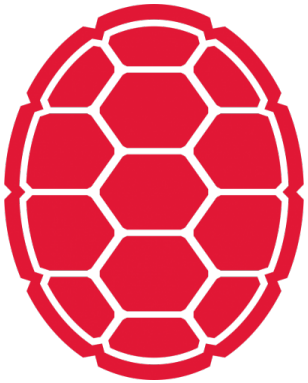

Slides adapted from William Cohen

Introduction to Machine Learning

Machine Learning: Jordan Boyd-Graber
University of Maryland

STOCASTIC GRADIENT DESCENT FOR LOGISTIC REGRESSION

Content Questions

Content Questions

Content Questions

Content Questions

Content Questions

Administrivia Questions

Administrivia Questions

Administrivia Questions

Reminder: Logistic Regression

$$P(Y = 0|X) = \frac{1}{1 + \exp\left[\beta_0 + \sum_i \beta_i X_i\right]} \tag{1}$$

$$P(Y = 1|X) = \frac{\exp\left[\beta_0 + \sum_i \beta_i X_i\right]}{1 + \exp\left[\beta_0 + \sum_i \beta_i X_i\right]} \tag{2}$$

- Discriminative prediction: $p(y|x)$
- Classification uses: ad placement, spam detection
- What we didn't talk about is how to learn β from data

Logistic Regression: Objective Function

$$\mathscr{L} \equiv \ln p(Y \mid X, \beta) = \sum_j \ln p(y^{(j)} \mid x^{(j)}, \beta) \tag{3}$$

$$= \sum_j y^{(j)} \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) - \ln \left[1 + \exp \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) \right] \tag{4}$$

Algorithm

1. Initialize a vector B to be all zeros
2. For $t = 1, \ldots, T$
   - For each example $\vec{x}_i, y_i$ and feature j:
     - Compute $\pi_i \equiv \Pr(y_i = 1 \mid \vec{x}_i)$
     - Set $\beta[j] = \beta[j]' + \lambda(y_i - \pi_i)x_i$
3. Output the parameters $\beta_1, \ldots, \beta_d$.

Example Documents

$$\beta[j] = \beta[j] + \lambda(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle \beta_{bias} = 0, \beta_A = 0, \beta_B = 0, \beta_C = 0, \beta_D = 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

(Assume step size $\lambda = 1.0$.)

You first see the positive example. First, compute π_1

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$$\pi_1 = \Pr(y_1 = 1 \mid \vec{x}_1) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp 0}{\exp 0 + 1} = 0.5$$

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$\pi_1 = 0.5$ What's the update for β_{bias}?

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What's the update for β_D?

Example Documents

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What's the update for β_D?

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$$\beta[j] = \beta[j] + \lambda(y_i - \pi_i)x_i$$
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(Assume step size $\lambda = 1.0$.)

Now you see the negative example. What's π_2?

Example Documents

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What's the update for β_{bias}?

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How does the gradient change with regularization?

- First

$$\beta_j = \beta_j' - \lambda 2 \mu \beta_j = \beta_j' \cdot (1 - 2\lambda\mu) \tag{5}$$

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- But difficult to update every feature every time (if there are many features)
- Following this up, we note that we can perform m successive "regularization" updates by letting $\beta_j = \beta_j' \cdot (1 - 2\lambda\mu)^{m_j}$

Basic Idea

Don't perform regularization updates for zero-valued x_j's, but instead to simply keep track of how many such updates would need to be performed to update β_j

Revised Algorithm

1. Initialize a vector β to be all zeros
2. Initialize a vector A to be all zeros
3. For $t = 1, \ldots, T$
   - For each example $\vec{x}_i, y_i$ and feature j:
     - Simulate regularization updates: $\beta[j] = \beta[j] \cdot (1 - 2\lambda\mu)^{k - A[j]}$
     - Compute $\pi_i \equiv \Pr(y_i = 1 \mid \vec{x}_i)$
     - Set $\beta[j] = \beta[j] + \lambda(y_i - \pi_i)x_i$
     - Keep track of last update for feature $A[j] = k$
4. For each paramter, catch up on missing updates
   $\beta[j] = \beta[j] \cdot (1 - 2\lambda\mu)^{T - A[j]}$
5. Output the parameters $\beta_1, \ldots, \beta_d$.

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

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Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

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Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

You first see the positive example. π_1 is still 0.5.

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What's the update for β_{bias}?

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Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_{bias}?

$$\beta_{bias} = \beta'_{bias}(1 - 2 \cdot \lambda \cdot \mu)^{m_{bias}} + \lambda(y_1 - \pi_1)x_{1,bias} =$$
$$0.0\left(1 - 2 \cdot 1.0 \cdot \tfrac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)1.0$$

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Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_{bias}?

$$\beta_{bias} = \beta'_{bias}(1 - 2 \cdot \lambda \cdot \mu)^{m_{bias}} + \lambda(y_1 - \pi_1)x_{1,bias} =$$
$$0.0\left(1 - 2 \cdot 1.0 \cdot \tfrac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)1.0 \quad = 2$$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A? $\beta_A = \beta'_A(1 - 2 \cdot \lambda \cdot \mu)^{m_A} + \lambda(y_1 - \pi_1)x_{1,A} =$
$0.0\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)4.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A? $\beta_A = \beta'_A(1 - 2 \cdot \lambda \cdot \mu)^{m_A} + \lambda(y_1 - \pi_1)x_{1,A} =$
$0.0\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)4.0 \quad = 1.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B? $\beta_B = \beta'_B(1 - 2 \cdot \lambda \cdot \mu)^{m_B} + \lambda(y_1 - \pi_1)x_{1,B} =$
$0.0\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)3.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B? $\beta_B = \beta'_B(1 - 2 \cdot \lambda \cdot \mu)^{m_B} + \lambda(y_1 - \pi_1)x_{1,B} =$
$0.0\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)3.0 \quad = 1.5$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C? $\beta_C = \beta'_C(1 - 2 \cdot \lambda \cdot \mu)^{m_C} + \lambda(y_1 - \pi_1)x_{1,C} =$
$0.0\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)1.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C? $\beta_C = \beta'_C(1 - 2 \cdot \lambda \cdot \mu)^{m_C} + \lambda(y_1 - \pi_1)x_{1,C} =$
$0.0\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)1.0 \quad = 0.5$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D?

We don't care: leave it for later.

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Now you see the negative example. What's π_2?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Now you see the negative example. What's π_2?

$$\pi_2 = \Pr(y_2 = 1 \mid \vec{x_2}) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp\{.5 + 1.5 + 1.5 + 0\}}{\exp\{.5 + 1.5 + 1.5 + 0\} + 1} =$$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Now you see the negative example. What's π_2?

$$\pi_2 = \Pr(y_2 = 1 \mid \vec{x_2}) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp\{.5 + 1.5 + 1.5 + 0\}}{\exp\{.5 + 1.5 + 1.5 + 0\} + 1} = 0.97$$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

$\pi_2 = 0.97$

Careful: You'd need to regularize β_D if it weren't already zero (multiply it by $(1 - 2\lambda\mu)^{m_j}$)

What's the update for β_{bias}?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_{bias}?

$$\beta_{bias} = \beta'_{bias}(1 - 2 \cdot \lambda \cdot \mu)^{m_{bias}} + \lambda(y_2 - \pi_2)x_{2,bias} =$$
$$0.5\left(1 - 2 \cdot 1.0 \cdot \tfrac{1}{4}\right)^1 + 1.0 \cdot (0.0 - 0.97)1.0$$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_{bias}?

$\beta_{bias} = \beta'_{bias}(1 - 2 \cdot \lambda \cdot \mu)^{m_{bias}} + \lambda(y_2 - \pi_2)x_{2,bias} =$

$0.5\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (0.0 - 0.97)1.0 \quad$ =-0.72

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A?

We don't care: leave it for later.

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B? $\beta_B = \beta'_B(1 - 2 \cdot \lambda \cdot \mu)^{m_B} + \lambda(y_2 - \pi_2)x_{2,B} =$
$1.5\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (0.0 - 0.97)1.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B? $\beta_B = \beta'_B(1 - 2 \cdot \lambda \cdot \mu)^{m_B} + \lambda(y_2 - \pi_2)x_{2,B} =$
$1.5\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (0.0 - 0.97)1.0$ =-0.22

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C? $\beta_C = \beta'_C(1 - 2 \cdot \lambda \cdot \mu)^{m_C} + \lambda(y_2 - \pi_2)x_{2,C} =$
$0.5\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (0.0 - 0.97)3.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C? $\beta_C = \beta'_C(1 - 2 \cdot \lambda \cdot \mu)^{m_C} + \lambda(y_2 - \pi_2)x_{2,C} =$
$0.5\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (0.0 - 0.97)3.0$ =-2.7

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D? $\beta_D = \beta'_D(1 - 2 \cdot \lambda \cdot \mu)^{m_D} + \lambda(y_2 - \pi_2)x_{2,D} =$
$0.0\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^2 + 1.0 \cdot (0.0 - 0.97)4.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$y_1 = 1$	$y_2 = 0$
A A A A B B B C	B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D? $\beta_D = \beta'_D(1 - 2 \cdot \lambda \cdot \mu)^{m_D} + \lambda(y_2 - \pi_2)x_{2,D} =$
$0.0\left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^2 + 1.0 \cdot (0.0 - 0.97)4.0$ =-3.9

If this were final iteration . . .

- Need to remember that β_A is still waiting for regularization

$$\beta_A^{\text{final}} = \beta_A \left(1 - 21.0 \frac{1}{4}\right)^1 = 1.0 \tag{6}$$

Next time . . .

- Multinomial logistic regression in sklearn (more than one option)
- Crafting effective features
- Preparation for third homework