



Boosting

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SLIDES ADAPTED FROM ROB SCHAPIRE

Motivating Example

Goal

Automatically categorize type of call requested by phone customer (Collect, CallingCard, PersonToPerson, etc.)

- yes I'd like to place a collect call long distance please (Collect)
- operator I need to make a call but I need to bill it to my office (ThirdNumber)
- yes I'd like to place a call on my master card please (CallingCard)
- I just called a number in sioux city and I musta rang the wrong number because I got the wrong party and I would like to have that taken off of my bill (BillingCredit)

Boosting Approach

- devise computer program for deriving rough rules of thumb
- apply procedure to subset of examples
- obtain rule of thumb
- apply to second subset of examples
- obtain second rule of thumb
- repeat T times

Details

- How to **choose** examples
- How to **combine** rules of thumb

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concentrate on *hardest* examples (those most often misclassified by previous rules of thumb)
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concentrate on *hardest* examples (those most often misclassified by previous rules of thumb)
- How to **combine** rules of thumb
take (weighted) majority vote of rules of thumb

Boosting

Definition

general method of converting rough rules of thumb into highly accurate prediction rule

- assume given *weak learning algorithm* that can consistently find classifiers (rules of thumb) at least slightly better than random, say, accuracy $\geq 55\%$ (in two-class setting)
- given sufficient data, a boosting algorithm can provably construct single classifier with very high accuracy, say, 99%

Formal Description

- Training set $(x_1, y_1) \dots (x_m, y_m)$
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 - Construct distribution D_t on $\{1, \dots, m\}$
 - Find weak classifier

$$h_t: \mathcal{X} \mapsto \{-1, +1\} \quad (1)$$

with small error ϵ_t on D_t :

$$\epsilon_t = \Pr_{i \sim D_t} [h_t(x_i) \neq y_i] \quad (2)$$

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- Output final classifier H_{final}

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$$D_{t+1}(i) \propto D_t(i) \cdot \exp\{-\alpha_t y_i h_t(x_i)\} \quad (3)$$

where $\alpha_t = \frac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right) > 0$

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Bigger if wrong, smaller if right

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Weight by how good the weak learner is

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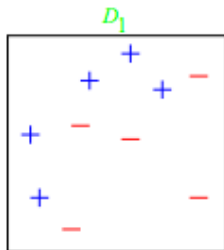
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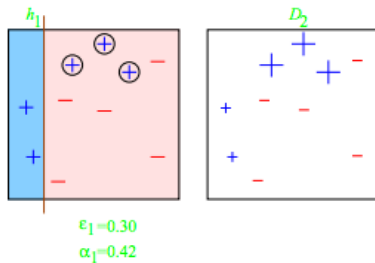
- Final classifier:

$$H_{fin}(x) = \text{sign}\left(\sum_t \alpha_t h_t(x)\right) \quad (4)$$

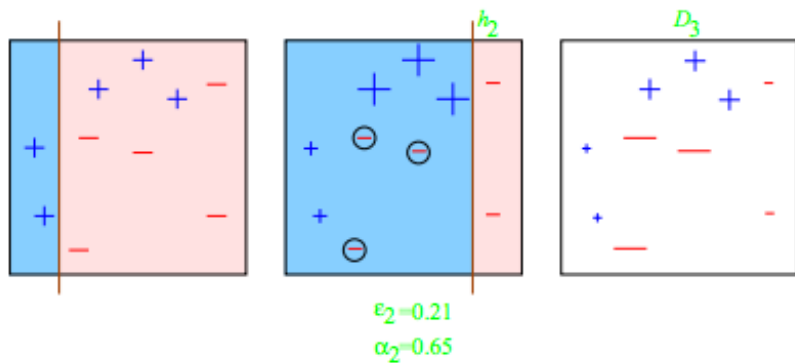
Toy Example



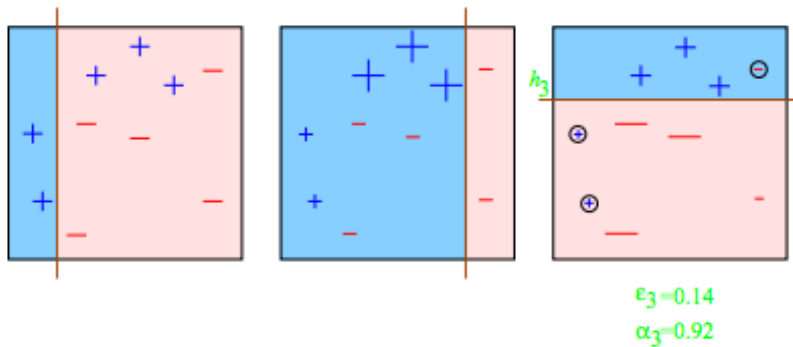
Round 1



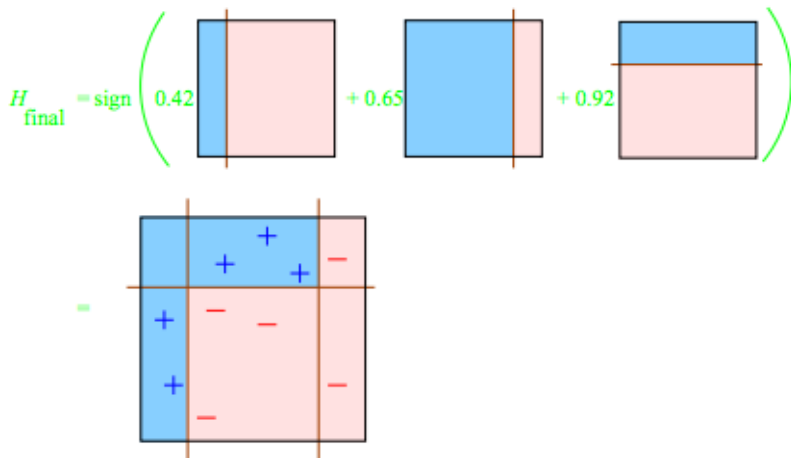
Round 2



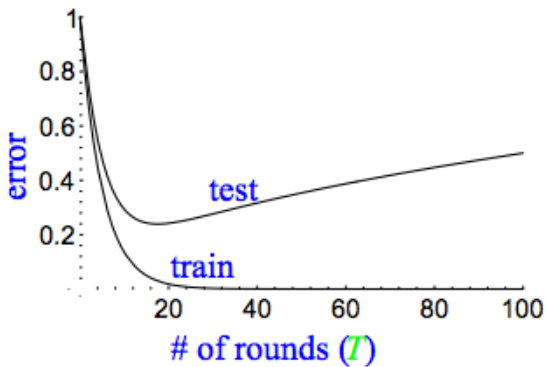
Round 3



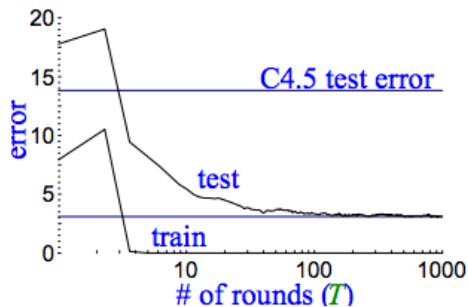
Final Classifier



Generalization



Generalization



(boosting C4.5 on
"letter" dataset)

Training Error

First, we can prove that the training error goes down. If we write the error at time t as $\frac{1}{2} - \gamma_t$,

$$\hat{R}(h) \leq \exp \left\{ -2 \sum_t \gamma_t^2 \right\} \quad (5)$$

- If $\forall t : \gamma_t \geq \gamma > 0$, then $\hat{R}(h) \leq \exp \{ -2\gamma^2 T \}$

Adaboost: do not need γ or T a priori

Training Error Proof: Preliminaries

Repeatedly expand the definition of the distribution.

$$D_{t+1}(i) = \frac{D_t(i) \exp \{-\alpha_t y_i h_t(x_i)\}}{Z_t} \quad (6)$$

$$\frac{D_{t-1}(i) \exp \{-\alpha_{t-1} y_i h_{t-1}(x_i)\} \exp \{-\alpha_t y_i h_t(x_i)\}}{Z_{t-1} Z_t} \quad (7)$$

$$\frac{\exp \{-y_i \sum_{s=1}^t \alpha_s h_s(x_i)\}}{m \prod_{s=1}^t Z_s} \quad (8)$$

Training Error Intuition

- On round t weight of examples incorrectly classified by h_t is increased
- If x_i incorrectly classified by H_T , then x_i wrong on (weighted) majority of h_t 's
 - If x_i incorrectly classified by H_T , then x_i must have large weight under D_T
 - But there can't be many of them, since total weight ≤ 1

Training Error Proof: It's all about the Normalizers

$$\hat{R}(h) = \frac{1}{m} \sum_{i=1}^m \mathbb{1} [y_i g(x_i) \leq 0] \quad (9)$$

(10)

Definition of training error

Training Error Proof: It's all about the Normalizers

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$$\leq \frac{1}{m} \sum_{i=1}^m \exp\{-y_i g(x_i)\} \quad (10)$$

$$(11)$$

$\mathbb{1}[u \leq 0] \leq \exp -u$ is true for all real u .

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Final distribution $D_{t+1}(i)$

$$D_{t+1}(i) = \frac{\exp\{-y_i \sum_{s=1}^t \alpha_s h_s(x_i)\}}{m \prod_{s=1}^t Z_s} \quad (12)$$

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$$= \frac{1}{m} \sum_{i=1}^m \left[m \prod_{t=1}^T Z_t \right] D_{T+1}(i) \quad (11)$$

$$(12)$$

m 's cancel, D is a distribution

Training Error Proof: It's all about the Normalizers

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Training Error Proof: Weak Learner Errors

Single Weak Learner

$$Z_t = \sum_{i=1}^m D_t(i) \exp \{ -\alpha_t y_i h_t(x_i) \} \quad (13)$$

$$= \quad (14)$$

$$= \quad (15)$$

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$$= (1 - \epsilon_t) \sqrt{\frac{\epsilon_t}{1 - \epsilon_t}} + \epsilon_t \sqrt{\frac{1 - \epsilon_t}{\epsilon_t}} \quad (16)$$

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Normalization Product

$$\prod_{t=1}^T Z_t = \prod_{t=1}^T 2\sqrt{\epsilon_t(1 - \epsilon_t)} = \sqrt{1 - 4\left(\frac{1}{2} - \epsilon_t\right)^2} \quad (14)$$

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$$=\exp\left\{-2\sum_{t=1}^T\left(\frac{1}{2}-\epsilon_t\right)^2\right\} \quad (15)$$

Generalization

VC Dimension

$$\leq 2(d+1)(T+1)\lg[(T+1)e]$$

Margin-based Analysis

AdaBoost maximizes a linear program maximizes an L_1 margin, and the weak learnability assumption requires data to be linearly separable with margin 2γ

Practical Advantages of AdaBoost

- fast
- simple and easy to program
- no parameters to tune (except T)
- flexible: can combine with any learning algorithm
- no prior knowledge needed about weak learner
- provably effective, provided can consistently find rough rules of thumb
 - shift in mind set: goal now is merely to find classifiers barely better than random guessing
- versatile
 - can use with data that is textual, numeric, discrete, etc.
 - has been extended to learning problems well beyond binary classification

Caveats

- performance of AdaBoost depends on data and weak learner
- consistent with theory, AdaBoost can fail if
- weak classifiers too complex
 - overfitting
- weak classifiers too weak ($\gamma_t \rightarrow 0$ too quickly)
 - underfitting
 - low margins \rightarrow overfitting
- empirically, AdaBoost seems especially susceptible to uniform noise