

# Fuzzy Logic Tutorial

2 Inputs, 9 rules

# Fuzzy Logic Tutorial

Using the given FAMM, output values and membership functions, calculate the crisp output of the fuzzy system when  $x = -0.7$  and  $y = -3$ .

Assume that the fuzzy combination operator used is the [Zadeh AND](#).

Outputs:

$NL = -5$

$NS = -2.5$

$ZE = 0$

$PS = 2.5$

$PL = 5.0$

x		
		N
y		ZE
S	NL	NS
M	NS	ZE
L	PS	PS

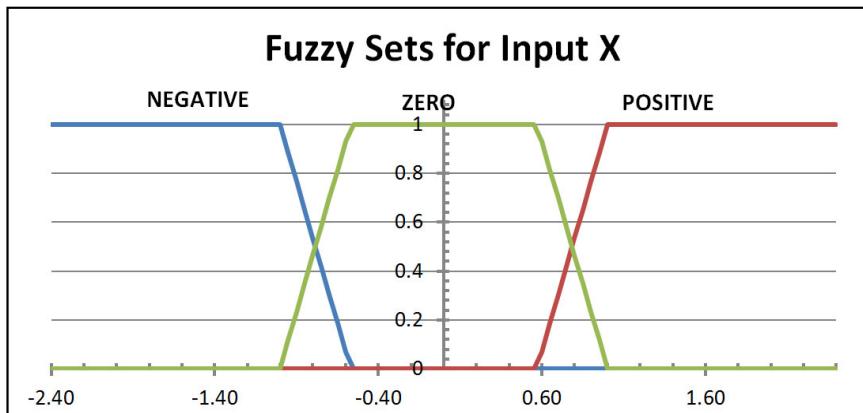
		x
		N
y		ZE
S	W1	W4
M	W2	W5
L	W3	W6

This is used only to refer to each of the rules individually

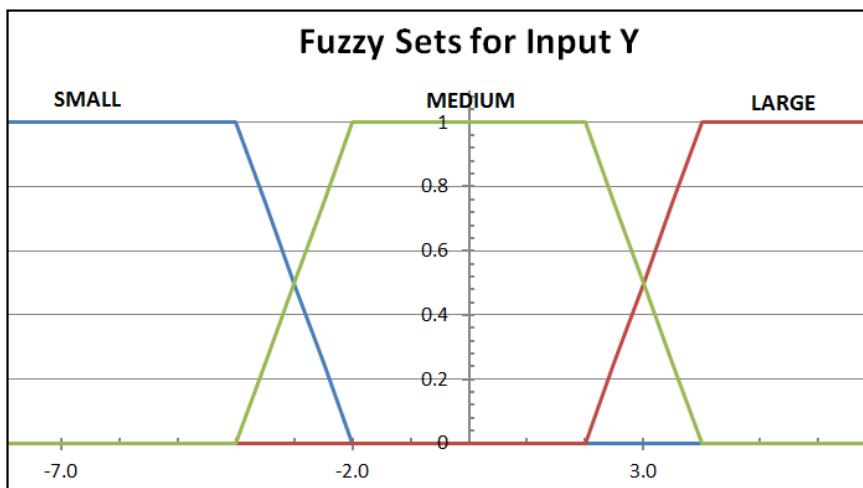
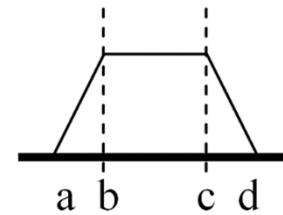
# Fuzzy Logic Tutorial

All fuzzy sets are of type trapezoidal.

		x		
		N	ZE	P
y	S	NL	NS	NS
	M	NS	ZE	PS
	L	PS	PS	PL



	a	b	c	d
NEG	0	0	-1	-0.57
ZE	-1	-0.57	0.57	1
POS	0.57	1	0	0



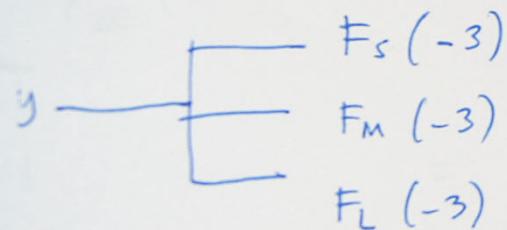
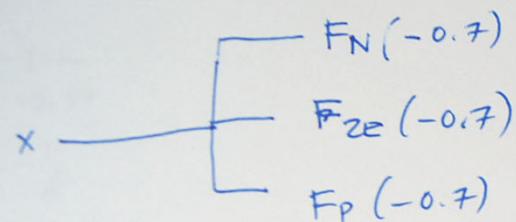
	a	b	c	d
S	0	0	-4	-2
M	-4	-2	2	4
L	2	4	0	0

# Fuzzy Logic Tutorial

## Fuzzify the inputs

$$x = -0.7$$

$$y = -3$$



$$= \max \left( \min \left( 1, \frac{x-a}{b-a}, \frac{x-d}{c-d} \right), 0 \right)$$

# Fuzzy Logic Tutorial

	a	b	c	d
NEG	0	0	-1	-0.57
ZE	-1	-0.57	0.57	1
POS	0.57	1	0	0

Fuzzify the inputs: **INPUT VARIABLE X**

$$F_{left\_trapezoid}(x) = \max\left(\min\left(1, \frac{x-d}{c-d}\right), 0\right)$$

~~Input~~

$$F_N(-0.7) = \max\left(\min\left(1, \frac{-0.7 - (-0.57)}{-1 - (-0.57)}\right), 0\right) = \frac{-0.13}{-0.43} = 0.30$$

$$F_{regular\_trapezoid}(x) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{x-d}{c-d}\right), 0\right)$$

$$F_{ZE}(x=-0.7) = \max\left(\min\left(1, \frac{-0.7 - (-1)}{-0.57 - (-1)}, \frac{-0.7 - 1}{0.57 - 1}\right), 0\right) = 0.7$$

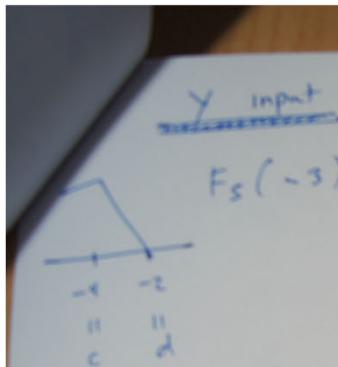
$$F_{POS}(x=-0.7) = \max\left(\min\left(1, \frac{-0.7 - 0.57}{1 - 0.57}\right), 0\right) = 0$$

$$F_{right\_trapezoid}(x) = \max\left(\min\left(1, \frac{x-a}{b-a}\right), 0\right)$$

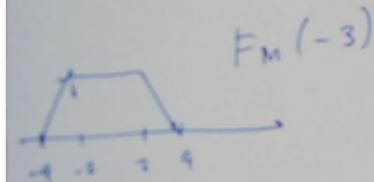
# Fuzzy Logic Tutorial

	a	b	c	d
S	0	0	-4	-2
M	-4	-2	2	4
L	2	4	0	0

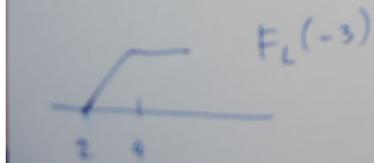
Fuzzify the inputs: **INPUT VARIABLE Y**



$$= \max \left( \min \left( 1, \frac{x-d}{c-d} \right), 0 \right) = \max \left( \min \left( 1, \frac{-3-(-2)}{-4-(-2)} \right), 0 \right) = \boxed{0.5}$$



$$= \max \left( \min \left( 1, \frac{-3-(-4)}{-2-(-4)} \right), \frac{-3-2}{2-4} \right), 0 \right) = \boxed{0.5}$$



$$= \max \left( \min \left( 1, \frac{-3-2}{4-2} \right), 0 \right) = \boxed{\emptyset}$$

$$F_{left\_trapezoid}(x) = \max \left( \min \left( 1, \frac{x-d}{c-d} \right), 0 \right)$$

$$F_{regular\_trapezoid}(x) = \max \left( \min \left( \frac{x-a}{b-a}, 1, \frac{x-d}{c-d} \right), 0 \right)$$

$$F_{right\_trapezoid}(x) = \max \left( \min \left( 1, \frac{x-a}{b-a} \right), 0 \right)$$

# Fuzzy Logic Tutorial

Compute degree of applicability of rules: **WEIGHTS**

DEFUZZIFICATION: compute crisp output using **CENTRE OF MASS**

$$F_{Neg}(X) = 0.3023256$$

$$F_{ZE}(X) = 0.6976745$$

$$F_{Pos}(X) = 0$$

$$F_{Small}(Y) = 0.5$$

$$F_{Med}(Y) = 0.5$$

$$F_{Large}(Y) = 0$$

$$W_1 = \min(0.3, 0.5) = 0.3$$

$$W_2 = \min(0.3, 0.5) = 0.3$$

$$W_3 = \min(0.3, \emptyset) = \emptyset$$

$$W_4 = \min(0.7, 0.5) = 0.5$$

$$W_5 = \min(0.7, 0.5) = 0.5$$

$$W_6 = \min(0.7, \emptyset) = \emptyset$$

$$W_7 = \min(\emptyset, 0.5) = \emptyset$$

$$W_8 = \min(\emptyset, 0.5) = \emptyset$$

$$W_9 = \min(\emptyset, 0) = 0$$

$$\text{Sum} = 1.6$$

Output =

$$R\#1 = (0.3 \times -5) = -1.5 \quad \text{certainty}$$

$$R\#2 = (0.3 \times -2.5) = -0.75 = \frac{-3.5}{1.6}$$

$$R\#3 = (\emptyset \times 2.5) = 0$$

$$R\#4 = (0.5 \times -2.5) = -1.25$$

$$R\#5 = (0.5 \times \emptyset) = 0$$

$$R\#6 = (\emptyset \times 2.5) = 0$$

$$R\#7 = (0 \times -2.5) = 0$$

$$R\#8 = (0 \times 2.5) = 0$$

$$R\#9 = (0 \times 5.0) = 0$$

x				
y	N	ZE		
	S	W1	W4	W7
	M	W2	W5	W8
L		W3	W6	W9

Outputs:  
NL=-5  
NS=-2.5  
ZE=0  
PS=2.5  
PL=5.0

x				
y	N	ZE		
	S	NL	NS	NS
	M	NS	ZE	PS
L		PS	PS	PL

# Fuzzy Logic Tutorial

A) Degree of Memberships of Input X

$F_{Neg}(X) = 0.3023256$

$F_{ZE}(X) = 0.6976745$

$F_{Pos}(X) = 0$

B) Degree of Memberships of Input Y

$F_{Small}(Y) = 0.5$

$F_{Med}(Y) = 0.5$

$F_{Large}(Y) = 0$

C) Weights

$Weight(1) = \text{Min}(F_{Neg}(X), F_{Small}(Y)) = \text{Min}(0.3023256, 0.5) = 0.3023256$

$Weight(2) = \text{Min}(F_{Neg}(X), F_{Med}(Y)) = \text{Min}(0.3023256, 0.5) = 0.3023256$

$Weight(3) = \text{Min}(F_{Neg}(X), F_{Large}(Y)) = \text{Min}(0.3023256, 0) = 0$

$Weight(4) = \text{Min}(F_{ZE}(X), F_{Small}(Y)) = \text{Min}(0.6976745, 0.5) = 0.5$

$Weight(5) = \text{Min}(F_{ZE}(X), F_{Med}(Y)) = \text{Min}(0.6976745, 0.5) = 0.5$

$Weight(6) = \text{Min}(F_{ZE}(X), F_{Large}(Y)) = \text{Min}(0.6976745, 0) = 0$

$Weight(7) = \text{Min}(F_{Pos}(X), F_{Small}(Y)) = \text{Min}(0, 0.5) = 0$

$Weight(8) = \text{Min}(F_{Pos}(X), F_{Med}(Y)) = \text{Min}(0, 0.5) = 0$

$Weight(9) = \text{Min}(F_{Pos}(X), F_{Large}(Y)) = \text{Min}(0, 0) = 0$

D) Crisp Output

Numerator/Denominator =  $-3.517442/1.604651$

CrispOutput = -2.192029

INPUT VALUES

X -0.7

Y -3

Summary of steps

