

# Part of Speech Tagging

Natural Language Processing: Jordan Boyd-Graber University of Maryland

Adapted from material by Jimmy Lin and Jason Eisner

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- So, we use dynamic programming to compute most likely tags for each token subsequence from 0 to t that ends in state k.
- Memoization: fill a table of solutions of sub-problems
- Solve larger problems by composing sub-solutions
- Base case:

$$\delta_1(k) = \pi_k \beta_{k, x_i} \tag{1}$$

$$\delta_n(k) = \max_{j} \left( \delta_{n-1}(j) \theta_{j,k} \right) \beta_{k,x_n}$$
 (2)

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- In class: example that shows why you need all O(KL) table cells (garden pathing)
- But just computing the max isn't enough. We also have to remember where we came from. (Breadcrumbs from best previous state.)

$$\Psi_n = \operatorname{argmax}_j \delta_{n-1}(j) \theta_{j,k}$$
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$$\Psi_n = \operatorname{argmax}_j \delta_{n-1}(j) \theta_{j,k}$$
 (3)

Let's do that for the sentence "come and get it"

POS	$\pi_k$	$\beta_{k,x_1}$	$\log \delta_1(k)$
MOD	0.234	0.024	-5.18
DET	0.234	0.032	-4.89
CONJ	0.234	0.024	-5.18
Ν	0.021	0.016	-7.99
PREP	0.021	0.024	-7.59
PRO	0.021	0.016	-7.99
V	0.234	0.121	-3.56

come and get it

## Why logarithms?

- More interpretable than a float with lots of zeros.
- Underflow is less of an issue
- Addition is cheaper than multiplication

$$log(ab) = log(a) + log(b)$$
 (4)

POS	$\log \delta_1(j)$	$\log \delta_2$	(CONJ)
MOD	-5.18		
DET	-4.89		
CONJ	-5.18		
N	-7.99		
PREP	-7.59		
PRO	-7.99		
V	-3.56		

POS	$\log \delta_1(j)$	$\log \delta_2({\sf CONJ})$
MOD	-5.18	
DET	-4.89	
CONJ	-5.18	???
N	-7.99	
PREP	-7.59	
PRO	-7.99	
V	-3.56	

POS	$\log \delta_1(j)$	$\log \delta_1(j)\theta_{j,CONJ}$	$\log \delta_2( extsf{CONJ})$
MOD	-5.18		
DET	-4.89		
CONJ	-5.18		???
N	-7.99		
PREP	-7.59		
PRO	-7.99		
V	-3.56		

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MOD	-5.18		
DET	-4.89		
CONJ	-5.18		???
N	-7.99		
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PRO	-7.99		
V	-3.56		

$$\log (\delta_0(V)\theta_{V, CONJ}) = \log \delta_0(k) + \log \theta_{V, CONJ} = -3.56 + -1.65$$

POS	$\log \delta_1(j)$	$\log \delta_1(j)\theta_{j,CONJ}$	$\log \delta_2( extsf{CONJ})$
MOD	-5.18		
DET	-4.89		
CONJ	-5.18		???
N	-7.99		
PREP	-7.59		
PRO	-7.99		
V	-3.56	-5.21	

come and get it

POS	$\log \delta_1(j)$	$\log \delta_1(j)\theta_{j,CONJ}$	$\log \delta_2( extsf{CONJ})$
MOD	-5.18		
DET	-4.89		
CONJ	-5.18		???
N	-7.99	≤-7.99	
PREP	-7.59	≤-7.59	
PRO	-7.99	≤-7.99	
V	-3.56	-5.21	

come and get it

POS	$\log \delta_1(j)$	$\log \delta_1(j)\theta_{j,CONJ}$	$\log \delta_2( extsf{CONJ})$
MOD	-5.18	-8.48	
DET	-4.89	-7.72	
CONJ	-5.18	-8.47	???
N	-7.99	≤-7.99	
PREP	-7.59	≤-7.59	
PRO	-7.99	≤-7.99	
V	-3.56	-5.21	

come and get it

POS	$\log \delta_1(j)$	$\log \delta_1(j)\theta_{j,CONJ}$	$\log \delta_2( extsf{CONJ})$
MOD	-5.18	-8.48	
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CONJ	-5.18	-8.47	???
N	-7.99	≤-7.99	
PREP	-7.59	≤-7.59	
PRO	-7.99	≤-7.99	
V	-3.56	-5.21	

come and get it

POS	$\log \delta_1(j)$	$\log \delta_1(j)\theta_{j,CONJ}$	$\log \delta_2( extsf{CONJ})$
MOD	-5.18	-8.48	
DET	-4.89	-7.72	
CONJ	-5.18	-8.47	
N	-7.99	≤-7.99	
PREP	-7.59	≤-7.59	
PRO	-7.99	≤-7.99	
V	-3.56	-5.21	

$$\log \delta_1(k) = -5.21 - \log \beta_{\text{CONJ, and}} =$$

POS	$\log \delta_1(j)$	$\log \delta_1(j)\theta_{j,CONJ}$	$\log \delta_2( extsf{CONJ})$
MOD	-5.18	-8.48	
DET	-4.89	-7.72	
CONJ	-5.18	-8.47	
N	-7.99	≤-7.99	
PREP	-7.59	≤-7.59	
PRO	-7.99	≤-7.99	
V	-3.56	-5.21	

$$\log \delta_1(k) = -5.21 - \log \beta_{\text{CONJ, and}} = -5.21 - 0.64$$

POS	$\log \delta_1(j)$	$\log \delta_1(j)\theta_{j,CONJ}$	$\log \delta_2( extsf{CONJ})$
MOD	-5.18	-8.48	
DET	-4.89	-7.72	
CONJ	-5.18	-8.47	-6.02
N	-7.99	≤-7.99	
PREP	-7.59	≤-7.59	
PRO	-7.99	≤-7.99	
V	-3.56	-5.21	

come and get it

POS	$\delta_1(k)$	$\delta_2(k)$	<i>b</i> <sub>2</sub>	$\delta_3(k)$	<i>b</i> <sub>3</sub>	$\delta_4(k)$	<i>b</i> <sub>4</sub>
MOD	-5.18						
DET	-4.89						
CONJ	-5.18	-6.02	V				
N	-7.99						
PREP	-7.59						
PRO	-7.99						
V	-3.56						
WORD	come	and	and		get		

POS	$\delta_1(k)$	$\delta_2(k)$	<i>b</i> <sub>2</sub>	$\delta_3(k)$	<i>b</i> <sub>3</sub>	$\delta_4(k)$	<i>b</i> <sub>4</sub>
MOD	-5.18	-0.00	Χ				
DET	-4.89	-0.00	Χ				
CONJ	-5.18	-6.02	V				
N	-7.99	-0.00	Χ				
PREP	-7.59	-0.00	Χ				
PRO	-7.99	-0.00	Χ				
V	-3.56	-0.00	Χ				
WORD	come	and		get		it	

POS	$\delta_1(k)$	$\delta_2(k)$	<i>b</i> <sub>2</sub>	$\delta_3(k)$	<i>b</i> <sub>3</sub>	$\delta_4(k)$	<i>b</i> <sub>4</sub>
MOD	-5.18	-0.00	Χ	-0.00	Χ		
DET	-4.89	-0.00	Χ	-0.00	X		
CONJ	-5.18	-6.02	V	-0.00	X		
N	-7.99	-0.00	Χ	-0.00	X		
PREP	-7.59	-0.00	Χ	-0.00 X			
PRO	-7.99	-0.00	Χ	-0.00	X		
V	-3.56	-0.00	Χ	-9.03	CONJ		
WORD	come	and		g	et	it	

POS	$\delta_1(k)$	$\delta_2(k)$	<i>b</i> <sub>2</sub>	$\delta_3(k)$ $b_3$		$\delta_4(k)$	<i>b</i> <sub>4</sub>
MOD	-5.18	-0.00	Χ	-0.00	Χ	-0.00	Χ
DET	-4.89	-0.00	Χ	-0.00	-0.00 X		Χ
CONJ	-5.18	-6.02	V	-0.00	Χ	-0.00	Χ
N	-7.99	-0.00	Χ	-0.00	Χ	-0.00	Χ
PREP	-7.59	-0.00	Χ	-0.00	Χ	-0.00	Χ
PRO	-7.99	-0.00	Χ	-0.00	Χ	-14.6	V
V	-3.56	-0.00	Χ	-9.03	CONJ	-0.00	Χ
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## What if you don't have training data?

- You can still learn a hmm
- Using a general technique called expectation maximization

### What if you don't have training data?

- You can still learn a hmm
- Using a general technique called expectation maximization
  - Take a guess at the parameters
  - Figure out latent variables
  - Find the parameters that best explain the latent variables
  - Repeat

Model Parameters

We need to start with model parameters

## **Model Parameters**

 $\pi$ ,  $\beta$ ,  $\theta$ 

We can initialize these any way we want

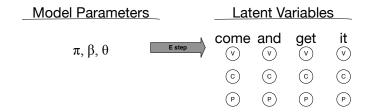
## **Model Parameters**

π, β, θ





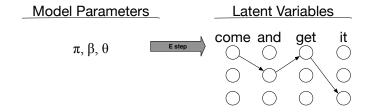
We compute the E-step based on our data



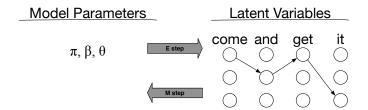
Each word in our dataset could take any part of speech

Model Parameters	Latent Variables				
$\pi, \beta, \theta$	$\Rightarrow$	come	and	get	it
		$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$
		$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$

But we don't know which state was used for each word



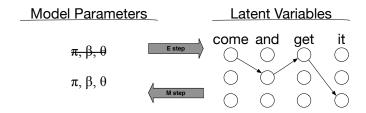
Determine the probability of being in each latent state using Forward / Backward



Calculate new parameters:

$$\theta_i = \frac{n_i + \alpha_i}{\sum_k \mathbb{E}_p[n_k] + \alpha_k} \tag{5}$$

Where the expected counts are from the lattice



Replace old parameters (and start over)

#### Hard vs. Full FM

### Hard EM

Train only on the most likely sentence (Viterbi)

- Faster: E-step is faster
- Faster: Fewer iterations

## Full EM

Compute probability of all possible sequences

More accurate: Doesn't get stuck in local optima as easily

### Warning about next homework(s)

- Kaggle competition
- Thus, late days not very useful
- Following homework is not computational

### **Garden Pathing**

What is the probability of the sequence "a/Det blue/Adj boat/N"?

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$$\pi_{d}\beta_{d,the}\theta_{d,a}\beta_{a,blue}\theta_{a,n}\beta_{n,boat} =$$
 (5)

$$0.3*0.6*0.4*0.3*0.5*0.1 = 0.00108$$
 (6)

### Garden Pathing

What is the probability of the sequence "a/Det blue/Adj boat/N"?

$$\pi_{d}\beta_{d,\text{the}}\theta_{d,a}\beta_{a,\text{blue}}\theta_{a,n}\beta_{n,\text{boat}} = \tag{5}$$

$$0.3*0.6*0.4*0.3*0.5*0.1 = 0.00108$$
 (6)

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3. 
$$\delta_1(d) = -1.7$$

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3. 
$$\delta_1(d) = -1.7$$

4. 
$$\delta_1(n) = -4.6$$

1. 
$$\delta_2(a) = \max\left(\underbrace{-5.8}_{a},\underbrace{-7.3}_{V},\underbrace{-2.6}_{d},\underbrace{-7.6}_{D}\right) + -1.2 = -2.6 + -1.2 = -3.8$$

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$$\delta_2(a) = \max\left(\underbrace{-5.8}_{a}, \underbrace{-7.3}_{v}, \underbrace{-2.6}_{d}, \underbrace{-7.6}_{n}\right) + -1.2 = -2.6 + -1.2 = -3.8$$

2. 
$$\delta_2(v) = \max\left(\underbrace{-6.9}_{a},\underbrace{-7.3}_{v},\underbrace{-4.7}_{d},\underbrace{-4.8}_{n}\right) + -2.3 = -4.7 + -2.3 = -7.0$$

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3. 
$$\delta_2(d) = \max\left(\underbrace{-6.9}_{\mathbf{a}}, \underbrace{-6.9}_{\mathbf{v}}, \underbrace{-4.0}_{\mathbf{d}}, \underbrace{-7.6}_{\mathbf{n}}\right) + -3.7 = -4.0 + -3.7 = -7.7$$

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$$\delta_2(d) = \max\left(\underbrace{-6.9}_{\mathbf{a}}, \underbrace{-6.9}_{\mathbf{v}}, \underbrace{-4.0}_{\mathbf{d}}, \underbrace{-7.6}_{\mathbf{n}}\right) + -3.7 = -4.0 + -3.7 = -7.7$$

4. 
$$\delta_2(n) = \max\left(\underbrace{-5.3}_{a}, \underbrace{-6.9}_{v}, \underbrace{-2.5}_{d}, \underbrace{-6.9}_{n}\right) + -1.9 = -2.5 + -1.9 = -4.4$$

1. 
$$\delta_3(a) = \max\left(\underbrace{-5.0}_{\mathbf{a}}, \underbrace{-8.6}_{\mathbf{V}}, \underbrace{-7.4}_{\mathbf{n}}\right) + -2.3 = -5.0 + -2.3 = -7.3$$

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2. 
$$\delta_3(v) = \max\left(\underbrace{-6.1}_{a}, \underbrace{-8.6}_{v}, \underbrace{-10.7}_{d}, \underbrace{-4.6}_{n}\right) + -0.9 = -4.6 + -0.9 = -5.5$$

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3. 
$$\delta_3(d) = \max\left(\underbrace{-6.1}_{\mathbf{a}}, \underbrace{-8.2}_{\mathbf{v}}, \underbrace{-10.0}_{\mathbf{d}}, \underbrace{-7.4}_{\mathbf{n}}\right) + -3.7 = -6.1 + -3.7 = -9.8$$

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$$\delta_3(a) = \max\left(\underbrace{-5.0}_{\mathbf{a}}, \underbrace{-8.6}_{\mathbf{v}}, \underbrace{-8.6}_{\mathbf{d}}, \underbrace{-7.4}_{\mathbf{n}}\right) + -2.3 = -5.0 + -2.3 = -7.3$$

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$$\delta_3(d) = \max\left(\underbrace{-6.1}_{\mathbf{a}}, \underbrace{-8.2}_{\mathbf{V}}, \underbrace{-10.0}_{\mathbf{d}}, \underbrace{-7.4}_{\mathbf{n}}\right) + -3.7 = -6.1 + -3.7 = -9.8$$

4. 
$$\delta_3(n) = \max\left(\underbrace{-4.5}_{\mathbf{a}}, \underbrace{-8.2}_{\mathbf{v}}, \underbrace{-8.5}_{\mathbf{d}}, \underbrace{-6.7}_{\mathbf{n}}\right) + -0.9 = -4.5 + -0.9 = -5.4$$

1. 
$$\delta_4(a) = \max\left(\underbrace{-8.5}_{a},\underbrace{-7.2}_{v},\underbrace{-10.7}_{d},\underbrace{-8.4}_{n}\right) + -3.4 = -7.2 + -3.4 = -10.6$$

1. 
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2. 
$$\delta_4(v) = \max\left(\underbrace{-9.6}_{a},\underbrace{-7.2}_{v},\underbrace{-12.8}_{d},\underbrace{-5.7}_{n}\right) + -3.4 = -5.7 + -3.4 = -9.1$$

1. 
$$\delta_4(a) = \max\left(\underbrace{-8.5}_{a}, \underbrace{-7.2}_{v}, \underbrace{-10.7}_{d}, \underbrace{-8.4}_{n}\right) + -3.4 = -7.2 + -3.4 = -10.6$$

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3. 
$$\delta_4(d) = \max\left(\underbrace{-9.6}_{a}, \underbrace{-6.8}_{v}, \underbrace{-12.1}_{d}, \underbrace{-8.4}_{n}\right) + -0.5 = -6.8 + -0.5 = -7.3$$

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$$\delta_4(a) = \max\left(\underbrace{-8.5}_{a}, \underbrace{-7.2}_{v}, \underbrace{-10.7}_{d}, \underbrace{-8.4}_{n}\right) + -3.4 = -7.2 + -3.4 = -10.6$$

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$$\delta_4(v) = \max\left(\underbrace{-9.6}_{a}, \underbrace{-7.2}_{v}, \underbrace{-12.8}_{d}, \underbrace{-5.7}_{n}\right) + -3.4 = -5.7 + -3.4 = -9.1$$

3. 
$$\delta_4(d) = \max\left(\underbrace{-9.6}_{\mathbf{a}}, \underbrace{-6.8}_{\mathbf{v}}, \underbrace{-12.1}_{\mathbf{d}}, \underbrace{-8.4}_{\mathbf{n}}\right) + -0.5 = -6.8 + -0.5 = -7.3$$

4. 
$$\delta_4(n) = \max\left(\underbrace{-8.0}_{a}, \underbrace{-6.8}_{v}, \underbrace{-10.6}_{d}, \underbrace{-7.7}_{n}\right) + -3.4 = -6.8 + -3.4 = -10.2$$

1. 
$$\delta_5(a) = \max\left(\underbrace{-11.8}_{a}, \underbrace{-10.7}_{v}, \underbrace{-8.2}_{d}, \underbrace{-13.2}_{n}\right) + -2.3 = -8.2 + -2.3 = -11$$

1. 
$$\delta_5(a) = \max\left(\underbrace{-11.8}_{\mathbf{q}}, \underbrace{-10.7}_{\mathbf{v}}, \underbrace{-8.2}_{\mathbf{q}}, \underbrace{-13.2}_{\mathbf{n}}\right) + -2.3 = -8.2 + -2.3 = -11$$

2. 
$$\delta_5(v) = \max\left(\underbrace{-12.9}_{a}, \underbrace{-10.7}_{v}, \underbrace{-10.3}_{d}, \underbrace{-10.4}_{n}\right) + -1.6 = -10.3 + -1.6 = -12$$

1. 
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2. 
$$\delta_5(v) = \max\left(\underbrace{-12.9}_{a}, \underbrace{-10.7}_{v}, \underbrace{-10.3}_{d}, \underbrace{-10.4}_{n}\right) + -1.6 = -10.3 + -1.6 = -12$$

3. 
$$\delta_5(d) = \max\left(\underbrace{-12.9}_{\mathbf{a}}, \underbrace{-10.3}_{\mathbf{v}}, \underbrace{-9.6}_{\mathbf{d}}, \underbrace{-13.2}_{\mathbf{n}}\right) + -3.7 = -9.6 + -3.7 = -13$$

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$$\delta_5(a) = \max\left(\underbrace{-11.8}_{a}, \underbrace{-10.7}_{v}, \underbrace{-8.2}_{d}, \underbrace{-13.2}_{n}\right) + -2.3 = -8.2 + -2.3 = -11$$

2. 
$$\delta_5(v) = \max\left(\underbrace{-12.9}_{a}, \underbrace{-10.7}_{v}, \underbrace{-10.3}_{d}, \underbrace{-10.4}_{n}\right) + -1.6 = -10.3 + -1.6 = -12$$

3. 
$$\delta_5(d) = \max\left(\underbrace{-12.9}_{\mathbf{d}}, \underbrace{-10.3}_{\mathbf{v}}, \underbrace{-9.6}_{\mathbf{d}}, \underbrace{-13.2}_{\mathbf{n}}\right) + -3.7 = -9.6 + -3.7 = -13$$

4. 
$$\delta_5(n) = \max\left(\underbrace{-11.3}_{a}, \underbrace{-10.3}_{v}, \underbrace{-8.1}_{d}, \underbrace{-12.5}_{n}\right) + -1.2 = -8.1 + -1.2 = -9.3$$

Reconstruction

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For "the old man the boats", the reconstruction starts with the best part of speech at Position 5, which is a noun (-9.3), which leads to the sequence "The/det old/n man/v the/det boats/n".