Language Models

Advanced Machine Learning for NLP

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FOUNDATIONS

Roadmap

After this class, you'll be able to:

- Give examples of where we need language models
- Evaluate language models
- Connection between Bayesian nonparametrics and backoff

Language models

- **Language models** answer the question: *How likely is a string of English words good English?*
- Autocomplete on phones and websearch
- Creating English-looking documents
- Very common in machine translation systems
    - Help with reordering / style

$$p_{\text{lm}}(\text{the house is small}) > p_{\text{lm}}(\text{small the is house})$$

    - Help with word choice

$$p_{\text{lm}}(\text{I am going home}) > p_{\text{lm}}(\text{I am going house})$$

N-Gram Language Models

- Given: a string of English words $W = w_1, w_2, w_3, ..., w_n$
- Question: what is $p(W)$?
- Sparse data: Many good English sentences will not have been seen before

→ Decomposing $p(W)$ using the chain rule:

$$
\begin{aligned}
&p(w_1, w_2, w_3, ..., w_n) = \\
&\qquad p(w_1)\, p(w_2|w_1)\, p(w_3|w_1, w_2) \dots p(w_n|w_1, w_2, ... w_{n-1})
\end{aligned}
$$

(not much gained yet, $p(w_n|w_1, w_2, ... w_{n-1})$ is equally sparse)

Markov Chain

- **Markov independence assumption**:
  - only previous history matters
  - limited memory: only last k words are included in history (older words less relevant)

⟶ k**th order Markov model**

- For instance 2-gram language model:

$$p(w_1, w_2, w_3, ..., w_n) \simeq p(w_1)\, p(w_2|w_1)\, p(w_3|w_2) ... p(w_n|w_{n-1})$$

- What is conditioned on, here w_{i-1} is called the **history**

How good is the LM?

- A good model assigns a text of real English W a high probability
- This can be also measured with **perplexity**

$$
\begin{aligned}
\text{perplexity}(W) &= P(w_1, \dots w_N)^{-\frac{1}{N}} \\
&= \sqrt[N]{\prod_i^N \frac{1}{P(w_i | w_1 \dots w_{i-1})}}
\end{aligned}
$$

Comparison 1–4-Gram

word	unigram	bigram	trigram	4-gram
i	6.684	3.197	3.197	3.197
would	8.342	2.884	2.791	2.791
like	9.129	2.026	1.031	1.290
to	5.081	0.402	0.144	0.113
commend	15.487	12.335	8.794	8.633
the	3.885	1.402	1.084	0.880
reporter	10.840	7.319	2.763	2.350
.	4.896	3.020	1.785	1.510
</s>	4.828	0.005	0.000	0.000
average				
perplexity	265.136	16.817	6.206	4.758

Example: 3-Gram

- Counts for trigrams and estimated word probabilities

the red (total: 225)

word	c.	prob.
cross	123	0.547
tape	31	0.138
army	9	0.040
card	7	0.031
,	5	0.022

  - ○ 225 trigrams in the Europarl corpus start with **the red**
  - ○ 123 of them end with **cross**

⟶ maximum likelihood probability is $\frac{123}{225} = 0.547$.

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- Can't use ML estimate

How do we estimate a probability?

- Assuming a **sparse Dirichlet** prior, $\alpha < 1$ to each count

$$\theta_i = \frac{n_i + \alpha_i}{\sum_k n_k + \alpha_k} \tag{1}$$

- α_i is called a smoothing factor, a pseudocount, etc.

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- What is a good value for α?
- Could be optimized on held-out set to find the "best" language model

Example: 2-Grams in Europarl

Count	Adjusted count		Test count
c	$(c+1)$	$(c+\alpha)$	t_c
0	0.00378	0.00016	0.00016
1	0.00755	0.95725	0.46235
2	0.01133	1.91433	1.39946
3	0.01511	2.87141	2.34307
4	0.01888	3.82850	3.35202
5	0.02266	4.78558	4.35234
6	0.02644	5.74266	5.33762
8	0.03399	7.65683	7.15074
10	0.04155	9.57100	9.11927
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Can we do better?

In higher-order models, we can learn from similar contexts!

Back-Off

- In given corpus, we may never observe
  - **Scottish beer drinkers**
  - **Scottish beer eaters**
- Both have count 0

  → our smoothing methods will assign them same probability
- Better: backoff to bigrams:
  - **beer drinkers**
  - **beer eaters**

Interpolation

- Higher and lower order n-gram models have different strengths and weaknesses
  - high-order n-grams are sensitive to more context, but have sparse counts
  - low-order n-grams consider only very limited context, but have robust counts
- Combine them

$$\begin{aligned} p_I(w_3|w_1,w_2) = & \quad \lambda_1 \, p_1(w_3) \\ & + \lambda_2 \, p_2(w_3|w_2) \\ & + \lambda_3 \, p_3(w_3|w_1,w_2) \end{aligned}$$

Back-Off

- Trust the highest order language model that contains n-gram

$$p_n^{BO}(w_i|w_{i-n+1},...,w_{i-1}) = \\ = \begin{cases} \alpha_n(w_i|w_{i-n+1},...,w_{i-1}) \\ \qquad\qquad \text{if count}_n(w_{i-n+1},...,w_i) > 0 \\ d_n(w_{i-n+1},...,w_{i-1})\, p_{n-1}^{BO}(w_i|w_{i-n+2},...,w_{i-1}) \\ \qquad\qquad \text{else} \end{cases}$$

- Requires
  - adjusted prediction model $\alpha_n(w_i|w_{i-n+1},...,w_{i-1})$
  - discounting function $d_n(w_1,...,w_{n-1})$

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- Requires
    - adjusted prediction model $\alpha_n(w_i|w_{i-n+1},...,w_{i-1})$
    - discounting function $d_n(w_1,...,w_{n-1})$
    - More next time

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- There are an infinite number of words
  - Possible to develop generative story of how new words are created
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- But how do you handle words outside of your vocabulary?

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- There are an infinite number of words
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- Defining a vocabulary (the event space)
- But how do you handle words outside of your vocabulary?
  - Ignore? You could win just by ignoring everything
  - Standard: replace with `<UNK>` token
- Next week: word representations!

Reducing Vocabulary Size

- For instance: each number is treated as a separate token
- Replace them with a number token num
  - but: we want our language model to prefer

$$p_{\text{lm}}(\text{I pay 950.00 in May 2007}) > p_{\text{lm}}(\text{I pay 2007 in May 950.00})$$

  - not possible with number token

$$p_{\text{lm}}(\text{I pay num in May num}) = p_{\text{lm}}(\text{I pay num in May num})$$

- Replace each digit (with unique symbol, e.g., **@** or **5**), retain some distinctions

$$p_{\text{lm}}(\text{I pay 555.55 in May 5555}) > p_{\text{lm}}(\text{I pay 5555 in May 555.55})$$