



Topic Models

Advanced Machine Learning for NLP

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SLIDES ADAPTED FROM DAVID MIMNO

Learning the Hidden Space

- Two major tools:
 - Gibbs Sampling: Easier to implement, easier to understand
 - Variational Inference: faster, harder to implement
- Variational shows the connections to “deep” models better, so it’s the focus
- However, would be injustice to not at least discuss Gibbs sampling

Inference

- We are interested in posterior distribution

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- Here, latent variables are topic assignments z and topics θ . X is the words (divided into documents), and Θ are hyperparameters to Dirichlet distributions: α for topic proportion, λ for topics.

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$$p(z, \beta, \theta | w, \alpha, \lambda) \tag{2}$$

$$p(w, z, \theta, \beta | \alpha, \lambda) = \prod_k p(\beta_k | \lambda) \prod_d p(\theta_d | \alpha) \prod_n p(z_{d,n} | \theta_d) p(w_{d,n} | \beta_{z_{d,n}})$$

Gibbs Sampling

- A form of Markov Chain Monte Carlo
- Chain is a sequence of random variable states
- Given a state $\{z_1, \dots, z_N\}$ given certain technical conditions, drawing $z_k \sim p(z_1, \dots, z_{k-1}, z_{k+1}, \dots, z_N | X, \Theta)$ for all k (repeatedly) results in a Markov Chain whose stationary distribution *is* the posterior.
- For notational convenience, call \mathbf{z} with $z_{d,n}$ removed $\mathbf{z}_{-d,n}$

Inference

computer,
technology,
system,
service, site,
phone,
internet,
machine

sell, sale,
store, product,
business,
advertising,
market,
consumer

play, film,
movie, theater,
production,
star, director,
stage

Hollywood studios are preparing to let people
download and buy electronic copies of movies over
the Internet, much as record labels now sell songs for
99 cents through Apple Computer's iTunes music store
and other online services ...

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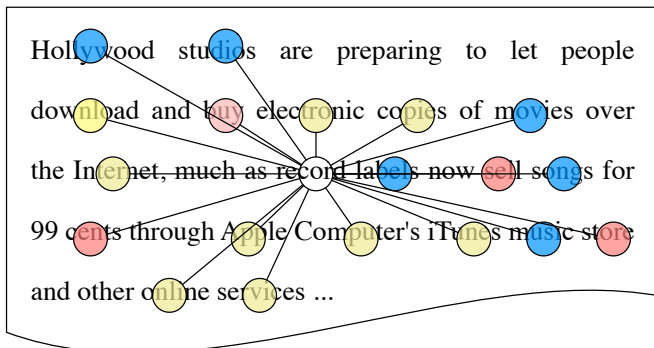
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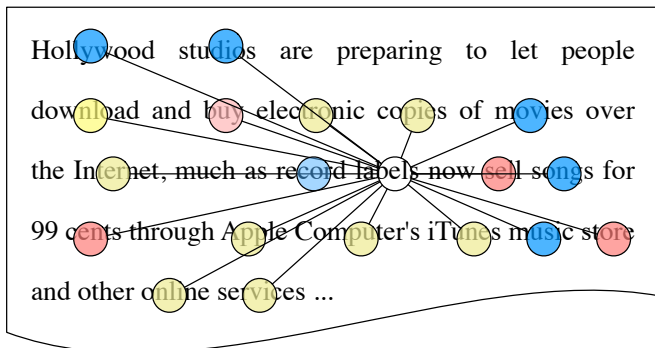
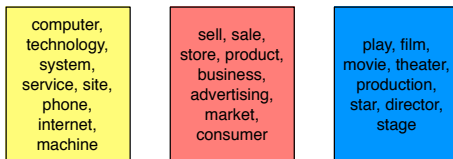
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Gibbs Sampling

- For LDA, we will sample the topic assignments
- Thus, we want:

$$p(z_{d,n} = k | \mathbf{z}_{-d,n}, \mathbf{w}, \alpha, \lambda) = \frac{p(z_{d,n} = k, \mathbf{z}_{-d,n} | \mathbf{w}, \alpha, \lambda)}{p(\mathbf{z}_{-d,n} | \mathbf{w}, \alpha, \lambda)}$$

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- The topics and per-document topic proportions are integrated out / marginalized
- Let $n_{d,i}$ be the number of words taking topic i in document d . Let $v_{k,w}$ be the number of times word w is used in topic k .

$$= \frac{\int_{\theta_d} \left(\prod_{i \neq k} \theta_d^{\alpha_i + n_{d,i} - 1} \right) \theta_d^{\alpha_k + n_{d,k}} d\theta_d \int_{\beta_k} \left(\prod_{i \neq w_{d,n}} \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) \beta_{k,w_{d,n}}^{\lambda_i + v_{k,w_{d,n}}} d\beta_k}{\int_{\theta_d} \left(\prod_i \theta_d^{\alpha_i + n_{d,i} - 1} \right) d\theta_d \int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k}$$

Gibbs Sampling

- Integral is normalizer of Dirichlet distribution

$$\int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k = \frac{\prod_i^V \Gamma(\beta_i + v_{k,i})}{\Gamma(\sum_i^V \beta_i + v_{k,i})}$$

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$$\frac{\frac{\Gamma(\alpha_k + n_{d,k} + 1)}{\Gamma(\sum_i^K \alpha_i + n_{d,i} + 1)} \prod_{i \neq k}^K \Gamma(\alpha_i + n_{d,i})}{\frac{\prod_i^K \Gamma(\alpha_i + n_{d,i})}{\Gamma(\sum_i^K \alpha_i + n_{d,i})}} \frac{\frac{\Gamma(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1)}{\Gamma(\sum_i^V \lambda_i + v_{k,i} + 1)} \prod_{i \neq w_{d,n}}^V \Gamma(\lambda_i + v_{k,i})}{\frac{\prod_i^V \Gamma(\lambda_i + v_{k,i})}{\Gamma(\sum_i^V \lambda_i + v_{k,i})}}$$

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$$\frac{\frac{\Gamma(\alpha_k + n_{d,k} + 1)}{\Gamma(\sum_i^K \alpha_i + n_{d,i} + 1)} \prod_{i \neq k}^K \Gamma(\alpha_i + n_{d,i})}{\frac{\prod_i^K \Gamma(\alpha_i + n_{d,i})}{\Gamma(\sum_i^K \alpha_i + n_{d,i})}} \frac{\frac{\Gamma(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1)}{\Gamma(\sum_i^V \lambda_i + v_{k,i} + 1)} \prod_{i \neq w_{d,n}}^V \Gamma(\lambda_i + v_{k,i})}{\frac{\prod_i^V \Gamma(\lambda_i + v_{k,i})}{\Gamma(\sum_i^V \lambda_i + v_{k,i})}}$$

Gamma Function Identity

$$z = \frac{\Gamma(z+1)}{\Gamma(z)} \quad (3)$$

$$\begin{aligned} & \frac{\frac{\Gamma(\alpha_k + n_{d,k} + 1)}{\Gamma(\sum_i^K \alpha_i + n_{d,i} + 1)} \prod_{i \neq k}^K \Gamma(\alpha_i + n_{d,i})}{\frac{\prod_i^K \Gamma(\alpha_i + n_{d,i})}{\Gamma(\sum_i^K \alpha_i + n_{d,i})}} \frac{\frac{\Gamma(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1)}{\Gamma(\sum_i^V \lambda_i + v_{k,i} + 1)} \prod_{i \neq w_{d,n}}^V \Gamma(\lambda_i + v_{k,i})}{\frac{\prod_i^V \Gamma(\lambda_i + v_{k,i})}{\Gamma(\sum_i^V \lambda_i + v_{k,i})}} \\ &= \frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i^V v_{k,i} + \lambda_i} \end{aligned}$$

Gibbs Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i} \quad (4)$$

- Number of times document d uses topic k
- Number of times topic k uses word type $w_{d,n}$
- Dirichlet parameter for document to topic distribution
- Dirichlet parameter for topic to word distribution
- How much this document likes topic k
- How much this topic likes word $w_{d,n}$

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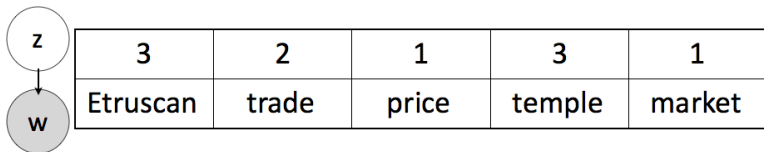
Sample Document

Etruscan	trade	price	temple	market

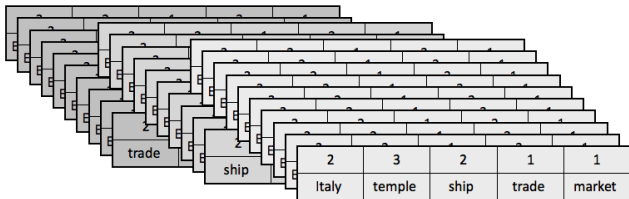
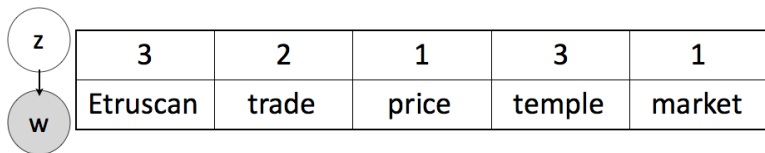
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Randomly Assign Topics



Randomly Assign Topics



Total Topic Counts

3	2	1	3	1
Etruscan	trade	price	temple	market

Total
counts
from **all**
docs

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	8	1
...			

Total Topic Counts

3	2	1	3	1
Etruscan	trade	price	temple	market

Total

	1	2	3
Etruscan	1	0	35
market	50	0	1

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

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...

We want to sample this word ...

3	2	1	3	1
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...			

Decrement its count

3	?	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	7	1
...			



What is the conditional distribution for this topic?

3	?	1	3	1
Etruscan	trade	price	temple	market

Part 1: How much does this document like each topic?

3	?	1	3	1
Etruscan	trade	price	temple	market

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Topic 1



Topic 2



Topic 3



Part 1: How much does this document like each topic?

3	?	1	3	1
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Topic 1 Topic 2 Topic 3

Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

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Part 2: How much does each topic like the word?

3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1



Topic 2



Topic 3



	1	2	3
trade	10	7	1

Part 2: How much does each topic like the word?

3	?	1	3	1
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Topic 1
Sampling Equation

Topic 2

Topic 3

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trade	10	/	1
-------	----	---	---

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Topic 1 Topic 2 Topic 3

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trade	10	/	1
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Geometric interpretation

3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1



Topic 2

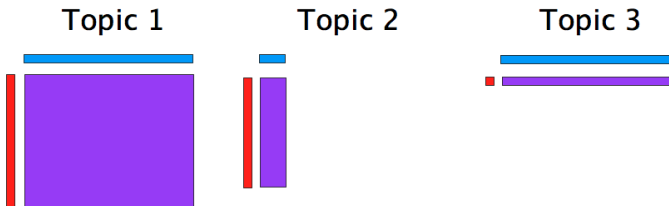


Topic 3



Geometric interpretation

3	?	1	3	1
Etruscan	trade	price	temple	market



Geometric interpretation

3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1



Topic 2




Topic 3



Update counts

3	?	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
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...			



Update counts

3	1	1	3	1
Etruscan	trade	price	temple	market

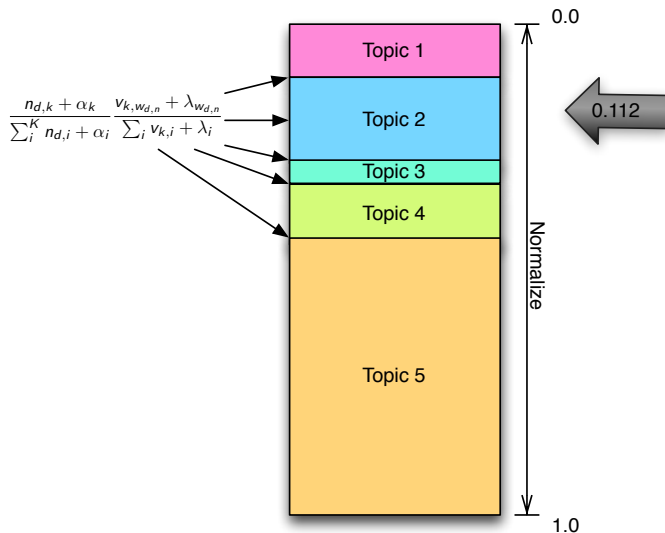
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Details: how to sample from a distribution



Algorithm

- ➊ For each iteration i :
 - ➊ For each document d and word n currently assigned to z_{old} :
 - ➊ Decrement $n_{d,z_{old}}$ and $v_{z_{old},w_{d,n}}$
 - ➋ Sample $z_{new} = k$ with probability proportional to
$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$
 - ➌ Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$

Implementation

Algorithm

- 1 For each iteration i :
 - 1 For each document d and word n currently assigned to z_{old} :
 - 1 Decrement $n_{d,z_{old}}$ and $v_{z_{old},w_{d,n}}$
 - 2 Sample $z_{new} = k$ with probability proportional to
$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$
 - 3 Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$

Desiderata

- Hyperparameters: Sample them too (slice sampling)
- Initialization: Random
- Sampling: Until likelihood converges
- Lag / burn-in: Difference of opinion on this
- Number of chains: Should do more than one

Available implementations

- Mallet (<http://mallet.cs.umass.edu>)
- LDAC (<http://www.cs.princeton.edu/~blei/lda-c>)
- Topicmod (<http://code.google.com/p/topicmod>)

