1.28

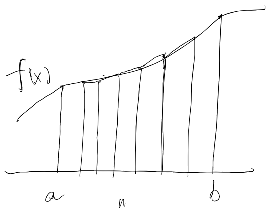
Thursday, January 28, 2021 8:19 PM

Module A4: Numerical Integration

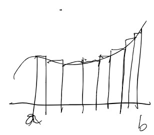
Show that, for any function, the average of the trapezoidal and midpoint approximations for n subintervals of equal length is **equivalent** to the trapezoidal approximation for $2n$ subintervals of equal length.

Module A4: Numerical Integration

Consider the function $f(x) = e^{\sin x}$. You would like to calculate $\int_{-\pi}^{\pi} f(x)dx$. What would be a least upper bound for the absolute error for this integral when using the trapezoidal and midpoint rules, assuming n subintervals for each?



T_n



M_n.

divide into n parts. on [a,b]

$$T_n = \left[\frac{1}{2}(f(a)+f(b)) + \sum_{k=1}^{n-1} f\left(a+k\cdot\frac{b-a}{n}\right)\right]\cdot\frac{b-a}{n}$$

$$M_n = \sum_{k=1}^{n} f\Big(\underbrace{a+\frac{2k-1}{2}\cdot\frac{b-a}{n}}_{\text{k-th midpoint}}\Big)\cdot\frac{b-a}{n}$$

$$\frac{M_n+T_n}{2} = \left[\frac{1}{2}(f(a)+f(b)) + \sum_{k=1}^{n-1} f\left(a+k\cdot\frac{b-a}{n}\right) + \sum_{k=1}^{n} f\left(a+\frac{2k-1}{2}\cdot\frac{b-a}{n}\right)\right]\frac{b-a}{2n}$$

$$= \left[\frac{1}{2}(f(a)+f(b)) + \sum_{k=1}^{n-1} f\Big(a+\underbrace{2k}_{\text{even}}\frac{b-a}{2n}\Big) + \sum f\Big(a+\underbrace{(2k-1)}_{\text{odd}}\frac{b-a}{2n}\Big)\right]\cdot\frac{b-a}{2n}$$

$$= T_{2n}.$$

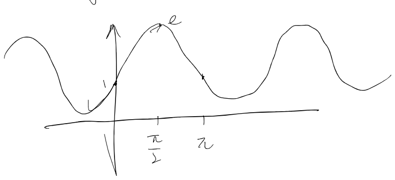
T_{2n} is a more accurate approximation.

$$\frac{M_n+T_n}{2} = T_{2n}.$$

Module A4: Numerical Integration

Consider the function $f(x) = e^{\sin x}$. You would like to calculate $\int_{-\pi}^{\pi} f(x)dx$. What would be a least upper bound for the absolute error for this integral when using the trapezoidal and midpoint rules, assuming n subintervals for each?

$$f(x) = e^{\sin x} \qquad \int_{-\pi}^{\pi} f(x)\,dx$$

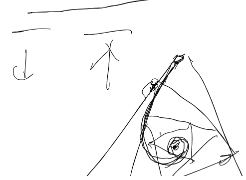


$$r\cdot\frac{d\theta}{dt} = \sin\varphi\cdot v_0.$$

$P_1 \to P_2 \quad P_2 \to P_3 \quad P_3 \to P_1.$

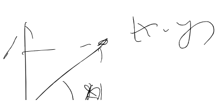
Q1: will they meet ?

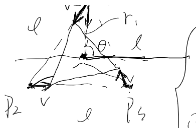
Q2: will they meet in finite time?



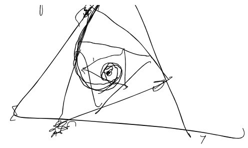
$x = r\cdot\cos\theta$

$y = r\cdot\sin\theta.$

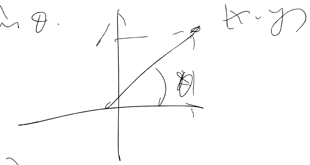




Q_1: will they meet?
Q_2: will they meet in finite time?
Q_3: How long it takes if so?
Q_4: What about the orbit? How it looks like?
Q_5: What about other n.

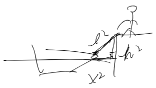


$y = r \cdot \sin\theta$.

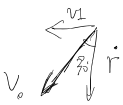


$\frac{1}{r}$ order

$|v|^2 = \frac{dx}{dt}^2 + \frac{dy}{dt}^2$



$x^2 + h^2 = l^2$



$P_1 = (r_1, \theta_1)$
$P_2 = (r_2, \theta_2)$
$P_3 = (r_3, \theta_3)$.

$$\begin{cases} \frac{dr}{dt} = -\cos\frac{\pi}{6} \cdot v_0 \\ r\frac{d\theta}{dt} = \sin\frac{\pi}{6} \cdot v_0 \end{cases} \Rightarrow \begin{matrix} -\cos\varphi \cdot v_0 \\ \sin\varphi \cdot v_0 \end{matrix}$$

$$\varphi = \frac{\pi - \frac{2\pi}{n}}{2}$$

$$|v|^2 = \left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2.$$

$$\boxed{r = -v_0 \cdot \frac{\sqrt{3}}{2} \cdot t + r_0}$$ will meet in finite time.

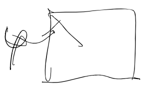
$$\frac{d\theta}{dt} = \frac{\sin\varphi \cdot v_0}{r_0 - \cos\varphi \cdot v_0 t}$$

$$\frac{d\theta}{dt} = \frac{\sin\varphi \cdot v_0}{r}$$ const

$\frac{d\theta}{dt} \uparrow$

$\theta = \int \cdots \; t$

periodic 2π.



$n = 4$ square.

$n \to \infty$ circle.