Let $\tilde{\mu}_{c,\alpha}$ be a gamma CRM, which is characterized by its Lévy intensity measure

$$\tilde{\nu}(\mathrm{d}v, \mathrm{d}y) = \frac{\mathrm{e}^{-cv}}{v}\mathrm{d}v\alpha(\mathrm{d}y)$$

with α a σ-finite measure and c a positive constant. Now consider a weighted gamma CRM $\tilde{\mu}^f_{c,\alpha}(\,\cdot\,) = \int_\cdot f(y)\tilde{\mu}_{c,\alpha}(\mathrm{d}y)$ for some measurable function f, which is characterized by its Laplace functional

$$\begin{aligned}\mathbb{E}\left[\exp\left\{-\int_{\mathbb{X}} g(y)f(y)\tilde{\mu}_{c,\alpha}(\mathrm{d}y)\right\}\right] &= \exp\left\{-\int_{\mathbb{R}^+\times\mathbb{X}}\left(1-\mathrm{e}^{-g(y)f(y)v}\right)\frac{\mathrm{e}^{-cv}}{v}\mathrm{d}v\,\alpha(\mathrm{d}y)\right\}\\ &= \exp\left\{-\int_{\mathbb{X}}\log\left(1+\frac{g(y)f(y)}{c}\right)\alpha(\mathrm{d}y)\right\}\end{aligned}$$

However the last expression coincides with the Laplace functional of a non-homogeneous gamma CRM $\tilde{\mu}_{c/f,\alpha}$ with Lévy intensity measure

$$\tilde{\nu}(\mathrm{d}v, \mathrm{d}y) = \frac{\exp\left\{-\frac{c}{f(y)}\,v\right\}}{v}\mathrm{d}v\alpha(\mathrm{d}y).$$

Hence, by the uniqueness of the Laplace transform one has the equality in distribution $\tilde{\mu}^f = \tilde{\mu}_{c/f,\alpha}$.

This result is implicitily established in Prop 3 of [1] and used and twisted also for special choices of f in [2] and [3]

[1] Nieto-Barajas, L.E., Pruenster, I. (2009). A sensitivity analysis for Bayesian nonparametric density estimators. Statistica Sinica, 19, 685-705

[2] Nieto-Barajas, L.E., Pruenster, I., Walker, S.G. (2004). Normalized Random Measures driven by Increasing Additive Processes. The Annals of Statistics, 32, 2343-2360

[3] Peccati, G., Pruenster, I. (2008). Linear and quadratic functionals of random hazard rates: an asymptotic analysis. The Annals of Applied Probability, 18, 1910-1943.