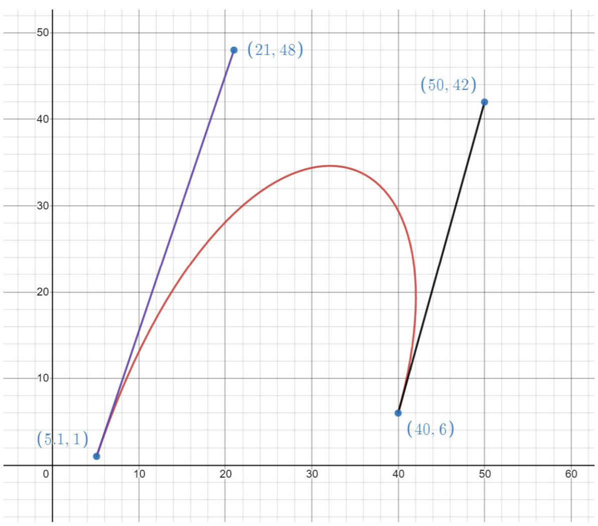
A Bézier curve is a family of parametric curves that can be used in computer aided design, computer graphics, animation, and other related fields which involve generating smooth curves. A cubic Bézier curve is defined as:

$$x = x_0(1-t)^3 + 3x_1 t(1-t)^2 + 3x_2 t^2(1-t) + x_3 t^3$$
$$y = y_0(1-t)^3 + 3y_1 t(1-t)^2 + 3y_2 t^2(1-t) + y_3 t^3$$

where $0 \le t \le 1$ and $P_0(x_0, y_0), P_1(x_1, y_1), P_2(x_2, y_2), P_3(x_3, y_3)$ are control points which determine the shape of the curve.

1. Graph a Bézier curve in Desmos using sliders for the control points.
2. Using the fact that the slope of a parametric curve is given by $\frac{y'(t)}{x'(t)}$, find the slope of the Bézier curve at $P_0(x_0, y_0)$ and $P_3(x_3, y_3)$.
3. Show that the tangent lines at $P_0(x_0, y_0)$ and $P_3(x_3, y_3)$ connect P_0P_1 and P_2P_3, respectively.



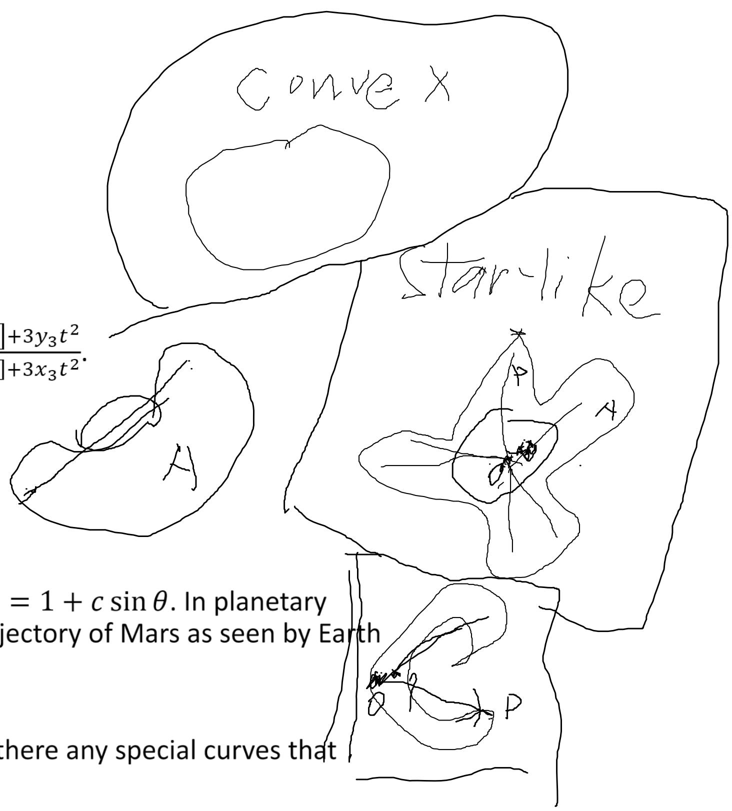
$$\frac{dy}{dx} = \frac{-3y_0(1-t)^2+3y_1[(1-t)^2-2t(1-t)]+3y_2[2t(1-t)-t^2]+3y_3t^2}{-3x_0(1-t)^2+3x_1[(1-t)^2-2t(1-t)]+3x_2[2t(1-t)-t^2]+3x_3t^2}.$$

t=0

$$\frac{y_1-y_0}{x_1-x_0}$$

t=1

$$\frac{y_3-y_2}{x_3-x_2}$$



A limaçon is a family of polar curves of the form $r = 1 + c\sin\theta$. In planetary motion studies, it has been used to model the trajectory of Mars as seen by Earth by setting $|c| > 1$!

1. Draw the limaçon for $c = -2, -1, 0, 1, 2$. Are there any special curves that arise from these cases?
2. Notice that at certain values of c, there is an inner loop at the origin. Determine which values of c exhibit this phenomenon.
3. Also notice that at other values of c, the limaçon has a dimple at $\theta = \frac{3\pi}{2}$, i.e., it bends into (rather than out from) the origin. Determine which values exhibit this phenomenon.

$2\cos 2\theta - \sin\theta$