2.10

Wednesday, February 10, 2021 8:05 PM

On the top of the screen, you can use "View Options" / "Annotate" to make annotation

I will answer your general questions at the beginning and then discuss some problems together.

Please feel free to ask any questions. You are encouraged to open your mic and camera to get to know each others!

The notes can be downloaded at https://enthazy.github.io/

Module B1: Riemann Sums and Applications of Integration

Suppose we have a Werther's Original which has a top-down shape given by the three regions bounded by $x = 0, x = \pi, y = \sin^2 x$, and $y = \cos^2 x$ and has densities ρ_a, ρ_b, ρ_c (see figure below). At which point $(\bar{x}, \bar{y})$ should you balance this candy such that it does not tip over (i.e., into/out of the page)?

Hint: Given an infinitesimal rectangle of width Δx, height h, density ρ, and centre located at (x, y), the following formulas hold:

Mass: $m = \rho h \Delta x$

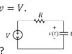
Moment about x-axis: $M_x = my$

Moment about y-axis: $M_y = mx$

Centre of mass: $(\bar{x}, \bar{y}) = (x, y) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$

Module C1: Introduction to ODEs RC circuit

The circuit below contains a battery with constant voltage $V = 40$ V, a resistor with resistance $R = 10\ \Omega$, and a capacitor with capacitance $C = 0.01$ F. The voltage $v(t)$ of the capacitor at time t is given by the ODE: $RC\frac{dv}{dt} + v = V$.



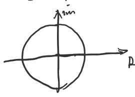
1. How many initial values must we know in order to solve for $v(t)$?
2. Find the value of $v(t)$ as $t \to \infty$.
3. Verify that $v(t) = V + (V_0 - V)e^{-\frac{t}{RC}}$ for some constant V_0 is a solution to the ODE. What is the initial value of $v(t)$ in terms of the variables in this solution?

Fundamental theorem of Algebra.

$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$

has n complex roots.

$z^3 - 1 = 0.$



Complex plane $\mathbb{C}$

$z \in \mathbb{C}.$

$z^3 - 1 = 0.$

let $z = r \cdot e^{i\theta}$

$e^{i\theta} = \cos\theta + i\sin\theta.$

$(re^{i\theta})^3 - 1 = 0$

$(x+iy)^3 - 1 = 0.$

$r^3 e^{i3\theta} = 1$

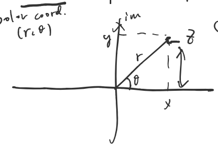
length direction

$r = 1$

$3\theta = 0 \pmod{2\pi}$

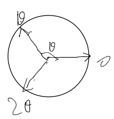
$re^{i\theta} = r\cos\theta + ir\sin\theta$ (x, y)

polar coord. (r, θ)



$\begin{cases} y = r\cdot\sin\theta \\ x = r\cdot\cos\theta \end{cases}$

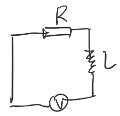
$\begin{cases} z = x + iy \\ z = re^{i\theta} \end{cases}$

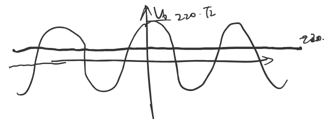


$L\frac{dI}{dt} + IR = V_0\cos\omega_0 t \leftarrow V_0 \cdot \mathrm{Re}(e^{i\omega_0 t})$

AC.

$I = Ce^{\lambda t}$





$C(L\lambda e^{\lambda t} + Re^{\lambda t}) = V_0 e^{i\omega_0 t}$

$Ce^{\lambda t}(L\lambda + R) = V_0 e^{i\omega_0 t}$

$\frac{C}{V_0} e^{(\lambda - i\omega_0)t}(L\lambda + R) = 1$

$x'' - 2x' + x = 0$

$\begin{cases} x(0) = 0 \\ x'(0) = 1 \end{cases}$

Assume: $x = C_1 e^{\lambda t}$.

$C\left[e^{\lambda t}(\lambda^2 - 2\lambda + 1)\right] = 0$

$\lambda_1 = 1, \lambda_2 = 1.$

$x = Ce^t$

Critical damping

$a\lambda^2 + b\lambda + c$

$\Delta = b^2 - 4ac > 0 \Rightarrow \lambda$ real $\leftarrow$ no oscillation

$\Delta < 0 \Rightarrow \lambda$ im $\leftarrow$ oscillation

$\Delta = 0 \Rightarrow \lambda$ degenerate

e^{iat}

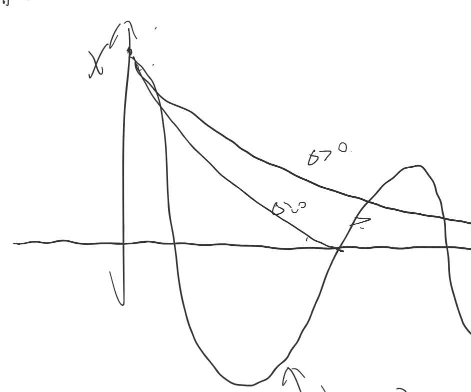
$\boxed{e^{it}} = \cos t + i\sin t.$

$t = 0$

$t = \frac{\pi}{2}$

$t = \pi$

$x = C_1 e^{\lambda t} + C_2 te^{\lambda t}$



Second order ODE: $x'' - 2x' + x = 0. \Rightarrow x'' = 2x' - x$

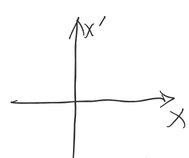
let $y = x'$

$y' = 2x' - x = 2y - x$

let $K = \begin{pmatrix} x \\ y \end{pmatrix}$

$K' = \begin{pmatrix} x' \\ y' \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$



phase diagram

let $A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$ then $K' = AK$.

first order ODE of two variables.

$\Rightarrow \quad K = Ce^{At}$ $\boxed{e^{At} = ?}$

$K \Rightarrow x$

$$e^{At} = \sum_{n=0}^{\infty} \frac{A^n}{n!} t^n$$