The previous NOTES are at enthazy.github.io

$M(t)$ is the percentage of people support the party.

ODE : $M' = 0.03 - 0.05 \cdot M$ (increasing, decreasing) Cont.

Iteration: $M_{n+1} = 0.03 - 0.05 M_n$ discrete

$\{M_1, M_2, M_3, \dots\}$

0.6, 1, 2, 3, 4 days

A particle is observed to be moving such that its trajectory satisfies the following ODEs: $y'' - 3y' + 2y = \cos 2t$ and $y'' + 4y = e^{2t} + e^t$. What must have been its initial position and velocity at time $t = 0$?

Q1.

① $y_1 = y_{c1} + y_{p1}$ ① $y_{c1} = Ae^{2t} + Be^{t}$, $y_{p1} = -\frac{1}{20}\cos 2t$

② $y_2 = y_{c2} + y_{p2}$ ② $y_{c2} = y_{c2} =$, $y_{p2} = \frac{1}{8}e^{2t}$

$y_1 = y_2$

$A = 1/8 \quad B = 1/5 \quad F = -$

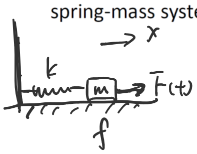
$y(0) = A + B + F = \frac{+1}{20}$

$y'(0) = 2A + B + 2G = \frac{3}{20}$

2.

Consider a damped spring-mass system with nonzero mass m, spring constant k, and damping coefficient b which is subjected to an external force $F(t) = F_0 \sin(\omega t)$.

a) Write a 2nd order ODE for the position x of the mass.
b) Is the characteristic solution always transient?
c) Find the particular solution. Is it transient or steady-state?
d) Convert the above into the form $C\sin(\omega t + \phi)$.
e) Set $m = k = 1$. Graph C as a function of ω for different values of b and comment on the effect of b and ω on the trajectory of the spring-mass system.



a) $m\ddot{x} + b\dot{x} + kx = F_0 \sin \omega t$

b) characteristic solution $\xrightarrow[t\to\infty]{} 0$ iff $mb > 0$, $mk > 0$

$\partial_t u = Au + f$ (linear op. on t, linear op., inhom.)

$e^{At} = \sum_{n=0}^{\infty} \frac{(At)^n}{n!} = 1 + At + \frac{A^2t^2}{2} \dots$

ODE $u(t) = e^{At} u_0 + \int_0^t e^{(t-s)A} f(s)\, ds$

$\{\sin, \cos\}$

$u'' + u' = e^t$

$A t e^t + B e^t$

$f := \sin$

$e^{(t-s)A} = \{(\partial_x)^n\}$

c) $x_p = A\cos\omega t + B\sin\omega t$

$$\begin{cases} -Am\omega^2 + Bb\omega + Ak = 0 & (\cos) \text{ even fun.} \\ -Bm\omega^2 - Ab\omega + Bk = F_0 & (\sin) \text{ odd fun.} \end{cases}$$

$$\begin{cases} A(k - m\omega^2) + B(b\omega) = 0 \\ B(k - m\omega^2) - A(b\omega) = F_0 \end{cases}$$

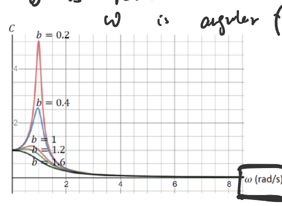
let $\alpha = k - m\omega^2$, $\beta = b\omega$

$$\Rightarrow \begin{cases} A = \dfrac{-F_0 \beta}{\alpha^2 + \beta^2} \\ B = \dfrac{-F_0 \alpha}{\alpha^2 + \beta^2} \end{cases}$$

$$x_p = \boxed{A\cos\omega t + B\sin\omega t = C\sin(\omega t + \phi)}$$

$$\boxed{C = \sqrt{A^2 + B^2} \quad \phi = \tan^{-1}\frac{A}{B}}$$

C is the amplitude
b is resistance const.
ω is angular freq. of driven force.



x_1

t

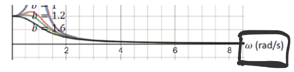
$- \frac{3}{20} \sin 2t,$

$F \cos 2t + G \sin 2t$

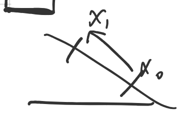
$+ \frac{1}{5} e^t$

$G = -\frac{3}{20}.$

6



$C \sin(\omega t + \phi)$



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