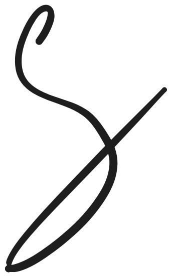
2-14 evens, 24
2,4,6,8,10,12,14
Name: I drew a self-portrait so you can know who made this, no need to thank my art skills

In Exercises 1 through 6, find all orbits of the given permutation.

2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 2 & 4 & 8 & 3 & 1 & 7 \end{pmatrix}$

The orbits are
(1,5,8,7,1)
(2,6,3)
(4)

4. $\sigma : \mathbb{Z} \to \mathbb{Z}$ where $\sigma(n) = n + 1$

$$\sigma = S_{\mathbb{Z}} \ : \ \sigma(n) = n+1, \quad \forall n \in \mathbb{Z}$$

which can be an isomorphism of $\mathbb{Z}$ therefore the orbit of σ

THE ORBIT IS $\mathbb{Z}$

6. $\sigma : \mathbb{Z} \to \mathbb{Z}$ where $\sigma(n) = n - 3$

So we can express the problem as the following

$$\begin{pmatrix} n & n-3\dots \\ n-3 & n-6\dots \end{pmatrix}$$

We can note that this is an isomorphism of $\mathbb{Z}$ and the following orbit is

$$3\mathbb{Z}$$

which can be extrapolated to be

$$\text{orbits of } \sigma \ : \ 3\mathbb{Z}, 1+3\mathbb{Z}, 2+3\mathbb{Z}$$

In Exercises 7 through 9, compute the indicated product of cycles that are permutations of $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

8. $(1, 3, 2, 7)(4, 8, 6)$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 7 & 2 & 8 & 5 & 4 & 1 & 6 \end{pmatrix}$$

In Exercises 10 through 12, express the permutation of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ as a product of disjoint cycles, and then as a product of transpositions.

10. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$

Product of Disjoint Cycles

$$(1,8)(3,6,4)(5,7)$$

Product of Transpositions

$$(1,8)(3,6)(6,4)(5,7)$$

12. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$

Product of Disjoint Cycles

$$(1,3,4,7,8,6,5,2)$$

Product of Transpositions

$$(1,3)(3,4)(4,7)(7,8)(8,6)(6,5)(5,2)$$

In Exercises 14 through 18, find the maximum possible order for an element of S_n for the given value of n.

14. $n = 5$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

This would result in

$(1,2,3)(4,5)\$\$ whichwouldresultinanorderof 6![[Pastedimage 20240305185317.png]]![[Pastedimage 20240306120852.png]]![[Pastedimage 2024030612085$

\rho_0 = (1,2)(2,3)

\rho_1 = (2,3)(3,1)

\rho_2 = (1,3)(3,2)

\mu_1 = (2,3)

\mu_2 = (1,3)

\mu_3 = (1,2)

$$So\$\rho_{0,1,2}\$arealleven![[Drawing 2024-03-0612.59.07.excalidraw]]$$