Midium Project 2
Galaxy Luminosity Function and Star Formation Efficiency

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November 10, 2020

Abstract

The luminosity of galaxy is strongly tight to its stellar mass by the mass-to-light ratio. However, the luminosity of the galaxy and the mass of its host dark matter halo are less correlated. Recent studies revealed that both Halo Mass Function (HMF) and galaxy Luminosity Function (LF) can be well described by the Press-Schechter theory but with different power-law indices. In this work, I take advantage of the New York University Value-Added Galaxy Catalog (NYU-VAGC) from Sloan Digital Sky Survey (SDSS) Data Release 2 to estimate the LF of low-redshift galaxies ($10\ h^{-1}\,\mathrm{Mpc} < d < 150\ h^{-1}\,\mathrm{Mpc}$). I find that the HMF and LF have similar slopes at $M_r - 5\log_{10} h = -20.5$, revealing a maximum star formation efficiency. For either fainter (less massive) or brighter (more massive) galaxies (halos), the HMF tend to overestimate the LF.

Keywords: galaxies: luminosity function, mass function – galaxies: haloes – dark matter

1 Introduction

As one of the fundamental parameters of galaxies, the luminosity function (LF) has been studied by many researchers via multiple methods (e.g., Press & Schechter, 1974; Schechter, 1976; Blanton et al., 2003, 2005b). Recent studies showed that both the LF of galaxies and the dark matter Halo Mass Function (HMF) can be well described by the Press & Schechter (1974) theory but with different slopes. This difference is caused by a non-constant mass-to-light ratio depending on halo mass (or luminosity of galaxy).

The Sloan Digital Sky Survey (SDSS, Abazajian et al., 2003) has has largely revolutionized our understanding of the extra galactic universe. In this work, I take advantage of the low-redshift data set from a subset of SDSS Data Release 2 Abazajian et al. (2004), the New York

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University Value-Added Galaxy Catalog (NYU-VAGC, Blanton et al., 2005a), to produce the LF of low-redshift galaxies with 10 h^{-1} Mpc $< d <$ 150 h^{-1} Mpc. The rest of the paper is organized as below. In Sec. 2, I describe my methodology in greater detail. I then post my major results in Sec. 3. Finally, I summarize the key points of this work in Sec. 4.

2 Method

2.1 Schechter Function

Based on the Press & Schechter (1974) theroy, the Schechter (1976) function takes the form,

$$\Phi(m)\ dm = \phi_* \left(\frac{m}{m_*}\right)^\alpha e^{-m/m_*}\ \frac{dm}{m_*}, \tag{1}$$

where m can be replaced by L for the LF. Such function comes from the Gaussian nature of matter density perturbation: the universe right after matter domination is almost uniform with Gaussian perturbations. Following Press & Schechter (1974), I assume the density perturbation of a patch of dark matter, $\delta(m)$ (where m denotes the mass of the patch), follows a Gaussian distribution, $\delta(m) \sim N[0, \sigma(m)]$, or $\nu(m) \sim N(0,1)$ for $\nu(m) = \delta(m)/\sigma(m)$. The probability of this patch with perturbation greater than a certain value, δ_c, is given by

$$P(\delta > \delta_c|m) = \frac{1}{\sqrt{2\pi}} \int_{\delta_c/\sigma(m)}^{\infty} e^{-x^2/2}\ dx. \tag{2}$$

Assuming a patch will collapse into a dark matter halo if $\delta(m) > \delta_c$, the total mass of halos between $m \sim (m + dm)$ is proportional to

$$\begin{aligned} F(m)\ dm \propto& \left|P(\delta > \delta_c|m + dm) - P(\delta > \delta_c|m)\right| & (3)\\ =&\left|\frac{dP(\delta > \delta_c|m)}{dm}\right| dm & (4)\\ =&\frac{1}{\sqrt{2\pi}}\left[\frac{d}{dm}\frac{\delta_c}{\sigma(m)}\right] e^{-\delta_c^2/2\sigma^2(m)}\ dm. & (5) \end{aligned}$$

Recent studies have suggested that the initial perturbation of matter follows a Gaussian random field with power-law spectrum, $\mathcal{P}(k) \propto k^n$. Since $\sigma^2(m)$ is the variance of a "low-pass filtered" Gaussian random field, we have

$$\sigma^2(m) = \xi_{k<k_{\rm max}}(0) = \frac{1}{(2\pi)^3}\int_{k<k_{\rm max}} \mathcal{P}(k)\ d^3\mathbf{k} \propto \frac{1}{(2\pi)^3}\int_0^{k_{\rm max}} k^{n+2}\ dk = \frac{k_{\rm max}^{n+3}}{(n+3)(2\pi)^3}, \tag{6}$$

for $n > -3$, where $\xi_{k<k_{\rm max}}$ is the auto-correlation function of the "low-pass filtered" field, and $k_{\rm max}$ is the upper limit of the "low-pass filter",

$$k_{\rm max} = \frac{2\pi}{R(m)} = 2\pi\left(\frac{4\pi\bar{\rho}}{3m}\right)^{1/3}. \tag{7}$$

We thus get

$$\sigma^2(m) \propto m^{-(n+3)/3}, \ \therefore \sigma^2(m) = \left(\frac{m}{m_*}\right)^{-(n+3)/3}. \tag{8}$$

Plugging this into Eq. (5) gives

$$F(m)\ dm \propto \left(\frac{m}{m_*}\right)^{(n-3)/6} \exp\left[-\frac{\delta_c^2}{2}\left(\frac{m}{m_*}\right)^{(n+3)/3}\right]\ dm. \tag{9}$$

Therefore, the HMF is given by

$$\Phi(m)\ dm = \frac{F(m)}{m}\ dm = \left(\frac{m}{m_*}\right)^{(n-9)/6} \exp\left[-\frac{\delta_c^2}{2}\left(\frac{m}{m_*}\right)^{(n+3)/3}\right]\ dm. \tag{10}$$

For $n > -3$, this form of HMF has a power-law region at low mass end and a exponential cutoff at high mass end. Schechter (1976) suggested that a simplified form, i.e., Eq. (1), is good enough to describe the realistic HMF and LF.

2.2 Data Specification

I collect data of low-redshift galaxies, including absolute magnitudes, Petrosian half-light surface brightness, etc., form the New York University Value-Added Galaxy Catalog (NYU-VAGC, Blanton et al., 2005a) [1]. The low-redshift dataset of NYU-VAGC covers 2221 $\deg^2$ of the sky in the range, $10\ h^{-1}\,\mathrm{Mpc} < d < 150\ h^{-1}\,\mathrm{Mpc}$. The detectability is limited to a maximum r-band apparent magnitude, $r < 17.7$. NYU-VAGC has 49967 galaxies in total with valid r-band absolute magnitudes, which are K-corrected to fix the bias caused by redshift. To give a intuition of the sample galaxies, I plot the color-magnitude diagram (CMD) in Fig. 1.

2.3 Volume Correction

As mentioned above, NYU-VAGC is an apparent magnitude limited sample of galaxies. Thus, the effective volume of galaxies with absolute magnitude M_r is

$$V_{\text{eff}}(M_r) = \frac{4\pi}{3}(R_{\text{eff}}^3 - R_{\text{min}}^3), \tag{11}$$

where $R_{\text{eff}} = \min(d_{\text{eff}}, R_{\text{max}})$, and d_{eff} is given by

$$\frac{d_{\text{eff}}(M_r)}{h^{-1}\,\mathrm{Mpc}} = 10^{0.2(17.7-M_r)-5}. \tag{12}$$

[1] `https://sdss.physics.nyu.edu/vagc/`

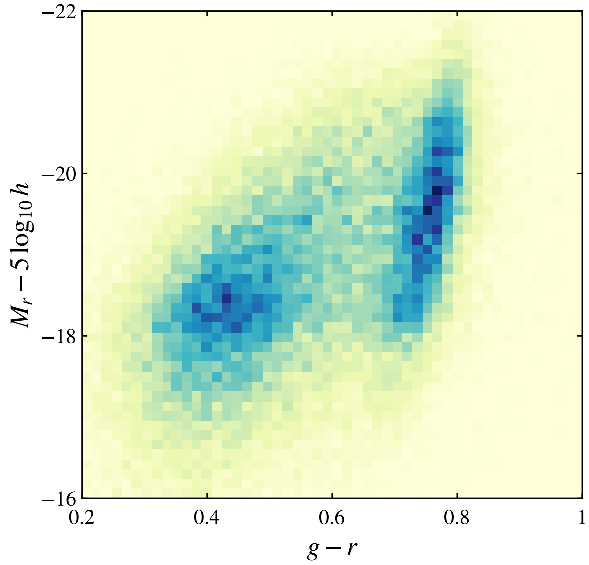

Figure 1: CMD of low-redshift galaxies with 10 h^{-1} Mpc $< d <$ 150 h^{-1} Mpc.

Note that I do not include K-correction, which is a fairly good approximation because the redshifts of all sample galaxies are low (less than 0.05), meaning that K-correction is not significant. Defining the volume correction factor,

$$\eta(M_r) = \frac{V}{V_{\rm eff}} = \frac{R_{\rm max}^3 - R_{\rm min}^3}{R_{\rm eff}^3 - R_{\rm min}^3}, \tag{13}$$

the number density of galaxies with absolute magnitude M_r is thus

$$\Phi(M_r)\ dM_r \propto \eta(M_r)N(M_r)\ dM_r = \frac{R_{\rm max}^3 - R_{\rm min}^3}{R_{\rm eff}^3 - R_{\rm min}^3}N(M_r)\ dM_r, \tag{14}$$

Where $R_{\rm min} = 10\ h^{-1}$ Mpc and $R_{\rm max} = 150\ h^{-1}$ Mpc limit the size of the range of observation. The lower limit, $R_{\rm min}$, also requires $M_r < -12.3$ for this analysis. Fainter galaxies will have $\eta < 0$ and be not included in the sample.

3 Results

3.1 Galaxy Luminosity Function

I apply two ways to produce the LF: 1) The histogram method and 2) the Gaussian kernel density estimation (KDE) method. The former method first bins all galaxies by M_r and then counts the number of galaxies within each bin. The final step is to correct the LF by the volume correction factor, η. The Gaussian KDE method view each galaxy as a Gaussian distribution in the M_r space, and the summation of all Gaussian distributions can be regarded as an estimation of the real LF of galaxies. In order to take into account the volume correction, the weights of each galaxy is set to η. Note that both the histogram and KDE methods have a free parameter: the width of bins/kernels: a larger width refers to a “smoother” LF curve. In this work, I simply select a bin/kernel width of 0.2 dex (the kernel width is the two times standard deviation of Gaussian distribution). The LFs obtained from the two methods are shown in Fig. 2. The two methods do not show significant difference, while the KDE curve is smoother than the histogram curve. Both galaxy LFs show power-law decline in faint end and exponential decline in bright end. The exponential cutoff is at $M_r \sim -20$. I also note that power-law range is divided into two parts: the curve at $M_r > -16$ is systematically steeper than $M_r < -16$.

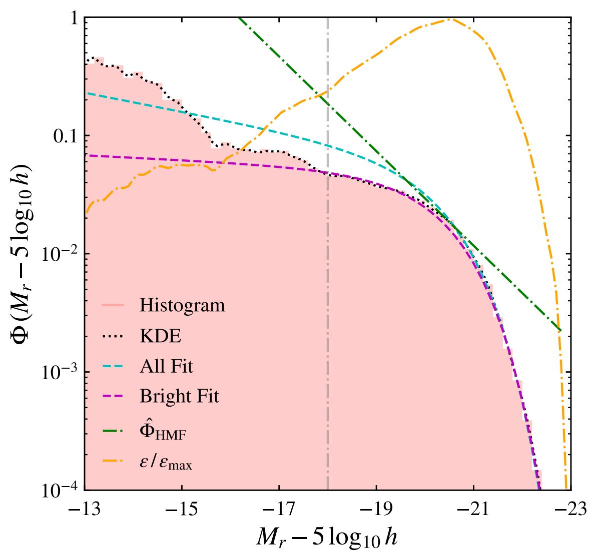

Figure 2: LFs of galaxies obtained by histogram (red shaded region) and KDE (black dotted) methods. Note that the variable here is magnitude M_r instead of luminosity. The best-fit Schechter function for all galaxies are shown as cyan dashed curve, while that for bright galaxies only (brighter than $M_r - 5\log_{10} h = -18$, i.e., the gray dash-dotted line) is shown as magenta dashed curve. The re-scaled HMF and SFE are plotted as green and orange curves, respectively. See main body for more details.

The best-fit Schechter functions (Eq. (1)) are over-plotted to Fig. 2. For all galaxies the best-fit parameters are

$$\alpha = -1.20,\ M_* = -20.4. \tag{15}$$

I also give here the best-fit parameters for bright galaxies with $M_r - 5\log_{10} h < -18$,

$$\alpha = -1.05,\ M_* = -20.4. \tag{16}$$

Both fitting parameters reveal $M_* = -20.4$, corresponding to the location of the exponential cutoff. Our results is consistent with (Blanton et al., 2003). They suggested $\alpha = -1.05$ and $M_* = -20.44$ for $z = 0.1$ galaxies in r-band. However, the Schechter function well describes the bright end of LF, but the faint end cannot be well fitted. One of the solutions is to use the double Schechter function to better characterize the faint end of LF (e.g. Blanton et al., 2005b).

3.2 Star Formation Efficiency

Many works suggested that the HMF also follows a Press-Schechter scheme, but with different power-law indices and exponential cutoffs. However, the power-law range for HMF is much broader than the LF. Therefore, the HMF can be simply seen as power-law function, corresponding to a Schechter function with $m_* \to \infty$. In this work, I will assume that slope of the HMF is $\alpha = -2$, and the stellar mass-to-light ratio (note, this is *not* the halo mass-to-light ratio) is constant.

The star formation efficiency (SFE, ϵ) is defined as the ratio of stellar mass to halo mass. However, the stellar mass-to-light ratio and the magnitude of HMF remain unknown. To solve the problems, 1) I convert halo mass to the re-scaled luminosity (absolute magnitude), which is defined as the halo mass divided by the stellar mass-to-light ratio. The physical meaning of the re-scaled luminosity is the luminosity of a halo if all matters are as luminous as stars. 2) I define the maximum SFE as $\epsilon_{\rm max}$, and define the re-scaled SFE as

$$\hat{\epsilon}(M_r) = \epsilon(M_r)/\epsilon_{\rm max}. \tag{17}$$

So that re-scaled HMF satisfies

$$\hat{\epsilon}(M_r)\hat{\Phi}_{\rm HMF}(M_r) = \Phi(M_r). \tag{18}$$

Through this definition, I can perform the following analysis in the luminosity space as well as get rid of the magnitude of HMF. The re-scaled HMF is over-plotted in Fig. 1. For either fainter (less massive) or brighter (more massive) galaxies (halos), the HMF tend to overestimate the LF. Additionally, the re-scaled SFE, estimated via Eq. (18), is also plotted in Fig. 1. The re-scaled SFE peaks at $M_r = 20.5$. Since the re-scaled SFE and the real SFE only differ by a constant $\epsilon_{\rm max}$, it is reasonable to say that the real SFE also peaks at $M_r = 20.5$.

4 Summary

In this work, I produce the LF of low-redshift galaxies with $10\ h^{-1}\,\rm Mpc < d < 150\ h^{-1}\,Mpc$ using NYU-VAGC. I find that the LF follows a Schechter function, which is power-law at faint end and exponential at bright end. The parameters of the Schechter function is $\alpha = -1.20,\ M_* = -20.4$ for all galaxies and $\alpha = -1.05,\ M_* = -20.4$ for bright galaxies only, which is consistent with previous works (e.g. Blanton et al., 2003). I also generate the re-scaled SFE at different halo masses by assuming a power-law HMF. I find that the SFE peaks at $M_r = 20.5$, corresponding the luminosity of a Milky Way-like galaxy.

For those who finally reach here: I am trying soooo hard to explain my ideas. Emmm, it's my second language you know. I'm quiet sure you find it painful reading this paper. If so, feel free to contact me!

5 Supplementary Materials

Source code of this work can be found in `https://github.com/EnthalpyBill/ASTRO533/tree/master/project_mid2`.

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