2.22

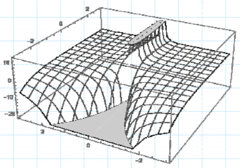
Monday, February 22, 2021 1:29 AM

There is a survey for the tutorial. Please spend 5 min to have a look on it.

In physics, an **equipotential line** is a curve which defines a constant potential (e.g., gravitational potential, electric potential) in 2D space. It is a well-known fact that its corresponding **field line** (e.g., gravitational field, electric field) is **always perpendicular** to it where they intersect in space under static conditions.

Now suppose we have a family of field lines of a system given by the equation $x = ky^2e^x$ for any constant $k$.

1. How would you relate the slopes of two perpendicular lines?
2. Give an expression for the slopes of the family of field lines in terms of $x$ and $y$.
3. Find an equation that represents the corresponding family of equipotential lines.



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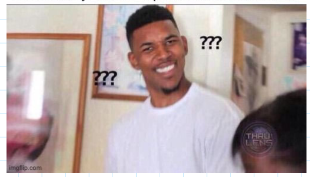
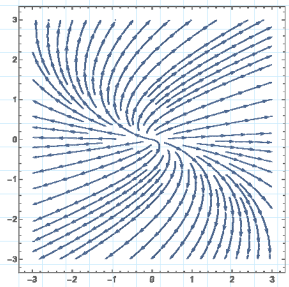




Suppose an object with mass $m$ is dropped from rest in an environment with an acceleration due to gravity $g$ and an air resistance force proportional to the object's velocity (assume a constant of proportionality $b$).

1. Find a differential equation for the velocity $v(t)$ of the object.
2. Solve the above differential equation for $v(t)$.
3. By considering $\frac{dv}{dm}$, conclude whether heavier objects fall faster or slower.

$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} \lambda & c \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \begin{cases} \lambda = 2 \\ c = 2. \end{cases}$



a linear ODE

Sheep vs. rabbit model.

$y$ $x$

nonlinear

$$\begin{cases} \dot{x} = x(3 - x - 2y) \\ \dot{y} = y(2 - y - x) \end{cases}$$

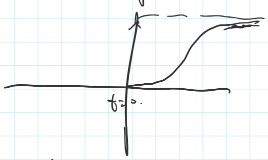
Linear ODE

$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

$\dot{X} = AX$ no

$\dot{x} = x(3 - x)$ logistic model for 1-dimension.

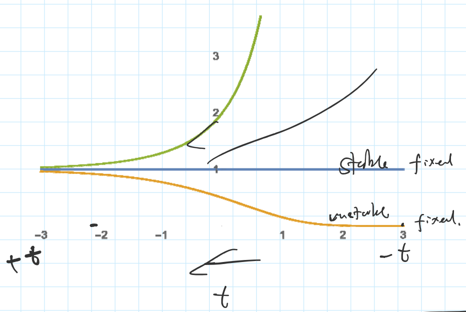
fixed point



for fixed point $x_0$

$x_0(3 - x_0) = 0.$ $\iff$ $\dot{x} = 0$ $x$ independents on $t$.

$\Rightarrow \begin{cases} x_0 = 0 & \leftarrow \text{unstable fixed pt.} \\ x_0 = 3 & \leftarrow \text{stable. fixed pt.} \end{cases}$




birth by rabbit by sheep

$$\begin{cases} \dot{x} = x(3 - x - 2y) \\ \dot{y} = y(2 - y - x) \end{cases}$$

① fixed pt. ② study fixed pt.

stable?

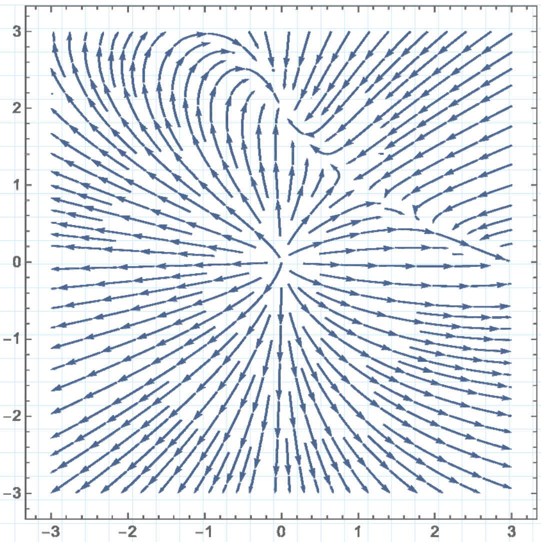
unstable?

saddle?

③ result. → one of them all dead.

only one group left.

— except only one special case. ↑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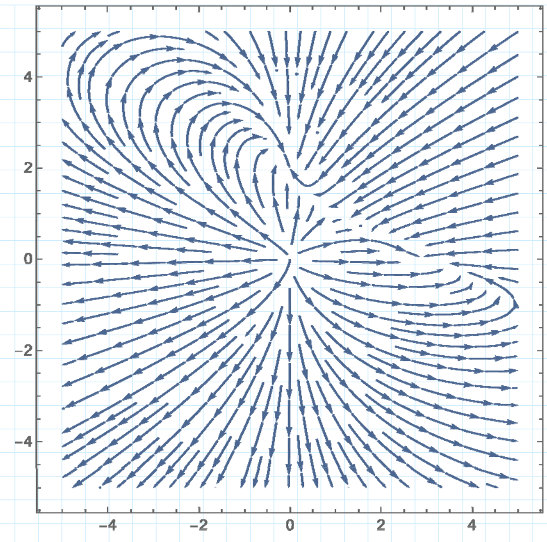
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