



Slides adapted from William Cohen

Introduction to Machine Learning

Machine Learning: Jordan Boyd-Graber

University of Maryland

STOCHASTIC GRADIENT DESCENT FOR LOGISTIC REGRESSION

Content Questions

Content Questions

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Content Questions

Administrivia Questions

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Reminder: Logistic Regression

$$P(Y = 0|X) = \frac{1}{1 + \exp[\beta_0 + \sum_i \beta_i X_i]} \quad (1)$$

$$P(Y = 1|X) = \frac{\exp[\beta_0 + \sum_i \beta_i X_i]}{1 + \exp[\beta_0 + \sum_i \beta_i X_i]} \quad (2)$$

- Discriminative prediction: $p(y|x)$
- Classification uses: ad placement, spam detection
- What we didn't talk about is how to learn β from data

Logistic Regression: Objective Function

$$\mathcal{L} \equiv \ln p(Y|X, \beta) = \sum_j \ln p(y^{(j)} | x^{(j)}, \beta) \quad (3)$$

$$= \sum_j y^{(j)} \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) - \ln \left[1 + \exp \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) \right] \quad (4)$$

Algorithm

1. Initialize a vector B to be all zeros
2. For $t = 1, \dots, T$
 - For each example \vec{x}_i, y_i and feature j :
 - Compute $\pi_i \equiv \Pr(y_i = 1 | \vec{x}_i)$
 - Set $\beta[j] = \beta[j]' + \lambda(y_i - \pi_i)x_i$
3. Output the parameters β_1, \dots, β_d .

Example Documents

$$\beta[j] = \beta[j] + \lambda(y_i - \pi_i)x_i$$

$$\vec{\beta} = \langle \beta_{bias} = 0, \beta_A = 0, \beta_B = 0, \beta_C = 0, \beta_D = 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

(Assume step size $\lambda = 1.0$.)

$$y_2 = 0$$

B C C C D D D D

You first see the positive example. First, compute π_1

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$$\pi_1 = \Pr(y_1 = 1 | \vec{x}_1) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp 0}{\exp 0 + 1} = 0.5$$

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Now you see the negative example. What's π_2 ?

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How does the gradient change with regularization?

- First

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- But difficult to update every feature every time (if there are many features)
- Following this up, we note that we can perform m successive “regularization” updates by letting $\beta_j = \beta'_j \cdot (1 - 2\lambda\mu)^{m_j}$

Basic Idea

Don't perform regularization updates for zero-valued x_j 's, but instead to simply keep track of how many such updates would need to be performed to update β_j

Revised Algorithm

1. Initialize a vector β to be all zeros
2. Initialize a vector A to be all zeros
3. For $t = 1, \dots, T$
 - For each example \vec{x}_i, y_i and feature j :
 - Simulate regularization updates: $\beta[j] = \beta[j] \cdot (1 - 2\lambda\mu)^{k-A[j]}$
 - Compute $\pi_i \equiv \Pr(y_i = 1 \mid \vec{x}_i)$
 - Set $\beta[j] = \beta[j] + \lambda(y_i - \pi_i)x_i$
 - Keep track of last update for feature $A[j] = k$
4. For each paramter, catch up on missing updates
$$\beta[j] = \beta[j] \cdot (1 - 2\lambda\mu)^{T-A[j]}$$
5. Output the parameters β_1, \dots, β_d .

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
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Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

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$$0.0 \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)1.0$$

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$$0.0 \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)1.0 = 2$$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A ? $\beta_A = \beta'_A (1 - 2 \cdot \lambda \cdot \mu)^{m_A} + \lambda(y_1 - \pi_1)x_{1,A} = 0.0 \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)4.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A ? $\beta_A = \beta'_A (1 - 2 \cdot \lambda \cdot \mu)^{m_A} + \lambda(y_1 - \pi_1)x_{1,A} = 0.0(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^1 + 1.0 \cdot (1.0 - 0.5)4.0 = 1.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ? $\beta_B = \beta_B' (1 - 2 \cdot \lambda \cdot \mu)^{m_B} + \lambda(y_1 - \pi_1)x_{1,B} = 0.0(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^1 + 1.0 \cdot (1.0 - 0.5)3.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ? $\beta_B = \beta_B' (1 - 2 \cdot \lambda \cdot \mu)^{m_B} + \lambda(y_1 - \pi_1)x_{1,B} = 0.0(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^1 + 1.0 \cdot (1.0 - 0.5)3.0 = 1.5$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ? $\beta_C = \beta'_C (1 - 2 \cdot \lambda \cdot \mu)^{m_C} + \lambda(y_1 - \pi_1)x_{1,C} = 0.0 \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (1.0 - 0.5)1.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ? $\beta_C = \beta'_C (1 - 2 \cdot \lambda \cdot \mu)^{m_C} + \lambda(y_1 - \pi_1)x_{1,C} = 0.0(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^1 + 1.0 \cdot (1.0 - 0.5)1.0 = 0.5$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_j$$
$$\vec{\beta} = \langle 0, 0, 0, 0, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D ?

We don't care: leave it for later.

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Now you see the negative example. What's π_2 ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Now you see the negative example. What's π_2 ?

$$\pi_2 = \Pr(y_2 = 1 | \vec{x}_2) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp\{.5 + 1.5 + 1.5 + 0\}}{\exp\{.5 + 1.5 + 1.5 + 0\} + 1} =$$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

Now you see the negative example. What's π_2 ?

$$\pi_2 = \Pr(y_2 = 1 | \vec{x}_2) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp\{.5 + 1.5 + 1.5 + 0\}}{\exp\{.5 + 1.5 + 1.5 + 0\} + 1} = 0.97$$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_j$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

$$\pi_2 = 0.97$$

Careful: You'd need to regularize β_D if it weren't already zero (multiply it by $(1 - 2\lambda\mu)^{m_j}$)

What's the update for β_{bias} ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_{bias} ?

$$\beta_{bias} = \beta'_{bias} (1 - 2 \cdot \lambda \cdot \mu)^{m_{bias}} + \lambda(y_2 - \pi_2)x_{2,bias} =$$
$$0.5 \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (0.0 - 0.97)1.0$$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_{bias} ?

$$\beta_{bias} = \beta'_{bias} (1 - 2 \cdot \lambda \cdot \mu)^{m_{bias}} + \lambda(y_2 - \pi_2)x_{2,bias} =$$
$$0.5 \left(1 - 2 \cdot 1.0 \cdot \frac{1}{4}\right)^1 + 1.0 \cdot (0.0 - 0.97)1.0 = -0.72$$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_j$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_A ?

We don't care: leave it for later.

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_j$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

$$y_2 = 0$$

B C C C D D D D

What's the update for β_B ? $\beta_B = \beta_B' (1 - 2 \cdot \lambda \cdot \mu)^{m_B} + \lambda(y_2 - \pi_2)x_{2,B} = 1.5(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^1 + 1.0 \cdot (0.0 - 0.97)1.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_B ? $\beta_B = \beta_B' (1 - 2 \cdot \lambda \cdot \mu)^{m_B} + \lambda(y_2 - \pi_2)x_{2,B} = 1.5(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^1 + 1.0 \cdot (0.0 - 0.97)1.0 = -0.22$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

$$y_2 = 0$$

B C C C D D D D

What's the update for β_C ? $\beta_C = \beta'_C (1 - 2 \cdot \lambda \cdot \mu)^{m_C} + \lambda(y_2 - \pi_2)x_{2,C} = 0.5(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^1 + 1.0 \cdot (0.0 - 0.97)3.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_C ? $\beta_C = \beta'_C (1 - 2 \cdot \lambda \cdot \mu)^{m_C} + \lambda(y_2 - \pi_2)x_{2,C} = 0.5(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^1 + 1.0 \cdot (0.0 - 0.97)3.0 = -2.7$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_j$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D ?

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

$$y_2 = 0$$

B C C C D D D D

What's the update for β_D ? $\beta_D = \beta_D' (1 - 2 \cdot \lambda \cdot \mu)^{m_D} + \lambda(y_2 - \pi_2)x_{2,D} = 0.0(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^2 + 1.0 \cdot (0.0 - 0.97)4.0$

Example Documents (Regularized)

$$\beta[j] = \beta[j]' \cdot (1 - 2\lambda\mu)^{m_j} + \lambda(y - p)x_i$$
$$\vec{\beta} = \langle .5, 2.0, 1.5, 0.5, 0 \rangle$$

$$y_1 = 1$$

A A A A B B B C

$$y_2 = 0$$

B C C C D D D D

Assume step size $\lambda = 1.0$ and $\mu = \frac{1}{4}$.

What's the update for β_D ? $\beta_D = \beta'_D (1 - 2 \cdot \lambda \cdot \mu)^{m_D} + \lambda(y_2 - \pi_2)x_{2,D} = 0.0(1 - 2 \cdot 1.0 \cdot \frac{1}{4})^2 + 1.0 \cdot (0.0 - 0.97)4.0 = -3.9$

If this were final iteration ...

- Need to remember that β_A is still waiting for regularization

$$\beta_A^{\text{final}} = \beta_A \left(1 - 21.0 \frac{1}{4} \right)^1 = 1.0 \quad (6)$$

Next time ...

- Multinomial logistic regression in sklearn (more than one option)
- Crafting effective features
- Preparation for third homework