Optimal Asset Allocations in both ALM and LDI Approaches of Pension Funds

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Abstract

When we consider pension fund management, asset only approach is traditionally used for optimal asset allocation. Recently, optimal asset allocation based on surplus framework is noteworthy. So, we investigate the feature of optimal asset allocation under the consideration of both asset and liability in pension fund. There are two surplus approaches such as ALM and LDI. We use mainly ALM approach which is often called as a balance sheet type ALM. Because, we apply this idea for Japanese government pension fund and the plan yield is given as 3.2 percent by Japanese government. However, the mainstream of pension fund management in U.K., Holland, and Denmark, etc. is LDI. The concept of LDI is also accepted and executed by the U.S.A..

So, we apply LDI's idea into this analysis. Namely, we execute the simulations by changing the volatility of liability and notice the difference of efficient frontier in both surplus and asset only approaches. Thus, as we increase the surplus return, the asset allocation of higher risk assets will increase and also increase the surplus risk simultaneously.

Key Words : Pension Fund, Surplus Approach, ALM, LDI, Asset Allocation
JEL Classification : G11, G23

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1. Introduction

The concept of surplus is utilized in pension fund management to grasp the future movement of funding ratio: $F_t = A_t / L_t$. Surplus is presented by market value and equals pension asset minus pension liability. In addition, the pension liability is represented by ABO: Accumulated Benefit Obligation, or PBO: Projected Benefit Obligation. The reason for us to use funding ratio in surplus approach, not pension asset minus pension liability is that there is a possibility of negative surplus in pension asset minus pension liability. And this is not convenient for the balance sheet type ALM (=Asset Liability Management). Especially, pension ALM focuses on the surplus. The purpose of it is to minimize the risk of decrease in surplus and figure out the policy asset mix to maximize the return. Here, we check the asset allocation of continuous time series model in surplus framework. By explaining the asset pricing process, we show the optimal asset allocation in asset only approach in multiple assets. Next, we introduce the asset allocation that the movement of liability is based on the Brownian motion. Furthermore, we compare the difference of optimal asset allocation between asset only approach and surplus approach under the specified power utility function. In the past literature review, Sharpe and Tint (1990) discuss the asset allocation in consideration of not only asset, but also liability and propose the idea of ALM in the corporate pension funds. Tanaka and Kitamura (2004) execute the survey of the methodology for pension fund risk control by using the idea of ALM.

While, the concept of LDI (=Liability Driven Investment) is diffused in U.K., Holland, Denmark, etc. now. The concept is also accepted and executed by the U.S.A.. Concerning LDI, Martellini (2006) discusses the difference between ALM and LDI from the perspectives both of practitioners and Academics. Roman (2008) introduces the examples of European countries that adopted LDI. Usuki (2007) proposes the introduction of LDI in Japan by indicating the merits. However, the infrastructure for introducing LDI has not yet completed in Japanese pension fund system including accounting standard based on market value.

Basically, the ideas of both ALM and LDI seem to be almost the same. But, the main difference between the two is the discount rate. Namely, LDI uses the rate of long term bond and ALM uses the rate of fixed plan yield. Furthermore, LDI focuses on the volatile risk of interest for pension liability from a short term perspective. ALM focuses on the attainment of target return under the premises that the pension plan continues for a long period of time. And if plan sponsor has a same duration's bond in asset side as compared with long term bond in liability side, it's relatively easy to match the fluctuation of asset and liability. This idea holds true to LDI, but it does not hold true to ALM due to the fixed plan yield. In other words, LDI are much more flexible than traditional ALM. Other differences between the two are that LDI considers the use of derivatives such as option and swap, and LDI adopts the idea of risk budgeting. On the other hand, The Japanese government pension fund case is cited in this case. Because Japanese government pension fund uses asset only approach, not surplus approach. The merit of asset only approach is to omit the complicated calculations of liability side and concentrates on the asset side problem. But, the demerit of asset only approach is that the weight of risk assets will become large due to the ignorance of correlations between asset and liability. While, surplus approach considers the correlations between asset and liability. Thus, this approach selects the asset allocation to minimize the surplus risk by considering a given target growth rate of surplus. As a result, generally speaking, investment weight of domestic bonds which have high correlations with liability will increase in this approach. We can also indicate the feature of ALM and LDI approaches from this analysis.

2. Model

At first, we examine the asset allocation used the continuous time series model in surplus framework. Price process of risk free bonds S^0 is written as $dS_t^0 = rS_t^0 dt$ under the conditions that risk free interest r is constant. Generally, stochastic process of risk assets S^i is written as

$$dS_t^i = S_t^i\left[(r_0 + \lambda_i)dt + \sum_{j=1}^{n} \sigma_{ij} dW_j\right] (\because i = 1,...,n) \ldots\ldots \quad (1)$$

while, $W^T = [W_1 W_2 ....W_n]$ means the *n* dimensional standard Brownian motion. And the numbers of Brownian motions and assets are the same. $\lambda^T = [\lambda_1 \lambda_2 ... \lambda_n]$ is the risk premium of risk assets. σ_{ij} is the volatility of asset for Brownian motion *j*. *C* is $n \times n$ matrix of volatility ; $C = \begin{bmatrix} \sigma_{11...}\sigma_{1n} \\ \sigma_{n1...}\sigma_{nn} \end{bmatrix}$.

For example, we introduce the asset allocation of multiple assets in asset only approach. Pension fund holds pension asset ; A_t. The price process of pension asset is expressed as $dA_t = A_t\left[\left(r+\phi^T\lambda\right)dt + \phi^T CdW\right]$...(2)

while, $\phi^T = \left[\phi_1\phi_2 ... \phi_n\right]$ means the asset allocation ratio of risk assets.

Suppose that the utility function is $U(A_T)$ under the terminal date; $t = T$. We select the optimal asset allocation ϕ_A^* to maximize the utility function. The value function is $V(t, A_t) = \max_{\phi} E\left[U(A_T)\right]$..(3)

HJB (=Hamilton-Jacobi-Bellman) formula is written as $\max_{\phi}\left[V_t + A\lambda\phi^T V_A + \frac{1}{2}A^2\phi^T D\phi V_{AA}\right] = 0$.. (4)

Here, $D = CC^T$. D is a covariance matrix among assets.
First order condition of optimal solution is $\lambda AV_A + D\phi A^2 V_{AA} = 0$.........................(5)
and optimal asset allocation ϕ_A^* is $\phi_A^* = -D^{-1}\lambda\dfrac{V_A}{AV_{AA}}$ .... (6)

We must specify the utility function to get an explicit asset allocation.
Suppose that the utility function of pension fund in terminal date is $U = A_T^r / \gamma$.

We can conjecture V as $V(t,T) = A_t^\gamma f(t)/\gamma$ and calculate V_A / V_{AA}.

Thus, optimal asset allocation is obtained by $\phi_A^* = \dfrac{1}{1-\gamma}D^{-1}\lambda$ ....(7)

Next, we introduce the pension liability and also deem that its movement is based on the Brownian motion. The market price of liability L_t is $dL_t = \mu_L L_t dt + \sigma_L L_t dW_1$ ....(8)
μ_L and σ_L are drift and volatility respectively and constant. W_1 is a standard Brownian motion. In addition, suppose that pension asset has a correlation with pension liability. The price process of pension asset ; A is $dA_t = A_t\left[\left(r+\phi_t^T\lambda\right)dt + \phi_t^T CdW\right]$..(9)

where, n dimensional vector $\phi^T = \left[\phi_1\phi_2 ... \phi_n\right]$ of each asset class S_i.

Generally, pension fund selects the optimal asset allocation ϕ^* which maximize the expected utility of funding ratio: $F_T = A_T / L_T$ under the terminal date $t = T$.

Namely, $\{\phi\} \in \arg\max_{\phi} H(t, A_t, L_t) = E_t[U(F_T)]$……………………………………(10)

Boundary conditions is based on the funding ratio $H(T, A_T, L_T) = U\left(\frac{A_T}{L_T}\right)$ in the terminal date. Likewise, the value function $V(t, A_t, L_t)$ is $V(t, A_t, L_t) = \max_{\phi} E_t[U(F_T)]$……………………………………………………..(11)

HJB formula of the value function is $\max_{\phi} V_t + \{\phi^T \lambda + r\} AV_A + \frac{1}{2}\phi^T D\phi A^2 V_{AA} + \mu_L LV_L + \frac{1}{2}\sigma_L^2 L^2 V_{LL} + \phi^T C_1 \sigma_L ALV_{AL} = 0$ ..(12)

where, C_1 is the first row of matrix C. Suppose that the existence of optimal solution ϕ^*, first order condition is $\lambda AV_A + D\phi A^2 V_{AA} + C_1 \sigma_L ALV_{AL} = 0$ ………..(13)

Thus, the optimal asset allocation ϕ_S^* in Surplus approach is

$$\phi_S^* = -\frac{1}{A}\frac{V_A}{V_{AA}} D^{-1}\left(\lambda + C_1 \sigma_L L \frac{V_{AL}}{V_A}\right). \quad \text{……………………………………………..(14)}$$

Furthermore, suppose that utility function is $U(F_T) = F^\gamma / \gamma$, we can conjecture the value function as $V(t, A_{t,} L_t) = \frac{1}{\gamma} A^\gamma L^{-\gamma}(f(t))^\gamma$, where $f(t)$ is a function of differentiable time *t*. The partial derivatives of value functions are $V_A = A^{\gamma-1} L^{-\gamma}(f(t))^\gamma$, $V_{AA} = (\gamma - 1)A^{\gamma-2} L^{-\gamma}(f(t))^\gamma$, $V_{AL} = -\gamma A^{\gamma-1} L^{-\gamma-1}(f(t))^\gamma$…………(15)

The ratios of partial derivatives of value functions in the optimal asset allocation become $\frac{V_A}{V_{AA}} = \frac{A^{\gamma-1} L^{-\gamma}(f(t))^\gamma}{(\gamma-1)A^{\gamma-2} L^{-\gamma}(f(t))^\gamma} = \frac{A}{\gamma - 1}$, $\frac{V_{AL}}{V_A} = \frac{-\gamma A^{\gamma-1} L^{-\gamma-1}(f(t))^\gamma}{A^{\gamma-1} L^{-\gamma}(f(t))^\gamma} = -\gamma \frac{1}{L}$ …(16)

Thus, the optimal asset allocation ϕ_S^* is calculated by substituting equation (16) to equation (14). Namely, $\phi_S^* = \frac{1}{1-\gamma} D^{-1}(\lambda - \gamma C_1 \sigma_L)$…………………………...…(17)

We can notice that there is a difference $D^{-1}\gamma C_1 \sigma_L / (1-\gamma)$ of optimal asset allocation in between asset only approach ϕ_A^* and surplus approach ϕ_S^*. This term is relied on the correlation and risk in both asset and liability, not relied on the expected return (plan yield) of liability. And optimal asset allocation is not relied on the initial funding ratio. We can show the instantaneous efficient frontier under the framework of surplus. Strictly speaking, surplus means asset minus liability, but we take up the funding ratio.

Stochastic process which pension asset A has is $dA_t = \mu\phi^T A_t dt + \phi^T C A_t dW_t$ ……..(18)

Here, ϕ is n dimensional vector, μ is n dimensional vector, C is $n \times n$ dimensional volatility matrix, and W is n dimensional standard Brownian motion. Likewise, stochastic process of liability L_t is $dL_t = \mu_L L dt + \sigma_L L dW_t^L$ …………(19)

μ_L is an expected growth rate of liability and σ_L is a volatility of liability. W^L is a standard Brownian motion. The inverse of liability is written as $G = 1/L$.

The process of G is given from Ito's lemma as

$$dG = \left[\frac{\partial G}{\partial t} + \mu_L L \frac{\partial G}{\partial L} + \frac{1}{2}\sigma_L^2 L^2 \frac{\partial^2 G}{\partial L^2}\right]dt + \sigma_L L \frac{\partial G}{\partial L} dW^L \text{ …………………………….(20)}$$

If we substitute $\partial G/\partial t = 0$, $\partial G/\partial L = -1/L^2$,and $\partial^2 G/\partial L^2 = -2/L^3$, into equation (20), the process of inverse of liability become $d\frac{1}{L} = -\frac{1}{L}\left\{\left(\mu_t - \sigma_L^2\right)dt + \sigma_L dW_t^L\right\}$.(21)

In addition, the process of funding ratio $d(A/L)$ is also given from equation (18) and (21) as

$$d\frac{A}{L} = \frac{A}{L}\left\{\left(\mu\phi^T - \mu_L + \sigma_L^2 - \phi^T C_1 \sigma_L\right)dt + \phi C dW_t - \sigma_L dW_t^L\right\} \text{ ………………..(22)}$$

Here, C_1 is represented as first column $C_1 = [\sigma_{11}\sigma_{21}...\sigma_{n1}]$ of matrix C by using Cholesky decomposition. In addition, D is decomposed as $D = CC^T$.

The instantaneous expected growth rate $m(\phi)$ under a certain asset allocation ϕ is given as $m(\phi) = \phi^T \mu - \mu_L + \sigma_L^2 - \phi^T C_1 \sigma_L$ ………………………………………….…………(23)

And instantaneous volatility $v(\phi)$ becomes $v(\phi) = \phi^T D \phi + \sigma_L^2 - 2\phi^T C_1 \sigma_L$ ....(24)

For example, asset allocation ϕ^* to minimize risk under a given return μ^* is given by solving the following optimal problem.

$$\min_{\phi} v(\phi)$$

subject to $m(\phi) = \mu^*$, $\phi^T e = 1$

Let Lagrange multipliers be θ and λ, Lagrange function $l(\cdot)$ becomes $l(\phi,\theta,\lambda) = \phi^T D\phi + \sigma_L^2 - 2\phi^T C_1\sigma_L + 2\theta(\phi^T\mu - \mu_L + \sigma_L^2 - \phi^T C_1\sigma_L) + 2\lambda(\phi^T e - 1)$.......(25)

First order conditions of optimality are $\frac{\partial l}{\partial \phi} = 2D\phi - 2C_1\sigma_L + 2\theta\mu - 2\theta C_1\sigma_L + 2\lambda e = 0$, $\frac{\partial l}{\partial \theta} = \phi^T\mu - \mu_L + \sigma_L - \phi^T C_1\sigma_L - \mu^* = 0$, $\frac{\partial l}{\partial \lambda} = \phi^T e - 1 = 0$ ....(26)

Thus, optimal asset allocation ϕ is $\phi = D^{-1}C_1\sigma_L - \theta D^{-1}\mu + \theta\sigma_L D^{-1}C_1 - \lambda D^{-1}e$ ...(27)

while, $\phi^T = \sigma_L C_1^T D^{-1} - \theta\mu^T D^{-1} + \theta\sigma_L C_1^T D^{-1} - \lambda e^T D^{-1}$ ....(28)

We define $L_D = C_1^T D^{-1}\mu$, $M_D = \mu^T D^{-1}\mu$, $N_D = e^T D^{-1}\mu$, $X_D = C_1^T D^{-1}C_1$, $Y_D = C_1^T D^{-1}e$, $Z_D = e^T D^{-1}e$. We also define $\alpha = 2\sigma_L L_D - M_D - \sigma_L^2 N_D$, $\beta = \sigma_L Y_D - N_D$, $\chi = \sigma_L L_D - \mu_L + \sigma_L^2 - \sigma_L^2 Y_D$, $\delta = Z_D$, $\eta = \sigma_L Y_D - 1$.

Here, we substitute equation (28) to equation (26). After that, we also substitute the former definitions to the substituted equation (26). Finally, if we solve the simultaneous equations $\alpha\theta + \beta\lambda + \chi - \mu^* = 0$, $\beta\theta - \delta\lambda + \eta = 0$, Lagrange multipliers becomes $\theta = -\frac{\delta(\chi - \mu^*) + \beta\eta}{\alpha\delta + \beta^2}$ and $\lambda = -\frac{\beta(\chi - \mu^*) - \alpha\eta}{\alpha\delta + \beta^2}$ ....(29)

If we substitute equation (29) to equation (27), we can get an optimal asset allocation ϕ under a certain expected return μ^*. In addition, we can draw an efficient frontier by changing μ^*.

3. Empirical Results

We consider Domestic Bond, Domestic Equity, Foreign Bond, Foreign Equity, and Liability for analysis. We use NIKKO J-MIX Indices in both Domestic Bond and Domestic Equity. We also use JP Morgan Government Bond Indices (GBI) for Foreign Bond, and MSCI-KOKUSAI for Foreign Equity. These data are from January 1988 to December 2007 on a monthly basis in Japanese Yen. Furthermore, efficient frontiers in the following analyses are instantaneous, not continuous. Suppose that we will predict the efficient frontier at the end of January 2008. Because, we usually use historical data to predict the next period. More specifically, the expected return in each asset in the past 20 years on a monthly basis will be used to predict the expected return on January 2008.

Table 1. shows the return and risk in each asset.

Table 1. Return and Risk in Each Asset

	Expected Return	Covariance Matrix			
Domestic Bonds	4.01%	0.0011341	-2.65464E-05	3.78998E-05	1.2817E-05
Domestic Equities	2.13%	-2.655E-05	0.036862893	-7.253E-05	0.00086801
Foreign Bonds	6.91%	3.79E-05	-7.253E-05	0.007457894	0.00059929
Foreign Equities	9.52%	1.282E-05	0.000868008	0.000599289	0.02674232

Table 2. shows the plan yield, change of liability's volatility with simulations, and change of related parameters. Especially, plan yield, in other words, expected growth rate of liability is determined as 3.2 percent by Japanese government. The basis of 3.2 percent is building block style. Table-2 also corresponds to ALM approach.

Table 2. Transition of Volatility and related Parameters (ALM)

Plan Yield* 3.20%

*Expected Growth Rate of Liability

Volatility of Liability	1.00%	3.00%	5.00%
L_D	1.19	1.19	1.19
M_D	2.34	2.34	2.34
N_D	47.73	47.73	47.73
X_D	1.00	1.00	1.00
Y_D	29.69	29.69	29.69
Z_D	1065.42	1065.42	1065.42
α	-2.32	-2.31	-2.34
β	-47.43	-46.84	-46.25
χ	-0.02	0.00	0.03
δ	1065.42	1065.42	1065.42
η	-0.70	-0.11	0.48

Glancing at Table 3-(a), we can notice that the proportion of riskless asset such as Domestic Bond will decrease with the increase of surplus return and risk. However, the proportion of decrease will decrease with the increase of liability's volatility.

While, the proportions of risk assets such as Foreign Bond and Foreign Equity will increase with the increase of surplus return and risk. The only exception is

Domestic(=Japanese) Equity, because it is not attractive for its high risk and low return tendency due to the collapse of bubble economy in 1990. Incidentally, the aftermath of it continued for more than a decade. Even if we change the volatility of liability, the same trend can be found out in Table 4-(a), and Table 5-(a). For reference, asset only frameworks corresponding to surplus frameworks are listed as Table 3-(b). We also list asset only frameworks in the following related Tables.

If we contrast the Figure 1-(a) and Figure 1-(b), we can find out that the efficient portfolio between the two are different from. Because, we must consider the volatility of liability in surplus approach. Thus, most riskless asset will become, not risk free assets but bonds which have the same duration of pension liability in this case. We can confirm this in Figure 2-(a), Figure 2-(b), Figure 3-(a), and Figure 3-(b),

In addition, we can also notice graphically that the percentage of Foreign Bond and Foreign Equity will increase and Domestic Bond and Domestic Equity will decrease with the increase of surplus risk from Figure 1-(c). Namely, Domestic Bond, Domestic Equity, Foreign Bond, and Foreign Equity from left to right are illustrated in Figure 1-(c). This holds true to Figure 2-(c) and 3-(c).

Table 3-(a). Asset Allocation in Surplus Framework (ALM) ; $\sigma=1.00\%$

Surplus Return	θ	λ	DB.	DE.	F.B.	F.E.	Surplus Risk
0.0%	0.0529	-0.0030	110.55%	5.34%	-8.30%	-7.59%	3.23%
0.2%	0.0433	-0.0026	106.37%	4.69%	-5.26%	-5.81%	2.92%
0.4%	0.0336	-0.0022	102.20%	4.05%	-2.22%	-4.03%	2.64%
0.6%	0.0240	-0.0017	98.02%	3.40%	0.82%	-2.24%	2.42%
0.8%	0.0144	-0.0013	93.84%	2.76%	3.86%	-0.46%	2.25%
1.0%	0.0047	-0.0009	89.67%	2.11%	6.90%	1.32%	2.16%
1.2%	-0.0049	-0.0004	85.49%	1.47%	9.94%	3.10%	2.17%
1.4%	-0.0145	0.0000	81.31%	0.82%	12.98%	4.88%	2.25%
1.6%	-0.0242	0.0004	77.14%	0.18%	16.02%	6.66%	2.42%
1.8%	-0.0338	0.0008	72.96%	-0.47%	19.06%	8.44%	2.65%
2.0%	-0.0435	0.0013	68.78%	-1.11%	22.10%	10.22%	2.92%
2.2%	-0.0531	0.0017	64.61%	-1.76%	25.14%	12.00%	3.24%
2.4%	-0.0627	0.0021	60.43%	-2.40%	28.18%	13.79%	3.58%
2.7%	-0.0724	0.0026	56.26%	-3.05%	31.22%	15.57%	3.94%
2.9%	-0.0820	0.0030	52.08%	-3.69%	34.27%	17.35%	4.31%
3.1%	-0.0916	0.0034	47.90%	-4.34%	37.31%	19.13%	4.69%
3.3%	-0.1013	0.0038	43.73%	-4.98%	40.35%	20.91%	5.09%
3.5%	-0.1109	0.0043	39.55%	-5.63%	43.39%	22.69%	5.49%
3.7%	-0.1206	0.0047	35.37%	-6.27%	46.43%	24.47%	5.89%
3.9%	-0.1302	0.0051	31.20%	-6.92%	49.47%	26.25%	6.30%
4.1%	-0.1398	0.0056	27.02%	-7.56%	52.51%	28.03%	6.72%
4.3%	-0.1495	0.0060	22.84%	-8.21%	55.55%	29.82%	7.14%
4.5%	-0.1591	0.0064	18.67%	-8.85%	58.59%	31.60%	7.55%
4.7%	-0.1687	0.0069	14.49%	-9.50%	61.63%	33.38%	7.98%
4.9%	-0.1784	0.0073	10.31%	-10.14%	64.67%	35.16%	8.40%
5.1%	-0.1880	0.0077	6.14%	-10.79%	67.71%	36.94%	8.82%
5.3%	-0.1977	0.0081	1.96%	-11.43%	70.75%	38.72%	9.25%
5.5%	-0.2073	0.0086	-2.21%	-12.08%	73.79%	40.50%	9.68%
5.7%	-0.2169	0.0090	-6.39%	-12.72%	76.83%	42.28%	10.10%
5.9%	-0.2266	0.0094	-10.57%	-13.37%	79.87%	44.07%	10.53%
6.1%	-0.2362	0.0099	-14.74%	-14.01%	82.91%	45.85%	10.96%
6.3%	-0.2458	0.0103	-18.92%	-14.66%	85.95%	47.63%	11.39%
6.5%	-0.2555	0.0107	-23.10%	-15.30%	88.99%	49.41%	11.83%
6.7%	-0.2651	0.0111	-27.27%	-15.95%	92.03%	51.19%	12.26%
6.9%	-0.2748	0.0116	-31.45%	-16.59%	95.07%	52.97%	12.69%
7.1%	-0.2844	0.0120	-35.63%	-17.24%	98.11%	54.75%	13.12%
7.3%	-0.2940	0.0124	-39.80%	-17.88%	101.15%	56.53%	13.55%

Table 3-(b). Asset Only Framework ; $\sigma=1.00\%$

Risk	Return
4.11%	3.23%
3.83%	3.43%
3.59%	3.63%
3.38%	3.84%
3.22%	4.04%
3.12%	4.24%
3.08%	4.44%
3.10%	4.65%
3.18%	4.85%
3.31%	5.05%
3.50%	5.25%
3.73%	5.46%
3.99%	5.66%
4.29%	5.86%
4.60%	6.07%
4.94%	6.27%
5.29%	6.47%
5.65%	6.67%
6.02%	6.88%
6.40%	7.08%
6.79%	7.28%
7.18%	7.49%
7.58%	7.69%
7.99%	7.89%
8.39%	8.09%
8.80%	8.30%
9.21%	8.50%
9.63%	8.70%
10.05%	8.90%
10.46%	9.11%
10.88%	9.31%
11.30%	9.51%
11.73%	9.72%
12.15%	9.92%
12.58%	10.12%
13.00%	10.32%
13.43%	10.53%

Figure 1-(a). Surplus Efficient Frontier (ALM) ; σ＝1.00%

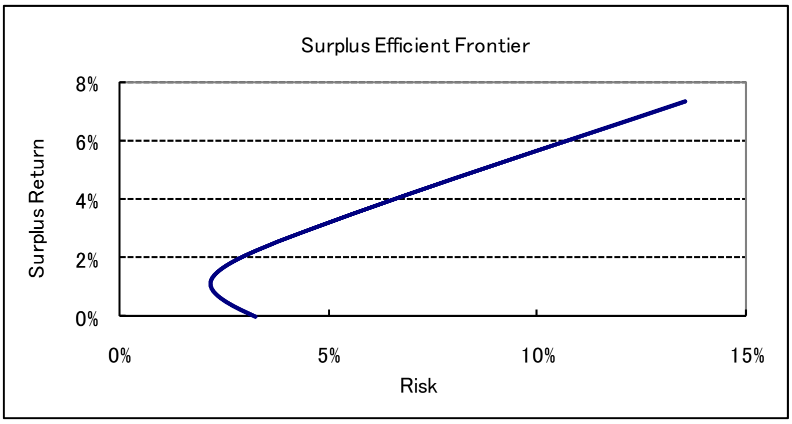


Figure 1-(b). Efficient Frontier of Asset Only ; σ＝1.00%

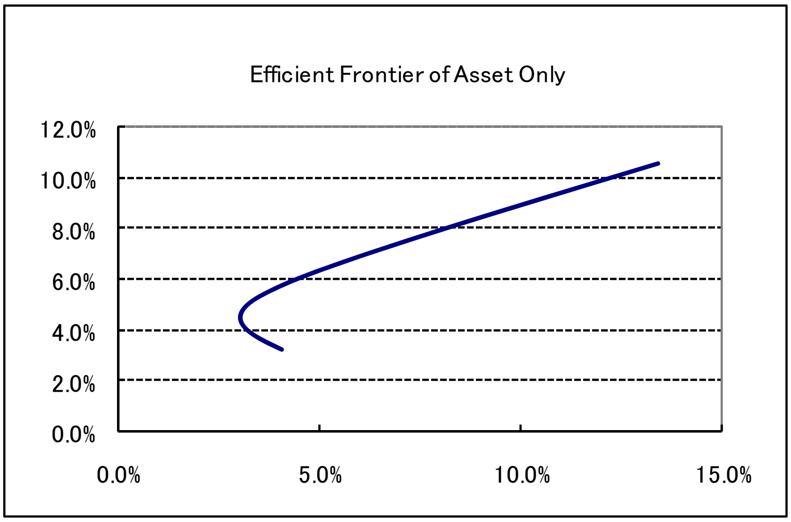


Figure 1-(c). Asset Allocation Ratio (ALM) ; σ＝1.00%

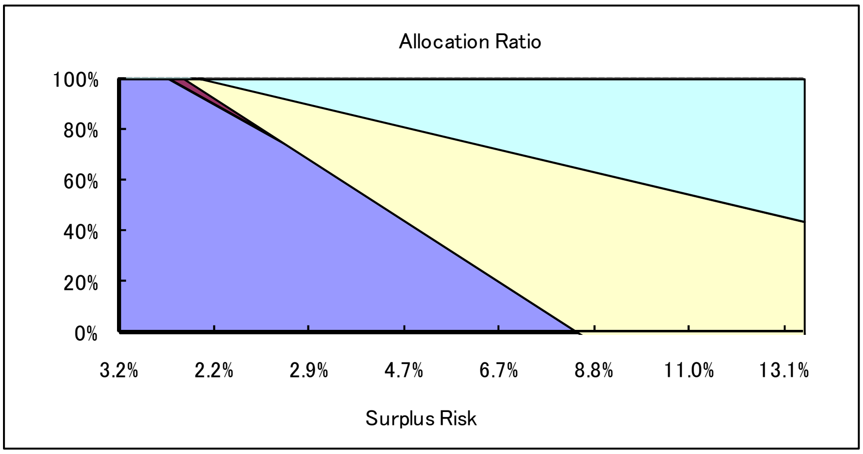


Table 4-(a). Asset Allocation in Surplus Framework (ALM) ; σ = 3.00%

Surplus Return	θ	λ	D.B.	D.E.	F.B.	F.E.	Surplus Risk
0.0%	0.0334	-0.0016	112.92%	2.47%	-9.48%	-5.90%	1.58%
0.2%	0.0254	-0.0012	109.37%	1.94%	-6.90%	-4.41%	1.22%
0.4%	0.0174	-0.0009	105.81%	1.42%	-4.32%	-2.91%	0.87%
0.6%	0.0094	-0.0005	102.26%	0.90%	-1.74%	-1.42%	0.55%
0.8%	0.0014	-0.0002	98.70%	0.37%	0.85%	0.08%	0.34%
1.0%	-0.0066	0.0002	95.15%	-0.15%	3.43%	1.57%	0.45%
1.2%	-0.0146	0.0005	91.60%	-0.68%	6.01%	3.07%	0.75%
1.4%	-0.0226	0.0009	88.04%	-1.20%	8.60%	4.56%	1.10%
1.6%	-0.0306	0.0012	84.49%	-1.72%	11.18%	6.06%	1.45%
1.8%	-0.0386	0.0016	80.93%	-2.25%	13.76%	7.55%	1.81%
2.0%	-0.0466	0.0019	77.38%	-2.77%	16.35%	9.05%	2.18%
2.2%	-0.0546	0.0023	73.83%	-3.30%	18.93%	10.54%	2.54%
2.4%	-0.0627	0.0027	70.27%	-3.82%	21.51%	12.04%	2.91%
2.7%	-0.0707	0.0030	66.72%	-4.34%	24.10%	13.53%	3.28%
2.9%	-0.0787	0.0034	63.16%	-4.87%	26.68%	15.02%	3.64%
3.1%	-0.0867	0.0037	59.61%	-5.39%	29.26%	16.52%	4.01%
3.3%	-0.0947	0.0041	56.06%	-5.92%	31.85%	18.01%	4.38%
3.5%	-0.1027	0.0044	52.50%	-6.44%	34.43%	19.51%	4.75%
3.7%	-0.1107	0.0048	48.95%	-6.96%	37.01%	21.00%	5.12%
3.9%	-0.1187	0.0051	45.39%	-7.49%	39.59%	22.50%	5.49%
4.1%	-0.1267	0.0055	41.84%	-8.01%	42.18%	23.99%	5.86%
4.3%	-0.1347	0.0058	38.28%	-8.53%	44.76%	25.49%	6.22%
4.5%	-0.1427	0.0062	34.73%	-9.06%	47.34%	26.98%	6.59%
4.7%	-0.1507	0.0065	31.18%	-9.58%	49.93%	28.48%	6.96%
4.9%	-0.1587	0.0069	27.62%	-10.11%	52.51%	29.97%	7.33%
5.1%	-0.1668	0.0072	24.07%	-10.63%	55.09%	31.47%	7.70%
5.3%	-0.1748	0.0076	20.51%	-11.15%	57.68%	32.96%	8.07%
5.5%	-0.1828	0.0079	16.96%	-11.68%	60.26%	34.46%	8.44%
5.7%	-0.1908	0.0083	13.41%	-12.20%	62.84%	35.95%	8.81%
5.9%	-0.1988	0.0086	9.85%	-12.73%	65.43%	37.45%	9.18%
6.1%	-0.2068	0.0090	6.30%	-13.25%	68.01%	38.94%	9.55%
6.3%	-0.2148	0.0093	2.74%	-13.77%	70.59%	40.44%	9.92%
6.5%	-0.2228	0.0097	-0.81%	-14.30%	73.18%	41.93%	10.28%
6.7%	-0.2308	0.0100	-4.36%	-14.82%	75.76%	43.43%	10.65%
6.9%	-0.2388	0.0104	-7.92%	-15.34%	78.34%	44.92%	11.02%
7.1%	-0.2468	0.0107	-11.47%	-15.87%	80.93%	46.42%	11.39%
7.3%	-0.2548	0.0111	-15.03%	-16.39%	83.51%	47.91%	11.76%

Table 4-(b). Asset Only Framework ; σ= 3.00%

Risk	Return
4.03%	3.22%
3.81%	3.42%
3.62%	3.62%
3.46%	3.83%
3.33%	4.03%
3.23%	4.23%
3.18%	4.43%
3.17%	4.63%
3.21%	4.83%
3.28%	5.03%
3.39%	5.23%
3.54%	5.43%
3.72%	5.63%
3.93%	5.83%
4.16%	6.03%
4.41%	6.23%
4.67%	6.43%
4.95%	6.63%
5.24%	6.84%
5.53%	7.04%
5.84%	7.24%
6.15%	7.44%
6.47%	7.64%
6.80%	7.84%
7.13%	8.04%
7.46%	8.24%
7.80%	8.44%
8.14%	8.64%
8.48%	8.84%
8.82%	9.04%
9.17%	9.24%
9.51%	9.44%
9.86%	9.64%
10.21%	9.85%
10.57%	10.05%
10.92%	10.25%
11.27%	10.45%

Figure 2-(a). Surplus Efficient Frontier (ALM) ; σ＝3.00％

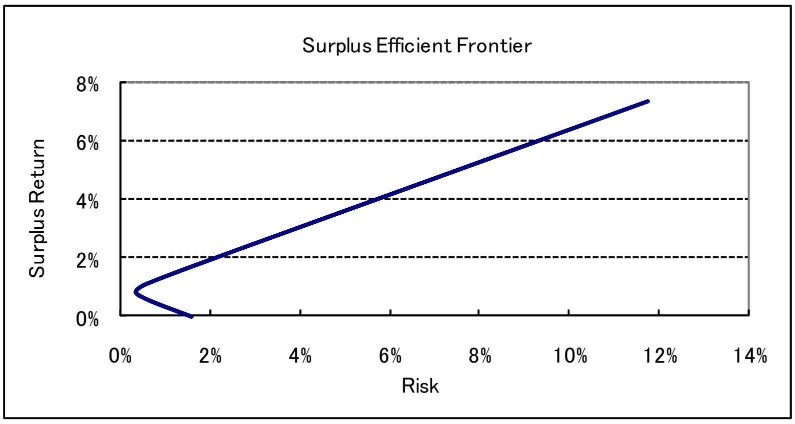


Figure 2-(b). Efficient Frontier of Asset Only ; σ=3.00%

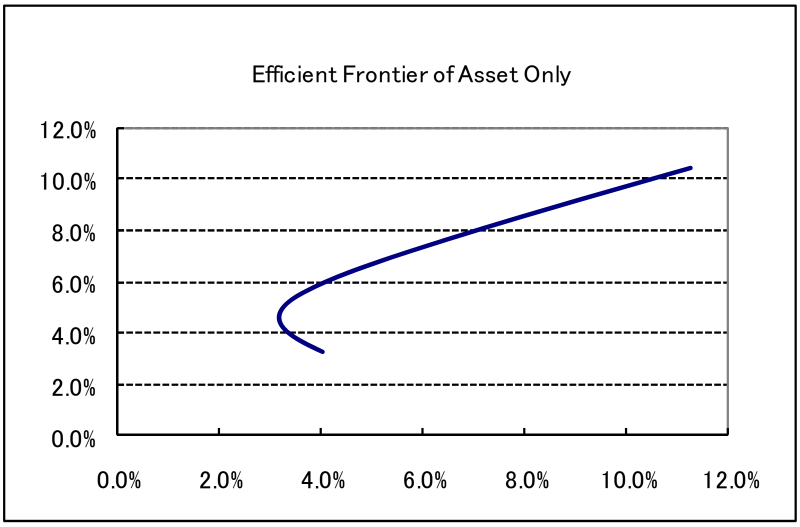


Figure 2-(c). Asset Allocation Ratio (ALM) ; σ=3.00%

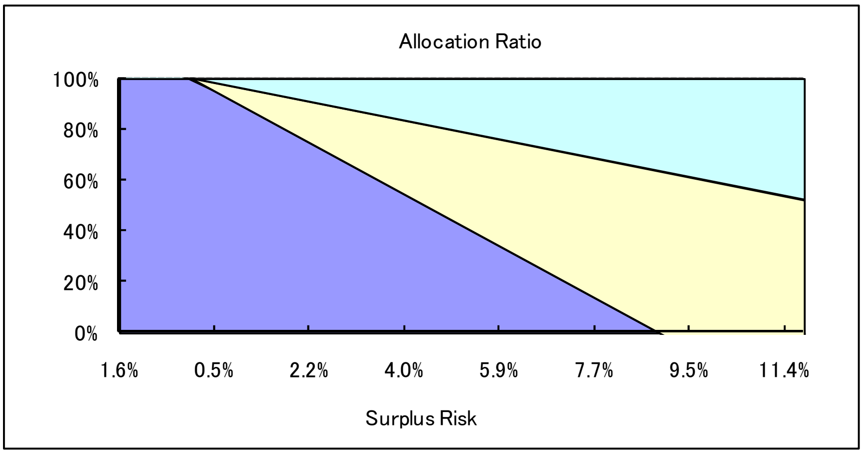


Table 5-(a). Asset Allocation in Surplus Framework (ALM) ; σ = 5.00%

Surplus Return	θ	λ	D.B.	D.E.	F.B.	F.E.	Surplus Risk
0.0%	0.0194	-0.0004	117.34%	0.00%	-12.18%	-5.17%	1.74%
0.2%	0.0133	-0.0001	114.58%	-0.39%	-10.17%	-4.02%	1.61%
0.4%	0.0072	0.0001	111.82%	-0.78%	-8.17%	-2.87%	1.52%
0.6%	0.0011	0.0004	109.05%	-1.17%	-6.16%	-1.73%	1.49%
0.8%	-0.0050	0.0007	106.29%	-1.56%	-4.15%	-0.58%	1.50%
1.0%	-0.0110	0.0009	103.52%	-1.94%	-2.15%	0.57%	1.57%
1.2%	-0.0171	0.0012	100.76%	-2.33%	-0.14%	1.72%	1.69%
1.4%	-0.0232	0.0015	98.00%	-2.72%	1.86%	2.86%	1.84%
1.6%	-0.0293	0.0017	95.23%	-3.11%	3.87%	4.01%	2.02%
1.8%	-0.0354	0.0020	92.47%	-3.50%	5.87%	5.16%	2.22%
2.0%	-0.0415	0.0023	89.71%	-3.89%	7.88%	6.30%	2.44%
2.2%	-0.0475	0.0025	86.94%	-4.28%	9.88%	7.45%	2.67%
2.4%	-0.0536	0.0028	84.18%	-4.67%	11.89%	8.60%	2.91%
2.7%	-0.0597	0.0030	81.42%	-5.05%	13.90%	9.74%	3.16%
2.9%	-0.0658	0.0033	78.65%	-5.44%	15.90%	10.89%	3.42%
3.1%	-0.0719	0.0036	75.89%	-5.83%	17.91%	12.04%	3.67%
3.3%	-0.0780	0.0038	73.12%	-6.22%	19.91%	13.18%	3.94%
3.5%	-0.0840	0.0041	70.36%	-6.61%	21.92%	14.33%	4.20%
3.7%	-0.0901	0.0044	67.60%	-7.00%	23.92%	15.48%	4.47%
3.9%	-0.0962	0.0046	64.83%	-7.39%	25.93%	16.62%	4.74%
4.1%	-0.1023	0.0049	62.07%	-7.78%	27.93%	17.77%	5.01%
4.3%	-0.1084	0.0052	59.31%	-8.16%	29.94%	18.92%	5.28%
4.5%	-0.1145	0.0054	56.54%	-8.55%	31.95%	20.07%	5.55%
4.7%	-0.1205	0.0057	53.78%	-8.94%	33.95%	21.21%	5.83%
4.9%	-0.1266	0.0060	51.01%	-9.33%	35.96%	22.36%	6.11%
5.1%	-0.1327	0.0062	48.25%	-9.72%	37.96%	23.51%	6.38%
5.3%	-0.1388	0.0065	45.49%	-10.11%	39.97%	24.65%	6.66%
5.5%	-0.1449	0.0067	42.72%	-10.50%	41.97%	25.80%	6.94%
5.7%	-0.1510	0.0070	39.96%	-10.89%	43.98%	26.95%	7.21%
5.9%	-0.1570	0.0073	37.20%	-11.27%	45.98%	28.09%	7.49%
6.1%	-0.1631	0.0075	34.43%	-11.66%	47.99%	29.24%	7.77%
6.3%	-0.1692	0.0078	31.67%	-12.05%	50.00%	30.39%	8.05%
6.5%	-0.1753	0.0081	28.90%	-12.44%	52.00%	31.53%	8.33%
6.7%	-0.1814	0.0083	26.14%	-12.83%	54.01%	32.68%	8.61%
6.9%	-0.1875	0.0086	23.38%	-13.22%	56.01%	33.83%	8.89%
7.1%	-0.1935	0.0089	20.61%	-13.61%	58.02%	34.98%	9.17%
7.3%	-0.1996	0.0091	17.85%	-14.00%	60.02%	36.12%	9.45%

Table 5-(b). Asset Only Framework ; σ= 5.00%

Risk	Return
4.17%	3.15%
4.01%	3.35%
3.86%	3.55%
3.72%	3.75%
3.61%	3.95%
3.51%	4.14%
3.44%	4.34%
3.38%	4.54%
3.35%	4.74%
3.35%	4.94%
3.36%	5.14%
3.41%	5.34%
3.47%	5.54%
3.56%	5.74%
3.66%	5.94%
3.79%	6.14%
3.93%	6.34%
4.09%	6.54%
4.26%	6.74%
4.44%	6.94%
4.63%	7.14%
4.83%	7.34%
5.04%	7.54%
5.26%	7.74%
5.48%	7.94%
5.71%	8.14%
5.94%	8.34%
6.18%	8.54%
6.42%	8.74%
6.66%	8.94%
6.91%	9.14%
7.16%	9.34%
7.41%	9.53%
7.67%	9.73%
7.92%	9.93%
8.18%	10.13%
8.44%	10.33%

Figure 3-(a). Surplus Efficient Frontier (ALM) ; σ＝5.00％

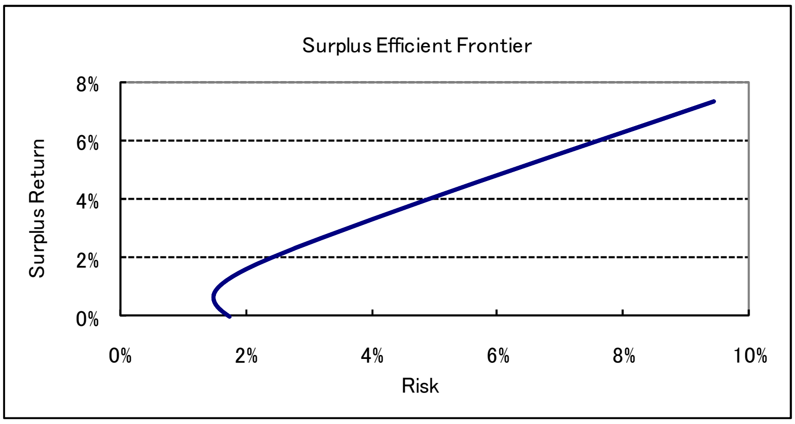


Figure 3-(b). Efficient Frontier of Asset Only ; σ＝5.00%

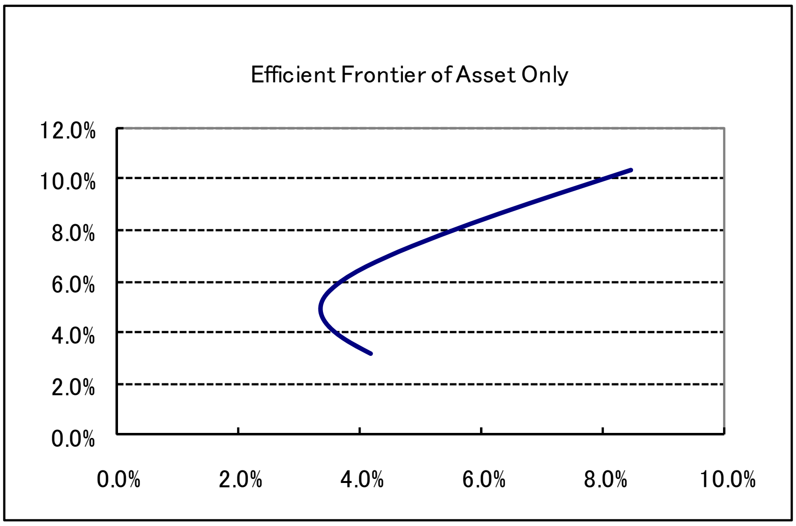


Figure 3-(c). Asset Allocation Ratio (ALM) ; σ＝5.00%

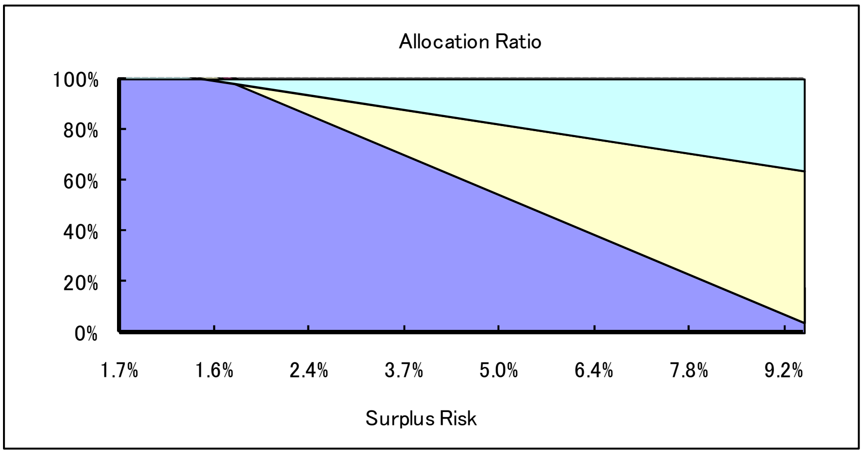


Table-6. shows the Yield on TSE Bonds (10years) with longest remaining maturity, change of liability's volatility with simulations, and change of related parameters. Especially, the Yield on TSE Bonds (10years); 1.44 % is derived from the data at the end of January 2008. Table-6 also corresponds to LDI approach. If we compare between the ALM and LDI, the main difference is in the expected growth rate of Liability. The results of following LDI's Tables are almost similar to the former ALM's ones. Likewise the Tables' results, the following LDI's Figures also show the similarity. However, if we compare the surplus efficient frontiers in both ALM and LDI, we notice that surplus returns in LDI are always higher than that of ALM under a given volatility of liability. Because, the discount rate, in other words, expected growth rate of liability in LDI is smaller than that of ALM. So, the pension liability of LDI at the end of January 2008 will become larger than that of ALM. Thus, higher surplus return is required in LDI to avoid the shortage of funding.

Table 6. Transition of Volatility and related Parameters (LDI)

Yields on TSE Bonds (10years) with longest remaining maturity: 1.44%

Volatility of Liability	1.00%	3.00%	5.00%
L_D	1.19	1.19	1.19
M_D	2.34	2.34	2.34
N_D	47.73	47.73	47.73
X_D	1.00	1.00	1.00
Y_D	29.69	29.69	29.69
Z_D	1065.42	1065.42	1065.42
α	-2.32	-2.31	-2.34
β	-47.43	-46.84	-46.25
χ	0.00	0.02	0.03
δ	1065.42	1065.42	1065.42
η	-0.70	-0.11	0.48

Table 7-(a). Asset Allocation in Surplus Framework (LDI) ; σ = 1.00%

Surplus Return	θ	λ	D.B.	D.E.	F.B.	F.E.	Surplus Risk
0.0%	0.1360	-0.0067	146.56%	10.90%	-34.52%	-22.94%	6.55%
0.2%	0.1264	-0.0063	142.38%	10.26%	-31.48%	-21.16%	6.14%
0.4%	0.1167	-0.0059	138.21%	9.61%	-28.44%	-19.38%	5.73%
0.6%	0.1071	-0.0054	134.03%	8.97%	-25.39%	-17.60%	5.33%
0.8%	0.0975	-0.0050	129.85%	8.32%	-22.35%	-15.82%	4.93%
1.0%	0.0878	-0.0046	125.68%	7.68%	-19.31%	-14.04%	4.54%
1.2%	0.0782	-0.0041	121.50%	7.03%	-16.27%	-12.26%	4.16%
1.4%	0.0686	-0.0037	117.32%	6.39%	-13.23%	-10.48%	3.79%
1.6%	0.0589	-0.0033	113.15%	5.74%	-10.19%	-8.70%	3.44%
1.8%	0.0493	-0.0029	108.97%	5.10%	-7.15%	-6.91%	3.11%
2.0%	0.0396	-0.0024	104.79%	4.45%	-4.11%	-5.13%	2.81%
2.2%	0.0300	-0.0020	100.62%	3.81%	-1.07%	-3.35%	2.55%
2.4%	0.0204	-0.0016	96.44%	3.16%	1.97%	-1.57%	2.34%
2.7%	0.0107	-0.0011	92.27%	2.51%	5.01%	0.21%	2.21%
2.9%	0.0011	-0.0007	88.09%	1.87%	8.05%	1.99%	2.15%
3.1%	-0.0085	-0.0003	83.91%	1.22%	11.09%	3.77%	2.19%
3.3%	-0.0182	0.0001	79.74%	0.58%	14.13%	5.55%	2.31%
3.5%	-0.0278	0.0006	75.56%	-0.07%	17.17%	7.33%	2.50%
3.7%	-0.0375	0.0010	71.38%	-0.71%	20.21%	9.12%	2.75%
3.9%	-0.0471	0.0014	67.21%	-1.36%	23.25%	10.90%	3.04%
4.1%	-0.0567	0.0019	63.03%	-2.00%	26.29%	12.68%	3.36%
4.3%	-0.0664	0.0023	58.85%	-2.65%	29.33%	14.46%	3.71%
4.5%	-0.0760	0.0027	54.68%	-3.29%	32.37%	16.24%	4.07%
4.7%	-0.0856	0.0032	50.50%	-3.94%	35.41%	18.02%	4.45%
4.9%	-0.0953	0.0036	46.32%	-4.58%	38.45%	19.80%	4.84%
5.1%	-0.1049	0.0040	42.15%	-5.23%	41.50%	21.58%	5.24%
5.3%	-0.1146	0.0044	37.97%	-5.87%	44.54%	23.36%	5.64%
5.5%	-0.1242	0.0049	33.79%	-6.52%	47.58%	25.15%	6.05%
5.7%	-0.1338	0.0053	29.62%	-7.16%	50.62%	26.93%	6.46%
5.9%	-0.1435	0.0057	25.44%	-7.81%	53.66%	28.71%	6.88%
6.1%	-0.1531	0.0062	21.27%	-8.45%	56.70%	30.49%	7.29%
6.3%	-0.1627	0.0066	17.09%	-9.10%	59.74%	32.27%	7.71%
6.5%	-0.1724	0.0070	12.91%	-9.74%	62.78%	34.05%	8.14%
6.7%	-0.1820	0.0074	8.74%	-10.39%	65.82%	35.83%	8.56%
6.9%	-0.1917	0.0079	4.56%	-11.03%	68.86%	37.61%	8.98%
7.1%	-0.2013	0.0083	0.38%	-11.68%	71.90%	39.39%	9.41%
7.3%	-0.2109	0.0087	-3.79%	-12.32%	74.94%	41.18%	9.84%

Table 7-(b). Asset Only Framework ; σ= 1.00%

Risk	Return
7.19%	1.48%
6.80%	1.68%
6.41%	1.88%
6.03%	2.09%
5.66%	2.29%
5.29%	2.49%
4.94%	2.70%
4.61%	2.90%
4.29%	3.10%
4.00%	3.30%
3.73%	3.51%
3.50%	3.71%
3.32%	3.91%
3.18%	4.11%
3.10%	4.32%
3.08%	4.52%
3.12%	4.72%
3.22%	4.93%
3.38%	5.13%
3.58%	5.33%
3.82%	5.53%
4.10%	5.74%
4.40%	5.94%
4.73%	6.14%
5.07%	6.35%
5.42%	6.55%
5.79%	6.75%
6.16%	6.95%
6.55%	7.16%
6.94%	7.36%
7.33%	7.56%
7.73%	7.76%
8.14%	7.97%
8.55%	8.17%
8.96%	8.37%
9.37%	8.58%
9.79%	8.78%

Figure 4-(a). Surplus Efficient Frontier (LDI) ; σ＝1.00%

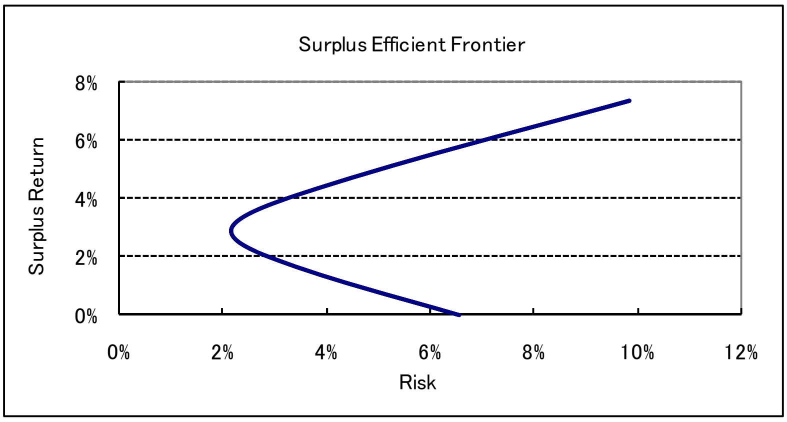


Figure 4-(b). Efficient Frontier of Asset Only ; σ＝1.00%

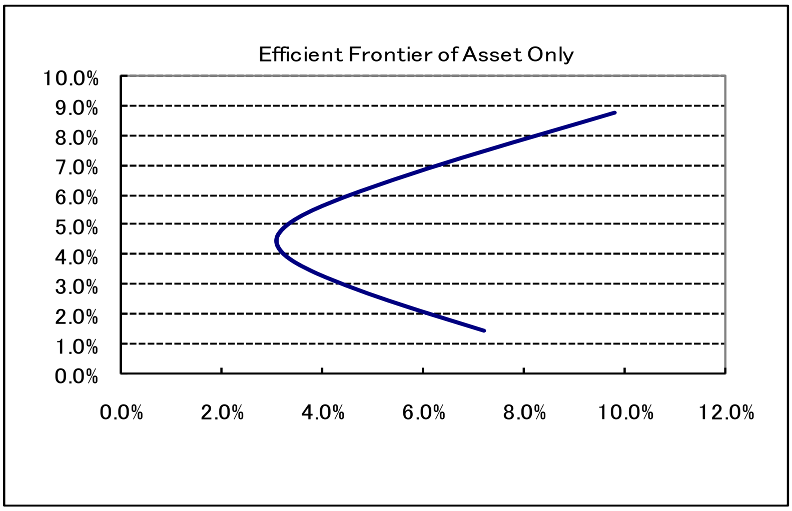


Figure 4-(c). Asset Allocation Ratio (LDI) ; σ=1.00%

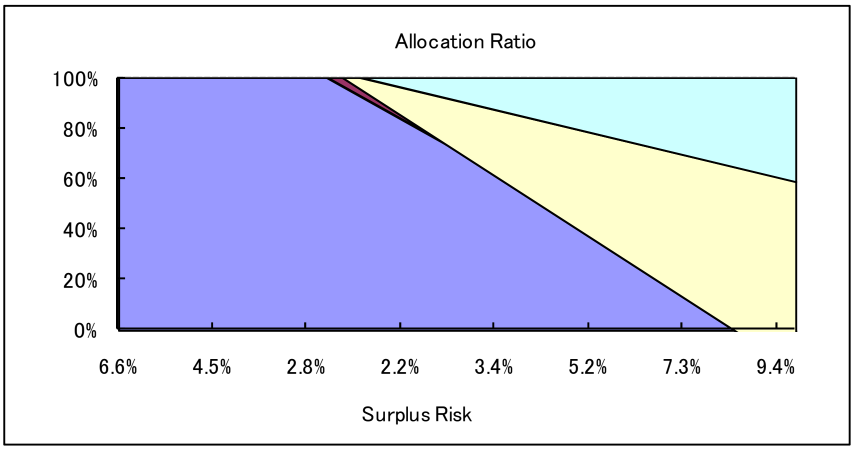


Table 8-(a). Asset Allocation in Surplus Framework (LDI) ; σ = 3.00%

Surplus Return	θ	λ	D.B.	D.E.	F.B.	F.E.	Surplus Risk
0.0%	0.1025	-0.0046	143.56%	6.98%	-31.76%	-18.79%	4.74%
0.2%	0.0945	-0.0043	140.01%	6.46%	-29.17%	-17.30%	4.37%
0.4%	0.0865	-0.0039	136.46%	5.94%	-26.59%	-15.80%	4.00%
0.6%	0.0785	-0.0036	132.90%	5.41%	-24.01%	-14.31%	3.64%
0.8%	0.0705	-0.0032	129.35%	4.89%	-21.42%	-12.81%	3.27%
1.0%	0.0625	-0.0028	125.79%	4.36%	-18.84%	-11.32%	2.90%
1.2%	0.0544	-0.0025	122.24%	3.84%	-16.26%	-9.82%	2.53%
1.4%	0.0464	-0.0021	118.69%	3.32%	-13.67%	-8.33%	2.17%
1.6%	0.0384	-0.0018	115.13%	2.79%	-11.09%	-6.83%	1.80%
1.8%	0.0304	-0.0014	111.58%	2.27%	-8.51%	-5.34%	1.44%
2.0%	0.0224	-0.0011	108.02%	1.75%	-5.93%	-3.84%	1.09%
2.2%	0.0144	-0.0007	104.47%	1.22%	-3.34%	-2.35%	0.74%
2.4%	0.0064	-0.0004	100.91%	0.70%	-0.76%	-0.85%	0.45%
2.7%	-0.0016	0.0000	97.36%	0.17%	1.82%	0.64%	0.34%
2.9%	-0.0096	0.0003	93.81%	-0.35%	4.41%	2.14%	0.56%
3.1%	-0.0176	0.0007	90.25%	-0.87%	6.99%	3.63%	0.88%
3.3%	-0.0256	0.0010	86.70%	-1.40%	9.57%	5.13%	1.23%
3.5%	-0.0336	0.0014	83.14%	-1.92%	12.16%	6.62%	1.59%
3.7%	-0.0417	0.0017	79.59%	-2.45%	14.74%	8.12%	1.95%
3.9%	-0.0497	0.0021	76.04%	-2.97%	17.32%	9.61%	2.32%
4.1%	-0.0577	0.0024	72.48%	-3.49%	19.91%	11.11%	2.68%
4.3%	-0.0657	0.0028	68.93%	-4.02%	22.49%	12.60%	3.05%
4.5%	-0.0737	0.0031	65.37%	-4.54%	25.07%	14.09%	3.42%
4.7%	-0.0817	0.0035	61.82%	-5.07%	27.66%	15.59%	3.78%
4.9%	-0.0897	0.0038	58.27%	-5.59%	30.24%	17.08%	4.15%
5.1%	-0.0977	0.0042	54.71%	-6.11%	32.82%	18.58%	4.52%
5.3%	-0.1057	0.0045	51.16%	-6.64%	35.40%	20.07%	4.89%
5.5%	-0.1137	0.0049	47.60%	-7.16%	37.99%	21.57%	5.26%
5.7%	-0.1217	0.0052	44.05%	-7.68%	40.57%	23.06%	5.63%
5.9%	-0.1297	0.0056	40.50%	-8.21%	43.15%	24.56%	5.99%
6.1%	-0.1377	0.0060	36.94%	-8.73%	45.74%	26.05%	6.36%
6.3%	-0.1458	0.0063	33.39%	-9.26%	48.32%	27.55%	6.73%
6.5%	-0.1538	0.0067	29.83%	-9.78%	50.90%	29.04%	7.10%
6.7%	-0.1618	0.0070	26.28%	-10.30%	53.49%	30.54%	7.47%
6.9%	-0.1698	0.0074	22.73%	-10.83%	56.07%	32.03%	7.84%
7.1%	-0.1778	0.0077	19.17%	-11.35%	58.65%	33.53%	8.21%
7.3%	-0.1858	0.0081	15.62%	-11.88%	61.24%	35.02%	8.58%

Table 8-(b). Asset Only Framework ; σ= 3.00%

Risk	Return
6.50%	1.49%
6.18%	1.69%
5.86%	1.89%
5.55%	2.10%
5.26%	2.30%
4.97%	2.50%
4.69%	2.70%
4.42%	2.90%
4.17%	3.10%
3.94%	3.30%
3.74%	3.50%
3.55%	3.70%
3.40%	3.90%
3.29%	4.10%
3.21%	4.30%
3.17%	4.50%
3.18%	4.70%
3.23%	4.90%
3.32%	5.11%
3.45%	5.31%
3.61%	5.51%
3.80%	5.71%
4.01%	5.91%
4.25%	6.11%
4.50%	6.31%
4.77%	6.51%
5.06%	6.71%
5.35%	6.91%
5.65%	7.11%
5.96%	7.31%
6.27%	7.51%
6.60%	7.71%
6.92%	7.91%
7.25%	8.12%
7.59%	8.32%
7.92%	8.52%
8.26%	8.72%

Figure 5-(a). Surplus Efficient Frontier (LDI) ; σ=3.00%

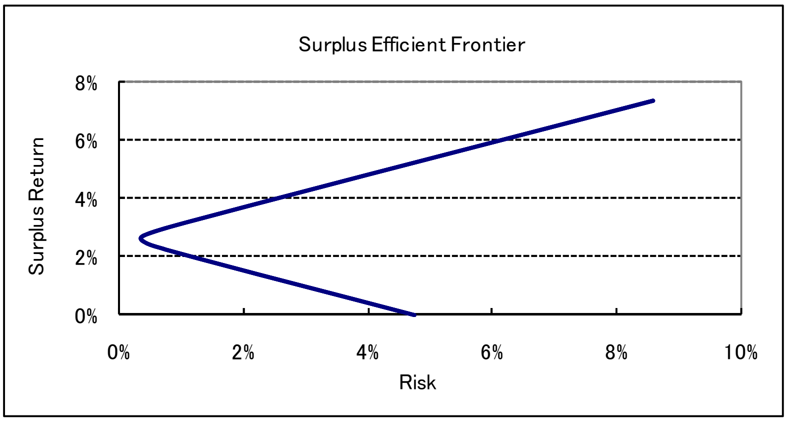


Figure 5-(b). Efficient Frontier of Asset Only ; σ＝3.00％

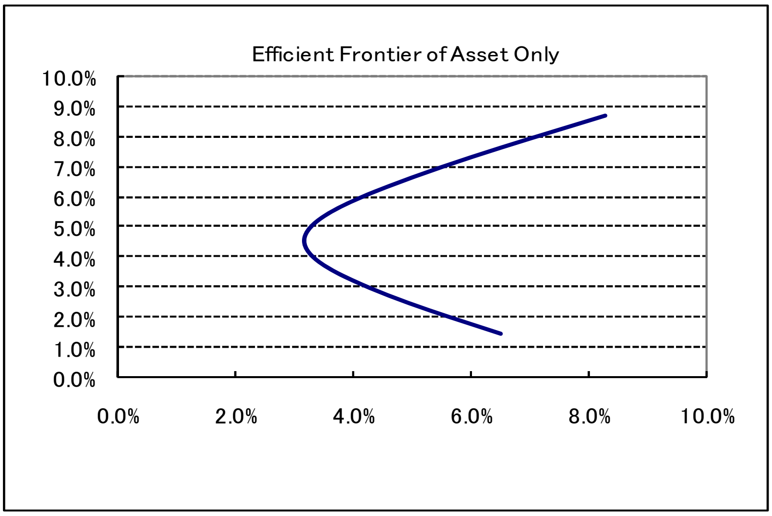


Figure 5-(c). Asset Allocation Ratio (LDI) ; σ＝3.00**%**

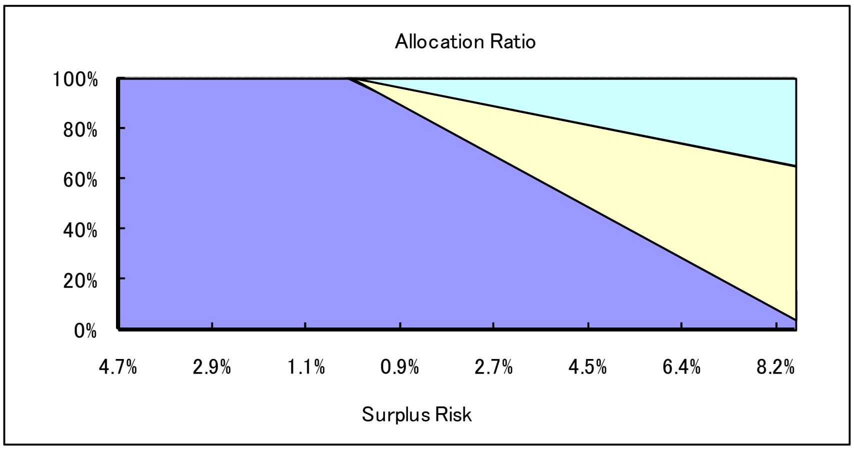


Table 9-(a). Asset Allocation in Surplus Framework (LDI) ; σ = 5.00%

Surplus Return	θ	λ	D.B.	D.E.	F.B.	F.E.	Surplus Risk
0.0%	0.0718	-0.0027	141.17%	3.35%	-29.47%	-15.05%	3.67%
0.2%	0.0658	-0.0024	138.41%	2.96%	-27.46%	-13.91%	3.41%
0.4%	0.0597	-0.0021	135.65%	2.57%	-25.46%	-12.76%	3.16%
0.6%	0.0536	-0.0019	132.88%	2.19%	-23.45%	-11.61%	2.91%
0.8%	0.0475	-0.0016	130.12%	1.80%	-21.45%	-10.47%	2.67%
1.0%	0.0414	-0.0013	127.35%	1.41%	-19.44%	-9.32%	2.44%
1.2%	0.0353	-0.0011	124.59%	1.02%	-17.44%	-8.17%	2.22%
1.4%	0.0293	-0.0008	121.83%	0.63%	-15.43%	-7.03%	2.02%
1.6%	0.0232	-0.0006	119.06%	0.24%	-13.42%	-5.88%	1.84%
1.8%	0.0171	-0.0003	116.30%	-0.15%	-11.42%	-4.73%	1.69%
2.0%	0.0110	0.0000	113.54%	-0.54%	-9.41%	-3.59%	1.57%
2.2%	0.0049	0.0002	110.77%	-0.92%	-7.41%	-2.44%	1.50%
2.4%	-0.0012	0.0005	108.01%	-1.31%	-5.40%	-1.29%	1.49%
2.7%	-0.0073	0.0008	105.24%	-1.70%	-3.40%	-0.15%	1.52%
2.9%	-0.0133	0.0010	102.48%	-2.09%	-1.39%	1.00%	1.61%
3.1%	-0.0194	0.0013	99.72%	-2.48%	0.61%	2.15%	1.74%
3.3%	-0.0255	0.0016	96.95%	-2.87%	2.62%	3.30%	1.90%
3.5%	-0.0316	0.0018	94.19%	-3.26%	4.63%	4.44%	2.09%
3.7%	-0.0377	0.0021	91.43%	-3.65%	6.63%	5.59%	2.30%
3.9%	-0.0438	0.0024	88.66%	-4.03%	8.64%	6.74%	2.53%
4.1%	-0.0498	0.0026	85.90%	-4.42%	10.64%	7.88%	2.76%
4.3%	-0.0559	0.0029	83.13%	-4.81%	12.65%	9.03%	3.01%
4.5%	-0.0620	0.0031	80.37%	-5.20%	14.65%	10.18%	3.26%
4.7%	-0.0681	0.0034	77.61%	-5.59%	16.66%	11.32%	3.51%
4.9%	-0.0742	0.0037	74.84%	-5.98%	18.66%	12.47%	3.77%
5.1%	-0.0803	0.0039	72.08%	-6.37%	20.67%	13.62%	4.04%
5.3%	-0.0863	0.0042	69.32%	-6.76%	22.68%	14.76%	4.30%
5.5%	-0.0924	0.0045	66.55%	-7.14%	24.68%	15.91%	4.57%
5.7%	-0.0985	0.0047	63.79%	-7.53%	26.69%	17.06%	4.84%
5.9%	-0.1046	0.0050	61.02%	-7.92%	28.69%	18.21%	5.11%
6.1%	-0.1107	0.0053	58.26%	-8.31%	30.70%	19.35%	5.38%
6.3%	-0.1168	0.0055	55.50%	-8.70%	32.70%	20.50%	5.66%
6.5%	-0.1228	0.0058	52.73%	-9.09%	34.71%	21.65%	5.93%
6.7%	-0.1289	0.0061	49.97%	-9.48%	36.71%	22.79%	6.21%
6.9%	-0.1350	0.0063	47.21%	-9.87%	38.72%	23.94%	6.49%
7.1%	-0.1411	0.0066	44.44%	-10.25%	40.73%	25.09%	6.76%
7.3%	-0.1472	0.0068	41.68%	-10.64%	42.73%	26.23%	7.04%

Table 9-(b). Asset Only Framework ; σ= 5.00%

Risk	Return
5.97%	1.43%
5.73%	1.63%
5.51%	1.82%
5.28%	2.02%
5.07%	2.22%
4.86%	2.42%
4.65%	2.62%
4.46%	2.82%
4.28%	3.02%
4.11%	3.22%
3.95%	3.42%
3.80%	3.62%
3.68%	3.82%
3.57%	4.02%
3.48%	4.22%
3.41%	4.42%
3.37%	4.62%
3.35%	4.82%
3.35%	5.02%
3.38%	5.22%
3.43%	5.42%
3.50%	5.62%
3.60%	5.82%
3.71%	6.02%
3.84%	6.22%
3.99%	6.42%
4.15%	6.62%
4.32%	6.82%
4.51%	7.02%
4.70%	7.21%
4.91%	7.41%
5.12%	7.61%
5.34%	7.81%
5.56%	8.01%
5.79%	8.21%
6.03%	8.41%
6.27%	8.61%

Figure 6-(a). Surplus Efficient Frontier (LDI) ; σ＝5.00％

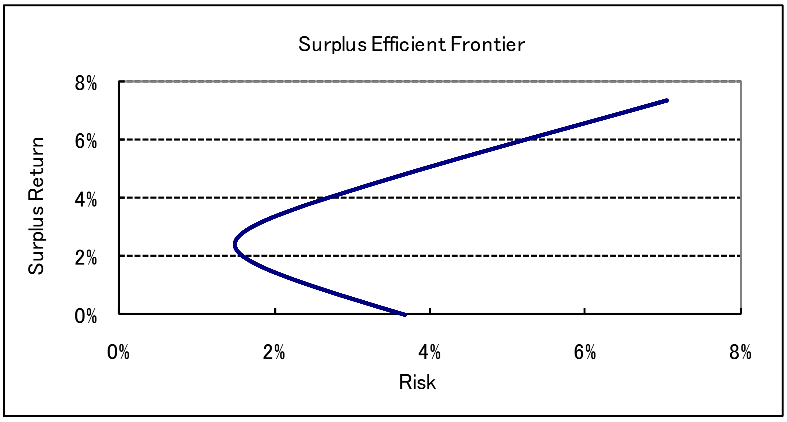


Figure 6-(b). Efficient Frontier of Asset Only ; σ=5.00%

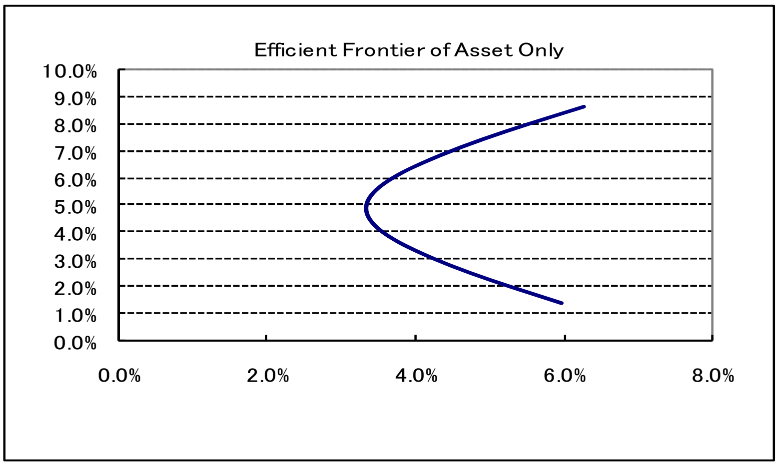
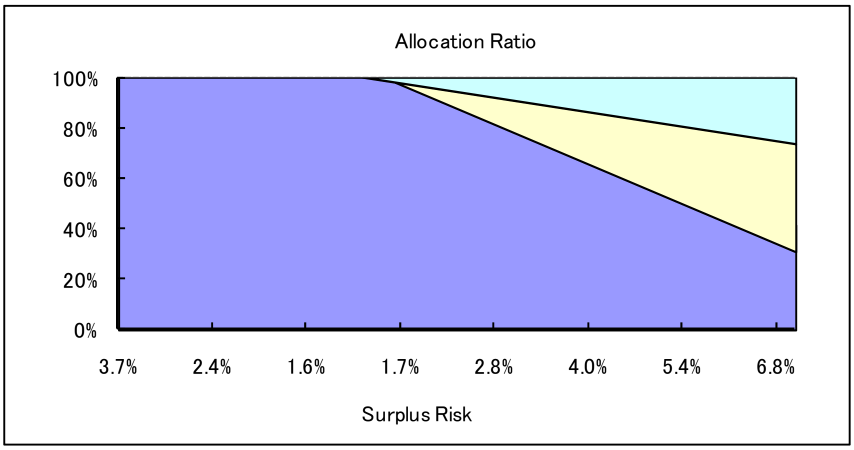


Figure 6-(c). Asset Allocation Ratio (LDI) ; σ=5.00%



4. Conclusion

We have compared surplus approach with asset only approach in this analysis. Then, we have noticed that if we consider liability with surplus approach, the shape of efficient frontier would much more different from the asset only approach.
The reason is that most riskless asset would become, not risk free assets but bonds in this case from the idea of duration matching. Therefore, the movement of efficient frontier would occur. Finally, LDI's framework would play an important role from now on. Because, the movement of evaluating the pension liability with market value are becoming stronger in Europe under the convergence of international accounting standard.

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