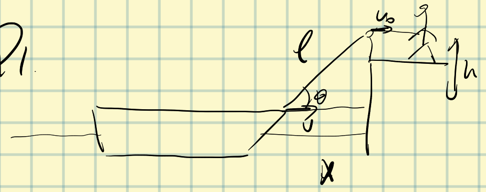
Q1



is $v = v_0 \cdot \cos\theta$? wrong

$x^2 + h^2 = l^2$

$2x \underbrace{\frac{dx}{dt}}_{v} + 2h \underbrace{\frac{dh}{dt}}_{0} = 2l \underbrace{\frac{dl}{dt}}_{v_0}$

1) $x \cdot v = l \, v_0$

$v = \frac{l}{x} v_0 = v_0 \cdot \frac{1}{\cos\theta}$

2) $\underbrace{\frac{dx}{dt}}_{v} v + \underbrace{\frac{dv}{dt}}_{a} \cdot x = \frac{dl}{dt} v_0 + \underbrace{\frac{dv_0}{dt}}_{0} \cdot l$

$v^2 + ax = v_0^2$

$a = (v_0^2 - v^2)/x = v_0^2 \left(1 - \frac{l^2}{x^2}\right)/x$

$= v_0^2 \left(\frac{-h^2}{x^3}\right)$

notice $v_0 < 0$, $v < 0$, $a < 0$

Q2. integrate $\int \frac{dx}{x^n (x-a)}$

partial fractional decomposition

let $\frac{1}{x^n (x-a)} = \sum_{i=1}^{n} \frac{A_i}{x^i} + \frac{B}{x-a}$

$1 = (x-a) \sum_{i=1}^{n} A_i x^{n-i} + B x^n$ for $\forall x$.

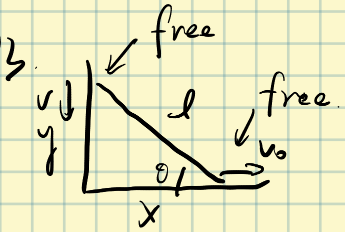
when $x = a$, $1 = B \cdot a^n \Rightarrow B = a^{-n}$

then $1 - \frac{x^n}{a^n} = (x-a) \sum_{i=1}^{n} A_i x^{n-i}$

LHS $\Rightarrow \frac{-1}{a^n} (x^n - a^n) = \frac{-1}{a^n} (x-a) \left(\sum_{i=0}^{n-1} a^{n-i-1} x^i \right) =$ RHS

then $A_i = -\frac{1}{a^{n-i+1}}$

Q3

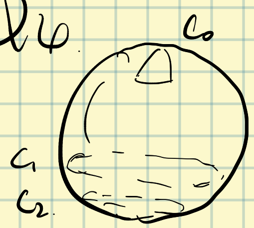


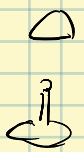
$x^2 + y^2 = l^2 \Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$\Rightarrow x \cdot v_0 = -y \cdot v$

$v = -v_0 \cdot \frac{1}{\tan\theta}$

Q4





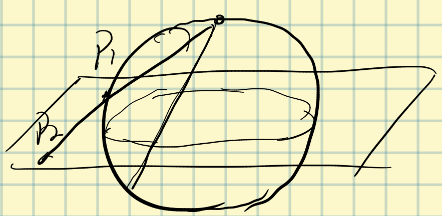
homotopy

$\begin{bmatrix} \downarrow S & 15km \\ \rightarrow E & 15km \\ \uparrow N & 15km \end{bmatrix} = A$

for $p \in C_k \subset S^2$

$A: S^2 \to S^2$

$A(p) = p$



Riemann Sphere

stereographic projection

Möbius transformation