



Slides adapted from William Cohen

Introduction to Machine Learning

Machine Learning: Jordan Boyd-Graber

University of Maryland

STOCHASTIC GRADIENT DESCENT FOR LOGISTIC REGRESSION

Content Questions

Content Questions

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Administrivia Questions

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Reminder: Logistic Regression

$$P(Y = 0|X) = \frac{1}{1 + \exp[\beta_0 + \sum_i \beta_i X_i]} \quad (1)$$

$$P(Y = 1|X) = \frac{\exp[\beta_0 + \sum_i \beta_i X_i]}{1 + \exp[\beta_0 + \sum_i \beta_i X_i]} \quad (2)$$

- Discriminative prediction: $p(y|x)$
- Classification uses: ad placement, spam detection
- What we didn't talk about is how to learn β from data

Logistic Regression: Objective Function

$$\mathcal{L} \equiv \ln p(Y|X, \beta) = \sum_j \ln p(y^{(j)} | x^{(j)}, \beta) \quad (3)$$

$$= \sum_j y^{(j)} \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) - \ln \left[1 + \exp \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) \right] \quad (4)$$

Algorithm

1. Initialize a vector B to be all zeros
2. For $t = 1, \dots, T$
 - For each example \vec{x}_i, y_i and feature j :
 - Compute $\pi_i \equiv \Pr(y_i = 1 | \vec{x}_i)$
 - Set $\beta[j] = \beta[j]' + \lambda(y_i - \pi_i)x_i$
3. Output the parameters β_1, \dots, β_d .

Example Documents

$$\beta[j] = \beta[j] + \lambda(y_i - \pi_i)x_i$$

$$\vec{\beta} = \langle \beta_{bias} = 0, \beta_A = 0, \beta_B = 0, \beta_C = 0, \beta_D = 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

(Assume step size $\lambda = 1.0$.)

$$y_2 = 0$$

B C C C D D D D

You first see the positive example. First, compute π_1

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$$\pi_1 = \Pr(y_1 = 1 | \vec{x}_1) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp 0}{\exp 0 + 1} = 0.5$$

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- But difficult to update every feature every time (if there are many features)
- Following this up, we note that we can perform m successive “regularization” updates by letting $\beta_j = \beta'_j \cdot (1 - 2\lambda\mu)^{m_j}$

Basic Idea

Don't perform regularization updates for zero-valued x_j 's, but instead to simply keep track of how many such updates would need to be performed to update β_j

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
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Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A ?

$$\beta_A = (\beta_A' + \lambda \cdot (y_1 - \pi_1) \cdot x_{1,A}) (1 - 2 \cdot \lambda \cdot \mu)^{m_A} =$$
$$(0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 4.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A ?

$$\beta_A = (\beta_A' + \lambda \cdot (y_1 - \pi_1) \cdot x_{1,A}) (1 - 2 \cdot \lambda \cdot \mu)^{m_A} =$$
$$(0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 4.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 = 1.0$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ?

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ?

$$\beta_B = (\beta_B' + \lambda \cdot (y_1 - \pi_1) \cdot x_{1,B}) (1 - 2 \cdot \lambda \cdot \mu)^{m_B} =$$
$$(0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 3.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ?

$$\beta_B = (\beta_B' + \lambda \cdot (y_1 - \pi_1) \cdot x_{1,B}) (1 - 2 \cdot \lambda \cdot \mu)^{m_B} =$$
$$(0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 3.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 = 0.75$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ?

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_j) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ?

$$\beta_C = (\beta_C' + \lambda \cdot (y_1 - \pi_1) \cdot x_{1,C}) (1 - 2 \cdot \lambda \cdot \mu)^{m_C} =$$
$$(0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 1.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ?

$$\beta_C = (\beta_C' + \lambda \cdot (y_1 - \pi_1) \cdot x_{1,C}) (1 - 2 \cdot \lambda \cdot \mu)^{m_C} =$$
$$(0.0 + 1.0 \cdot (1.0 - 0.5) \cdot 1.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 = 0.25$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D ?

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_j) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D ?

We don't care: leave it for later.

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_j) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Now you see the negative example. What's π_2 ?

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_j) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Now you see the negative example. What's π_2 ?

$$\pi_2 = \Pr(y_2 = 1 | \vec{x}_2) = \frac{\exp \beta^T x_j}{1 + \exp \beta^T x_j} = \frac{\exp\{.25 + 0.75 + 0.75 + 0\}}{\exp\{.25 + 0.75 + 0.75 + 0\} + 1} =$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_j) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Now you see the negative example. What's π_2 ?

$$\pi_2 = \Pr(y_2 = 1 | \vec{x}_2) = \frac{\exp \beta^T x_j}{1 + \exp \beta^T x_j} = \frac{\exp\{.25 + 0.75 + 0.75 + 0\}}{\exp\{.25 + 0.75 + 0.75 + 0\} + 1} = 0.85$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_j) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

$$\pi_2 = 0.85$$

What's the update for β_{bias} ?

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_{bias} ?

$$\beta_{bias} = (\beta'_{bias} + \lambda \cdot (y_2 - \pi_2) \cdot x_{2,bias}) (1 - 2 \cdot \lambda \cdot \mu)^{m_{bias}} =$$
$$(0.25 + 1.0 \cdot (0.0 - 0.85) \cdot 1.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_{bias} ?

$$\beta_{bias} = (\beta'_{bias} + \lambda \cdot (y_2 - \pi_2) \cdot x_{2,bias}) (1 - 2 \cdot \lambda \cdot \mu)^{m_{bias}} =$$
$$(0.25 + 1.0 \cdot (0.0 - 0.85) \cdot 1.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 = -0.30$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A ?

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_j) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A ?

We don't care: leave it for later.

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ?

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ?

$$\beta_B = (\beta_B' + \lambda \cdot (y_2 - \pi_2) \cdot x_{2,B}) (1 - 2 \cdot \lambda \cdot \mu)^{m_B} =$$
$$(0.75 + 1.0 \cdot (0.0 - 0.85) \cdot 1.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ?

$$\beta_B = (\beta_B' + \lambda \cdot (y_2 - \pi_2) \cdot x_{2,B}) (1 - 2 \cdot \lambda \cdot \mu)^{m_B} =$$
$$(0.75 + 1.0 \cdot (0.0 - 0.85) \cdot 1.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 = -0.05$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ?

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ?

$$\beta_C = (\beta_C' + \lambda \cdot (y_2 - \pi_2) \cdot x_{2,C}) (1 - 2 \cdot \lambda \cdot \mu)^{m_C} =$$
$$(0.25 + 1.0 \cdot (0.0 - 0.85) \cdot 3.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ?

$$\beta_C = (\beta_C' + \lambda \cdot (y_2 - \pi_2) \cdot x_{2,C}) (1 - 2 \cdot \lambda \cdot \mu)^{m_C} =$$
$$(0.25 + 1.0 \cdot (0.0 - 0.85) \cdot 3.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 = -1.15$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D ?

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D ?

$$\beta_D = (\beta_D' + \lambda \cdot (y_2 - \pi_2) \cdot x_{2,D}) (1 - 2 \cdot \lambda \cdot \mu)^{m_D} =$$
$$(0.0 + 1.0 \cdot (0.0 - 0.85) \cdot 4.0) (1 - 2 \cdot 1.0 \cdot \frac{1}{4})^2$$

Example Documents (Regularized)

$$\beta[j] = (\beta[j]' + \lambda(y - p)x_i) \cdot (1 - 2\lambda\mu)^{m_j}$$
$$\vec{\beta} = \langle .25, 1, 0.75, 0.25, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D ?

$$\beta_D = (\beta_D' + \lambda \cdot (y_2 - \pi_2) \cdot x_{2,D}) (1 - 2 \cdot \lambda \cdot \mu)^{m_D} =$$
$$(0.0 + 1.0 \cdot (0.0 - 0.85) \cdot 4.0) \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^2 = -0.85$$

Next time ...

- Multinomial logistic regression in sklearn (more than one option)
- Crafting effective features
- Preparation for third homework