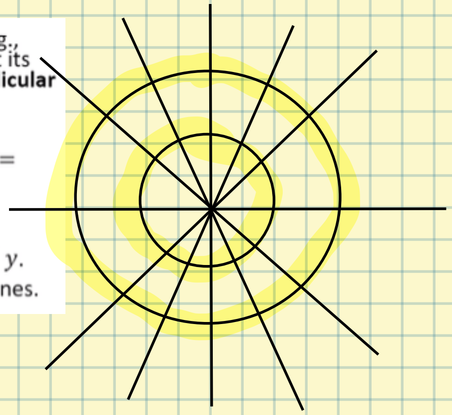
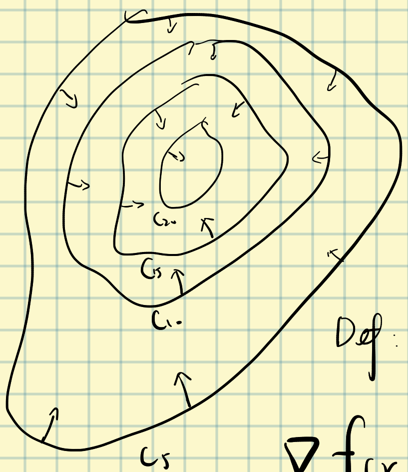
In physics, an **equipotential line** is a curve which defines a constant potential (e.g., gravitational potential, electric potential) in 2D space. It is a well-known fact that its corresponding **field line** (e.g., gravitational field, electric field) is **always perpendicular** to it where they intersect in space under static conditions.

dense!

Now suppose we have a family of field lines of a system given by the equation $x = ky^2e^x$ for any constant k.

1. How would you relate the slopes of two perpendicular lines?
2. Give an expression for the slopes of the family of field lines in terms of x and y.
3. Find an equation that represents the corresponding family of equipotential lines.





$x = ky^2e^x$

$xy^{-2}e^{-x} = k$

let $f(x,y) = xy^{-2}e^{-x}$ ← potential

Def: $C(k) := \{(x,y) \mid f(x,y) = k\}$

$\nabla f(x,y) = \left(\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y}\right)$ is the vector

$\frac{\partial f(x,y)}{\partial x}$ partial derivative

regard y as const.

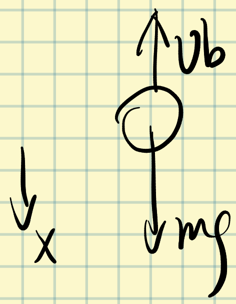
$\frac{\partial f}{\partial y}$: regard x as const.

or $\frac{dy}{dx} = \frac{y(1-x)}{2x}$

$y^2 = 4(x + \ln|x-1|) + C$

Suppose an object with mass m is dropped from rest in an environment with an acceleration due to gravity g and an air resistance force proportional to the object's velocity (assume a constant of proportionality b).

1. Find a differential equation for the velocity $v(t)$ of the object.
2. Solve the above differential equation for $v(t)$.
3. By considering $\frac{dv}{dm}$, conclude whether heavier objects fall faster or slower.



$a = \frac{dv}{dt} = \frac{F}{m} = \frac{1}{m}(mg - vb)$

$\Rightarrow \dot{v} = g - \frac{b}{m}v$ ①

Set IVP : $v(0) = 0$

$\frac{dv}{g - \frac{b}{m}v} = dt$

$v(t) = \frac{mg}{b}\left(1 - e^{-\frac{ct}{m}}\right)$

$\to 0$ as $t \to \infty$

faster, by check ① s.t.

$\dot{v} = g + \underbrace{\left(-\frac{b}{m}v\right)}_{\text{resistance } f/m}$

$m\uparrow$, $f/m \downarrow$, then $\dot{v}\uparrow$.