Topic Models

Advanced Machine Learning for NLP

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SLIDES ADAPTED FROM DAVID MIMNO

Learning the Hidden Space

- Two major tools:
  - Gibbs Sampling: Easier to implement, easier to understand
  - Variational Inference: faster, harder to implement
- Variational shows the connections to “deep” models better, so it’s the focus
- However, would be injustice to not at least discuss Gibbs sampling

Inference

- We are interested in posterior distribution

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$$p(\boldsymbol{z}, \boldsymbol{\beta}, \boldsymbol{\theta} | \boldsymbol{w}, \alpha, \lambda) \tag{2}$$

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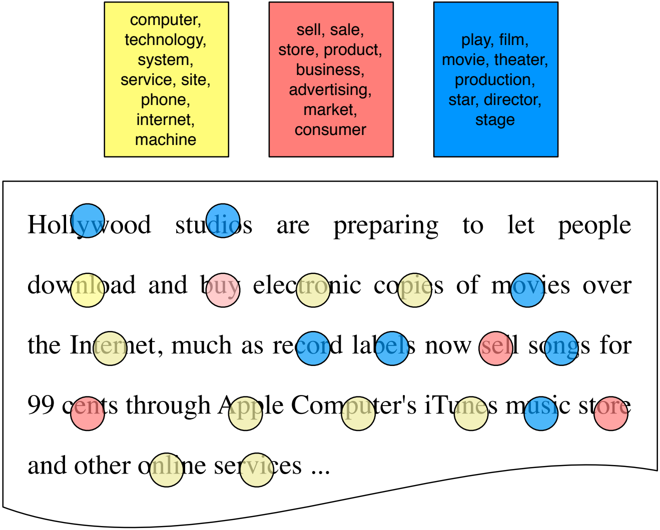
$$p(\boldsymbol{z},\boldsymbol{\beta},\boldsymbol{\theta}|\boldsymbol{w},\alpha,\lambda) \tag{2}$$

$$p(\boldsymbol{w},\boldsymbol{z},\boldsymbol{\theta},\boldsymbol{\beta}|\alpha,\lambda) = \prod_k p(\beta_k|\lambda)\prod_d p(\theta_d|\alpha)\prod_n p(z_{d,n}|\theta_d)p(w_{d,n}|\beta_{z_{d,n}})$$

Gibbs Sampling

- A form of Markov Chain Monte Carlo
- Chain is a sequence of random variable states
- Given a state $\{z_1, \dots z_N\}$ given certain technical conditions, drawing $z_k \sim p(z_1, \dots z_{k-1}, z_{k+1}, \dots z_N | X, \Theta)$ for all k (repeatedly) results in a Markov Chain whose stationary distribution *is* the posterior.
- For notational convenience, call $\boldsymbol{z}$ with $z_{d,n}$ removed $\boldsymbol{z}_{-d,n}$

Inference


computer, technology, system, service, site, phone, internet, machine
sell, sale, store, product, business, advertising, market, consumer
play, film, movie, theater, production, star, director, stage
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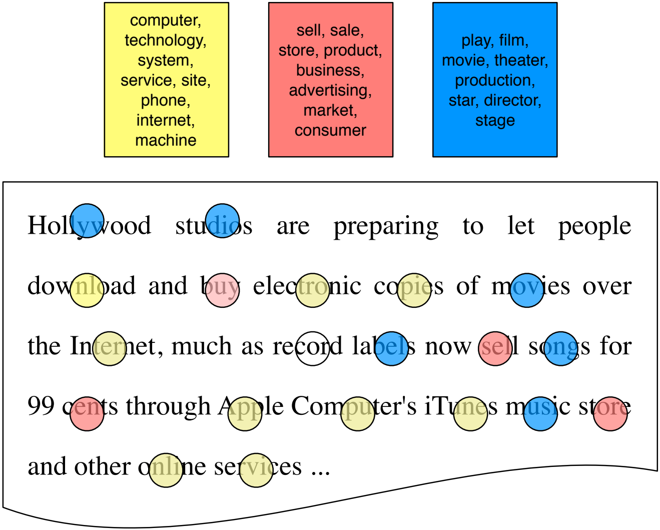
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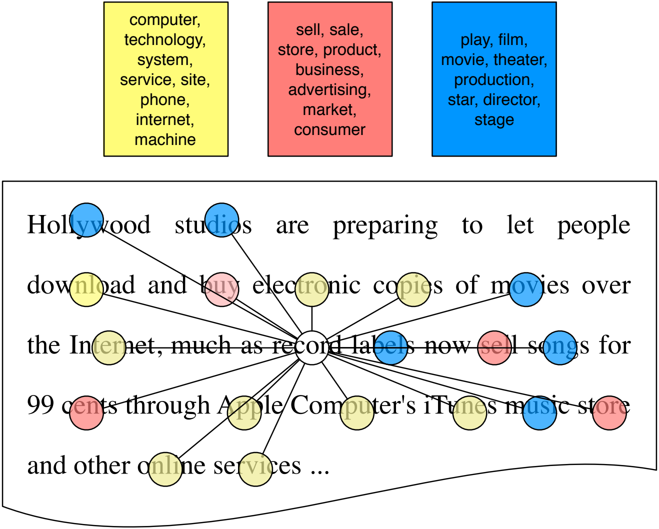
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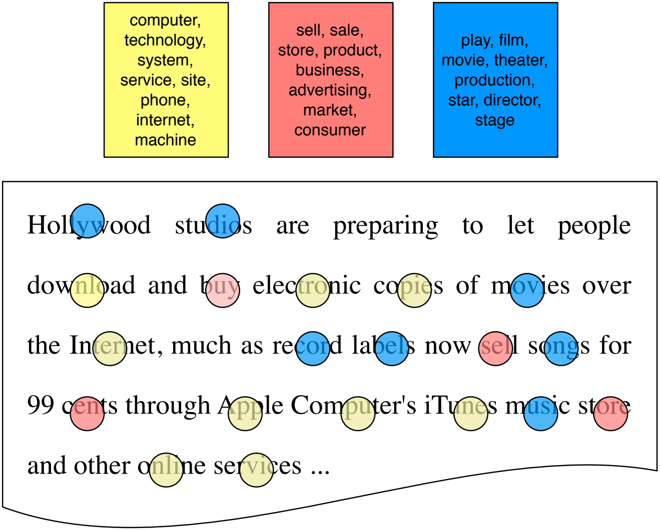
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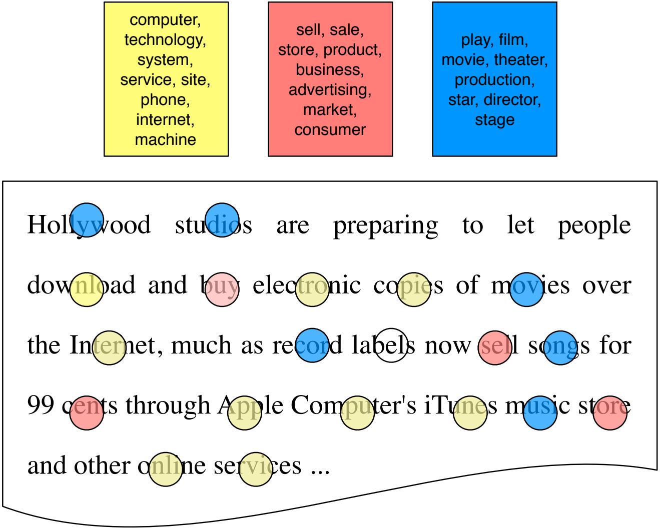
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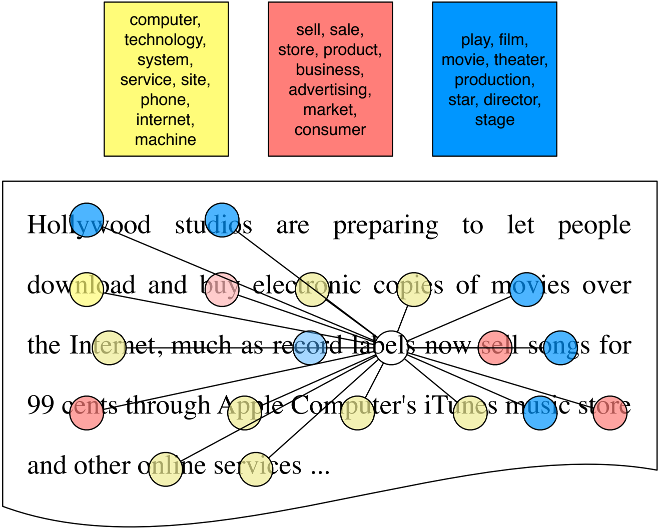

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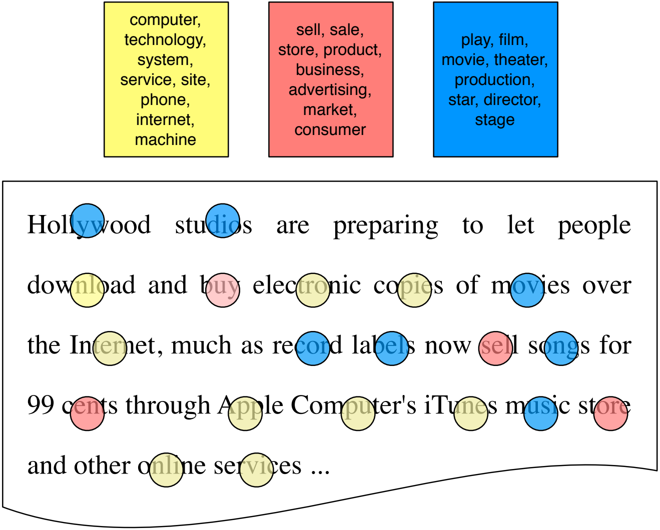
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Gibbs Sampling

- For LDA, we will sample the topic assignments
- Thus, we want:

$$p(z_{d,n}=k|\boldsymbol{z}_{-d,n},\boldsymbol{w},\alpha,\lambda)=\frac{p(z_{d,n}=k,\boldsymbol{z}_{-d,n}|\boldsymbol{w},\alpha,\lambda)}{p(\boldsymbol{z}_{-d,n}|\boldsymbol{w},\alpha,\lambda)}$$

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- The topics and per-document topic proportions are integrated out / marginalized
- Let $n_{d,i}$ be the number of words taking topic i in document d. Let $v_{k,w}$ be the number of times word w is used in topic k.

$$=\frac{\int_{\theta_d}\left(\prod_{i\neq k}\theta_d^{\alpha_i+n_{d,i}-1}\right)\theta_d^{\alpha_k+n_{d,k}}d\theta_d\int_{\beta_k}\left(\prod_{i\neq w_{d,n}}\beta_{k,i}^{\lambda_i+v_{k,i}-1}\right)\beta_{k,w_{d,n}}^{\lambda_i+v_{k,w_{d,n}}}d\beta_k}{\int_{\theta_d}\left(\prod_i\theta_d^{\alpha_i+n_{d,i}-1}\right)d\theta_d\int_{\beta_k}\left(\prod_i\beta_{k,i}^{\lambda_i+v_{k,i}-1}\right)d\beta_k}$$

Gibbs Sampling

- Integral is normalizer of Dirichlet distribution

$$\int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k = \frac{\prod_i^V \Gamma\left(\beta_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \beta_i + v_{k,i}\right)}$$

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$$\frac{\int_{\theta_d} \left(\prod_{i \neq k} \theta_d^{\alpha_i + n_{d,i} - 1}\right) \theta_d^{\alpha_k + n_{d,k}} \, d\theta_d \int_{\beta_k} \left(\prod_{i \neq w_{d,n}} \beta_{k,i}^{\lambda_i + v_{k,i} - 1}\right) \beta_{k,w_{d,n}}^{\lambda_i + v_{k,w_{d,n}}} d\beta_k}{\int_{\theta_d} \left(\prod_i \theta_d^{\alpha_i + n_{d,i} - 1}\right) d\theta_d \int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1}\right) d\beta_k} =$$

$$\frac{\frac{\Gamma(\alpha_k + n_{d,k} + 1)}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i} + 1\right)} \prod_{i \neq k}^K \Gamma\left(\alpha_k + n_{d,k}\right)}{\frac{\prod_i^K \Gamma(\alpha_i + n_{d,i})}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i}\right)}} \frac{\frac{\Gamma\left(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i} + 1\right)} \prod_{i \neq w_{d,n}}^V \Gamma\left(\lambda_k + v_{k,w_{d,n}}\right)}{\frac{\prod_i^V \Gamma\left(\lambda_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i}\right)}}$$

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Gamma Function Identity

$$z = \frac{\Gamma(z+1)}{\Gamma(z)} \tag{3}$$

$$\frac{\frac{\Gamma(\alpha_k + n_{d,k} + 1)}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i} + 1\right)} \prod_{i \neq k}^K \Gamma\left(\alpha_k + n_{d,k}\right)}{\frac{\prod_i^K \Gamma(\alpha_i + n_{d,i})}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i}\right)}} \frac{\frac{\Gamma\left(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i} + 1\right)} \prod_{i \neq w_{d,n}}^V \Gamma\left(\lambda_k + v_{k,w_{d,n}}\right)}{\frac{\prod_i^V \Gamma\left(\lambda_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i}\right)}}$$

$$= \frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

Gibbs Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i} \tag{4}$$

- Number of times document d uses topic k
- Number of times topic k uses word type $w_{d,n}$
- Dirichlet parameter for document to topic distribution
- Dirichlet parameter for topic to word distribution
- How much this document likes topic k
- How much this topic likes word $w_{d,n}$

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Sample Document

Etruscan	trade	price	temple	market

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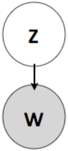
Randomly Assign Topics



3	2	1	3	1
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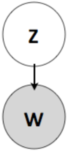

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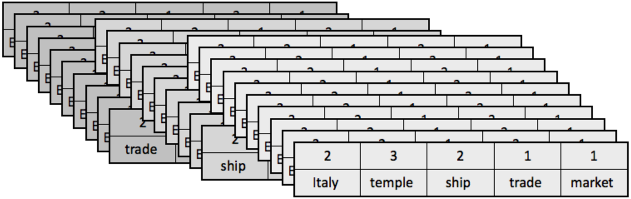

Total Topic Counts

3	2	1	3	1
Etruscan	trade	price	temple	market

Total counts from **all** docs

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	8	1
...			

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We want to sample this word . . .

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Decrement its count

3	?	1	3	1
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	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	10	**7**	1
...			

What is the conditional distribution for this topic?

3	?	1	3	1
Etruscan	trade	price	temple	market

Part 1: How much does this document like each topic?

3	?	1	3	1
Etruscan	trade	price	temple	market

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Topic 1 Topic 2 Topic 3

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Part 2: How much does each topic like the word?

3	?	1	3	1
Etruscan	trade	price	temple	market

Topic 1 Topic 2 Topic 3

	1	2	3
trade	10	7	1

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Geometric interpretation

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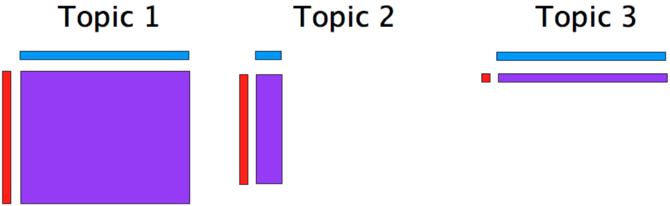

Geometric interpretation

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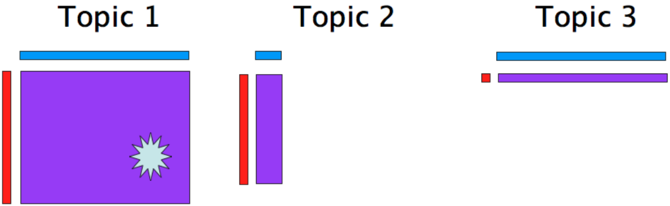

Update counts

3	?	1	3	1
Etruscan	trade	price	temple	market

	1	2	3
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Update counts

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Etruscan	trade	price	temple	market

	1	2	3
Etruscan	1	0	35
market	50	0	1
price	42	1	0
temple	0	0	20
trade	**11**	7	1
...			

Update counts

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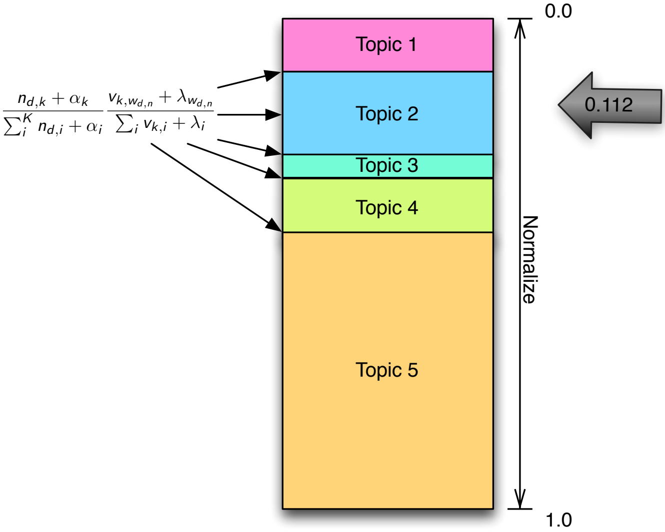
Etruscan	trade	price	temple	market

Details: how to sample from a distribution



Algorithm

1. For each iteration i:
   1. For each document d and word n currently assigned to z_{old}:
      1. Decrement $n_{d,z_{old}}$ and $v_{z_{old},w_{d,n}}$
      2. Sample $z_{new} = k$ with probability proportional to $\frac{n_{d,k}+\alpha_k}{\sum_i^K n_{d,i}+\alpha_i} \frac{v_{k,w_{d,n}}+\lambda_{w_{d,n}}}{\sum_i v_{k,i}+\lambda_i}$
      3. Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$

Implementation

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      3. Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$

Desiderata

- Hyperparameters: Sample them too (slice sampling)
- Initialization: Random
- Sampling: Until likelihood converges
- Lag / burn-in: Difference of opinion on this
- Number of chains: Should do more than one

Available implementations

- Mallet (http://mallet.cs.umass.edu)
- LDAC (http://www.cs.princeton.edu/ blei/lda-c)
- Topicmod (http://code.google.com/p/topicmod)