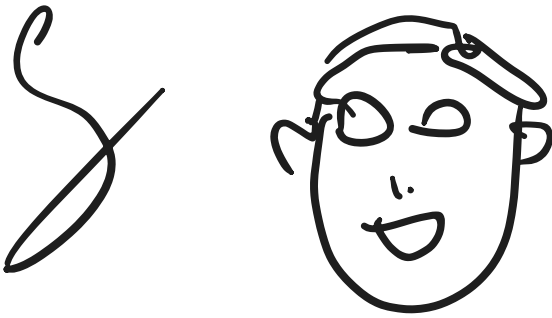


2-14 evens, 24  
2,4,6,8,10,12,14

Name: I drew a self-portrait so you can know who made this, no need to thank my art skills



In Exercises 1 through 6, find all orbits of the given permutation.

2. 
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 2 & 4 & 8 & 3 & 1 & 7 \end{pmatrix}$$

The orbits are

(1,5,8,7,1)

(2,6,3)

(4)

4.  $\sigma : \mathbb{Z} \rightarrow \mathbb{Z}$  where  $\sigma(n) = n + 1$

$$\sigma = S_{\mathbb{Z}} : \sigma(n) = n + 1, \quad \forall n \in \mathbb{Z}$$

which can be an isomorphism of  $\mathbb{Z}$  therefore the orbit of  $\sigma$

THE ORBIT IS  $\mathbb{Z}$

6.  $\sigma : \mathbb{Z} \rightarrow \mathbb{Z}$  where  $\sigma(n) = n - 3$

So we can express the problem as the following

$$\begin{pmatrix} n & n-3 & \dots \\ n-3 & n-6 & \dots \end{pmatrix}$$

We can note that this is an isomorphism of  $\mathbb{Z}$  and the following orbit is

$$3\mathbb{Z}$$

which can be extrapolated to be

$$\text{orbits of } \sigma : 3\mathbb{Z}, 1 + 3\mathbb{Z}, 2 + 3\mathbb{Z}$$

In Exercises 7 through 9, compute the indicated product of cycles that are permutations of  $\{1, 2, 3, 4, 5, 6, 7, 8\}$ .

8.  $(1, 3, 2, 7)(4, 8, 6)$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 7 & 2 & 8 & 5 & 4 & 1 & 6 \end{pmatrix}$$

In Exercises 10 through 12, express the permutation of  $\{1, 2, 3, 4, 5, 6, 7, 8\}$  as a product of disjoint cycles, and then as a product of transpositions.

10.  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$

Product of Disjoint Cycles

$$(1, 8)(3, 6, 4)(5, 7)$$

Product of Transpositions

$$(1, 8)(3, 6)(6, 4)(5, 7)$$

12.  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$

Product of Disjoint Cycles

$$(1, 3, 4, 7, 8, 6, 5, 2)$$

Product of Transpositions

$$(1, 3)(3, 4)(4, 7)(7, 8)(8, 6)(6, 5)(5, 2)$$

In Exercises 14 through 18, find the maximum possible order for an element of  $S_n$  for the given value of  $n$ .

14.  $n = 5$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

This would result in

$(1, 2, 3)(4, 5)$  which would result in an order of 6!

$$\rho_0 = (1, 2)(2, 3)$$

$$\rho_1 = (2, 3)(3, 1)$$

$$\rho_2 = (1, 3)(3, 2)$$

$$\mu_1 = (2, 3)$$

$$\mu_2 = (1, 3)$$

$$\mu_3 = (1, 2)$$

So  $\rho_{0,1,2}$  are all even!