CS184A Homework Report

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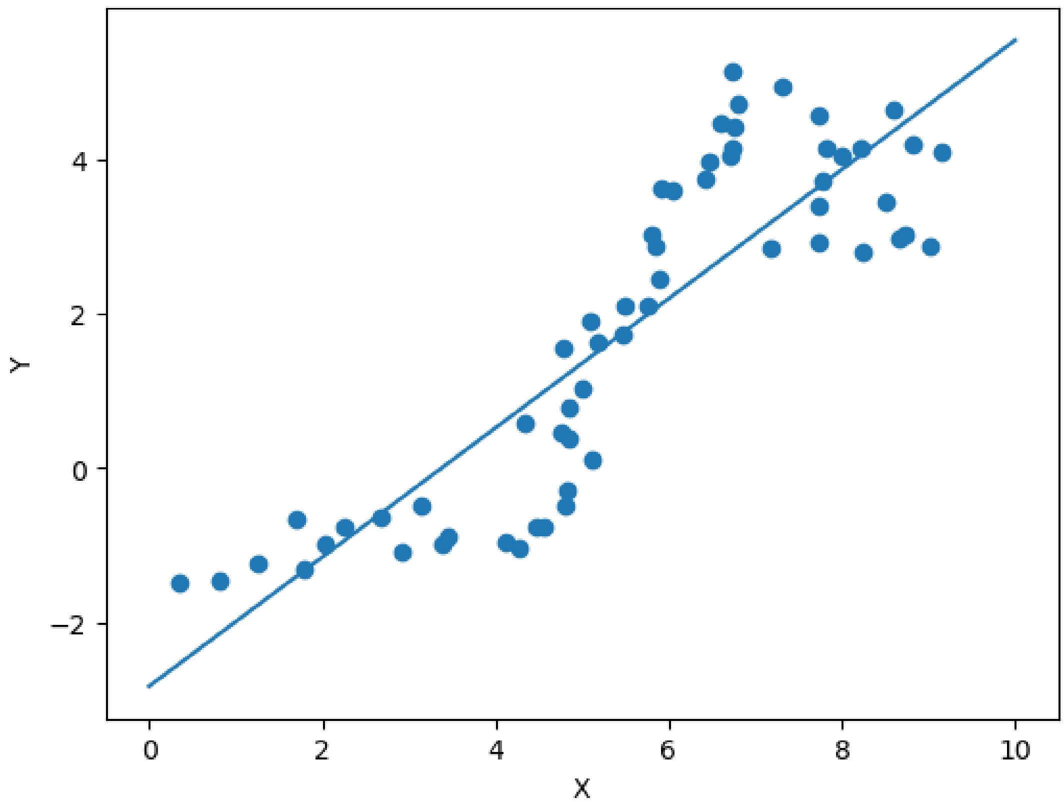
Problem 1: Linear Regression

[1]:
```
import numpy as np
import mltools as ml
import matplotlib.pyplot as plt

data = np.genfromtxt("data/curve80.txt", delimiter=None)

X = data[:,0]
X = np.atleast_2d(X).T
Y = data[:,1]
Xtr,Xte,Ytr,Yte = ml.splitData(X,Y,0.75)

print('Shapes for the objects:\n',
      Xtr.shape,Xte.shape,Ytr.shape,Yte.shape)
```

```
Shapes for the objects:
 (60, 1) (20, 1) (60,) (20,)
```

[2]:
```
lr = ml.linear.linearRegress(Xtr, Ytr)
xs = np.linspace(0,10,200)
xs = xs[:,np.newaxis]
ys = lr.predict(xs)

plt.scatter(Xtr,Ytr)
plt.plot(xs,ys)
plt.xlabel('X')
plt.ylabel('Y')
```

[2]:
```
Text(0, 0.5, 'Y')
```



[3]:
```
lr.theta
```

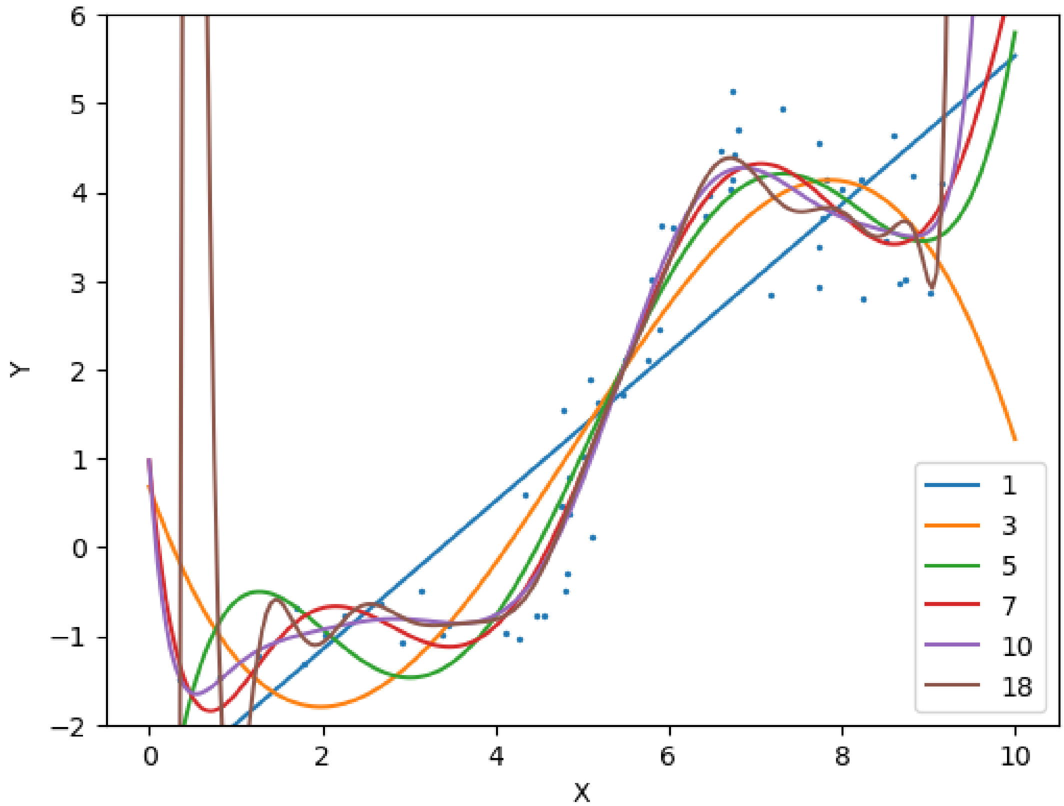
[3]:
```
array([[-2.82765049,  0.83606916]])
```

[4]:
```
YhatTr = lr.predict(Xtr)
mseTr = np.mean(
      np.square(np.subtract(YhatTr,Ytr[:,np.newaxis])))

YhatTe = lr.predict(Xte)
mseTe = np.mean(
      np.square(np.subtract(YhatTe,Yte[:,np.newaxis])))

print('Training MSE:',mseTr,'\nTest MSE:',mseTe)
```

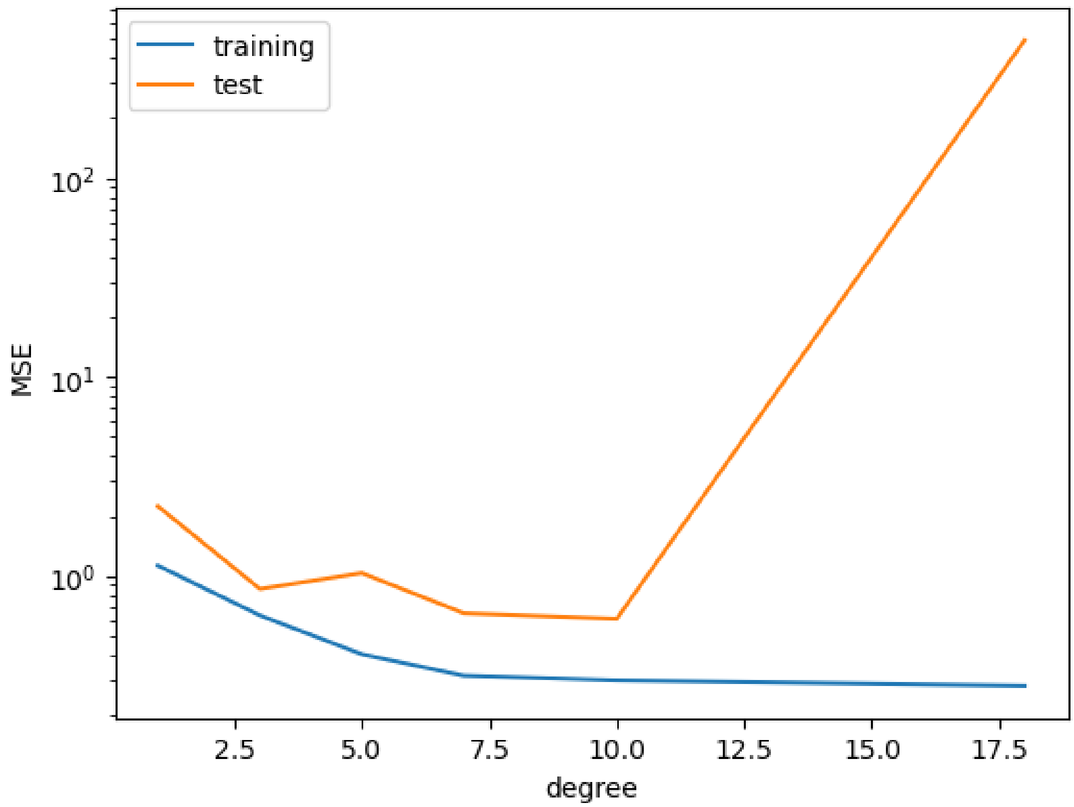
```
Training MSE: 1.127711955609391
Test MSE: 2.242349203010125
```

[5]:
```
xs = np.linspace(0,10,200)[:,np.newaxis]

def train_lr(degree,xs):
      XtrP = ml.transforms.fpoly(Xtr, degree, bias=False)
```

```
    XtrP,params = ml.transforms.rescale(XtrP)
    lr = ml.linear.linearRegress(XtrP, Ytr)

    XteP,_ = ml.transforms.rescale(ml.transforms.fpoly(Xte,degree,False),␣
↪params)

    YhatTr = lr.predict(XtrP)
    YhatTe = lr.predict(XteP)

    mseTr = np.mean(
    np.square(np.subtract(YhatTr,Ytr[:,np.newaxis])))

    mseTe = np.mean(
    np.square(np.subtract(YhatTe,Yte[:,np.newaxis])))

    xsP,_ = ml.transforms.rescale(
        ml.transforms.fpoly(xs, degree, bias=False),
        params)
    ys = lr.predict(xsP)

    return ys, mseTr, mseTe

ys1, mseTr1, mseTe1 = train_lr(1,xs)
ys3, mseTr3, mseTe3 = train_lr(3,xs)
ys5, mseTr5, mseTe5 = train_lr(5,xs)
ys7, mseTr7, mseTe7 = train_lr(7,xs)
ys10, mseTr10, mseTe10 = train_lr(10,xs)
ys18, mseTr18, mseTe18 = train_lr(18,xs)

plt.plot(xs,np.concatenate((
    ys1,ys3,ys5,ys7,ys10,ys18
),axis=1),label = (1,3,5,7,10,18))
plt.scatter(Xtr,Ytr,s=2)
plt.legend(loc='lower right')
plt.ylim((-2,6))
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```



[6]:
```
plt.semilogy((1,3,5,7,10,18),
             (mseTr1,mseTr3,mseTr5,mseTr7,mseTr10,mseTr18),
             label = 'training')
plt.semilogy((1,3,5,7,10,18),
             (mseTe1,mseTe3,mseTe5,mseTe7,mseTe10,mseTe18),
             label = 'test')
plt.xlabel('degree')
plt.ylabel('MSE')
plt.legend(loc='upper left')
plt.show()
```



I recommend degree=10, which makes the model to have the lowest test error.

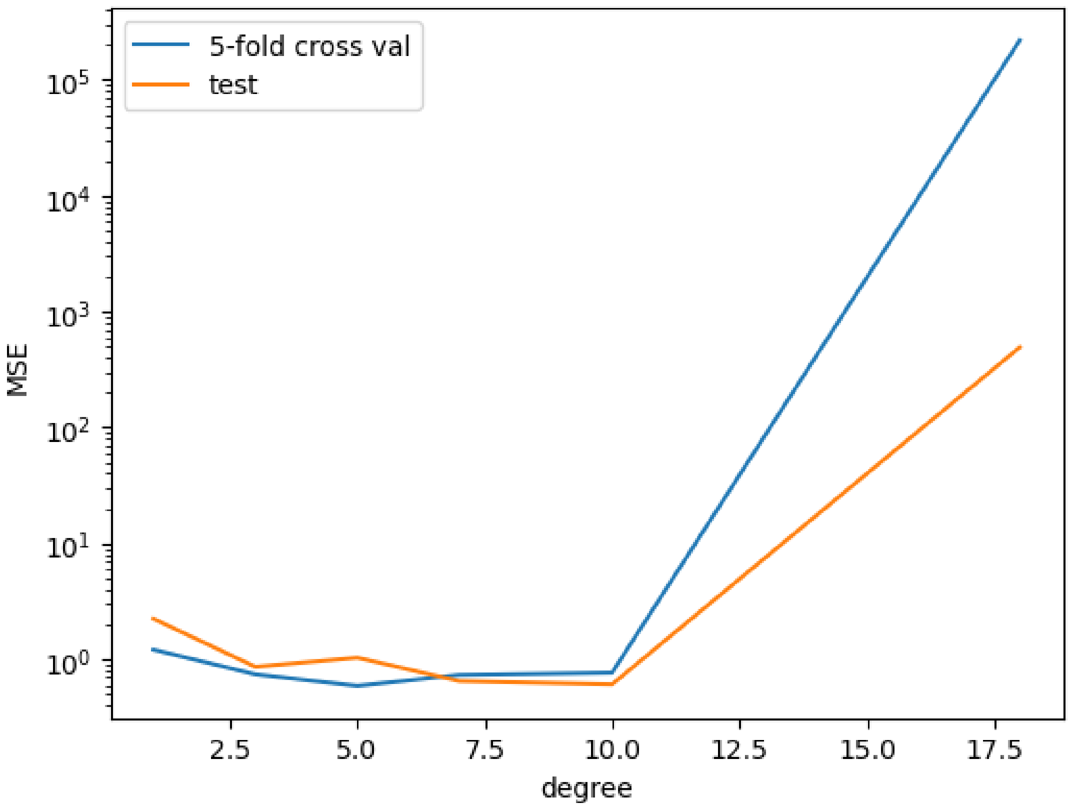
Problem 2: Cross-validation

[7]:
```
nFolds = 5;
J = np.zeros(5)
for iFold in range(nFolds):
      Xti,Xvi,Yti,Yvi = ml.crossValidate(Xtr,Ytr,nFolds,iFold)
      learner = ml.linear.linearRegress(Xti,Yti)
      YhatVi = learner.predict(Xvi)
      J[iFold] = np.mean(
      np.square(np.subtract(YhatVi,Yvi[:,np.newaxis])))
print (np.mean(J))
```

```
1.2118626629641982
```

[8]:
```
def train_lr_cv(degree,nFolds):
      J = np.zeros(nFolds)

      for iFold in range(nFolds):
            Xti,Xvi,Yti,Yvi = ml.crossValidate(Xtr,Ytr,nFolds,iFold)
```

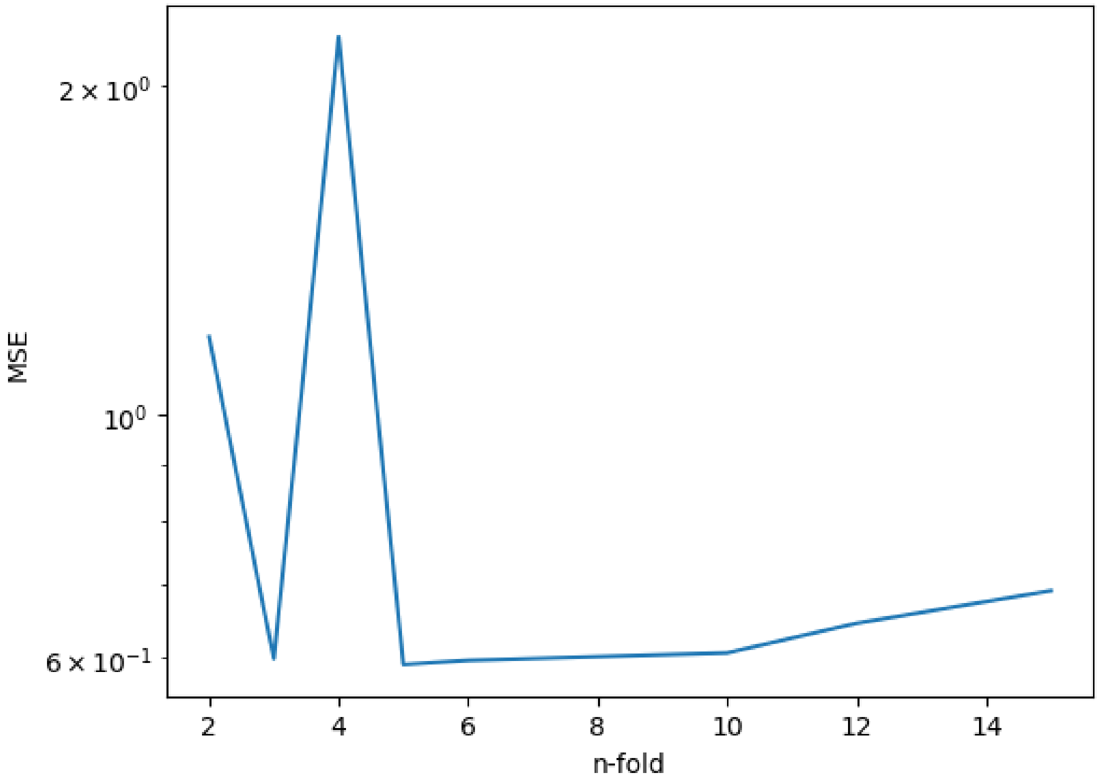
```
            XtiP = ml.transforms.fpoly(Xti, degree, bias=False)
            XtiP,params = ml.transforms.rescale(XtiP)
            lr = ml.linear.linearRegress(XtiP,Yti)

            XviP,_ = ml.transforms.rescale(ml.transforms.
↪fpoly(Xvi,degree,False), params)
            YhatVi = lr.predict(XviP)

            J[iFold] = np.mean(
            np.square(np.subtract(YhatVi,Yvi[:,np.newaxis])))

      return np.mean(J)

mseVi1 = train_lr_cv(1,5)
mseVi3 = train_lr_cv(3,5)
mseVi5 = train_lr_cv(5,5)
mseVi7 = train_lr_cv(7,5)
mseVi10 = train_lr_cv(10,5)
mseVi18 = train_lr_cv(18,5)

plt.semilogy((1,3,5,7,10,18),
             (mseVi1,mseVi3,mseVi5,mseVi7,mseVi10,mseVi18),
             label = '5-fold cross val')
plt.semilogy((1,3,5,7,10,18),
             (mseTe1,mseTe3,mseTe5,mseTe7,mseTe10,mseTe18),
             label = 'test')
plt.xlabel('degree')
plt.ylabel('MSE')
plt.legend(loc='upper left')
plt.show()
```



5-fold cross validation error and test error share similar behavior, both decrease firstly and then increase quickly once passed degree=10.

Based on 5-fold cross validation error, degree=5 is mostly recommended.

[9]:
```
mseVi5f2 = train_lr_cv(5,2)
mseVi5f3 = train_lr_cv(5,3)
mseVi5f4 = train_lr_cv(5,4)
mseVi5f5 = train_lr_cv(5,5)
mseVi5f6 = train_lr_cv(5,6)
mseVi5f10 = train_lr_cv(5,10)
mseVi5f12 = train_lr_cv(5,12)
mseVi5f15 = train_lr_cv(5,15)

plt.semilogy((2,3,4,5,6,10,12,15),
             (mseVi5f2,mseVi5f3,mseVi5f4,mseVi5f5,
              mseVi5f6,mseVi5f10,mseVi5f12,mseVi5f15))
plt.xlabel('n-fold')
plt.ylabel('MSE')
plt.show()
```



The n-fold cross validation error has a fluctuating behavior when n<5, then it increases slightly. The error becomes stable as n becomes larger because more data can be used for training purpose in each fold but eventually increases because fewer data are assigned to be validation data, because of higher variance and lower confidence.

Statement of Collaboration

This homework was done completely by Jiachen Sun without collaboration.