



Support Vector Machines

Computational Linguistics: Jordan Boyd-Graber
University of Maryland

AUGUST 22, 2018

Slides adapted from Tom Mitchell, Eric Xing, and Lauren Hannah

Roadmap

- Classification: machines labeling data for us
- Previously: naïve Bayes and logistic regression
- This time: SVMs
 - (another) example of linear classifier
 - Good classification accuracy
 - Good theoretical properties

Thinking Geometrically

- Suppose you have two classes: vacations and sports
- Suppose you have four documents

Sports

Doc₁: {ball, ball, ball, travel}

Doc₂: {ball, ball}

Vacations

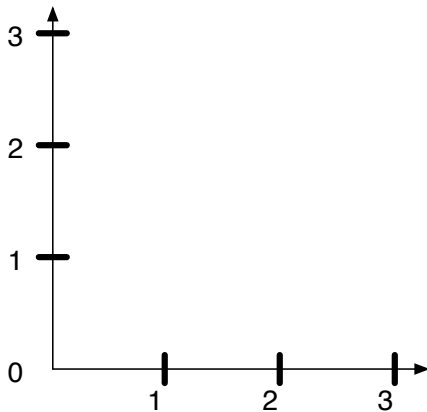
Doc₃: {travel, ball, travel}

Doc₄: {travel}

- What does this look like in vector space?

Put the documents in vector space

Travel



Ball

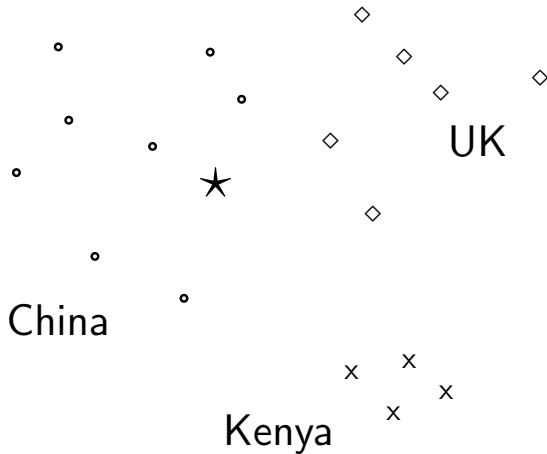
Vector space representation of documents

- Each document is a vector, one component for each term.
- Terms are axes.
- High dimensionality: 10,000s of dimensions and more
- How can we do classification in this space?

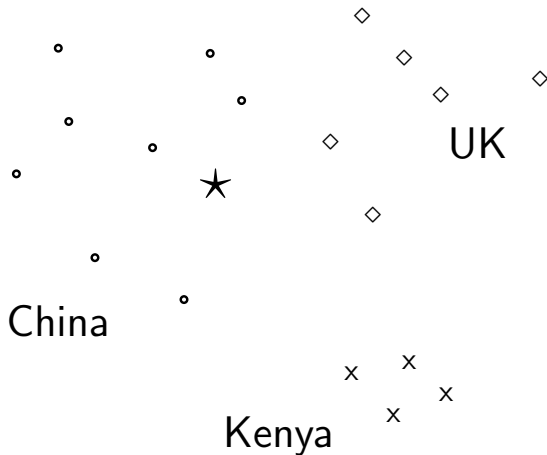
Vector space classification

- As before, the training set is a set of documents, each labeled with its class.
- In vector space classification, this set corresponds to a labeled set of points or vectors in the vector space.
- Premise 1: Documents in the same class form a **contiguous region**.
- Premise 2: Documents from different classes **don't overlap**.
- We define lines, surfaces, hypersurfaces to divide regions.

Classes in the vector space

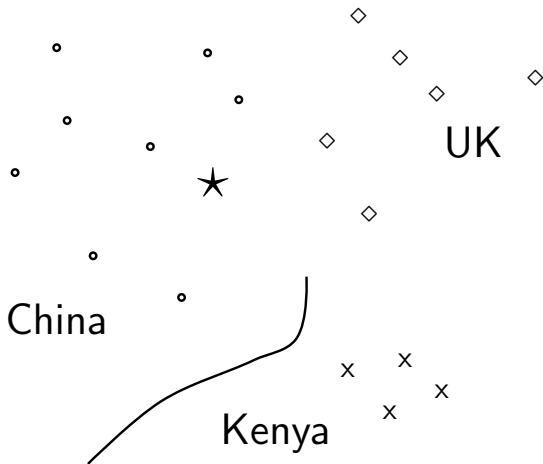


Classes in the vector space



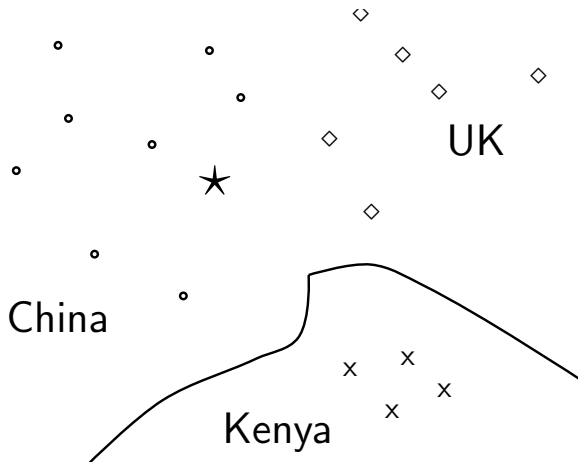
Should the document \star be assigned to China, UK or Kenya?

Classes in the vector space



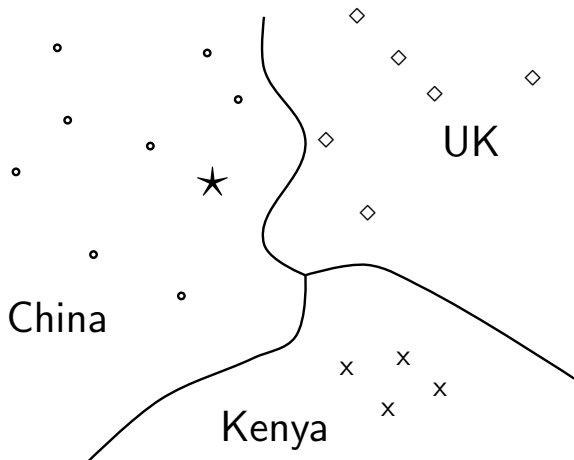
Find separators between the classes

Classes in the vector space



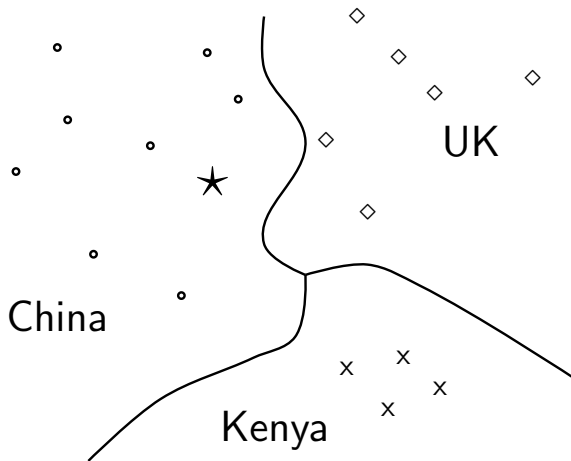
Find separators between the classes

Classes in the vector space



Based on these separators: ★ should be assigned to China

Classes in the vector space



How do we find separators that do a good job at classifying new documents like ★? – Main topic of today

Linear classifiers

- Definition:
 - A linear classifier computes a linear combination or weighted sum $\sum_i \beta_i x_i$ of the feature values.
 - Classification decision: $\sum_i \beta_i x_i > \beta_0$? (β_0 is our bias)
 - ... where β_0 (the threshold) is a parameter.
- We call this the **separator** or **decision boundary**.
- We find the separator based on training set.
- Methods for finding separator: logistic regression, naïve Bayes, linear SVM
- Assumption: The classes are **linearly separable**.

Linear classifiers

- Definition:
 - A linear classifier computes a linear combination or weighted sum $\sum_i \beta_i x_i$ of the feature values.
 - Classification decision: $\sum_i \beta_i x_i > \beta_0$? (β_0 is our bias)
 - ... where β_0 (the threshold) is a parameter.
- We call this the **separator** or **decision boundary**.
- We find the separator based on training set.
- Methods for finding separator: logistic regression, naïve Bayes, linear SVM
- Assumption: The classes are **linearly separable**.
- Before, we just talked about equations. What's the geometric intuition?

A linear classifier in 1D



- A linear classifier in 1D is a point x described by the equation $\beta_1 x_1 = \beta_0$

A linear classifier in 1D



- A linear classifier in 1D is a point x described by the equation $\beta_1 x_1 = \beta_0$
- $x = \beta_0 / \beta_1$

A linear classifier in 1D



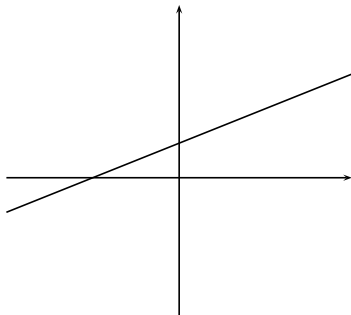
- A linear classifier in 1D is a point x described by the equation $\beta_1 x_1 = \beta_0$
- $x = \beta_0 / \beta_1$
- Points (x_1) with $\beta_1 x_1 \geq \beta_0$ are in the class c .

A linear classifier in 1D



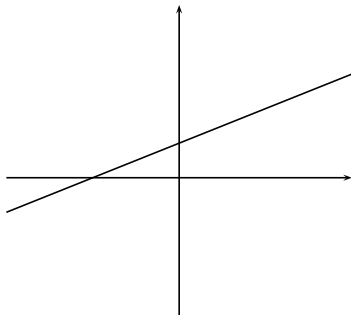
- A linear classifier in 1D is a point x described by the equation $\beta_1 x_1 = \beta_0$
- $x = \beta_0 / \beta_1$
- Points (x_1) with $\beta_1 x_1 \geq \beta_0$ are in the class c .
- Points (x_1) with $\beta_1 x_1 < \beta_0$ are in the complement class \bar{c} .

A linear classifier in 2D



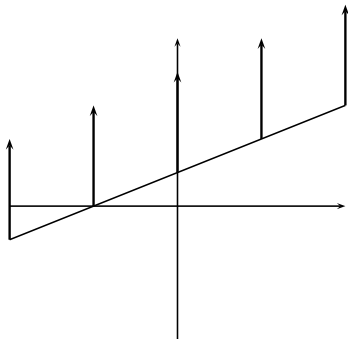
- A linear classifier in 2D is a line described by the equation $\beta_1 x_1 + \beta_2 x_2 = \beta_0$

A linear classifier in 2D



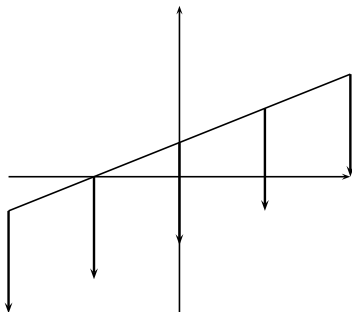
- A linear classifier in 2D is a line described by the equation $\beta_1 x_1 + \beta_2 x_2 = \beta_0$
- Example for a 2D linear classifier

A linear classifier in 2D



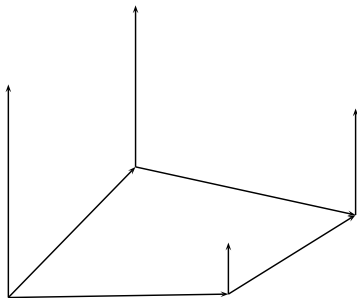
- A linear classifier in 2D is a line described by the equation $\beta_1 x_1 + \beta_2 x_2 = \beta_0$
- Example for a 2D linear classifier
- Points $(x_1 \ x_2)$ with $\beta_1 x_1 + \beta_2 x_2 \geq \beta_0$ are in the class c .

A linear classifier in 2D



- A linear classifier in 2D is a line described by the equation $\beta_1 x_1 + \beta_2 x_2 = \beta_0$
- Example for a 2D linear classifier
- Points $(x_1 \ x_2)$ with $\beta_1 x_1 + \beta_2 x_2 \geq \beta_0$ are in the class c .
- Points $(x_1 \ x_2)$ with $\beta_1 x_1 + \beta_2 x_2 < \beta_0$ are in the complement class \bar{c} .

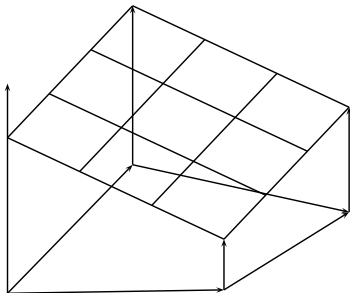
A linear classifier in 3D



- A linear classifier in 3D is a plane described by the equation

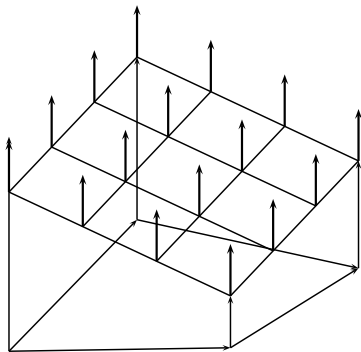
$$\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = \beta_0$$

A linear classifier in 3D



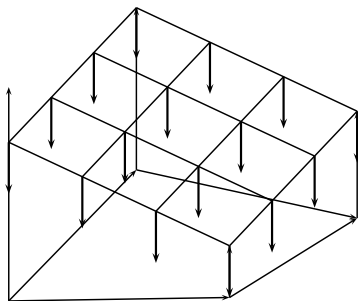
- A linear classifier in 3D is a plane described by the equation
$$\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = \beta_0$$
- Example for a 3D linear classifier

A linear classifier in 3D



- A linear classifier in 3D is a plane described by the equation
$$\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = \beta_0$$
- Example for a 3D linear classifier
- Points $(x_1 \ x_2 \ x_3)$ with
$$\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \geq \beta_0$$
are in the class c .

A linear classifier in 3D



- A linear classifier in 3D is a plane described by the equation
$$\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = \beta_0$$
- Example for a 3D linear classifier
- Points $(x_1 \ x_2 \ x_3)$ with
$$\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \geq \beta_0$$
are in the class c .
- Points $(x_1 \ x_2 \ x_3)$ with
$$\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 < \beta_0$$
are in the complement class \bar{c} .

Naive Bayes and Logistic Regression as linear classifiers

Multinomial Naive Bayes is a linear classifier (in log space) defined by:

$$\sum_{i=1}^M \beta_i x_i = \beta_0$$

where $\beta_i = \log[\hat{P}(t_i|c)/\hat{P}(t_i|\bar{c})]$, x_i = number of occurrences of t_i in d , and $\beta_0 = -\log[\hat{P}(c)/\hat{P}(\bar{c})]$. Here, the index i , $1 \leq i \leq M$, refers to terms of the vocabulary.

Logistic regression is the same (we only put it into the logistic function to turn it into a probability).

Naive Bayes and Logistic Regression as linear classifiers

Multinomial Naive Bayes is a linear classifier (in log space) defined by:

$$\sum_{i=1}^M \beta_i x_i = \beta_0$$

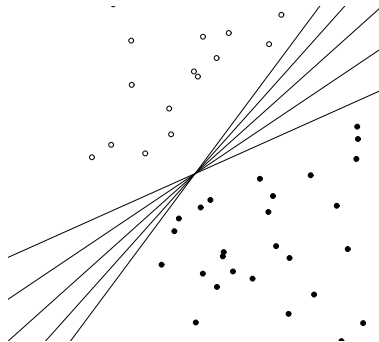
where $\beta_i = \log[\hat{P}(t_i|c)/\hat{P}(t_i|\bar{c})]$, x_i = number of occurrences of t_i in d , and $\beta_0 = -\log[\hat{P}(c)/\hat{P}(\bar{c})]$. Here, the index i , $1 \leq i \leq M$, refers to terms of the vocabulary.

Logistic regression is the same (we only put it into the logistic function to turn it into a probability).

Takeway

Naïve Bayes, logistic regression and SVM are all linear methods. They choose their hyperplanes based on different objectives: joint likelihood (NB), conditional likelihood (LR), and the margin (SVM).

Which hyperplane?



Which hyperplane?

- For linearly separable training sets: there are **infinitely** many separating hyperplanes.
- They all separate the training set perfectly . . .
- . . . but they behave differently on test data.
- Error rates on new data are low for some, high for others.
- How do we find a low-error separator?

Support vector machines

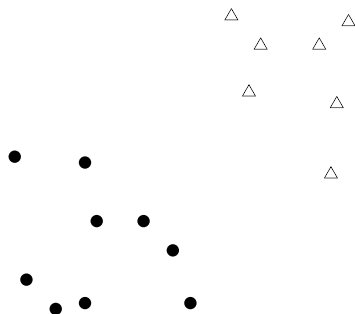
- Machine-learning research in the last two decades has improved classifier effectiveness.
- New generation of state-of-the-art classifiers: support vector machines (SVMs), boosted decision trees, regularized logistic regression, neural networks, and random forests
- Applications to IR problems, particularly text classification

SVMs: A kind of large-margin classifier

Vector space based machine-learning method aiming to find a decision boundary between two classes that is maximally far from any point in the training data (possibly discounting some points as outliers or noise)

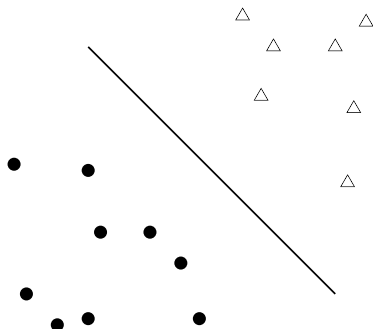
Support Vector Machines

- 2-class training data



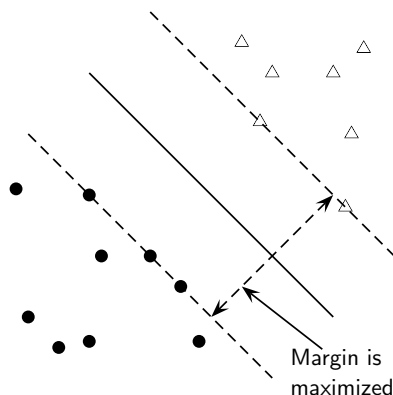
Support Vector Machines

- 2-class training data
- decision boundary →
linear separator



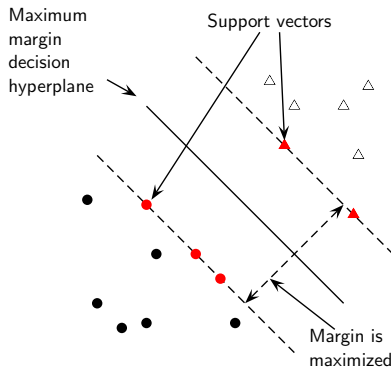
Support Vector Machines

- 2-class training data
- decision boundary → **linear separator**
- criterion: being maximally far away from any data point → determines classifier **margin**



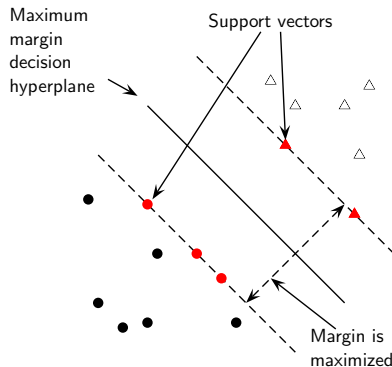
Support Vector Machines

- 2-class training data
- decision boundary → **linear separator**
- criterion: being maximally far away from any data point → determines classifier **margin**
- linear separator position defined by **support vectors**



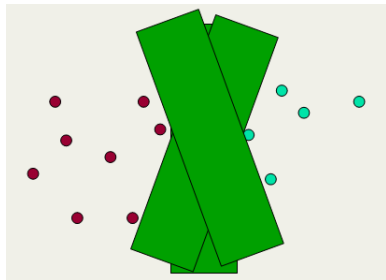
Why maximize the margin?

- Points near decision surface \rightarrow uncertain classification decisions
- A classifier with a large margin is always confident
- Gives classification safety margin (measurement or variation)



Why maximize the margin?

- SVM classifier: large margin around decision boundary
- compare to decision hyperplane: place fat separator between classes
 - unique solution
- decreased memory capacity
- increased ability to correctly generalize to test data



Equation

- Equation of a hyperplane

$$\vec{w} \cdot x_i + b = 0 \quad (1)$$

- Distance of a point to hyperplane

$$\frac{|\vec{w} \cdot x_i + b|}{\|\vec{w}\|} \quad (2)$$

- The margin ρ is given by

$$\rho \equiv \min_{(x,y) \in S} \frac{|\vec{w} \cdot x_i + b|}{\|\vec{w}\|} = \frac{1}{\|\vec{w}\|} \quad (3)$$

Equation

- Equation of a hyperplane

$$\vec{w} \cdot x_i + b = 0 \quad (1)$$

- Distance of a point to hyperplane

$$\frac{|\vec{w} \cdot x_i + b|}{\|\vec{w}\|} \quad (2)$$

- The margin ρ is given by

$$\rho \equiv \min_{(x,y) \in S} \frac{|\vec{w} \cdot x_i + b|}{\|\vec{w}\|} = \frac{1}{\|\vec{w}\|} \quad (3)$$

- This is because for any point on the marginal hyperplane, $\vec{w} \cdot x + b = \pm 1$

Optimization Problem

We want to find a weight vector \vec{w} and bias b that optimize

$$\min_{\vec{w}, b} \frac{1}{2} \|\vec{w}\|^2 \quad (4)$$

subject to $y_i(\vec{w} \cdot x_i + b) \geq 1, \forall i \in [1, m]$.

Choosing what kind of classifier to use

When building a text classifier, first question: **how much training data is there currently available?**

- None?
- Very little?
- A fair amount?
- A huge amount

Choosing what kind of classifier to use

When building a text classifier, first question: **how much training data is there currently available?**

- None? **Hand write rules or use active learning**
- Very little?
- A fair amount?
- A huge amount

Choosing what kind of classifier to use

When building a text classifier, first question: **how much training data is there currently available?**

- None? **Hand write rules or use active learning**
- Very little? **Naïve Bayes**
- A fair amount?
- A huge amount

Choosing what kind of classifier to use

When building a text classifier, first question: **how much training data is there currently available?**

- None? **Hand write rules or use active learning**
- Very little? **Naïve Bayes**
- A fair amount? **SVM**
- A huge amount

Choosing what kind of classifier to use

When building a text classifier, first question: **how much training data is there currently available?**

- None? **Hand write rules or use active learning**
- Very little? **Naïve Bayes**
- A fair amount? **SVM**
- A huge amount **Doesn't matter, use whatever works**

SVM extensions: What's next

- Finding solutions
- Slack variables: not perfect line
- Kernels: different geometries

