to the realized shortfall under respectively constant and actual discounting. The certainty equivalents are scaled by the funding ratio at time zero, denoted by S_0.

$\lambda = 2$								
	constant discounting				actual discounting			
S_0	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled
0.90	1.00	0.00	0.00	1.0526	0.78	0.00	0.22	1.0606
0.92	0.97	0.00	0.03	1.0533	0.61	0.00	0.39	1.0628
0.95	0.81	0.00	0.19	1.0568	0.42	0.00	0.58	1.0691
0.97	0.75	0.00	0.25	1.0603	0.38	0.00	0.62	1.0734
0.98	0.72	0.00	0.28	1.0622	0.40	0.00	0.60	1.0752
0.99	0.70	0.00	0.30	1.0642	0.41	0.00	0.59	1.0766
1.00	0.67	0.01	0.32	1.0661	0.43	0.00	0.57	1.0778
1.01	0.66	0.02	0.33	1.0679	0.44	0.00	0.56	1.0788
1.02	0.65	0.00	0.35	1.0697	0.46	0.00	0.54	1.0796
1.05	0.62	0.00	0.38	1.0742	0.51	0.00	0.49	1.0814
1.08	0.61	0.00	0.39	1.0776	0.55	0.00	0.45	1.0823
1.10	0.61	0.00	0.39	1.0793	0.57	0.00	0.43	1.0827
1.12	0.61	0.00	0.39	1.0806	0.59	0.00	0.41	1.0830
1.15	0.61	0.00	0.39	1.0818	0.60	0.00	0.40	1.0832
1.20	0.62	0.00	0.38	1.0827	0.62	0.00	0.38	1.0834
≥ 1.50	0.62	0.00	0.38	1.0834	0.62	0.00	0.38	1.0834

$\lambda = 5$								
	constant discounting				actual discounting			
S_0	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled
0.90	0.69	0.28	0.03	0.9626	0.37	0.00	0.63	0.9788
0.92	0.48	0.49	0.03	0.9798	0.21	0.00	0.79	1.0058
0.95	0.13	0.86	0.01	1.0189	0.03	0.00	0.97	1.0586
0.97	0.16	0.78	0.06	1.0389	0.14	0.00	0.86	1.0671
0.98	0.21	0.70	0.09	1.0441	0.18	0.00	0.82	1.0697
0.99	0.26	0.62	0.12	1.0484	0.23	0.00	0.77	1.0719
1.00	0.29	0.56	0.15	1.0521	0.26	0.00	0.74	1.0736
1.01	0.33	0.50	0.17	1.0533	0.30	0.00	0.70	1.0751
1.02	0.36	0.45	0.19	1.0582	0.33	0.00	0.67	1.0764
1.05	0.43	0.32	0.25	1.0652	0.41	0.00	0.59	1.0792
1.08	0.49	0.21	0.30	1.0706	0.47	0.00	0.53	1.0810
1.10	0.51	0.15	0.34	1.0735	0.51	0.00	0.49	1.0818
1.12	0.55	0.07	0.38	1.0760	0.53	0.00	0.47	1.0823
1.15	0.58	0.00	0.42	1.0790	0.57	0.00	0.43	1.0828
1.20	0.61	0.00	0.39	1.0816	0.60	0.00	0.40	1.0833
≥ 1.50	0.62	0.00	0.38	1.0834	0.62	0.00	0.38	1.0834

Table 4: Portfolio weights and standardized certainty equivalents for a 10-year CRRA portfolio optimization problem without liabilities, VaR constraint or sponsor contributions. Due to the power utility specification over wealth, the weights are independent of the wealth level at time zero. The portfolio weights are rebalanced annually. We run 50 simulations and report averages and standard deviations (between brackets). In the last column (gain in basis points per year) we take the ratio of the certainty equivalents, subtract one and multiply by 10,000 to get the gains to dynamic (strategic) investment compared to myopic (tactical) investment in basis points per year.

	Myopic				Dynamic				
γ	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled	Gains (bp/a)
1	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	3.266 (0.012)	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	3.266 (0.012)	0.0 (0.0)
5	0.85 (0.021)	0.15 (0.021)	0.00 (0.011)	2.549 (0.010)	0.94 (0.047)	0.01 (0.025)	0.05 (0.039)	2.560 (0.011)	4.4 (1.1)
8	0.53 (0.012)	0.47 (0.012)	0.00 (0.006)	2.235 (0.012)	0.61 (0.070)	0.20 (0.087)	0.19 (0.068)	2.255 (0.014)	8.8 (3.2)

Table 5: Portfolio weights and standardized certainty equivalents for a 10-period CRRA ALM portfolio optimization without VaR constraint or sponsor contributions. Due to the power utility specification over the funding ratio, the weights are independent of the funding ratio at time zero. The portfolio weights are rebalanced annually. We run 50 simulations and report averages and standard deviations (between brackets). In the last column (gain in basis points per year) we take the ratio of the certainty equivalents, subtract one and multiply by 10,000 to get the gains to dynamic (strategic) investment compared to myopic (tactical) investment in basis points per year.

	Myopic				Dynamic				
γ	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled	Gains (bp/a)
1	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	3.365 (0.012)	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	3.365 (0.012)	0.0 (0.0)
5	0.61 (0.017)	0.00 (0.000)	0.39 (0.017)	2.651 (0.010)	0.85 (0.041)	0.00 (0.000)	0.15 (0.041)	2.676 (0.012)	9.5 (1.2)
8	0.37 (0.012)	0.00 (0.000)	0.63 (0.012)	2.355 (0.012)	0.57 (0.056)	0.00 (0.000)	0.43 (0.056)	2.377 (0.016)	9.3 (3.3)

Table 6: Dynamic and myopic portfolio weights and certainty equivalents for a 10-period CRRA ($\gamma = 5$) ALM problem (no sponsor contributions) with a VaR constraint imposed on the actual (actual discounting) funding ratio (δ=0.025). The certainty equivalents are scaled by the funding ratio at time zero (S_0). We run 50 simulations and report averages and standard deviations (between brackets). In the last column (gain in basis points per year) we take the ratio of the certainty equivalents, subtract one and multiply by 10,000 to get the gains to dynamic (strategic) investment compared to myopic (tactical) investment in basis points per year.

$\gamma = 5,\ \delta = 0.025$									
	Myopic				Dynamic				
S_0	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled	Gains (bp/a)
0.10	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.226 (0.006)	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.226 (0.006)	0.0 (0.0)
0.75	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.406 (0.007)	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.406 (0.007)	0.0 (0.2)
0.95	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.523 (0.009)	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.527 (0.009)	1.3 (0.4)
0.99	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.549 (0.009)	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.553 (0.009)	1.6 (0.4)
1.00	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.554 (0.009)	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.559 (0.009)	1.7 (0.4)
1.01	0.33 (0.000)	0.00 (0.000)	0.67 (0.000)	2.566 (0.009)	0.33 (0.000)	0.00 (0.000)	0.67 (0.000)	2.571 (0.010)	1.8 (0.4)
1.03	0.44 (0.013)	0.00 (0.000)	0.56 (0.013)	2.583 (0.010)	0.44 (0.013)	0.00 (0.000)	0.56 (0.013)	2.588 (0.010)	2.0 (0.5)
1.10	0.61 (0.019)	0.00 (0.000)	0.39 (0.019)	2.612 (0.011)	0.62 (0.030)	0.00 (0.000)	0.38 (0.030)	2.618 (0.011)	2.5 (0.5)
1.20	0.61 (0.019)	0.00 (0.000)	0.39 (0.019)	2.628 (0.011)	0.71 (0.031)	0.00 (0.000)	0.29 (0.031)	2.639 (0.012)	3.9 (0.7)
2.00	0.61 (0.019)	0.00 (0.000)	0.39 (0.019)	2.648 (0.010)	0.84 (0.040)	0.00 (0.000)	0.16 (0.040)	2.670 (0.012)	8.4 (1.1)
3.00	0.61 (0.017)	0.00 (0.000)	0.39 (0.017)	2.651 (0.010)	0.85 (0.041)	0.00 (0.000)	0.15 (0.041)	2.676 (0.012)	9.5 (1.2)

Table 7: Dynamic and myopic portfolio weights and certainty equivalents for a 10-period CRRA ALM problem (no sponsor contributions) with a Value-at-Risk imposed on the smoothed (constant discounting) funding ratio (δ=0.025) and risk aversion $\gamma = 5$. The certainty equivalents are scaled by the funding ratio at time zero (S_0). The portfolio weights are rebalanced annually. We run 50 simulations and report averages and standard deviations (between brackets). In the last column (gain in basis points per year) we take the ratio of the certainty equivalents, subtract one and multiply by 10,000 to get the gains to dynamic (strategic) investment compared to myopic (tactical) investment in basis points per year.

$\gamma = 5,\ \delta = 0.025$									
	Dynamic				Myopic				
S_0	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled	Gains (bp/a)
0.10	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	1.867 (0.007)	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	1.867 (0.007)	0.0 (0.1)
0.75	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	2.255 (0.010)	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	2.255 (0.009)	0.0 (0.4)
0.95	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	2.468 (0.011)	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	2.468 (0.011)	0.0 (0.7)
0.99	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	2.504 (0.010)	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	2.505 (0.011)	0.6 (0.7)
1.00	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	2.511 (0.010)	0.24 (0.016)	0.60 (0.000)	0.16 (0.016)	2.513 (0.011)	0.8 (0.7)
1.01	0.29 (0.019)	0.50 (0.010)	0.21 (0.023)	2.527 (0.010)	0.31 (0.017)	0.51 (0.016)	0.18 (0.030)	2.530 (0.010)	1.1 (0.7)
1.03	0.41 (0.027)	0.33 (0.013)	0.27 (0.036)	2.552 (0.010)	0.43 (0.019)	0.34 (0.016)	0.23 (0.031)	2.556 (0.011)	1.5 (0.8)
1.10	0.61 (0.019)	0.00 (0.000)	0.39 (0.019)	2.590 (0.011)	0.69 (0.031)	0.00 (0.008)	0.31 (0.031)	2.597 (0.012)	2.7 (0.8)
1.20	0.61 (0.019)	0.00 (0.000)	0.39 (0.019)	2.607 (0.011)	0.77 (0.033)	0.00 (0.000)	0.23 (0.033)	2.620 (0.012)	4.9 (0.9)
2.00	0.61 (0.019)	0.00 (0.000)	0.39 (0.019)	2.643 (0.011)	0.84 (0.040)	0.00 (0.000)	0.16 (0.040)	2.665 (0.012)	8.5 (1.1)
3.00	0.61 (0.017)	0.00 (0.000)	0.39 (0.017)	2.651 (0.010)	0.85 (0.041)	0.00 (0.000)	0.15 (0.041)	2.676 (0.012)	9.5 (1.2)

Table 8: Dynamic and myopic portfolio weights and standardized (scaled) certainty equivalents for a 10-period ALM problem with sponsor contributions and reputation loss determined under actual discounting (no VaR constraint) for risk aversion $\gamma = 5$ and $\lambda = 2$. The portfolio weights are rebalanced annually. The certainty equivalents are scaled by the funding ratio at time zero (S_0). The portfolio weights are rebalanced annually. We run 50 simulations and report averages and standard deviations (between brackets). In the last column (gain in basis points per year) we take the ratio of the certainty equivalents, subtract one and multiply by 10,000 to get the gains to dynamic (strategic) investment compared to myopic (tactical) investment in basis points per year.

$\gamma = 5,\ \lambda = 2$									
	Dynamic				Myopic				
S_0	Stocks	Riskfree	Bonds	CE scaled	Stocks	Riskfree	Bonds	CE scaled	Gains (bp/a)
0.40	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	1.361 (0.000)	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	1.362 (0.000)	0.9 (0.0)
0.75	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	0.993 (0.001)	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	0.996 (0.001)	2.9 (0.1)
0.80	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	1.012 (0.002)	1.00 (0.000)	0.00 (0.000)	0.00 (0.000)	1.016 (0.002)	3.9 (0.1)
0.85	0.84 (0.011)	0.00 (0.000)	0.16 (0.011)	1.054 (0.002)	0.70 (0.000)	0.00 (0.000)	0.30 (0.000)	1.062 (0.002)	7.6 (0.2)
0.90	0.40 (0.000)	0.00 (0.000)	0.60 (0.000)	1.182 (0.002)	0.30 (0.000)	0.00 (0.000)	0.70 (0.000)	1.200 (0.002)	15.2 (0.2)
0.95	0.03 (0.000)	0.00 (0.000)	0.97 (0.000)	1.899 (0.004)	0.00 (0.000)	0.00 (0.000)	1.00 (0.000)	2.170 (0.005)	134.2 (1.9)
0.99	0.19 (0.014)	0.00 (0.000)	0.81 (0.014)	2.013 (0.017)	0.07 (0.011)	0.00 (0.000)	0.93 (0.011)	2.355 (0.006)	158.3 (9.1)
1.00	0.21 (0.017)	0.00 (0.000)	0.79 (0.017)	2.042 (0.022)	0.10 (0.000)	0.00 (0.000)	0.90 (0.000)	2.368 (0.006)	149.1 (11.6)
1.01	0.24 (0.011)	0.00 (0.000)	0.76 (0.011)	2.063 (0.013)	0.13 (0.000)	0.00 (0.000)	0.87 (0.000)	2.379 (0.006)	143.7 (5.5)
1.02	0.27 (0.000)	0.00 (0.000)	0.73 (0.000)	2.071 (0.003)	0.14 (0.011)	0.00 (0.000)	0.86 (0.011)	2.388 (0.006)	143.5 (2.3)
1.05	0.33 (0.000)	0.00 (0.000)	0.67 (0.000)	2.109 (0.004)	0.20 (0.000)	0.00 (0.000)	0.80 (0.000)	2.415 (0.007)	136.5 (2.3)
1.10	0.41 (0.017)	0.00 (0.000)	0.59 (0.017)	2.170 (0.014)	0.28 (0.017)	0.00 (0.000)	0.72 (0.017)	2.450 (0.007)	122.2 (7.2)
1.25	0.56 (0.016)	0.00 (0.000)	0.44 (0.016)	2.307 (0.007)	0.46 (0.016)	0.00 (0.000)	0.54 (0.016)	2.521 (0.009)	89.1 (3.0)
1.50	0.60 (0.011)	0.00 (0.000)	0.40 (0.011)	2.442 (0.009)	0.62 (0.018)	0.00 (0.000)	0.38 (0.018)	2.577 (0.010)	53.7 (1.4)
2.00	0.61 (0.014)	0.00 (0.000)	0.39 (0.014)	2.581 (0.011)	0.72 (0.022)	0.00 (0.000)	0.28 (0.022)	2.635 (0.011)	20.7 (0.9)
2.50	0.61 (0.014)	0.00 (0.000)	0.39 (0.014)	2.636 (0.011)	0.79 (0.027)	0.00 (0.000)	0.21 (0.027)	2.661 (0.012)	9.7 (0.8)
3.00	0.61 (0.014)	0.00 (0.000)	0.39 (0.014)	2.644 (0.012)	0.83 (0.026)	0.00 (0.000)	0.17 (0.026)	2.667 (0.013)	8.6 (0.8)