1.18

Monday, January 18, 2021 7:54 PM

Welcome to the TUT0105B
We will start at 8:10.
I am going to answer your questions, so please come with questions prepared.

An Old Question:

A man on the earth moves toward south 10km, then east 10km, then north 10km. Finally he moves back to the start position. Where is this start position on the earth?

$$\boxed{\frac{3x^2}{x^3-1}} = \frac{3x^2}{(x-1)(x^2+x+1)}$$

1. first we consider x^3-1

$x=1$ as a root.

therefore, $x^3-1 = (x-1)(x^2+x+1)$

$f(x) = x^2+x+1$ has two roots.

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ for ax^2+bx+c

$x_1 = \frac{-1+\sqrt{1-4}}{2} = \frac{-1+\sqrt{3}i}{2}$ $\quad i = \sqrt{-1}$

$x_2 = \frac{-1-\sqrt{3}i}{2}$

$$\frac{3\sqrt{x}}{(x^3+2x+3)^2}\,dx$$

Q2. $I = \int \frac{2x}{(x^2+2x+4)^2}\,dx =$

$2x\,dx \Rightarrow dx^2$

$\frac{(\ N\ )}{(\ D\ )}$

x^2+1, x^2-1, $\sqrt{x^2-1}$, $\sqrt{x^2+1}$

$u = x^2+2x+4 \qquad du = (2x+2)\,dx$

$$I = \int \frac{2x+2}{(\underbrace{x^2+2x+4}_{u})^2}\,dx - \int \frac{2}{(x^2+2x+4)^2}\,dx$$

arcsin
sin

$$= \int \frac{du}{u^2} - \int \frac{2}{(x^2+2x+4)^2}\,dx$$

x^2+2x+4
$c(s^2+1)$

$$= -\frac{1}{u} - 2I_2$$

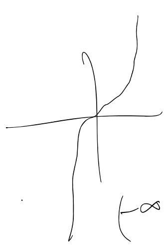
$$I_2 = \int \frac{1}{(x^2+2x+4)^2}\,dx = \int \frac{1}{((x+1)^2+3)^2}\,dx$$

let

$t = (x+1)$

$$I_2 = \int \frac{1}{(t^2+3)^2}\,dt$$

$t = \sqrt{3}\cdot\tan(s)$



$(-\infty, \infty)$

$$I_2 = \int \frac{\sqrt{3}\cdot\frac{1}{\cos^2 s}}{(3\tan^2 s+3)^2}\,ds$$

$\frac{dt}{ds} = \frac{\sqrt{3}}{\cos^2 s}$

$$= \frac{\sqrt{3}}{9}\int \frac{1}{\cos^2 s\,(\underbrace{\tan^2 s+1}_{\frac{1}{\cos^2 s}})^2}\,ds$$

$$= \frac{\sqrt{3}}{9}\int \cos^2 s\,ds$$

$\tan^2 s + 1 = \frac{\sin^2 s+\cos^2 s}{\cos^2 s} = \frac{1}{\cos^2 s}$

$$= \frac{\sqrt{3}}{9}\int \frac{\cos 2s+1}{2}\,ds$$

$\cos^2 + \sin^2 = 1$

$\cos 2x = \cos^2 - \sin^2$

$\frac{d\tan}{dx}$

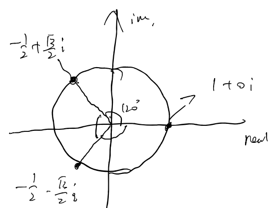
$\cos^2 = \frac{\cos 2x+1}{2}$

$$= \frac{\sqrt{3}}{18}\int \cos 2s+1\,ds$$

Q 3. $\frac{3x^2}{x^3-1} =$ $f(x) = x^3 - 1$

$x^3 = 1 \Rightarrow f(x) = x^3 - 1 \;;\; f(x) = 0$

$\boxed{e^{ix} = \cos x + i \sin x}$ euler equation.

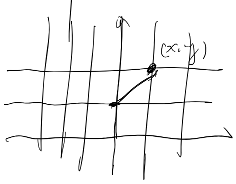


$x^3 = 1$

$(x-1)(x^2+x+1)$

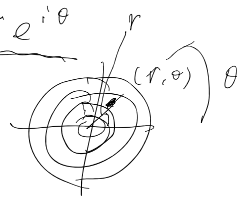
$\begin{cases} x_1 = 1 \\ x_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i \\ x_3 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i \end{cases}$ $\sin 0 = 0$, $\cos 0$

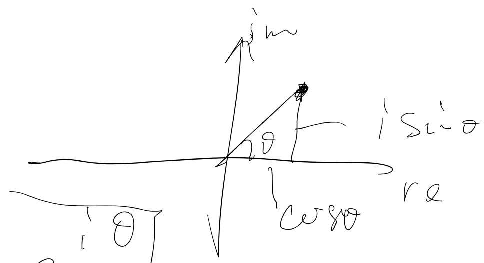
$x^3 - 1 = 0$



$x = re^{i\theta}$

$\Rightarrow$





$\boxed{e^{i\theta}} = \cos\theta + i \sin\theta$

$(re^{i\theta})^3 - 1 = 0$

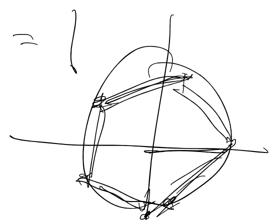
$(re^{i\theta})^3 = 1$ $\quad r \neq 0$

$\boxed{r^3} e^{i3\theta} = 1 \quad r^3 = 1 \Rightarrow r = 1$

$e^{i3\theta} = 1 \Rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{6\pi}{3}$

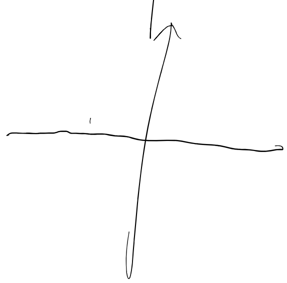
$e^{i\theta} = 1$

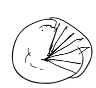
$x^5 = 1$ $\quad x^5 = 1$



$\boxed{x^5 - 2x = 1}$

$x(x^4 - 2) = 1$





17 equally

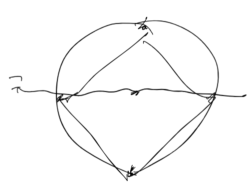
17 - polygon

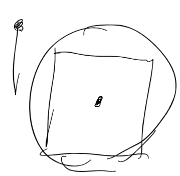
$\boxed{x^{17} = 1}$

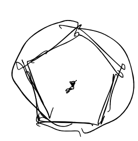
$\boxed{17}$ - polygon

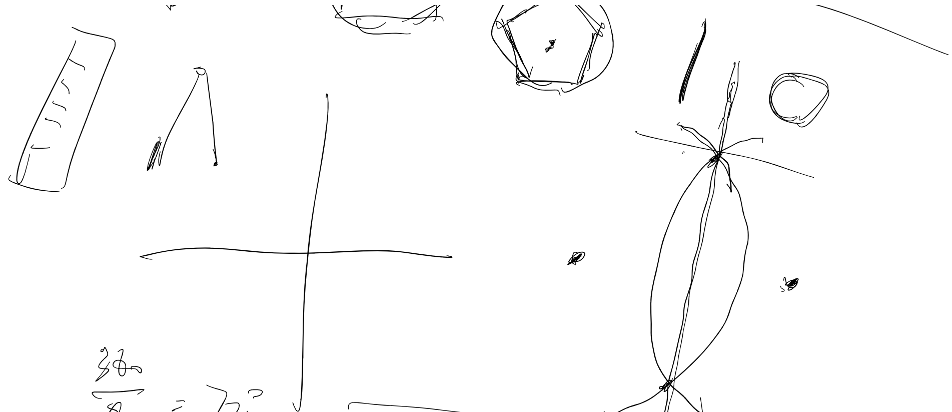
Gauss

$\frac{360}{17} =$









$$\frac{360}{5} = 72^\circ$$

$$\boxed{\cos 72^\circ + i \sin 72^\circ}$$

1.25

Monday, January 25, 2021 7:44 PM

Welcome to the TUT0105B

Hanchun Wang
Stephen
Mathematics Physics Specialist

The Tut will start at 8:10.

On the top of the screen, you can use "View Options" / "Annotate" to make annotation

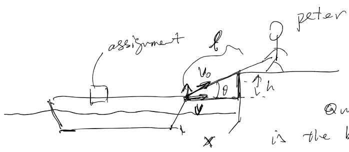
I will answer your general questions at the beginning and then discuss some problems together.

Please feel free to ask any questions. You are encouraged to open your mic and camera to get to know each others!

There is a feedback form at the end that you can provide me any of your thought about this tutorial.

	3							
		6		1		2		
7		8	5	9	4			3
								6
8		2	3	6				
				7	5			
	6		4			3	7	
5			7	3		4		
		3	9	8	2			1

1.



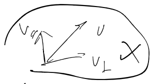
V_0, h const.

Question : v?

is the boat accelerating?

get slower?

constant speed?



is $v = v_0 \cdot \cos\theta$?

no true

find some conserved quantity relastion

in the triangle.

$x^2 + h^2 = l^2$

$\frac{dx^2}{dt} = \frac{dx}{dt} \cdot \frac{dx^2}{dx} = 2x \cdot \frac{dx}{dt}$

Take derivative over t on both side.

$2x \frac{dx}{dt} + 2h\frac{dh}{dt} = 2l \frac{dl}{dt}$ (1) $\frac{dh}{dt} = 0$

$x^2 + h^2 - l^2 = 0$

$x \frac{dx}{dt} = l \frac{dl}{dt}$ (2)

$\frac{dx}{dt} = v$ $\frac{dl}{dt} = v_0$

$x \cdot v = l \cdot v_0$

$v = \frac{l}{x} v_0$

$v = \frac{1}{\cos\theta} v_0$. not $v = \cos\theta\, v_0$.

when $x \downarrow$, $\theta \uparrow$, $\cos\theta \downarrow$, $\frac{1}{\cos\theta} \uparrow$. $v \uparrow$.

2.

Module A3: Partial Fractions

Calculate the integral of $f(x) = \frac{1}{x^n(x-a)}$ where n is a positive integer and $a \neq 0$.

Hint 1: Try this out for various values of n and see if you notice a pattern.

Hint 2: Find the coefficient A of $\frac{1}{x-a}$, subtract $\frac{A}{x-a}$ from $f(x)$, simplify the resulting difference, and use the fact that $x^n - a^n = (x-a)(x^{n-1} + x^{n-2}a + \cdots + xa^{n-2} + a^{n-1})$.

$f(x) = \frac{1}{x^n(x-a)}$

$\int dx\, f(x)$. or $\int f(x)\, dx$

$\int \frac{1}{x^n(x-a)} dx$.

partial fraction decomposition

$\int \frac{1}{x^n(x-a)}\,dx.$

$= \underline{\frac{1}{x^n(x-a)}}$ partial fraction decomposition

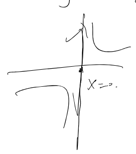
fundamental theorem of Algebra.

$= \frac{B}{x-a} + \sum_{i=1}^{n} \frac{A_i}{x^i}$

$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0.$

get n complex roots.

Singularity.



$\frac{1}{x}$ $\frac{1}{x-a}$

$$\frac{1}{x^n(x-a)} = \frac{B}{x-a} + \sum_{i=1}^{n} \frac{A_i}{x^i}$$

$1 = Bx^n + \underbrace{\sum_{i=1}^{n} A_i x^{n-i}(x-a)}_{0}$ holds for $\forall x$.

when $x=a$.

$1 = Ba^n$ $\Rightarrow B = \frac{1}{a^n}$

$$1 = \frac{x^n}{a^n} + \sum_{i=1}^{n} A_i x^{n-i}(x-a)$$

$\int_a^{\infty} f(x)\,dx$ absolutely convergent.?

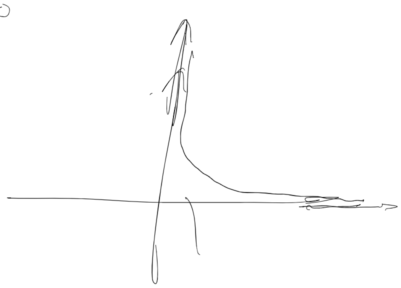
convergent.?

$\lim_{x\to\infty} f(x) = 0.$

$\lim_{x\to\infty} \int f(x)dx = C$

$\int_a^b f(x)\,dx$

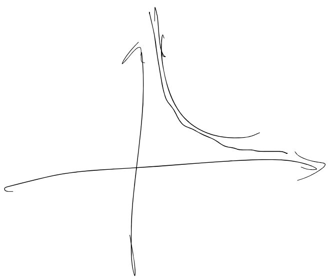
$\lim\limits_{x\to\infty} \int f(x)dx = c$



$\int_a f(x)\,dx$

$$\int_a^b \frac{1}{x^2}\,dx$$ well-posed?

$f \quad \lim\limits_{\substack{a\to 0\\ b\to\infty}} \int_a^b \frac{1}{x^2}\,dx = \frac{1}{a} - \frac{1}{}$



$\int_a^c \frac{1}{x^2}\,dx +$